disha Publication Inc

NDA/ NA 16 years MATHEMATICS

Topic-wise Solved Papers (2006 - 2021)

EBD_7346

Corporate Office

DISHA PUBLICATION
45, 2nd Floor, Maharishi Dayanand Marg,
Corner Market, Malviya Nagar, New Delhi - 110017
Tel : 49842349 / 49842350

Typeset by Disha DTP Team

CONTENTS

NATIONAL DEFENCE ACADEMY EXAMINATION

The National Defence Academy (NDA) founded in 1957, is a premier Inter Service training institution where future cadets are trained. The training involves an exacting schedule of 3 years before the cadets join their respective Service Academies, viz. Indian Military Academy, Naval Academy and Air Force Academy.

The examination of the National Defence Academy is conducted by UPSC twice a year, after which the selected candidates are sent to National Defence Academy for training.

SYLLABUS

1. **Algebra :** Concept of a set, operations on sets, Venn diagrams. De Morgan laws. Cartesian product, relation, equivalence relation. Representation of real numbers on a line. Complex numbers – basic properties, modulus, argument, cube roots of unity. Binary system of numbers. Conversion of a number in decimal system to binary system and vice-versa. Arithmetic, Geometric and Harmonic progressions. Quadratic equations with real coefficients. Solution of linear inequations of two variables by graphs. Permutation and Combination. Binomial theorem and its application. Logarithms and their applications.

2. **Matrices and Determinants :** Types of matrices, operations on matrices Determinant of a matrix, basic properties of determinant. Adjoint and inverse of a square matrix, Applications – Solution of a system of linear equations in two or three unknowns by Cramer's rule and by Matrix Method.

3. **Trigonometry :** Angles and their measures in degrees and in radians. Trigonometrical ratios. Trigonometric identities Sum and difference formulae. Multiple and Sub-multiple angles. Inverse trigonometric functions. Applications – Height and distance, properties of triangles.

4. **Analytical Geometry of two and three dimensions :** Rectangular Cartesian Coordinate system. Distance formula. Equation of a line in various forms. Angle between two lines. Distance of a point from a line. Equation of a circle in standard and in general form. Standard forms of parabola, ellipse and hyperbola. Eccentricity and axis of a conic.

5. **Differential Calculus :** Concept of a real valued function - domain, range and graph of a function. Composite functions, one to one, onto and inverse functions. Notion of limit, Standard limits - examples. Continuity of functions - examples, algebraic operations on continuous functions. Derivative of a function at a point, geometrical and physical interpreatation of a derivative - applications. Derivatives of sum, product and quotient of functions, derivative of a function with respect of another function, derivative of a composite function. Second order derivatives. Increasing and decreasing functions. Application of derivatives in problems of maxima and pminima.

6. **Integral Calculus and Differential Equations :** Integration as inverse of differentiation, integration by substitution and by parts, standard integrals involving algebraic expressions, trigonometric, exponential and hyperbolic functions. Evaluation of definite integrals - determination of areas of plane regions bounded by curves - applications. Definition of order and degree of a differential equation, formation of a differential equation by examples. General and particular solution of a differential equation, solution of first order and first degree differential equations of various types - examples. Application in problems of growth and decay.

7. **Vector Algebra:** Vectors in two and three dimensions, magnitude and direction of a vector. Unit and null vectors, addition of vectors, scalar multiplication of vector, scalar product or dot product of two-vectors. Vector product and cross product of two vectors. Applications-work done by a force and moment of a force, and in geometrical problems.

8. **Statistics:** Classification of data, Frequency distribution, cumulative frequency distribution - examples Graphical representation – Histogram, Pie Chart, Frequency Polygon - examples. Measures of Central tendency – mean, median and mode. Variance and standard deviation - determination and comparison. Correlation and regression.

9. **Probability :** Random experiment, outcomes and associated sample space, events, mutually exclusive and exhaustive events, impossible and certain events. Union and Intersection of events. Complementary, elementary and composite events. Definition of probability - classical and statistical - examples. Elementary theorems on probability - simple problems. Conditional probability, Bayes' theorem - simple problems. Random variable as function on a sample space. Binomial distribution, examples of random experiments giving rise to Binominal distribution.

DETAILED BREAKUP OF QUESTIONS (2007-21)

	2007		2008		2009		2010		2011		2012		2013		2014		2015		2016		2017		2018		2019		2020	2021
CHAPTERS NAME	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	I
Set, Relation, Function and Number System	10	13	11	9	13	18	9	9	16	5	14	7	13	12	13	8	8	8	17	13	7	5	11	8	10	12	10	7
Polynomial, Quadratic Equation & Inequalities	11	11	6	9	7	7	7	5	11	11	7	3	8	7	2	2	5	5	6	7	5	7	2	4	14	14	3	5
Sequence and Series	3	10	4	9	4	5	6	3	7	7	7	7	2	3	2	5	2	2	5	9	8	6	5	7	4	9	1	7
Complex Numbers	4	2	6	3	3	5	5	5	3	8	2	3	2	2	4	3	5	6	4	2	5	3	7	2	5	2	2	5
Binomial Theorem, Mathematical Induction	2	2	4	1	1	2	1	4	1	2	2	0	1	2	4	5	1	3	1	2	2	4	5	2	3	2	3	4
Permutation and Combination	1	3	3	3	4	3	3	3	6	2	1	1	2	3	5	0	4	2	5	4	1	2	3	4	3	3	3	2
Cartesian Co-ordinate System, Straight Line	6	3	4	3	3	3	3	6	8	6	6	10	8	9	0	4	7	7	4	4	6	4	8	5	8	3	6	6
Pair of Straight Lines	0	0	0	1	0	0	0	0	0	0	0	1	0	5	0	1	2	1	0	2	1	1	1	0	0	1	1	1
Circles	2	2	3	2	1	1	1	2	1	1	0	0	1	2	0	0	0	0	2	3	2	1	0	2	2	-	1	2
Conics - Parabola, Ellipse & Hyperbola	2	2	1	3	3	2	5	2	2	2	2	2	2	4	5	5	2	2	4	2	1	3	1	0	3	1	2	2
Trigonometry : Ratio & Identity, Trigonometric Equation	10	8	10	7	13	14	7	12	4	10	19	8	9	7	11	11	5	6	8	8	8	4	8	7	18	11	19	11
Properties of Triangle, Inverse Trigonometric Function	3	6	5	3	5	4	5	2	7	6	5	1	2	1	4	6	9	4	5	4	1	4	5	3	4	3	4	4
Height & Distance	1	1	1	1	2	1	1	2	1	4	4	2	3	2	2	1	2	2	1	2	1	2	3	2	1	1	-	2
Functions, Limits, Continuity and Differentiability	7	3	8	9	5	4	10	7	3	0	5	8	7	6	11	9	7	9	9	9	12	17	10	11	6	6	8	7
Derivatives	0	3	7	2	5	8	4	4	1	2	3	4	6	4	0	2	2	2	5	5	2	3	2	2	2	3	4	3
Application of Derivatives	7	4	3	5	7	4	6	5	3	6	5	3	2	3	5	4	8	6	4	3	2	5	2	4	1	6	6	5
Indefinite Integration	2	1	2	3	3	2	4	3	3	1	2	3	2	3	0	0	4	6	1	2	3	2	1	3	2	2	4	3
Definite Integration & its applications	3	5	2	3	2	3	3	6	4	4	3	5	6	7	5	6	8	5	3	3	4	4	10	3	3	3	4	3
Differential Equation	6	5	2	6	2	3	5	6	5	2	3	7	5	3	13	4	6	6	5	4	6	4	5	7	5	6	4	4
Matrices and Determinants	8	11	11	7	10	8	9	9	8	7	11	8	7	9	6	10	7	8	11	11	12	9	3	15	7	5	8	11
Probability and Probability Distribution	6	4	6	7	8	6	8	4	6	5	3	10	11	10	5	13	8	8	2	6	9	11	8	7	16	8	10	5
Vectors	9	6	9	10	7	7	8	8	8	6	7	8	6	6	2	10	7	7	12	6	6	6	5	8	5	5	5	5
3D-Geometry	6	7	5	7	6	6	8	2	5	4	5	8	7	1	13	7	4	8	2	8	6	5	5	4	4	5	5	5
Statistics	7	6	6	7	6	6	4	10	7	19	5	11	8	9	8	4	7	7	4	5	10	8	10	10	4	9	10	11

TOPICWISE SOLVED PAPER FOR NDA/NA MATHEMATICS 2019-21

Sets, Relations, Functions and Number System

1. If A, B and C are subsets of a given set, then which one of the following relations is ***not*** correct? **[NDA 2019-II]**

(a) $A \cup (A \cap B) = A \cup B$

(b) $A \cap (A \cup B) = A$

(c) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

(d) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

2. If a set A contains 3 elements and another set B contains 6 elements, then what is the minimum number of elements that $(A \cup B)$ can have? **[NDA 2019-II]**

(a) 3 (b) 6 (c) 8 (d) 9

3. In a school, 50% students play cricket and 40% play football. If 10% of students play both the games, then what per cent of students play neither cricket nor football? **[NDA 2019-II]**

(a) 10% (b) 15% (c) 20% (d) 25%

4. If $A = [x\ ;\ 0 \le x \le 2]$ and $B = [y, y$ is a prime number$]$, then what is $A \cap B$ equal to ? **[NDA 2019-II]**

(a) ϕ (b) [1] (c) [2] (d) [1, 2]

5. What is the value of **[NDA 2019-II]**

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\infty}}}?$$

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$

(c) 3 (d) 4

6. If $n!$ has 17 zeros, then what is the value of n? **[NDA 2019-II]**

(a) 95

(b) 85

(c) 80

(d) No such value of n exists

7. Let $S = (2, 4, 6, 8, \ldots\ldots\ldots\ldots 20)$. **[NDA 2019-II]**

What is the maximum number of subsets does S have?

(a) 10 (b) 20 (c) 512 (d) 1024

8. A binary number is represented by $(cdccddcccddd)_2$, where $c > d$. What is its decimal equivalent? **[NDA 2019-II]**

(a) 1848 (b) 2048 (c) 2842 (d) 2872

Directions for the following two (02) items : *Read the following information and answer the two items that follow:*

Let $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \ln x$.

9. For $x = \frac{\sqrt{\pi}}{2}$, what is the value of $[ho(gof)](x)$? **[NDA 2019-II]**

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

10. What is $[fo(fof)]$ (2) equal to? **[NDA 2019-II]**

(a) 2 (b) 8 (c) 16 (d) 256

11. A car travels first 60 km at a speed of 3v km/hr and travels next 60 km at 2v km/hr. What is the average speed of the car? **[NDA 2019-II]**

(a) 2.5 v km/hr (b) 2.4 v km/hr

(c) 2.2 v km/hr (d) 2.1 v km/hr

12. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls is 55 kg. What are the number of boys and girls respectively in the class? **[NDA 2019-II]**

(a) 75 and 75 (b) 50 and 100

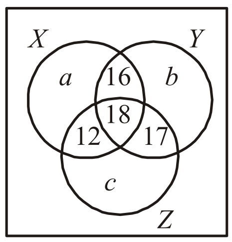
(c) 70 and 80 (d) 100 and 50

13. Consider the proper subsets of {1, 2, 3, 4}. How many of these proper subsets are superset of the set {3}? **[NDA 2020-I]**

(a) 5 (b) 6 (c) 7 (d) 8

DIRECTIONS (Qs. 14-16) : *Read the following information and answer the three items that follow:*

Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by $n(Z)$ which is equal to 90.



14. If the number of elements in Y and Z are in the ratio 4 : 5, then what is the value of b? **[NDA 2020-I]**

(a) 18 (b) 19 (c) 21 (d) 23

15. What is the value of **[NDA 2020-I]**

$$n(X)+n(Y)+n(Z)-n(X\cap Y)-n(Y\cap Z)-n(X\cap Z)$$
$$+n(X\cap Y\cap Z)?$$

(a) $a + b + 43$ (b) $a + b + 63$

(c) $a + b + 96$ (d) $a + b + 106$

16. If the number of elements belonging to neither X nor Y, nor Z is equal to p, then what is the number of elements in the complement of X? **[NDA 2020-I]**

(a) $p+b+60$ (b) $p+b+40$
(c) $p+a+60$ (d) $p+a+40$

17. The number $(1101101+1011011)_2$ can be written in decimal system as **[NDA 2020-I]**

(a) $(198)_{10}$ (b) $(199)_{10}$
(c) $(200)_{10}$ (d) $(201)_{10}$

18. What is the value of

$\frac{1}{10}\log_5 1024 - \log_5 10 + \frac{1}{5}\log_5 3125$? **[NDA 2020-I]**

(a) 0 (b) 1 (c) 2 (d) 3

19. If $x=\log_c(ab),\ y=\log_a(bc),\ z=\log_b(ca)$, then which of the following is correct? **[NDA 2020-I]**

(a) $xyz=1$
(b) $x+y+z=1$
(c) $(1+x)^{-1}+(1+y)^{-1}+(1+z)^{-1}=1$
(d) $(1+x)^{-2}+(1+y)^{-2}+(1+z)^{-2}=1$

20. Let $S=\{1, 2, 3, ...\}$. A relation R on $S \times S$ is defined by xRy if $\log_a x > \log_a y$ when $a=\frac{1}{2}$. Then the relation is **[NDA 2020-I]**

(a) reflexive only
(b) symmetric only
(c) transitive only
(d) both symmetric and transitive

21. If $f(x)=3x^2-5x+p$ and $f(0)$ and $f(1)$ are opposite in sign, then which of the following is correct? **[NDA 2020-I]**

(a) $-2<p<0$ (b) $-2<p<2$
(c) $0<p<2$ (d) $3<p<5$

22. If $f(x)=2x-x^2$, then what is the value of $f(x+2)+f(x-2)$ when $x=0$? **[NDA 2020-I]**

(a) –8 (b) –4 (c) 8 (d) 4

23. A chord subtends an angle 120° at the centre of a unit circle. What is the length of the chord? **[NDA 2021-I]**

(a) $\sqrt{2}-1$ units (b) $\sqrt{3}-1$ units
(c) $\sqrt{2}$ units (d) $\sqrt{3}$ units

24. What is the interior angle of a regular octagon of side length 2 cm ? **[NDA 2021-I]**

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{3\pi}{5}$ (d) $\frac{3\pi}{8}$

25. Consider the following statements :
1. $A=(1, 3, 5)$ and $B=(2, 4, 7)$ are equivalent sets.
2. $A=(1, 5, 9)$ and $B=(1, 5, 5, 9, 9)$ are equal sets.

Which of the above statements is/are correct? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

26. Consider the following statements :
1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct? **[NDA 2021-I]**

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

27. Let R be a relation defined as xRy if and only if $2x+3y=20$, where $x, y \in N$. How many elements of the form (x, y) are there in R ? **[NDA 2021-I]**

(a) 2 (b) 3 (e) 4 (d) 6

28. Consider the following statements :
1. A function $f: \mathbb{Z} \to \mathbb{Z}$, defined by $f(x)=x+1$, is one-one as well as onto.
2. A function $f: \mathbb{N} \to \mathbb{N}$, defined by $f(x)=x+1$, is one-one but not onto.

Which of the above statements is/are correct? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

29. A 24 cm long wire is bent to form a triangle with one of the angles as 60°. What is the altitude of the triangle having the greatest possible area? **[NDA 2021-I]**

(a) $4\sqrt{3}$ cm (b) $2\sqrt{3}$ cm
(c) 6 cm (d) 3 cm

30. The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is **[NDA 2021-I]**

(a) reduced by $\frac{1}{3}$ (b) increased by $\frac{10}{3}$
(c) reduced by $\frac{10}{3}$ (d) reduced by 50

Polynomial, Quadratic Equation & Inequalities

1. If both p and q belong to the set (1, 2, 3, 4), then how many equations of the form $px^2+qx+1=0$ will have real roots? **[NDA 2019-II]**

(a) 12 (b) 10 (c) 7 (d) 6

2. What is the value of k for which the sum of the squares of the roots of $2x^2-2(k-2)x-(k+1)=0$ is minimum? **[NDA 2019-II]**

(a) –1 (b) 1
(c) $\frac{3}{2}$ (d) 2

3. If $|x^2 - 3x + 2| > x^2 - 3x + 2$, then which one of the following is correct? **[NDA 2019-II]**
(a) $x < 1$ or $x > 2$
(b) $1 \le x \le 2$
(c) $1 < x < 2$
(d) x is any real value except 3 and 4

4. Under which one of the following conditions will the quadratic equation $x^2 + mx + 2 = 0$ always have real roots? **[NDA 2019-II]**
(a) $2\sqrt{3} \le m^3 < 8$ (b) $\sqrt{3} \le m^2 < 4$
(c) $m^2 > 8$ (d) $m^2 \le \sqrt{3}$

5. If α and β are the roots of $x^2 + x + 1 = 0$, then what is $\sum_{j=0}^{3}(\alpha^j + \beta^j)$ equal to? **[NDA 2019-II]**
(a) 8 (b) 6 (c) 4 (d) 2

6. How many terms are there in the expansion of $(1 + 2x + x^2)^5 + (1 + 4y + 4y^2)^5$? **[NDA 2019-II]**
(a) 12 (b) 20 (c) 21 (d) 22

7. Let $A \cup B = \{x|(x - a)(x - b) > 0$, where $a < b\}$.
What are A and B equal to? **[NDA 2019-II]**
(a) $A = \{x|x > a\}$ and $B = \{x|x > b\}$
(b) $A = \{x|x < a\}$ and $B = \{x|x > b\}$
(c) $A = \{x|x < a\}$ and $B = \{x|x < b\}$
(d) $A = \{x|x > a\}$ and $B = \{x|x < b\}$

8. What is the solution of $x \le 4$, y^3 0 and $x \le -4$, $y \le 0$? **[NDA 2019-II]**
(a) $x^3 - 4, y \le 0$ (b) $x \le 4, y^3\ 0$
(c) $x \le -4, y = 0$ (d) $x^3 - 4, y = 0$

9. If $x^{\log_7 x} > 7$ where $x > 0$, then which one of the following is correct? **[NDA 2019-II]**
(a) $x \in (0, \infty)$ (b) $x \in \left(\frac{1}{7}, 7\right)$
(c) $x \in \left(0, \frac{1}{7}\right) \cup (7, \infty)$ (d) $x \in \left(\frac{1}{7}, \infty\right)$

10. How many real roots does the equation $x^3 + 3|x| + 2 = 0$ have? **[NDA 2019-II]**
(a) Zero (b) One (c) Two (d) Four

11. Consider the following statements in respect to the quadratic equation **[NDA 2019-II]**
$4(x - p)(x - q) - r^2 = 0$,
where p, q and r are real numbers :
1. The roots are real
2. The roots are equal if $p = q$ and $r = 0$
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

12. What is the area of the region bounded by $|x| < 5$, $y = 0$ and $y = 8$? **[NDA 2019-II]**
(a) 40 square units (b) 80 square units
(c) 120 square units (d) 160 square units

13. Which one of the following is the second degree polynomial function $f(x)$ where $f(0) = 5$, $f(-1) = 10$ and $f(1) = 6$? **[NDA 2019-II]**
(a) $5x^2 - 2x + 5$ (b) $3x^2 - 2x - 5$
(c) $3x^2 - 2x + 5$ (d) $3x^2 - 10x + 5$

14. If p and q are the roots of the equation $x^2 - 30x + 221 = 0$, what is the value of $p^3 + q^3$? **[NDA 2019-II]**
(a) 7010 (b) 7110 (c) 7210 (d) 7240

15. If cot α and cot β are the roots of the equation $x^2 - 3x + 2 = 0$, then what is $\cot(\alpha + \beta)$ equal to ? **[NDA 2020-I]**
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3

16. The roots α and β of a quadratic equation, satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is the number of such quadratic equations ? **[NDA 2020-I]**
(a) 0 (b) 2 (c) 3 (d) 4

17. If $1.5 \le x \le 4.5$, then which one of the following is correct? **[NDA 2020-I]**
(a) $(2x - 3)(2x - 9) > 0$
(b) $(2x - 3)(2x - 9) < 0$
(c) $(2x - 3)(2x - 9) \ge 0$
(d) $(2x - 3)(2x - 9) \le 0$

18. The number of integer values of k, for which the equation $2\sin x = 2k + 1$ has a solution, is **[NDA 2021-I]**
(a) zero (b) one (e) two (d) four

19. If the roots of the quadratic equation $x^2 + 2x + k = 0$ are real, then **[NDA 2021-I]**
(a) $k < 0$ (b) $k \le 0$ (c) $k < 1$ (d) $k \le 1$

20. If α and β are the roots of the equation $4x^2 + 2x - 1 = 0$, then which one of the following is correct? **[NDA 2021-I]**
(a) $\beta = -2\alpha^2 - 2\alpha$ (b) $\beta = 4\alpha^2 - 3\alpha$
(c) $\beta = \alpha^2 - 3\alpha$ (d) $\beta = -2\alpha^2 - 2\alpha$

21. If one root of $5x^2 + 26x + k = 0$ is reciprocal of the other, then what is the value of k? **[NDA 2021-I]**
(a) 2 (b) 3 (c) 5 (d) 8

22. If k is one of the roots of the equation $x(x + 1) + 1 = 0$, then what is its other root? **[NDA 2021-I]**
(a) 1 (b) $-k$ (c) k^2 (d) $-k^2$

Sequence and Series

1. What is the value of **[NDA 2019-II]**
$1 - 2 + 3 - 4 + 5 - ... + 101$?
(a) 51 (b) 55 (c) 110 (d) 111

2. If the sum of first n terms of a series is $(n + 12)$, then what is its third term? **[NDA 2019-II]**
(a) 1 (b) 2 (c) 3 (d) 4

3. If the roots of the equation **[NDA 2019-II]**
$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$
are equal, then which one of the following is correct?
(a) a, b and c are in AP (b) a, b and c are in GP
(c) a, b and c are in HP
(d) a, b and c do not follow any regular pattern

4. A geometric progression (GP) consists of 200 terms. If the sum of odd terms of the GP is m, and the sum of even terms of the GP is n, then what is its common ratio? **[NDA 2019-II]**

(a) m/n (b) n/m
(c) $m + (n/m)$ (d) $n + (m/n)$

5. Let m and n $(m < n)$ be the roots of the equation $x^2 - 16x + 39 = 0$. If four terms p, q, r and s are inserted between m and n to form an AP, then what is the value of $p + q + r + s$? **[NDA 2019-II]**

(a) 29 (b) 30 (c) 32 (d) 35

6. Let a, b, c be in AP and $k \neq 0$ be a real number. Which of the following are correct? **[NDA 2019-II]**

1. ka, kb, kc are in AP 2. $k - a, k - b, k - c$ are in AP
3. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

7. How many two-digit numbers are divisible by 4? **[NDA 2019-II]**

(a) 21 (b) 22 (c) 24 (d) 25

8. Let S_n be the sum of the first n terms of an AP. If $S_{2n} = 3n + 14n^2$, then what is the common difference? **[NDA 2019-II]**

(a) 5 (b) 6 (c) 7 (d) 9

9. If 3rd, 8th and 13th terms of a GP are p, q and r respectively, then which one of the following is correct? **[NDA 2019-II]**

(a) $q^2 = pr$ (b) $r^2 = pq$
(c) $pqr = 1$ (d) $2q = p + r$

10. If p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP, then which of the following is/are correct? **[NDA 2020-I]**

1. p, q and r are in GP.
2. $\ln p$, $\ln q$ and $\ln r$ are in AP.

Select the correct answer using the code given below : **[NDA 2020-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

11. If $n = 100!$, then what is the value of the following?

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{100} n}$$ **[NDA 2021-I]**

(a) 0 (b) 1 (e) 2 (d) 3

12. If the first term of an AP is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms, then what is the sum of the first ten terms? **[NDA 2021-I]**

(a) -500 (b) -250 (c) 500 (d) 250

13. Consider the following statements :

1. If each term of a GP is multiplied by same non-zero number, then the resulting sequence is also a GP.
2. If each term of a GP is divided by same non-zero number, then the resulting sequence is also a GP.

Which of the above statements is/are correct? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

14. If x^2, x, -8 are in AP, then which one of the following is correct? **[NDA 2021-I]**

(a) $x \in \{-2\}$ (b) $x \in \{4\}$
(c) $x \in \{-2, 4\}$ (d) $x \in \{-4, 2\}$

15. The third term of a GP is 3. What is the product of its first five terms? **[NDA 2021-I]**

(a) 81
(b) 243
(c) 729
(d) Cannot be detennined due to insufficient data

16. Let x be the HM and y be the GM of two positive numbers m and n. If $5x = 4y$, then which one of the following is correct? **[NDA 2021-I]**

(a) $5m = 4n$ (b) $2m = n$
(c) $4m = 5n$ (d) $m = 4n$

17. The geometric mean of a set of observations is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is **[NDA 2021-I]**

(a) 810 (b) 900 (c) 30000 (d) 81000

Complex Numbers

1. What is the value of **[NDA 2019-II]**

$$\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019} ?$$

(a) 1 (b) -1 (c) $2i$ (d) $-2i$

2. If $x = 1 + i$, then what is the value of $x^6 + x^4 + x^2 + 1$? **[NDA 2019-II]**

(a) $6i - 3$ (b) $-6i + 3$ (c) $-6i - 3$ (d) $6i + 3$

3. What is the modulus of the complex number $\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$, where $i = \sqrt{-1}$? **[NDA 2020-I]**

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

4. What is the argument of the complex number $\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$, where $i = \sqrt{-1}$? **[NDA 2020-I]**

(a) 240° (b) 210° (c) 120° (d) 60°

5. The smallest positive integer n for which $\left(\frac{1-i}{1+i}\right)^{n^2}$ where $i = \sqrt{-1}$, is **[NDA 2021-I]**

(a) 2 (b) 4 (c) 6 (d) 8

6. If $Z = 1 + i$, where $i = \sqrt{-1}$, then what is the modulus of $Z + \frac{2}{Z}$? **[NDA 2021-I]**

(a) 1 (b) 2 (c) 3 (d) 4

7. Consider the following in respect of a complex number Z:

1. $\overline{(Z^{-1})} = (\overline{Z})^{-1}$
2. $ZZ^{-1} = |Z|^2$

Which of the above is/are correct? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

8. Consider the following statements in respect of an arbitrary complex number Z:

1. The difference of Z and its conjugate is an imaginary number.
2. The sum of Z and its conjugate is a real number.

Which of the above statements is/are correct? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

9. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$, where $n \in N$ and $i = \sqrt{-1}$? **[NDA 2021-I]**

(a) -1 (b) 1 (c) $\sqrt{2}$ (d) 2

Binomial Theorem, Mathematical Induction

1. If the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2x}$ is $184756x^{10}$, then what is the value of n? **[NDA 2019-II]**

(a) 10 (b) 8 (c) 5 (d) 4

2. If the constant term in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then what can be the values of k? **[NDA 2019-II]**

(a) ± 2 (b) ± 3 (c) ± 5 (d) ± 9

3. What is the sum of the last five coefficients in the expansion of $(1 + x)^9$ when it is expanded in ascending powers of x? **[NDA 2020-I]**

(a) 256 (b) 512 (c) 1024 (d) 2048

4. The term independent of x in the binomial expansion of $\left(\frac{2}{x^2} - \sqrt{x}\right)^{10}$ is equal to **[NDA 2020-I]**

(a) 180 (b) 120 (c) 90 (d) 72

5. If $(1+2x-x^2)^6 = a_0 + a_1x + a_2x^2 + ... + a_{12}x^{12}$, then what is $a_0 - a_1 + a_2 - a_3 + a_4 - .... + a_{12}$ equal to? **[NDA 2020-I]**

(a) 32 (b) 64 (c) 2048 (d) 4096

6. If $C_0, C_1, C_2, \ldots, C_n$ are the coefficients in the expansion of $(1 + x)^n$, then what is the value of $C_1 + C_2 + C_3 + \ldots + C_n$? **[NDA 2021-I]**

(a) 2^n (b) $2^n - 1$ (c) 2^{n-1} (d) $2^n - 2$

7. What is the coefficient of the middle term in the expansion of $(1 + 4x + 4x^2)^5$? **[NDA 2021-I]**

(a) 8064 (b) 4032 (c) 2016 (d) 1008

8. What is $C(n, 1) + C(n, 2) + \ldots + C(n, n)$ equal to? **[NDA 2021-I]**

(a) $2 + 2^2 + 2^3 + \ldots + 2^n$
(b) $1 + 2 + 2^2 + 2^3 + \ldots + 2^n$
(c) $1 + 2 + 2^2 + 2^3 + \ldots + 2^{n-1}$
(b) $2 + 2^2 + 2^3 + \ldots + 2^{n-1}$

9. What is the sum of the coefficients of first and last terms in the expansion of $(1 + x)^{2n}$, where n is a natural number? **[NDA 2021-I]**

(a) *1* (b) *2* (c) n (d) $2n$

Permutation and Combination

1. What is the number of diagonals of an octagon? **[NDA 2019-II]**

(a) 48 (b) 40 (c) 28 (d) 20

2. If $P(n, r) = 2520$ and $C(n, r) = 21$, then what is the value of $C(n + 1, r + 1)$? **[NDA 2019-II]**

(a) 7 (b) 14 (c) 28 (d) 56

3. What is $C(47, 4) + C(51, 3) + C(50, 3) + C(49, 3) + C(48, 3) + C(47, 3)$ equal to ? **[NDA 2019-II]**

(a) $C(47, 4)$ (b) $C(52, 5)$
(c) $C(52, 4)$ (d) $C(47, 5)$

4. If $C(20, n + 2) = C(20, n - 2)$, then what is n equal to? **[NDA 2020-I]**

(a) 18 (b) 25 (c) 10 (d) 12

5. What is the number of ways in which the letters of the word 'ABLE' can be arranged so that the vowels occupy even places? **[NDA 2020-I]**

(a) 2 (b) 4 (c) 6 (d) 8

6. What is the maximum number of points of intersection of 5 non-overlapping circles? **[NDA 2020-I]**

(a) 10 (b) 15 (c) 20 (d) 25

7. In how many ways can a team of 5 players be selected from 8 players so as not to include a particular player? **[NDA 2021-I]**

(a) 42 (b) 35 (c) 21 (d) 20

8. How many 5-digit prime numbers can be formed using the digits 1, 2, 3, 4, 5 if the repetition of digits is not allowed? **[NDA 2021-I]**

(a) 5 (b) 4 (c) 3 (d) 0

Cartesian Coordinate System and Straight Line

1. The equation $ax + by + c = 0$ represents a straight line **[NDA 2019-II]**

(a) for all real numbers, a, b and c
(b) only when $a \neq 0$
(c) only when $b \neq 0$
(d) only when at least one of a and b is non-zero

2. What is the distance between the points **[NDA 2019-II]** $P(m \cos 2\alpha, m \sin 2\alpha)$ and $Q(m\cos \beta, m \sin 2\beta)$?

(a) $|2m \sin (\alpha - \beta)|$ (b) $|2m \cos (\alpha - \beta)|$
(c) $|m \sin (2\alpha - 2\beta)|$ (d) $|m \sin (2\alpha - 2\beta)|$

3. An equilateral triangle has one vertex at (–1, – 1) and another vertex at $\left(-\sqrt{3}, \sqrt{3}\right)$. The third vertex may lie on **[NDA 2019-II]**

(a) $\left(-\sqrt{2}, \sqrt{2}\right)$ (b) $\left(\sqrt{2}, -\sqrt{2}\right)$
(c) $(1, 1)$ (d) $(1, -1)$

4. The point (1, –1) is one of the vertices of a square. If $3x + 2y = 5$ is the equation of one diagonal of the square, then what is the equation of the other diagonal? **[NDA 2020-I]**

(a) $3x - 2y = 5$ (b) $2x - 3y = 1$
(c) $2x - 3y = 5$ (d) $2x + 3y = -1$

5. If the circumcentre of the triangle formed by the lines $x + 2 = 0$, $y + 2 = 0$ and $kx + y + 2 = 0$ is (–1, –1), then what is the value of k ? **[NDA 2020-I]**

(a) –1 (b) –2 (c) 1 (d) 2

6. Under which condition, are the points (a, b), (c, d) and $(a - c, b - d)$ collinear? **[NDA 2020-I]**

(a) $ab = cd$ (b) $ac = bd$
(c) $ad = bc$ (d) $abc = d$

7. Let ABC be a triangle. If $D(2, 5)$ and $E(5, 9)$ are the mid-points of the sides AB and AC respectively, then what is the length of the side BC? **[NDA 2020-I]**

(a) 8 (b) 10 (c) 12 (d) 14

8. If the foot of the perpendicular drawn from the point $(0, k)$ to the line $3x - 4y - 5 = 0$ is (3, 1), then what is the value of k? **[NDA 2020-I]**

(a) 3 (b) 4 (c) 5 (d) 6

9. If $3x - 4y - 5 = 0$ and $3x - 4y + 15 = 0$ are the equations of a pair of opposite sides of a square, then what is the area of the square? **[NDA 2020-I]**

(a) 4 square units (b) 9 square units
(c) 16 square units (d) 25 square units

10. A parallelogram has three consecutive vertices (– 3, 4), (0, – 4) and (5, 2). The fourth vertex is **[NDA 2021-I]**

(a) (2, 10) (b) (2, 9) (c) (3, 9) (d) (4, 10)

11. If the lines $y + px = 1$ and $y - qx = 2$ are perpendicular, then which one of the following is correct? **NDA 2021-I]**

(a) $pq + 1 = 0$ (b) $p + q + 1 = 0$
(c) $pq - 1 = 0$ (d) $p - q + 1 = 0$

12. If A, B and C are in AP, then the straight line $Ax + 2By + C = 0$ will always pass through a fixed point. The fixed point is **[NDA 2021-I]**

(a) (0, 0) (b) (– 1, 1) (c) (1, –2) (d) (1, – 1)

13. If the image of the point (– 4, 2) by a line mirror is (4, – 2), then what is the equation of the line mirror? **[NDA 2021-I]**

(a) $y = x$ (b) $y = 2x$ (c) $4y = x$ (d) $y = 4x$

14. Consider the following statements in respect of the points $(p, p - 3)$, $(q + 3, q)$ and (6, 3) :

1. The points lie on a straight line.
2. The points always lie in the first quadrant only for any value of p and q.

Which of the above statements is/are correct ? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

15. The point of intersection of diagonals of a square $ABCD$ is at the origin and one of its vertices is at A {4, 2}. What is the equation of the diagonal BD? **[NDA 2021-I]**

(a) $2x + y = 0$ (b) $2x - y = 0$
(c) $x + 2y = 0$ (d) $x - 2y = 0$

Pair of Straight Lines

1. What is the angle between the lines **[NDA 2019-II]**
$x\cos\alpha + y\sin\alpha = a$ and
$x\sin\beta - y\cos\beta = a$

(a) $\beta - \alpha$ (b) $\pi + \beta - \alpha$
(c) $\frac{(\pi + 2\beta + 2\alpha)}{2}$ (d) $\frac{(\pi - 2\beta + 2\alpha)}{2}$

2. What is the obtuse angle between the lines whose slopes are $2 - \sqrt{3}$ and $2 + \sqrt{3}$? **[NDA 2020-I]**

(a) 105° (b) 120° (c) 135° (d) 150°

3. What is the acute angle between the lines $x - 2 = 0$ and $\sqrt{3x} - y - 2 = 0$? **[NDA 2021-I]**

(a) 0° (b) 30° (e) 45° (d) 60°

Circles

1. The center of the circle
$(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$ is **[NDA 2020-I]**

(a) $(2a, 2c)$ (b) $(2b, 2d)$
(c) $(a + b, c + d)$ (d) $(a - b, c - d)$

2. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$? **[NDA 2021-I]**

(a) 14 units (b) 10.5 units
(c) 7 units (d) 3.5 units

CONICS- Parabola, Ellipse & Hyperbola

1. If the angle between the lines joining the end points of minor axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with one of its foci is $\frac{\pi}{2}$, then what is the eccentricity of the ellipse? **[NDA 2019-II]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

2. Let $P(x, y)$ be any point on the ellipse $25x^2 + 16y^2 = 400$. If $Q(0, 3)$ and $R(0, -3)$ are two points, then what is $(PQ + PR)$ equal to ? **[NDA 2020-I]**

(a) 12 (b) 10 (c) 8 (d) 6

3. In the parabola, $y^2 = x$, what is the length of the chord passing through the vertex and inclined to the x-axis at an angle θ? **[NDA 2020-I]**

(a) $\sin\theta \cdot \sec^2\theta$ (b) $\cos\theta \cdot \text{cosec}^2\theta$

(c) $\cot\theta \cdot \sec^2\theta$ (d) $2\tan\theta \cdot \text{cosec}^2\theta$

4. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$, then what is the eccentricity of the hyperbola ? **[NDA 2021-I]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{\sqrt{11}}{2}$ (d) $\frac{\sqrt{13}}{2}$

5. Consider the following with regard to eccentricity (e) of a conic section :

1. $e = 0$ for circle
2. $e = 1$ for parabola
3. $e < 1$ for ellipse

Which of the above are correct? **[NDA 2021-I]**

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

TRIGONOMETRY- Ratio & Identity, Trigonometric Equations

1. If $\text{cosec}\ \theta = \frac{29}{21}$ where $0 < \theta < 90°$, then what is the value of $4\sec\theta + 4\tan\theta$? **[NDA 2019-II]**

(a) 5 (b) 10 (c) 15 (d) 20

2. Consider the following statements: **[NDA 2019-II]**

1. $\cos\theta + \sec\theta$ can never be equal to 1.5.
2. $\tan\theta + \cot\theta$ can never be less than 2.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

3. What is the length of the chord of a unit circle which subtends an angle θ at the centre? **[NDA 2019-II]**

(a) $\sin\left(\frac{\theta}{2}\right)$ (b) $\cos\left(\frac{\theta}{2}\right)$

(c) $2\sin\left(\frac{\theta}{2}\right)$ (d) $2\cos\left(\frac{\theta}{2}\right)$

4. What is the minimum value of $\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$ where $a > 0$ and $b > 0$? **[NDA 2019-II]**

(a) $(a+b)^2$ (b) $(a-b)^2$

(c) $a^2 + b^2$ (d) $|a^2 + b^2|$

5. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then what is the value of $\cot(A - B)$? **[NDA 2019-II]**

(a) $\frac{1}{x} + \frac{1}{y}$ (b) $\frac{1}{y} - \frac{1}{x}$

(c) $\frac{xy}{x+y}$ (d) $1 + \frac{1}{xy}$

6. What is $\sin(\alpha + \beta) - 2\sin\alpha\cos\beta + \sin(\alpha - \beta)$ equal to ? **[NDA 2019-II]**

(a) 0 (b) $2\sin\alpha$

(c) $2\sin\beta$ (d) $\sin\alpha + \sin\beta$

7. If $2\tan A - 3\tan B - 1$, then what is $\tan(A - B)$ equal to? **[NDA 2019-II]**

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{7}$ (d) $\frac{1}{9}$

8. What is $\cos 80° + \cos 40° - \cos 20°$ equal to? **[NDA 2019-II]**

(a) 2 (b) 1 (c) 0 (d) –19

9. What is $\cot\left(\frac{A}{2}\right) - \tan\left(\frac{A}{2}\right)$ equal to? **[NDA 2019-II]**

(a) tanA (b) cotA (c) 2tanA (d) 2cotA

10. What is cotA + cosecA equal to? **[NDA 2019-II]**

(a) $\tan\left(\frac{A}{2}\right)$ (b) $\cot\left(\frac{A}{2}\right)$

(c) $2\tan\left(\frac{A}{2}\right)$ (d) $2\cot\left(\frac{A}{2}\right)$

11. What is $\tan 25° \tan 15° + \tan 15° \tan 50° + \tan 25° \tan 50°$ equal to? **[NDA 2019-II]**

(a) 0 (b) 1 (c) 2 (d) 4

DIRECTIONS (Qs. 12-14) : *Read the following information and answer the three items that follow:*

Let $a\sin^2 x + b\cos^2 x = c$; $b\sin^2 y + a\cos^2 y = d$ and $p\tan x = q\tan y$.

12. What is $\tan^2 x$ equal to ? **[NDA 2020-I]**

(a) $\frac{c-b}{a-c}$ (b) $\frac{a-c}{c-b}$

(c) $\frac{c-a}{c-b}$ (d) $\frac{c-b}{c-a}$

13. What is $\frac{d-a}{b-d}$ equal to? **[NDA 2020-I]**

(a) $\sin^2 y$ (b) $\cos^2 y$ (c) $\tan^2 y$ (d) $\cot^2 y$

14. What is $\frac{p^2}{q^2}$ equal to? **[NDA 2020-I]**

(a) $\frac{(b-c)(b-d)}{(a-d)(a-c)}$ (b) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$

(c) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$ (d) $\frac{(b-c)(b-d)}{(c-a)(a-d)}$

DIRECTIONS (Qs. 15-17) : *Read the following information and answer the three items that follow:*

Let $t_n = \sin^n\theta + \cos^n\theta$.

15. What is $\frac{t_3 - t_5}{t_5 - t_7}$ equal to? **[NDA 2020-I]**

(a) $\frac{t_1}{t_3}$ (b) $\frac{t_3}{t_5}$ (c) $\frac{t_5}{t_7}$ (d) $\frac{t_1}{t_7}$

16. What is $t_1^2 - t_2$ equal to? **[NDA 2020-I]**

(a) $\cos 2\theta$ (b) $\sin 2\theta$

(c) $2\cos\theta$ (d) $2\sin\theta$

17. What is the value of t_{10} where $\theta = 45°$? **[NDA 2020-I]**

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$

DIRECTIONS (Qs. 18-20) : *Read the following information and answer the three items that follow:*

Let $\alpha = \beta = 15°$.

18. What is the value of $\sin\alpha + \cos\beta$? **[NDA 2020-I]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

19. What is the value of $\sin 7\alpha - \cos 7\beta$? **[NDA 2020-I]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

20. What is $\sin(\alpha + 1°) + \cos(\beta + 1°)$ equal to? **[NDA 2020-I]**

(a) $\sqrt{3}\cos 1° + \sin 1°$

(b) $\sqrt{3}\cos 1° - \frac{1}{2}\sin 1°$

(c) $\frac{1}{\sqrt{2}}(\sqrt{3}\cos 1° - \sin 1°)$

(d) $\frac{1}{2}(\sqrt{3}\cos 1° + \sin 1°)$

21. If $\sin x + \sin y = \cos y - \cos x$, where $0 < y < x < \frac{\pi}{2}$, then what is $\tan\left(\frac{x-y}{2}\right)$ equal to? **[NDA 2020-I]**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

DIRECTIONS (Qs. 22-23) : *Read the following information and answer the two items that follow:*

Let $\frac{\tan 3A}{\tan A} = K$, where $\tan A \neq 0$ and $K \neq \frac{1}{3}$.

22. What is $\tan^2 A$ equal to? **[NDA 2020-I]**

(a) $\frac{K+3}{3K-1}$ (b) $\frac{K-3}{3K-1}$ (c) $\frac{3K-3}{K-3}$ (d) $\frac{K+3}{3K+1}$

23. For real values of tan A, K **cannot** lie between **[NDA 2020-I]**

(a) $\frac{1}{3}$ and 3 (b) $\frac{1}{2}$ and 2

(c) $\frac{1}{5}$ and 5 (d) $\frac{1}{7}$ and 7

24. If $\tan\theta = \frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$, then what is the value of θ? **[NDA 2020-I]**

(a) 0° (b) 28° (c) 38° (d) 52°

25. A and B are positive acute angles such that $\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$. What is the value of $(A + 2B)$? **[NDA 2020-I]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

26. What is

$\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$

equal to ? **[NDA 2020-I]**

(a) 0 (b) 1 (c) $2\sin 2x$ (d) $4\cos 4x$

27. The value of ordinate of the graph of $y = 2 + \cos x$ lies in the interval **[NDA 2020-I]**

(a) [0, 1] (b) [0, 3] (c) [–1, 1] (d) [1, 3]

28. What is the value of $8\cos 10° \cdot \cos 20° \cdot \cos 40°$? **[NDA 2020-I]**

(a) tan 10° (b) cot 10° (c) cosec 10° (d) sec 10°

29. What is the value of cos 48° – cos 12°? **[NDA 2020-I]**

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{1-\sqrt{5}}{4}$ (c) $\frac{\sqrt{5}+1}{2}$ (d) $\frac{1-\sqrt{5}}{8}$

30. The value of x, satisfying the equation $\log_{\cos x} \sin x = 1$, where $0 < x < \frac{\pi}{2}$, is **[NDA 2021-I]**

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

31. What is $\cot 2x \cot 4x - \cot 4x \cot 6x - \cot 6x \cot 2x$ equal to? **[NDA 2021-I]**

(a) – 1 (b) 0 (c) 1 (d) 2

32. If $\tan x = -\frac{3}{4}$ and x is in the second quadrant, then what is the value of $\sin x . \cos x$? **[NDA 2021-I]**

(a) $\frac{6}{25}$ (b) $\frac{12}{25}$ (c) $-\frac{6}{25}$ (d) $-\frac{12}{25}$

33. What is the value of the following? **[NDA 2021-I]**

$\text{cosec}\left(\frac{7\pi}{6}\right) \sec\left(\frac{5\pi}{3}\right)$

(a) $\frac{4}{3}$ (b) 4 (c) – 4 (d) $-\frac{4}{\sqrt{3}}$

34. What is the value of the following?

tan 31° tan 33° tan 35° .... tan 57° tan 59° **[NDA 2021-I]**

(a) – 1 (b) 0 (c) 1 (d) 2

35. What is the value of the following?

(sin 24° + cos 66°) (sin 24° – cos 66°) **[NDA 2021-I]**

(a) – 1 (b) 0 (c) 1 (d) 2

36. What is (1 + cot θ – cosec θ) (1 + tan θ + sec θ) equal to? **[NDA 2021-I]**

(a) 1 (b) 2 (c) 3 (d) 4

37. What is $\frac{1+\tan^2\theta}{1+\cot^2\theta}-\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$ equal to? **[NDA 2021-I]**

(a) 0 (b) 1 (c) 2 tan θ (d) 2 cot θ

38. If 7 sin θ + 24 cos θ = 25, then what is the value of (sin θ + cos θ)? **[NDA 2021-I]**

(a) 1 (b) $\frac{26}{25}$ (c) $\frac{6}{5}$ (d) $\frac{31}{25}$

39. If 3 cos θ = 4 sin θ, then what is the value of tan (45° + θ)? **[NDA 2021-I]**

(a) 10 (b) 7 (c) $\frac{7}{2}$ (d) $\frac{7}{4}$

40. If $\tan A = \frac{1}{7}$, then what is cos 2*A* equal to ? **[NDA 2021-I]**

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{12}{25}$ (d) $\frac{6}{25}$

Properties of Triangle, Inverse Trigonometric Function

1. What is $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$ equal to? **[NDA 2019-II]**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{9}$

2. If the angles of a triangles ABC are in AP and $b : c = \sqrt{3} : \sqrt{2}$, then what is the measure of angle A? **[NDA 2019-II]**

(a) 30° (b) 45° (c) 60° (d) 75°

3. If angle C of a triangle ABC is a right angle, then what is tanA + tanB equal to? **[NDA 2019-II]**

(a) $\frac{a^2-b^2}{ab}$ (b) $\frac{a^2}{bc}$ (c) $\frac{b^2}{ca}$ (d) $\frac{c^2}{ab}$

DIRECTIONS (Qs. 4-5) : *Read the following information and answer the two items that follow:*

ABCD is a trapezium such that *AB* and *CD* are parallel and *BC* is perpendicular to them. Let $\angle ADB = \theta$, $\angle ABD = \alpha$, $BC = p$ and $CD = q$ and $CD = q$.

4. Consider the following : **[NDA 2020-I]**

1. $AD\sin\theta = AB\sin\alpha$
2. $BD\sin\theta = AB\sin(\theta+\alpha)$

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

5. What is *AB* equal to? **[NDA 2020-I]**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (b) $\frac{(p^2-q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

(c) $\frac{(p^2+q^2)\sin\theta}{q\cos\theta+p\sin\theta}$ (d) $\frac{(p^2-q^2)\cos\theta}{q\cos\theta+p\sin\theta}$

6. Consider the following statements : **[NDA 2020-I]**

1. If *ABC* is a right-angled triangle, right-angled at *A* and if $\sin B = \frac{1}{3}$, then cosec *C* = 3.
2. If *b* cos *B* = *c* cos *C* and if the triangle *ABC* is not right-angled, then *ABC* must be isosceles.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

7. Consider the following statements : **[NDA 2020-I]**

1. If in a triangle *ABC*, *A* = 2*B* and *b* = *c*, then it must be an obtuse-angled triangle.
2. There exists no triangle *ABC* with *A* = 40°, *B* = 65° and $\frac{a}{c}$ = sin 40° cosec 15°.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

8. The equation $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$ has **[NDA 2021-I]**

(a) no solution
(b) unique solution
(c) two solutions
(d) infinite number of solutions

9. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ holds, when **[NDA 2021-I]**

(a) $x \in R$ (b) $x \in R - (-1, 1)$ only
(c) $x \in R - \{0\}$ only (d) $x \in R - [-1, 1]$ only

10. The sides of a triangle are *m*, *n* and $\sqrt{m^2+n^2+mn}$. What is the sum of the acute angles of the triangle? **[NDA 2021-I]**

(a) 45° (b) 60° (c) 75° (d) 90°

11. What is the area of the triangle *ABC* with sides *a* = 10 cm, *c* = 4 cm and angle *B* = 30° ? **[NDA 2021-I]**

(a) 16 cm^2 (b) 12 cm^2 (c) 10 cm^2 (d) 8 cm^2

Height & Distance

1. A ladder 9 m long reaches a point 9 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the flagstaff is 60°. What is the height of the flagstaff? **[NDA 2019-II]**

(a) 9m (b) 10.5m (c) 13.5m (d) 15m

2. A ladder 6 m long reaches a point 6 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the top of the flagstaff is 75°. What is the height of the flagstaff ? **[NDA 2021-I]**

(a) 12 m (b) 9 m

(c) $(6+\sqrt{3})$ m (d) $(6+3\sqrt{3})$ m

3. The shadow of a tower is found to be x metre longer, when the angle of elevation of the sun changes from 60° to 45°. If the height of the tower is $5(3+\sqrt{3})$ m, then what is x equal to? **[NDA 2021-I]**

(a) 8 m (b) 10 m (c) 12 m (d) 15 m

Functions, Limit, Continuity and Differentiability

1. Consider the following statements in respect of the function $f(x) = \sin\left(\frac{1}{x}\right)$ for x^1 0 and $f(0) = 0$; **[NDA 2019-II]**

1. $\lim_{x\to 0} f(x)$ exists
2. $f(x)$ is continuous at $x = 0$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

2. What is the value of $\lim_{x\to 0}\frac{\sin x^\circ}{\tan 3x^\circ}$? **[NDA 2019-II]**

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

Directions for the following three (03) items: *Read the following information and answer the three items that follow:*

Consider the function $f(x) = g(x) + h(x)$

where $g(x) = \sin\left(\frac{x}{4}\right)$ and $h(x) = \cos\left(\frac{4x}{5}\right)$

3. What is the period of the function $g(x)$? **[NDA 2019-II]**

(a) π (b) 2π (c) 4π (d) 8π

4. What is the period of the function $h(x)$? **[NDA 2019-II]**

(a) π (b) $\frac{4p}{5}$ (c) $\frac{5p}{2}$ (d) $\frac{3p}{2}$

5. What is the period of the function $f(x)$? **[NDA 2019-II]**

(a) 10π (b) 20π (c) 40π (d) 80π

6. For what value of k is the function **[NDA 2019-II]**

$$f(x) = \begin{cases} 2x + \frac{1}{4}, & x < 0 \\ k, & x = 0 \text{ continuous?} \\ \left(x + \frac{1}{2}\right)^2, & x > 0 \end{cases}$$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

7. Consider the following statements for $f(x) = e^{-|x|}$: **[NDA 2020-I]**

1. The function is continuous at $x = 0$.
2. The function is differentiable at $x = 0$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

8. What is $\lim_{x\to 0}\frac{3^x + 3^{-x} - 2}{x}$ equal to? **[NDA 2020-I]**

(a) 0 (b) –1

(c) 1 (d) Limit does not exist

9. Which one of the following is correct in respect of the graph of $y = \frac{1}{x-1}$? **[NDA 2020-I]**

(a) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of reals.

(b) The domain is $\{x \in R \mid x \neq 1\}$, the range is $\{y \in R \mid y \neq 0\}$ and the graph intersects y-axis at $(0, -1)$.

(c) The domain is the set of reals and the range is the singleton set $\{0\}$.

(d) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of points on the y-axis.

10. If $f(x) = \frac{\sin x}{x}$, where $x \in R$, is to be continuous at $x = 0$, then the value of the function at $x = 0$ **[NDA 2020-I]**

(a) should be 0 (b) should be 1

(c) should be 2 (d) cannot be determined

11. What is the domain of the function $f(x) = \cos^{-1}(x-2)$? **[NDA 2020-I]**

(a) [–1, 1] (b) [1, 3] (c) [0, 5] (d) [–2, 1]

12. What is $\lim_{x\to 1}\frac{x + x^2 + x^3 - 3}{x-1}$ equal to ? **[NDA 2020-I]**

(a) 1 (b) 2 (c) 3 (d) 6

13. If $\lim_{x\to 1}\frac{x^4 - 1}{x-1} = \lim_{x\to k}\frac{x^3 - k^3}{x^2 - k^2}$, where $k \neq 0$, then what is the value of k? **[NDA 2020-I]**

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) 4

14. What is $\lim_{x\to 0}\frac{\sin x \log(1-x)}{x^2}$ equal to? **[NDA 2020-I]**

(a) –1 (b) Zero (c) $-e$ (d) $-\frac{1}{e}$

15. If $f(x+1) = x^2 - 3x + 2$, then what is $f(x)$ equal to? **[NDA 2021-I]**

(a) x^2-5x+4 (b) x^2-5x+6
(c) x^2+3x+3 (d) x^2-3x+1

16. If $\lim_{x\to a}\frac{a^x-x^a}{x^a-a^a}=-1$ then what is the value of a? **[NDA 2021-I]**

(a) -1 (b) 0 (c) 1 (d) 2

17. What is $\lim_{x\to -1}\frac{x^3+x^2}{x^2+3x+2}$ equal to? **[NDA 2021-I]**

(a) 0 (b) 1 (c) 2 (d) 3

18. If a differentiable function $f(x)$ satisfies

$\lim_{x\to -1}\frac{f(x)+1}{x^2-1}=-\frac{3}{2}$ then what is $\lim_{x\to -1} f(x)$ equal to? **[NDA 2021-I]**

(a) $-\frac{3}{2}$ (b) -1 (c) 0 (d) 1

19. If the function $f(x)=\begin{cases} a+bx, & x<1 \\ 5, & x=1 \\ b-ax, & x>1 \end{cases}$ is continuous, then what is the value of $(a+b)$? **[NDA 2021-I]**

(a) 5 (b) 10 (c) 15 (d) 20

20. What is the domain of the function $f(x)=3^x$? **[NDA 2021-I]**

(a) $(-\infty,\infty)$ (b) $(0,\infty)$
(c) $[0,\infty)$ (d) $(-\infty,\infty)-\{0\}$

Derivatives

Directions for the following two (02) items: *Read the following information and answer the two items that follow:*

Consider the equation $x^y=e^{x-y}$

1. What is $\frac{dy}{dx}$ at $x=1$ equal to? **[NDA 2019-II]**

(a) 0 (b) 1 (c) 2 (d) 4

2. What is $\frac{d^2y}{dx^2}$ at $x=1$ equal to? **[NDA 2019-II]**

(a) 0 (b) 1 (c) 2 (d) 4

3. What is the derivative of $2^{(\sin x)^2}$ with respect to $\sin x$? **[NDA 2019-II]**

(a) $\sin x 2^{(\sin x)^2}\ln 4$ (b) $2\sin x 2^{(\sin x)^2}\ln 4$

(c) $\text{In}(\sin x)2^{(\sin x)^2}$ (d) $2\sin x\cos x 2^{(\sin x)^2}$

4. What is the derivative of $\tan^{-1}x$ with respect to $\cot^{-1}x$? **[NDA 2020-I]**

(a) -1 (b) 1 (c) $\frac{1}{x^2+1}$ (d) $\frac{x}{x^2+1}$

5. If $e^{\theta\varphi}=c+4\theta\varphi$, where c is an arbitrary constant and φ is a function of θ, then what is $\varphi\, d\theta$ equal to? **[NDA 2020-I]**

(a) $\theta\, d\varphi$ (b) $-\theta\, d\varphi$ (c) $4\theta\, d\varphi$ (d) $-4\theta\, d\varphi$

6. If $x^m y^n=a^{m+n}$, then what is $\frac{dy}{dx}$ equal to? **[NDA 2020-I]**

(a) $\frac{my}{nx}$ (b) $-\frac{my}{nx}$ (c) $\frac{mx}{ny}$ (d) $-\frac{ny}{mx}$

7. What is the minimum value of $|x-1|$, where $x\in R$? **[NDA 2020-I]**

(a) 0 (b) 1 (c) 2 (d) -1

8. What is the derivative of sin (ln x) + cos (ln x) with respect to x at $x=e$? **[NDA 2021-I]**

(a) $\frac{\cos 1-\sin 1}{e}$ (b) $\frac{\sin 1-\cos 1}{e}$

(c) $\frac{\cos 1+\sin 1}{e}$ (d) 0

9. If $x=e^t\cos t$ and $y=e^t\sin t$, then what is $\frac{dx}{dy}$ at $t=0$ equal to? **[NDA 2021-I]**

(a) 0 (b) 1 (c) $2e$ (d) -1

10. What is the derivative of e^x with respect to x^e? **[NDA 2021-I]**

(a) $\frac{xe^x}{ex^e}$ (b) $\frac{e^x}{x^e}$ (c) $\frac{xe^x}{x^e}$ (d) $\frac{e^x}{ex^e}$

Application of Derivatives

Directions for the following three (03) items : *Read the following information and answer the three items that follow :*

A curve $y=me^{mx}$ where $m>0$ intersects y-axis at a point P.

1. What is the slope of the curve at the point of intersection P? **[NDA 2019-II]**

(a) m (b) m^2 (c) $2m$ (d) $2m^2$

2. How much angle does the tangent at P make with y-axis? **[NDA 2019-II]**

(a) $\tan^{-1} m^2$ (b) $\cot^{-1}(1+m^2)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+m^4}}\right)$ (d) $\sec^{-1}\sqrt{1+m^4}$

3. What is the equation of tangent to the curve at P? **[NDA 2019-II]**

(a) $y=mx+m$ (b) $y=-mx+2m$
(c) $y=m^2x+2m$ (d) $y=m^2x+m$

Directions for the following two (02) items: *Read the following information and answer the two items that follow:*

Consider the function

$f(x)=3x^4-20x^3-12x^2+288x+1$

4. In which one of the following intervals is the function increasing? **[NDA 2019-II]**
(a) $(-2, 3)$ (b) $(3, 4)$ (c) $(-3, -2)$ (d) $(-4, -3)$

5. In which one of the following intervals is the function decreasing? **[NDA 2019-II]**
(a) $(-2, 3)$ (b) $(3, 4)$ (c) $(4, 6)$ (d) $(6, 9)$

6. If $f(x)=\frac{x^3}{3}-\frac{5x^2}{2}+6x+7$ increases in the interval T and decreases in the interval S, then which one of the following is correct? **[NDA 2019-II]**
(a) $T=(-\infty, 2)\cup(3,\infty)$ and $S=(2,3)$
(b) $T=\phi$ and $S=(-\infty,\infty)$
(c) $T=(-\infty,\infty)$ and $S=\phi$
(d) $T=(2,3)$ and $S=(-\infty,2)\cup(3,\infty)$

7. What is the maximum value of $\sin x\cdot\cos x$? **[NDA 2020-I]**
(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $2\sqrt{2}$

8. What is the minimum value of $3\cos\left(A+\frac{\pi}{3}\right)$ where $A\in R$? **[NDA 2020-I]**
(a) –3 (b) –1 (c) 0 (d) 3

9. Consider the following statements : **[NDA 2020-I]**
1. The function $f(x)=\ln x$ increases in the interval $(0,\infty)$.
2. The function $f(x)=\tan x$ increases in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

10. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference? **[NDA 2020-I]**
(a) 4.4 cm/sec (b) 8.4 cm/sec
(c) 8.8 cm/sec (d) 15.4 cm/sec

11. A particle starts from origin with a velocity (in m/s) given by the equation $\frac{dx}{dt}=x+1$. The time (in second) taken by the particle to traverse a distance of 24 m is **[NDA 2021-I]**
(a) ln 24 (b) ln 5 (c) 2 ln 5 (d) 2 ln 4

12. The curve $y=-x^3+3x^2+2x-27$ has the maximum slope at **[NDA 2021-I]**
(a) $x=-1$ (b) $x=0$ (c) $x=1$ (d) $x=2$

13. If $x+y=20$ and $P=xy$, then what is the maximum value of P? **[NDA 2021-I]**
(a) 100 (b) 96 (c) 84 (d) 50

14. What is the maximum value of $\sin 2x\cdot\cos 2x$? **[NDA 2021-I]**
(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

15. Consider the following statements in respect of the function $f(x)=\sin x$: **[NDA 2021-I]**
1. $f(x)$ increases in the interval $(0,\pi)$.
2. $f(x)$ decreases in the interval $\left(\frac{5\pi}{2},3\pi\right)$.

Which of the above statements is/are correct? **[NDA 2021-I]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Indefinite Integration

1. What is $\int\frac{dx}{2x^2-2x+1}$ equal to? **[NDA 2019-II]**
(a) $\frac{\tan^{-1}(2x-1)}{2}+c$ (b) $2\tan^{-1}(2x-1)+c$
(c) $\frac{\tan^{-1}(2x+1)}{2}+c$ (d) $\tan^{-1}(2x-1)+c$

2. What is $\int\frac{dx}{x(1+\ell nx)^n}$ equal to $(n^1\ 1)$? **[NDA 2019-II]**
(a) $\frac{1}{(n-1)(1+\ell nx)^{n+1}}+c$ (b) $\frac{1-n}{(1+\ell nx)^{n-1}}+c$
(c) $\frac{n+1}{(1+\ell nx)^{n-1}}+c$ (d) $\frac{1}{(n-1)(1+\ell nx)^{n-1}}+c$

3. If $p(x)=(4e)^{2x}$, then what is $\int p(x)\,dx$ equal to ? **[NDA 2020-I]**
(a) $\frac{p(x)}{1+2\ln 2}+c$ (b) $\frac{p(x)}{2(1+2\ln 2)}+c$
(c) $\frac{2p(x)}{1+\ln 4}+c$ (d) $\frac{p(x)}{1+\ln 2}+c$

4. What is $\int(e^{\log x}+\sin x)\cos x\,dx$ equal to ? **[NDA 2020-I]**
(a) $\sin x+x\cos x+\frac{\sin^2 x}{2}+c$
(b) $\sin x-x\cos x+\frac{\sin^2 x}{2}+c$
(c) $x\sin x+\cos x+\frac{\sin^2 x}{2}+c$
(d) $x\sin x-x\cos x+\frac{\sin^2 x}{2}+c$

5. What is $\int \frac{dx}{x(x^n+1)}$ equal to? **[NDA 2020-I]**

(a) $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right)+c$ (b) $\ln\left(\frac{x^n+1}{x^n}\right)+c$

(c) $\ln\left(\frac{x^n}{x^n+1}\right)+c$ (d) $\frac{1}{n}\ln\left(\frac{x^n+1}{x^n}\right)+c$

6. What is the value of k such that integration of $\frac{3x^2+8-4k}{x}$ with respect to x, may be a rational function? **[NDA 2020-I]**

(a) 0 (b) 1 (c) 2 (d) –2

7. What is $\int \frac{dx}{\sec x+\tan x}$ equal to ? **[NDA 2021-I]**

(a) $\ln(\sec x)+\ln|\sec x+\tan x|+c$
(b) $\ln(\sec x)-\ln|\sec x+\tan x|+c$
(c) $\sec x\tan x-\ln|\sec x-\tan x|+c$
(d) $\ln|\sec x+\tan x|-\ln|\sec x|+c$

8. What is $\int \frac{dx}{\sec^2(\tan^{-1}x)}$ equal to ? **[NDA 2021-I]**

(a) $\sin^{-1}x+c$ (b) $\tan^{-1}x+c$
(c) $\sec^{-1}x+c$ (d) $\cos^{-1}x+c$

9. What is $\int e^{(2\ln x+\ln x^2)}dx$ equal to? **[NDA 2021-I]**

(a) $\frac{x^4}{4}+c$ (b) $\frac{x^3}{3}+c$

(c) $\frac{2x^5}{5}+c$ (d) $\frac{x^5}{5}+c$

Definite Integration & Its Application

Directions for the following two (02) items: *Read the following information and answer the two items that follow:*

Consider the integrals

$$I_1=\int_0^{x}\frac{xdx}{1+\sin x} \text{ and } I_2=\int_0^{\pi}\frac{(\pi-x)dx}{1-\sin(\pi+x)}$$

1. What is the value of I_1? **[NDA 2019-II]**

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

2. What is the value of I_1+I_2? **[NDA 2019-II]**

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0

3. What is the area of the region enclosed between the curve $y^2=2x$ and the straight line $y=x$? **[NDA 2019-II]**

(a) $\frac{2}{3}$ square units (b) $\frac{4}{3}$ square units

(c) $\frac{1}{3}$ square units (d) 1 square units

4. Let l be the length and b be the breadth of a rectangle such that $l+b=k$. What is the maximum area of the rectangle? **[NDA 2020-I]**

(a) $2k^2$ (b) k^2 (c) $\frac{k^2}{2}$ (d) $\frac{k^2}{4}$

5. What is the value of $\int_0^{\pi/4}(\tan^3x+\tan x)dx$? **[NDA 2020-I]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

6. Let $y=3x^2+2$. If x changes from 10 to 10.1, then what is the total change in y? **[NDA 2020-I]**

(a) 4.71 (b) 5.23 (c) 6.03 (d) 8.01

7. What is the area of the region enclosed between the curve $y^2=2x$ and the straight line $y=x$? **[NDA 2020-I]**

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) 2

8. What is $\int_0^{a}\frac{f(a-x)}{f(x)+f(a-x)}dx$ equal to? **[NDA 2021-I]**

(a) a (b) $2a$ (c) 0 (d) $\frac{a}{2}$

9. If $\int_0^{a}[f(x)+f(-x)]dx=\int_{-a}^{a}g(x)dx$ then what is $g(x)$ equal to? **[NDA 2021-I]**

(a) $f(x)$ (b) $f(-x)+f(x)$
(c) $-f(x)$ (d) None of the above

10. What is the area bounded by $y=\sqrt{16-x^2}$, $y\geq 0$ and the x-axis ? **[NDA 2021-I]**

(a) 16π square units (b) 8π square units
(c) 4π square units (d) 2π square units

Differential Equation

1. What is the degree of the differential equation $\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2-x^2\left(\frac{d^4y}{dx^4}\right)=0$? **[NDA 2019-II]**

(a) 1 (b) 2 (c) 3 (d) 4

2. Which one of the following is the differential equation that represents the family of curves $y=\frac{1}{2x^2-c}$ where c is an arbitrary constant? **[NDA 2019-II]**

(a) $\frac{dy}{dx} = 4xy^2$ (b) $\frac{dy}{dx} = \frac{1}{y}$

(c) $\frac{dy}{dx} = x^2 y$ (d) $\frac{dy}{dx} = -4xy^2$

Directions for the following three (03) items: *Read the following information and answer the three items that follow:*

Let $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$

3. What are the roots of the equation $g[f(x)] = 0$? **[NDA 2019-II]**

(a) 1, –1 (b) –1, –1 (c) 1, 1 (d) 0, 1

4. Consider the following statements: **[NDA 2019-II]**

1. $f[g(x)]$ is a polynomial of degree 3.
2. $g[g(x)]$ is a polynomial of degree 2.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

5. If $h(x) = 5f(x) - xg(x)$, then what is the derivative of $h(x)$? **[NDA 2019-II]**

(a) –40 (b) –20 (c) –10 (d) 0

6. The differential equation which represents the family of curves given by $\tan y = c(1 - e^x)$ is **[NDA 2019-II]**

(a) $e^x \tan y dx + (1 - e^x) dy = 0$

(b) $e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0$

(c) $e^x (1 - e^x) dx + \tan y dy = 0$

(d) $e^x \tan y dy + (1 - e^x) dx = 0$

7. The function $u(x, y) = c$ which satisfies the differential equation $x(dx - dy) + y(dy - dx) = 0$, is **[NDA 2020-I]**

(a) $x^2 + y^2 = xy + c$ (b) $x^2 + y^2 = 2xy + c$

(c) $x^2 - y^2 = xy + c$ (d) $x^2 - y^2 = 2xy + c$

8. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) = x$? **[NDA 2020-I]**

(a) $y = e^x + c$ (b) $y = e^{-x} + c$

(c) $y = \ln x + c$ (d) $y = 2\ln x + c$

9. The solution of the differential equation $dy = (1 + y^2)dx$ is **[NDA 2020-I]**

(a) $y = \tan x + c$ (b) $y = \tan(x + c)$

(c) $\tan^{-1}(y + c) = x$ (d) $\tan^{-1}(y + c) = 2x$

10. The order and degree of the differential equation $k\frac{dy}{dx} = \int\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{2}{3}} dx$ are respectively **[NDA 2020-I]**

(a) 1 and 1 (b) 2 and 3

(c) 2 and 4 (d) 1 and 4

11. If $f(x) = e^{|x|}$, then which one of the following is correct? **[NDA 2021-I]**

(a) $f'(0) = 1$ (b) $f'(0) = -1$

(c) $f'(0) = 0$ (d) $f'(0)$ does not exist

12. If the general solution of a differential equation is $y^2 + 2cy - cx + c^2 = 0$, where c is an arbitrary constant, then what is the order of the differential equation? **[NDA 2021-I]**

(a) 1 (b) 2 (c) 3 (d) 4

13. What is the degree of the following differential equation? $x = \sqrt{1 + \frac{d^2y}{dx^2}}$ **[NDA 2021-I]**

(a) 1 (b) 2

(c) 3 (d) Degree is not defined

14. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$? **[NDA 2021-I]**

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + y = 1$ (d) $\frac{dy}{dx} - y = 0$

15. What is the solution of the following differential equation?

$\ln\left(\frac{dy}{dx}\right) + y = x$ **[NDA 2021-I]**

(a) $e^x + e^y = c$ (b) $e^{x+y} = c$

(c) $e^x - e^y = c$ (d) $e^{x-y} = c$

Matrices & Determinants

1. What is the value of the determinant **[NDA 2019-II]**

$$\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} ?$$

(a) 0 (b) 12 (c) 24 (d) 36

2. What are the values of x that satisfy the equation **[NDA 2019-II]**

$$\begin{vmatrix} x & 0 & 2 \\ 2x & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 0 & 2 \\ x^2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0 ?$$

(a) $-2 \pm \sqrt{3}$ (b) $-1 \pm \sqrt{3}$

(c) $-1 \pm \sqrt{6}$ (d) $-2 \pm \sqrt{6}$

3. If $x + a + b + c = 0$, then what is the value of

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} ?$$ **[NDA 2019-II]**

(a) 0 (b) $(a + b + c)^2$

(c) $a^2 + b^2 + c^2$ (d) $a + b + c - 2$

4. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then the expression $A^3 - 2A^2$ is **[NDA 2019-II]**

(a) a null matrix (b) an identity matrix
(c) equal to A (d) equal to –A

5. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, then which one of the following is correct? **[NDA 2019-II]**

(a) Both AB and BA exist
(b) Neither AB nor BA exists
(c) AB exists but BA does not exist
(d) AB does not exist but BA exists

6. Let p, q and r be three distinct positive real numbers. If $D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$, then which one of the following is correct? **[NDA 2020-I]**

(a) $D < 0$ (b) $D \le 0$
(c) $D > 0$ (d) $D \ge 0$

7. Consider the following in respect of a non-singular matrix of order 3 : **[NDA 2020-I]**

1. A (adj A) = (adj A) A
2. | adj A | = | A |

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

8. If A is a matrix of order 3 × 5 and B is a matrix of order 5 × 3, then the order of AB and BA will respectively be **[NDA 2020-I]**

(a) 3 × 3 and 3 × 3 (b) 3 × 5 and 5 × 3
(c) 3 × 3 and 5 × 5 (d) 5 × 3 and 3 × 5

9. If matrix $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$ where $i = \sqrt{-1}$, then which one of the following is correct? **[NDA 2020-I]**

(a) A is hermitian
(b) A is skew-hermitian
(c) $(\bar{A})^T + A$ is hermitian
(d) $(\bar{A})^T + A$ is skew-hermitian

10. For how many values of k, is the matrix $\begin{bmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{bmatrix}$ singular? **[NDA 2020-I]**

(a) Only one (b) Only two
(c) Only four (d) Infinite

11. Let $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. If $AB = C$, then what is the value of the determinant of the matrix A? **[NDA 2020-I]**

(a) –10 (b) –14 (c) –24 (d) –34

12. What is the value of the determinant $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$ where $i = \sqrt{-1}$? **[NDA 2020-I]**

(a) 0 (b) –2 (c) $4i$ (d) $-4i$

13. Let $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then what is AB equal to? **[NDA 2020-I]**

(a) $\begin{bmatrix} ax+hy+gz \\ y \\ z \end{bmatrix}$

(b) $\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ z \end{bmatrix}$

(c) $\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$

(d) $\begin{bmatrix} ax+hy+gz & hx+by+fz & gx+fy+cz \end{bmatrix}$

14. If Δ is the value of the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then what is the value of the following determinant?

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

($p \ne 0$ or 1, $q \ne 0$ or 1) **[NDA 2021-I]**

(a) $p\Delta$ (b) $q\Delta$
(c) $(p+q)\Delta$ (d) $pq\Delta$

15. If $a + b + c = 4$ and $ab + bc + ca = 0$, then what is the value of the following determinant? **[NDA 2021-I]**

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(a) 32 (b) – 64 (c) – 128 (d) 64

16. If $a_1, a_2, a_3, \ldots, a_9$ are in GP, then what is the value of the following determinant? **[NDA 2021-I]**

$$\begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$$

(a) 0 (b) 1 (c) 2 (d) 4

17. If A and B are two matrices such that AB is of order $n \times n$, then which one of the following is correct? **[NDA 2021-I]**

(a) A and B should be square matrices of same order.
(b) Either A or B should be a square matrix.
(c) Both A and B should be of same order.
(d) Orders of A and B need not be the same.

18. How many matrices of different orders are possible with elements comprising all prime numbers less than 30? **[NDA 2021-I]**

(a) 2 (b) 3 (c) 4 (d) 6

19. Let $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$ where p, q, r and s are any four different prime numbers less than 20. What is the maximum value of the determinant? **[NDA 2021-I]**

(a) 215 (b) 311 (c) 317 (d) 323

20. If A and B are square matrices of order 2 such that $\det(AB) = \det(BA)$, then which one of the following is correct? **[NDA 2021-I]**

(a) A must be a unit matrix.
(b) B must be a unit matrix.
(c) Both A and B must be unit matrices.
(d) A and B need not be unit matrices.

21. If the determinant $\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$ then what is x equal to? **[NDA 2021-I]**

(a) -2 or 2 (b) -3 or 3 (c) -1 or 1 (d) 3 or 4

22. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

then what is $f(-1) + f(0) + f(1)$ equal to? **[NDA 2021-I]**

(a) 0 (b) 1
(c) 100 (d) -100

23. The element in the i^{th} row and the j^{th} column of a determinant of third order is equal to $2(i + j)$. What is the value of the determinant ? **[NDA 2021-I]**

(a) 0 (b) 2 (c) 4 (d) 6

24. With the numbers 2, 4, 6, 8, all the possible determinants with these four different elements are constructed. What is the sum of the values of all such determinants? **[NDA 2021-I]**

(a) 128 (b) 64 (c) 32 (d) 0

Probability and Probability Distribution

1. A coin is biased so that heads comes up thrice as likely as tails. For three independent losses of a coin, what is the probability of getting at most two tails? **[NDA 2019-II]**

(a) 0.16 (b) 0.48
(c) 0.58 (d) 0.98

2. A bag contains 20 books out of which 5 are defective. If 3 of the books are selected at random and removed from the bag in succession without replacement, then what is the probability that all three books are defective? **[NDA 2019-II]**

(a) 0.009 (b) 0.016 (c) 0.026 (d) 0.047

3. If a coin is tossed till the first head appears, then what will be the sample space? **[NDA 2019-II]**

(a) {H}
(b) {TH}
(c) {T, HT, HHT, HHHT, ...}
(d) {H, TH, TTH, TTTH, ...}

4. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on them is a prime number? **[NDA 2019-II]**

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{7}{12}$ (d) $\frac{2}{3}$

5. If 5 of a Company's 10 delivery trucks do not meet emission standards and 3 of them are chosen for inspection, then what is the probability that none of the trucks chosen will meet emission standards? **[NDA 2019-II]**

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{12}$ (d) $\frac{1}{4}$

6. There are 3 coins in a box. One is a two-headed coin; a fair coin another is; and third is biased coin that comes up heads 75% of time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin? **[NDA 2019-II]**

(a) $\frac{2}{9}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

7. Consider the following statements: **[NDA 2019-II]**

1. If A and B are mutually exclusive events, then it is possible that $P(A) = P(B) = 0.6$.
2. If A and B are any two events such that $P(A|B) = 1$, then $P(\overline{B}|\overline{A}) = 1$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

8. If a fair die is rolled 4 times, then what is the probability that there are exactly 2 sixes? **[NDA 2019-II]**

(a) $\frac{5}{216}$ (b) $\frac{25}{216}$
(c) $\frac{125}{216}$ (d) $\frac{175}{216}$

9. What is the probability that February of a leap year selected at random, will have five Sundays? **[NDA 2020-I]**

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) 1

10. A husband and wife appear in an interview for two vacancies for the same post. The probability of the husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. If the events are independent, then the probability of which one of the following is $\frac{11}{35}$? **[NDA 2020-I]**

(a) At least one of them will be selected
(b) Only one of them will be selected
(c) None of them will be selected
(d) Both of them will be selected

11. A dealer has a stock of 15 gold coins out of which 6 are counterfeits. A person randomly picks 4 of the 15 gold coins. What is the probability that all the coins picked will be counterfeits? **[NDA 2020-I]**

(a) $\frac{1}{91}$ (b) $\frac{4}{91}$ (c) $\frac{6}{91}$ (d) $\frac{15}{91}$

12. A committee of 3 is to be formed from a group of 2 boys and 2 girls. What is the probability that the committee consists of 2 boys and 1 girl? **[NDA 2020-I]**

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

13. In a lottery of 10 tickets numbered 1 to 10, two tickets are drawn simultaneously. What is the probability that both the tickets drawn have prime numbers? **[NDA 2020-I]**

(a) $\frac{1}{15}$ (b) $\frac{1}{2}$ (c) $\frac{2}{15}$ (d) $\frac{1}{5}$

14. Consider a random variable X which follows Binomial distribution with parameters n = 10 and $p = \frac{1}{5}$. Then $Y = 10 - X$ follows Binomial distribution with parameters n and p respectively given by **[NDA 2020-I]**

(a) $5, \frac{1}{5}$ (b) $5, \frac{2}{5}$ (c) $10, \frac{3}{5}$ (d) $10, \frac{4}{5}$

15. If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, then consider the following statements : **[NDA 2020-I]**

1. $P(\bar{A} \cup B) = 0.9$.
2. $P(\bar{B} \mid \bar{A}) = 0.6$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

16. Three cooks X, Y and Z bake a special kind of cake, and with respective probabilities 0.02, 0.03 and 0.05, it fails to rise. In the restaurant where they work, X bakes 50%, Y bakes 30% and Z bakes 20% of cakes. What is the proportion of failures caused by X? **[NDA 2020-I]**

(a) $\frac{9}{29}$ (b) $\frac{10}{29}$ (c) $\frac{19}{29}$ (d) $\frac{28}{29}$

17. If three dice are rolled under the condition that no two dice show the same face, then what is the probability that one of the faces is having the number 6? **[NDA 2020-I]**

(a) $\frac{5}{6}$ (b) $\frac{5}{9}$ (c) $\frac{1}{2}$ (d) $\frac{5}{12}$

18. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\text{not } A) = \frac{1}{2}$, then which one of the following is **not** correct? **[NDA 2020-I]**

(a) $P(B) = \frac{2}{3}$
(b) $P(A \cap B) = P(A)P(B)$
(c) $P(A \cup B) > P(A) + P(B)$
(d) $P(\text{not } A \text{ and not } B) = P(\text{not } A)\, P(\text{not } B)$

19. Let two events A and B be such that $P(A) = L$ and $P(B) = M$. Which one of the following is correct? **[NDA 2021-I]**

(a) $P(A \mid B) < \frac{L+M-1}{M}$ (b) $P(A \mid B) > \frac{L+M-1}{M}$

(c) $P(A \mid B) \geq \frac{L+M-1}{M}$ (d) $P(A \mid B) = \frac{L+M-1}{M}$

20. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$, then which of the following is/are correct?

1. A and B are independent events.
2. A and B are mutually exclusive events.

Select the correct answer using the code given below. **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

21. A coin is tossed twice. If E and F denote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to? **[NDA 2021-I]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

22. In a binomial distribution, the mean is $\frac{2}{3}$ and variance is $\frac{5}{9}$. What is the probability that random variable $X = 2$? **[NDA 2021-I]**

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$ (c) $\frac{25}{54}$ (d) $\frac{25}{216}$

23. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then consider the following statements : **[NDA 2021-I]**

1. The minimum value of $P(A \cup B)$ is $\frac{3}{4}$.
2. The maximum value of $P(A \cap B)$ is $\frac{5}{8}$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Vectors

1. What is the scalar projection of $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ on $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$? **[NDA 2019-II]**

(a) $\frac{\sqrt{6}}{9}$ (b) $\frac{19}{9}$ (c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

2. If the magnitude of the sum of two non-zero vectors is equal to the magnitude of their difference, then which one of the following is correct? **[NDA 2019-II]**

(a) The vectors are parallel
(b) The vectors are perpendicular
(c) The vectors are anti-parallel
(d) The vectors must be unit vectors

3. Consider the following equations for two vectors $\vec{a}$ and $\vec{b}$: **[NDA 2019-II]**

1. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$
2. $(|\vec{a}+\vec{b}|)(|\vec{a}-\vec{b}|) = |\vec{a}|^2 - |\vec{b}|^2$
3. $|\vec{a}\cdot\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$

Which of the above statements are correct?

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

4. Consider the following statements:

1. The magnitude of $\vec{a}\times\vec{b}$ is same as the area of a triangle with sides $\vec{a}$ and $\vec{b}$
2. If $\vec{a}\times\vec{b} = \vec{0}$ where $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$, then $\vec{a} = \lambda\vec{b}$

Which of the above statements is/are correct? **[NDA 2019-II]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

5. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then what is $\sin^2\left(\frac{\theta}{2}\right)$ equal to? **[NDA 2019-II]**

(a) $\frac{|\vec{a}+\vec{b}|^2}{4}$ (b) $\frac{|\vec{a}-\vec{b}|^2}{4}$ (c) $\frac{|\vec{a}+\vec{b}|^2}{2}$ (d) $\frac{|\vec{a}-\vec{b}|^2}{2}$

6. If $\hat{a}$ is a unit vector in the xy-plane making an angle 30° with the positive x-axis, then what is $\hat{a}$ equal to ? **[NDA 2020-I]**

(a) $\frac{\sqrt{3}\hat{i}+\hat{j}}{2}$ (b) $\frac{\sqrt{3}\hat{i}-\hat{j}}{2}$
(c) $\frac{\hat{i}+\sqrt{3}\hat{j}}{2}$ (d) $\frac{\hat{i}-\sqrt{3}\hat{j}}{2}$

7. Let A be a point in space such that $|\overrightarrow{OA}| = 12$, where O is the origin. If $\overrightarrow{OA}$ is inclined at angles 45° and 60° with x-axis and y-axis respectively, then what is $\overrightarrow{OA}$ equal to ? **[NDA 2020-I]**

(a) $6\hat{i}+6\hat{j}\pm\sqrt{2}\hat{k}$ (b) $6\hat{i}+6\sqrt{2}\hat{j}\pm 6\hat{k}$
(c) $6\sqrt{2}\hat{i}+6\hat{j}\pm 6\hat{k}$ (d) $3\sqrt{2}\hat{i}+3\hat{j}\pm 6\hat{k}$

8. Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. What is the magnitude of dot product of vectors which represent its diagonals? **[NDA 2020-I]**

(a) 21 (b) 25
(c) 31 (d) 36

9. If $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then what is $|\vec{b}|$ equal to ? **[NDA 2020-I]**

(a) 3 (b) 4 (c) 6 (d) 8

10. If the vectors $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c} = \hat{j}+p\hat{k}$ are coplanar, then what is the value of p? **[NDA 2020-I]**

(a) 1 (b) –1 (c) 5 (d) –5

11. A vector $\vec{r} = a\hat{i} + b\hat{j}$ is equally inclined to both x and y axes. If the magnitude of the vector is 2 units, then what are the values of a and b respectively? **[NDA 2021-I]**

(a) $\frac{1}{2}, \frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(c) $\sqrt{2}, \sqrt{2}$ (d) 2, 2

12. Consider the following statements in respect of a vector $\vec{c} = \vec{a} + \vec{b}$, where $|\vec{a}| = |\vec{b}| \neq 0$:

1. $\vec{c}$ is perpendicular to $(\vec{a}-\vec{b})$.
2. $\vec{c}$ is perpendicular to $(\vec{a}\times\vec{b})$.

Which of the above statements is/are correct? **[NDA 2021-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

13. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| = 4$, then which one of the following is correct? **[NDA 2021-I]**

(a) $\vec{a}$ and $\vec{b}$ must be unit vectors.

(b) $\vec{a}$ must be parallel to $\vec{b}$.

(c) $\vec{a}$ must be perpendicular to $\vec{b}$.

(d) $\vec{a}$ must be equal to $\vec{b}$.

14. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, then what is $(2\vec{a} \times 3\vec{b}) \cdot 4\vec{c} + (5\vec{b} \times 3\vec{c}) \cdot 6\vec{a}$ equal to? **[NDA 2021-I]**

(a) 114 (b) 66 (c) 0 (d) -66

15. Consider the following statements : **[NDA 2021-I]**

1. The cross product of two unit vectors is always a unit vector.
2. The dot product of two unit vectors is always unity.
3. The magnitude of sum of two unit vectors is always greater than the magnitude of their difference.

Which of the above statements are ***not*** correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

3-D Geometry

1. A point on a line has coordinates $(p + 1, p - 3, \sqrt{2}p)$ where p is any real number. What are the direction cosines of the line?

(a) $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}, \frac{1}{2}, -\frac{1}{2}$

(d) Cannot be determined due to insufficient date

2. A point on the line $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+2}{7}$ has coordinates **[NDA 2019-II]**

(a) (3, 5, 4) (b) (2, 5, 5)

(c) (–1, –1, 5) (d) (2, –1, 0)

3. If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$, then what is the value of k? **[NDA 2019-II]**

(a) 2 (b) 3

(c) 5 (d) 7

4. A straight line passes through the point (1, 1, 1) makes an angle 60° with the positive direction of z-axis, and the cosine of the angles made by it with the positive directions of the y-axis and the x-axis are in the ratio $\sqrt{3}:1$. What is the acute angle between the two possible positions of the line? **[NDA 2019-II]**

(a) 90° (b) 60° (c) 45° (d) 30°

5. If the points $(x, y, -3)$, $(2, 0, -1)$ and $(4, 2, 3)$ lie on a straight line, then what are the values of x and y respectively? **[NDA 2019-II]**

(a) $1, -1$ (b) $-1, 1$ (c) $0, 2$ (d) $3, 4$

6. What is the length of the diameter of the sphere whose centre is at (1, –2, 3) and which touches the plane $6x - 3y + 2z - 4 = 0$? **[NDA 2020-I]**

(a) 1 unit (b) 2 units (c) 3 units (d) 4 units

7. What is the perpendicular distance from the point (2, 3, 4) to the line $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$? **[NDA 2020-I]**

(a) 6 units (b) 5 units (c) 3 units (d) 2 units

8. If a line has direction ratios $< a+b, b+c, c+a >$, then what is the sum of the squares of its direction cosines? **[NDA 2020-I]**

(a) $(a + b + c)^2$ (b) $2(a + b + c)$

(c) 3 (d) 1

9. Into how many compartments do the coordinate planes divide the space? **[NDA 2020-I]**

(a) 2 (b) 4 (c) 8 (d) 16

10. What is the equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane? **[NDA 2020-I]**

(a) $x + y = 5$ (b) $z = 5$

(c) $z = 0$ (d) $x + y + z = 5$

11. What is the angle between the two lines having direction ratios (6, 3, 6) and (3, 3, 0)? **[NDA 2021-I]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

12. If l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to? **[NDA 2021-I]**

(a) 1 (b) $\frac{11}{27}$ (c) $\frac{13}{27}$ (d) 4

13. What is the projection of the line segment joining $A(1, 7, -5)$ and $B(-3, 4, -2)$ on y-axis ? **[NDA 2021-I]**

(a) 5 (b) 4

(c) 3 (d) 2

14. What is the number of possible values of k for which the line joining the points $(k, 1, 3)$ and $(1, -2, k+1)$ also passes through the point $(15, 2, -4)$? **[NDA 2021-I]**

(a) Zero (b) One

(c) Two (d) Infinite

15. The foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ is **[NDA 2021-I]**

(a) (0, 1, 2) (b) (0, 0, 3)

(c) (1, 1, 1) (d) (–1, 1, 3)

Statistics

1. The median of the observations 22, 24, 33, 37, $x+1$, $x+3$, 46, 47, 57, 58 in ascending order is 42. What are the values of 5th and 6th observations respectively? **[NDA 2019-II]**
(a) 42, 45 (b) 41, 43
(c) 43, 46 (d) 40, 40

2. Arithmetic mean of 10 observations is 60 and sum of squares of deviations from 50 is 5000. What is the standard deviation of the observations? **[NDA 2019-II]**
(a) 20 (b) 21
(c) 22.36 (d) 24.70

3. For the variables x and y, the two regression lines are $6x + y = 30$ and $3x + 2y = 25$. What are the values of $\bar{x}$, $\bar{y}$ and r respectively? **[NDA 2019-II]**
(a) $\frac{20}{3}, \frac{35}{9}, -0.5$ (b) $\frac{20}{3}, \frac{35}{9}, 0.5$
(c) $\frac{35}{9}, \frac{20}{3}, -0.5$ (d) $\frac{35}{9}, \frac{20}{3}, 0.5$

4. The class marks in a frequency table are given to be 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. The class limits of the first five classes are **[NDA 2019-II]**
(a) 3–7, 7–13, 13–17, 17–23, 23–27
(b) 2.5–7.5, 7.5–12.5, 12.5–17.5, 17.5–22.5, 22.5–27.5
(c) 1.5–8.5, 8.5–11.5, 11.5–18.5, 18.5–21.5, 21.5–28.5
(d) 2–8, 8–12, 12–18, 18–22, 22–28

5. The mean of 5 observations is 4.4 and variance is 8.24. If three of the five observations are 1, 2 and 6, then what are the other two observations? **[NDA 2019-II]**
(a) 9, 16 (b) 9, 4 (c) 81, 16 (d) 81, 4

6. Consider the following discrete frequency distribution:

x	1	2	3	4	5	6	7	8
f	3	15	45	57	50	36	25	9

What is the value of median of the distribution? **[NDA 2019-II]**
(a) 4 (b) 5 (c) 6 (d) 7

7. Mean of 100 observations is 50 and standard deviation is 10. If 5 is added to each observation, then what will be the new mean and new standard deviation respectively? **[NDA 2019-II]**
(a) 50, 10 (b) 50, 15 (c) 55, 10 (d) 55, 15

8. If the range of a set of observations on a variable X is known to be 25 and if Y = 40 + 3X, then what is the range of the set of corresponding observations on Y? **[NDA 2019-II]**
(a) 25 (b) 40 (c) 75 (d) 115

9. If V is the variance and M is the mean of first 15 natural numbers, then what is $V + M^2$ equal to? **[NDA 2019-II]**
(a) $\frac{124}{3}$ (b) $\frac{148}{3}$
(c) $\frac{248}{3}$ (d) $\frac{124}{9}$

DIRECTIONS (Qs. 10-12) : *Read the following information and answer the three items that follow:*

Marks	Number of students	
	Physics	Mathematics
10 - 20	8	10
20 - 30	11	21
30 - 40	30	38
40 - 50	26	15
50 - 60	15	10
60 - 70	10	6

10. The difference between number of students under Physics and Mathematics is largest for the interval **[NDA 2020-I]**
(a) 20 - 30 (b) 30 - 40
(c) 40 - 50 (d) 50 - 60

11. Consider the following statements : **[NDA 2020-I]**
1. Modal value of the marks in Physics lies in the interval 30 - 40.
2. Median of the marks in Physics is less than that of marks in Mathematics.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

12. What is the mean of marks in Physics? **[NDA 2020-I]**
(a) 38.4 (b) 39.4
(c) 40.9 (d) 41.6

13. What is the standard deviation of the observations
$-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}$? **[NDA 2020-I]**
(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 4

14. If $\Sigma x_i = 20$, $\Sigma x_i^2 = 200$ and $n = 10$ for an observed variable x, then what is the coefficient of variation? **[NDA 2020-I]**
(a) 80 (b) 100 (c) 150 (d) 200

15. The arithmetic mean of 100 observations is 40. Later, it was found that an observation '53' was wrongly read as '83'. What is the correct arithmetic mean? **[NDA 2020-I]**
(a) 39.8 (b) 39.7
(c) 39.6 (d) 39.5

16. Let X and Y represent prices (in ₹) of a commodity in Kolkata and Mumbai respectively. It is given that $X = 65$, $Y = 67$, $\sigma_X = 2.5$, $\sigma_Y = 3.5$ and $r(X, Y) = 0.8$. What is the equation of regression of Y on X? **[NDA 2020-I]**
(a) $Y = 0.175X - 5$ (b) $Y = 1.12X - 5.8$
(c) $Y = 1.12X - 5$ (d) $Y = 0.17X + 5.8$

17. The numbers 4 and 9 have frequencies x and $(x - 1)$ respectively. If their arithmetic mean is 6, then what is the value of x? **[NDA 2020-I]**
(a) 2 (b) 3 (c) 4 (d) 5

18. The sum of deviations of n number of observations measured from 2.5 is 50. The sum of deviations of the same set of observations measured from 3.5 is –50. What is the value of n? **[NDA 2020-I]**
(a) 50 (b) 60 (c) 80 (d) 100

19. A data set of n observations has mean $2M$, while another data set of $2n$ observations has mean M. What is the mean of the combined data sets? **[NDA 2020-I]**
(a) M (b) $\frac{3M}{2}$ (c) $\frac{2M}{3}$ (d) $\frac{4M}{3}$

20. Consider the following measures of central tendency for a set of N numbers : **[NDA 2021-I]**
1. Arithmetic mean 2. Geometric mean
Which of the above uses/use all the data?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

21. The numbers of Science, Arts and Commerce graduates working in a company are 30, 70 and 50 respectively. If these figures are represented by a pie chart, then what is the angle corresponding to Science graduates? **[NDA 2021-I]**
(a) 36° (b) 72° (c) 120° (d) 168°

22. For a histogram based on a frequency distribution with unequal class intervals, the frequency of a class should be proportional to **[NDA 2021-I]**
(a) the height of the rectangle
(b) the area of the rectangle
(c) the width of the rectangle
(d) the perimeter of the rectangle

23. The coefficient of correlation is independent of **[NDA 2021-I]**
(a) change of scale only
(b) change of origin only
(c) both change of scale and change of origin
(d) neither change of scale nor change of origin

24. The following table gives the frequency distribution of number of peas per pea pod of 198 pods :

Number of peas	1	2	3	4	5	6	7
Frequency	4	33	76	50	26	8	1

What is the median of this distribution? **[NDA 2021-I]**
(a) 3 (b) 4 (c) 5 (d) 6

25. If M is the mean of n observations $x_1 - k, x_2 - k, x_3 - k, ..., x_n - k$, where k is any real number, then what is the mean of $x_1, x_2, x_3, ..., x_n$? **[NDA 2021-I]**
(a) M (b) $M + k$ (c) $M - k$ (d) kM

26. What is the sum of deviations of the variate values 73, 85, 92, 105, 120 from their mean? **[NDA 2021-I]**
(a) – 2 (b) – 1 (c) 0 (d) 5

27. If the mean of a frequency distribution is 100 and the coefficient of variation is 45%, then what is the value of the variance? **[NDA 2021-I]**
(a) 2025 (b) 450 (c) 45 (d) 4·5

28. For which of the following sets of numbers do the mean, median and mode have the same value? **[NDA 2021-I]**
(a) 12, 12, 12, 12, 24 (b) 6, 18, 18, 18, 30
(c) 6, 6, 12, 30, 36 (d) 6, 6, 6, 12, 30

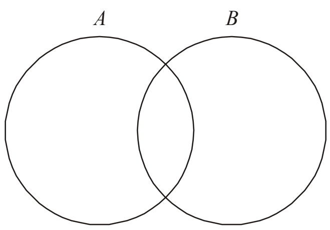
29. The mean of 12 observations is 75. If two observations are discarded, then the mean of the remaining observations is 65. What is the mean of the discarded observations? **[NDA 2021-I]**
(a) 250
(b) 125
(c) 120
(d) Cannot be determined due to insufficient data

30. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x? **[NDA 2021-I]**
(a) 10 (b) 12 (c) 13 (d) 15

HINTS & SOLUTIONS

Sets, Relations, Functions and Number System

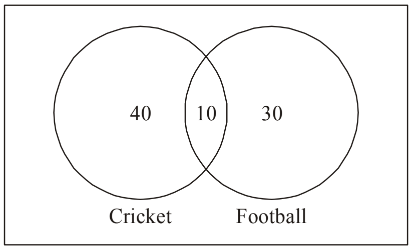
1. (a)



$A \cup (A \cap B) = A$
$\therefore$ option (a) is wrong.

2. (b) $\because$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ...(1)
$\because$ $n(A) = 6$
$n(B) = 3$
$\therefore$ $[n(A \cap B)]_{max} = 3$
$\therefore$ by eq. (1)
$[n(A \cup B)]_{min} = 6 + 3 - 3 = 6$

3. (c) Given: n(Cricket) = 50
n(Football) = 40
n(Football $\cap$ Cricket) = 10



$\therefore$ Total players = 80
$\therefore$ Non players = 100 – 80 = 20

4. (c) $A=\{x: 0 \le x \le 2\}$
$B=\{y: y \text{ is a prime}\}$
$\therefore A \cap B=\{2\}$

5. (*) Let $y=2+\cfrac{1}{2+\cfrac{1}{2+....}}$

$\therefore \quad y=2+\frac{1}{y}$

$\Rightarrow \quad y^2=2y+1 \Rightarrow y^2-2y-1=0$

$\Rightarrow \quad y=\frac{2\pm\sqrt{(-2)^2-4(-1)}}{2} \Rightarrow y=\frac{2\pm 2\sqrt{2}}{2}=1\pm\sqrt{2}$

$\because \quad y>2$

$\therefore \quad y=1\pm\sqrt{2}$

6. (d) $\because$ number of zeroes = Highest power of 5.
$\therefore$ by using options
highest power in 95 is

$\left[\frac{95}{5}\right]+\left[\frac{95}{25}\right]+\left[\frac{95}{125}\right]=21$

highest power in 80 is

$\left[\frac{80}{5}\right]+\left[\frac{80}{25}\right]=19$ highest power in 85 is

$\left[\frac{85}{5}\right]+\left[\frac{85}{25}\right]=19$

$\therefore$ option (D) no such value of n exists.

7. (d) $S=\{2, 4, 6, 8, ..., 20\}$
$n(S)=10$
$\therefore$ Total subsets $=2^{10}=1024$.

8. (d) $(cd\ cc\ dd\ ccc\ ddd)_2$
$\because c>d$
$\therefore c=1$ and $d=0$
$\therefore$ no $=(10\ 11\ 00\ 111\ 000)_2$
$\therefore$ decimal equivalent $=2^{11}+2^9+2^8+2^5+2^4+2^3$
$=2048+512+256+32+16+8$
$=2872$

9. (a) Given: $f(x)=x^2; f(x)=\tan x$
$h(x)=\ell n\, x$

$\therefore \quad (h\, o\, (g\, o f)\, x$ at $x=\frac{\sqrt{\pi}}{2}$

$= \quad \ell n(\tan(x^2)=\ell n\left(\tan\frac{\sqrt{\pi}}{4}\right)=\ell n\, 1=0$

10. (d) $[f o (f o f)](2)$
$=((x^2)^2)^2=x^8$ at $x=2$, we get $x^8=2^8=256$

11. (b) For first 60 km, speed = $3v$ kmph
for next 60 km speed = $2v$ kmph

avg. speed $=\frac{\text{Total distance}}{\text{Total time}}$

$=\frac{120}{\frac{60}{3v}+\frac{60}{v}}=\frac{2}{\frac{1}{3v}+\frac{1}{2v}}=\frac{2v}{5}\times 6=\frac{12v}{5}=2.4\, v$ kmph

12. (b) Let no. of boys be x.

$\therefore \quad 60=\frac{70x+55(150-x)}{150}$

$\Rightarrow \quad 15x+8250=9000 \Rightarrow 15x=750 \Rightarrow x=50$

$\therefore$ Boys = 50 and girls = 100.

13. (c) Number of proper subset of any set of n elements $=2^n-1$
Here given set $=\{1, 2, 3, 4\}$
Number of proper subset $=2^4-1=16-1=15$.
Proper subset = {(1), (2), (3), (4), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4) (ϕ)}
Now, A is superset of B, if B is proper set of A, but B is not proper set of A.
i.e. $B \le A$ but $A \not\subset B$. Then $A \ge B$.
So, superset of {3} are {(3), (1, 3), (2, 3), (3, 4), (1, 2, 3), (1, 3, 4), (2, 3, 4)}
Hence, number of superset of {3} = 7.

14. (c) $n(z)=90$
$12+18+17+C=90 \Rightarrow C=43$.

From question, $\frac{n(y)}{n(z)}=\frac{4}{5}$

$\frac{16+18+17+b}{90}=\frac{4}{5}$

$b=72-51=21$.

15. (d) $n(X)+n(Y)+n(Z)-n(X\cap Y)$
$-n(Y\cap Z)-n(X\cap Z)+n(X\cap Y\cap Z)$
$=n(X\cup Y\cup Z)$
$=a+b+90+16$
$=a+b+106$.

16. (a) $n(X\cup Y\cup Z)^1=P$
and $n(Z)=90$ (given)
$\therefore n(X)^1=P+90-12-18+b=p+b+60$.

17. (c) $(1101101)_2+(1011011)_2$
$=(1\times 2^6+1\times 2^5+0+1\times 2^3+1\times 2^2+0+1\times 2^0)_{10}$
$+(1\times 2^6+0+1\times 2^4+1\times 2^3+0+1\times 2^1+1\times 2^0)_{10}$
$=(64+32+8+4+1)_{10}+(64+0+16+8+2+1)_{10}$
$=(109+91)_{10}=(200)_{10}$.

18. (a) $\frac{1}{10}\log_5 1024-\log_5 10+\frac{1}{5}\log_5 3125$

$=\log_5(1025)^{1/10}-\log_5 10+\log_5(3125)^{1/5}$

$=\log_5(2^{10})^{\frac{1}{10}}-\log_5 10+\log_5(5^5)^{\frac{1}{5}}$

$=\log_5(2)-\log_5 10+\log_5 5$

$=\log_5\left(\frac{2\times 5}{10}\right)=\log_5 1=0$.

19. (c) $1+x=\log_c(ab)+1$
$=\log_c(ab)+\log_c c=\log_c(abc)$

$(1+x)^{-1} = \dfrac{1}{\log_c(abc)} = \log_{(abc)} c$

Similarly, $(1+y)^{-1} = \log_{(abc)} a$

and $(1+z)^{-1} = \log_{(abc)} b$

Now, $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

$= \log_{(abc)} c + \log_{(abc)} a + \log_{(abc)} b$

$= \log_{(abc)}(cab) = 1.$

20. (c) Give set $S = \{1, 2, 3, ...\}$

For xRy, $\log_a x > \log_a y$

$\Rightarrow x > y$

As xRx, $\log_a x > \log_a x$ is not valid.
Hence, relation is not reflexive.

For xRy, $\log_a x > \log_a y \Rightarrow x > y$

yRx, $\log_a y > \log_a x \Rightarrow y > x$

This is also not valid. Hence, relation is not symmetric also.

For xRy, $\log_a x > \log_a y \Rightarrow x > y$

For yRz, $\log_a y > \log_a z \Rightarrow y > z$

So, xRz, $\log_a x > \log_a z \Rightarrow x > z$

This is a valid relation. Hence, relation is only transitive.

21. (c) $f(0) = 3(0)^2 - 5(0) + P = P$

$f(1) = 3(1)^2 - 5(1) + P = P - 2$

Clearly, $f(0)$ and $f(1)$ are opposite in sign.

$\therefore P > 0$ and $P - 2 < 0$.

$\Rightarrow 0 < P < 2.$

22. (a) $f(x) = 2x - x^2$

$f(x+2) + f(x-2)$

$= 2(x+2) - (x+2)^2 + 2(x-2) - (x-2)^2$

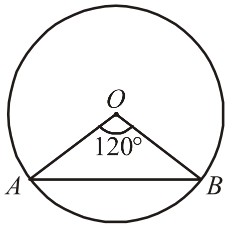
$= 4 + 4x - x^2 - 4 - 4x - x^2 - 4 + 4x - 4$

$= 8x - 2x^2 - 8$

When $x = 0$, then, $f(x+2) + f(x-2) = -8$

23. (c) Here AB is the chord and $\angle AOB = 120°$

$AO = OB = 1$ units (Radius)



From cosine rule,

$$\cos(\angle AOB) = \frac{(OA)^2 + (OB)^2 - (AB)^2}{2(AO)\cdot(OB)}$$

$$\Rightarrow \cos(120°) = \frac{1^2 + 1^2 - (AB)^2}{2(1)\cdot(1)} \Rightarrow \frac{1}{2} = \frac{2 - (AB)^2}{2}$$

$\therefore AB = \sqrt{2}$ units.

24. (b) Number of sides, $n = 8$

Interior angle of n-sides polygon

$= \dfrac{(n-2)\times\pi}{n} = \dfrac{(8-2)\times\pi}{8} = \dfrac{3\pi}{4}.$

25. (c)

1. $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$ are equivalent sets, because number of elements are equal.
2. $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\} = \{1, 5, 9\}$ are equal sets, as A and B have all common elements.

26. (d) All three statements are true.

27. (b) $2x + 3y = 20 \Rightarrow y = \dfrac{20 - 2x}{3}$

As $(x, y) \in N$

For $x = 1$, $y = \dfrac{20-2}{3} = 6$, $(x, y) = (1, 6) \in N$

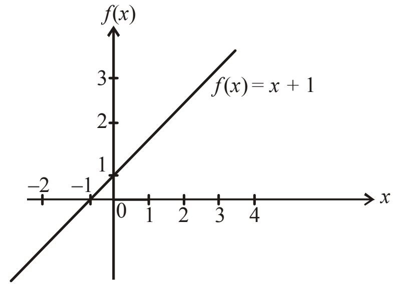
For $x = 4$, $y = \dfrac{20-8}{3} = 4$, $(x, y) = (4, 4) \in N$

For $x = 7$, $y = \dfrac{20-14}{3} = 2$, $(x, y) = (7, 2) \in N$

$\therefore$ Number of elements $(x, y) = 3$.

28. (a) $f(x) = x + 1$

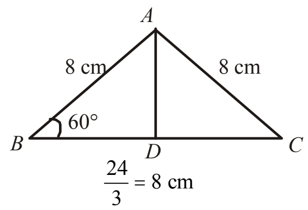
Graph :



$f(x) = x + 1$ is defined for all $x \in R$.

Hence, $f(x) = x + 1$ is *one-one* and *onto*.

29. (a) Among all triangles that have same perimeter, equilateral triangles are one that enclose maximum area.



Here, $AB = BC = CA = \dfrac{24}{3} = 8$ cm

Area of $\Delta ABC = \dfrac{\sqrt{3}}{4}(BC)^2 = \dfrac{\sqrt{3}}{4}(8)^2 = 16\sqrt{3}$ cm^2

AD is the altitude of the ΔABC.

Then, Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$\Rightarrow 16\sqrt{3} = \frac{1}{2} \times 8 \times AD$

$\therefore AD = 4\sqrt{3}$ cm.

30. (c) Sum of 15 observations $= 15 \times A$ (where A is average)
Reduce in value after correction.
$8x - 3x = 50$ {where x = unit digit of wrong number}

$\therefore$ Reduction in new average $= \frac{50}{15} = \frac{10}{3}$

Polynomial, Quadratic Equation & Inequalities

1. (c) $p, q \in \{1, 2, 3, 4\}$
For real roots
$q^2 \geq 4p$
If $q = 1, p = \phi$
If $q = 2, p = 1$
If $q = 3, p = 1, 2$
If $q = 4, p = 1, 2, 3, 4$
$\therefore$ 7 such pairs exist.

2. (c) For $2x^2 - 2(k-2)x - (k+1) = 0$
Let roots be α and β.

$\therefore \quad \alpha + \beta = k - 2; \alpha\beta = -\frac{k+1}{2}$

$\because \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$\Rightarrow \quad \alpha^2 + \beta^2 = (k-2)^2 + (k+1)$
$\Rightarrow \quad \alpha^2 + \beta^2 = k^2 - 4k + 4 + k + 1$
$\Rightarrow \quad \alpha^2 + \beta^2 = k^2 - 3k + 5$

$\Rightarrow \quad \alpha^2 + \beta^2 = \left(k - \frac{3}{2}\right)^2 + \left(5 - \frac{9}{4}\right)$

for minimum value $k = \frac{3}{2}$

3. (c) $|x^2 - 3x + 2| > x^2 - 3x + 2$
$\Rightarrow \quad |(x-1)(x-2)| > (x-1)(x-2)$
by options, if we put $x = 1$
then we get $0 > 0$
$\therefore$ options (a), (b) and (d) are not correct.

4. (c) $x^2 + mx + 2 = 0$
for real roots
$m^2 \geq 4(2)(1)$
$m^2 \geq 8$

5. (d) $\because x^2 + x + 1 = 0$
$\Rightarrow \alpha = \omega$ and $\beta = \omega^2$

$\therefore \sum_{j=0}^{3}(\alpha^j + \beta^j) = (\alpha^0 + \alpha^1 + \alpha^2 + \alpha^3) + (\beta^0 + \beta^1 + \beta^2 + \beta^3)$

$\because$ sum of 3 consecutive powers of α and $\beta = 0$

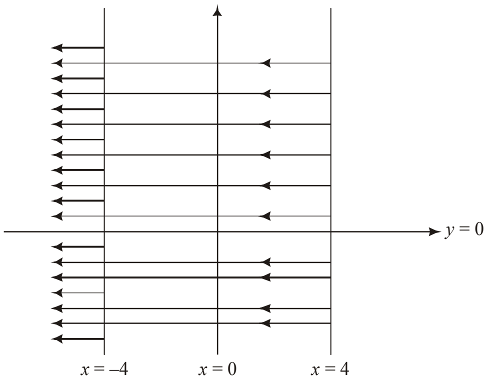
$\therefore \sum_{j=0}^{3}(\alpha^j + \beta^j) = \alpha^0 + \beta^0 = 2$

6. (c) $(1 + 2x + x^2)^5 + (1 + 4y + 4y^2)^5$
$= [(1+x)^2]^5 + [(1+2y)^2]^5 = (1+x)^{10} + (1+xy)^{10}$
$\therefore$ Total numbers of terms in each of them is 11 but constant term will be added
$\therefore \quad$ Total terms $= 11 + 11 - 1 = 21$

7. (b) $A \cup B = \{x : (x-a) \cdot (x-b) > 0; a < b\}$

eleminating options
(a) $x > a$ has two signs in range (a, ∞)
$\therefore x > a$ is not possible
(b) $x < a$ satisfies and $x > b$ has only one sign in range (b, ∞)
$\therefore x > b$ also satisfies.
$\therefore A = \{x : x < a\}$
and $B = \{x : x > b\}$

8. (c) $x \leq 4, y \geq 0$
and $x \leq -4, y \leq 0$



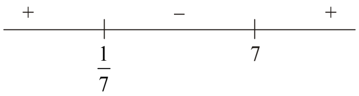
by observing graph
we get $x \leq -4$ and $y = 0$

9. (c) $x^{\log_7^x} > 7; x > 0$
taking log both sides

$\Rightarrow \quad \log_7^x \cdot \log x > \log 7 \Rightarrow \frac{\log x}{\log 7} \cdot \log x > \log 7$

$\Rightarrow \quad (\log x)^2 > (\log 7)^2 \Rightarrow (\log x)^2 - (\log 7)^2 > 0$

$\Rightarrow \quad (\log x + \log 7)(\log x - \log 7) > 0 \Rightarrow x = \frac{1}{7}, 7$



$\therefore \quad x \in \left(0, \frac{1}{7}\right) \cup (7, \infty)$

10. (a) $x^2 + 3|x| + 2 = 0$
$= (|x|)^2 + 3|x| + 2 = 0$
$\Rightarrow \quad (|x| + 1)(|x| + 2) = 0 \Rightarrow |x| = -1, -2$
$\therefore$ No real roots exist.

11. (c) $4(x-p)(x-q) - r^2 = 0$

$= (x-p)(x-q) - \frac{r^2}{4} = 0 = x^2 - (p+q)x + pq - \frac{r^2}{4} = 0$

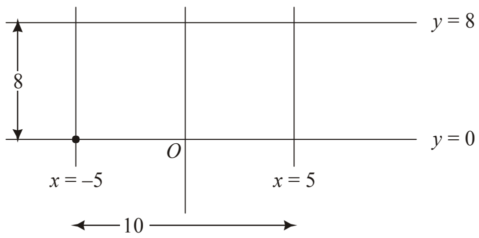
Discriminat,

$D = [-(p+q)]^2 - 4\left(pq - \frac{r^2}{4}\right) = 0$

$D = (p+q)^2 - 4pq + r^2 = 0$
$D = (p-q)^2 + r^2 = 0$
We have $D \geq 0 \Rightarrow$ real roots.

and if $p = q$, $r = 0$
the D = 0
$\therefore$ equal roots $\therefore$ both statements are correct.

12. (b) Given $|x| < 5 \Rightarrow -5 < x < 5$
$y = 0$ & $y = 8$



$\therefore$ Area = $8 \times 10 = 80$ sq. units

13. (c) $f(x) = ax^2 + bx + c$
$f(0) = 5 \Rightarrow c = 5$ ...(i)
$f(-1) = 10 \Rightarrow a - b = 5$ ...(ii)
$f(1) = 6 \Rightarrow a + b = 6$ ...(iii)
from (ii) & (iii)
we get, $a = 3$
$b = -2$
$\therefore$ quadratic equation is
$3x^2 - 2x + 5 = 0$

14. (b) $x^2 - 30x + 221 = 0$
$x^2 - 13x - 17x + 17 \times 13 = 0$
$\Rightarrow (x - 13)(x - 17) = 0$
$\therefore p = 13, q = 17$
$\therefore p^3 + q^3 = 2197 + 4913 = 7110$

15. (b) Given equation : $x^2 - 3x + 2 = 0$
Sum of roots, $\cot\alpha + \cot\beta = -(-3) = 3$ ...(i)
Product of roots, $\cot\alpha \cdot \cot\beta = 2$ ...(ii)
Now, $\cot(\alpha + \beta) = \dfrac{\cot\alpha \cdot \cot\beta - 1}{\cot\alpha + \cot\beta} = \dfrac{2 - 1}{3} = \dfrac{1}{3}$.

16. (b) $\alpha\beta = \alpha^2\beta^2 \Rightarrow \alpha\beta(1 - \alpha\beta) = 0$
$\therefore \alpha\beta = 0$ or 1
When $\alpha\beta = 1$ then $\alpha = \dfrac{1}{\beta}$
Again from $\alpha + \beta = \alpha^2 + \beta^2$
$\Rightarrow \dfrac{1}{\beta} + \beta = \dfrac{1}{\beta^2} + \beta^2 \Rightarrow \beta^2 - \beta = \dfrac{1}{\beta} - \dfrac{1}{\beta^2}$
$\Rightarrow \beta(\beta - 1) = \dfrac{(\beta - 1)}{\beta^2} \Rightarrow (\beta - 1)\left(\beta - \dfrac{1}{\beta^2}\right) = 0$
$\Rightarrow (\beta - 1)(\beta^3 - 1) = 0 \Rightarrow (\beta - 1)^2(\beta^2 + \beta + 1) = 0$
$\therefore \beta = 1$ and $\beta = \dfrac{-1 \pm \sqrt{3}i}{2}$
Again, when $\beta = 1$, then $\alpha = \dfrac{1}{\beta} = 1$, roots $(\alpha, \beta) = (1, 1)$
When $\beta = \dfrac{-1 + \sqrt{3}i}{2}$, then $\alpha = \dfrac{-1 - \sqrt{3}i}{2}$
Roots $(\alpha, \beta) = \left(\dfrac{-1 \pm \sqrt{3}i}{2}, \dfrac{-1 \mp \sqrt{3}i}{2}\right)$
Thus, number of different quadratic equations = 2.

17. (d) $1.5 \le x \le 4.5$
$\dfrac{3}{2} \le x \le \dfrac{9}{2} \Rightarrow 3 \le 2x \le 9$
$\therefore (2x - 3) \ge 0$ and $(2x - 9) \le 0$
Hence, $(2x - 3)(2x - 9) \le 0$

18. (c) $2\sin x = 2k + 1$
$\sin x = k + \dfrac{1}{2}$ As $\sin x \in [-1, 1]$
$\therefore -1 \le k + \dfrac{1}{2} \le 1$ $\Rightarrow -\dfrac{3}{2} \le k \le \dfrac{1}{2}$
Hence, number of Integer values of k that satisfy are 2 and that are (– 1 and 0)

19. (d) Roots are real, if $D \ge 0$
$(-2)^2 - 4k \ge 0$
$\Rightarrow (1 - k) \ge 0 \Rightarrow k \le 1$.

20. (a) α and β are roots of $4x^2 + 2x - 1 = 0$.
$\therefore$ Sum of roots $(\alpha + \beta) = -\dfrac{2}{4} = -\dfrac{1}{2}$
$\therefore \beta = -\dfrac{1}{2} - \alpha = \dfrac{-1 - 2\alpha}{2}$ and $4\alpha^2 + 2\alpha - 1 = 0$
$\Rightarrow 4\alpha^2 = 1 - 2\alpha$.
Now, $-2\alpha^2 - 2\alpha = \dfrac{2\alpha - 1}{2} - 2\alpha = \dfrac{-2\alpha - 1}{2} = \beta$.

21. (c) As one root is reciprocal of the other.
$\therefore$ Product of roots = 1.
$\therefore \dfrac{K}{5} = 1 \Rightarrow K = 5$.

22. (c) Given equation : $x(x + 1) + 1 = 0$
$D = (-1)^2 - 4(1)(1) = -3 < 0$.
Two roots of the equation are same as cube root of unity (ω and ω^2).
Hence, other roots = K^2.

Sequence and Series

1. (a) $S = (1 - 2) + (3 - 4) + ... + 101$
$\Rightarrow -1 - 1 - 1 ... + 101$
$\Rightarrow -50 + 101$
$\Rightarrow S = 51$

2. (a) Given: $S_n = n + 12$
$\because T_n = S_n - S_{n-1}$
$\therefore T_3 = S_3 - S_2$
$\Rightarrow T_3 = (3 + 12) - (2 + 12) \Rightarrow T_3 = 1$

3. (c) $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$
has coefficients in cyclic order

$\therefore$ its one root will be 1.

$\because$ both the roots are equal

$\therefore$ other root = 1

$\therefore$ Product of roots = 1

$\Rightarrow \dfrac{c(a-b)}{a(b-c)} = 1 \Rightarrow ac - bc = ab - ac$

$\Rightarrow 2ac = ab + bc \Rightarrow \dfrac{2}{b} = \dfrac{1}{a} + \dfrac{1}{c} \Rightarrow a, b, c$ are in HP.

4. (b) $\because S_n = \dfrac{a(1-r^n)}{1-r}$

given: $a_1 + a_3 + ... + a_{199} = m$

$\therefore \dfrac{a[(r^2)^{100} - 1]}{r^2 - 1} = m$ ...(1)

also

$a_2 + a_4 + ... + a_{200} = n$

$\therefore \dfrac{ar[(r^2)^{100} - 1]}{r^2 - 1} = n$ ...(2)

eq. (2) ÷ (1) $\Rightarrow r = \dfrac{n}{m}$

5. (c) $x^2 - 16x + 39 = 0$

$\Rightarrow x^2 - 13x - 3x + 39 = 0 \Rightarrow (x-13)(x-3) = 0$

$\Rightarrow$ Roots are 3 and 13.

AP between 3 and 13 has common difference = $\dfrac{b-a}{n+1}$

$\Rightarrow d = \dfrac{13-3}{5} = 2$

$\therefore AP$ between 3 and 13 is 5, 7, 9, 11

$\therefore$ sum = 5 + 7 + 9 + 11 = 32

6. (d) AP remains an AP if it is multiplied, divided and subtracted by a constant number.

$\therefore$ All three statements are correct.

7. (b) First 2 digit number divisible by 4 is = 12

Last 2 digit number divisible by 4 is = 96.

$\therefore$ by AP concept

$96 = 12 + (n-1)4$

$\Rightarrow n = \dfrac{96-12}{4} + 1 = 22$

8. (c) $S_{2n} = 3n + 14n^2$

$\therefore S_n = 3\dfrac{n}{2} + 14\left(\dfrac{n}{2}\right)^2 \therefore S_n = \dfrac{3n + 7n^2}{2}$

$\therefore S_1 = \dfrac{3+7}{2} = 5 = T_1$

$S_2 = \dfrac{6+28}{2} = 17 = T_1 + T_2$

$\therefore (T_1 + T_2) - (T_1) = 17 - 5 = 12$

$\Rightarrow T_2 = 12$

$\therefore d = T_2 - T_1 = 12 - 5 = 7.$

9. (*) $T_3 = AR^2 = P$

$T_8 = AR^7 = q$

$T_{13} = AR^{12} = r$

here $q^2 = P.r.$

i.e. $(AR^7)^2 = (AR^2)(AR^{12})$

10. (c)

(1) p^2, q^2 and r^2 in G.P.

Then, $(q^2)^2 = p^2 \cdot r^2 \Rightarrow q^2 = (p^2 \cdot r^2)^{\frac{1}{2}}$

$\Rightarrow q^2 = p \cdot r$

Hence, p, q and r in G.P.

(2) As, $q^2 = p \cdot r$

Taking log on both sides, we have

$\ln(q^2) = \ln(p \cdot r)$

$2\ln(q) = \ln p + \ln r$

Hence, $\ln p$, $\ln q$ and $\ln r$ are in A.P.

11. (b) From $\dfrac{1}{\log_a b} = \log_b a$

$\dfrac{1}{\log_2 n} + \dfrac{1}{\log_3 n} + \dfrac{1}{\log_4 n} + \dfrac{1}{\log_5 n} + ... + \dfrac{1}{\log_{100} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + ... + \log_n 100$

$= \log_n (2 \cdot 3 \cdot 4 \cdot 5 \cdot ..... 100) = \log_n 100!$

As $n = 100!$ (given)

$\therefore \log_{100!} 100! = 1.$

12. (b) Let common difference = d.

Then, sum of first 5 terms

$= \dfrac{\text{Sum of next 5 terms}}{4}$

$\Rightarrow \dfrac{1}{2} \cdot 5(4 + 4 \cdot d) = \dfrac{1}{4} \cdot \dfrac{1}{2} \cdot 5\{(2 + 5 \cdot d) + 4 \cdot d\}$

$\Rightarrow 4 \times 4(1 + d) = 4 + 14d$

$\therefore d = -6.$

$\therefore$ Sum of first ten terms

$= \dfrac{1}{2} \times 10\{(2 \times 2 + 9(-6)\} = -250.$

13. (c) Both (1) and (2) are true.

14. (c) $x^2, x, -8$ are in A.P.

$\therefore 2x = x^2 - 8$

$\Rightarrow x^2 - 2x + 1 - 9 = 0 \qquad \Rightarrow (x-1)^2 = 9$

$\therefore (x-1) = \pm 3 \qquad \therefore x \in \{-2, 4\}$

15. (b) Let G.P. is $\dfrac{3}{r^2}, \dfrac{3}{r}, 3, 3r$ and $3r^2$

Product $= \dfrac{3}{r^2} \cdot \dfrac{3}{r} \cdot 3 \cdot 3r \cdot 3r^2 = (3)^5 = 243.$

16. (d) $x = \frac{2m \cdot n}{m+n}$ and $y = \sqrt{m \cdot n}$

$5x = 4y$

$A.M.$ of m and $n = \frac{m+n}{2}$, As $5.\frac{(2m \cdot n)}{m+n} = 4 . \sqrt{m \cdot n}$

$\frac{5m \cdot n}{2\sqrt{m \cdot n}} = m + n$ $\qquad \frac{5}{2}\sqrt{m \cdot n} = (m+n)$

$\frac{5}{2} = \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$

Let $\sqrt{\frac{m}{n}} = Z$, then $\frac{5}{2} = Z + \frac{1}{Z}$

$2(Z^2+1) = 5Z$

$2Z^2 - 5Z + 2 = 0$

$(Z-2)(2Z-1) = 0$

$\therefore \quad Z = 2$ or $\frac{1}{2}$

$\sqrt{\frac{m}{n}} = 2 \Rightarrow m = 4n$ or, $\sqrt{\frac{m}{n}} = \frac{1}{2}$ $\qquad n = 4m$

17. (c) Geometric mean = 10

when, each observation is replaced by $3x_i^4$

Now $g.m. = 3.(10)^4 = 30000$

Complex Numbers

1. (c) $\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019}$

$\because \quad -i \times i = 1$

$\therefore \quad \left[\frac{-i(-1+i\sqrt{3})}{2}\right]^{2019} + \left[\frac{-i(-1-i\sqrt{3})}{2}\right]^{2019}$

$= -i^{2019}\left[\omega^{2019} + (\omega^2)^{2019}\right]$

$= -i^3\,[1+i] = -(-i)2 = 2i$

2. (c) Given $x = 1 + i$

squaring both sides

$\Rightarrow x^2 = 1 + i^2 + 2i$

$\Rightarrow x^2 = 2i$

$\therefore x^6 + x^4 + x^2 + i \quad = (x^2)^3 + (x^2)^2 + x^2 + 1$

$= (2i)^3 + (2i)^2 + 2i + 1$

$= 8i^3 + 4i^2 + 2i + 1$

$= -8i - 4 + 2i + 1$

$= -6i - 3$

3. (b) $Z = \frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$

$= \frac{(\cos\theta + i\sin\theta)}{(\cos\theta - i\sin\theta)} \times \frac{(\cos\theta + i\sin\theta)}{(\cos\theta + i\sin\theta)}$

$= \frac{(\cos\theta + i\sin\theta)^2}{\cos^2\theta - (i\sin\theta)^2}$

$= \frac{\cos 2\theta + i\sin 2\theta}{\cos^2\theta + \sin^2\theta} = \cos 2\theta + i\sin 2\theta.$

Modulus of $Z = |Z| = \sqrt{\cos^2(2\theta) + \sin^2(2\theta)} = 1$

4. (a) Let $x + iy = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$

$= \frac{(1-i\sqrt{3})(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} = \frac{(1-i\sqrt{3})^2}{1^2 - (i\sqrt{3})^2}$

$= \frac{1 - 3 - i2\sqrt{3}}{1+3} = -\left(\frac{1+i\sqrt{3}}{2}\right) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$\therefore x = -\frac{1}{2},\ y = -\frac{\sqrt{3}}{2}$

Argument $= \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi + \tan^{-1}(\sqrt{3}) = 180° + 60° = 240°.$

5. (a) $\left(\frac{1-i}{1+i}\right)^{n^2} = 1$

$\Rightarrow \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^{n^2} = 1 \Rightarrow \left(\frac{(1)^2 + (i)^2 - 2i}{1-(i)^2}\right)^{n^2} = 1$

$\Rightarrow \left(\frac{-2i}{2}\right)^{n^2} = 1 \Rightarrow (-i)^{n^2} = 1$

For $n = 2$, $(-i)^{2^2} = (-i)^4 = 1$

Hence, $n = 2$.

6. (b) $\left|Z + \frac{2}{Z}\right| = \left|(1+i) + \frac{2}{(1+i)}\right| = \left|(1+i) + \frac{2(1-i)}{(1+i)(1-i)}\right|$

$= \left|(1+i) + \frac{2(1-i)}{2}\right| = |1 + i + 1 - i| = 2$

7. (c) Let $z = x + iy$, $\bar{z} = x - iy$

and $z^{-1} = \frac{1}{z} = \frac{1}{x+iy}$

1. $\overline{(z^{-1})} = \overline{\left(\frac{1}{x+iy}\right)} = \overline{\left(\frac{x-iy}{x^2+y^2}\right)} = \left(\frac{x+iy}{x^2+y^2}\right)$

$(\bar{z})^{-1} = (x - iy)^{-1} = \frac{1}{x-iy} = \frac{x+iy}{x^2+y^2}$

$\therefore \overline{(z^{-1})} = (\bar{z})^{-1}.$

2. $z \cdot z^{-1} = (x+iy)\left(\frac{1}{x-iy}\right)$

$$= \frac{(x+iy)(x+iy)}{(x-iy)(x+iy)} = \frac{(x+iy)^2}{x^2+y^2} = |z|^2$$

Hence, both (1) and (2) are true.

8. (c) Let $z = x + iy$ conjugate of $z' = x - iy$

Now, $z + z' = x + iy + x - iy = 2x$ (Real)

$z - z' = x + iy - x + iy = (2y)i$ (Imaginery)

Hence, both (1) and (2) are true.

9. (b) $(i)^{2n+1}(-i)^{2n-1} = (-1)^{2n-1}\left\{(i)^{2n+1+2n-1}\right\}$

$$= (-1)^{2n-1} \cdot (i)^{4n} = (-1)^{2n-1} \cdot (1)$$

Now, modulus of $(-1)^{2n-1} = 1$.

Binomial Theorem, Mathematical Induction

1. (a) $\left(x^2 + \frac{1}{x}\right)^{2n}$

Its index is even

$\therefore$ Middle term $= T_{n+1}$

$T_{n+1} = {}^{2n}C_n (x^2)^n \left(\frac{1}{x}\right)^n$

$= {}^{2n}C_n x^n = 184756x^{10}$

$\therefore$ comparing power of x we get $n = 10$

2. (b) $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$

according to question. Term independent of x can be calculated as

$${}^{10}C_r \left(\sqrt{x}\right)^{10-r} . \left(-\frac{k}{x^2}\right)^r$$

$\Rightarrow x^{5-r/2-2r} = x^0 \Rightarrow 10 - 4r - r = 0 \Rightarrow r = 2$

$\therefore {}^{10}C_2 \cdot (\sqrt{x})^8 . \left(\frac{-k}{x^2}\right)^2 = 405$

$\Rightarrow {}^{10}C_2 \cdot k^2 = 405 \Rightarrow \frac{10!}{2!.8!} \cdot k^2 = 405 \Rightarrow k^2 = 9 \Rightarrow k = \pm 3$

3. (a) Expansion of

$$(1+x)^9 = {}^9C_0 + {}^9C_1 x + {}^9C_2 x^2 + {}^9C_3 x^3 + {}^9C_4 x^4 + {}^9C_5 x^5 + {}^9C_6 x^6 + {}^9C_7 x^7 + {}^9C_8 x^8 + {}^9C_9 x^9$$

Sum of last five co-efficient

$= {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$

$= 126 + 84 + 36 + 9 + 1 = 256.$

4. (a) Let $(r+1)^{\text{th}}$ term is independent of x.

$(r+1)^{\text{th}}$ term $= {}^{10}C_r \cdot \left(\frac{2}{x^2}\right)^{10-r} \cdot (-\sqrt{x})^r$

Exponent of x in independent term = 0

i.e. $\frac{r}{2} - 2(10-r) = 0$

$r - 4(10-r) = 0 \Rightarrow 5r - 40 = 0 \Rightarrow r = 8.$

Thus, 9th term is independent of x.

9th term $= {}^{10}C_8 (2)^{10-8} \cdot (-1)^8 = {}^{10}C_8 (2)^2$

$$= \frac{10 \times 9}{2} \times 4 = 180.$$

5. (b) $(1 + 2x - x^2)^6 = a_0 + a_1 \cdot x + a_2 \cdot x^2 + ... + a_{12} \cdot x^{12}$

Putting $x = -1$,

$a_0 - a_1 + a_2 + ... + a_{12} = (1 + 2(-1) - (-1)^2)^6$

$$= (1 - 2 - 1)^{12} = (-2)^6 = 64.$$

6. (b) $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + ... + C_n x^n$

For, $x = 1$,

$(1+1)^n = C_0 + C_1 + C_2 + C_3 + ... + C_n$

$\Rightarrow 2^n = 1 + C_1 + C_2 + C_3 + ... + C_n \{\because {}^nC_0 = {}^nC_n = 1\}$

$\therefore C_1 + C_2 + C_3 + ... + C_n = 2^n - 1$

7. (a) $(1 + 4x + 4x^2)^5 = (1+2x)^{2\times5} = (1+2x)^{10}$.

$\therefore$ Middle term $= {}^{10}C_5 (2x)^5 = \frac{10!}{5!\,5!} \cdot 2^5 \cdot x^5$.

Co-efficient of middle term = 8064.

8. (c) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n$

Putting $x = 1$,

$(1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$\Rightarrow 2^n - 1 = {}^nC_1 + {}^nC_2 + ... + {}^nC_n$ $(\because {}^nC_0 = 1)$

Now, $2^n - 1 = 1 + 2 + 2^2 + ... + 2^{n-1}$

$\therefore \Sigma C(n, r) = 1 + 2 + 2^2 + ... + 2^{n-1}$

9. (b) $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 2x + ... + {}^{2n}c_{2n} x^{2x}$

Sum of co-efficient of first and last term

$= {}^{2n}C_0 + {}^{2n}C_{2n} = 1 + 1 = 2.$

Permutation and Combination

1. (d) $\because$ Numbers of diagonals $= {}^nC_2 - n$

$\therefore$ diagonals in octagon $= {}^8C_2 = -8$

$= \frac{8 \times 7}{2} - 8 = 28 - 8 = 20$

2. (c) ${}^nP_r = 2520$; ${}^nC_r = 21$

${}^nP_r = \frac{n!}{(n-r)!} = 2520$ ...(1)

${}^nC_r = -\frac{n!}{(n-r)!.r!} = 21$ ...(2)

Divide (1) by (2)

$\Rightarrow r! = \frac{2520}{21} = 120$

$\Rightarrow r! = 5! \Rightarrow r = 5 \because {}^nC_r = 21$

$\Rightarrow \ ^nC_5 = 21 \Rightarrow n = 7$

$\therefore \ C(n+1, r+1) = C(8, 6)$

$= {}^8C_6 = 28$

3. (c) ${}^{47}C_4 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$\because \ {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore$ we get

${}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{49}C_4 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 = {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$

4. (c) ${}^{20}C_{(n+2)} = {}^{20}C_{(n-2)}$

$$\Rightarrow \frac{20!}{(n+2)!(20-n-2)!} = \frac{20!}{(n-2)!(20-n+2)!}$$

$$\Rightarrow \frac{(22-n)!}{(18-n)!} = \frac{(n+2)!}{(n-2)!}$$

$\Rightarrow (22-n)(21-n)(20-n)(19-n)$

$= (n+2)(n+1)\cdot n\cdot(n-1)$

For $n = 10$

$(22-10)(21-10)(20-10)(19-10)$

$= (10+2)(10+1)\cdot 10\cdot(10-1)$

$\Rightarrow 12\cdot 11\cdot 10\cdot 9 = 12\cdot 11\cdot 10\cdot 9$

Hence, $n = 10$.

5. (b) 2nd and 4th place are even place, so vowel '*A*' and '*E*' arrange either 2nd or 4th place in 2! = 2 × 1 = 2 ways
Consonent letter, '*B*' and '*L*' arrange at 1st and 3rd place in 2! = 2 × 1 = 2 ways.
Total number of arrangement = 2 × 2 = 4.

6. (c) Maximum number of points of intersection of 2 non-overlapping circles = 2.
So, maximum number of points of intersection of 5 non-overlapping circles = 8 + 6 + 4 + 2 = 20.

7. (c) Total number of players (excluding one particular player) = 8 – 1 = 7.
$\therefore$ Required number of ways of selection = ${}^7C_5 = 21$.

8. (d) Any 5-digits number formed by the digits 1, 2, 3, 4 and 5 (without repetition) will be always divisible by number 3, because sum of digits of any number, thus formed 1 + 2 + 3 + 4 + 5 = 15, is divisible by 3.
Hence, prime number of 5 digits can not be obtained by using the digits 1, 2, 3, 4, 5.

Cartesian Coordinate System and Straight Line

1. (d) $ax + by + c = 0$ represents a straight line only when at least one of a and b is non zero.

2. (a) $P = (m\cos 2\alpha, m\sin 2\alpha)$
$Q = (m\cos 2\beta, m\sin 2\beta)$

$$PQ = \sqrt{(m\cos 2\alpha - m\cos^2\beta)^2 + (m\sin 2\alpha - m\sin 2\beta)^2}$$

$$\Rightarrow PQ = \sqrt{m^2.2.[1-\cos(2\alpha-2\beta)]}$$

$$\Rightarrow PQ = m\sqrt{2.2.\sin^2(\alpha-\beta)}$$

$\Rightarrow PQ = |2m\sin(\alpha-\beta)|$

3. (c) Distance between

$(-1, -1)$ and $(-\sqrt{3}, \sqrt{3})$

is $\sqrt{(-1+\sqrt{3})^2 + (-1-\sqrt{3})^2}$

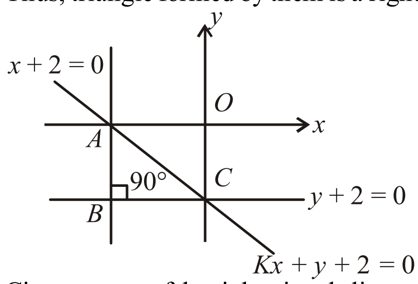
$= \sqrt{1+3-2\sqrt{3}+1+3+2\sqrt{3}} = \sqrt{8}$

by options, distance between (–1, –1) and (1, 1)

is $\sqrt{2^2+2^2} = \sqrt{8}$

4. (c) We know that diagonal of a square bisect each other perpendicularly.
Equation of a diagonal : $3x + 2y = 5$ (given).
Now, equation of other diagonal that is perpendicular to the given diagonal = $2x - 3y = K$.
As vertex point (1, –1) does not lies on $3x + 2y = 5$
$\{\because 3(1) + 2(-1) \neq 5\}$
Then, point (1, –1), must be on the diagonal
$2x - 3y = K$
Then, $2(1) - 3(-1) = K$.
$\therefore K = 5$.
Hence, equation of other diagonal : $2x - 3y = 5$.

5. (c) Equation of two sides of the triangle are $x + 2 = 0$ and $y + 2 = 0$.
They intersect at right angle.
Thus, triangle formed by them is a right angle triangle.



Circumcentre of the right triangle lies on its hypotaneous.
So, circumcentre (–1, –1) must lies on the line
$Kx + y + 2 = 0$
$\therefore K(-1) + (-1) + 2 = 0 \Rightarrow K = 1$.

6. (c) Given points (a, b), (c, d) and $(a-c, b-d)$ are collinear, if

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

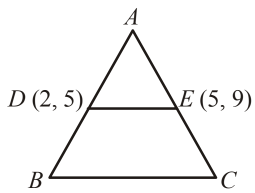
$a(d-b+d) + b(a-c-c) + 1(c(b-d) - d(a-c)) = 0$

$2ad - ab + ab - 2bc + bc - ad = 0$

$\Rightarrow ad - bc = 0$

$\therefore ad = bc$.

7. (b) As point $D(2, 5)$ and point $E(5, 9)$ are mid point of side AB and AC, then
$2 \times DE = BC$



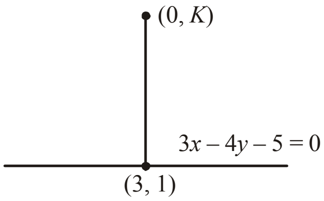
Now,

$BC = 2 \times \sqrt{(5-2)^2 + (9-5)^2}$

$= 2 \times \sqrt{3^2 + 4^2} = 2 \times 5 = 10.$

8. (c) Slope of the line, $3x-4y-5=0$ is $m=\frac{3}{4}$

Slope of any line perpendicular to $3x-4y-5=0$ is $m'=-\frac{4}{3}$.



Now, required line passes through the points $(0, K)$ and $(3, 1)$.

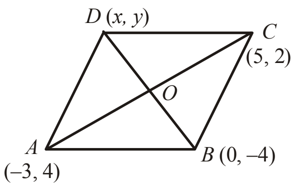
$\therefore \frac{K-1}{0-3}=-\frac{4}{3} \Rightarrow K=5.$

9. (c) We know that pair of opposite sides of my square are parallel.
So, distance between two parallel sides
= Side length of the square
$\therefore$ Side length of the square

$$=\left|\frac{15-(-5)}{\sqrt{3^2+4^2}}\right|=\frac{20}{5}=4 \text{ units}$$

Area of the square = 4 × 4 = 16 units square.

10. (a) Let $ABCD$ is a parallelogram as shown in figure.
Point O is the point of intersection of two diagonals.



Point 'O' $=\left(\frac{-3+5}{2}, \frac{4+2}{2}\right)=(1, 3)$.

Now, point 'O' bisect diagonal BD.

$1=\frac{x+0}{2} \Rightarrow x=2$, $3=\frac{y-4}{2} \Rightarrow y=10$.

$\therefore$ Fourth vertices $D(x, y)=(2, 10)$.

11. (c) Two lines are perpendicular, if $a_1a_2+b_1b_2=0$.

$1-p \cdot q=0 \Rightarrow pq-1=0.$

12. (d) As A, B and C are in A.P.

$\therefore 2B=A+C \Rightarrow A-2B+C=0$

From given equation $Ax+2By+C=0$.

For $(x, y)=(1, -1)$, $A-2B+C=0$

Hence, line always passes through $(1, -1)$.

13. (b) Let Point $A=(-4, 2)$ and $A'=(4, -2)$ then, equation of line mirror passes through the mid-point of line AA' and also perpendicular to the line AA'

Mid point of line $AA'=\left(\frac{-4+4}{2}, \frac{2-2}{2}\right)=(0,0)$

Slope of AA' $=\frac{2-(-2)}{-4-4}=\frac{4}{-8}=\frac{-1}{2}$

$=\frac{2-(-2)}{-4-4}=\frac{4}{-8}=-\frac{1}{2}$

$\therefore$ Slope of line mirror = 2
and equation of line mirror will slope $m = 2$ and passes through $(0, 0)$ is $y = 2x$.

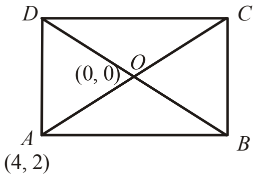
14. (a) Point $(p, p-3)$, $(q+3, q)$ and $(6, 3)$ are collinear, if

$$\begin{vmatrix} p & p-3 & 1 \\ q+3 & q & 1 \\ 6 & 3 & 1 \end{vmatrix}=0$$

$\Rightarrow p(q-3)+(p-3)(6-q-3)+1(3(q+3)-6q)=0$

Hence, given points are collinear.
Here (p, q) can take +ve or –ve values.
So, statement (2) is incorrect.

15. (a)



Slope of diagonal $BD=\frac{-(4-0)}{(2-0)}=-2.$

Now, equation of line passes through origin and slope $m=-2$.

$y=mx \Rightarrow y=-2x \Rightarrow 2x+y=0.$

Pair of Straight Lines

1. (d) for : $x\cos\alpha+y\sin\alpha=a$
we get, $m_1=-\cot\alpha$
for: $x\sin\beta-y\cos\beta=a$
we get, $m_2=\tan\beta$

$\therefore \tan\theta=\left|\frac{m_2-m_1}{1+m_1 \cdot m_2}\right|$

$\Rightarrow \tan\theta=\left|\frac{\tan\beta+\cot\alpha}{1-\tan\beta.\cot\alpha}\right|$

$\Rightarrow \tan\theta=\left|\frac{1+\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}\right|$

$\Rightarrow \tan\theta=\frac{1}{\tan(\alpha-\beta)}$

$\Rightarrow \tan\theta=\cot(\alpha-\beta)$

$\Rightarrow \theta=\frac{\pi}{2}+(\alpha-\beta)$

$\Rightarrow \theta=\frac{\pi+2\alpha-2\beta}{2}$

2. (b) Here, $m_1=(2-\sqrt{3})$ and $m_2=(2+\sqrt{3})$.
Obtuse angle between them,

$\theta=\tan^{-1}\left(\frac{m_1-m_2}{1+m_1 \cdot m_2}\right)$

$=\tan^{-1}\left(\frac{2-\sqrt{3}-2-\sqrt{3}}{1+(2-\sqrt{3})(2+\sqrt{3})}\right)$

$=\tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right)=\tan^{-1}(-\sqrt{3})=120°.$

3. (b) Given equation $x-2=0$

Slope, $m_1 = \tan\theta_1 = \frac{1}{0} = \infty$.

$\therefore \tan\theta_1 = \tan 90° \Rightarrow \theta_1 = 90°$.

Other equation :

$\sqrt{3}x - y - q = 0$

$\Rightarrow$ Slope $m_2 = \tan\theta_2 = \frac{\sqrt{3}}{1}$

$\tan\theta_2 = \tan 60° \Rightarrow \theta_2 = 60°$

Angle between them $= 90° - 60° = 30°$.

Circles

1. (c) $(x-2a)(x-2b)+(y-2c)(y-2d)=0$

$x^2 - 2(a+b)x + 4ab + y^2 - 2(c+d)y + 4cd = 0$

$x^2 + y^2 - 2(a+b)x - 2(c+d)y + 4(ab+cd) = 0$

From general equation of circle,

$x^2 + y^2 + 2gx + 2fy + c = 0$

Center $= (-g, -f)$ and radius $= \sqrt{(-g)^2 + (-f)^2 + C}$

$\therefore$ Here $g = -(a+b)$ and $f = -(c+d)$.

Hence, center $= (-g, -f) = ((a+b), (c+d))$.

2. (d) Circle equation :

$4x^2 + 4y^2 - 20x + 12y - 15 = 0$

$\Rightarrow x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$

Radius $= \sqrt{(-g)^2 + (-f)^2 - C}$

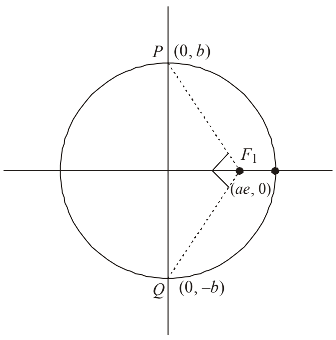
Here, $g = -\frac{5}{2}$, $f = \frac{3}{2}$ and $C = -\frac{15}{4}$

$\therefore$ Radius $= \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{3}{2}\right)^2 - \left(-\frac{15}{4}\right)}$

$= \sqrt{\frac{25+9+15}{4}} = \frac{7}{2} = 3.5$ units.

CONICS- Parabola, Ellipse & Hyperbola

1. (b) Given, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



given: $\angle PF_1Q = 90°$

$\therefore \quad m_{PF_1} \times m_{QF_1} = -1$

$\Rightarrow \quad \frac{-b}{ae} \times \frac{b}{ae} = -1$

$\Rightarrow \quad b^2 = a^2 . e^2$ ...(i)

$\because \quad b^2 = a^2(1-e^2)$ ...(ii)

From (i) and (ii) we get,

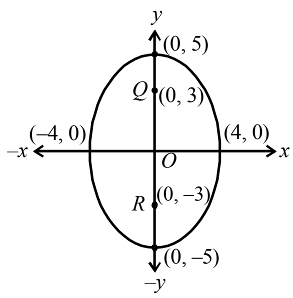
$e^2 = 1 - e^2$

$\Rightarrow \quad 2e^2 = 1$

$\Rightarrow \quad e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$

2. (b) Equation of the Ellipse : $25x^2 + 16y^2 = 400$.



$\frac{x^2}{16} + \frac{y^2}{25} = 1$

Here $a^2 = 16$, $b^2 = 25$

Focci points $= (0, \pm be) = (0, \pm\sqrt{b^2 - a^2})$

$= (0, \pm\sqrt{25-16}) = (0, \pm 3)$

Point $Q = (0, 3)$, and $R = (0, -3)$.

Now, from question point $P(x, y)$ lies on the ellipse, then sum of its distance from two fixed points is always constant, equal to length of its major axis.

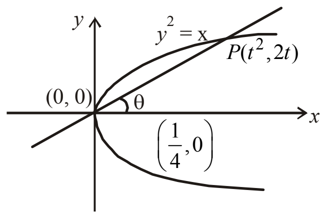
Thus, $(PQ + PR) = 2 \times 5 = 10$.

3. (b) Given parabola : $y^2 = x$.

From standard equation of parabola $y^2 = 4ax$

Focus $= (a, 0)$

Here $a = \frac{1}{4}$



$\therefore$ Focus of parabola $y^2 = x = \left(\frac{1}{4}, 0\right)$

Vertex of the parabola = (0, 0)

Let $P\left(\frac{t^2}{4}, \frac{t}{2}\right)$ lies on the parabola

then, its slope = tan (θ)   {given}

$\therefore \frac{\frac{t}{2}}{\frac{t^2}{4}} = \tan\theta \Rightarrow t = 2\cot\theta$

So, point $P\left(\frac{t^2}{4}, \frac{t}{2}\right) = (\cot^2\theta, \cot\theta)$

Its distance from vertex (0, 0)

$= \sqrt{(\cot^2\theta - 0)^2 + (\cot\theta - 0)^2}$

$= \cot\theta\sqrt{\cot^2\theta + 1} = \cot\theta \cdot \operatorname{cosec}\theta$

$= \cos\theta \cdot \operatorname{cosec}^2\theta$

4. (d) Let equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

then eccentricity $e = \sqrt{1 + \frac{a^2}{b^2}}$

Now, as point (3 tan θ, 2 sec θ) lies on hyperbola

then, $\frac{3^2\tan^2\theta}{a^2} - \frac{2^2\sec^2\theta}{b^2} = 1$

$\frac{9\tan^2\theta}{a^2} - \frac{4\sec^2\theta}{b^2} = 1.$

This is true for $a^2 = 9$ and $b^2 = 4$

$\therefore$ eccentricity $(e) = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

5. (d) All three statements are correct.

TRIGONOMETRY- Ratio & Identity, Trigonometric Equations

1. (b) Given: $\operatorname{cosec}\theta = \frac{29}{21} = \frac{H}{P}$

$\because$ we have Pythagorean triplet of (20, 21, 29)

$\therefore\ P = 21$

$B = 20$

$H = 29$

$\therefore\ 4(\sec\theta + \tan\theta) = 4\left(\frac{29}{21} + \frac{21}{20}\right) \approx \frac{200}{20} = 10$

2. (a) (i) $\cos\theta + \sec\theta = \frac{3}{2}$

$\Rightarrow \frac{1 + \cos^2\theta}{\cos\theta} = \frac{3}{2}$

$\Rightarrow 2\cos^2\theta - 3\cos\theta + 2 = 0$

here discriminant is negative

$\therefore$ No real roots

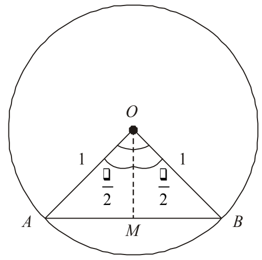
$\therefore$ No solution.

(ii) In second quadrant both tan θ and cot θ are negative

$\therefore$ In second quadrant value is less than 2.

So only statement 1 is true.

3. (c)



In ΔOAM,

$\sin\left(\frac{\theta}{2}\right) = AM$

$\because\ AB = 2AM$

$\therefore\ AB = 2\sin\left(\frac{\theta}{2}\right)$

4. (a) For minimum value

$\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$

$= a^2\sec^2 x + b^2\operatorname{cosec}^2 x$

$= a^2(1 + \tan^2 x) + b^2(1 + \cot^2 x)$

$= a^2 + b^2 + Q^2\tan^2 x + b^2\cot^2 x$

$\because\ a^2\tan^2 x + b^2\cot^2 x \geq 2ab$

$\therefore$ minimum value $= a^2 + b^2 + 2ab$

$= (a + b)^2$

5. (a) Given:

$\tan A - \tan B = x$

and $\cot B - \cot A = y$

$\Rightarrow \frac{\tan A - \tan B}{\tan A \,.\, \tan B} = y \Rightarrow \tan A \,.\, \tan B = \frac{x}{y}$ for $\cot(A - B)$,

$\cot(A - B) = \frac{1 + \tan A\tan B}{\tan A - \tan B} = \frac{1 + \frac{x}{y}}{x} = \left(\frac{1}{x} + \frac{1}{y}\right)$

6. (a) $\sin(\alpha + \beta) - 2\sin\alpha\cos\beta + \sin(\alpha - \beta)$

by applying formula

$\sin C + \sin D = 2\sin\left(\frac{C + D}{2}\right) \cdot \cos\left(\frac{C - D}{2}\right)$

$= 2\sin\alpha \,.\, \cos\beta - 2\sin\alpha \,.\, \cos\beta$

$= 0$

7. (c) Given : $2\tan A = 1 \Rightarrow \tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

$\because\ \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan \cdot A\tan B}$

$\Rightarrow\ \tan(A - B) = \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{1}{6}}{\frac{7}{6}} = \frac{1}{7}$

8. (c) $\because \cos C + \cos D$

$= 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

$\Rightarrow \cos 80° + \cos 40° = 2\cos 60° . \cos 20°$

$= 2 . \frac{1}{2} . \cos 20° = \cos 20°$

$\therefore \cos 80° + \cos 40° - \cos 20° = 0$

9. (d) $\cot\frac{A}{2} - \tan\frac{A}{2}$

$= \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} - \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \frac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{\sin\frac{A}{2}.\cos\frac{A}{2}} \times \frac{2}{2}$

$= 2.\frac{\cos A}{\sin A} = 2\cot A$

10. (b) $\cot A + \operatorname{cosec} A$

$= \frac{\cos A}{\sin A} + \frac{1}{\sin A} = \frac{1+\cos A}{\sin A} = \frac{2\cos^2\frac{A}{2}}{2\sin\frac{A}{2}.\cos\frac{A}{2}} = \cot\left(\frac{A}{2}\right)$

11. (b) $\tan 25° . \tan 15° + \tan 15° . \tan 50° + \tan 50° . \tan 25°$
$= \tan 15°(\tan 15° + \tan 50°) + \tan 50° . \tan 25°$ ...(1)

$\because \tan 75° = \frac{\tan 25° + \tan 50°}{1 - \tan 25° . \tan 50°} = \cot 15°$

$\Rightarrow \tan 25° + \tan 50° = \cot 15°(1 - \tan 25° . \tan 50°)$

Put in (1)

$\Rightarrow \tan 15° . \cot 15° . (1 - \tan 25° . \tan 50°) + \tan 50° . \tan 25°$
$= 1 - \tan 25° . \tan 50° + \tan 50° . \tan 25°$
$= 1$

12. (a) From $a\sin^2 x + b\cos^2 x = c$

$a\sin^2 x + b\cos^2 x = c(\sin^2 x + \cos^2 x)$

$(a-c)\sin^2 x = (c-b)\cos^2 x$

$\Rightarrow \frac{\sin^2 x}{\cos^2 x} = \frac{(c-b)}{(a-c)}$

$\Rightarrow \tan^2 x = \frac{(c-b)}{(a-c)}$

13. (c) From $b\sin^2 y + a\cos^2 y = d$

$b\sin^2 y + a\cos^2 y = d(\sin^2 y + \cos^2 y)$

$(b-d)\sin^2 y = (d-a)\cos^2 y$

$\Rightarrow \frac{\sin^2 y}{\cos^2 y} = \frac{(d-a)}{(b-d)} \Rightarrow \tan^2 y = \frac{(d-a)}{(b-d)}$

14. (b) From, $p \cdot \tan x = q \cdot \tan y$

$p^2 \cdot \tan^2 x = q^2 \cdot \tan^2 y$

$\Rightarrow \frac{p^2}{q^2} = \frac{\tan^2 y}{\tan^2 x} = \frac{(d-a)}{(b-d)} \cdot \frac{(a-c)}{(c-b)} = \frac{(a-d)(c-a)}{(b-c)(d-b)}.$

15. (a) $t_n = \sin^n\theta + \cos^n\theta$

Now, $\frac{t_3 - t_5}{t_5 - t_7} = \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}$

$= \frac{\sin^3\theta(1-\sin^2\theta) + \cos^3\theta(1-\cos^2\theta)}{\sin^5\theta(1-\sin^2\theta) + \cos^7\theta(1-\cos^2\theta)}$

$= \frac{\sin^3\theta \cdot \cos^2\theta + \cos^3\theta \cdot \sin^2\theta}{\sin^5\theta \cdot \cos^2\theta + \cos^7\theta \cdot \sin^2\theta}$

$= \frac{\sin^2\theta \cdot \cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta \cdot \cos^2\theta(\sin^3\theta + \cos^3\theta)}$

$= \frac{(\sin\theta + \cos\theta)}{(\sin^3\theta + \cos^3\theta)} = \frac{t_1}{t_3}$

16. (b) $t_1^2 - t_2 = (\sin\theta + \cos\theta)^2 - (\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta - (\sin^2\theta + \cos^2\theta)$

$= 1 + \sin 2\theta - 1 = \sin 2\theta.$

17. (c) $t_{10} = \sin^{10}(\theta) + \cos^{10}(\theta)$

When $\theta = 45°$,

$t_{10} = \sin^{10}(45°) + \cos^{10}(45°)$

$= \left(\frac{1}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}\right)^{10} = 2 \cdot \left(\frac{1}{\sqrt{2}}\right)^{10}$

$= 2 \times \frac{1}{32} = \frac{1}{16}.$

18. (d) $\alpha = \beta = 15°$ (given)

Now, $\sin\alpha + \cos\beta$

$= \sqrt{2}.\left(\frac{1}{\sqrt{2}} \cdot \sin\alpha + \frac{1}{\sqrt{2}} \cdot \cos\alpha\right)$ $\{\because \alpha = \beta\}$

$= \sqrt{2}(\sin\alpha \cdot \cos 45° + \cos\alpha \cdot \sin 45°)$

$= \sqrt{2}(\sin(\alpha + 45°)) = \sqrt{2} \cdot \sin(15° + 45°)$

$= \sqrt{2} \cdot \sin 60° = \sqrt{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}.$

19. (d) $\sin 7\alpha - \cos 7\beta$

$= \sin 7(15°) - \cos 7(15°) = \sin(105°) - \cos(105°)$

$= \sin(90° + 15°) - \cos(90° + 15°)$

$= \cos 15° - (-\sin(15°)) = \cos 15° + \sin 15° = \frac{\sqrt{3}}{\sqrt{2}}$

20. (None) $\sin(\alpha + 1°) + \cos(\beta + 1°)$

$= \sqrt{2}\left\{\frac{1}{\sqrt{2}} \cdot \sin(\alpha + 1°) + \frac{1}{\sqrt{2}} \cdot \cos(\beta + 1°)\right\}$

$= \sqrt{2}\{\cos 45° \cdot \sin(\alpha + 1) + \sin 45° \cdot \cos(\alpha + 1)\}$ $(\because \alpha = \beta)$

$= \sqrt{2}\{\sin(\alpha + 1 + 45°)\} = \sqrt{2} \cdot \sin(15° + 1° + 45°)$

$= \sqrt{2} \cdot \sin 61°$ (i)

Again

$\frac{1}{\sqrt{2}}(\sqrt{3} \cdot \cos 1° + \sin 1°) \quad = \frac{2}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} \cdot \cos 1° + \frac{1}{2} \sin 1°\right)$

$= \sqrt{2} \cdot (\sin 60° \cdot \cos 1° + \cos 60° \cdot \sin 1°)$

$= \sqrt{2}(\sin 61°).$ (ii)

Thus (i) = (ii)

But none of the option have $\frac{1}{\sqrt{2}}\left(\sqrt{3} \cdot \cos 1° + \sin 1°\right).$

21. (c) $\sin x + \sin y = \cos y - \cos x$

$$\frac{\sin x + \sin y}{\cos y - \cos x} = 1$$

$$\frac{2\sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)} = 1$$

$$\cot\left(\frac{x-y}{2}\right) = 1 \Rightarrow \tan\left(\frac{x-y}{2}\right) = 1.$$

22. (b) $\frac{\tan 3A}{\tan A} = K$

$$\frac{3\tan A - \tan^3 A}{\tan A \cdot (1 - 3\tan^2 A)} = K$$

$$\frac{(3 - \tan^2 A)}{(1 - 3\tan^2 A)} = K \Rightarrow 3 - \tan^2 A = K - 3K\tan^2 A$$

$$(3K - 1)\tan^2 A = K - 3 \Rightarrow \tan^2 A = \frac{K - 3}{(3K - 1)}$$

23. (a, b) $\tan A = \sqrt{\frac{(K-3)}{(3K-1)}}$

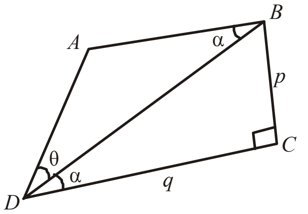
For real value of tan$A$, $\left(\frac{K-3}{3K-1}\right) > 0.$

$\therefore$ For, $\frac{1}{3} < K < 3.$

tan A is not real.

Also, for $\frac{1}{2} <$ k $<$ 2, tan A is not real.

24. (c)



(1) Applying Sine ryke in ΔABD

$$\frac{AD}{\sin\alpha} = \frac{AB}{\sin\theta}$$

$\Rightarrow AD \cdot \sin\theta = AB \cdot \sin\alpha$

Hence, (1) is correct.

(2) $\frac{AB}{\sin\theta} = \frac{BD}{\sin(180° - (\alpha + \theta))}$

$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\alpha + \theta)}$$

$AB \cdot \sin(\theta + \alpha) = BD \cdot \sin\theta$

Hence, (2) is correct.

25. (a) From ΔABD,

$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(180° - (\alpha + \theta))}$$

$$\Rightarrow AB = \frac{BD \cdot \sin\theta}{\sin(\alpha + \theta)} = \frac{BD \cdot \sin\theta}{\sin\alpha \cdot \cos\theta + \cos\alpha \cdot \sin\theta}$$

Again from ΔBCD,

$$BD = \sqrt{(BC)^2 + (CD)^2} = \sqrt{p^2 + q^2}$$

$$\sin\alpha = \frac{BC}{BD} = \frac{p}{\sqrt{p^2 + q^2}}$$

$$\cos\alpha = \frac{CD}{BD} = \frac{q}{\sqrt{p^2 + q^2}}$$

$$\therefore AB = \frac{\sqrt{p^2 + q^2} \cdot \sin\theta}{\frac{p \cdot \cos\theta}{\sqrt{p^2 + q^2}} + \frac{q \cdot \sin\theta}{\sqrt{p^2 + q^2}}}$$

$$= \frac{(p^2 + q^2) \cdot \sin\theta}{p \cdot \cos\theta + q \cdot \sin\theta}$$

26. (b) $\tan\theta = \frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°} \quad = \frac{1 - \frac{\sin 17°}{\cos 17°}}{1 + \frac{\sin 17°}{\cos 17°}} = \frac{1 - \tan 17°}{1 + \tan 17°}$

$$\tan\theta = \frac{\tan 45° - \tan 17°}{1 + \tan 45° \cdot \tan 17°} \quad \tan\theta = \tan(45° - 17°)$$

$\tan\theta = \tan(28°) \Rightarrow \theta = 28°.$

27. (d) From question, we have

$\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$

Now,

$$\cos(A + 2B) = \cos A \cdot \cos 2B - \sin A \cdot \sin 2B$$

$$= \cos A \cdot 3\sin^2 A - \sin A \cdot \frac{3}{2}\sin 2A$$

$$= 3\cos A \cdot \sin^2 A - \frac{3}{2} \cdot \sin A \cdot (2\sin A - \cos A)$$

$$= 3\cos A \cdot \sin^2 A - 3\cos A \cdot \sin^2 A = 0.$$

$\therefore \cos(A+2B) = \cos\left(\frac{\pi}{2}\right)$

$\therefore A+2B = \frac{\pi}{2}.$

28. (a) $\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$

$= \sin 3x + \cos 3x - \sin 3x - \cos 3x = 0.$

29. (d) $y = 2 + \cos x$

Range of $\cos x = [-1, 1]$

$y_{min} = 2 - 1 = 1.$

$y_{max} = 2 + 1 = 3$

Thus, ordinate of the graph = [1, 3].

30. (b) $8 \cdot \cos 10° \cdot \cos 20° \cdot \cos 40°$

$= \frac{4}{\sin 10°}[2 \cdot \cos 10° \cdot \sin 10°] \cdot \cos 20° \cdot \cos 40°$

$= \frac{4}{\sin 10°}[\sin 20°] \cdot \cos 20° \cdot \cos 40°$

$= \frac{2}{\sin 10°}[2\sin 20° \cdot \cos 20°] \cdot \cos 40°$

$= \frac{2}{\sin 10°}[\sin 40°] \cdot \cos 40°$

$= \frac{1}{\sin 10°}[2\sin 40° \cdot \cos 40°] = \frac{\sin 80°}{\sin 10°}$

$= \frac{\sin(90° - 10°)}{\sin 10°} = \frac{\cos 10°}{\sin 10°} = \cot 10°.$

31. (b) $\cos 48° - \cos 12°$

$= 2 \cdot \sin\left(\frac{48° + 12°}{2}\right) \cdot \sin\left(\frac{12° - 48°}{2}\right)$

$= 2 \cdot \sin 30° \cdot \sin(-18°) = -\sin 18°$

$= -\left(\frac{\sqrt{5}-1}{4}\right) = \left(\frac{1-\sqrt{5}}{4}\right).$

32. (c) $\log_{\cos x} \sin x = 1 \quad \Rightarrow \frac{\log \sin x}{\log \cos x} = 1$

For $x = \frac{\pi}{4}$, $\frac{\log \sin \frac{\pi}{4}}{\log \cos \frac{\pi}{4}} = \frac{\log\left(\frac{1}{\sqrt{2}}\right)}{\log\left(\frac{1}{\sqrt{2}}\right)} = 1.$

33. (c) $\cot(6x) = \cot(2x + 4x)$

$\Rightarrow \cot 6x = \frac{\cot 2x \cdot \cot 4x - 1}{\cot 2x + \cot 4x}$

$\therefore \cot 2x \cdot \cot 6x + \cot 6x \cdot \cot 4x = \cot 2x \cdot \cot 4x - 1$

or, $\cot 2x \cdot \cot 4x - \cot 4x \cdot \cot 6x - \cot 6x \cdot \cot 2x = 1.$

34. (d) $\tan x = -\frac{3}{4}$ (Here $90° < x < 180°$)

$\sin x = \frac{3}{5}$ and $\cos x = -\frac{4}{5}$

Now,

$\sin x \cdot \cos x = \frac{3}{5} \cdot \left(-\frac{4}{5}\right) = -\frac{12}{25}.$

35. (c) $\text{cosec}\left(\frac{7\pi}{6}\right) \cdot \sec\left(\frac{5\pi}{3}\right)$

$= \text{cosec}\left(\pi + \frac{\pi}{6}\right) \cdot \sec\left(2\pi - \frac{\pi}{3}\right)$

$= -\text{cosec}\frac{\pi}{6} \cdot \sec\frac{\pi}{3} = (-2)(2) = -4.$

36. (c)

$\tan 31° \cdot \tan 33° \cdot \tan 35° \ldots\ldots\ldots \tan 55° \cdot \tan 57° \cdot \tan 59°$

$= \tan 31° \cdot \tan 33° \cdot \tan 35° \ldots\ldots\ldots$

$\tan(90° - 35°) \cdot \tan(90° - 33°) \cdot \tan(90° - 31°)$

$= \tan 31° \cdot \tan 33° \cdot \tan 35° \ldots\ldots\ldots \tan 45° \ldots\ldots\ldots$

$\cot 35° \cdot \cot 33° \cdot \cot 31°$

$= 1.$

37. (b) $(\sin 24° + \cos 66°)(\sin 24° - \cos 66°)$

$= \sin^2 24° - \cos^2 66°$

$= \sin^2 24° - \cos^2(90° - 24°)$

$= \sin^2 24° - \sin^2 24° = 0.$

38. (b) $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)$

$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta \cdot \cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta - 1}{\sin\theta \cdot \cos\theta}$

$= \frac{1 + 2\sin\theta \cdot \cos\theta - 1}{\sin\theta \cdot \cos\theta} = 2.$

39. (a) $\frac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\frac{1 - \tan\theta}{1 - \cot\theta}\right)^2 = \frac{1 + \tan^2\theta}{\frac{\tan^2\theta + 1}{\tan^2\theta}} - \left(\frac{1 - \tan\theta}{\frac{\tan\theta - 1}{\tan\theta}}\right)^2$

$= \tan^2\theta - (-\tan\theta)^2 = 0.$

40. (d) $7\sin\theta + 24\cos\theta = 25$

$\Rightarrow \frac{7}{25} \cdot \sin\theta + \frac{24}{25} \cdot \cos\theta = 1$

Again, $(7)^2 + (24)^2 = (25)^2$

$\therefore$ Let $\frac{7}{25} = \cos\alpha$ and $\frac{24}{25} = \sin\alpha$

Then, $\sin\theta \cdot \cos\alpha + \cos\theta \cdot \sin\alpha = 1$

$$\Rightarrow \sin(\theta+\alpha) = \sin\frac{\pi}{2}$$

$$\Rightarrow \theta+\alpha = \frac{\pi}{2} \Rightarrow \theta = \left(\frac{\pi}{2}-\alpha\right)$$

Now, $\sin\theta + \cos\theta$

$$= \sin\left(\frac{\pi}{2}-\alpha\right) + \cos\left(\frac{\pi}{2}-\alpha\right)$$

$$= \cos\alpha + \sin\alpha = \frac{7}{25} + \frac{24}{25} = \frac{31}{25}.$$

41. (b) $3\cos\theta = 4\sin\theta$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{3}{4} \Rightarrow \tan\theta = \frac{3}{4}$$

$$\therefore \tan(45+\theta) = \frac{\tan 45° + \tan\theta}{1 - \tan 45° \cdot \tan\theta}$$

$$= \frac{1+\frac{3}{4}}{1-\frac{3}{4}} = \frac{7}{1} = 7.$$

42. (a) $\tan A = \frac{1}{7}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2} = \frac{24}{25}.$

Properties of Triangle, Inverse Trigonometric Function

1. (b) $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$

$$\because \quad 2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}$$

$$\therefore \quad 2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\frac{2/3}{1-1/9} = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\therefore \quad \tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\} = \tan\left(\tan^{-1}\frac{3}{4}\right) = \frac{3}{4}$$

2. (d) Since, A, B, C are in AP
$\therefore \quad 2B = A + C$ ...(1)
also, $A + B + C = 180°$
$\Rightarrow \quad A + C = 180° - B$ ...(2)
from (1) and (2)
$B = 60°$

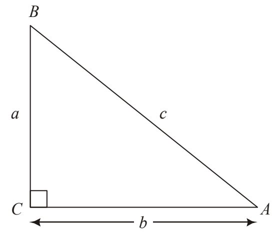
also given that $b : c = \sqrt{3} : \sqrt{2}$
$\therefore$ by sine rule

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\Rightarrow \quad \frac{\sin 60°}{\sqrt{3}} = \frac{\sin C}{\sqrt{2}} \Rightarrow \sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45°$$

$\therefore \quad A = 75°$

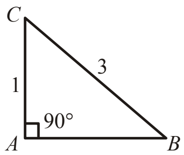
3. (d)



we have $c^2 = a^2 + b^2$

$$\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab} = \frac{c^2}{ab}$$

4. (b)



(1) ΔABC is a right angled triangle.

$$\sin B = \frac{AC}{BC} = \frac{1}{3}$$

$\therefore AC = 1,\ BC = 3.$

Now,

$$AB = \sqrt{(BC)^2 - (AC)^2} = \sqrt{(3)^2 - (1)^2} = 2\sqrt{2}.$$

Now, $\sin C = \frac{AB}{BC} = \frac{2\sqrt{2}}{3}$

$$\therefore \text{cosec } C = \frac{3}{2\sqrt{2}}.$$

Hence, (1) is not correct.

(2) $b \cdot \cos B = c \cdot \cos C$

$$b \cdot \frac{(a^2+c^2-b^2)}{2ac} = c \cdot \frac{(a^2+b^2-c^2)}{2ab}$$

$$b^2(a^2+c^2-b^2) = c^2(a^2+b^2-c^2)$$

$$a^2b^2 - b^4 - a^2c^2 + c^4 = 0$$

$$a^2(b^2-c^2) - (b^4-c^4) = 0$$

$$(b^2-c^2)(a^2-b^2-c^2) = 0$$

Either $b^2+c^2-a^2 = 0$ or $(b^2-c^2) = 0$

When, $b^2+c^2-a^2 = 0$

$b^2 + c^2 = a^2$

Hence, ΔABC is a right angle triangle.

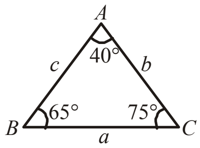
And when, $b^2 - c^2 = 0 \Rightarrow b = c$

Hence, ΔABC is an isosceles triangle.
From question ΔABC is not right angle triangle.
Hence, ΔABC must be an isosceles triangle.

5. (b)
(1) Consider a right angle triangle ABC, right angle at A and $B = C = 45°$.
Then, $b = c$ is also true.

Hence, for the given condition, ΔABC must not be an obtuse-angled triangle.

(2)



In ΔABC, $\angle A = 40°$, $\angle B = 65°$

$\therefore \angle C = 180° - 40° - 65° = 75°$.

From sine rule,

$$\frac{a}{\sin 40°} = \frac{c}{\sin 75°}$$

$$\therefore \frac{a}{c} = \sin 40° \cdot \text{cosec}75°$$

So, $\frac{a}{c} \neq \sin 40° \cdot \text{cosec}15°$

Hence, ΔABC is not possible.

Thus, statement (2) is correct.

6. (b) $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ ...(i)

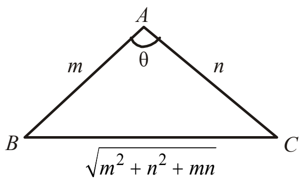
We know that, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ ...(ii)

On adding, $\sin^{-1} x = \frac{\pi}{3}$

7. (a) $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$

This hold for all $x \in R$.

8. (b)



From cosine rule in ΔABC,

$$\cos(\angle BAC) = \frac{(AB)^2 + (AC)^2 - (BC)^2}{2 \cdot (AB) \cdot (AC)}$$

$$= \frac{m^2 + n^2 - (m^2 + n^2 + mn)}{2 \cdot m \cdot n}$$

$$\therefore \cos(\angle BAC) = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$$

$$\therefore \angle BAC = \frac{2\pi}{3}$$

Then, sum of other two acute angle

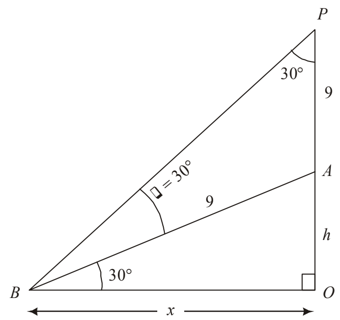
$$= \pi - \frac{2\pi}{3} = \frac{\pi}{3} = 60°.$$

9. (c) Area of $\Delta ABC = \frac{1}{2} \cdot a \cdot c \cdot \sin(\angle B)$

$$= \frac{1}{2} \cdot 10 \cdot 4 \cdot \sin(30°) = \frac{1}{2} \cdot 10 \cdot 4 \cdot \frac{1}{2} = 10 \text{ cm}^2.$$

Height & Distance

1. (c)



ΔPAB is isosceles Δ

$\Rightarrow \theta = 30°$

In ΔABD

$$\tan 30° = \frac{h}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = h\sqrt{3}$$

In ΔPBO,

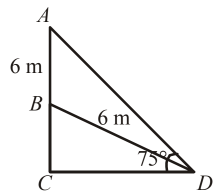
$$\tan 60° = \sqrt{3} = \frac{9 + h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{9+h}{h\sqrt{3}} \Rightarrow 3h = q + h \Rightarrow h = \frac{9}{2} m$$

$\therefore$ Total height = 13.5 m

2. (d) Let AC is a flagstaff of length h m and BD is a ladder of length 6 m. Point B is below the top of the flagstaff, such that $AB = 6$ m.

Then, $\angle ADC = 75°$.



From ΔACD, $\angle CAD + \angle ADC + \angle ACD = 180°$

$\therefore \angle CAD + 75° + 90° = 180°$

$\therefore \angle CAD = 180° - 90° - 75° = 15°$.

As, $AB = BD$.

$\therefore \angle CAD = \angle BDA = 15°$ and

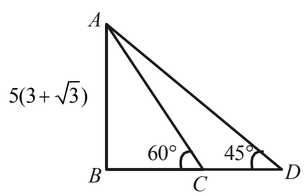
$\angle BDC = 75° - 15° = 60°$.

From ΔBCD, $\sin(60°) = \frac{BC}{BD} = \frac{BC}{6}$

$$\therefore BC = 6 \cdot \sin(60°) = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}.$$

Hence, height of flagstaff $= (6 + 3\sqrt{3})$ m.

3. (b) Shadow length when elevation of Sun is 60° = BC



From ΔABC,

$$BC = AB \cdot \cot(60°) = 5(3+\sqrt{3}) \cdot \frac{1}{\sqrt{3}} = 5(\sqrt{3}+1) \text{ m}$$

Shadow length when elevation of Sun is 45° = BD

From ΔABD,

$$BD = AB \cdot \cot 45° = 5(3+\sqrt{3}) \cdot 1 = 5(3+\sqrt{3}) \text{ m}$$

$$\therefore x = BD - BC = 5(3+\sqrt{3}) - 5(\sqrt{3}+1) = 10 \text{ m}.$$

Functions, Limit, Continuity and Differentiability

1. (d) $f(0) = 0$ and, $f(x) = \sin\left(\frac{1}{x}\right)$

(i) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \sin\left(\frac{1}{x}\right)$

if $x \to 0^+$; $\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right) = \sin\infty$

if $x \to 0^-$; $\lim_{x\to 0^-} \sin\left(\frac{1}{x}\right) = -\sin\infty$

$\because \sin\infty \neq -\sin\infty$

$\therefore$ LHL $\neq$ RHL

(ii) at $x = 0$, LHL $\neq$ RHL

$\therefore f(x)$ is not continous

$\therefore$ both statements are wrong.

2. (d) $\lim_{x\to 0} \frac{\sin x°}{\tan 3x°} \times \frac{3x°}{x°} \times \frac{1}{3}$

$= \frac{1}{3} \lim_{x\to 0}\left(\frac{\sin x°}{x°}\right).\left(\frac{3x°}{\tan 3x°}\right) = \frac{1}{3}$

3. (d) If $f(x)$ has period T then,

$a f(bx+c)$ has period $\frac{T}{|b|}$

$\therefore$ period of $g(x) = \frac{2\pi}{1/4} = 8\pi$

4. (c) Period of $h(x) = \frac{2\pi}{4/5} = \frac{5\pi}{2}$

5. (c) Period of $f(x)$ = LCM of period of $g(x)$ & $h(x)$

$\Rightarrow$ LCM$\left[8\pi, \frac{5\pi}{2}\right]$

$= 40\pi$

6. (a) $f(x) = \begin{cases} 2x + \frac{1}{4} & ; \quad x < 0 \\ k & ; \quad x = 0 \\ \left(x + \frac{1}{2}\right)^2 & ; \quad x > 0 \end{cases}$

at $x = 0$,

LHL $= \frac{1}{4}$ and RHL $= \frac{1}{4}$

$\therefore$ Function is continous at $x = 0$

$\therefore \quad k = \frac{1}{4}$

7. (a)

(1) $f(x) = e^{-|x|}$, $f(0) = e^{-|0|} = 1$.

$f(x) = e^{-x}$, for $x \geq 0$

$= e^x$, for $x < 0$.

LHL $= \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^x = 1$

RHL $= \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{-x} = 1$

As, $f(0)$ = LHL = RHL

Thus $f(x)$ is continuous at $x = 0$.

(2) LHD $= \lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} e^x = e^0 = 1$

RHD $= \lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} -e^{-x} = -e^0 = -1$

As LHD $\neq$ RHD

Hence, $f(x)$ is not differentiable at $x = 0$.

8. (a) $\lim_{x\to 0} \frac{3^x + 3^{-x} - 2}{x} = \lim_{x\to 0} \frac{3^x \cdot \log 3 - 3^{-x} \cdot \log 3}{1}$

(By L'Hospital rule)

$$= \frac{3^0 \cdot \log 3 - 3^0 \cdot \log 3}{1} = 0$$

9. (b) Given graph $Y = \frac{1}{x-1}$

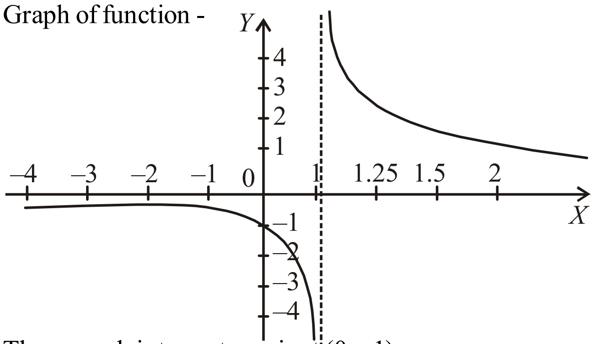
This is defined for all real x, except $x = 1$.

Range of the graph is $y \in R \mid y \neq 0$

Table for the graph :

x	–4	–3	–1	0	0.25	0.5	0.75	1.25	1.5	2
y	–0.2	–0.25	–0.5	–1	–1.33	–2	–4	4	2	1

Graph of function -



Thus, graph intersect y-axis at $(0, -1)$.

10. (b) As the given function is continuous at $x = 0$, then LHL = RHL

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin x}{x} = \lim_{x\to 0^-}\left[\frac{\cos x}{1}\right] = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sin x}{x} = \lim_{x\to 0^+}\left[\frac{\cos x}{1}\right] = 1$

$\therefore \frac{\sin x}{x}$ at $x = 0$, should be 1.

11. (b) $f(x) = \cos^{-1}(x-2)$

Domain of $\cos^{-1}(x-2) = [-1, 1]$

$\therefore -1 \le x-2 \le 1$

$-1+2 \le x \le 1+2$

$1 \le x \le 3$

$\therefore$ Domain $= [1, 3]$

12. (d) $\lim_{x\to1} \frac{x+x^2+x^3-3}{x-1}$

This is in $\frac{0}{0}$ form, so we can use L' Hospital rule.

$$\lim_{x\to1} \frac{x+x^2+x^3-3}{x-1} = \lim_{x\to1} \frac{1+2x+3x^2}{1} = \frac{1+2+3}{1} = 6$$

13. (c) $\lim_{x\to1} \frac{x^4-1}{x-1} = \lim_{x\to1} \frac{(x-1)(x+1)(x^2+1)}{(x-1)}$

$$= \lim_{x\to1}(x+1)(x^2+1) = (1+1)(1+1) = 4.$$

Now, $\lim_{x\to k} \frac{x^3-k^3}{x^2-k^2} = 4$

$$\lim_{x\to k} \frac{(x-k)(x^2+k^2+xk)}{(x-k)(x+k)} = 4$$

$$\lim_{x\to k} \frac{(x^2+k^2+xk)}{(x+k)} = 4$$

$$\frac{k^2+k^2+k\cdot k}{k+k} = 4 \Rightarrow \frac{3}{2}k = 4 \Rightarrow k = \frac{8}{3}$$

14. (a) $\lim_{x\to0} \frac{\sin x \cdot \log(1-x)}{x^2}$

This is in $\frac{0}{0}$ form, by using L' Hospital rule.

$$\lim_{x\to0} \frac{\sin x \cdot \log(1-x)}{x^2}$$

$$= \lim_{x\to0} \frac{\cos x \cdot \log(1-x) + \sin x \cdot \frac{1}{(1-x)}(-1)}{2x}$$

$$= \lim_{x\to0} \frac{\log(1-x)\cdot(-\sin x) - \frac{\cos x}{(1-x)} - \frac{1}{(1-x)}\cdot\cos x - \frac{\sin x}{(1-x)^2}}{2}$$

$$= \frac{0-1-1-0}{2} = \frac{-2}{2} = -1.$$

Image formed virtual, erect, magnified and behind the mirror.

15. (b) For $f(x+1) = x^2-3x+2$

$$= (x^2+2x+1) - 5x - 5 + 6$$

$$= (x+1)^2 - 5(x+1) + 6$$

$\therefore f(x) = x^2 - 5x + 6$.

16. (c) $\lim_{x\to a} \frac{a^x - x^a}{x^a - a^a}$

This is in $\frac{0}{0}$ form, by L'Hospital rule

$$\lim_{x\to a} \frac{a^x - x^a}{x^a - a^a} = \lim_{x\to a} \frac{a^x \cdot \ln a - a\cdot x^{a-1}}{a\cdot x^{a-1} - 0}$$

$$= \frac{a^a \cdot \ln a - a^a}{a^a} = \ln a - 1.$$

$\therefore \ln a - 1 = -1 \Rightarrow \ln a = 0$

$\therefore a = (e)^0 = 1.$

17. (b) $\lim_{x\to-1} \frac{x^3+x^2}{x^2+3x+2}$

By L'Hospital rule,

$$\lim_{x\to-1} \frac{3x^2+2x}{2x+3} = \frac{3(-1)^2+2(-1)}{2(-1)+3} = \frac{3-2}{-2+3} = 1.$$

18. (a) $\lim_{x\to-1} \frac{f(x)+1}{x^2-1} = \frac{-3}{2}$

$$\left[\frac{f'(x)}{2x}\right] = \frac{-3}{2}$$

$f'(x) = -3x$

On integrating both sides,

$f(x) = \frac{-3}{2}x^2 + c$ where c = Constant

$$\lim_{x\to-1} f(x) = \lim_{x\to-1}\left[\frac{-3}{2}x^2 + c\right] = \frac{-3}{2}$$

19. (a) $f(x) = \begin{cases} a+bx, & x<1 \\ 5, & x=1 \\ b-ax, & x>1 \end{cases}$, is continuers

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$$

$$\lim_{x\to1^-}(a+bx) = \lim_{x\to1^+}(b-ax) = 5$$

$a+b=5$ .... (i) and $b-a=5$........ (ii)

From (i) and (ii) $b=5$ and $a=0$

$\therefore a+b = 0+5 = 5$

20. (a) $y = f(x) = 3^x$

Domain $= (-\infty, \infty)$

Derivatives

1. (a) $x^y = e^{x-y}$

$$\left(\frac{dy}{dx}\right)_{x=1} = ?$$

at $x=1$; $x^y = e^{x-y}$

$\Rightarrow 1 = e^{1-y}$

$\Rightarrow y = 1$

$\because x^y = e^{x-y}$

$\Rightarrow y\,.\,\ell n\, x = x - y$

On differentiating w.r.t. x.

$\Rightarrow y.\frac{1}{x} = 1 - y^1 - \ell n\, x\,.\, y^1$

$\Rightarrow 1 - \frac{y}{x} = y^1(1 + \ell nx)$

$\Rightarrow y^1 = \frac{x - y}{x(1 + \ell nx)}$

$\therefore y^1$ at $x = 1$

$= \frac{1-1}{1(1+0)} = 0$

2. (b) $\because y^1 = \frac{x - y}{x(1 + \ell nx)}$

$\Rightarrow y^{11} = \frac{x(1+\ell nx)(1-y^1) - (x-y)(1+1+\ell nx)}{x^2(1+\ell nx)^2}$

$\Rightarrow y^{11}$ at $x = 1$

$\Rightarrow y^{11} = \frac{1(1)\,(1) - 0}{1(1)}$

$\Rightarrow y^{11} = 1$

3. (b) $y = 2^{\sin^2 x}$

$\Rightarrow \ell n\, y = \sin^2 x\,.\,\ell n\, 2$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = 2\sin x\,.\,\cos x\,.\,\ell n\, 2$

$\Rightarrow \frac{dy}{dx} = y\,.\,\sin x\,.\,\cos x\,.\,\ell n\, 4$

Let $\sin x = 4$

$\therefore \frac{dy}{dx} = \cos x$

$\therefore \frac{dy}{dx} = \frac{\ell n 4.\sin x.2^{\sin^2 x}.\cos x}{\cos x}$

$\Rightarrow \frac{dy}{dx} = 2^{\sin^2 x}.\sin x.\ell n 4$

4. (a) Let $y = \tan^{-1} x$; and $z = \cot^{-1} x$

Then, $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $\frac{dz}{dx} = -\frac{1}{1+x^2}$

Now, $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{\frac{1}{1+x^2}}{-\left(\frac{1}{1+x^2}\right)} = -1.$

5. (b) $e^{\theta\phi} = C + 4\theta\cdot\phi$

$(\theta\cdot\phi)\ln e = \ln(C + 4\theta\cdot\phi)$

$\theta\cdot\phi = \ln(C + 4\theta\cdot\phi)$

$\Rightarrow \theta\cdot g\phi = 4\{\ln(\theta) + \ln(\phi)\} + \ln C$

Differentiating both sides with respect to 'θ'.

$\phi + \theta\cdot\frac{d\phi}{d\theta} = 4\left(\frac{1}{\theta} + \frac{1}{\phi}\cdot\frac{d\phi}{d\theta}\right)$

$\phi\cdot d\theta + \theta\cdot d\phi = 4\phi\cdot d\theta + 4\cdot\theta\cdot d\phi$

$\Rightarrow 3\theta\cdot d\phi = -3\phi\cdot d\theta$

$\therefore \phi\cdot d\theta = -\theta\cdot d\phi$

6. (b) $x^m y^n = a^{m+n}$

$\frac{x^m}{a^m} = \frac{a^n}{y^n}$ ...(i)

Differentiating equation (i) w.r.t. x, we get

$\frac{m\cdot x^{(m-1)}}{a^m} = \frac{a^n\cdot(-n)}{y^{n+1}}\cdot\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{m\cdot y^{(n+1)}\cdot(x)^{(m-1)}}{-n\cdot a^m\cdot a^n}$

$= -\frac{my}{n\cdot x}\cdot\left(\frac{x^m\cdot y^n}{a^{m+n}}\right) = -\frac{my}{nx}(1) = -\frac{my}{nx}$

7. (a) Minimum value of any modulus is 0.

8. (a) $y = \sin(l\,n\,x) + \cos(l\,n\,x)$

$\frac{dy}{dx} = \frac{d}{dx}(\sin(i\,n\,x)) + \frac{d}{dx}(\cos(i\,n\,x))$

$= \cos(l\,n\,x)\cdot\frac{1}{x} + \{-\sin(l\,n\,x)\}\cdot\frac{1}{x}$

$= \frac{\cos(\ln x) - \sin(\ln x)}{x}$

$\left.\frac{dy}{dx}\right|_{x=e} = \frac{\cos(l\,n\,e) - \sin(l\,n\,e)}{e} = \frac{\cos(1) - \sin(1)}{e}$

9. (b) $x = e^t\cdot\cos t \Rightarrow \frac{dx}{dt} = e^t\cdot\cos t - e^t\cdot\sin t$

$y = e^t\cdot\sin t \Rightarrow \frac{dy}{dt} = e^t\cdot\sin t + e^t\cdot\cos t$

$\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{e^t(\cos t - \sin t)}{e^t(\sin t + \cos t)} = \frac{\cos t - \sin t}{\sin t + \cos t}$

$\left.\frac{dx}{dy}\right|_{t=0} = \frac{\cos 0 - \sin 0}{\sin 0 + \cos 0} = \frac{1-0}{0+1} = 1.$

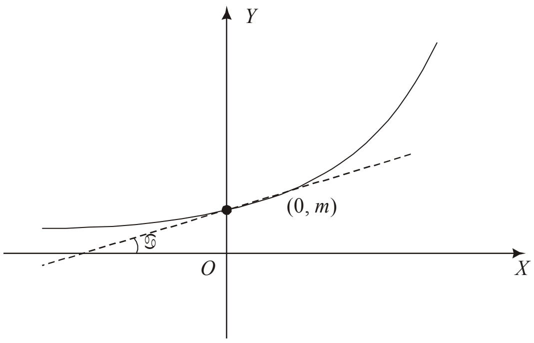
10. (a) Let $y = e^x$, $\frac{dy}{dx} = e^x$

and $z = x^e \Rightarrow \frac{dz}{dx} = e\,.\,x^{(e-1)}$

Now, $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{e^x}{e.x^{(e-1)}} = \frac{x.\,e^x}{e.\,x^e}$

Application of Derivatives

1. (b) Given $y = me^{mx}, m > 0$



for slope, differentiate w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(m \,.\, e^{mx})$$

$$\frac{dy}{dx} = m^2 \,.\, e^{mx}$$

$$\left(\frac{dy}{dx}\right)_{(0,m)} = m^2$$

2. (a) $\tan\alpha = \frac{dy}{dx} = m^2$

$\therefore\ \alpha = \tan^{-1}(m^2)$

3. (d) For equation of tangent

$y - y_1 = \tan\alpha\,(x - x_1)$

$\Rightarrow\ y - m = m^2(x - 0)$

$\Rightarrow\ y = m^2x + m$

4. (*) $f(x) = 3x^4 - 20x^3 - 12x^2 + 288x + 1$

$\therefore\ f'(x) = 12x^3 - 60x^2 - 24x + 288$

$\Rightarrow\ f'(x) = (x+2)(x'-3)(x-4)$

$f'(x) > 0$

$\Rightarrow\ (x+2)(x-3)(x-4) > 0$

$\therefore\ x \in (-2, 3) \cup (4, \infty)$

5. (*) $f'(x) < 0$

$\Rightarrow\ (x+2)(x-3)(x-4) < 0$

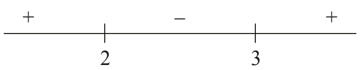
$\Rightarrow\ x \in (-\infty, -2) \cup (3, 4)$

6. (a) $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$

$$f'(x) = \frac{3x^2}{3} - \frac{5.2.x}{2} + 6$$

$\Rightarrow\ f'(x) = x^2 - 5x + 6$

$\Rightarrow\ f'(x) = (x-3)(x-2)$



$\therefore\ T \in (-\infty, 2) \cup (3, \infty)$

and $S \in (2, 3)$

7. (c) $y = \sin x \cdot \cos x$

$$= \frac{2\sin x \cdot \cos x}{2} = \frac{\sin 2x}{2}$$

Now, maximum value of $\sin 2x = 1$.

$\therefore\ y_{\max} = \frac{1}{2}$.

8. (a) Let $y = 3\cdot\cos\left(A + \frac{\pi}{3}\right),\ y' = -3\cdot\sin\left(A + \frac{\pi}{3}\right)$

For extremum value, $y' = 0$

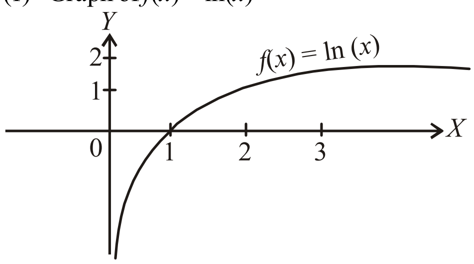
$$\sin\left(A + \frac{\pi}{3}\right) = 0 \Rightarrow A + \frac{\pi}{3} = 0 \Rightarrow A = -\frac{\pi}{3}$$

$$y'' = -3\cdot\cos\left(A + \frac{\pi}{3}\right)$$

$$= -3\cos\left(-\frac{\pi}{3} + \frac{\pi}{3}\right) = -3\cos(0) = -3.$$

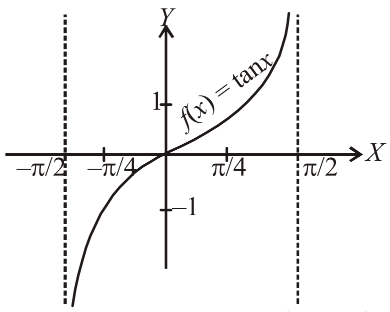
9. (c)

(1) Graph of $f(x) = \ln(x)$



This is increasing function in the interval $(0, \infty)$.

(2) Graph of $f(x) = \tan x\cdot$



This is also increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

10. (a) Circumference of a circle $C = 2\pi r$

Differentiating both sides w.r.t. time (t).

$$\frac{dc}{dt} = 2\pi\cdot\frac{dr}{dt} = 2\pi\cdot(0.7) = 1.4\pi = 1.4\times\frac{22}{7} = 4.4 \text{ cm/ sec.}$$

11. (c) $\frac{dx}{dt} = x + 1$

$$\Rightarrow \frac{dx}{x+1} = dt$$

Integrating both sides, we get $\ln(x+1) = t + c$.

At $t = 0, x = 0$ (origin point)

Then, $\ln(0+1) = 0 + c \Rightarrow c = 0$

$\therefore \ln(x+1) = t$

when $x = 24,\ t = \ln(24+1) = \ln(25) = 2\ln(5)$.

12. (c) Curve $y = -x^3 + 3x^2 + 2x - 27$

Slope $= \frac{dy}{dx} = -3x^2 + 6x + 2$

Again, $\frac{d^2y}{dx^2} = -6x + 6$

For maximum slope $\frac{d^2y}{dx^2} = 0 \Rightarrow 6(-x+1) = 0$

$\therefore x = 1$

Hence, the curve has maximum slope at $x = 1$.

13. (a) $x + y = 20$ and $P = xy$

A.M. of x and y $= \frac{x+y}{2} = \frac{20}{2} = 10.$

G.M. of x and $y = \sqrt{x.y} = \sqrt{P}$

As, A.M. $\geq$ G.M.

$\therefore 10 \geq \sqrt{P}$

Hence, $P_{\max.} = (10)^2 = 100.$

14. (a) $y = \sin 2x \cdot \cos 2x$

$\frac{dy}{dx} = \frac{d}{dx}(\sin 2x)\cdot \cos 2x + \frac{d}{dx}(\cos 2x)\cdot \sin 2x$

$= \cos^2 2x - \sin^2 2x = \cos 4x.$

For maximum or minimum value

$\frac{dy}{dx} = 0 \Rightarrow \cos 4x = 0$

$\therefore 4x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2\times 4}$

Now, $\frac{d^2y}{dx^2} = -4\cdot \sin 4x$

At $x = \frac{\pi}{2\times 4}, \frac{d^2y}{dx^2} = -4\cdot \sin\frac{\pi}{2} = -4.$

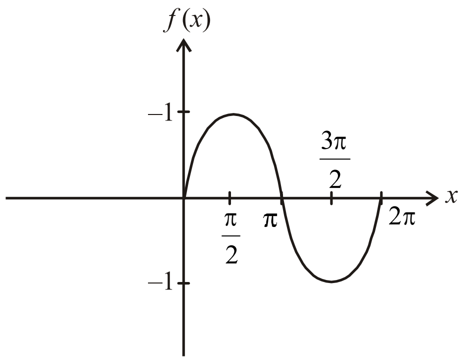
$\therefore y$ is max. at $x = \frac{\pi}{2\times 4}$

$y_{\max.} = \sin 2\left(\frac{\pi}{2\times 4}\right)\cdot \cos 2\left(\frac{\pi}{2\times 4}\right)$

$= \sin\frac{\pi}{4}\cdot \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2}.$

15. (b) $f(x) = \sin x$

Graph of $f(x) = \sin x$ from $x \in [0, 2\pi]$



Here, $f(x) = \sin x$, Increases from $\left[0, \frac{\pi}{2}\right]$ and $\left[\frac{3\pi}{2}, 2\pi\right]$

decreases from $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

Hence (1) is not correct.

Domain $= \left[\frac{5\pi}{2}, 3\pi\right]$

$= \left[2\pi + \frac{\pi}{2}, 2\pi + \pi\right] = \left[\frac{\pi}{2}, \pi\right]$

From graph, sin (x) decreases from $\left[\frac{\pi}{2}, \pi\right]$

Hence, (2) is correct.

Indefinite Integration

1. (d) $\int \frac{dx}{2x^2 - 2x + 1} = \frac{1}{2}\int \frac{dx}{x^2 - x + \frac{1}{2}}$

$= \frac{1}{2}\int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \frac{1}{2}\left[\frac{1}{\frac{1}{2}}\cdot \tan^{-1}\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right)\right] + C$

$= \frac{1}{2}.2 : \tan^{-1}(2x-1) + C$

$= \tan^{-1}(2x-1) + C$

2. (a) $\int \frac{dx}{x(1 + \ell nx)^n}$

Let $1 + \ell n\, x = t$

$\Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{dt}{t^n} = \frac{t^{-n} + 1}{-n + 1}$

$$= \frac{(1+\ell\, nx)^{-n+1}}{1-n}$$

$$= \frac{1}{(n-1)(1+\ \ell \text{nx})^{n-1}} + C$$

3. (b) $\int P(x)\cdot dx = \int (4\cdot e)^{2x}\cdot dx = \int e^{2x(\ln e + \ln 4)}\cdot dx$

$$= \int e^{2x(1+\ln 4)}\cdot dx = \frac{e^{2x(1+\ln 4)}}{2(1+\ln 4)} + C \quad \text{where } C = \text{constant}$$

$$= \frac{(4e)^{2x}}{2(1+2\ln 2)} + C$$

$$\therefore \int P(x)\cdot dx = \frac{P(x)}{2(1+2\ln(2))} + C.$$

4. (c) $\int (e^{\log x} + \sin x)\cdot \cos x \cdot dx = \int (x+\sin x)\cdot \cos x\cdot dx$

$$= \int (x\cdot \cos x + \sin x\cdot \cos x)\cdot dx$$

$$= \int x\cdot \cos x\cdot dx + \int \sin x\cdot \cos x\cdot dx$$

$$= x\cdot \int \cos x\cdot dx - \int\left(\frac{dx}{dx}\cdot \int (\cos x)\cdot dx\right)\cdot dx$$

$$+ \int \sin x\cdot \cos x\cdot dx$$

$$= x\cdot \sin x - \int \sin x\cdot dx + \int \sin x\cdot \cos x\cdot dx$$

$$= x\cdot \sin x - (-\cos x) + \int \sin x\cdot \cos x\cdot dx$$

$$= x\sin x + \cos x + \int \sin x\cdot \cos x\cdot dx$$

Now, $\int \sin x\cdot \cos x\cdot dx;$

Let $\sin x = z,\ dz = \cos x\cdot dx$

$$\int \sin x\cdot \cos x\, dx = \int z\cdot dz = \frac{z^2}{2} = \frac{\sin^2 x}{2}$$

$$\therefore I = x\cdot \sin x + \cos x + \frac{\sin^2 x}{2} + C.$$

5. (a) $\int \frac{dx}{x(x^n+1)} = \int \frac{x^{n-1}}{x^n(x^n+1)}\cdot dx$

Let $z = x^n \Rightarrow dz = nx^{n-1}dx$

$$\therefore \int \frac{dx}{x(x^n+1)} = \int \frac{x^{n-1}}{x^n(x^n+1)}\cdot dx = \int \frac{1}{z(z+1)}\cdot \frac{dz}{n}$$

$$= \frac{1}{n}\left[\int \frac{1}{z}\cdot dz - \int \frac{dz}{z+1}\right]$$

$$= \frac{1}{n}\left[\ln z - \ln(z+1)\right] + C \quad \text{where } C = \text{constant}$$

$$= \frac{1}{n}\cdot \ln\left(\frac{z}{z+1}\right) + C = \frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right) + C$$

6. (c) $I = \int \left(\frac{3x^2+8-4k}{x}\right)\cdot dx$

$$= \int 3x\ dx + \int \frac{8-4k}{x}\cdot dx$$

$$= \frac{3}{2}x^2 + (8-4k)\cdot \ln(x) + C \quad \text{where } C = \text{constant}$$

To get integration as rational function,

$$(8-4k)\cdot \ln(x) = 0 \Rightarrow 8-4k = 0 \Rightarrow k = \frac{8}{4} = 2$$

7. (d) $I = \int \frac{dx}{\sec x + \tan x} = \int \frac{(\sec x - \tan x)\cdot dx}{(\sec^2 x - \tan^2 x)}$

$$= \int \sec x\cdot dx - \int \tan x\cdot dx$$

$$= l\,\text{n}\left|(\sec x + \tan x)\right| - (-l\,\text{n}\,|\cos x|) + C$$

$$= l\,\text{n}\left|\sec x + \tan x\right| - l\,\text{n}\,|\cos x|^{-1} + C$$

$$= i\,\text{n}\left|\sec x + \tan x\right| - l\,\text{n}\,|\sec x| + C$$

8. (b) $\int \frac{dx}{\sec^2(\tan^{-1} x)}$

Let $\tan^{-1} x = y \Rightarrow \tan y = x$ and

$$\sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$\therefore \int \frac{dx}{\sec^2(\tan^{-1} x)} = \int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

9. (d) $\text{I} = \int e^{(2l\,\text{n}\,x + l\,\text{n}\,x^2)}\cdot dx$

$$= \int e^{(l\,\text{n}\,x + l\,\text{n}\,x^2)}\cdot dx = \int e^{(2\cdot\ln x^2)}\cdot dx$$

$$= \int e^{(\ln x^4)}\cdot dx = \int x^4\cdot dx = \frac{x^5}{5} + c$$

Definite Integration & Its Application

1. (c) $I_1 = \int_0^{\pi} \frac{x dx}{1+\sin x} = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)} dx$

$$\Rightarrow I_1 = \int_0^{\pi} \frac{\pi - x}{1+\sin x}.dx$$

$$\Rightarrow 2I_1 = \int_0^{\pi} \frac{\pi}{1+\sin x}.dx$$

$$\Rightarrow I_1 = \frac{\pi}{2}\int_0^{\pi}\frac{1}{1+\cos(\frac{\pi}{2}-x)}.dx$$

$$\Rightarrow I_1 = \frac{\pi}{2}\int_0^{\pi}\frac{1}{1+2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)-1}.\,dx$$

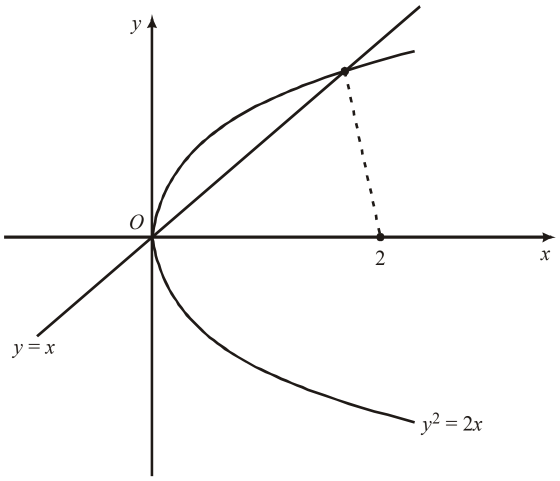
$$\Rightarrow I_1 = \frac{\pi}{2}\int_0^{\pi}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right).dx$$

$$\Rightarrow I_1 = \frac{\pi}{2}\left[\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-1}\right]_0^{\pi}$$

$$\Rightarrow I_1 = \frac{-\pi}{2}(-1-1) = \pi.$$

2. (a) $\because I_1 = I_2$
$\therefore I_1 + I_2 = \pi + \pi = 2\pi$

3. (a) $y^2 = 2x, y = x$



$\because\ y^2 = 2x$ & $y = x$
$\Rightarrow\ x^2 = 2x$
$\Rightarrow\ x = 0, 2$

$$\therefore \text{ Area} = \int_0^2(\sqrt{2x}-x).dx$$

$$= \left[\frac{\sqrt{2}.\sqrt{x^3}}{-\frac{3}{2}} - \frac{x^2}{2}\right]_0^2$$

$$= \left(\frac{4}{3/2}-\frac{4}{2}\right) - 0$$

$$= \frac{8}{3} - 2 = \frac{2}{3} \text{ sq. unit}$$

4. (d) $l + b = K$, where l = length, b = breadth.
Area of the rectangle $A = l \cdot b$.

$A = l(K-l) = lK - l^2$

For maximum area, $\frac{dA}{dl} = 0$

$\frac{d}{dl}(lk - l^2) = 0$

$K - 2l = 0 \Rightarrow l = \frac{K}{2}$

Now, $\frac{K}{2} + b = K \Rightarrow b = K - \frac{K}{2} = \frac{K}{2}$

Area, $A = l \cdot b = \frac{K}{2}\cdot\frac{K}{2} = \frac{K^2}{4}$

5. (b) $I = \int_0^{\frac{\pi}{4}}(\tan^3 x + \tan x)\cdot dx$

$$= \int_0^{\frac{\pi}{4}}\tan^3 x\cdot dx + \int_0^{\frac{\pi}{4}}\tan x\cdot dx$$

$$= \int_0^{\frac{\pi}{4}}\tan x\cdot\tan^2 x\cdot dx + \int_0^{\frac{\pi}{4}}\tan x\cdot dx$$

$$= \int_0^{\frac{\pi}{4}}\tan x\cdot(\sec^2 x - 1)\cdot dx + \int_0^{\frac{\pi}{4}}\tan x\cdot dx$$

$$= \int_0^{\frac{\pi}{4}}\tan x\cdot\sec^2 x\cdot dx - \int_0^{\frac{\pi}{4}}\tan x\cdot dx + \int_0^{\frac{\pi}{4}}\tan x\cdot dx$$

$$= \int_0^{\frac{\pi}{4}}\tan x\cdot\sec^2 x\cdot dx$$

Let $\tan x = z$, $dz = \sec^2 x\cdot dx$

$\therefore \int\tan x\cdot\sec^2 x\cdot dx = \int z\cdot dz = \frac{z^2}{2} + C$

where C = constant

$$\therefore I = \left[\frac{\tan^2 x}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2} - 0 = \frac{1}{2}.$$

6. (c) From question, $dx = 10.1 - 10 = 0.1$
At, $x = 10$

$y = (3)(10)^2 + 2 = 302.$

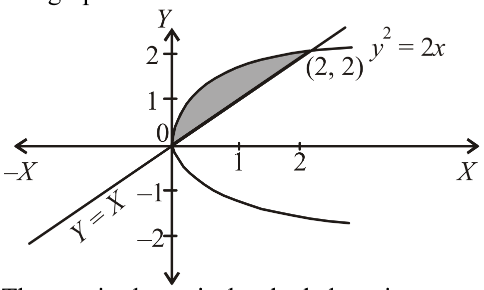
At $x = 10.1$, $y = 3(10.1)^2 + 2 = 308.03$.
Total change in $y = 308.03 - 302 = 6.03$.

7. (c) Two given graphs are $y^2 = 2x$ and $y = x$.
Point of intersections are :

$x^2 = 2x \Rightarrow x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ and 2 & $y = 0$ and 2.
So, points of intersections are (0, 0) and (2, 2).
The graph are



The required area is the shaded portion

$$A = \left|\int_0^2 (y_1 - y_2)\cdot dx\right|$$

$$= \left|\int_0^2 (\sqrt{2x} - x)\cdot dx\right| = \left[\frac{2\sqrt{2}(x)^{3/2}}{3} - \frac{x^2}{2}\right]_0^2$$

$$= \frac{2\sqrt{2}}{3}(2)^{\frac{3}{2}} - 2 = \frac{8}{3} - 2 = \frac{2}{3}$$

8. (d) $I = \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\cdot dx$

$$= \int_0^a \frac{f(a+0-a+x)}{f(a+0-x)+f(a+0-a+x)}\cdot dx$$

$$= \int_0^a \frac{f(x)}{f(a-x)+f(x)}\cdot dx$$

$$2I = \int_0^a \frac{f(a-x)+f(x)}{f(a-x)+f(x)}\cdot dx = [x]_0^a$$

$$\therefore I = \frac{a}{2}$$

9. (a) $\int_0^a [f(x)+f(-x)]\cdot dx = \int_{-a}^a g(x)\cdot dx$

$$= \int_{-a}^0 g(x)\cdot dx + \int_0^a g(x)\cdot dx$$

$$= \int_0^a g(-x)\cdot dx + \int_0^a g(x)\cdot dx$$

$$= \int_0^a [g(x)+g(-x)]\cdot dx$$

$\therefore f(x) = g(x)$.

10. (b) $y = \sqrt{16 - x^2}$

As, $y \ge 0,\ 16 - x^2 \ge 0$

$\therefore x \in [-4, 4]$

Now, from $I = \int \sqrt{a^2 - x^2}\cdot dx$

$$= \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

Here $a^2 = 16$
$\therefore$ Area bounded by the curve

$$I = \int_{-4}^4 \sqrt{(4)^2 - x^2}\cdot dx$$

$$= \left[\frac{x\sqrt{16-x^2}}{2}\right]_{-4}^4 + \frac{16}{2}\left[\sin^{-1}\left(\frac{x}{4}\right)\right]_{-4}^4$$

$$= 0 + 8\cdot\left[\sin^{-1}\left(\frac{4}{4}\right) - \sin^{-1}\left(-\frac{4}{4}\right)\right]$$

$$= 8\left[\sin^{-1}(1) + \sin^{-1}(1)\right]$$

$$= 16\cdot\sin^{-1}(1) = 16\cdot\frac{\pi}{2} = 8\pi \text{ square units}$$

Differential Equation

1. (a) Degree is 1.

2. (d) $y = \frac{1}{2x^2 - c}$

$$\Rightarrow \frac{dy}{dx} = \frac{0-(4x)}{(2x^2-c)^2}$$

$$= -4x \times \frac{1}{(2x^2-c)^2}$$

$= -4x \cdot y^2$

$$\Rightarrow \frac{dy}{dx} = -4xy^2$$

3. (a) $f(x) = x^2 + 2x - 5$
$g(x) = 5x + 30$
$g[f(x)] = 0$
$\Rightarrow 5f(x) + 30 = 0$
$\Rightarrow f(x) + 6 = 0$
$\Rightarrow x^2 + 2x - 5 + 6 = 0$
$\Rightarrow x^2 + 2x + 1 = 0$
$\Rightarrow x = -1, -1$

4. (d) (i) $f(g(x)) = (5x+30)^2 + 2(5x+30) - 5$
$\Rightarrow f(g(x))$ is second degree.
(ii) $g(g(x)) = 5(5x+30) + 30$

$\Rightarrow g(g(x))$ is first degree

$\therefore$ both statements are wrong.

5. (b) $h(x) = 5f(x) - x\,g(x)$

$= 5[x^2 + 2x - 5] - x[5x + 30]$

$= -20x - 25$

$\therefore \quad h'(x) = -20$

6. (b) $\tan y = c(1 - e^x)$

$\Rightarrow \sec^2 y \dfrac{dy}{dx} = c(-e^x)$

$\Rightarrow c = \dfrac{1}{e^x} \sec^2 y \dfrac{dy}{dx}$

$\therefore \tan y = \dfrac{-1}{e^x} \sec^2 y . \dfrac{dy}{dx} (1 - e^x)$

$\Rightarrow e^x . \tan y = \sec^2 y(1 - e^x) \dfrac{dy}{dx}$

$\Rightarrow e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

7. (b) $x(dx - dy) + y(dy - dx) = 0$

On integrating both sides, we have

$\int x(dx - dy) + \int y(dy - dx) = C$ (where C = constant)

$\dfrac{x^2}{2} - xy + \dfrac{y^2}{2} - xy = C$

$x^2 + y^2 - 2xy = C$

Or, $x^2 + y^2 = 2xy + C$.

8. (a) $\ln\left(\dfrac{dy}{dx}\right) = x$

$\dfrac{dy}{dx} = e^x$

Integrating both sides, we get

$\int dy = \int e^x .dx$

$y = e^x + C$, where C = integration constant.

9. (b) $dy = (1 + y^2) \cdot dx$

$\dfrac{1}{(1 + y^2)} \cdot dy = dx$

Integrating both sides, we get

$\int \dfrac{dy}{(1 + y^2)} = \int dx$

$\tan^{-1}(y) = x + C$, where C = constant.

$\therefore y = \tan(x + C)$.

10. (b) Given differential equation is

$k \cdot \dfrac{dy}{dx} = \int \left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{2/3} \cdot dx$

On differentiating both sides, we get

$k \cdot \dfrac{d^2y}{dx^2} = \left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{2/3}$

$\Rightarrow \left(k \cdot \dfrac{d^2y}{dx^2}\right)^3 = \left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^2$

Now, order of the D.E. = 2

Degree of the D.E. = 3.

11. (d) $f(x) = e^{|x|}$

$f'(x) = e^x,\ x > 0$

$= -e^{|x|},\ x < 0$

$\lim_{x \to 0^+} f'(x) \neq \lim_{x \to 0^-} f'(x)$

$\therefore f'(x)$ does not exist at $x = 0$.

12. (b) $y^2 + 2cy - cx + c^2 = 0$

$2y . \dfrac{dy}{dx} + 2c . \dfrac{dy}{dx} - c = 0$

$2\left(\dfrac{dy}{dx}\right)^2 + 2y. \dfrac{d^2y}{dx^2} + 2c. \dfrac{d^2y}{dx^2} = 0$

$(y + c). \dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx}$

13. (a) $x = \sqrt{1 + \dfrac{dy^2}{dx^2}} \Rightarrow x^2 = 1 + \dfrac{d^2y}{dx^2}$

$\therefore$ degree of the D.E. is 1

14. (b) $y = a.e^{x} + b.e^{-x}$

$\dfrac{dy}{dx} = a.e^{x} - b.e^{-x}$

$\dfrac{d^2y}{dx^2} = a.e^x + b.e^{-x} \Rightarrow \dfrac{d^2y}{dx^2} = y$

$\therefore \dfrac{d^2y}{dx^2} - y = 0$

15. (c) $\ln\left(\dfrac{dy}{dx}\right) = x - y$

$e^{x - y} = \dfrac{dy}{dx}$

$e^x . dx = e^y . dy$

On integrating both sides, we get,

$\int e^x \cdot dx = \int e^y \cdot dy$

$e^x = e^y + c$

$e^x - e^y = c$

Matrices & Determinants

1. (c) $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$

Taking 2! common from C_2 and 3! common from C_3

$$\Rightarrow\ 2!\cdot 3!\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 4\\ 6 & 12 & 20\end{vmatrix}$$

On expanding, we get

$12\times[1(60-48)-1(40-24)+1(24-18)]$

$=12\times[12-16+6]$

$=12\times 2=24$

2. (a) On expanding both the determinants we get

$[n(1)+2(2x-2)]+[3x+2x^2]=0$

$= \quad 2x^2+8x-4=0$

$\Rightarrow\ x^2+4x-2=0$

$\Rightarrow\ x=-2\pm\sqrt{6}$

3. (a) Given: $x+a+b+c=0$

$$\Delta=\begin{vmatrix}x+a & b & c\\ a & x+b & c\\ a & b & x+c\end{vmatrix}$$

Replace c, by $\rightarrow c_1+c_2+c_3$

$$\Rightarrow\ \begin{vmatrix}x+a+b+c & b & c\\ x+a+b+c & x+b & c\\ x+a+b+c & b & x+c\end{vmatrix}$$

$\because\ x+a+b+c=0$

$\therefore\ \Delta=0$

4. (a) $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$$\therefore\ A^2=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}=2A$$

$\therefore\ A^3-2A^2=A\cdot A^2-2A^2$

$=A(2A)-2(2A)$

$=2A^2-4A$

$=2(2A)-4A$

$=4A-4A$

$=0$

5. (c) $A=\begin{bmatrix}1 & 2\\ 2 & 3\\ 3 & 4\end{bmatrix};\ B=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$

Order of $A=3\times 2$

Order of $B=2\times 2$

$\therefore$ by fundamental properties we can say

AB exists while BA does not exist.

6. (a) $D=\begin{vmatrix}p & q & r\\ q & r & p\\ r & p & q\end{vmatrix}$

$=p(rq-p^2)+q(pr-q^2)+r(pq-r^2)$

$=3pqr-p^3-q^3-r^3$

$=-(p^3+q^3+r^3-3pqr).$

For real and distinct positive real value of p, q and r.

$p^3+q^3+r^3-3pqr>0$

$\therefore D<0.$

7. (a)

(1) We know that,

$A\cdot(\text{adj}\,A)=(\text{adj}\,A)\cdot A=|A|\cdot I$

Hence, statement (1) is correct.

(2) $|\text{adj}\,(A)|=|A^{-1}\cdot|A||$

Hence, statement (2) is not correct.

8. (c) $A=[i\times j]_{3\times 5},\ B=[i\times j]_{5\times 3}$

Now, $AB=[i\times j]_{3\times 3}$ and $BA=[j\times i]_{5\times 5}$

9. (c) A square M matrix is said to be Hermitian (or self-adjoint) if it is equal to its. Own Hermitian conjugate, i.e.

$(\bar{M})^T=M$

Given Matrix $A=\begin{bmatrix}1-i & i\\ -i & 1-i\end{bmatrix}$

$(\bar{A})^T=\begin{bmatrix}1+i & i\\ -i & 1+i\end{bmatrix}$

Now, $A+(\bar{A})^T=\begin{bmatrix}1-i & i\\ -i & 1-i\end{bmatrix}+\begin{bmatrix}1+i & i\\ -i & 1+i\end{bmatrix}$

$=\begin{bmatrix}2 & 2i\\ -2i & 2\end{bmatrix}=2\begin{bmatrix}1 & i\\ -i & 1\end{bmatrix}$

Conjugate transpose of $(A+(\bar{A})^T)=2\begin{bmatrix}1 & i\\ -i & 1\end{bmatrix}$

Hence, $(A+(\bar{A})^T)$ is hermitian.

10. (d) For singular matrix,

$$\begin{bmatrix}0 & K & 4\\ -K & 0 & -5\\ -K & K & -1\end{bmatrix}=0$$

$K(5K-K)+4(-K^2)=0$

$4K^2-4K^2=0$

Hence, for all values of K, the given matrix is singular matrix.

11. (b) $AB=\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix}$

$=\begin{bmatrix}2(x+y)-y\\ 4x-x+y\end{bmatrix}=\begin{bmatrix}2x+y\\ 3x+y\end{bmatrix}$

As $AB=C$

$\therefore\begin{bmatrix}2x+y\\ 3x+y\end{bmatrix}=\begin{bmatrix}3\\ 2\end{bmatrix}$

$2x + y = 3$ ...(i)

and $3x + y = 2$ ...(ii)

From equation (i) and (ii), we get
$x = -1, y = 5$.

$\therefore A = \begin{bmatrix} -1+5 & 5 \\ -2 & -1-5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

$= 4(-6) - 5(-2) = -14.$

12. (d) $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix} = \begin{vmatrix} i & -1 & -i \\ 1 & -1 & 1 \\ i & 1 & -i \end{vmatrix}$

$= i(i-1) - 1(i-(-i)) - i(1+i)$

$= i^2 - i - 2i - i - i^2 = -4i.$

13. (c) $AB = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$

14. (d) Here, $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Now,

$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix} = p \cdot q \cdot \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = p \cdot q \cdot \Delta$

15. (b) $(a + b + c) = 4$ (Given)
Now,

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc - a^2) + b(ac - b^2) + c(ab - c^2)$

$= 3abc - a^3 - b^3 - c^3$

$= -(a^3 + b^3 + c^3 - 3abc) = -(a+b+c)^3$

$= -(4)^3 = -64.$

16. (a) As $a_1, a_2, a_3 ....a_9$ are in G.P.

$\therefore \frac{a_4}{a_1} = \frac{a_5}{a_2} = \frac{a_6}{a_3} ..... = r^3$ (where r = common ratio)

Now, $\begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$

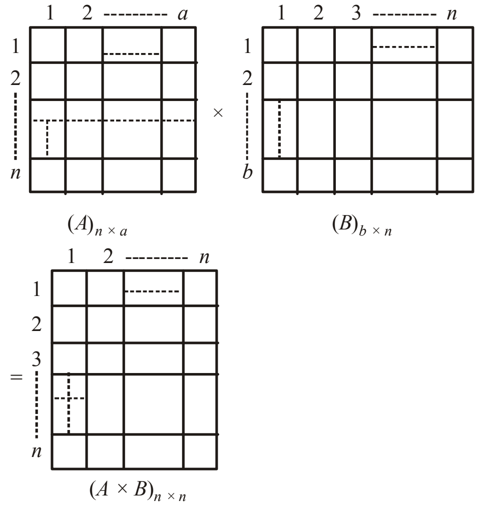
$= \begin{vmatrix} \ln a_4 - \ln a_1 & \ln a_5 - \ln a_2 & \ln a_6 - \ln a_3 \\ \ln a_7 - \ln a_4 & \ln a_8 - \ln a_5 & \ln a_9 - \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$

(Applying $R_1 \to R_2 - R_1$ and $R_2 \to R_3 - R_2$)

$= \begin{vmatrix} \ln \frac{a_4}{a_1} & \ln \frac{a_5}{a_2} & \ln \frac{a_6}{a_3} \\ \ln \frac{a_7}{a_4} & \ln \frac{a_8}{a_5} & \ln \frac{a_9}{a_6} \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$

$= \begin{vmatrix} \ln r^3 & \ln r^3 & \ln r^3 \\ \ln r^3 & \ln r^3 & \ln r^3 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix} = 0$

17. (d) Let A is matrix of order $n \times a$
and B is matrix of order $b \times n$.
Now,



Hence, order of A and B need not to be same.

18. (c) Prime number less than 30 are
2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.
Number of prime number = 10.
Different order of matrices :
1 × 10, 10 × 1, 2 × 5 and 5 × 2.

19. (c) $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$, where (p, q, r, s) prime number < 20

$\therefore (p, q, r, s) = (2, 3, 5, 7, 11, 13, 17 \text{ and } 19)$

Now, $A = p \cdot s - q \cdot r$.

For A_{max}, product $(p.s)$ should be max. and product $(q.r)$ should be min.

$\therefore A_{max.} = p \cdot s - q \cdot r = 19 \times 17 - 2 \times 3 = 317.$

20. (d) Let $A = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}_{2\times2}$ and $B = \begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix}_{2\times2}$

$|A| = 4$ and $|B| = 4$.

$\det(A \cdot B) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 8 \end{vmatrix} = 16.$

$$\det(B\cdot A)=\begin{vmatrix}1&0\\0&4\end{vmatrix}\cdot\begin{vmatrix}2&0\\0&2\end{vmatrix}=\begin{vmatrix}2&0\\0&8\end{vmatrix}=16.$$

Here $\det(A\cdot B)=\det(B\cdot A)$, but A and B are not unit matrices.

21. (c) $\begin{vmatrix}x&1&3\\0&0&1\\1&x&4\end{vmatrix}=0$

$\Rightarrow x(0-x)+1(1-0)+3(0-0)=0$

$\Rightarrow -x^2+1=0$

$\therefore x=\pm 1.$

22. (a) $f(x)=\begin{vmatrix}1&x&x+1\\2x&x(x-1)&x(x+1)\\3x(x-1)&2(x-1)(x-2)&x(x+1)(x-1)\end{vmatrix}$

For $x=-1$, $C_3\to\begin{bmatrix}-1+1\\-1(-1+1)\\-1(-1+1)(-1-1)\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

For $x=1$,

$R_3\to[3(1-1)\quad 2(1-1)(1-2)\quad (1+1)(1-1)]=[0\quad 0\quad 0]$

$\therefore f(-1)+f(0)+f(1)=0.$

23. (a) Determinant

$$=\begin{vmatrix}4&6&8\\6&8&10\\8&10&12\end{vmatrix}=2\begin{vmatrix}2&3&4\\3&4&5\\4&5&6\end{vmatrix}$$

$=2[2(24-25)+3(20-18)+4(15-16)]=0.$

24. (d) All possible determinants from 2, 4, 6 and 8

$$\begin{vmatrix}8&6\\4&2\end{vmatrix}+\begin{vmatrix}8&4\\2&6\end{vmatrix}+\begin{vmatrix}2&4\\8&6\end{vmatrix}+\begin{vmatrix}4&2\\6&8\end{vmatrix}+\begin{vmatrix}4&8\\6&2\end{vmatrix}+\begin{vmatrix}6&8\\2&4\end{vmatrix}$$

$=(16-24)+(48-8)+(12-32)+(32-12)$
$+(8-48)+(24-16)\quad=0.$

Probability and Probability Distribution

1. (d) If $P(\text{Tails})=x$

$\therefore\ P(\text{Head})=3x$

$\because\ P(T)+P(H)=1\Rightarrow x=\frac{1}{4}$

Probability of at most 2 tails

$=1-P(\text{all tail})=1-\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{63}{64}\ =.98$

2. (a) Probability that all are defective

$=\frac{5}{20}\times\frac{4}{19}\times\frac{3}{18}=\frac{1}{114}$

$\approx .009$

3. (d) It can be observed that sample space will be {H, TH, TTH, TTTH, ...}

4. (a) Sum as prime number on 2 dices are: 2, 3, 5, 7, 11

no. of times each prime no. as sum occurs =

No.	Times
2	1
3	2
5	4
7	6
11	2
	15

$\therefore\ P(E)=\frac{15}{36}=\frac{5}{12}$

5. (c) P(None meets emission standard)

$=\frac{5}{10}\times\frac{4}{9}\times\frac{3}{8}=\frac{1}{12}$

6. (c) $P(T)=\frac{1}{3}$; $P\left(\frac{H}{T}\right)=1$ $P(F)=\frac{1}{3}, P\left(\frac{H}{F}\right)=\frac{1}{2}$

$P(B)=\frac{1}{3}, P\left(\frac{H}{B}\right)=\frac{3}{4}$

$$P\left(\frac{T}{H}\right)=\frac{P(T).P\left(\frac{H}{T}\right)}{P(T).P\left(\frac{H}{T}\right)+P(F).P\left(\frac{H}{F}\right)+P(B).P\left(\frac{H}{B}\right)}$$

$$\Rightarrow P\left(\frac{T}{H}\right)=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}+\frac{1}{4}}=\frac{\frac{1}{9}}{\frac{9}{4}}=\frac{4}{9}$$

7. (b) (i) Since A & B are mutually exclusive.

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=.6+.6-0$

$\Rightarrow P(A\cup B)=1.2$

$\therefore$ statement 1 is wrong.

(ii) $\because\ P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow 1=\frac{P(A\cap B)}{P(B)}\Rightarrow P(B)=P(A\cap B)\Rightarrow B\subseteq A$

now,

$P\left(\frac{\bar{B}}{\bar{A}}\right)=\frac{P(\bar{B}\cap\bar{A})}{P(\bar{A})}=\frac{1-P(A\cup B)}{1-P(A)}$

$(\because A\cup B=A)=\frac{1-P(A)}{1-P(A)}=1$

$\therefore$ statement 2 is correct.

8. (b) Given,

$n=4, p=\frac{1}{6}, q=\frac{5}{6}$ $P(x=2)={}^4C_2\,.\,p^2\,.\,q^2$

$=6.\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2=\frac{25}{216}$

9. (b) Number of days in February, when year is a leap year $= 29$ days $= 4$ weeks $+ 1$ odd day
4 weeks have 4 sundays.

Probability that 1 odd day is sunday $= \frac{1}{7}$.

10. (a) Probability for Husband's selection $P(H) = \frac{1}{7}$

Probability when Husband is not selection

$P(H') = 1 - \frac{1}{7} = \frac{6}{7}$

Probability for wife's selection $P(W) = \frac{1}{5}$

Probability when Wife is not selected $P(W') = 1 - \frac{1}{5} = \frac{4}{5}$

As both are independent event.
So, probability for atleast one of them will be selected

$= P(H)P(W') + P(H')P(W)$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{11}{35}$.

11. (a) Number of ways of picking 4 counterfeits gold coins out of 6 counterfeits coins $= {}^6C_4$
Number of ways of picking 4 coins out of 15 gold coins $= {}^{15}C_4$

$\therefore$ Required probability $= \frac{{}^6C_4}{{}^{15}C_4} = \frac{15}{15 \times 7 \times 13} = \frac{1}{91}$.

12. (d) Number of ways of selecting 2 boys out of 2 boys $= {}^2C_2 = 1$.
Number of ways of selecting 1 girl out of 2 girls $= {}^2C_1 = 2$.
Number of ways of selecting 3 out of 4 persons $= {}^4C_3 = 4$.

$\therefore$ Required probability $= \frac{{}^2C_1 \times {}^2C_2}{{}^4C_3} = \frac{2 \times 1}{4} = \frac{1}{2}$.

13. (c) Prime number between 1 to 10 are 2, 3, 5, 7.
Now, number of ways of selecting 2 prime number out of 4 prime number $= {}^4C_2 = 6$.
Number of ways of selecting 2 numbers out of 10 numbers $= {}^{10}C_2 = 45$.

$\therefore$ Required probability $= \frac{6}{45} = \frac{2}{15}$.

14. (d) From question $P = \frac{1}{5}$

Now, $P + q = 1$

$\Rightarrow \frac{1}{5} + q = 1 \Rightarrow q = 1 - \frac{1}{5} = \frac{4}{5}$.

and $n = 10$.

15. (d) $P(\bar{A}) = 1 - P(A) = 1 - 0.6 = 0.4$.

$P(\bar{B}) = 1 - P(B) = 1 - 0.5 = 0.5$.

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.5 - 0.4 = 0.7$.

(1) Now $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$

$= P(\bar{A}) + P(B) - \{P(B) - P(A \cap B)\}$

$= 0.4 + 0.5 - \{0.5 - 0.4\} = 0.8$.

Hence, statement (1) is not correct.

(2) $P(\bar{B} \mid \bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = \frac{P(B \cup A)'}{P(\bar{A})} = \frac{1 - P(A \cup B)}{P(\bar{A})}$

$= \frac{1 - 0.7}{0.4} = 0.75$.

Hence, statement (2) is not correct.

16. (b) From question, we have

$P(F \mid X) = 0.02,\ P(F \mid Y) = 0.03,\ P(F \mid Z) = 0.05$

and

$P(F) = P(X) \cdot P(F \mid X) + P(Y) \cdot P(F \mid Y)$

$+ P(Z) \cdot P(F \mid Z)$

$= 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.05$

$= 0.01 + 0.009 + 0.01 = 0.029$.

Now,

$P(X \cap F) = P(X) \cdot P(F \mid X) = 0.5 \times 0.02 = 0.01$.

$\therefore P(X \mid F) = \frac{P(X \cap F)}{P(F)} = \frac{0.01}{0.029} = \frac{10}{29}$

17. (c) Number of ways in which one of the face having the number 6 and no two dice show the same number.
(1, 2, 6), (1, 3, 6), (1, 4, 6), (1, 5, 6), (2, 3, 6), (2, 4, 6), (2, 5, 6), (3, 4, 6), (3, 5, 6)......
Total favourable case $= 20 + 20 + 20 = 60$.
Number of total output when three top faces of three dice shows different number $= 6 \times 5 \times 4 = 120$.

$\therefore$ Required probability $= \frac{60}{120} = \frac{1}{2}$.

18. (c) $P(A \cup B) = \frac{5}{6},\ P(A \cap B) = \frac{1}{3},\ P(A') = \frac{1}{2}$

$P(A) = 1 - P(A') = 1 - \frac{1}{2} = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$

(a) $P(B) = \frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{5 - 3 + 2}{6} = \frac{4}{6} = \frac{2}{3}$

(b) $P(A \cap B) = \frac{1}{3} = \frac{1}{2} \cdot \frac{2}{3} = P(A) \cdot P(B)$

$\therefore P(A \cap B) = P(A) \cdot P(B)$.

(c) $P(A \cup B) = \frac{5}{6}$

$P(A) + P(B) = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

$\therefore P(A \cup B) < P(A) + P(B)$

Hence, option (c) is not correct.

(d) $P(A' \cap B') = P(A') \cdot P(B')$

$1 - P(A \cup B) = 1 - \frac{5}{6} = \frac{1}{6}$

and $P(A') \cdot P(B') = (1 - P(A)) \cdot (1 - P(B))$

$= \left(1-\frac{1}{2}\right)\cdot\left(1-\frac{2}{3}\right) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}.$

Hence, $P(A' \cap B') = P(A') \cdot P(B')$.

19. (c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\therefore$ min. ($P(A \cap B) = L + M - 1$ {Here $P(A \cup B)$ $= 1$ (max. value)

$\therefore P(A|B) \geq \dfrac{L+M-1}{M}$

20. (a) $P(\bar{A}) = \frac{1}{2}$ $\therefore P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$

and $P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= \frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{2}{3}$

Now, $P(A \cap B) = P(A) . P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

$\therefore$ A and B are independent events.

21. (c) $P(E) = \frac{1}{2}, P(F) = \frac{1}{2}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

As E & F are two independent event

$\therefore P(E \cap F) = P(E) . P(F)$

$P(E \cup F) = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} . \frac{1}{2} = \frac{3}{4}$

22. (d) Mean $(xp) = \frac{2}{3}$ Variance $(npq) = \frac{5}{9}$

$\therefore \dfrac{\text{Variance}(npq)}{\text{mean}(np)} = q = \dfrac{\frac{5}{9}}{\frac{2}{3}} = \frac{5}{6}$

$p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$

Here number of trial $n \times \frac{1}{6} = \frac{2}{3}$ $n = 4$

Random Variable $X = 2$

$\therefore$ Probability $= {}^4C_2 (P)^{4-2} Q^2 = {}^4C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2$

$= \dfrac{25}{216}$

23. (c) $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B)$ is min when $P(A \cap B)$ is max i.e. $\frac{5}{8}$

$P(A \cup B)_{\min} = \frac{3}{4} + \frac{5}{8} - \frac{5}{8} = \frac{3}{4}$

Hence, both (1) and (2) are correct.

Vectors

1. (b) $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,

$\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

$\because$ projection of $\vec{a}$ on $\vec{b}$ is $= \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + \hat{k}).(4\hat{i} - 4\hat{j} + 7\hat{k})$

$= 4 + 8 + 7 = 19$

$|\vec{b}| = \sqrt{4^2 + 4^2 + 7^2} = \sqrt{81} = 9$

$\therefore$ projection $= \frac{19}{9}$

2. (b) Given: $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow a^2 + b^2 + 2ab \cos\theta = a^2 + b^2 - 2ab\cos\theta$

$\Rightarrow 4ab\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90°$

$\therefore$ Vectors are perpendicular.

3. (a) (i) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b}$

$= |\vec{a}|^2 - |\vec{b}|^2$

(ii) $(|\vec{a} + \vec{b}|) \cdot (|\vec{a} - \vec{b}|)$

$= \sqrt{a^2 + b^2 + 2ab} \cdot \sqrt{a^2 + b^2 - 2ab}$

$= \sqrt{(a^2 + b^2)^2 - 4a^2b^2} = \sqrt{(a^2 - b^2)^2} = a^2 - b^2$

$= |\vec{a}|^2 - |\vec{b}|^2$

(iii) $|\vec{a} \cdot \vec{b}|^2 + |\vec{a} \times \vec{b}|^2$

$= a^2b^2 \cos^2\theta + a^2b^2 \sin^2\theta$

$= a^2b^2(\cos^2\theta + \sin^2\theta)$

$= a^2b^2$

$= |\vec{a}|^2 |\vec{b}|^2$

$\therefore$ All 3 statements are correct.

4. (b) (i) Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$\therefore$ statement (i) is incorrect

(ii) $\vec{a} \times \vec{b} = 0$

$\Rightarrow |\vec{a}|.|\vec{b}|. \sin\theta = 0$

$\Rightarrow \sin\theta = 0$

$\therefore \vec{a} \parallel \vec{b} \Rightarrow \vec{a} = \lambda\vec{b}$

$\therefore$ Statement (ii) is correct.

5. (b) Given:

$|\vec{a}| = |\vec{b}| = 1$

$\because \quad |\vec{a}-\vec{b}|^2 = a^2 + b^2 - 2ab\cos\theta$

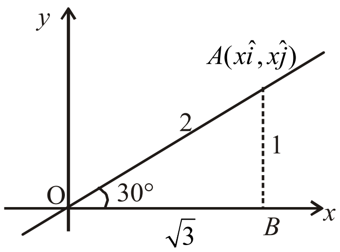
$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2 - 2\cos\theta$

$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2(1-\cos\theta)$

$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2\,.\,2\sin^2\left(\frac{\theta}{2}\right)$

$= 4\sin^2\frac{\theta}{2} \Rightarrow \sin^2\left(\frac{\theta}{2}\right) = \frac{|\vec{a}-\vec{b}|^2}{4}$

6. (a)



Let $A(x\hat{i},\ y\hat{j})$ is a point in the xy-plane.
From question,

$\angle AOB = 30°$ and $\overline{OA} = 1$

$x = |\overrightarrow{OA}|\cdot\cos 30° = 1\times\frac{\sqrt{3}}{2}\hat{i} = \frac{\sqrt{3}}{2}\hat{i}$

$y = |\overrightarrow{OA}|\cdot\sin 30° = 1\times\frac{1}{2}\hat{j} = \frac{\hat{j}}{2}$

$\therefore \vec{a} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j} = \frac{\sqrt{3}\hat{i}+1\hat{j}}{2}$

7. (c) Let $A = (x, y, z)$

Then, $|\overrightarrow{OA}| = \sqrt{x^2+y^2+z^2} = 12$

Also, $x = |\overrightarrow{OA}|\cdot\cos 45° = 12\cdot\frac{1}{\sqrt{2}} = 6\sqrt{2}\hat{i}.$

$y = |OA|\cdot\cos 60° = 12\cdot\frac{1}{2} = 6\hat{j}.$

Hence, $\overrightarrow{OA} = 6\sqrt{2}\hat{i} + 6\hat{j} \pm 6\hat{k}$

8. (c) Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$
Two diagonals of the parallelograms are given by

$(\vec{a}+\vec{b}) = (2+1)\hat{i} - (4+2)\hat{j} + (5-3)\hat{k}$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

and $(\vec{a}-\vec{b}) = (2-1)\hat{i} - (4-2)\hat{j} + (5-(-3))\hat{k}$

$= \hat{i} - 2\hat{j} + 8\hat{k}$

Dot products of the diagonals

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (3\hat{i}-6\hat{j}+2\hat{k})\cdot(\hat{i}-2\hat{j}+8\hat{k})$
$= (3 + 12 + 16) = 31$ units

9. (a) $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 144$

$\{|\vec{a}|\cdot|\vec{b}|\cdot\sin\theta\}^2 + \{|\vec{a}|\cdot|\vec{b}|\cdot\cos\theta\}^2 = 144$

$(|\vec{a}|\cdot|\vec{b}|)^2(\sin^2\theta + \cos^2\theta) = 144$

$|\vec{a}|\cdot|\vec{b}| = \sqrt{144} = 12$

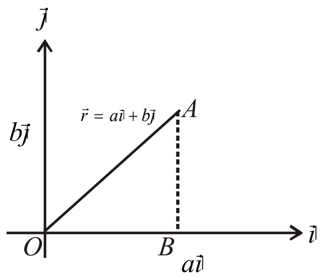
$4\cdot|\vec{b}| = 12 \Rightarrow |\vec{b}| = \frac{12}{4} = 3.$

10. (b) As the given vectors are coplanar, then

$$\vec{c}\times\vec{a}\times\vec{b} = 0 \Rightarrow \begin{vmatrix} 0 & 1 & p \\ 2 & -3 & 1 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$\Rightarrow 0 + 1(1+6) + p(4+3) = 0 \quad \Rightarrow 7 + 7p = 0 \Rightarrow p = -1.$

11. (c)



$|\vec{r}| = \sqrt{a^2+b^2} = 2$

$\Rightarrow a^2 + b^2 = 4$ ...(i)

As $\vec{r}$ is equally inclined to both axis,
$\therefore a = b.$
Again from equation (i),

$a^2 + a^2 = 4 \Rightarrow a = \sqrt{2} = b.$

12. (c) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

$= 0 \qquad (\because |\vec{a}| = |\vec{b}|)$

$\therefore$ Vector $\vec{c} = (\vec{a}+\vec{b})$ is perpendicular to $(\vec{a}-\vec{b})$.

Also, $\vec{c}\cdot(\vec{a}-\vec{b}) = \begin{bmatrix}\vec{c} & \vec{a} & \vec{b}\end{bmatrix}$

$\therefore \vec{c}$ is perpendicular to $(\vec{a}\times\vec{b})$.

13. (c) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$
On squaring both sides, we get

$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b} = 0 \qquad \therefore \quad \vec{a}\perp\vec{b}$

14. (a) $(2\vec{a}\times 3\vec{b})\cdot 4\vec{c} = (5\vec{b}\times 3\vec{c})\cdot 6\vec{a} = 0 + 0$

$\{\because \vec{a}, \vec{b}$ and $\vec{c}$ are coplaner $= 0\}$

15. (a) Let $\vec{a}$ and $\vec{b}$ are two unit vectors then cross product

$\vec{a}\times\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\sin\theta\cdot\hat{n}$
(i) This is always a unit vector.
(ii) Dot product $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$
This is always equal to one.

(iii) $|\vec{a}+\vec{b}| > |\vec{a}-\vec{b}|$ squaring on both sides

$|\vec{a}+\vec{b}| > |\vec{a}-\vec{b}|^2$

$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} > |\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b} > 0$

$|\vec{a}|\cdot|\vec{b}|\cdot\cos\theta > 0$

But if $\theta \in [90°, 180°]$, $\cos\theta < 0$

Hence, statement i and ii are correct, but Statement iii is not correct.

3D Geometry

1. (d) given point is

$(P+1, P-3, \sqrt{2}p)$

to determine direction ratios of a line we require 2 points.

$\therefore$ DR's can't be determined as infinite lines will pass through this point.

Thus direction cosines can not be obtained.

2. (b) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+2}{7}$

Verify from options

Only (2, 5, 5) satisfies the above equation.

3. (d) Line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$

passes through (4, 2, k) are lies in plane

$2x-4y+z=7$

so it must satisfy

$2(4)-4(2)+k=7 \Rightarrow k=7$

4. (b) given angle from z-axis = 60°

$\therefore \quad n=\cos 60° = \frac{1}{2}$

Let the cosines of angle made by line with y-axis and x-axis be $\sqrt{3}x$ and x

$\because \quad l^2+m^2+n^2=1$

$\Rightarrow \quad 3x^2+x^2+\frac{1}{4}=1 \quad x=\pm\frac{\sqrt{3}}{4} \quad \therefore \sqrt{3}x=\frac{3}{4}$

$x=\frac{\sqrt{3}}{4} \therefore (l, m, n)=\left(\frac{3}{4},\frac{\sqrt{3}}{4},\frac{1}{2}\right)$ or $\left(\frac{-3}{4},\frac{-\sqrt{3}}{4},\frac{1}{2}\right)$

for angle

$|l_1 l_2+m_1 m_2+n_1 n_2|$

$=\left|\frac{-9}{16}-\frac{3}{16}+\frac{1}{4}\right| = \left|\frac{-1}{2}\right| = \cos\theta$

$\Rightarrow$ Angle $=\theta=60°$

5. (a) Given points are

$(x, y, -3)$; $(2, 0, -1)$; $(4, 2, 3)$

since they lie on same line

$\therefore$ their DR's must be same

DR's $=(2-x, -y, 2)$

$(2, 2, 4)$

$\therefore \quad \frac{2-x}{2}=\frac{-y}{2}=\frac{2}{4}$

$\Rightarrow \quad x=1\ ;\ y=-1$

6. (d) Radius of the sphere

$=\frac{6(1)-3(-2)+2(3)-4}{\sqrt{(6)^2+(-3)^2+(2)^2}}=\frac{6+6+6-4}{7}=\frac{14}{7}=2$

Diameter of the sphere $=2\times 2=4$ units

7. (b) Perpendicular distance $=\sqrt{(4)^2+(3)^2}=5$ units.

8. (d) Direction ratios are <$a+b$, $b+c$, $c+a$>

Then, direction cosine,

$l=\frac{(a+b)}{\sqrt{(a+b)^2+(b+c)^2+(c+a)^2}}$

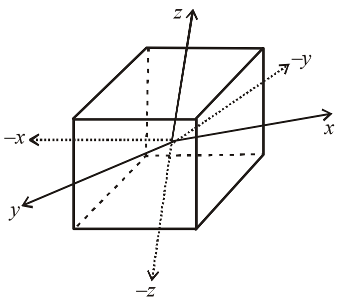
$m=\frac{(b+c)}{\sqrt{(a+b)^2+(b+c)^2+(c+a)^2}}$

$n=\frac{(c+a)}{\sqrt{(a+b)^2+(b+c)^2+(c+a)^2}}$

Sum of squares of direction cosines

$l^2+m^2+n^2=\frac{(a+b)^2+(b+c)^2+(c+a)^2}{(a+b)^2+(b+c)^2+(c+a)^2}=1.$

9. (c)



Co-ordinate plane divide the space into 8 octanes.

10. (b) Equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane, is

$z=5, y=0, x=0.$

11. (b) Angle between two lines

$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}.\sqrt{a_2^2+b_2^2+c_2^2}}$

Here, $(a_1, b_1, c_1)=(6, 3, 6)$

and $(a_2, b_2, c_2)=(,3\ 3, 0)$

$\therefore \cos\theta=\frac{6\times 3+3\times 3+0}{\sqrt{6^2+3^2+6^2}\cdot\sqrt{3^2+3^2+0}}$

$=\frac{27}{9\cdot 3\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\therefore \theta=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$

12. (b) Let $x-1=(y-3)=1-z=k$ (Say)

$\therefore l=k, m=\frac{k}{2}$ and $n=k$

From $l^2+m^2+n^2=1$

$k^2+\left(\frac{k}{2}\right)^2+k^2=1 \Rightarrow k=\frac{2}{3}$

$\therefore\ l = \frac{2}{3}, m = \frac{1}{3}$ and $n = \frac{2}{3}$

$\therefore\ l^4 + m^4 + n^4$

$= \left(\frac{2}{3}\right)^4 + \left(\frac{1}{3}\right)^4 + \left(\frac{2}{3}\right)^4 = \frac{11}{27}$

13. (c) Point $A = (1, 7, -5)$ and $B = (-3, 4, -2)$
Equation of plane $AB : al + bm + cn = 0$
$a = (1-(-3)) = 4$
$b = (7-4) = 3$
$c = (-5-(-2)) = -3$
$\therefore$ Projection on y-axis = 3.

14. (c) As three points are collinear.

So, $\Delta = \begin{vmatrix} k & 1 & 3 \\ 1 & -2 & k+1 \\ 15 & 2 & -4 \end{vmatrix} = 0$

$\Rightarrow k(8-2(k+1)) + 1(15(k+1)+4) + 3(2+30) = 0$

$\Rightarrow k(6-2k) + (15k+19) + 96 = 0$

$\Rightarrow 2k^2 - 21k - 105 = 0$

On solving, we will get two different values of k.

15. (c) Given equation of plane $x + y + z = 3$
Let point P is (0, 0, 0) and point Q is the foot of perpendicular drawn from point P on the plane.
Since, PQ is perpendicular to the plane, so direction ratio of line PQ < 1, 1, 1 >.
$\therefore$ Equation of line PQ,

$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1} = \lambda$

$\therefore (x, y, z) = (\lambda, \lambda, \lambda)$
As point Q lies on the given plane.
$\therefore \lambda + \lambda + \lambda = 3 \Rightarrow \lambda = 1$
Hence, Co-ordinate of point Q = (1, 1, 1).

Statistics

1. (b) $22, 24, 33, 37, x+1, x+3, 46, 47, 57, 58$
given median = 42

$\frac{(x+1)+(x+3)}{2} = 42$

$\Rightarrow \frac{2x+4}{2} = 42 \Rightarrow x + 2 = 42 \Rightarrow x = 40$

$\therefore\ x + 1 = 41\ \ x + 3 = 43$

2. (a) Given, $\bar{x} = 60$ and $n = 10$

and $\sum(\bar{x}-50)^2 = 5000$

$\Rightarrow \sum(x^2 - 100x + 2500) = 5000$

$\Rightarrow \sum x^2 - 100\sum x = -20000 \Rightarrow \sum x^2 = 60000 - 20000$

$\Rightarrow \sum x^2 = 40{,}000$

now,

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

$\Rightarrow \sigma = \sqrt{4000 - 3600} = 20$

3. (d) Given regression lines
$6x + y = 30$
and $3x + 2y = 25$
point of intersection of both lines

$= (\bar{x}, \bar{y}) = \left(\frac{35}{9}, \frac{20}{3}\right)$ for $6x = -y + 30$

$\Rightarrow x = -\frac{1}{6}y + 5$ and for $2y = -3x + 25$

$y = -\frac{3}{2}x + \frac{25}{2}$

$\therefore\ r^2 = \left(-\frac{1}{6}\right).\left(-\frac{3}{2}\right) \Rightarrow r = \pm\frac{1}{2}$

$\because$ sign of $\bar{x}, \bar{y}$ & r is same.

$\therefore\ r = \frac{1}{2}$

4. (b) It can be clearly seen from options that class limits are 2.5–7.5, 7.5–12.5, ...

5. (b) Given: $n = 5$,
$\bar{x} = 4.4$
$\sigma^2 = 8.24$
$\because\ \Sigma x = \bar{x}.x$
$\Rightarrow\ \Sigma x = 5 \times 4.4 = 22$
also $1 + 2 + 6 + p + q = 22$
$\Rightarrow\ p + q = 13$
$\therefore\ 9, 4 = p, q.$

6. (b)

x	f	cumulative frequency
1	3	3
2	15	18
3	45	63
4	57	120
5	50	170
6	36	206
7	25	231
8	9	240

$\Sigma f = 240 = N$

Median = $\left(\frac{N+1}{2}\right)^{th}$ data

= 120.5^{th} data
= 5

7. (c) Given: $n = 100$
$\bar{x} = 50, \sigma = 10$
5 is added to each observation
$\therefore$ new mean = 55
& standard deviation remains same.

8. (c) Given: Range of $x = 25$
and $y = 40 + 3x$
Range of $y = 3 \times 25$
= 75

9. (c) For first 15 natural numbers.

Mean $(M) = \frac{15+1}{2} = 8$ and $V = \frac{n^2-1}{12} = \frac{224}{12} = \frac{56}{3}$

$\therefore\ V + M^2 = \frac{56}{3} + 64 = \frac{248}{3}$

10. (c)

Number of students		Marks Range	Difference of number of students
Physics	Maths		
11	21	20 – 30	21 – 11 = 10
30	38	30 – 40	38 – 30 = 8
26	15	40 – 50	26 – 15 = 11 ←Maximum
15	10	50 – 60	15 – 10 = 5

Hence, difference is largest for the interval (40-50).

11. (a)

(1) Modal value of the marks of Physics is the interval in which maximum number of students got his marks. In the marks interval of (30–40), number of students in Physics is 30, which is largest number of students in any interval. Hence, modal values of marks in Physics is (30–40). Statement (1) is correct.

(2) Median class is given by $\left(\frac{N}{2}\right)^{th}$ item i.e. $\left(\frac{100}{2}\right)^{th}$ item which is 50th item. This corresponds to the class interval of (40–50) for Physics and (30–40) for Mathematics.

Marks	Number of Physics students	Cumulative Frequency	Number of Maths student	Cumulative Frequency
10-20	8	8	10	10
20-30	11	19	21	31
30-40	30	49	38	69
40-50	26	75	15	84
50-60	15	90	10	94
60-70	10	100	6	100

Medium $= l_1 + \dfrac{\frac{N}{2} - C.f.}{f} \times i$

$\therefore$ Median for Physics $= 40 + \dfrac{\frac{100}{2} - 49}{26} \times 10$

$= 40 + \dfrac{50-49}{26} \times 10 = 40.385$

Median for Maths $= 30 + \dfrac{\frac{100}{2} - 31}{38} \times 10$

$= 30 + \dfrac{50-31}{38} \times 10 = 35$

Thus, median of the marks in Physics is more than median of the marks in Mathematics.
Hence, statement (2) is not correct.

12. (c) For physics

Marks	c_i	f_i	$c_i f_i$
10–20	15	8	120
20–30	25	11	275
30–40	35	30	1050
40–50	45	26	1170
50–60	55	15	825
60–70	65	10	650
		$\sum_i = 100$	$\sum_i = 4090$

Mean of marks of physics $= \dfrac{4090}{100} = 40.9.$

13. (b) Standard deviation $= \sqrt{\dfrac{(x-\bar{x})^2}{N}}$

Given data: $-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}.$

$\bar{x} = \dfrac{\text{Sum of datas}}{\text{Number of data}} = 0$

$(x-\bar{x})^2 = \left(-\sqrt{6}-0\right)^2 + \left(-\sqrt{5}-0\right)^2 + \left(-\sqrt{4}-0\right)^2 +$
$\left(-1-0\right)^2 + \left(1-0\right)^2 + \left(\sqrt{4}-0\right)^2 + \left(\sqrt{5}-0\right)^2 + \left(\sqrt{6}-0\right)^2$
$= 6+5+4+1+1+4+5+6 = 32$

$\text{S.D.} = \sqrt{\dfrac{(x-\bar{x})^2}{N}} = \sqrt{\dfrac{32}{8}} = \sqrt{4} = 2$

14. (d) Coefficient of Variation (C.V.)

$= \dfrac{\text{Standard Deviation }(\sigma)}{\text{Mean }(\mu)} \times 100$

Now, standard deviation $(\sigma) = \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2}$

$= \sqrt{\dfrac{200}{10} - \left(\dfrac{20}{10}\right)^2} = 4$ Mean $(\mu) = \dfrac{\Sigma x_i}{n} = \dfrac{20}{10} = 2.$

$\therefore$ Co-efficient of variation (C.V.) $= \dfrac{4}{2} \times 100 = 200.$

15. (b) Correct arithmetic mean

$= 40 + \dfrac{(-83+53)}{100} = 40 + \dfrac{(-30)}{100} = 39.7.$

16. (b) Regression of Y on X:

$Y = \left\{\dfrac{\sigma_y}{\sigma_x} \times r(x,y)\right\} X - a$ (where a = constant term)

$\Rightarrow Y = \left(\dfrac{3.5}{2.5} \times 0.8\right) X - 5.8$

$\therefore Y = 1.12X - 5.8.$

17. (b) Arithmetic mean $= \dfrac{\text{Sum of data}}{\text{Number of data}}$

$6 = \dfrac{4 \times x + 9(x-1)}{(x+x-1)}$

$6(2x-1) = 4x + 9x - 9$

$12x - 6 = 13x - 9$

$x = 9 - 6 = 3.$

18. (d) Number of data set = n.
Mean = 2.5
Sum of deviations = 50
Again, sum of deviations, when mean = 3.5 is –50.
So, $(3.5 - 2.5) \times n = 50 - (-50)$
$\therefore n = 100$

19. (d) Sum of n observation = $2M \times n$
Sum of $2n$ observation = $M \times 2n$

Mean of combined data sets $= \dfrac{2Mn + 2Mn}{(n + 2n)} = \dfrac{4}{3}M.$

20. (a) (i) Arithmetic mean $= \dfrac{\text{Sum of observation}}{\text{Number of observation}}$

(ii) Geometric mean $= \sqrt[n]{\text{Product of n observation}}$
Here, only Arithmetic mean measures central tendency.

21. (b) Science graduate (angle) $= \left(\dfrac{30}{30 + 70 + 50}\right) \times 360° = 72°$

22. (b) 23. (c)

24. (a)

Number of Peas	Frequency	Cummulative frequency
1	4	4
2	33	37
3	76	113
4	50	163
5	26	189
6	8	197
7	1	198

Group of S.D. $= \dfrac{198}{2} = 99$

$\therefore$ S.D. = 3.

25. (b) $M = \dfrac{\sum_{i=1}^{n}(x_i - k)}{n}$ $\qquad M + K = \dfrac{\sum_{i=1}^{n} xi}{n}$

$\therefore$ Meen = M + K

26. (c) Mean $= \dfrac{73 + 85 + 92 + 105 + 120}{5} = 95$

$\therefore$ Sum of the deviation from the mean
$= (95 - 73) + (95 - 85) + (95 - 92) + (95 - 105)$
$+ (95 - 180)$
$= 22 + 10 + 3 - 10 - 25 = 0$

27. (a) Co-efficient of variation $(C.V.) = \dfrac{S.D.}{\text{Mean}}$

$\dfrac{45}{100} = \dfrac{S.D.}{100} \Rightarrow S.D. = 45$

Then, variance $= (S.D.)^2 = (45)^2 = 2025$

28. (b) For set of numbers : 6, 18, 18, 18, 30
Mean = 18, Median = 18, Mode = 18.

29. (b) Mean of discarded observation

$= \dfrac{\text{Sum of 12 observation - Sum of 10 observation}}{2}$

$= \dfrac{12 \times 75 - 10 \times 65}{2} = 125$

30. Mode = Data with highest frequency As mode is 15, So x = 15.

Sets, Relations, Functions and Number System

1

1. Universal set,
   $U = \{x \mid x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$
   $A = \{x \mid x^2 - 5x + 6 = 0\}$
   $B = \{x \mid x^2 - 3x + 2 = 0\}$
   What is $(A \cap B)'$ equal to ? *[2006-I]*
   (a) $\{1, 3\}$ (b) $\{1, 2, 3\}$
   (c) $\{0, 1, 3\}$ (d) $\{0, 1, 2, 3\}$
2. Suppose that A denotes the collection of all complex numbers whose square is a negative real number. Which one of the following statements is correct ? *[2006-I]*
   (a) $A \subseteq R$
   (b) $A \supseteq R$
   (c) $A = \{x + iy | x^2 \in R | x, y \in R\}$
   (d) $A = \{iy | y \in R\}$
3. A relation R is defined on the set Z of integers as follows :
   $mRn \Leftrightarrow m + n$ is odd.
   Which of the following statements is/are true for R ?
   1. R is reflexive 2. R is symmetric
   3. R is transitive
   Select the correct answer using the code given below :
   (a) 2 only (b) 2 and 3
   (c) 1 and 2 (d) 1 and 3 *[2006-I]*
4. Let A and B be two non-empty subsets of a set X. If $(A - B) \cup (B - A) = A \cup B$, then which one of the following is correct?
   (a) $A \subset B$ (b) $A \subset (X - B)$
   (c) $A = B$ (d) $B \subset A$ *[2006-I]*
5. Let $A = \{(n, 2n) : n \in N\}$ and $B = \{(2n, 3n) : n \in N\}$. What is $A \cap B$ equal to ?
   (a) $\{(n, 6n) : n \in N\}$ (b) $\{(2n, 6n) : n \in N\}$
   (c) $\{(n, 3n) : n \in N\}$ (d) ϕ *[2006-I]*
6. Which one of the following operations on sets is not correct where B' denotes the complement of B? *[2006-I]*
   (a) $(B' - A') \cup (A' - B') = (A \cup B) - (A \cap B)$
   (b) $(A - B) \cup (B - A) = (A' \cup B') - (A' \cap B')$
   (c) $(B' - A') \cap (A' - B') = (B - A) \cap (A - B)$
   (d) $(B' - A') \cap (A' - B') = (B - A') \cup (A' - B)$
7. Which one of the following sets has all elements as odd positive integers ? *[2006-I]*
   (a) $S = \{x \in R | x^3 - 8x^2 + 19x - 12 = 0\}$
   (b) $S = \{x \in R | x^3 - 9x^2 + 23x - 15 = 0\}$
   (c) $S = \{x \in R | x^3 - 7x^2 + 14x - 8 = 0\}$
   (d) $S = \{x \in R | x^3 - 12x^2 + 44x - 48 = 0\}$
8. Which of the following statements is not correct for the relation R defined by aRb if and only if b lives within one kilometer from a? *[2006-I]*
   (a) R is reflexive (b) R is symmetric
   (c) R is not anti-symmetric (d) None of the above
9. Let X be any non-empty set containing n elements. Then what is the number of relations on X ? *[2006-I]*
   (a) 2^{n^2} (b) 2^n
   (c) 2^{2n} (d) n^2
10. What is the region that represents $A \cap B$ if *[2006-I]*
    $A = \{(x, y) \mid x + y \le 4\}$ and $B = \{(x, y) \mid x + y \le 0\}$?
    (a) $\{(x, y) \mid x + y \le 2\}$ (b) $\{(x, y) \mid 2x + y \le 4\}$
    (c) $\{(x, y) \mid x + y \le 0\}$ (d) $\{(x, y) \mid x + y \le 4\}$
11. In a group of 500 students, there are 475 students who can speak Hindi and 200 can speak Bengali. What is the number of students who can speak Hindi only ? *[2006-I]*
    (a) 275 (b) 300
    (c) 325 (d) 350
12. Let X and Y be two non-empty sets and let R_1 and R_2 be two relations from X into Y. Then, which one of the following is correct ? *[2006-I]*
    (a) $(R_1 \cap R_2)^{-1} \subset R_1^{-1} \cap R_2^{-1}$
    (b) $(R_1 \cap R_2)^{-1} \supset R_1^{-1} \cap R_2^{-1}$
    (c) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$
    (d) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$
13. What is the value of
    $$\frac{(1001)_2^{(11)_2} - (101)_2^{(11)_2}}{(1001)_2^{(10)_2} + (1001)_2^{(01)_2}(101)_2^{(01)_2} + (101)_2^{(10)_2}}\ ?$$
    (a) $(1001)_2$ (b) $(101)_2$
    (c) $(110)_2$ (d) $(100)_2$ *[2006-I]*
14. Let $x > y$ be two real numbers and $z \in R$, $z \neq 0$. Consider the following :
    1. $x + z > y + z$ and $xz > yz$
    2. $x + z > y - z$ and $x - z > y - z$
    3. $xz > yz$ and $\frac{x}{z} > \frac{y}{z}$
    4. $x - z > y - z$ and $\frac{x}{z} > \frac{y}{z}$

    Which of the above is/are correct ? *[2006-I]*
    (a) 1 only (b) 2 only
    (c) 1 and 2 only (d) 1, 2, 3 and 4

15. If A, B and C are any three arbitrary events then which one of the following expressions shows that both A and B occur but not C ? *[2006-I]*
    (a) $A \cap \overline{B} \cap \overline{C}$ (b) $A \cap B \cap \overline{C}$
    (c) $\overline{A} \cap \overline{B} \cap \overline{C}$ (d) $(A \cup B) \cap \overline{C}$

16. Let $P = \{p_1, p_2, p_3, p_4\}$
    $Q = \{q_1, q_2, q_3, q_4\}$ and
    $R = \{r_1, r_2, r_3, r_4\}$.
    If $S_{10} = \{(p_i, q_j, r_k) : i + j + k = 10\}$,
    how many elements does S_{10} have ? *[2006-I]*
    (a) 2 (b) 4
    (c) 6 (d) 8

17. Which one of the following is correct ? *[2006-I]*
    (a) $A \cup (B - C) = A \cap (B \cap C')$
    (b) $A - (B \cup C) = (A \cap B') \cap C'$
    (c) $A - (B \cap C) = (A \cap B') \cap C$
    (d) $A \cap (B - C) = (A \cap B) \cap C$

18. The maximum three digit integer in the decimal system will be represented in the binary system by which one of the following ? *[2006-II]*
    (a) 1111110001 (b) 1111111110
    (c) 1111100111 (d) 1111000111

19. What is the difference between the smallest five digit binary integer and the largest four digit binary integer ? *[2006-II]*
    (a) The smallest four digit binary integer
    (b) The smallest one digit binary integer
    (c) The greatest one digit binary integer
    (d) The greatest three digit binary integer.

20. If F(n) denotes the set of all divisors of n except 1, what is the least value of y satisfying $[F(20) \cap F(16)] \subseteq F(y)$? *[2006-II]*
    (a) 1 (b) 2
    (c) 4 (d) 8

21. On the set Z of integers, relation R is defined as "a R b $\Leftrightarrow$ a + 2b is an integral multiple of 3". Which one of the following statements is correct for R ? *[2006-II]*
    (a) R is only reflexive
    (b) R is only symmetric
    (c) R is only transitive
    (d) R is an equivalence relation

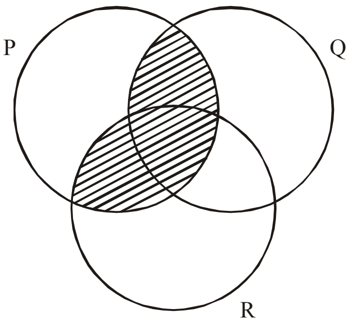
22. For non-empty sets A, B and C, the following two statements are given:
    Statement P : $A \cap (B \cup C) = (A \cap B) \cup C$
    Statement Q : C is a subset of A
    Which one of the following is correct ? *[2006-II]*
    (a) $P \Leftarrow Q$
    (b) $P \Leftrightarrow Q$
    (c) $P \Rightarrow Q$
    (d) Nothing can be said about the correctness of the above three with certainty

23. If $X = \{x : x > 0, x^2 < 0\}$, and Y = {flower, Churchill, moon, Kargil), then which one of the following is a correct statement?
    (a) X is well defined but Y is not a well defined set
    (b) Y is well defined but X is not a well defined set
    (c) Both X and Y are well defined sets
    (d) Neither X nor Y is a well defined set *[2006-II]*

24. Consider the following for any three non-empty sets A, B and C.
    1. $A - (B \cup C) = (A - B) \cup (A - C)$
    2. $A - B = A - (A \cap B)$
    3. $A = (A \cap B) \cup (A - B)$
    Which of the above is/are correct ?
    (a) Only 1 (b) 2 and 3
    (c) 1 and 2 (d) 1 and 3 *[2006-II]*

25. Consider the following statements :
    There are infinitely many rational numbers between two distinct
    1. integers.
    2. rational numbers.
    3. real numbers.
    Which of the statements above are correct ?
    (a) Only 1 and 2 (b) Only 2 and 3
    (c) Only 1 and 3 (d) 1, 2 and 3 *[2006-II]*

26. What does the shaded region represent in the figure given below ?



    (a) $(P \cup Q) - (P \cap Q)$ (b) $P \cap (Q \cap R)$
    (c) $(P \cap Q) \cap (P \cap R)$ (d) $(P \cap Q) \cup (P \cap R)$ *[2006-II]*

27. If $a^x = b$, $b^y = c$, $c^z = a$, then what is the value of
    $$\frac{1}{(xy + yz + zx)}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)?$$
    (a) 0 (b) abc
    (c) 1 (d) – 1 *[2006-II]*

28. If $2^x = 3^y = 12^z$, then what is $(x + 2y)/(xy)$ equal to ?
    (a) z (b) $\frac{1}{z}$
    (c) 2z (d) $\frac{z}{2}$ *[2006-II]*

29. If a set X contains n (n > 5) elements, then what is the number of subsets of X containing less than 5 elements ?
    (a) C (n, 4) (b) C (n, 5)
    (c) $\sum_{r=0}^{5} C(n, r)$ (d) $\sum_{r=0}^{4} C(n, r)$ *[2006-II]*

30. Which one of the following is an infinite set ?
    (a) The set of human beings on the earth
    (b) The set of water drops in a glass of water
    (c) The set of trees in a forest
    (d) The set of all primes *[2006-II]*

31. What is the value of $0.\overline{2} + 0.\overline{23}$?
    (a) $0.\overline{43}$ (b) $0.\overline{45}$
    (c) $0.\overline{223}$ (d) $0.2\overline{23}$ *[2006-II]*

32. If $3^{(x-1)} + 3^{(x+1)} = 30$, then what is the value of $3^{(x+2)} + 3^x$?
(a) 30 (b) 60
(c) 81 (d) 90 *[2007-I]*

33. Let f: $[-100\pi, 100\pi] \to [-1, 1]$ be defined by $f(\theta) = \sin\theta$. Then what is the number of values of $\theta \in [-100\pi, 1000\pi]$ for which $f(\theta) = 0$?
(a) 1000 (b) 1101
(c) 1100 (d) 1110 *[2007-I]*

34. For non-empty subsets A,B and C of a set X such that $A \cup B = B \cap C$, which one of the following is the strongest inference that can be derived?
(a) $A = B = C$ (b) $A \subseteq B = C$
(c) $A = B \subseteq C$ (d) $A \subseteq B \subseteq C$ *[2007-I]*

35. If μ is the universal set and P is a subset of μ, then what is $P \cap (P - \mu) \cup (\mu - P)\}$ equal to ?
(a) ϕ (b) P'
(c) μ (d) P *[2007-I]*

36. let μ = the set of all triangles, P = the set of all isosceles triangles, Q = the set of all equilateral triangles, R = the set of all right-angled triangles. What do the sets $P \cap Q$ and $R - P$ represents respectively ?
(a) The set of isosceles triangles; the set of non- isosceles right angled triangles
(b) The set of isosceles triangles; the set of right angled triangles
(c) The set of equilateral triangles; the set of right angled triangles
(d) The set of isosceles triangles; the set of equilateral triangles *[2007-I]*

37. Consider the following statements:
For non empty sets A, B and C
1. $A - (B - C) = (A - B) \cup C$
2. $A - (B \cup C) = (A - B) - C$
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1nor 2 *[2007-I]*

38. A relation R is defined over the set of non-negative integers as $xRy \Rightarrow x^2 \quad y^2 \quad 36$ what is R?
(a) $\{(0, 6)\}$
(b) $\{(6,0), (\sqrt{11},5), (3,3,\sqrt{3})$
(c) $\{(6, 0), (0, 6)\}$
(d) $(\sqrt{11},5), (2,4\sqrt{2}), (5\sqrt{11}), (4\sqrt{2},2)\}$ *[2007-I]*

39. Consider the following statements:
1. Parallelism of lines is an equivalence relation.
2. x R y, if x is a father of y, is an equivalence relation.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2 *[2007-I]*

40. Which one of the following binary numbers is the prime number?
(a) 1 11101 (b) 1 11010
(c) 1 11111 (d) 1 00011 *[2007-I]*

41. What is the product of the binary numbers 1001.01 and 11.1?
(a) 101110.011 (b) 1 00000.011
(c) 101110.101 (d) 100000.101 *[2007-I]*

42. Among the following equations, which are linear?
1. $2x + y - z = 5$
2. $\pi x + y - ez = \log 3$
3. $3^x + 2y = 7$
4. $\sin x - y - 5z = 4$
Select the correct answer using the code given below
(a) 1 only (b) 1 and 2 only
(c) 3 and 4 (d) 1, 2 and 4 *[2007-II]*

43. The multiplication of the number $(10101)_2$ by $(1101)_2$ yields which one of the following ?
(a) $(100011001)_2$ (b) $(100010001)_2$
(c) $(110010011)_2$ (d) $(100111001)_2$ *[2007-II]*

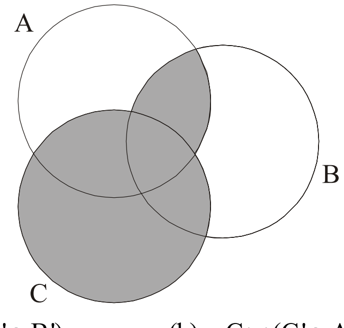
44. If A and B are two sets satisfying $A - B = B - A$, then which one of the following is correct?
(a) $A = \phi$ (b) $A \cap B = \phi$
(c) $A = B$ (d) None of these *[2007-II]*

45. Which one of the following is correct? The real number $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$ is :
(a) an integer
(b) a rational number but not an integer
(c) an irrational number
(d) none of the above *[2007-II]*

46. If $(A - B) \cup (B - A) = A$ for subsets A and B of the universal set U, then which one of the following is correct?
(a) B is proper non-empty subset of A
(b) A and B are non-empty disjoint sets
(c) $B = \phi$
(d) None of the above *[2007-II]*

47. If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then what is the value of $(A' \cap B')$?
(a) 100 (b) 200
(c) 300 (d) 400 *[2007-II]*

48. What does the shaded region in the Venn diagram given below represent ? *[2007-II]*



(a) $C \cap (A' \cap B')$ (b) $C \cup (C' \cap A \cap B)$
(c) $C \cup (C \cap A) \cup (C \cap B)$ (d) $C \cup (A / B)$

49. Let N be the set of integers. A relation R on N is defined as $R = \{(x, y) : xy > 0, x, y, \in N\}$. Then, which one of the following is correct? *[2007-II]*
(a) R is symmetric but not reflexive
(b) R is reflexive but not symmetric
(c) R is symmetric and reflexive but not transitive
(d) R is an equivalence relation

50. What is the value of

$$\frac{\log_{27} 9 \times \log_{16} 64}{\log_4 \sqrt{2}} ?$$

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$
(c) 8 (d) 4 *[2007-II]*

51. Elements of a population are classified according to the presence or absence of each of 3 attributes A, B and C. What is the number of smallest ultimate classes into which the population is divided?
(a) 5 (b) 6
(c) 8 (d) 9 *[2007-II]*

53. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.
Assertion (A) : If events, A, B, C, D are mutually exhaustive, then $(A \cup B \cup C)^C = D$.
Reason (R) : $(A \cup B \cup C)^C = D$ implies if any element is excluded from the sets A, B and C, then it is included in D.
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2007-II]*

54. For what value (s) of x is $\log_{10} \{999 + \sqrt{x^2 - 3x + 3}\} = 3$?
(a) 0 (b) 1 only
(c) 2 only (d) 1, 2 *[2007-II]*

55. Which one of the following is correct? The function $f : A \to R$ where $A = \left\{x \in R, -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$ defined by $f(x) = \tan x$.
(a) Injective (b) Not injective
(c) Bijective (d) Not bijective *[2008-I]*

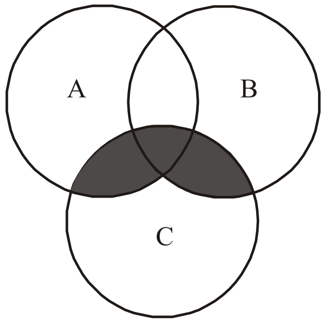
56. Which one of the following real valued functions is never zero?
(a) Polynomial function
(b) Trigonometric function
(c) Logarithmic function
(d) Exponential function *[2008-I]*

57. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*
Assertion (A) : $\{x \in R \mid x^2 < 0\}$ is not a set. Here R is the set of real numbers.
Reason (R) : For every real number x, $x^2 > 0$.
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2008-I]*

58. Let R be the relation defined on the set of natural number N as aRb; a, b $\in$ N, if a divides b. Then, which one of the following is correct ?
(a) R is reflexive only
(b) R is symmetric only
(c) R is transitive only
(d) R is reflexive and transitive *[2008-I]*

59. If $10^{(\log 10 \ |x|)} = 2$, what is the value of x ?
(a) 2 only (b) – 2 only
(c) 2 or – 2 (d) 1 or – 1 *[2008-I]*

60. Consider the following statements
1. $\phi \in \{\phi\}$ 2. $\{\phi\} \subseteq \phi$
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2 *[2008-I]*

61.


What does the shaded region in the above diagram represent? *[2008-I]*
(a) $(A \cap B) \cap C$ (b) $(A \cup B) \cap C$
(c) $(A \cup B) - C$ (d) None of the above

62. The binary number 0.111111 .... (where the digit 1 is recurring) is equivalent in decimal system to which one of the following?
(a) $\frac{1}{10}$ (b) $\frac{11}{10}$
(c) 1 (d) $\frac{10}{11}$ *[2008-I]*

63. The difference of two numbers 10001100 and 1101101 in binary system is expressed in decimal system by which one of the following?
(a) 27 (b) 29
(c) 31 (d) 33 *[2008-I]*

64. Let $A = \{x \in R \mid -9 \le x < 4\}$; $B = \{x \in R \mid -13 < x \le 5\}$ and $C = \{x \in R \mid -7 \le x \le 8\}$.
Then, which one of the following is correct? *[2008-I]*
(a) $-9 \in (A \cap B \cap C)$ (b) $-7 \in (A \cap B \cap C)$
(c) $4 \in (A \cap B \cap C)$ (d) $5 \in (A \cap B \cap C)$

65. Which one of the following is correct? *[2008-I]*
(a) $A \cup P(A) = P(A)$ (b) $A \cap P(A) = A$
(c) $A - P(A) = A$ (d) $P(A) - \{A\} = P(A)$
Here P(A) denotes the power set of a set A.

66. A function f is defined by $f(x) = x + \frac{1}{x}$. Consider the following. [2008-II]
(1) $(f(x))^2 = f(x^2) + 2$
(2) $(f(x))^3 = f(x^3) + 3f(x)$
Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

67. If a set A contains 4 elements, then what is the number of elements in $A \times P(A)$? [2008-II]
(a) 16 (b) 32
(c) 64 (d) 128

68. If A, B, C are three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then which one of the following is correct? [2008-II]
(a) $A = B$ only (b) $B = C$ only
(c) $A = C$ only (d) $A = B = C$

69. The number $\left(2+\sqrt{2}\right)^2$ is [2008-II]
(a) a natural number (b) an irrational number
(c) a rational number (d) a whole number

70. If A and B are disjoint sets, then $A \cap (A' \cup B)$ is equal to which one of the following? [2008-II]
(a) ϕ (b) A
(c) $A \cup B$ (d) $A - B'$

71. If A, B, C are three sets, then what is A – (B – C) equal to? [2008-II]
(a) $A - (B \cap C)$ (b) $(A - B) \cup C$
(c) $(A - B) \cup (A \cap C)$ (d) $(A - B) \cup (A - C)$

72. If A and B are two subsets of a set X, then what is $A \cap (A \cup B)'$ equal to? [2008-II]
(a) A (b) B
(c) ϕ (d) A'

73. $f : \{1, 2, 3\} \to \{4, 5\}$ is not a function if it is defined by which one of the following? [2008-II]
(a) $\{(2, 4), (3, 5), (1, 5)\}$
(b) $\{(1, 4), (2, 4), (3, 4)\}$
(c) $\{(1, 4), (2, 5), (3, 4)\}$
(d) $\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

74. If A = {1, 2, 3, 4} and R = {(1, 1), (1, 3), (2, 2), (3, 1), (3, 4), (4, 3), (4, 4)} is a relation on $A \times A$, then which one of the following is correct? [2008-II]
(a) R is reflexive
(b) R is symmetric and transitive
(c) R is transitive but not reflexive
(d) R is neither reflexive nor transitive

75. If X and Y are any two non-empty sets, then what is $(X - Y)'$ equal to? [2009-I]
(a) $X' - Y'$ (b) $X' \cap Y$
(c) $X' \cup Y$ (d) $X - Y'$

76. What is the binary equivalent of decimal number $(0.8125)_{10}$? [2009-I]
(a) $(0.1101)_2$ (b) $(0.1001)_2$
(c) $(0.1111)_2$ (d) $(0.1011)_2$

77. What is the number of proper subsets of a given finite set with n elements? [2009-I]
(a) $2n-1$ (b) $2n-2$
(c) 2^n-1 (d) 2^n-2

78. If A, B and C are three finite sets, then what is $[(A \cup B) \cap C]'$ equal to? [2009-I]
(a) $A' \cup B' \cap C'$ (b) $A' \cap B' \cap C'$
(c) $A' \cap B' \cup C'$ (d) $A \cap B \cap C$

79. If A and B are subsets of a set X, then what is $\{A \cap (X - B)\} \cup B$ equal to? [2009-I]
(a) $A \cup B$ (b) $A \cap B$
(c) A (d) B

80. The total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B. What is the number of elements in the set A? [2009-I]
(a) 5 (b) 6
(c) 7 (d) 8

81. Which one of the following is correct? [2009-I]
(a) $A \times (B - C) = (A - B) \times (A - C)$
(b) $A \times (B - C) = (A \times B) - (A \times C)$
(c) $A \cap (B \cup C) = (A \cap B) \cup C$
(d) $A \cup (B \cap C) = (A \cup B) \cap C$

82. In an examination out of 100 students, 75 passed in English 60 passed in Mathematics and 45 passed in both English and Mathematics. What is the number of students passed in exactly one of the two subjects? [2009-I]
(a) 45 (b) 60
(c) 75 (d) 90

83. Let $R = \{x \mid x \in N, x \text{ is a multiple of 3 and } x \le 100\}$
$S = \{x \mid x \in N, x \text{ is a multiple of 5 and } x \le 100\}$
What is the number of elements in $(R \times S) \cap (S \times R)$? [2009-I]
(a) 36 (b) 33
(c) 20 (d) 6

84. If $A = \{a, b, c\}$ and $R = \{(a, a), (a, b), (b, c), (b, b), (c, c), (c, a)\}$ is a binary relation of A, then which one of the following is correct? [2009-I]
(a) R is reflexive and symmetric, but not transitive
(b) R is reflexive and transitive, but not symmetric
(c) R is reflexive, but neither symmetric nor transitive
(d) R is reflexive, symmetric and transitive

85. If $\log_{10}(x + 1) + \log_{10} 5 = 3$, then what is the value of x? [2009-I]
(a) 199 (b) 200
(c) 299 (d) 300

86. What is the value of $2\log_8 2 - \frac{\log_3 9}{3}$? [2009-I]
(a) 0 (b) 1
(c) 8/3 (d) 16/3

87. What is the decimal equivalent of $(101\,.\,101)_2$? [2009-I]
(a) $(5.225)_{10}$ (b) $(5.525)_{10}$
(c) $(5.625)_{10}$ (d) $(5.65)_{10}$

88. Let $A = \{x \mid x \le 9, x \in N\}$. Let $B = \{a, b, c\}$ be the subset of A where $(a + b + c)$ is a multiple of 3. What is the largest possible number of subsets like B? [2009-II]
(a) 12 (b) 21 (c) 27 (d) 30

89. Let $A = \{-1, 2, 5, 8\}$, $B = \{0, 1, 3, 6, 7\}$ and R be the relation 'is one less than' from A to B, then how many elements will R contain? [2009-II]
(a) 2 (b) 3
(c) 5 (d) 9

90. Natural numbers are divided into groups as (1), (2, 3), (4, 5, 6), (7, 8, 9, 10) and so on. What is the sum of the numbers in the 11th group? [2009-II]
(a) 605 (b) 615
(c) 671 (d) 693

91. What is the value of $\frac{\log_{27} 9 \ \log_{16} 64}{\log_4 \sqrt{2}}$? [2009-II]
(a) 1 (b) 2
(c) 4 (d) 8

92. If $x = (1\,1\,0\,1)_2$ and $y = (1\,1\,0)_2$, then what is the value of $x^2 - y^2$? [2009-II]
(a) $(1\,0\,0\,0\,1\,0\,1)_2$ (b) $(1\,0\,0\,0\,0\,1\,0\,1)_2$
(c) $(1\,0\,0\,0\,1\,1\,0\,1)_2$ (d) $(1\,0\,0\,1\,0\,1\,0\,1)_2$

93. If $(1\,0\,x\,0\,1\,0)_2 - (1\,1\,y\,1)_2 = (1\,0\,z\,1\,1)_2$, then what are the possible values of the binary digits x, y, z respectively? [2009-II]
(a) 0, 0, 1 (b) 0, 1, 0
(c) 1, 1, 0 (d) 0, 0, 0

94. The number 0.0011 in binary system represents [2009-II]
(a) rational number 3/8 in decimal system
(b) rational number 1/8 in decimal system
(c) rational number 3/16 in decimal system
(d) rational number 5/16 in decimal system

95. If $n(A) = 115$, $n(B) = 326$, $n(A - B) = 47$, then what is $n(A \cup B)$ equal to? [2009-II]
(a) 373 (b) 165
(c) 370 (d) 394

96. If $P(A)$ denotes the power set of A and A is the void set, then what is number of elements in $P\{P\{P\{P(A)\}\}\}$? [2009-II]
(a) 0 (b) 1
(c) 4 (d) 16

97. During a certain plane period a state out of a total budget of Rs 1400 crores had spent 28% of the total amount on Agriculture, 35% on Industry, 12% on Energy and 8% on Social Welfare, 105 crores on Education and the balance amount on Transport. What is the amount spent on Transport in crores of rupees? [2009-II]
(a) 123 (b) 145
(c) 165 (d) 133

98. In a town 35.4% of the people are not literates, 27% have education up to primary school, 18.6% have education up to middle school. The people with education up to high school are twice the number of people with education up to Pre-University. Of the remaining, 660 are graduates. If the population of the town is 15000, then what is the number of people with education up to high school? [2009-II]
(a) 3120 (b) 1560
(c) 1460 (d) None of these

99. If $(\log_x x)(\log_3 2x)(\log_{2x} y) = \log_x x^2$, then what is the value of y? [2009-II]
(a) 9/2 (b) 9
(c) 18 (d) 27

100. If $\log_k x \log_5 k = 3$, then what is x equal to? [2009-II]
(a) k^5 (b) $5k^3$
(c) 243 (d) 125

101. If $N_a = \{ax \mid x \in N\}$, then what is $N_{12} \cap N_8$ equal to? [2009-II]
(a) N_{12} (b) N_{20}
(c) N_{24} (d) N_{48}

102. If $X = \{(4^n - 3n - 1) \mid n \in N\}$ and $Y = \{9(n-1) \mid n \in N\}$, then what is $X \cup Y$ equal to? [2009-II]
(a) X (b) Y
(c) N (d) A null set

103. Sets A and B have n elements in common. How many elements will $(A \times B)$ and $(B \times A)$ have in common? [2009-II]
(a) 0 (b) 1
(c) n (d) n^2

104. Let $f: R \to R$ be defined by $f(x) = |x|/x, x \neq 0, f(0) = 2$. What is the range of f? [2009-II]
(a) $\{1, 2\}$ (b) $\{1, -1\}$
(c) $\{-1, 1, 2\}$ (d) $\{1\}$

105. What is the equivalent binary number of the decimal number 13.625? [2010-I]
(a) 1101.111 (b) 1111.101
(c) 1101.101 (d) 1111.111

106. The order of a set A is 3 and that of a set B is 2. What is the number of relations from A to B? [2010-I]
(a) 4 (b) 6 (c) 32 (d) 64

107. What is the value of $\frac{\log_{\sqrt{\alpha\beta}} H}{\log_{\sqrt{\alpha\beta\gamma}} H}$? [2010-I]
(a) $\log_{\alpha\beta}(\alpha)$ (b) $\log_{\alpha\beta\gamma}(\alpha\beta)$
(c) $\log_{\alpha\beta}(\alpha\beta\gamma)$ (d) $\log_{\alpha\beta}(\beta)$

108. For a set A, consider the following statements: [2010-I]
1. $A \cup P(A) = P(A)$
2. $\{A\} \cap P(A) = A$
3. $P(A) - \{A\} = P(A)$

where P denotes power set. [2010-I]
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) 3 only (d) 1, 2 and 3

109. If $A = P(\{1, 2\})$ where P denotes the power set, then which one of the following is correct? [2010-I]
(a) $\{1, 2\} \subset A$ (b) $1 \in A$
(c) $\phi \notin A$ (d) $\{1, 2\} \in A$

110. Let X be the set of all graduates in India. Elements x and y in X are said to be related if they are graduates of the same university. Which one of the following statements is correct? [2010-I]
(a) Relation is symmetric and transitive only
(b) Relation is reflexive and transivtive only
(c) Relation is reflexive and symmetric only
(d) Relation is reflexive symmetric and transitive

111. What is the value of [2010-I]

$$\frac{(0.101)_2^{(11)_2} + (0.011)_2^{(11)_2}}{(0.101)_2^{(10)_2} - (0.101)_2^{(01)_2}(0.011)_2^{(01)_2} + (0.011)_2^{(10)_2}}$$

(a) $(0.001)_2$ (b) $(0.01)_2$
(c) $(0.1)_2$ (d) $(1)_2$

112. If $A = \{a, b, c, d\}$, then what is the number of proper subsets of A? *[2010-I]*
(a) 16 (b) 15
(c) 14 (d) 12

113. Out of 32 persons, 30 invest in National Savings Certificates and 17 invest in shares. What is the number of persons who invest in both? *[2010-I]*
(a) 13 (b) 15 (c) 17 (d) 19

114. What is $(1111)_2 + (1001)_2 - (1010)_2$ equal to ? *[2010-II]*
(a) $(111)_2$ (b) $(1100)_2$
(c) $(1110)_2$ (d) $(1010)_2$.

115. The relation $R = \{(1, 1), (2, 2), (3, 3)\ (1, 2), (2, 3), (1, 3)\}$ on a set $A = \{1, 2, 3\}$ is *[2010-II]*
(a) reflexive, transitive but not symmetric
(b) reflexive, symmetric but not transitive
(c) symmetric, transitive but not reflexive
(d) reflexive but neither symmetric nor transitive

116. If $\log_3 [\log_3 [\log_3 x]] = \log_3 3$, then what is the value of x? *[2010-II]*
(a) 3 (b) 27
(c) 3^9 (d) 3^{27}

117. What is the binary number equivalent of the decimal number 32.25? *[2010-II]*
(a) 100010.10 (b) 100000.10
(c) 100010.01 (d) 100000.01

118. If A and B are two disjoint sets, then which one of the following is correct? *[2010-II]*
(a) $A - B = A - (A \cap B)$ (b) $B - A' = A \cap B$
(c) $A \cap B = (A - B) \cap B$ (d) All of these

119. Let N denote the set of natural numbers and $A = \{n^2 : n \in N\}$ and $B = \{n^3 : n \in N\}$. Which one of the following is incorrect? *[2010-II]*
(a) $A \cup B = N$
(b) The complement of $(A \cup B)$ is an infinite set
(c) $A \cap B$ must be a finite set
(d) $A \cap B$ must be a proper subset of $\{m^6 : m \in N\}$

120. If $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$, then what is the number of elements of $A \times (B \cap C)$? *[2010-II]*
(a) 2 (b) 4
(c) 6 (d) 8

121. Let $U = \{1, 2, 3,.....,20\}$. Let A, B, C be the subsets of U. Let A be the set of all numbers, which are perfect squares, B be the set of all numbers which are multiples of 5 and C be the set of all numbers, which are divisible by 2 and 3. Consider the following statements : *[2010-II]*
I. A, B, C are mutually exclusive.
II. A, B, C are mutually exhaustive.
III. The number of elements in the complement set of $A \cup B$ is 12.
Which of the statements given above are the correct?
(a) I and II only (b) I and III only
(c) II and III only (d) I, II and III

122. If the cardinality of a set A is 4 and that of a set B is 3, then what is the cardinality of the set $A \Delta B$? *[2010-II]*
(a) 1
(b) 5
(c) 7
(d) Cannot be determined as the sets A and B are not given

123. What is the range of $f(x) = \cos 2x - \sin 2x$? *[2011-I]*
(a) $[2, 4]$ (b) $[-1, 1]$
(c) $\left[-\sqrt{2}, \sqrt{2}\right]$ (d) $\left(-\sqrt{2}, 2\right)$

124. If $A = \{1, 2, 5, 6\}$ and $B = \{1, 2, 3\}$, then what is $(A \times B) \cap (B \times A)$ equal to? *[2011-I]*
(a) $\{(1, 1), (2, 1), (6, 1), (3, 2)\}$
(b) $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$
(c) $\{(1, 1), (2, 2)\}$
(d) $\{(1, 1), (1, 2), (2, 5), (2, 6)\}$

DIRECTIONS (Qs. 125-129) : *Read the following passage and give answer.*

The students of a class are offered three languages (Hindi, English and French). 15 students learn all the three languages whereas 28 students do not learn any language. The number of students learning Hindi and English but not French is twice the number of students learning Hindi and French but not English. The number of students learning English and French but not Hindi is thrice the number of students learning Hindi and French but not English. 23 students learn only Hindi and 17 students learn only English. The total number of students learning French is 46 and the total number of students learning only French is 11. *[2011-I]*

125. How many students learn precisely two languages?
(a) 55 (b) 40
(c) 30 (d) 13

126. How many students learn at least two languages?
(a) 15 (b) 30
(c) 45 (d) 55

127. What is the total strength of the class?
(a) 124 (b) 100
(c) 96 (d) 66

128. How many students learn English and French?
(a) 30 (b) 43
(c) 45 (d) 73

129. How many students learn at least one languages?
(a) 45 (b) 51
(c) 96 (d) None of these

130. What is

$$\log\left(a + \sqrt{a^2 + 1}\right) + \log\left(\frac{1}{a + \sqrt{a^2 + 1}}\right)$$ equal to? *[2011-I]*

(a) 1 (b) 0
(c) 2 (d) $\frac{1}{2}$

131. Consider the following with regard to a relation R on a set of real numbers defined by xRy if and only if $3x + 4y = 5$
I. $0\,R\,1$ II. $1\,R\,\frac{1}{2}$
III. $\frac{2}{3}\,R\,\frac{3}{4}$
Which of the above are correct? *[2011-I]*
(a) I and II (b) I and III
(c) II and III only (d) I, II and III

132. What is the value of

$\log_{10}\left(\frac{9}{8}\right)-\log_{10}\left(\frac{27}{32}\right)+\log_{10}\left(\frac{3}{4}\right)$? *[2011-I]*

(a) 3 (b) 2
(c) 1 (d) 0

133. In a binary number system, assume that a = 00111 and b = 01110, then in a decimal system $\frac{b}{a}$, which is equal to

(a) 1 (b) 2 *[2011-I]*
(c) 4 (d) 5

134. Let M be the set of men and R is a relation 'is son of' defined on M. Then, R is *[2011-I]*

(a) an equivalence relation
(b) a symmetric relation only
(c) a transitive relation only
(d) None of the above

135. The number 10101111 in binary system is represented in decimal system by which one of the following numbers? *[2011-I]*

(a) 157 (b) 175
(c) 571 (d) 751

136. If A, B and C are non-empty sets such that $A\cap C=\phi$, then what is $(A\times B)\cap(C\times B)$ equal to?

(a) $A\times C$ (b) $A\times B$ *[2011-I]*
(c) $B\times C$ (d) ϕ

137. If A = {$4n + 2$| n is a natural number} and B = {$3n$ | n is a natural number}, then what is $(A\cap B)$ equal to? *[2011-I]*

(a) $\{12n^2+6n \mid n$ is a natural number$\}$
(b) $\{24n-12 \mid n$ is a natural number$\}$
(c) $\{60n+30 \mid n$ is a natural number$\}$
(d) $\{12n-6 \mid n$ is a natural number$\}$

138. If P, Q and R are three non-collinear points, then what is $PQ\cap PR$ equal to? *[2011-I]*

(a) Null set (b) $\{P\}$
(c) $\{P,Q,R\}$ (d) $\{Q,R\}$

139. In binary system the decimal number 0.3 can be expressed as *[2011-II]*

(a) $(0.01001\ldots\ldots)_2$ (b) $(0.10110\ldots\ldots)_2$
(c) $(0.11001\ldots\ldots)_2$ (d) $(0.10101\ldots\ldots)_2$

140. If $\tan\theta=\sqrt{m}$, where m is non-square natural number, then sec 2θ is *[2011-II]*

(a) a negative number
(b) a transcendental number
(c) an irrational number
(d) a rational number

141. If A = {a, b, c}, then what is the number of proper subsets of A? *[2011-II]*

(a) 5 (b) 6 (c) 7 (d) 8

142. What is the value of $\log_2(\log_3 81)$? *[2011-II]*

(a) 2 (b) 3 (c) 4 (d) 9

143. If ϕ is a null set, then which one of the following is correct? *[2011-II]*

(a) $\phi=0$ (b) $\phi=\{0\}$
(c) $\phi=\{\phi\}$ (d) $\phi=\{\ \}$

144. Out of 500 first year students, 260 passed in the first semester and 210 passed in the second semester. If 170 did not pass in either semester, how many passed in both semesters?

(a) 30 (b) 40 *[2012-I]*
(c) 70 (d) 140

145. What is the decimal number representation of the binary number $(1\,1\,1\,0\,1.0\,0\,1)_2$? *[2012-I]*

(a) 30.125 (b) 29.025
(c) 29.125 (d) 28.025

146. Let U = $\{x\in N:1\le x\le 10\}$ be the universal set, N being the set of natural numbers. If A = {1, 2, 3, 4} and B = {2, 3, 6, 10} then what is the complement of (A – B)? *[2012-I]*

(a) {6, 10} (b) {1, 4}
(c) {2, 3, 5, 6, 7, 8, 9, 10} (d) {5, 6, 7, 8, 9, 10}

147. Let A= {x : x is a square of a natural number and x is less than 100} and B is a set of even natural numbers. What is the cardinality of $A\cap B$? *[2012-I]*

(a) 4 (b) 5
(c) 9 (d) None of the above

148. The number 292 in decimal system is expressed in binary system by *[2012-I]*

(a) 100001010 (b) 100010001
(c) 100100100 (d) 101010000

149. The set A = {x : x + 4 = 4} can also be represented by:

(a) 0 (b) φ *[2012-I]*
(c) {φ} (d) {0}

DIRECTIONS (Qs. 150-153) : *In a city, three daily newspapers A, B, C are published, 42% read A; 51% read B; 68% read C; 30% read A and B; 28% read B and C; 36% read A and C; 8% do not read any of the three newspapers.*

150. What is the percentage of persons who read all the three papers? *[2012-I]*

(a) 20% (b) 25%
(c) 30% (d) 40%

151. What is the percentage of persons who read only two papers? *[2012-I]*

(a) 19% (b) 31%
(c) 44% (d) None of the above

152. What is the percentage of persons who read only one paper?

(a) 38% (b) 48% *[2012-I]*
(c) 51% (d) None of the above

153. What is the percentage of persons who read only A but neither B nor C? *[2012-I]*

(a) 4% (b) 3%
(c) 1% (d) None of the above

154. What is the value of $2\log_8 2-\frac{1}{3}\log_3 9$? *[2012-I]*

(a) 0 (b) 1
(c) 2 (d) 1/3

155. If A = {0, 1} and B = {1, 0}, then what is A × B equal to? *[2012-I]*

(a) {(0, 1), (1, 0)} (b) {(0, 0), (1, 1)}
(c) {(0, 1), (1, 0), (1, 1)} (d) A × A

156. If A and B are two non-empty sets having n elements in common, then what is the number of common elements in the sets A × B and B × A? *[2012-I]*

(a) n (b) n^2
(c) 2n (d) zero

157. If A and B are any two sets, then what is $A \cap (A \cup B)$ equal to? *[2012-I]*
(a) Complement of A (b) Complement of B
(c) B (d) A

158. The relation 'has the same father as' over the set of children is: *[2012-II]*
(a) only reflexive (b) only symmetric
(c) only transitive (d) an equivalence relation

159. The decimal representation of the number $(1011)_2$ in binary system is: *[2012-II]*
(a) 5 (b) 7
(c) 9 (d) 11

160. The decimal number $(57.375)_{10}$ when converted to binary number takes the form: *[2012-II]*
(a) $(111001.011)_2$ (b) $(100111.110)_2$
(c) $(110011.101)_2$ (d) $(111011.011)_2$

161. If $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$, then what is y equal to? *[2012-II]*
(a) 4.5 (b) 9
(c) 18 (d) 27

162. Let P = {1, 2, 3} and a relation on set P is given by the set R = {(1, 2), (1, 3), (2, 1), (1, 1), (2, 2), (3, 3), (2, 3)}. Then R is: *[2012-II]*
(a) Reflexive, transitive but not symmetric
(b) Symmetric, transitive but not reflexive
(c) Symmetric, reflexive but not transitive
(d) None of the above

163. If a non-empty set A contains n elements, then its power set contains how many elements ? *[2012-II]*
(a) n^2 (b) 2^n
(c) 2n (d) n + 1

164. Let A = {x ∈ W, the set of whole numbers and x < 3}, B = {x ∈ N, the set of natural numbers and $2 \le x < 4$} and C = {3, 4}, then how many elements will $(A \cup B) \times C$ contain? *[2012-II]*
(a) 6 (b) 8
(c) 10 (d) 12

165. What is the range of the function $f(x) = \frac{|x|}{x}, x \neq 0$? *[2013-I]*
(a) Set of all real numbers (b) Set of all integers
(c) {–1, 1} (d) {–1, 0, 1}

166. The binary representation of the decimal number 45 is *[2013-I]*
(a) 110011 (b) 101010
(c) 1101101 (d) 101101

167. If d is the number of degrees contained in an angle, m is the number of minutes and s is the number of seconds, then the value of (s – m)/(m – d) is: *[2013-I]*
(a) 1 (b) 60
(c) $\frac{1}{60}$ (d) None of these

DIRECTIONS (Qs. 168-173) : *For the next six (06) questions that follow :*

In a state with a population of 75×10^6, 45% of them know Hindi, 22% know English, 18% know Sanskrit, 12% know Hindi and English, 8% know English and Sanskrit, 10% know Hindi and Sanskrit and 5% know all the three languages.

168. What is the number of people who do not know any of the above three languages ? *[2013-I]*
(a) 3×10^6 (b) 4×10^6
(c) 3×10^7 (d) 4×10^7

169. What is the number of people who know Hindi only? *[2013-I]*
(a) 21×10^6 (b) 25×10^6
(c) 28×10^6 (d) 3×10^7

170. What is the number of people who know Sanskrit only ? *[2013-I]*
(a) 5×10^6 (b) 45×10^5
(c) 4×10^6 (d) None of the above

171. What is the number of people who know English only ? *[2013-I]*
(a) 5×10^6 (b) 45×10^5
(c) 4×10^6 (d) None of the above

172. What is the number of people who know only one language ? *[2013-I]*
(a) 3×10^6 (b) 4×10^6
(c) 3×10^7 (d) 4×10^7

173. What is the number of people who know only two languages ? *[2013-I]*
(a) 11.25×10^5 (b) 11.25×10^6
(c) 12×10^5 (d) 12.5×10^5

174. Which one of the following is a null set ? *[2013-I]*
(a) {0} (b) {{{}}}
(c) {{}} (d) $\{x \mid x^2 + 1 = 0, x \in R\}$

175. If A = {x, y}, B = {2, 3}, C = {3, 4}, then what is the number of elements in $A \times (B \cup C)$? *[2013-I]*
(a) 2 (b) 4 (c) 6 (d) 8

176. What is the value of $\log_y x^5 \log_x y^2 \log_z z^3$? *[2013-I]*
(a) 10 (b) 20 (c) 30 (d) 60

177. If A is a relation on a set R, then which one of the following is correct ? *[2013-I]*
(a) $R \subseteq A$ (b) $A \subseteq R$
(c) $A \subseteq (R \times R)$ (d) $R \subseteq (A \times A)$

178. If *A* = {1, 2}, *B* = { 2, 3} and *C* = { 3, 4}, then what is the cardinality of $(A \times B) \cap (A \times C)$? *[2013-II]*
(a) 8 (b) 6 (c) 2 (d) 1

179. If *A* is a finite set having *n* elements, then the number of relations which can be defined in *A* is *[2013-II]*
(a) 2^n (b) n^2
(c) 2^{n^2} (d) n^n

180. Which one of the following is an example of non-empty set ? *[2013-II]*
(a) Set of all even prime numbers
(b) $(x : x^2 - 2 = 0$ and x is rational)
(c) {x : x is a natural number, $x < 8$ and simultaneously $x > 12$}
(d) {x : x is a point common to any two parallel lines}

181. The number 83 is written in the binary system as *[2013-II]*
(a) 100110 (b) 101101
(c) 1010011 (d) 110110

182. The relation *R* in the set *Z* of integers given by *R* = {(*a*, *b*) : *a* – *b* is divisible by 5}is *[2013-II]*
(a) reflexive
(b) reflexive but not symmetric
(c) symmetric and transitive
(d) an equivalence relation

183. In a group of 50 people, two tests were conducted, one for diabetes and one for blood pressure. 30 people were diagnosed with diabetes and 40 people were diagnosed with high blood pressure. what is the minimum number of people who were having diabetes and high blood pressure ? *[2013-II]*
(a) 0 (b) 10
(c) 20 (d) 30

184. Let $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$. What is the number of elements in $A \times B$? *[2013-II]*
(a) 6 (b) 7
(c) 12 (d) 64

185. If A is a subset of B, then which one of the following is correct ? *[2013-II]*
(a) $A^c \subseteq B^c$ (b) $B^c \subseteq A^c$
(c) $A^c \quad B^c$ (d) $A \subseteq A \cap B$

186. What is the angle (in circular measure) between the hour hand and the minute hand of a clock when the time is half past 4 ? *[2013-II]*
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) None of these

187. Consider the following : *[2013-II]*
1. $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$
2. $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$
Which of the above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

188. A number in binary system is 110001. It is equal to which one of the following numbers in decimal system ? *[2013-II]*
(a) 45 (b) 46
(c) 48 (d) 49

189. If $A = \{1, 3, 5, 7\}$, then what is the cardinality of the power set $P(A)$? *[2013-II]*
(a) 8 (b) 15 (c) 16 (d) 17

190. What is $\log_{81} 243$ equal to ? *[2013-II]*
(a) 0.75 (b) 1.25
(c) 1.5 (d) 3

191. Let X be the set of all citizens of India. Elements x, y in X are said to be related if the difference of their age is 5 years. Which one of the following is correct ? *[2014-I]*
(a) The relation is an equivalence relation on X.
(b) The relation is symmetric but neither reflexive nor transitive.
(c) The relation is reflexive but neither, symmetric nor transitive.
(d) None of the above

192. Consider the following relations from A to B where $A = \{u, v, w, x, y, z\}$ and $B = \{p, q, r, s\}$. *[2014-I]*
1. $\{(u, p), (v, p), (w, p), (x, q), (y, q), (z, q)\}$
2. $\{(u, p), (v, q), (w, r), (z, s)\}$
3. $\{(u, s), (v, r), (w, q), (u, p), (v, q), (z, q),\}$
4. $\{(u, q), (v, p), (w, s), (x, r), (y, q), (z, s),\}$
Which of the above relations are not functions ?
(a) 1 and 2 (b) 1 and 4
(c) 2 and 3 (d) 3 and 4

193. Let S denote set of all integers. Define a relation R on S as 'aRb if $ab \geq 0$ where $a, b \in S$'. Then R is : *[2014-I]*
(a) Reflexive but neither symmetric nor transitive relation
(b) Reflexive, symmetric but not transitive relation
(c) An equivalence relation
(d) Symmetric but neither reflexive nor transitive relation

194. What is the sum of the two numbers $(11110)_2$ and $(1010)_2$? *[2014-I]*
(a) $(101000)_2$ (b) $(110000)_2$
(c) $(100100)_2$ (d) $(101100)_2$

195. $p, q, r, s, t,$ are five numbers such that the average of p, q and r is 5 and that of s and t is 10. What is the average of all the five numbers ? *[2014-I]*
(a) 7.75 (b) 7.5
(c) 7 (d) 5

196. The number 251 in decimal system is expressed in binary system by : *[2014-I]*
(a) 11110111 (b) 11111011
(c) 11111101 (d) 11111110

DIRECTIONS (Qs. 197-199) : *For the next three (03) items that follow:*

In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. *[2014-I]*

197. The number of students who had taken only physics is :
(a) 2 (b) 3
(c) 5 (d) 6

198. The number of students who had taken only two subjects is :
(a) 7 (b) 8
(c) 9 (d) 10

199. Consider the following statements :
1. The number of students who had taken only one subject is equal to the number of students who had taken only two subjects.
2. The number of students who had taken at least two subjects is four times the number of students who had taken all the three subjects.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

200. Consider the following statements : *[2014-I]*
1. The function $f(x) = \sin x$ decreases on the interval $(0, \pi/2)$.
2. The function $f(x) = \cos x$ increases on the interval $(0, \pi/2)$.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

201. The relation S is defined on the set of integers Z as xSy if integer x devides integer y. Then *[2014-II]*
(a) s is an equivalence relation.
(b) s is only reflexive and symmetric.
(c) s is only reflexive and transitive.
(d) s is only symmetric and transitive.

202. What is $(1001)_2$ equal to ? *[2014-II]*
(a) $(5)_{10}$ (b) $(9)_{10}$
(c) $(17)_{10}$ (d) $(11)_{10}$

203. A and B are two sets having 3 elements in common. If n(A) = 5, n(B) = 4, then what is n(A×B) equal to ? *[2014-II]*
(a) 0 (b) 9 (c) 15 (d) 20

204. Let X be the set of all persons living in a city. Persons x, y in X are said to be related as $x < y$ if y is at least 5 years older than x. Which one of the following is correct? *[2015-I]*
(a) The relation is an equivalence relation on X
(b) The relation is transitive but neither reflexive nor symmetric
(c) The relation is reflexive but neither transitive nor symmetric
(d) The relation is symmetric but neither transitive nor reflexive

205. In a class of 60 students, 45 students like music, 50 students like dancing, 5 students like neither. Then the number of students in the class who like both music and dancing is *[2015-I]*
(a) 35 (b) 40 (c) 50 (d) 55

206. If $\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are three consecutive terms of an A.P, then the value of x is *[2015-I]*
(a) 1 (b) $\log_5 2$
(c) $\log_2 5$ (d) $\log_{10} 5$

207. Let Z be the set of integers and aRb, where $a, b \in Z$ if and only if $(a - b)$ is divisible by 5. *[2015-I]*
Consider the following statements:
1. The relation R partitions Z into five equivalent classes.
2. Any two equivalent classes are either equal or disjoint.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

208. Let A = {1, 2 , 3, 4, 5, 6, 7, 8, 9, 10}. Then the number of subsets of A containing exactly two elements is *[2015-I]*
(a) 20 (b) 40
(c) 45 (d) 90

209. The decimal number $(127.25)_{10}$, when converted to binary nunber, takes the form *[2015-I]*
(a) $(1111111.11)_2$ (b) $(1111110.01)_2$
(c) $(1110111.11)_2$ (d) $(1111111.01)_2$

210. If $A = \{x : x$ is a multiple of 3$\}$ and
$B = \{x : x$ is a multiple of 4$\}$ and
$C = \{x : x$ is a multiple of 12$\}$, then which one of the following is a null set? *[2015-I]*
(a) $(A/B) \cup C$ (b) $(A/B)/C$
(c) $(A \cap B) \cap C$ (d) $(A \cap B)/C$

211. If $(11101011)_2$ is converted decimal system, then the resulting number is *[2015-I]*
(a) 235 (b) 175
(c) 160 (d) 126

212. For each non-zero real number x, let $f(x) = \dfrac{x}{|x|}$. The range of f is *[2015-I]*
(a) a null set
(b) a set consisting of only one element
(c) a set consisting of two elements
(d) a set consisting of infinitely many elements

213. Let X be the set of all persons living in Delhi. The persons a and b in X are said to be related if the difference in their ages is at most 5 years. The relation is *[2015-II]*
(a) an equivalence relation
(b) reflexive and transitive but not symmetric
(c) symmetric and transitive but not reflexive
(d) retlexive and symmetric but not transitive

214. What is $(1000000001)_2 - (0.0101)_2$ equal to ? *[2015-II]*
(a) $(512.6775)_{10}$ (b) $(512.6875)_{10}$
(c) $(512.6975)_{10}$ (d) $(512.0909)_{10}$

215. If $A = |x \in IR : x^2 + 6x - 7 < 0\}$ and
$B = \{x \in IR : x^2 + 9x + 14 > 0\}$, then which of the following is/ are correct? *[2015-II]*
1. $(A \cap B) = (-2, 1)$
2. $(A \backslash B) = (-7, -2)$
Select the correct answer using the code given below:
(a) 1 only (b) 2 Only
(c) Both 1 and 2 (d) Neither 1 nor 2

216. A, B, C and D are four sets such that $A \cap B = C \cap D = \phi$. Consider the following : *[2015-II]*
1. $A \cup C$ and $B \cup D$ are always disjoint.
2. $A \cap C$ and $B \cap D$ are always disjoint
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

217. If $\log_8 m + 1\log_8 \dfrac{1}{6} \quad \dfrac{2}{3}$, then m is equal to *[2015-II]*
(a) 24 (b) 18 (c) 12 (d) 4

218. f(xy) = f(x) + f(y) is true for all *[2015-II]*
(a) Polynomial functions f
(b) Trigonometric functions f
(c) Exponential functions f
(d) Logarithmic functions f

219. Suppose there is a relation $*$ between the positive numbers x and y given by x $*$ y if and only if $x \le y^2$. Then which one of the following is correct? *[2016-I]*
(a) $*$ is reflexive but not transitive and symmetric
(b) $*$ is transitive but not reflexive and symmetric
(c) $*$ is symmetric and reflexive but not transitive
(d) $*$ is symmetric and but not reflexive and transitive

220. If $f(x_1) - f(x_2) = f\left(\dfrac{x_1 - x_2}{1 - x_1 x_2}\right)$ for $x_1, x_2 \in (-1, 1)$, then what is f(x) equal to? *[2016-I]*
(a) $\ln\left(\dfrac{1-x}{1+x}\right)$ (b) $\ln\left(\dfrac{2+x}{1-x}\right)$
(c) $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)$ (d) $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$

221. What is the range of the function $y = \dfrac{x^2}{1+x^2}$, where $x \in \mathbf{R}$? *[2016-I]*
(a) [0, 1) (b) [0, 1] (c) (0, 1) (d) (0, 1]

222. What is the binary equivalent of the decimal number 0.3125? *[2016-I]*
(a) 0.0111 (b) 0.1010
(c) 0.0101 (d) 0.1101

223. Let R be a relation on the set N of natural numbers defined by 'nRM $\Leftrightarrow$ n is a factor of m'. Then which one of the following is correct? *[2016-I]*
(a) R is reflexive, symmetric but not transitive
(b) R is transitive, symmetric but not reflexive
(c) R is reflexive, transitive but not symmetric
(d) R is an equivalence relation

224. What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25? *[2016-I]*
(a) 860 (b) 854 (c) 840 (d) 824

225. If $\log_a(ab) = x$, then what is $\log_b(ab)$ equal to? *[2016-I]*
(a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$
(c) $\frac{x}{1-x}$ (d) $\frac{x}{x-1}$

226. Let *S* be a set of all distinct numbers of the form $\frac{P}{Q}$, where $p, q \in \{1, 2, 3, 4, 5, 6\}$. What is the cardinality of the set S? *[2016-II]*
(a) 21 (b) 23 (c) 32 (d) 36

227. If $A = \{x \in R : x^2 + 6x - 7 < 0\}$ and $B = \{x \in R : x^2 + 9x + 14 > 0\}$, then which of the following is/are correct? *[2016-II]*
1. $A \cap B = \{X \in R : -2 < x < 1\}$
2. $A \setminus B = \{x \in R : -7 < x < -2\}$
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

228. Let R be a relation from A = {1, 2, 3, 4} to B = {1, 3, 5} such that R = [(a, b) :a < b, where a $\in$ A and b $\in$ B]. What is RoR^{-1} equal to? *[2016-II]*
(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}
(b) {(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}
(c) {(3, 3), (3, 5), (5, 3), (5, 5)}
(d) {(3, 3), (3, 4), (4, 5)}

229. If the number 235 in decimal system is converted into binary system, then what is the resulting number ? *[2016-II]*
(a) $(11110011)_2$ (b) $(11101011)_2$
(c) $(11110101)_2$ (d) $(11011011)_2$

230. In an examination, 70% students passed in Physics, 80% students passed in Chemistry, 75% students passed in Mathematics and 85% students passed in Biology, and x% students failed in all the four subjects. What is the minimum value of x ? *[2016-II]*
(a) 10 (b) 12
(c) 15 (d) None of the above

231. A coin is tossed three times. Consider the following events:
A: No head appears
B: Exactly one head appears
C: At least two heads appear
Which one of the following is correct? *[2016-II]*
(a) $A \cup B \cap A \cup C = B \cup C$
(b) $A \cap B' \cup A \cap C' = B' \cup C'$
(c) $A \cap B' \cup C' = A \cup B \cup C$
(d) $A \cap B' \cup C' = B' \cap C'$

232. Let S be the set of all persons living in Delhi. We say that x, y in S are related if they were born in Delhi on the same day. Which one of the following is correct? *[2017-I]*
(a) The relation is an equivalent relation
(b) The relation is not reflexive but it is symmetric and transitive
(c) The relation is not symmetric but it is reflexive and transitive
(d) The relation is not transitive but it is reflexive and symmetric

233. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Then the number of subsets of A containing two or three elements is *[2017-I]*
(a) 45 (b) 120 (c) 165 (d) 330

234. Three-digit numbers are formed from the digits 1, 2 and 3 in such a way that the digits are not repeated. What is the sum of such three-digit numbers? *[2017-I]*
(a) 1233 (b) 1322 (c) 1323 (d) 1332

235. Consider the following in respect of sets A and B : *[2017-I]*
1. $(A - B) \cup B = A$
2. $(A - B) \cup A = A$
3. $(A - B) \cap B = \phi$
4. $A \subseteq B \Rightarrow A \cup B = B$
Which of the above are correct?
(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 3 and 4 (d) 1, 2 and 4

236. In the binary equation *[2017-I]*
$(1p101)_2 + (10ql)_2 = (100r00)_2$
where p, q and r are binary digits, what are the possible values of p, q and r respectively?
(a) 0, 1, 0 (b) 1, 1, 0
(c) 0, 0, 1 (d) 1, 0, 1

237. If $S = (x : x^2 + 1 = 0, x \text{ is real})$, then S is *[2017-I]*
(a) {–1} (b) {0}
(c) {1} (d) an empty set

238. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys in the class is 70 kg and that of girls is 55 kg. What is the number of boys in the class? *[2017-I]*
(a) 50 (b) 55 (c) 60 (d) 100

239. If $x + \log_{10}\left(1 + 2^x\right) = x\log_{10} 5 + \log_{10} 6$ then x is equal to *[2017-II]*
(a) 2, –3 (b) 2 only
(c) 1 (d) 3

240. The remainder and the quotient of the binary division $(101110)_2 \div (110)_2$ are respectively *[2017-II]*
(a) $(111)_2$ and $(100)_2$ (b) $(100)_2$ and $(111)_2$
(c) $(101)_2$ and $(101)_2$ (d) $(100)_2$ and $(100)_2$

241. If E is the universal set and $A = B \cup C$, then the set $E - (E - (E - (E - (E - A))))$ is same as the set *[2017-II]*
(a) $B' \cup C'$ (b) $B \cup C$
(c) $B' \cap C'$ (d) $B \cap C$

242. If $A = \{x : x \text{ is a multiple of } 2\}$, $B = \{x : x \text{ is a multiple of } 5\}$ and $C = \{x : x \text{ is a multiple of } 10\}$, then $A \cap (B \cap C)$ is equal to *[2017-II]*
(a) A (b) B
(c) C (d) $\{x : x \text{ is a multiple of } 100\}$

243. If we define a relation R on the set $N \times N$ as $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$, then the relation is: *[2017-II]*
(a) symmetric only
(b) symmetric and transitive only
(c) equivalence relation
(d) reflexive only

244. If $n = (2017)!$, then what is *[2018-I]*
$$\frac{1}{\log_2 n} \quad \frac{1}{\log_3 n} \quad \frac{1}{\log_4 n} \quad .... \quad \frac{1}{\log_{2017} n}$$
equal to?
(a) 0 (b) 1
(c) $\frac{n}{2}$ (d) n

245. Let A and B be subsets of X and $C = (A \cap B') \cup (A' \cap B)$, where A' and B' are complements of A and B respectively in X. What is C equal to? *[2018-I]*
(a) $(A \cup B') - (A \cap B')$ (b) $(A' \cup B) - (A' \cap B)$
(c) $(A \cup B) - (A \cap B)$ (d) $(A' \cup B') - (A' \cap B')$

246. If $x + \log_{15}(1 + 3^x) = x \log_{15} 5 + \log_{15} 12$, where x is an integer, then what is x equal to? *[2018-I]*
(a) –3 (b) 2 (c) 1 (d) 3

DIRECTION (Qs. 247-248) : *Consider the information given below and answer the two items (02) that follow:*

In a class, 54 students are good in Hindi only, 63 students are good in Mathematics only and 41 students are good in English only. There are 18 students who are good in both Hindi and Mathematics. 10 students are good in all three subjects. *[2018-I]*

247. What is the number of students who are good in either Hindi or Mathematics but not in English?
(a) 99 (b) 107 (c) 125 (d) 130

248. What is the number of students who are good in Hindi and Mathematics but not in English?
(a) 18 (b) 12 (c) 10 (d) 8

249. The binary number expression of the decimal number 31 is *[2018-I]*
(a) 1111 (b) 10111 (c) 11011 (d) 11111

250. What is $\frac{1}{\log_2 N} \quad \frac{1}{\log_3 N} \quad \frac{1}{\log_4 N} \quad .... \quad \frac{1}{\log_{100} N}$ equal to $(N \neq 1)$? *[2018-I]*
(a) $\frac{1}{\log_{100!} N}$ (b) $\frac{1}{\log_{99!} N}$
(c) $\frac{99}{\log_{100!} N}$ (d) $\frac{99}{\log_{99!} N}$

251. What is the greatest integer among the following by which the number $5^5 + 7^5$ is divisible? *[2018-I]*
(a) 6 (b) 8
(c) 11 (d) 12

252. A survey of 850 students in a University yields that 680 students like music and 215 like dance. What is the least number of students who like both music and dance? *[2018-I]*
(a) 40 (b) 45 (c) 50 (d) 55

253. If $0 < a < 1$, the value of $\log_{10} a$ is negative. This is justified by *[2018-I]*
(a) Negative power of 10 is less than 1
(b) Negative power of 10 is between 0 and 1
(c) Negative power of 10 is positive
(d) Negative power of 10 is negative

254. A train covers the first 5 km of its journey at a speed of 30 km/hr and the next 15 km at a speed of 45 km/hr. What is the average speed of the train? *[2018-I]*
(a) 35 km/hr (b) 37.5 km/hr
(c) 39.5 km/hr (d) 40 km/hr

255. What is the value of $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$ equal to? *[2018-II]*
(a) $3 \log_2 7$ (b) $1 - 3 \log_2 7$
(c) $1 - 3 \log_7 2$ (d) $\frac{7}{8}$

256. If A, B and C are subsets of a Universal set, then which one of the following is not correct? *[2018-II]*
(a) $A \cup B \cap C = A \cup B \cap A \cup C$
(b) $A' \cup A \cup B = B' \cap A' \cup A$
(c) $A' \cup B \cup C = C' \cap B' \cap A$
(d) $A \cap B \cup C = A \cup C \cap B \cup C$
Where A' is the complement of A.

257. Let x be the number of integers lying between 2999 and 8001 which have at least two digits equal. Then x is equal to *[2018-II]*
(a) 2480 (b) 2481 (c) 2482 (d) 2483

DIRECTION (Qs. 258-259) : *Consider the information given below and answer the two (02) items that follow:*

A survey was conducted among 300 students. If was found that 125 students like to play cricket, 145 students like to play football and 90 students like to play tennis, 32 students like to play exactly two games out of the three games. *[2018-II]*

258. How many students like to play all the three games?
(a) 14 (b) 21 (c) 28 (d) 35

259. How many students like to play exactly only one game?
(a) 196 (b) 228 (c) 254 (d) 268

260. What is the value of $\log_9 27 + \log_8 32$? *[2018-II]*
(a) $\frac{7}{2}$ (b) $\frac{19}{6}$
(c) 4 (d) 7

261. The sum of the binary numbers $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is the binary number $(101101101)_2$. What are the values of x and y? *[2018-II]*
(a) x = 1, y = 1 (b) x = 1, y = 0
(c) x = 0, y = 1 (d) x = 0, y = 0

262. If $(0.2)^x = 2$ and $\log_{10} 2 = 0.3010$, then what is the value of x to the nearest tenth? *[2018-II]*
(a) – 10.0 (b) – 0.5 (c) – 0.4 (d) – 0.2

263. Suppose X = {1, 2, 3, 4} and R is a relation on X. If R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3). (3, 2)}, then which one of the following is correct? *[2019-I]*
(a) R is reflexive and symmetric, but not transitive
(b) R is symmetric and transitive, but not reflexive

(c) R is reflexiveand transitive, but not symmetric
(d) R is neither reflexive nor transitive, but symmetric

264. A relation R is defined on the set N of natural numbers as $xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$, Then which one of the following is correct ? *[2019-I]*
(a) R is reflexive and symmetric, but not transitive
(b) R is reflexive and transitive, but not symmetric
(c) R is reflexive, symmetric and transitive
(d) R is reflexive, but neither symmetric nor transitive

265. Consider the following statements for the two non-empty sets A and B :
(1) $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup B$
(2) $(A \cup (\overline{A} \cap \overline{B})) = A \cup B$
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

266. Let X be a non-empty set and let A, B, C be subsets of X.
Consider the following statments :
(1) $A \subset C \Rightarrow (A \cap B) \subset (C \cap B)(A \cup B) \subset (C \cap B)$
(2) $(A \cup B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$
(3) $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$
Which of the above statements are correct ?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

267. If $A = \{\lambda, (\lambda, \mu)\}$, then the power set of A is *[2019-I]*
(a) $\{\phi, \{\phi\}, \{\lambda\}, \{\lambda, \mu\}\}$
(b) $\{\phi, \{\lambda\}, \{\{\lambda, \mu\}\}, \{\lambda, \{\lambda, \mu\}\}\}$
(c) $\{\phi, \{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$
(d) $\{\{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$

DIRECTION (Qs. 268-269) : *Consider the following for the next 02 (two) items that follow:*

In a school, all the students play at least one of three indoor games - chess, carrom and table tennis, 60 play chess, 50 play table tennis, 48 play carrom, 12 play chess and carrom, 15 play carrom and table tennis, 20 play table tennis and chess.

268. What can be the minimum number of students in the school? *[2019-I]*
(a) 123 (b) 111 (c) 95 (d) 63

269. What can be the maximum number of students in the school ? *[2019-I]*
(a) 111 (b) 123 (c) 125 (d) 135

270. If $f(x) = \log_{10}(1 + x)$, then what is $4 f(4) + 5f(1) - \log_{10} 2$ equal to ? *[2019-I]*
(a) 0 (b) 1 (c) 2 (d) 4

271. For $r > 0$, f(r) is the ratio of perimeter to area of a circle of radius r. Then $f(1) + f(2)$ is equal to *[2019-I]*
(a) 1 (b) 2 (c) 3 (d) 4

272. In a circle of diameter 44 cm, the length of a chord is 22 cm. What is the length of minor arc of the chord ? *[2019-I]*
(a) $\frac{484}{21}$ cm (b) $\frac{242}{21}$ cm
(c) $\frac{121}{21}$ cm (d) $\frac{44}{7}$ cm

ANSWER KEY

1	(c)	24	(b)	47	(c)	70	(a)	93	(b)	116	(d)	139	(a)	162	(a)	185	(b)	208	(c)	231	(d)	254	(d)
2	(d)	25	(d)	48	(b)	71	(c)	94	(c)	117	(d)	140	(a)	163	(b)	186	(b)	209	(d)	232	(a)	255	(c)
3	(a)	26	(d)	49	(d)	72	(c)	95	(a)	118	(a)	141	(c)	164	(b)	187	(d)	210	(d)	233	(c)	256	(c)
4	(b)	27	(c)	50	(d)	73	(d)	96	(d)	119	(a)	142	(a)	165	(c)	188	(d)	211	(a)	234	(d)	257	(b)
5	(d)	28	(b)	51	(c)	74	(d)	97	(d)	120	(a)	143	(d)	166	(d)	189	(c)	212	(c)	235	(b)	258	(a)
6	(d)	29	(d)	52	(b)	75	(c)	98	(c)	121	(b)	144	(d)	167	(c)	190	(b)	213	(d)	236	(a)	259	(c)
7	(b)	30	(d)	53	(a)	76	(a)	99	(b)	122	(d)	145	(c)	168	(c)	191	(b)	214	(b)	237	(d)	260	(b)
8	(b)	31	(b)	54	(d)	77	(c)	100	(d)	123	(c)	146	(c)	169	(a)	192	(c)	215	(a)	238	(a)	261	(b)
9	(a)	32	(d)	55	(a)	78	(c)	101	(c)	124	(b)	147	(a)	170	(d)	193	(c)	216	(b)	239	(c)	262	(c)
10	(c)	33	(b)	56	(d)	79	(a)	102	(b)	125	(c)	148	(c)	171	(d)	194	(a)	217	(a)	240	(b)	263	(d)
11	(b)	34	(d)	57	(a)	80	(b)	103	(d)	126	(c)	149	(d)	172	(c)	195	(c)	218	(d)	241	(c)	264	(d)
12	(d)	35	(a)	58	(d)	81	(b)	104	(c)	127	(a)	150	(b)	173	(b)	196	(b)	219	(a)	242	(c)	265	(a)
13	(d)	36	(a)	59	(c)	82	(a)	105	(c)	128	(a)	151	(d)	174	(d)	197	(a)	220	(a)	243	(c)	266	(b)
14	(d)	37	(b)	60	(d)	83	(a)	106	(b)	129	(c)	152	(b)	175	(c)	198	(c)	221	(a)	244	(b)	267	(b)
15	(b)	38	(c)	61	(b)	84	(c)	107	(c)	130	(b)	153	(c)	176	(c)	199	(b)	222	(c)	245	(c)	268	(b)
16	(c)	39	(a)	62	(c)	85	(a)	108	(a)	131	(c)	154	(a)	177	(c)	200	(d)	223	(c)	246	(c)	269	(b)
17	(b)	40	(a)	63	(c)	86	(a)	109	(d)	132	(d)	155	(d)	178	(c)	201	(c)	224	(b)	247	(c)	270	(d)
18	(c)	41	(b)	64	(b)	87	(c)	110	(d)	133	(b)	156	(b)	179	(c)	202	(b)	225	(d)	248	(d)	271	(c)
19	(c)	42	(b)	65	(a)	88	(d)	111	(d)	134	(d)	157	(d)	180	(a)	203	(d)	226	(b)	249	(d)	272	(a)
20	(b)	43	(b)	66	(c)	89	(b)	112	(b)	135	(b)	158	(a)	181	(c)	204	(b)	227	(c)	250	(a)		
21	(d)	44	(c)	67	(c)	90	(c)	113	(b)	136	(d)	159	(d)	182	(d)	205	(b)	228	(c)	251	(d)		
22	(b)	45	(b)	68	(b)	91	(c)	114	(c)	137	(d)	160	(a)	183	(c)	206	(c)	229	(c)	252	(b)		
23	(c)	46	(c)	69	(b)	92	(b)	115	(a)	138	(b)	161	(b)	184	(c)	207	(c)	230	(d)	253	(b)		

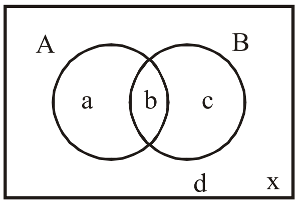
HINTS & SOLUTIONS

1. (c) $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$
Solving for values of x, we get
$= \{0, 1, 2, 3\}$
$A = \{x : x^2 - 5x + 6 = 0\}$
Solving for values of x, we get
$= \{2, 3\}$
and $B = \{x : x^2 - 3x + 2 = 0\}$
Solving for values of x, we get
$= \{2, 1\}$
$A \cap B = \{2\}$
$\therefore$ $(A \cap B)' = U - (A \cap B)$
$= \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$

2. (d) If A denotes the collection of all complex number whose square is a negative real number, then
Square of a complex number is a negative real number only if it has no real part and has only imaginary part.
Hence, $A = \{iy : y \in R\}$

3. (a) $\because$ R is a relation defined on the set Z of integers as follows:
mRn $\Leftrightarrow$ m + n is odd
(1) Then, mRm = 2m and nRn = 2n are not odd multiples of 2 are not odd. Thus, it is not reflexive.
(2) If m and n are numbers such that mRn $\Leftrightarrow$ m + n is odd.
Thus, nRm $\Leftrightarrow$ n + m is odd.
$\therefore$ This relation is symmetric.
(3) mRn = m + n, if there is third number p and nRp = n + p is odd. (for ex: 2 + 3 = 5 is odd 3 + 4 = 7 is odd. But, 2 + 4 = 6 is not odd.) Then mRp = m + p may not be odd. So, this relation is not transitive.

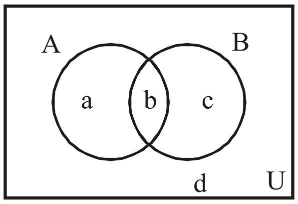
4. (b) A and B are subsets of X and its, Venn diagram is shown.



A – B indicates region a, B – A indicates c, $A \cup B$ indicates a, b, c.
If a, c ≡ a, b, c indicates, the b is zero. A and B are mutually exclusive. So, when
$(A - B) \cup (B - A) = A \cup B$
$A \subset (X - B)$

5. (d) $A = \{(n, 2n) : n \in N\}$ and $B = \{(2n, 3n)\} : n \in N$
Listing few members of each set
A = {(1, 2), (2, 4), (3, 6),....}
B = {(2, 3), (4, 6), (6, 9)......}
There is no member common to both these sets, hence.
$A \cap B = \phi$

6. (d) Let there be two sets A and B and universal set of A and B, be U.
Then drawing these sets on a Venn-diagram, four regions are created as shown in the figure :



B ≡ regions b, c
B' ≡ regions a, d
A ≡ regions a, b
A' ≡ regions c, d
A – B ≡ region a
B – A ≡ region c
B' – A' ≡ region a
A' – B' ≡ region c
B – A' ≡ region b
A' – B ≡ region d
From these we check the operations given in the choice.
choices (a), (b) and (c) are correct
(d) LHS = region $a \cap c = \phi$
RHS = region $b \cup$ region d = b, d.
So, for (d) LHS ≠ RHS

7. (b) We take option (a) : $x^3 - 8x^2 + 19x - 12 = 0$
$\Rightarrow$ $(x - 1)(x^2 - 7x + 12) = 0$
$\Rightarrow$ $(x - 1)(x - 3)(x - 4) = 0$
$\Rightarrow$ $x = 1, 3, 4$
Thus, it is not a set of elements as odd positive integers.
(b) $x^3 - 9x^2 + 23x - 15 = 0$
$\Rightarrow$ $(x - 1)(x^2 - 8x + 15) = 0$
$\Rightarrow$ $(x - 1)(x - 3)(x - 5) = 0$
$\Rightarrow$ $x = 1, 3, 5$
Thus, S will be a set of elements as odd positive integers.

8. (b) aRb means b lives with one km from a, and bRa means a lives within one km from b but distance from a to b = distance from b to a.
So, R is symmetric.

9. (a) Number of elements in X, is n, then the number of relations on X means, number of elements of cartesian product X × X.
Since, n (X) = n.
So, n (X × X) = n.n
then the total number of relations is $2^{n.n} = 2^{n^2}$

10. (c) As given $A = \{(x, y) | x + y \le 4\}$
and $B = \{(x, y) | x + y \le 0\}$
Set A contains all the pairs in the interval $(-\infty, 2)$ and set B contains all the pairs in the interval $(-\infty, 0)$ so, $A \cap B$ shows a set containing all the pairs in the interval $[-\infty, 0]$
So, $A \cap B = \{(x, y) | x + y \le 0\}$

11. (b) Total number of students = 500
Let H be the set showing number of students who can speak Hindi = 475 and B be the set showing number of students who can speak Bengali = 200
So, n (H) = 475 and n (B) = 200 and given that $n(B \cup H) = 500$
we have
$n(B \cup H) = n(B) + n(H) - n(B \cap H)$
$\Rightarrow 500 = 200 + 475 - n(B \cap H)$
so, $n(B \cap H) = 175$
Hence, persons who speak Hindi only $= n(H) - n(B \cap H)$ $= 475 - 175 = 300$

12. (d) The correct relations as per De Morgan's theorem is $(R_1 \cap R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$

13. (d) Converting from binary to decimal
$(1001)_2 = 1 \times 2^3 + 2^0 = 8 + 1 = 9$
$(11)_2 = 2^1 + 2^0 = 2 + 1 = 3$
$(101)_2 = 2^2 + 2^0 = 4 + 1 = 5$
$(10)_2 = 2^1 = 2$
and $(01)_2 = 1$

$$\therefore \frac{(1001)_2^{(11)_2} - (101)_2^{(11)_2}}{(1001)_2^{(10)_2} \quad (1001)_2^{(01)_2}(101)_2^{(01)_2} \quad (101)_2^{(10)_2}}$$

$$= \frac{9^3 - 5^3}{9^2 + 9 \times 5 + 5^2}$$

$$= \frac{(9-5)(9^2 + 9 \times 5 \quad 5^2)}{(9^2 + 9 \times 5 \quad 5^2)} = \frac{4 \times (81 + 45 + 25)}{(81 + 45 + 25)}$$

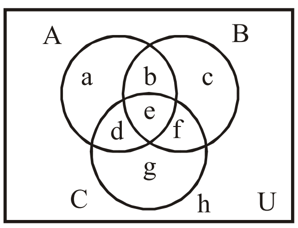
$= 4 = (100)_2$ [Converting from decimal to binary]

14. (d) All statements are correct.

15. (b) If A, B and C are any three arbitrary events occurrence of both A and B is given by $A \cap B$ and non-occurrence of C as $\overline{C}$ then the event both A and B occur but not C is represented by $A \cap B \cap \overline{C}$.

16. (c) Given that $P = \{p_1, p_2, p_3, p_4\}$
$Q = \{q_1, q_2, q_3, q_4\}$
and $R = \{r_1, r_2, r_3, r_4\}$
$S_{10} = \{p_2, q_4, r_4), (p_3, q_3, r_4), (p_3, q_4, r_3), (p_4, q_2, r_4),$
$(p_4, q_3, r_3), (p_4, q_4, r_2)\}$
$\therefore$ Total number of elements in S_{10} are 6.

17. (b) Let a Venn-diagram be drawn taking three intersecting sets A, B and C under a universal set U. This makes 8 regions a to h as shown.



A has regions a, b, d, e
B has regions b, c, e, f
C has regions d, e, f, g
C' has regions a, b, c, h
B' has regions a, d, g, h
Statement (a) : $A \cup (B - C) = A \cap (B \cap C)$
LHS ≡ (a, b, e, d) ∪ b, c ≡ a, b, c, d, e.
RHS ≡ a, b, d, e ∩ e, f ≡ e,
So, statement (a) is not correct.
Statement (b) : $A - (B \cup C) = (A \cap B') \cap C'$
LHS ≡ (a, b, d, e) – (b, c, d, e, f, g) ≡ a.
RHS ≡ (a, b, d, e ∩ a, d, g, h) ∩ (a, b, c, h) ≡ a,
So, statement (b) is correct.
Correct statement is :
$A - (B \cup C) = (A \cap B') \cap C'$

18. (c) The maximum three digit integer in decimal system = 999. We go on dividing till we get a dividend < 2 and write remainders from last to first as shown below:

2	999	
2	499	1
2	249	1
2	124	1
2	62	0
2	31	0
2	15	1
2	7	1
2	3	1
	1	1

Hence, $(999)_{10} = (1111100111)_2$

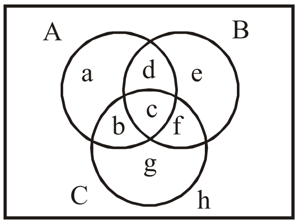
19. (c) The largest four digit number in binary system is 1111 which is equivalent to 15 in decimal system and smallest five digit number in binary system is 10000 which is equivalent to 16 in decimal system.
Difference between numbers = 16 – 15 = 1
Which is the greatest one digit binary integer.

20. (b) Given that F(n) = set of all divisors of n except 1
$\therefore$ F(20) = {2, 4, 5, 10, 20}
and F(16) = {2, 4, 8, 16}
$\therefore$ $F(20) \cap F(16) = \{2, 4, 5, 10, 20\} \cap \{2, 4, 8, 16\}$ $= \{2, 4\}$
Also, $\{F(20) \cap F(16)\} \subseteq F(y)$
So, least value of y = 2

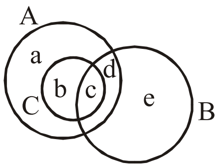
21. (d) The given relation is
aRb ⇔ a + 2b, is an integral multiple of 3.
In this relation
aRa ⇔ a + 2a = 3a, an integral multiple of 3. So, it is reflexive
aRb ⇔ a + 2b and
bRa = b + 2a + 4b – 4b
= 2(a + 2b) – 3b is also an integral multiple of 3.
So, it is symmetric.
Let there be another value c, bRc = b + 2c, be an integral multiple of 3.
Then aRc = a + 2c
So, aRb + bRc = a + 2b + b + 2c = a + 2c + 3b is integral

multiple of 3, hence, $a + 2c$ is also integral multiple of 3 so, aRb and bRc $\Rightarrow$ aRc. So, it is transitive.
Therefore, relation is reflexive, symmetric as well as transitive. Hence, R is an equivalence relation.

22. (b) A Venn diagram is drawn for 3 intersecting set A, B, C under a universal set U; creating 8 regions in total named, a to h as shown



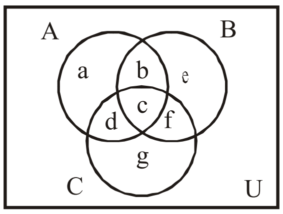
Statement P: $A \cap (B \cup C) = (A \cap B) \cup C$
LHS $\equiv$ a, b, c, d, $\cap$ (b, c, d, e, f, g,) $\equiv$ b, c, d.
RHS $\equiv$ (c, d) $\cup$ (b, c, f, g) = b, c, d, f, g
If P is correct then region f, g do not lie in set C and set C has regions b, c only.
This follows that C is subset of A. Since, Set A has regions a, b, c, d and C has regions b, c.
Thus, P $\Rightarrow$ Q.
Also, if C is a subset of A, Q is true, then the Venn diagram appears as below:
$\{A \cap (B \cup C)\}$



LHS of P statement gives
region a, b, c, d $\cap$ region b, c, d, e $\equiv$ b, c, d.
RHS: $\{(A \cap B) \cup C\}$ gives : region c, d, $\cup$ region b,c.
$\equiv$ b, c, d
and LHS = RHS shows Q $\Rightarrow$ P,
Comparing both gives P $\Leftrightarrow$ Q.

23. (c) $X = \{x : x > 0, x^2 < 0\}$
We know that the square of each number greater than zero is always greater than zero. So, X contains no member and so, X is null set but a well defined set.
Also, Y = {flower, Churchill, Moon, Kargil} is well defined. So, Y is also a well defined set.

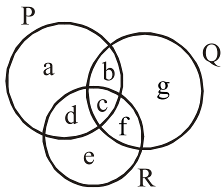
24. (b) A Venn diagram of the three non-empty and intersecting sets is drawn, dividing into 7 regions, a to g as shown below and consider statements one by one.
Statement (1): $A - (B \cup C) = (A - B) \cup (A - C)$



LHS $\equiv$ regions a, b, c, d – b, c, d, e, f, g $\equiv$ a.
RHS $\equiv$ a, d, $\cup$ a, b $\equiv$ a, b, d.
So, statement (1) is not correct.
Statement (2) : $A - B = A - (A \cap B)$
LHS $\equiv$ region a, d.
RHS $\equiv$ region a, b, c, d – (region b, c,) = a, d.
So, statement (2) is correct.
Statement (3): $A = (A \cap B) \cup (A - B)$
LHS $\equiv$ regions a, b, c, d.
RHS = regions b, c, $\cup$ region a, d $\equiv$ a, b, c, d so, statement (3) is correct.

25. (d) There are infinitely many rational numbers between two distinct integers, so, statement 1 is correct. Same is true in case of two distinct rational numbers and real numbers. So, statement (2) and (3) are also correct.

26. (d) The shaded region represents $(P \cap Q) \cup (P \cap R)$.
Let the intersecting sets P, Q, R divide it into 7 regions marked, a to g as shown below.



The shaded part contains regions b, c, and d.
(a) $(P \cup Q) - (P \cap Q)$ $\equiv$ regions a, b, c, d, f, g, – b, c
$\equiv$ a, d, f, g, . not correct.
(b) $(P \cap (Q \cap R)$ $\equiv$ a, b, c, d, $\cap$ c, f $\equiv$ c not correct.
(c) $(P \cap Q) \cap (P \cap R)$ $\equiv$ regions b, c, $\cap$ region, c, d $\equiv$ c, so not correct
(d) $(P \cap Q) \cup (P \cap R)$ $\equiv$ regions b, c, $\cup$ c, d $\equiv$ b, c, d so correct.

27. (c) Given that $a^x = b$, $b^y = c$, $c^z = a$
$\Rightarrow$ $c^z = b^{yz} = a$ $\Rightarrow b^{yz} = a^{xyz} = a$
$\Rightarrow$ xyz=1

$$\text{Now, } \frac{1}{(xy + yz + zx)}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$$= \frac{1}{(xy + yz + zx)}\left(\frac{xy + yz + zx}{xyz}\right) = \frac{1}{xyz} = 1$$

28. (b) Given that $2^x = 3^y = 12^z = k$
Taking $\log_2$ on both the sides
$x = \log_2 k$, $y = \log_3 k$ and $z = \log_{12} k$

$$\frac{x + 2y}{xy} = \frac{\log_2 k + 2\log_3 k}{\log_2 k \log_3 k}$$

$$= \frac{1}{\log_3 k} + \frac{2}{\log_2 k}$$

$$= \log_k 3 + 2\log_k 2 = \log_k 3 + \log_k 4$$

$$= \log_k 12 = \frac{1}{\log_{12} k} = \frac{1}{z}$$

29. (d) Number of subsets of X containing less than 5 elements is given by

$^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4$

$$\sum_{r=0}^{4} {}^nC_r = \sum_{r=0}^{4} C(n, r)$$

30. (d) In the given sets, the set of all primes is an infinite set.

31. (b) $0.\overline{2} + 0.\overline{23}$

$$= \frac{2}{9} + \frac{23}{99} = \frac{22+23}{99} = \frac{45}{99} = 0.\overline{45}$$

32. (d) Given: $3^{(x-1)} + 3^{(x+1)} = 30$

$\Rightarrow \frac{3^x}{3} + 3.3^x = 30$ ...(i)

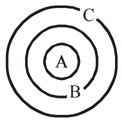
Multiplying both the sides by 3 in equation (i)

$3^x + 3^2 .3^x = 90$

$\Rightarrow 3^x + 3^{x+2} = 90$

33. (b) $f(\theta) = 0$, if θ is an integral multiple of π. From -100π to 0π there are 101 values of θ for which $f(\theta) = 0$.
From π to 1000π, there are 1000 values for which $f(\theta) = 0$
so, total values number is 101 + 1000 = 1101

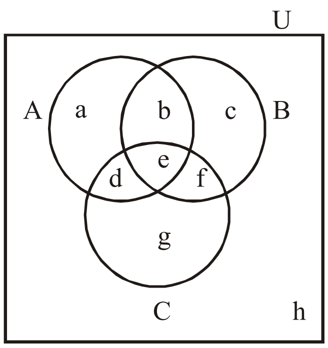
34. (d) $\because A \cup B = B \cap C$
Since union of $A \cup B$ is same as infersection $B \cap C$ where ABC are non-empty subsets of X, strongest inference is $A \subseteq B \subseteq C$ which can be shown in Venn diagram as below :



35. (a) Since μ is universal set
and $P \subseteq \mu$, $P - \mu = \phi$ and $\mu - P = P'$
So, $(P - \mu) \cup (\mu - P) = \phi \cup P' = P'$
Now, $P \cap \{P - \mu) \cup (\mu - P)\} = P \cap P' = \phi$

36. (a) As given : μ = the set of all triangles
P = the set of all isosceles triangles
Q = the set of all equilateral triangles
R = the set of all right angled triangles
$\therefore$ $P \cap Q$ represents the set of isosceles triangles and $R - P$ represents the set of non-isosceles right angled triangles.

37. (b) Let there be three non empty, non overlapping sets; inside a universal set U. This creates 8 regions marked as: a, b, c, d, e, f, g, h.



Statement 1: $A - (B - C) = (A - B) \cup C$
LHS represent region a, RHS represents a, d, g.
Hence, this is not correct.
Statement 2: $A - (B \cup C) = (A - B) - C$
LHS represents, region 'a' RHS also represents a.
Hence, only statement 2 is correct.

38. (c) R is defined over the set of non negative integers, $x^2 + y^2 = 36$

$\Rightarrow y = \sqrt{36 - x^2} = \sqrt{(6-x)(6+x)}, x = 0 \text{ or } 6$
for x = 0, y = 6 and for x = 6, y = 0
So, y is 6 or 0
so, R = {(6, 0), (0, 6)}

39. (a) Statement 1: Let l, m, n are parallel line and R is a relation.
$\therefore$ $l \parallel l$, then R is reflexive.
and $l \parallel m$ and $m \parallel l$, the R is symmetric.
also $l \parallel m$, $m \parallel n \Rightarrow l \parallel n$, then R is transitive.
Hence, R is an equivalence relation.
Statement 2: x is father of y then x is not the father of x, so relation is not reflexive.
Also, x is father of y but y is not father of x, so it is not symmetric.
And x is father of y and y is father of z does not imply that x is father of z so, it is not transitive too. So, this is not an equivalence relation. so, only statement 1 is correct.

40. (a) $111101 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$
$= 32 + 16 + 8 + 4 + 1 = 61$
Which is a prime number.

(b) $111010 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1$
$= 32 + 16 + 8 + 2 = 58$
Which is not a prime number.

(c) $111111 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$
$= 32 + 16 + 8 + 4 + 2 + 1 = 63$
Which is not a prime number.

(d) $100011 = 1 \times 2^5 + 1 \times 2^1 + 1 \times 2^0$
$= 32 + 2 + 1 = 35$
Which is not a prime number
Thus, option (a) is correct.

41. (b) $1001.01 = 1 \times 2^3 + 1 \times 2^0 + 1 \times 2^{-2}$,
corresponding to number of base 10.

$$= 8 + 1 + \frac{1}{4} = \frac{37}{4} = 9.25$$

and $11.1 = 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1}$

$$= 2 + 1 + \frac{1}{2} = \frac{7}{2} = 3.5$$

Corresponding to the number of base 10.
$\therefore$ $1001.01 \times 11.1 = 9.25 \times 3.5 = 32.375$
From decimal to binary
$(32)_{10} = (100000)_2$
and $(.375)_{10} = 0.25 + 0.125$

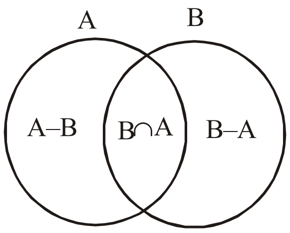
$$= \frac{1}{4} \; \frac{1}{8} = 1 \times 2^{-2} + 1 \times 2^{-3} = (0.011)_2$$

$\therefore$ $(32.375)_{10} = (100000.011)_2$

42. (b) An equation of the form ax +by + cz = d, where a, b, c, d are real number, not all zero, is linear.
$\Rightarrow$ $2x + y - z = 5$
and $\pi x + y - ez = \log 3$ are linear.

43. (b) $\because\ (10101)_2 = 2^4 \times 1 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2 + 1 \times 2^0$
$= 16 + 4 + 1 = 21$
and $(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 8 + 4 + 1 = 13$
$\therefore\ (10101)_2 \times (1101)_2 = 21 \times 13$
$= 273 = 256 + 16 + 1 = 2^8 + 2^4 + 2^0$
So, there will be 1 at 9th, 5th and first place from right and zero at other places
So, $(273)_{10} = (100010001)_2$

44. (c) We draw the Venn diagram,



$A - B = A - (A \cap B)$ and $B - A = B - (A \cap B)$
$A - B = B - A$
$\Rightarrow A - (A \cap B) = B - (A \cap B)$
$\Rightarrow A = B$

45. (b) The given number
$\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$
can be written as :
$(2 \quad \sqrt{5})^{1/3} \quad (2-\sqrt{5})^{1/3}$
$= 2^{1/3}\left[1 \quad \frac{1}{2}\sqrt{5}\right]^{1/3} \quad 2^{1/3}\left[1-\frac{1}{2}\sqrt{5}\right]^{1/3}$
$= 2^{1/3}\left[1 \quad \frac{1}{6}\sqrt{5} \quad \ldots \quad 1-\frac{1}{6}\sqrt{5} \quad \ldots\right]$
Thus the given number is a rational number but not an integer.

46. (c) For subsets A and B of U,
If $(A - B) \cup (B - A) = A$,
$\Rightarrow B = \phi$.

47. (c) From the given data
n (U) = 700, n (A) = 200, n (B) = 300 and
$n(A \cap B) = 100$.
We know that,
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 200 + 300 - 100 = 400$
Now, $n(A' \cap B') = \cap(A \cup B)' = n(U) - n(A \cup B)$
$= 700 - 400 = 300$

48. (b) In the given Venn diagram shaded region is
$C \cup (C' \cap A \cap B)$

49. (d) A relation is equivalent if it is
(i) Reflexive
(ii) Symmetric and
(iii) Transitive
We check for the same, one by -one
$x, y \in N \Rightarrow x > 0, y > 0$
$R = \{(x, y) \mid xy > 0, x, y, \in N\}$
(i) Reflexive
$\because$ $x, y \in N$
$\therefore$ $x, x, \in N \Rightarrow x^2 > 0$
$\therefore$ R is reflexive
(ii) Symmetric
$\because$ $x, y \in N$
and $xy > 0 \Rightarrow yx > 0$
$\therefore$ R is also symmetric
(iii) Transitive
$\because$ $x, y, z \in N$
$\Rightarrow$ $xy > 0, yz > 0 \quad \Rightarrow \quad xz > 0$
$\therefore$ R is also transtive.
Conclusion : R is an equivalence relation.

50. (d) The given logarithm expression
$$\frac{\log_{27} 9 \log_{16} 64}{\log_4 \sqrt{2}}$$
is simplified as :
$$\frac{\log 9}{\log 27} \times \frac{\log 64}{\log 16} \times \frac{\log 4}{\log \sqrt{2}}$$
$$= \frac{2\log 3}{3\log 3} \times \frac{6\log 2}{4\log 2} \times \frac{2\log 2}{\frac{1}{2}\log 2}$$
$$= \frac{2}{3} \times \frac{6}{4} \times 4 \quad 4$$

51. (c) Elements of a population are classified according to the presence or absence of each of 3 attributes A, B and C. Then, the smallest number of smallest ultimate classes into which the population is divided, is $2^3 = 8$

53. (a) Both (A) and (R) are true and R is the correct explanation of A.

54. (d) Given equation is :
$\log_{10}\{999 + \sqrt{x^2 - 3x \quad 3}\} \quad 3$
$\Rightarrow$ $999 + \sqrt{x^2 - 3x + 3} = 10^3 = 1000$
$\Rightarrow$ $\sqrt{x^2 - 3x \quad 3} \quad 1$
$\Rightarrow$ $x^2 - 3x + 3 = 1$
$x^2 - 3x + 2 = 0$
$\Rightarrow$ $x^2 - 2x - x + 2 = 0$
$\Rightarrow$ $x(x-2) - 1(x-2) = 0$
$\Rightarrow$ $(x-1)(x-2) = 0$
$\Rightarrow$ $x = 1, 2.$

55. (a) $\because$ $f(x) = \tan x$
f(x) is increasing in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Hence, f(x) is injective in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

56. (d) When $f(x) = e^x$

$f(x) \neq 0, \ \forall x \in R$

$\Rightarrow$ An exponential function is never zero.

57. (a) Since $x^2 < 0$ is not possible for real numbers. A is true.

Since $x^2 > 0$ for $\forall x \in R$. Both (A) and (R) are true and (R) is the correct explanation of (A).

58. (d) For reflexive :

aRa $\Rightarrow$ a divides a

$\therefore$ R is reflexive.

For symmetric :

aRb $\Rightarrow$ a divides b

bRa $\Rightarrow$ b divides a

which may not be true

$\Rightarrow$ R is not symmetric.

For transitive

aRB $\Rightarrow$ a divides b $\Rightarrow$ b = ka

bRc $\Rightarrow$ b divides c $\Rightarrow$ c = lb

Now, c = lka

$\Rightarrow$ a divides c

$\Rightarrow$ a Rc

$\Rightarrow$ aRb, bRc $\Rightarrow$ c R a

$\Rightarrow$ R is transitive.

59. (c) Given equation is :

$10^{\log_{10}|x|=2}$

Taking $\log_{10}$ on both sides

$\Rightarrow \log_{10}|x| = \log_{10} 2$

$\Rightarrow |x| = 2$

$\Rightarrow x = 2$ or -2

60. (d) Both statements are incorrect.

61. (b) In the given Venn diagram, shaded region shows $(A \cup B) \cap C$.

62. (c) Let binary number 0.1111111... = x

$$\Rightarrow x = 2^{-1} + 2^{-2} + 2^{-3} + ... \infty = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\infty$$

This is an infinite G.P. series with first term = $\frac{1}{2}$ and common ratio = $\frac{1}{2}$

$$\Rightarrow x = \frac{1/2}{1 - \frac{1}{2}} = \frac{1/2}{1/2} = 1$$

63. (c) The given binary number $10001100 = 1 \times 2^7 + 1 \times 2^3 + 1 \times 2^2$

$= 128 + 8 + 4 = 140$ (decimal numbers)

and $1101101 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$

$= 64 + 32 + 8 + 4 + 1 = 109$

their difference = 140 – 109 = 31

64. (b) Given sets in set builder form are :

$A = \{x \in R : -9 \leq x < 4\}$

$B = \{x \in R : -13 < x 5\}$

and $C = \{x \in R : -7 \leq x \leq 8\}$

$\Rightarrow A \cap B \cap C = \{x \in R : -7 \leq x < 4\}$

Hence, $-7 \in (A \cap B \cap C)$

65. (a) $A \cup P(A) = P(A)$ is correct.

Since A is a subset of its power set.

66. (c) $f(x^2) + 2 = x^2 + \frac{1}{x^2} + 2$

$$= \left(x + \frac{1}{x}\right)^2 = \{f(x)\}^2$$

and $f(x^3) + 3f(x)$

$$= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 = \{f(x)\}^3$$

Thus, both 1 and 2 are correct.

67. (c) Since, set A contains 4 elements, then number of elements in $P(A) = 2^4 = 16$

So, the number of elements in $A \times P(A) = 4 \times 16 = 64$

68. (b) Let A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.

Let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ ...(1)

Since, $A \cup B = A \cup C$ there fore

$x \in A$ or $x \in C$ ...(2)

Also, Given $A \cap B = A \cap C$

$\therefore$ $x \in A \cap B \Rightarrow x \in A$ and $x \in B$ ...(3)

and $x \in A$ and $x \in C$ ($\because A \cap B = A \cap C$) ...(4)

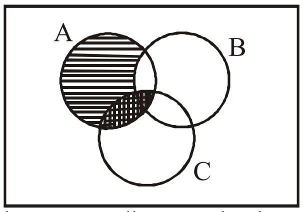
Thus, from (1), (2), (3), and (4), we have $B = C$ only

69. (b) $(2 + \sqrt{2})^2 = 4 + 2 + 4\sqrt{2} = 6 + 4\sqrt{2}$

So, it is an irrational number

70. (a) $A \cap (A' \cup B) = A \cap A' = \phi$

71. (c) Following venn diagram shows the relation A – (B–C)



In the above venn diagram, horizontal lines shows (A–B) and vertical lines show $(A \cap C)$

$\therefore (A - B) \cup (A \cap C) = A - (B - C)$

72. (c) On applying Demorgan's law, we get

$A \cap (A \cup B)' = A \cap (A' \cap B')$

On applying associative law, we get,

$= (A \cap A') \cap B' = \phi \cap B' = \phi$

73. (d) If the relation is defined by option (d), then each 1, 2 and 3 has two images. So, it is not a function.

74. (d) Since, $3 \in A$

but $(3, 3) \notin R$

So, it is not reflexive.

and $(3, 4) \in R$ and $(4, 3) \in R$

but (3, 3) $\notin$ R
So, it is also not transitive.
Hence, R is neither reflexive nor transitive.

75. (c) We know

$$X - Y = X \cap Y'$$
$$(X - Y)' = (X \cap Y')'$$
$$= X' \cup (Y')' = X' \cup Y$$

76. (a) Work with option

$(0.1101)_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-4}$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = \frac{13}{16} = (0.8125)_{10}.$$

Hence option (a) gives the binary equivalent of given decimal number.

77. (c) We know that Total number of proper subsets of a finite set with n elements is $2^n - 1$

78. (c) We know that

$$[(A \cup B) \cap C]' = A' \cap B' \cup C'$$

79. (a) Since, A and B are subsets of set X therefore $A \subseteq X$ and $B \subseteq X$

Consider $\{(A \cap (X - B)\} \cup B$

$= (A \cap B') \cup B$

$= A \cup B \qquad (\because B' \cap B \quad B)$

80. (b) Let sets A and B have m and n elements respectively. The set made by subsets of a finite sets A and B is known as power set i.e. P(A) and P(B).

We know, if set A has n elements then P(A) has 2^n elements.

Thus, The total no. of subsets of a finite set A = 2^m and set B = 2^n

So, According to the question

$2^m - 2^n = 56$

$\Rightarrow 2^n(2^{m-n} - 1) = 8 \times 7 = 2^3 \times (2^3 - 1)$

On comparing the powers, both side

$n = 3$ and $m - n = 3$

$\Rightarrow m = 6$ and $n = 3$

81. (b) We know that

$A \times (B - C) = (A \times B) - (A \times C)$

82. (a) Total no. of students = 100

Let E denote the students who have passed in English.
Let M denote the students who have passed in Maths.

$\therefore \; n(E) = 75, n(M) = 60$ and $n(E \cap M) = 45$

we know $n(E \cup M) = n(E) + n(M) - n(E \cap M)$

$= 75 + 60 - 45 = 90$

Required number of students = 90 – 45 = 45

83. (a) Let $R = \; x : x \in N, x$ is a multiple of 3 and $x \leq 100$

and $S = \; x : x \in N, x$ is a multiple of 5 and $x \leq 100$

$\therefore \; R = \{3, 6, 9, 12, 15, \ldots, 99\}$

and $S = \{5, 10, 15, \ldots, 95, 100\}$

Now, $(R \times S) \cap (S \times R) = (R \cap S) \times (S \cap R)$

$= (15, 30, 45, 60, 75, 90) \times (15, 30, 45, 60, 75, 90)$

$\therefore$ Number of elements in $(R \times S) \cap (S \times R)$

$= 6 \times 6 = 36$

84. (c) Let A = $\{a, b, c\}$ and

R = $\{(a, a), (a, b), (b, c), (b, b), (c, c), (c, a)\}$

Since, $(a, a), (b, b), (c, c) \in R$

$\therefore$ R is reflexive relation.

But $(a, b) \in R$ and $(b, a) \notin R$.

$\therefore$ R is not symmetric relation.

Also, $(a, b), (b, c) \in R$

$\Rightarrow$ $(c, a) \in R$ But $(a, c) \notin R$

$\therefore$ R is not transitive relations.

85. (a) Let $\log_{10}(x + 1) + \log_{10} 5 = 3$

$\Rightarrow \log_{10} 5(x + 1) = 3 \quad (\because \log m + \log n = \log mn)$

$\Rightarrow 5(x + 1) = 10^3$

$\Rightarrow (x \quad 1) \quad \frac{1000}{5} \quad 200$

$\Rightarrow x = 200 - 1 = 199$

86. (a) $2\log_8 2 - \frac{\log_3 9}{3} = 2\log_{2^3} 2 - \frac{\log_3 3^2}{3}$

$= \frac{2}{3}\log_2 2 - 2\frac{\log_3 3}{3} = \frac{2}{3} - \frac{2}{3} \quad 0 \quad (\because \log_a a = 1)$

87. (c) Consider $(101.101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$4 \quad 1 \quad \frac{1}{2} \quad \frac{1}{8} \quad \frac{40 \quad 4 \quad 1}{8} \quad \frac{45}{8} \quad (5.625)_{10}$

88. (d) Given

A = $\{x : x \leq 9, x \in N\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Total possible multiple of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27

But 3 and 27 are not possible because 3 and 27 can not be express as such that a + b + c is multiple of 3

$6 \rightarrow 1 \quad 2 \quad 3$

$9 \rightarrow 2 \quad 3 \quad 4, 5 \quad 3 \quad 1, 6 \quad 2 \quad 1$

$12 \rightarrow 9 \quad 2 \quad 1, 8 \quad 3 \quad 1, 7 \quad 1 \quad 4, 7 \quad 2 \quad 3,$

$6 \quad 4 \quad 2, 6 \quad 5 \quad 1, 5 \quad 4 + 3$

$15 \rightarrow 9 \quad 4 \quad 2, 9 \quad 5 \quad 1, 8 \quad 6 \quad 1, 8 \quad 5 \quad 2,$

$8 \quad 4 \quad 3, 7 \quad 6 \quad 2, 7 \quad 5 \quad 3, 6 \quad 5 \quad 4$

$18 \rightarrow 9 \quad 8 \quad 1, 9 \quad 7 \quad 2, 9 \quad 6 \quad 3,$

$9 \quad 5 \quad 4, 8 \quad 7 \quad 3, 8 \quad 6 \quad 4, 7 \quad 6 + 5$

$21 \rightarrow 9 \quad 8 \quad 4, 9 \quad 7 \quad 5, 8 \quad 7 \quad 6$

$24 \rightarrow 9 \quad 8 \quad 7$

Hence, total largest possible subsets are 30.

89. (b) Given, A = {–1, 2, 5, 8} and B = {0, 1, 3, 6, 7}

Since, R be the relation 'is one less than' from A to B

$\therefore$ R = {(–1, 0), (2, 3), (5, 6)}

Hence, R contains 3 elements.

90. (c) The 11th term of the group is

(56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66)

$$\text{sum} = \frac{n(n^2 + 1)}{2} = \frac{11(121 + 1)}{2} = 11 \times 61 = 671$$

Their sum is 671

91. (c) Consider $\dfrac{(\log_{27} 9)(\log_{16} 64)}{\log_4 \sqrt{2}}$

$= \dfrac{\log_{3^3}(3^2)\log_{4^2}(4)^3}{\log_{2^2}(2^{1/2})}$

$= \dfrac{\frac{2}{3}\log_3 3 \times \frac{3}{2}\log_4 4}{\frac{1}{2\times 2}\log_2 2} = \dfrac{1}{\frac{1}{4}} = 4$

92. (b) Given, $x = (1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2$
$= 8 + 4 + 1 = 13$
and $y = (110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
$= 4 + 2 = 6$
$\therefore x^2 - y^2 = (13)^2 - (6)^2 = 169 - 36 = 133$

Now,

2	133	
2	66	1
2	33	0
2	16	1
2	8	0
2	4	0
2	2	0
	1	0

$\therefore\ 133 = (10000101)_2$

93. (b) $(10x010)_2 - (11y1)_2 = (10z11)_2$
$\Rightarrow (2^5 \times 1 + 0 + x \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0)$
$\quad - (2^3 \times 1 + 2^2 \times 1 + y \times 2^1 + 1 \times 2^0)$
$= 2^4 \times 1 + 0 + 2^2 \times z + 2^1 \times 1 + 2^0$
$\Rightarrow (34 + 8x) - (13 + 2y) = 19 + 4z$
$\Rightarrow 2 = -8x + 2y + 4z$
$\Rightarrow x = 0, y = 1, z = 0$

94. (c) $(0.0011) = 0 \times \frac{1}{2} + 0 \times \frac{1}{2^2} + 1 \times \frac{1}{2^3} + 1 \times \frac{1}{2^4}$

$= 0 + 0 + \frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

Hence, option (c) is correct.

95. (a) We know, for two sets A and B
$A - B = A - (A \cap B)$
$\therefore\ n(A - B) = n(A) - n(A \cap B)$
Given, $n(A) = 115$, $n(B) = 326$ and $n(A - B) = 47$.
$\Rightarrow 47 = 115 - n(A \cap B)$
$\Rightarrow n(A \cap B) = 68$
Consider $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 115 + 326 - 68 = 373$

96. (d) Since, A is void set therefore the number of elements in power set of A is 1.
$\therefore\ P\{P(A)\} = 2^1 = 2$
because If set A has n elements then $P(A)$ has 2^n elements.

$\Rightarrow P\ \ P\{P(A)\}\quad 2^2\quad 4$

$\Rightarrow P\{P\{P\{P(A)\}\}\} = 2^4 = 16$

97. (d) Total Budget = 1400 crores
Total expenditure = 28% + 35% + 12% + 8% + 105 cr.
= 83% + 105 crores

Thus, total expenditure = $\left(1400 \times \frac{83}{100}\quad 105\right)$ cr.
= (1162 + 105) cr.
= 1267 cr.
Now, Balance amount = (1400 – 1267) cr. = 133 cr.
Hence, Amount spent an Transport in crores of rupees is 133.

98. (c) No. of people who are illiterates = $15000 \times \frac{34.5}{100} = 5175$
No. of people who have education up to primary school

$= 15000 \times \frac{27}{100}\quad 4050$

Similarly, No. of people who have education upto middle school = 2790
Let the no. of people who have education upto high school = x.
$\therefore$ According to the question.
No. of people who have education upto pre-university

$\frac{x}{2}$.

So, Total no. of people who are not graduates

$= 5175 + 4050 + 2790 + \boldsymbol{x} + \frac{x}{2} = 12150 + \boldsymbol{x} + \frac{x}{2}$

Since, 660 are graduates, therefore

$15000 - (12150 + x + \frac{x}{2}) = 660$

$\Rightarrow 15000 - 660 = 12150 + \frac{3x}{2}$
$\Rightarrow 2(15000 - 660) = 24300 + 3x$
$\Rightarrow 28680 - 24300 = 3x$

$\Rightarrow \frac{4380}{3}\quad x \Rightarrow x = 1460$

Hence, 1460 students have education upto high school.

99. (b) $(\log_x x)(\log_3 2x)(\log_{2x} y) = \log_x x^2$
$\Rightarrow 1(\log_3 2x)(\log_{2x} y) = 2\ (\because \log_x x^2)\quad 2\log_x x)$

$\Rightarrow \left(\frac{\log 2x}{\log 3}\right)\left(\frac{\log y}{\log 2x}\right) = 2$

$\Rightarrow \frac{\log y}{\log 3} = 2 \Rightarrow \log y = 2\log 3$

$\Rightarrow \log y = \log 3^2 \Rightarrow y = 3^2 \Rightarrow y = 9$

100. (d) Given, $\log_5 k \log_k x = 3$

$\frac{\log k}{\log 5} \cdot \frac{\log x}{\log k}\quad 3 \Rightarrow \frac{\log x}{\log 5} = 3$

$\Rightarrow \log x = 3\log 5 \Rightarrow \log x = \log 5^3$
$\Rightarrow x = 5^3 \Rightarrow x = 125$

101. (c) Given, $N_a = \{ax | x \in N\}$
$\therefore N_{12} = \{12, 24, 36, 48, ...\}$
and $N_8 = \{8, 16, 24, ...\}$
$\therefore N_{12} \cap N_8 = \{24, 48, ...\}$
$= N_{24}$

102. (b) Let $X = \{(4^n - 3n - 1) | n \in N\}$
and $Y = \{9(n-1) | n \in N\}$
$\Rightarrow X = \{0, 9, 54, ...\}$ (By putting $n = 1, 2, ...$)
and $Y = \{0, 9, 18, 27, 36, 54, ...\}$
(By putting $n = 1, 2, ...$)
$\therefore X \cup Y = \{0, 9, 18, 27, 36, 54, ...\} = Y$

103. (d) The total number of elements common in $(A \times B)$ and $(B \times A)$ is n^2.

104. (c) Let $f : R \to R$ be defined as $f(x) = \dfrac{|x|}{x}, x \neq 0$.
Also, $f(0) = 2$
ie. value of function at $x = 0$ is 2.

$$\text{Consider, } f(x) = \begin{cases} \dfrac{x}{x} = 1 \text{ if } x > 0 \\ \dfrac{-x}{x} = -1 \text{ if } x < 0 \end{cases}$$

$$\text{because we have } |x| \begin{cases} x, & x \geq 0 \\ -x, & x \quad 0 \end{cases}$$

Thus, Range of $f(x) = \{1, -1, 2\}$

105. (c) 13.625

2	13	
2	6	1
2	3	0
	1	1

$0.625 \times 2 = 1.250$ 1
$0.250 \times 2 = 0.5$ 0
$0.5 \times 2 = 1$ 1
$\therefore 13.625 = 1101.101$

106. (b) Since, order of a set A is 3 and order of set B is 2 therefore
$n(A) = 3$ and $n(B) = 2$
$\therefore$ Number of relations from A to B
$= n(A) \times n(B) = 3 \times 2 = 6$

107. (c) Consider $\dfrac{\log_{\sqrt{\alpha\beta}} H}{\log_{\sqrt{\alpha\beta\gamma}} H} = \dfrac{\log_H \sqrt{\alpha\beta\gamma}}{\log_H \sqrt{\alpha\beta}}$

$= \log_{\sqrt{\alpha\beta}} \sqrt{\alpha\beta\gamma} = \log_{\alpha\beta}(\alpha\beta\gamma)$

108. (a) Since, Power set is the collection of all the subsets of the set A therefore
A U P (A) = P(A)
$\therefore$ statement (1) is correct.

109. (d) $A = P(\{1, 2\}) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$
($\because$ Power set is the collection of all subsets of the set A)
From above it is clear that $\{1, 2\} \in$ A

110. (d) $xRy \Leftrightarrow x$ and y are graduates of the same university.
Reflexive $x R x \Leftrightarrow x$ and x are graduates of the same university.
$\therefore$ Relation is reflexive.
Symmetric $x R y \Leftrightarrow x$ and y are graduates of the same university
$\Rightarrow yRx \Leftrightarrow y$ and x are graduates of the same university.
$\therefore$ Relation is symmetric.
Transitive xRy, $yRz \Leftrightarrow xRz$
It means x and y, y and z are graduates of the same university, then x and z are also graduates of the same university.
$\therefore$ Relation is transitive.
Hence, relation is reflexive, symmetric and transitive.

111. (d) $(0.101)_2 = 2^{-1} \times 1 + 2^{-2} \times 0 + 2^{-3} \times 1$

$= \dfrac{1}{2} + 0 + \dfrac{1}{8} = \dfrac{5}{8}$

and $(0.011)_2 = 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$

$= 0 + \dfrac{1}{4} + \dfrac{1}{8} = \dfrac{3}{8}$

Also, $(11)_2 = 1 \times 2^1 + 1 \times 2^0 = 3$

and $(01)_2 = 0 \times 2^1 \times 1 \times 2^0 = 1$

$$\therefore \frac{(0.101)_2^{(11)} + (0.011)_2^{(11)_2}}{(0.101)_2^{(10)_2} - (0.101)_2^{(01)_2}(0.011)_2^{(01)} + (0.011)_2^{(10)_2}}$$

$$= \frac{\left(\frac{5}{8}\right)^3 + \left(\frac{3}{8}\right)^3}{\left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)\left(\frac{3}{7}\right) + \left(\frac{3}{8}\right)^2} = \frac{5}{8} + \frac{3}{8} = \frac{8}{8} = 1 = (1)_2$$

112. (b) Let A = $\{a, b, c, d\}$
Let n = no. of elements in A = n(A) = 4
Now, number of subsets = $2^n = 2^4 = 16$
As we know that no. of proper subsets = $2^n - 1$
$\therefore$ Number of proper subsets = $2^4 - 1 = 16 - 1 = 15$

113. (b) Let N = National savings certificates
S = Shares
Total no. of persons = 32
No. of persons who invest in National savings certificates = 30
No. of persons who invest in shares = 17
Therefore $n(N \cup S) = 32$, $n(N) = 30$, $n(S) = 17$
We know that,
$n(N \cup S) = n(N) + n(S) - n(N \cap S)$

$\Rightarrow \quad 32 = 30 + 17 - n(N \cap S)$

$\Rightarrow \quad n(N \cap S) = 47 - 32 = 15$

114. (c) Since, $(1111)_2 = 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0$
$= 8 + 4 + 2 + 1 = 15$
$(1001)_2 = 1\times 2^3 + 0\times 2^2 + 0\times 2^1 + 1\times 2^0 = 8 + 1 = 9$
and $(1010)_2 = 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0 = 8 + 2 = 10$
$\therefore \quad (1111)_2 + (1001)_2 - (1010)_2 = 15 + 9 - 10 = 14$
$\therefore \quad (14)_{10} = (1110)_2$

115. (a) Let R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}
Reflexive
Since, 1R1, 2R2, 3R3 in the set R
$\therefore$ R is reflexive relation.
Symmetric
Since 1R2 but 2 is not related to 1 in R
$\therefore$ R is not symmetric relation.
Transitive
1R2, 2R3 $\Rightarrow$ 1R3
$\therefore$ R is transitive relation.
Hence, R is reflexive and transitive only.

116. (d) Consider $\log_3 [\log_3[\log_3 x]] = \log_3 3$
$\Rightarrow \quad \log_3 [\log_3 x] = 3$
$\Rightarrow \quad \log_3 x = 3^3$
$\Rightarrow \quad \log_3 x = 27 \Rightarrow x = 3^{27}$

117. (d) Remainder

2	32	0	0.25	
2	16	0	×2	
2	8	0		
2	4	0	0.5	0
2	2	0	×2	
2	1	1	1	1

$\therefore$ Required binary number equivalent to 32.25 is 100000.01

118. (a) Since, A and B are two disjoints therefore $A \cap B = \phi$

$\therefore \quad A - B = A - (A \cap B)$

119. (a) Let A = $\{n^2 : n \in N\}$ and B = $\{n^3 : n \in N\}$
A = {1, 4, 9, 16, ...}
and B = {1, 8, 27, 64, ...}
Now, $A \cap B = \{1\}$ which is a finite set.
Also, $A \cup B = \{1, 4, 8, 9, 27, ...\}$
So, complement of $A \cup B$ is infinite set.
Hence, $A \cup B \neq N$

120. (a) Given A = {2,3}, B = {4,5}, C = {5,6}
$\therefore \quad B \cap C = \{5\}$
$\Rightarrow \quad A \times (B \cap C) = \{2,3\} \times \{5\} = \{(2,5), (3,5)\}$
Hence, required number of elements in
$A \times (B \cap C) = 2$

121. (b) Let, U = {1,2,3,....,20}
A = Set of all numbers which are perfect square = {1,4,9,16}
B = Set of all numbers which are multiples of 5 = {5,10,15,20}
C = Set of all numbers which are divisible by 2 and 3 = {6,12,18}
$A \cup B = \{1,4,9,16,5,10,15,20\}$
$\Rightarrow \quad n(A \cup B) = 8$
$\Rightarrow \quad n(A \cup B)' = 20 - 8 = 12$
Also, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$\because \quad A \cup B \cup C = \{1,4,9,16,5,10,15,20,6,12,18\}$
$n(A \cup B \cup C) = 11$
and $n(A) + n(B) + n(C) = 4 + 4 + 3 = 11$
Hence, only statement I and III are correct.

122. (d) Given, n (A) = 4, n (B) = 3
Since, the sets A and B are not known, then cardinality of the set A Δ B cannot be determined.

123. (c) Let $f(x) = \cos 2x - \sin 2x$

$$f(x) = \frac{1}{\sqrt{2}}\left[\sqrt{2}\cos 2x - \sin 2x\right]$$

$$f(x) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos 2x - \frac{1}{\sqrt{2}}\sin 2x\right]$$

$$f(x) = \sqrt{2}\left[\cos\frac{\pi}{4}\cos 2x - \sin\frac{\pi}{4}\sin 2x\right]$$

$$f(x) = \sqrt{2}\left[\cos\left(\frac{\pi}{4} \quad 2x\right)\right]$$

We know,

$$-1 \le \cos\left(\frac{\pi}{4} + 2x\right) \le 1$$

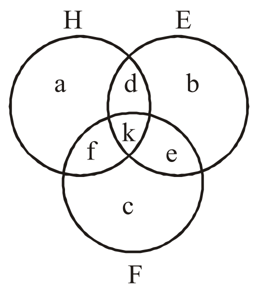
$$\Rightarrow -\sqrt{2} \le \sqrt{2}\cos\left(\frac{\pi}{4} + 2x\right) \le \sqrt{2}$$

$$\Rightarrow -\sqrt{2} \le f(x) \le \sqrt{2}$$

$\therefore \quad$ Range of $f(x) = [-\sqrt{2}, \sqrt{2}]$

124. (b) Let A = {1, 2, 5, 6} and B = {1, 2, 3}
$\therefore \quad$ A × B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)}
and $\quad$ B × A = {(1, 1), (1, 2), (1, 5), (1, 6), (2, 1), (2, 2), (2, 5), (2, 6), (3, 1), (3, 2), (3, 5), (3, 6)}
$\Rightarrow \quad (A \times B) \cap (B \times A)$
= {(1, 1), (1, 2), (2, 1), (2, 2)}

(125-129).
Given :
k = 15, c + f + k + e = 46
c = 11, a = 23, b = 17
d = 2f and e = 3f



$\Rightarrow \quad d \quad \frac{2}{3}e$

on solving above these we get
e = 15, d = 10, f = 5

125. (c) Required number of students = d + e + f
= 10 + 15 + 5 = 30

126. (c) Number of students learn at least two languages
= d + e + k + f = 10 + 15 + 15 + 5 = 45

127. (a) Total number of students in class = 96 + 28 = 124

128. (a) Required number of students = e + k = 15 + 15 = 30

129. (c) Number of students learn at least one languages
= 23+ 10+ 17 +15 +15 +5 +11= 96

130. (b) Let $\log(a+\sqrt{a^2+1})+\log\left(\frac{1}{a+\sqrt{a^2+1}}\right)$

$= \log(a+\sqrt{a^2+1})+\log 1-\log(a+\sqrt{a^2+1})$

$= \log(a+\sqrt{a^2+1})-\log(a+\sqrt{a^2+1})$

$= 0$

131. (c) Let on the set of real numbers, R is a relation defined by xRy if and only if $3x+4y=5$
Consider, $3x+4y=5$
(I) Put $x = 0$ and $y = 1$, we get
LHS = $3(0)+4(1)=4\neq 5$ (= RHS)
Hence 0 is not related to 1.
(II) Now, Put $x = 1$ and $y = \frac{1}{2}$, we get
LHS = $3(1)+4\times\frac{1}{2}=5=5$ (= RHS)
Hence 1 is related to $\frac{1}{2}$.
(III) Similarly, $\frac{2}{3}$ is related to $\frac{3}{4}$.
Hence, both statements II and III are correct.

132. (d) Consider, $\log\frac{9}{8}-\log\frac{27}{32}+\log\frac{3}{4}$

$= \log\frac{9}{8}+\log\frac{32}{27}+\log\frac{3}{4}$

$= \log\left(\frac{9}{8}\times\frac{32}{27}\right)+\log\frac{3}{4}$

$= \log\left(\frac{4}{3}\right)+\log\frac{3}{4} = \log\left(\frac{4}{3}\times\frac{3}{4}\right) = \log 1 = 0$

133. (b) Let $a = 00111 = 2^2\times 1+2^1\times 1+2^0\times 1$
$= 4+2+1=7$
Let $b = 01110 = 2^3\times 1+2^2\times 1+2^1\times 1+2^0\times 0$
$= 8+4+2=14$
$\therefore\quad \frac{b}{a}=\frac{14}{7}=2$

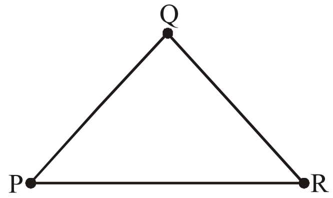
134. (d) Let M = Set of men and R is a relation 'is son of' defined on M.
Reflexive : aRa
($\because$ a can not be a son of a)
Symmetric : $aRb \Rightarrow bRa$
which is not also possible.
($\because$ If a is a son of b then b can not be a son of a)
Transitive : $aRb, bRc \Rightarrow aRc$
which is not possible.

135. (b) The number 10101111 can be rewritten as
$10101111 = 2^7\times 1+2^6\times 0+2^5\times 1+2^4\times 0+2^3\times 1 + 2^2\times 1+2^1\times 1+2^0\times 1$
$= 128+32+8+4+2+1=175$

136. (d) Let A, B and C are non-empty sets such that
$A\cap C=\phi$
Consider, $(A\times B)\cap(C\times B)=(A\cap C)\times(B\cap B)$
$=(A\cap C)\times B=\phi\times B=\phi$

137. (d) Let $A=\{4n+2: n\in N\}$
and $B=\{3n: n\in N\}$
$\Rightarrow$ $A=\{6, 10, 14, 18, 22, 26, 30, 34, 38, 42, ...\}$
and $B=\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...\}$
$\therefore$ $A\cap B=\{6, 18, 30, 42, ...\}$
$=6+12n-12=12n-6.$
Hence, $A\cap B=\{12n-6 : n$ is a natural number$\}$.

138. (b) Since, P, Q and R are three non-collinear points.
$\therefore$ We have P, Q and R are like as



Now, only P is common between PQ and PR.
Hence, $PQ\cap PR=\{P\}$

139. (a) Consider $0.3\times 2=0.6\times 2=1.2\times 2$
Now, treated 1 as 0
So, $0.2\times 2=0.4\times 2=0.8\times 2=1.6\times 2$
Again, $1.6\times 2=0.6\times 2=1.2$
Thus,

$0.3\times 2=\boxed{0}.6\times 2=\boxed{1}.2\times 2=\boxed{0}.4\times 2$

$=\boxed{0}.8\times 2=\boxed{1}.6\times 2=\boxed{1}.2$ -------- so on

Hence, $0.3 = (0.0\,1001\,\text{-----})_2$.

140. (a) Let $\tan\theta=\sqrt{m}$, where m is a non-square natural number.

$\Rightarrow \sin\theta=\sqrt{m}\cos\theta$

Consider, $\sec 2\theta=\frac{1}{\cos 2\theta}=\frac{1}{\cos^2\theta-\sin^2\theta}$

$=\frac{1}{\cos^2\theta-m\cos^2\theta}=\frac{1}{\cos^2\theta(1-m)}$

$=\frac{\sec^2\theta}{1-m}=\frac{1+\tan^2\theta}{1-m}=\frac{1+m}{1-m}$

$=\frac{(1+m)(1-m)}{(1-m)(1-m)}=\frac{(1-m^2)}{(1-m)^2}$

Numerator will always be negative and denominator will always be positive.

Hence, $\sec 2\theta=\frac{1-m^2}{(1-m)^2}$ is a negative number.

141. (c) Let A = {a, b, c} $\Rightarrow$ O(A) = 3
Now, number of proper subsets of
A = $2^{O(A)} - 1 = 2^3 - 1 = 7$

142. (a) Let $\log_2 (\log_3 81) = x$
$\Rightarrow \log_2 (\log_3 3^4) = x$
$\Rightarrow \log_2 (4 \log_3 3) = x$
$\Rightarrow \log_2(4) = x \qquad (\because \log_a a = 1)$
$\Rightarrow 4 = 2^x$
$\Rightarrow 2^2 = 2^x \Rightarrow x = 2$

143. (d) Since ϕ is null set therefore $\phi = \{\ \}$

144. (d) Let A = no. of students passed in the first semester.
B = no. of students passed in second semester.
Given, n(A) = 260, n(B) = 210
$\therefore$ $n(\overline{A})$ = no. of students did not pass in first semester.
$= 500 - 260 = 240$
Similarly, $n(\overline{B}) = 500 - 210 = 290$
Thus, we have $n(\overline{A} \cup \overline{B}) = 170$,
$n(\overline{A}) = 240$, $n(\overline{B}) = 290$
$\therefore\ n(\overline{A} \cup \overline{B}) = n(\overline{A}) + n(\overline{B}) - n(\overline{A} \cap \overline{B})$
$\Rightarrow 170 = 240 + 290 - n(\overline{A} \cap \overline{B})$
$\Rightarrow n(\overline{A} \cap \overline{B}) = 360$
So, n (students passed in both semester)
$= 500 - 360 = 140$

145. (c) 1 1 1 0 1 . 0 0 1
$= 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 . 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$
$= (16 + 8 + 4 + 0 + 1).\left(0 + 0 + \frac{1}{8}\right) = 29.125$

146. (c) A – B = {1, 4}
$(A - B)^c = U - (A - B) = \{2, 3, 5, 6, 7, 8, 9, 10\}$

147. (a) Here A = {1, 4, 9, 16, 25, 36, 49, 64, 81}
B = {even natural numbers}
$A \cap B = \{4, 16, 36, 64\}$
So, cardinality of $A \cap B = 4$

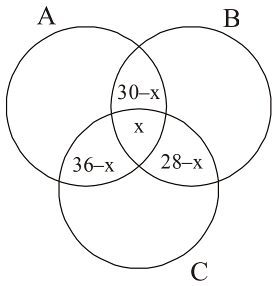
148. (c) Remainder

2	292	0
2	146	0
2	73	1
2	36	0
2	18	0
2	9	1
2	4	0
2	2	0
	1	

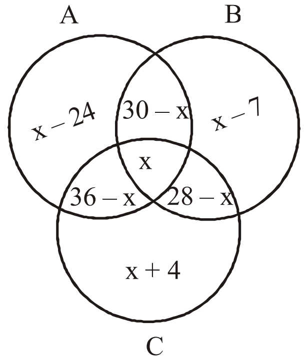
Required answer = 100 100 100

149. (d) Let
A = {x : x + 4 = 4} = {x : x = 4 – 4}
= {x : x = 0} = {0}

(150-153):
Let the people who read all three papers A, B, C = x%
So, people who read only A and B not C = (30 – x)%
People who read only B and C, not A = (28 – x)%
People who read only A and C, not B = (36 – x)%
Venn diagram representing these is shown below.



Remaining numbers in circles are filled as shown below.
People who read only A + 30 – x + x + 36 – x = 42
$\Rightarrow$ People reading only A = 42 – 30 – 36 + x = (x – 24)%
Similarly, people who read only B = 51 – (30 – x + x + 28 – x)
= 51 – (58 – x) = 51 – 58 + x = (x – 7)%
People who read only C = 68 – (36 – x + x + 28 – x)
= 68 – (64 – x) = (x + 4)%



Let x % people read all the three newspapers.
Since 8% people do not read any newspapers.
$\therefore$ (x–24) + (x – 7) + (x + 4) + (30 – x) + (36 – x) + (28 – x) + x = 92
$\Rightarrow$ x + 98 – 31 = 92
$\Rightarrow$ x = 92 – 67 = 25

150. (b) Hence people who read all the three newspapers = 25%

151. (d) (30 – x) + (36 – x) + (28 – x) = 94 – 3x
= 98 – 3 × 25 = 23
Hence percentage of people who read only two newspapers = 23%

152. (b) (x – 24) + (x – 7) + (x + 4) = 3x – 27
= 3 × 25 – 27 = 48
Hence percentage of people who read only one newspaper = 48%

153. (c) x – 24 = 25 – 24 = 1
Hence percentage of people who read only Newspaper A but neither B nor C = 1%

154. (a) Consider $2\log_8 2 - \frac{1}{3}\log_3 9$

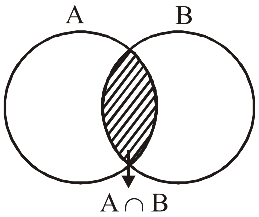
$$= 2.\frac{\log 2}{\log 8} - \frac{1}{3}.\frac{\log 9}{\log 3} = 2.\frac{\log 2}{\log 2^3} - \frac{1}{3}.\frac{\log 3^2}{\log 3}$$

$$= 2.\frac{\log 2}{3\log 2} - \frac{1}{3}2.\frac{\log 3}{\log 3} = \frac{2}{3} - \frac{2}{3} = 0$$

155. (d) Let $A = \{0, 1\}, B = \{1, 0\}$
$A \times B = \{(0, 1), (1, 1), (0, 0), (1, 0)\}$.
$A \times A = \{(0, 0), (0, 1), (1, 1), (1, 0)\}$.
$\therefore A \times B = A \times A$

156. (b) Since A and B have n elements in common so, A × B and B × A have n^2 elements in common.

157. (d)



$$A \cap (A \cup B) = (A \cap A) \cup (A \cap B) = A \cup (A \cap B)$$
$$= A \text{ (By diagram)}$$

Thus, $A \cap (A \cup B) = A$

158. (a) The relation 'has the same father as' over the set of children is only reflexive.
($\because$ In reflexivity an element is related to itself)

159. (d) $(1011)_2 = 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$
$= 8 + 0 + 2 + 1 = (11)_{10}$.

160. (a) $(111001.011)_2 = [2^5 \times 1 + 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 0 + 2^0 \times 1]. [2^{-1} \times 0 + 2^{-2} \times 1 + 2^{-3} \times 1] = (57.375)_{10}$

161. (b) $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$,

$$\frac{\log x}{\log 3} \times \frac{\log 2x}{\log x} \times \frac{\log y}{\log 2x} = \frac{\log x^2}{\log x}$$

$$\frac{\log y}{\log 3} = \frac{2\log x}{\log x}$$

$\log y = 2\log 3$
$\log y = \log 9$

$\boxed{y = 9}$

162. (a) Given relation
$R = \{(1, 2), (1, 3), (2, 1), (1, 1), (2, 2), (3, 3), (2, 3)\}$ is reflexive and transitive only but not symmetric
$\because (3, 1)$ and $(3, 2) \notin R$.

163. (b) No. of elements in power set of $A = 2^n$.

164. (b) We have
$A = \{0, 1, 2\}$
$B = \{2, 3\}$
$C = \{3, 4\}$
$A \cup B = \{0, 1, 2, 3\}$
$(A \cup B) \times C = \{(0, 3), (0, 4), (1, 3); (1, 4); (2, 3), (2, 4), (3, 3); (3, 4)\}$

$\therefore n[(A \cup B) \times C] = 8$

165. (c) As we know

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\therefore f(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} & \text{if } x > 0 \\ \frac{-x}{x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Hence, Range = {–1, 1}.

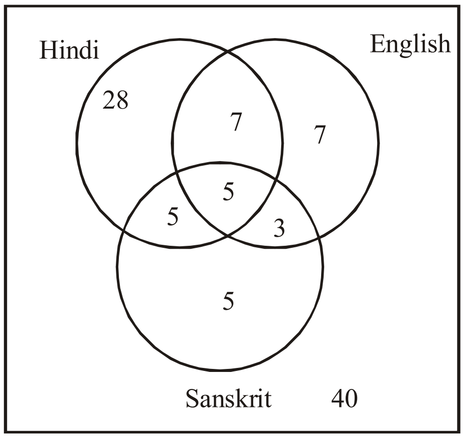
166. (d) $\therefore$ Binary form of 45 = 101101

2	45	1
2	22	0
2	11	1
	5	1
2	2	0
2	1	

167. (c) $d = 60 \times 60s$
$m = 60s$

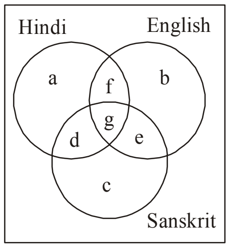
$$\frac{s - m}{m - d} = \frac{s - 60s}{60s - 60 \times 60s} = \frac{-59}{60 \times (-59)} = \frac{1}{60}$$

167. (c)



40% of total population $= 0.4 \times 75 \times 10^6 = 3 \times 10^7$.

(168-173):
Given, $P = 75 \times 10^6$



45% population know Hindi
i.e., $a + f + d + g = 45\%$ of P

$$= \frac{45}{100} \times 75 \times 10^6 = 33.75 \times 10^6 \quad ...(i)$$

22% population know English
$f + b + g + e = 22\%$ of P

$$= \frac{22}{100} \times 75 \times 10^6 = 16.50 \times 10^6 \quad ...(2)$$

18% population know Sanskrit.

$d + g + e + c = 18\%$ of P

$= \frac{18}{100} \times 75 \times 10^6 = 13.5 \times 10^6$ ...(3)

12% population knows Hindi and English

$f + g = 12\%$ of $P = \frac{12}{100} \times 75 \times 10^6 = 9 \times 10^6$ ...(4)

8% population knows English and Sanskrit

$g + e = 8\%$ of $P = \frac{8}{100} \times 75 \times 10^6 = 6 \times 10^6$ ...(5)

10% population Hindi and Sanskrit

$g + d = 10\%$ of $P = \frac{10}{100} \times 75 \times 10^6 = 7.5 \times 10^6$ ...(6)

5% population knows all three languages

$g = 5\%$ of $P = \frac{5}{100} \times 75 \times 10^6 = 3.75 \times 10^6$ ...(7)

From (6), (7) $3.75 \times 10^6 + d = 7.5 \times 10^6$

$\Rightarrow d = 10^6(7.5 - 3.75) = 3.75 \times 10^6$

From (5), (7) $3.75 \times 106 + e = 6 \times 10^6$

$\Rightarrow e = 10^6(6 - 3.75) = 2.25 \times 106$

From (4), (7) $\Rightarrow f + 3.75 \times 106 = 9 \times 10^6$

$\Rightarrow f = 10^6(9 - 3.75) = 5.25 \times 10^6$

From (2), $b = 16.5 \times 10^6 - (f + g + e)$

$= 16.5 \times 10^6 - (5.25 \times 10^6 + 3.75 \times 10^6 + 2.25 \times 10^6)$

$= 10^6[16.5 - 5.25 - 3.75 - 2.25] = 5.25 \times 10^6$

From (3), $c = 13.5 \times 10^6 - (d + g + e)$

$= 13.5 \times 10^6 - 9.75 \times 10^6 = 3.75 \times 10^6$

From (4), $a = 33.75 \times 10^6 - (f + g + d)$

$= 33.75 \times 10^6 - 12.75 \times 10^6 = 21 \times 10^6$

168. (a) Now, Number of people who don't know any of three languauges

$=$ Total population $- (a + b + c + d + e + f + g)$

$= 75 \times 10^6 - (21 + 5.25 + 3.75 + 3.75 + 2.25 + 5.25 + 375)10^6$

$= 75 \times 10^6 - 45 \times 10^6 = 30 \times 10^6 = 3 \times 10^7$

169. (d) Number of people who know only Hindi $= a = 21 \times 10^6$.

170. (d) Number of people who know only Sanskrit $= c = 3.75 \times 10^6$.

171. (c) Number of people who know only English $= b = 5.25 \times 10^6$.

172. (b) Number of people who know only one language

$= a + b + c = 21 \times 10^6 + 5.25 \times 10^6 + 3.75 \times 10^6$

$= 30 \times 10^6 = 3 \times 10^7$.

173. (c) Number of people who know only two language

$= d + e + f = 3.75 \times 10^6 + 2.25 \times 10^6 + 5.25 \times 10^6$

$= 11.25 \times 10^6$

174. (d) Consider the set given in option 'd'.

$\{x \mid x^2 + 1 = 0, x \in R\}$

Let $x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \pm i$ which is complex.

But $x \in R$. Hence for, any $x \in R$, $x^2 + 1$ can not be zero.

175. (c) Let $B = \{2, 3\}$ and $C = \{3, 4\}$

Now, $B \cup C = \{2, 3, 4\}$ consider $A \times (B \cup C)$

$= \{x, y\} \times \{2, 3, 4\}$

$= \{(x, 2), (x, 3), (x, 4), (y, 2), (y, 3), (y, 4)\}$

Hence, number of element in $A \times (B \cup C) = 6$.

176. (c) $\log_y x^5 \times \log_x y^2 \log_z z^3$

$= \frac{5 \ln x}{\ln y} \times \frac{2 \ln y}{\ln x} \times \frac{3 \ln z}{\ln z} = 30$

177. (c) Since A is a relation on a set R

$\therefore A \subseteq (R \times R)$

178. (c) $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$

$A \times C = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

Number of elements in $(A \times B) \cap (A \times C) = 2$

179. (c) If A is a finite set having n elements, then the number of relations which can be defined in A set is $2^{n \times n} = 2^{n^2}$.

180. (a) S be the set of all even prime numbers

S = 2 in an even prime number = (Non-empty set)

181. (c)

2	83	1
2	41	1
2	20	0
2	10	0
2	5	1
2	2	0
	1	

Therefore, $(83)_{10} = (1010011)_2$

182. (d) **For reflexive :**

$(a, a) = a - a = 0$ is divisible by 5.

For symmetric :

If $(a - b)$ is divisible by 5, then $b - a = -(a - b)$ is also divisible by 5.

Thus relation is symmetric.

For transitive

If $(a - b)$ and $(b - c)$ is divisible by 5.

Then $(a - c)$ is also divisible by 5.

Thus relation is transitive.

$\therefore$ R is an equivalence relation.

183. (c) n (T) = 50

n (D) = 30

n (H) = 40

$n(T) = n(D) + n(H) - n(DnH)$

$50 = 30 + 40 - n(D \cap H)$

$n(D \cap H) = 70 - 50 = 20$

Number of people having diabetes and high blood pressure = 20

184. (c) n(A) = 4 and n (B) = 3

Number of elements in $n(A \times B) = 4 \times 3 = 12$

185. (b) Suppose U = {a, b, c, d, e, f, g, h}

A = {a, b, c, d}

B = {a, b, c, d, e} Given $A \subseteq B$

$A^c = \{e, f, g, h\}$, $B^C = \{f, g, h\}$

Hence, $B^C \subseteq A^C$

186. (b) Angle traced by the hour hand in 12 hours = 360°
Angle traced by it in 4 hr 30 min $\left(4h \quad \frac{30}{60}hr\right) = \frac{9}{2}$ hr
$= \frac{9}{2} \times \frac{360}{12} \quad 135$
Angle traced by minute hand is 60° min = 360°
Angle traced by it in 30 min = $\frac{30}{60} \times 360 = 180°$
Required Angle = $180° - 135° = 45° \Rightarrow 45 \times \frac{\pi}{100} \quad \frac{\pi}{4}$ radian

187. (d) According to 'Distribution law' in set theorry the given both statements are wrong.
1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

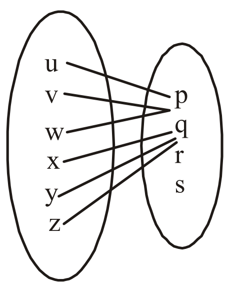
188. (d) $(110001)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 32 + 16 + 0 + 0 + 0 + 1 = (49)_{10}$

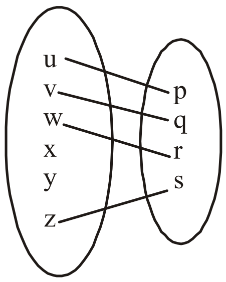
189. (c) Number of element in set A is 4.
Cordinality of the power set P(A) = $2^4 = 16$

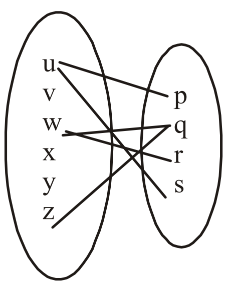
190. (b) $\log_{81} 243 = \log_{3^5} 3^6 = \frac{\log 3^6}{\log 3^5} = \frac{6}{5} = 1.25$

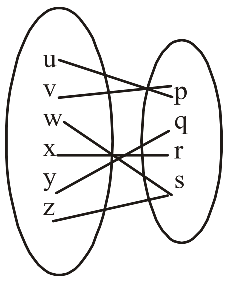
191. (b) X = Set of all citizens of India
R = {(x, y) : x, y ∈ X, |x – y| = 5}
$|x - x| = 0 \neq 5$ (R is not reflexive)
$xRy \Rightarrow |x - y| = 5$
$\Rightarrow |y - x| = 5$ (R is symmetric)
$xRy \Rightarrow |x - y| = 5$
$yRz \Rightarrow |y - z| = 5$
But $|x - z| \neq 5$ (R is not transitive)

192. (c) Given that, A = {u, v, x, y, z}; B = {p, q, r, s}
As we know, a mapping f : x → y is said to be a function, if each element in the set x has its image in set y. It is also possible that these are few elements in set y which are not the image of any element in set x. Every element in set x should have one and only one image.

(i)


(ii)


(iii)


(iv)


(ii) and (iii) are not function.

193. (c) S = Set of all integers and
R = {(a, b), a, b ∈ S and ab ≥ 0}
For reflexive : $aRa \Rightarrow a.a = a^2 \geq 0$
for all integers a. a ≥ 0
For symmetric : $aRb \Rightarrow ab \geq 0 \; \forall \; a, b \in S$
If ab ≥ 0, then ba ≥ 0 ⇒ bRa
For transitive :
If ab ≥ 0 , bc ≥ 0, then also ac ≥ 0
Relation R is reflexive, symmetric and transitive. Therefore relation is equivalence.

194. (a) $(11110)_2 = 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 0$
$= 16 + 8 + 4 + 2 + 0 = 30$
$(1010)_2 = (2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 0 = 8 + 0 + 2 + 0) = 10$
Sum = 30 + 10 = 40
$= (101000)_2$

2	40	
2	20	0
2	10	0
2	5	0
2	2	1
	1	0

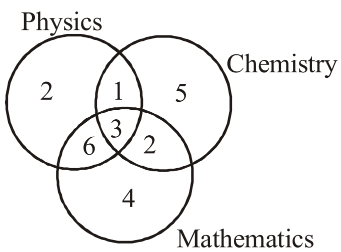
195. (c) According to question p + q + r = 5 × 3 = 15 ...(i)
s + t = 10 × 2 = 20 ...(ii)
From equations (i) and (ii), p + q + r + s + t = 15 + 20 = 35
Average p, q, r, s and t = $\frac{35}{5} = 7$

196. (b)

2	251	1
2	125	1
2	62	0
2	31	1
2	15	1
2	7	1
2	3	1
	1	

Therefore, $(251)_{10} = (11111011)_2$

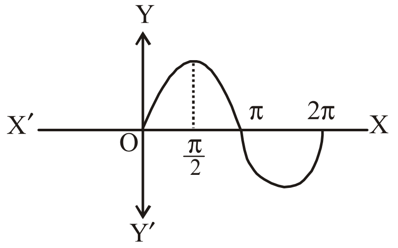
Sol. (197-199)



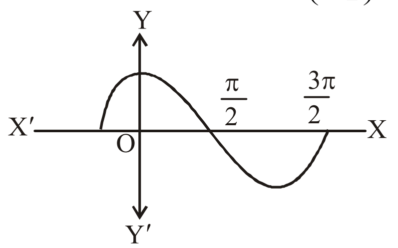
197. (a) Only Physics = 12 – (1 + 3 + 6) = 2
198. (c) Only two subjects = 6 + 2 + 1 = 9
199. (b) **Statement 1 :**
Students, who had taken only one subject
= 2 + 5 + 4 = 11
Students, who had taken only two subjects
= 6 + 2 + 1 = 9
$1 \neq 9$

Statement 2 :
Stude nts who had taken atleast two subject
$= 1 + 2 + 6 + 3 = 12$
Students who had taken all three subjects
$= 4 \times 3 = 12$

200. (d) sin x increases on the interval $\left(0, \frac{\pi}{2}\right)$



cos x decreases on the interval $\left(0, \frac{\pi}{2}\right)$



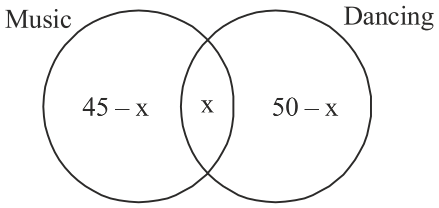
201. (c) The relation S is defined on the set of integers Z and xSy, if integer x divides integer y.
Reflexive : Since, every integer divides itself
$\therefore$ integer x divides integer x
$\Rightarrow xSx$
Hence, S is reflexive.
Symmetric : Let $x, y \in Z$ such that xSy
i.e., integer x divides integer y
Now, this does not implies that integer y divides integer x.
e.g. Take $x = 2$ and $y = 4$
Then, 2 divides 4 but 4 does not divides 2.
Thus, S is not symmetric.
Transitive : Let $x, y, z \in Z$ such that xSy and ySz.
$\Rightarrow$ integer x divides integer y and integer y divides integer z
$\Rightarrow$ integer x divides integer z
$\Rightarrow xSz$
Hence, S is transitive.

202. (b) $(1001)_2 = (2^3 \times 1 + 2^2 \times 0 + 2^1 \times 0 + 2^0 \times 1)_{10}$
$= (8 + 1)_{10} = (9)_{10}$

203. (d) Here, $n(A) = 5$ and $n(B) = 4$
$\therefore n(A \times B) = 5 \times 4 = 20$
$[\because n(A) = m, n(B) = n \Rightarrow n(A \times B) = mn]$

204. (b) Given that $x < y$ if $y \geq x + 5$
For Reflexive:
$x \nless x$
Hence, relation is not reflexive.
For Symmetry:
if $x < y$,
then $y \nless x$
Hence, relation is not symmetry.
For Transitive:
if $x < y$ and $y < z$,
then $x < z$
Hence, relation is transitive.

205. (b)



Let 'x' be the number of students who likes both music and dance.
5 students likes neither music nor dancing.
Hence, total number of remaining students
$= 60 - 5 = 55$
Now from Venn diagram,
$45 - x + x + 50 - x = 55$
$\therefore 95 - x = 55$
$\therefore x = 95 - 55 = 40.$

206. (c) $\log_{10} 2, \log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in A.P.
Hence, common difference will be same.
$\therefore \log_{10}(2^x - 1) - \log_{10} 2 = \log(2^x + 3) - \log_{10}(2^x - 1)$

$$\therefore \log_{10}\left(\frac{2^x - 1}{2}\right) = \log_{10}\left(\frac{2^x + 3}{2^x - 1}\right)$$

$$\Rightarrow \frac{2^x - 1}{2} = \frac{2^x + 3}{2^x - 1}$$

$(2^x - 1)^2 = 2(2^x + 3)$
$2^{2x} - 2^{x+1} + 1 = 2^{x+1} + 6$
$2^{2x} - 2^{x+2} = 5$
Let $2^x = y$, then
$y^2 - 4y - 5 = 0$
$y^2 - 5y + y - 5 = 0$
$y(y - 5) + 1(y - 5) = 0$
$y = -1, y = 5$
Therefore, $2^x = 5$
$x = \log_2 5.$

207. (c)

208. (c) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
Number of subsets of A containing two elements
$= 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$

$$\frac{9(9+1)}{2} = \frac{90}{2} = 45$$

$\therefore$ Option (c) is correct
Alternate Method
The number of subsets of A containing exactly two elements is:

$$^{10}C_2 = \frac{10 \times 9}{2 \times 1} = 45$$

209. (d) $(127.25)_{10}$
Now,

2	127
2	63 – 1
2	31 – 1
2	15 – 1
2	7 – 1
2	3 – 1
	1 – 1

$127 = (1111111)_2$
Now,

$0.25 \times 2 = 0.5 \rightarrow 0$

$0.5 \times 2 = 1.0 \rightarrow 1$

$\therefore (127.25)_{10} = (1111111.01)_2$
∴ Option (d) is correct.

210. (d) A = {x : x is a multiple of 3}
∴ A = {3, 6, 9, 12, 15, 18, 24, ...}

∴ $\frac{A \cap B}{C}$

B = {x : x is a multiple of 4}
∴ B = {4, 8, 12, 16, 20, 24, 28, 32, ...}
C = {x : x is a multiple of 12}
∴ C = {12, 24, 36, 48, 60, 72, 84, 96, ...}

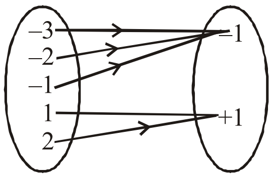
$A \cap B = \{12, 24, ...\} = \frac{A \cap B}{C}$.

211. (a) $(11101011)_2$
$= (1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0)_{10}$
$= (128 + 64 + 32 + 8 + 2 + 1)_{10} = (235)_{10}$
∴ Option (a) is correct.

212. (c) For a non-zero real number x

$f(x) = \frac{x}{|x|}$

domain $f(x) = \frac{x}{|x|}$ range



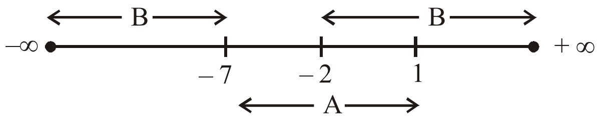
Here we can not take value of 'x' as zero as (x) is non zero real number

for any value of x in domain a set consisting of two elements i.e. (– 1, + 1)
∴ Option (c) is correct.

213. (d) a – b ≤ 5
Let b – c ≤ 5
so a – c ≤ 10
as (a – b) ∈ R and (b – c) ∈ R
but (a – c) ∉ R
So it is not a transitive relation.
By considering all the options, we come to the conclusion that only option (d) is correct.

214. (b) $(1000000001)_2$
$= 1 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + ... + 1 \times 2^9$
$= 1 + 0 + 0 + ... + 512$
$= (513)_{10}$
$(0.0101)_2 = 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$
$= \frac{1}{4} + \frac{1}{16} = \frac{5}{16} = (0.3125)_{10}$
$(1000000001)_2 - (0.0101)_2 = 513 - 0.312$
$= (512.6875)_{10}$

215. (a) $x^2 + 6x - 7 < 0$
$\Rightarrow (x + 7)(x - 1) < 0$
$\Rightarrow x = (-7, 1)$
$\Rightarrow A = \{-6, -5, -4, -3, -2, -1, 0\}$
$\Rightarrow x^2 + 9x + 14 > 0$
$\Rightarrow (x + 7)(x + 2) > 0$
$\Rightarrow x = (-\infty, -7) \cup (-2, \infty)$
$\Rightarrow B = R - \{-7, -6, -5, -4, -3, -2\}$



So $A \cap B = (-2, 1)$

216. (b) Let A = {1, 2}
B = {3, 4, 0}
C = {5, 6, 0}
D = {7, 8}
Such that $(A \cap B) = (C \cap D) = \phi$
$\Rightarrow (A \cup C) = \{1, 2, 5, 6, 0\}$
$\Rightarrow (B \cup D) = \{3, 4, 7, 8, 0\}$
$\Rightarrow (A \cup C) \cap (B \cup D) = \{0\}$
So $(A \cup C)$ and $(B \cup D)$ are not always dispoint
$\Rightarrow (A \cap C) = \phi$ and $(B \cap D) = \phi$
So $(A \cap C)$ and $(B \cap D)$ are always disjoint.

217. (a) $\log_8 m \quad \log_8 \frac{1}{6} \quad \frac{2}{3}$

$\Rightarrow \log_8 \left(m.\frac{1}{6} \right) = \frac{2}{3}$

$\Rightarrow (8)^{\frac{2}{3}} = m.\frac{1}{6}$

$\Rightarrow m = 24$

218. (d) Let f(x) = log x
∴ f(y) = log y
& f(xy) = log (xy) = log x + log y
⇒ f(xy) = f(x) + f(y)

219. (a) x and y are positive numbers.

$x \le y^2$

Reflexive -

$x \le x^2 \ \forall$ positive numbers.

Hence relation is reflexive.

Transitive -

$x \le y^2 \qquad y \le z^2$

Let $x = 5, y = 3, z = 2$

$5 \le (3)^2 \qquad (3) \le (2)^2$

but $5 \not\le (2)^2$

Hence, $x \le y^2 \qquad y \le z^2$

but $x \not\le z^2$

Thus relation is not transitive.

Symmetric

$1 \le (2)^2$ while $2 \not\le (1)^2$

Hence relation is not symmetric.

Thus $x \le y^2 \ \forall$ positive numbers is reflexive, but not transitive and symmetric.

220. (a) $f(x_1) - f(x_2) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$

$x_1, x_2 \in (-1, 1)$

then $f(x) = \log\frac{(1-x)}{(1+x)}$

$f(x_1) = \log\frac{1-x_1}{1+x_1} \qquad f(x_2) = \log\frac{1-x_2}{1+x_2}$

$$f(x_1) - f(x_2) = \log\frac{1-x_1}{1+x_1} - \log\frac{1-x_2}{1+x_2}$$

$$= \log\frac{(1-x_1)}{(1+x_1)} \times \frac{(1+x_2)}{(1-x_2)}$$

$$= \log\frac{(1 - x_1 + x_2 - x_1x_2)}{(1 + x_1 - x_2 - x_1x_2)}$$

$$\log\frac{(1-x_1x_2)-(x_1-x_2)}{(1-x_1x_2)+(x_1-x_2)}$$

$$= \log\frac{1-\left(\frac{x_1-x_2}{1-x_1x_2}\right)}{1+\left(\frac{x_1-x_2}{1-x_1x_2}\right)}$$

$$f(x_1) - f(x_2) = f\left(\frac{x_1-x_2}{1-x_1x_2}\right)$$

221. (a) Function $y = \frac{x^2}{1+x^2} \quad x \in R$

$x = 0$	$y = 0$
$x = 1, -1$	$y = \frac{1}{2}$
$x = 2, -2$	$y = \frac{4}{5}$
$x = 3, -3$	$y = \frac{9}{10}$
.	.
.	.
.	.

Clearly $0 \le y \quad 1$

$\Rightarrow y \in, [0, 1)$

Hence Range of $y = [0, 1)$

222. (c)

$0.3125 \times 2 = 0.6250$
$0.6250 \times 2 = 1.2500$
$0.2500 \times 2 = 0.5000$
$0.5000 \times 2 = 1.0000$

$(0.3125)_{10} = (0.0101)_2$

223. (c) $nRm \Leftrightarrow n$ is a factor of m.

$\Rightarrow m$ is divisible by n.

Reflexivity

We know that

n is divisible by $n \forall n \in N$

$(n, n) \in R \ \forall \ n \in N$

R is reflexive.

Symmetric

$n, m \in N$

Let $n = 2, m = 6$

m is divisible by n but n is not divisible by m. Hence R is not symmetric.

Transitivity

Let $(n, m) \in R$ and $(m, p) \in R$ then $(n, m) \in R$ and $(m, p) \in R \Rightarrow (n, p) \in R$

or If m is divisible by n and p is divisible by m. Hence p is divisible by n.

$(n, p) \in R \ \forall \ n, p \in N$

R is transitive relation on N.

Hence R is reflexive, transitive but not symmetric.

$$\therefore \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \ \ if\ f(2a-x) \quad fx$$

224. (b) Let A, B & C be the sets of numbers divisible by 10, 15 & 25 respectively

No. divisible by 10 = 100 = n(A)

No. divisible by 15 = 66 = n (B)

No. divisible by 25 = 40 = n (C)

No. divisible by (10 & 15) = 33 = n (A $\cap$ B)

No. divisible by (15 & 25) = 13 = n (B ∩ C)
No. divisible by (25 & 10) = 20 = n (A ∩ C)
No. divisible by (10, 15 & 25) = 6 = n (A ∩ B ∩ C)
No. divisible by 10, 15 and 25 = n (A ∪ B ∪ C)
= 100 + 66 + 40 – 33 – 13 – 20 + 6 = 146
Thus, no. which are neither divisible by 10 nor 15 nor 25 = 1000 – 146 = 854.

225. (d) $log_a ab = x$

$log_a a + log_a b = x$

$$\frac{1}{\log_b a} = x - 1$$

$$\log_b a = \frac{1}{x-1} \quad ...(1)$$

$\log_b ab = \log_b a + \log_b b$

$$= \frac{1}{x-1} + 1 \quad \text{(From (1))}$$

$$= \frac{1 + x - 1}{x - 1}$$

$$\boxed{\log_b ab = \frac{x}{x-1}}$$

226. (b) Given that p, q: (1, 2, 3, 4, 5, 6,)

For $\frac{p}{q}$ form, when p = 1, q = 1, 2, 3, 4, 5, 6

thus, $\frac{p}{q} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$

$n = \left(\frac{p}{q}\right) = 6$

When p = 2, q = 1, 3, 5

thus $\frac{p}{q} = 2, \frac{2}{3}, \frac{2}{5}$ and $n\left(\frac{p}{q}\right) = 3$

When p = 3, q = 1, 2, 4, 5

thus $\frac{p}{q} = 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{5}$ and $n\left(\frac{p}{q}\right) = 4$

When p = 4, q = 1, 3, 5

thus $\frac{p}{q} = 4, \frac{4}{3}, \frac{4}{5}$ and $n\left(\frac{p}{q}\right) = 3$

When p = 5, q = 1, 2, 3, 4, 6

thus $\left(\frac{p}{q}\right) = 5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}$ and $\frac{5}{6}$ and $n\left(\frac{p}{q}\right) = 5$

When P = 6, q = 1, 5

thus $\left(\frac{p}{q}\right) = 6, \frac{6}{5}$ and $n\left(\frac{p}{q}\right) = 2$

Hence, cardinality of the set (s)
= 6 + 3 + 4 + 3 + 5 + 2 = 23.

227. (c) We have :

$x^2 + 6x - 7 < 0$ & $x^2 + 9x + 14 > 0$

$\Rightarrow (x-1)(x+7) < 0$ & $\Rightarrow (x+2)(x+7) > 0$

$\Rightarrow x \in (-7, 1)$ & $\Rightarrow x \in (-\infty, -7) \cup (-2, \infty)$

$\therefore A \cap B = \{x \in R : -2 < x < 1\} \rightarrow$ It is true.

$A \setminus B = A - B = \{x \in R : -7 < x < -2\} \rightarrow$ It is also true.

228. (c) R = {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}.
$\Rightarrow R^{-1}$ = {(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}.
$\Rightarrow RoR^{-1}$ = {(3, 3), (3, 5), (5, 3), (5, 5)}.

229. (b)

2	235	1
2	117	1
2	58	0
2	29	1
2	14	0
2	7	1
2	3	1
	1	

So, $(235)_{10} = (11101011)_2$

230. (d) Here, maximum number of students failed in all the four subjects = 15%
But, minimum number of students failed in all the four subjects varies from 0 to 15%. So, correct option is (d).

231. (d) $U = \{(HHH)(HHT)(HTH)(HTT)(THH)(THT)(TTH)(TTT)\}$

$A = \{(TTT)\}$

$B = \{(HTT)(THT)(TTH)\}$

$C = \{(HHH)(HHT)(HTH)(THH)\}$

By checking the options

(d) $A \cap (B' \cup C') = B' \cap C'$ is correct.

232. (a) S = {All persons living in Delhi}
A relationship is said to be equivalence relation if it is reflexive, symmetric and transitive.
⇒ Here, (x, y) ∈ R ⇒ reflexive relation.
⇒ Since, x ε y are born on the same day,
x R y ⇒ y R x
So, it is symmeitric relation.
⇒ x R y, y R Z ⇒ x R Z (Date of births are same)
So, it is transitive relation.
So, the given relation is an equivalent relation.

233. (c) Given, A {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
Set A has 10 elements.
Number of sub sets Containing 2 and 3 elements is

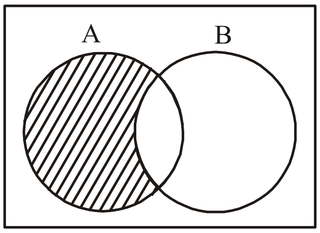
$10_{c_2} + 10_{c_3}$.

$$10_{c_2} + 10_{c_3} = \frac{10 \times 9}{2} + \frac{10 \times 9 \times 8}{3 \times 2}$$

= 45 + 120 = 165

234. (d) Sum of the numbers = Sum of given numbers. $(n-1)!\,[10^0 + 10^1 + 10^2 + ...]$
Here, sum of three digit numbers = Sum of the numbers $(3-1)!\,[10^0 + 10^1 + 10^2]$
$= (1 + 2 + 3)(3 - 1)!\,[10^0 + 10^1 + 10^2]$
$= 6 \times 2 \times 1\,[1 + 10 + 100]$
$= 12 \times 111$
= 1332

235. (b) Venn diagram



from, Venn diagram we can observe that A – B is the shaded part.

$(A-B)\cup A = A$

$(A-B)\cap B = \phi$

$A \subseteq B \Rightarrow A \cup B = B$

236. (a) $(1p101)_2 + (10q1)_2 = (100r00)_2$

$\Rightarrow (1\times 2^4 + p\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0)$

$+ (1\times 2^3 + 0\times 2^2 + q\times 2^1 + 1\times 2^0)$

$= 1\times 2^5 + 0 + 0 + r\times 2^2 + 0 + 0$

$\Rightarrow 16 + 8p + 4 + 1 + 8 + 2q + 1 = 32 + 4r$

$\Rightarrow 30 + 8p + 2q = 32 + 4r$

$\Rightarrow 8p + 2q = 2 + 4r$

from options, substitute p = 0, q = 1, r = 0 we get

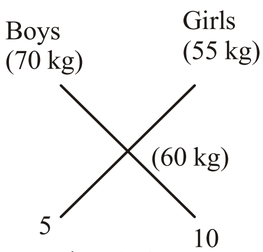
$0 + 2(1) = 2 + 0 \Rightarrow 2 = 2.$

237. (d) $S = \{x : x^2 + 1 = 0, x : 5 \text{ real}\}$

$x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \sqrt{-1} \rightarrow$ complex number

No real numbers. So, S is empty set

238. (a) Number of students = 150.



By aligation, ratio = 1 : 2

$\therefore$ No. of boys $= \frac{1}{3}\times 150 = 50$

239. (c) $x + \log_{10}(1+2^x) = x\log_{10}5 + \log_{10}6$

$\Rightarrow x - x\log_{10}5 = \log_{10}6 - \log_{10}(1+2^x)$

$\Rightarrow x(1-\log_{10}5) = \log_{10}6 - \log_{10}(1+2^x)$

$\Rightarrow x\left(\log_{10}10 - \log_{10}5\right) = \log_{10}\left(\frac{6}{1+2^x}\right)$

$\Rightarrow x\left(\log_{10}\left(\frac{10}{5}\right)\right) \quad \log_{10}\left(\frac{6}{1 \quad 2^x}\right)$

$\Rightarrow x\log_{10}2 = \log_{10}\left(\frac{6}{1+2^x}\right)$

This is possible only when x = 1.

240. (b) $(101110)_2 = 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0$

$= 32 + 0 + 8 + 4 + 2 + 0$

$= (46)_{10}$

Similarly, $(110)_2 = 1\times 2^2 + 1\times 2^1 + 0\times 2^0$

$= 4 + 2$

$= (6)_{10}$

Quotient = 7

Remainder = 4

$(7)_{10} = (111)_2$ and $(4)_{10} = (100)_2$

241. (c) E is the universal set and $A = B \cup C$.

Since, E is the universal set, $E - A = A'$

$\therefore E-(E-(E-(E-(E-A))))$

$= E-(E-(E-(E-A')))$

$= E-(E-(E-A))$

$= E-(E-A')$

$= E-A$

$= A'$

$= (B\cup C)'$

$= B^1 \cap C'$

242. (c) $A = \{x : x \text{ is multiple of } 2\}$

$B = \{x : x \text{ is multiple of } 5\}$

$C = \{x : x \text{ is multiple of } 10\}$

We know, multiples of 2 include multiples of 10.

$\therefore C \subset A$

Also, multiples of 5 include multiples of 10.

$\therefore C \subset B$

Also, $C = A \cap B$

But $B = A \cap C$, $B \cap C = B$

$\therefore A \cap (B \cap C) = A \cap B = C.$

243. (c) $(a, b)\,R\,(c, d) \Leftrightarrow a + d = b + c$

(i) $a + a = a + a$.

$\therefore$ (a, a) R (a, a) $\Rightarrow$ R is reflexive.

(ii) (a, b) R (c, d) $\Rightarrow a + d = b + c$

(c, d) R (a, b) $\Rightarrow c + b = d + a$

$\therefore$ R is symmetric.

(iii) Let (a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow a + d = b + c$ and $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$

$\Rightarrow a + f = b + e$

$\Rightarrow$ (a, b) R (e, f)

$\therefore$ R is transitive.

from (i), (ii), (iii) R is an equivalence relation.

244. (b) $n = (2017)!$

$\frac{1}{\log_2 n} \quad \frac{1}{\log_3 n} \quad \frac{1}{\log_4 n} \quad .... \quad \frac{1}{\log_{2017} n}$

$\frac{1}{\frac{\log_n}{\log_2}} \quad \frac{1}{\frac{\log_n}{\log_3}} \quad \frac{1}{\frac{\log_n}{\log_4}} \quad .... \quad \frac{1}{\frac{\log_n}{\log_{2017}}}$

$\left(\because \log_a b \quad \frac{\log_b}{\log_a}\right)$

$$= \frac{\log_2}{\log_n} + \frac{\log_3}{\log_n} + \frac{\log_4}{\log_n} + ... + \frac{\log_{2017}}{\log_n}$$

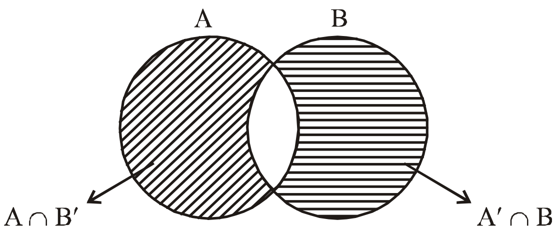
$$= \frac{\log_2 + \log_3 + \log_4 + ... + \log_{2017}}{\log_n}$$

$$= \frac{\log(2.3.4....2017)}{\log_n}$$

$(\because \log a + \log b + \log c + ... = \log a.b.c.....)$

$$= \frac{\log(2017!)}{\log_n} = \frac{\log_n}{\log_n} = 1$$

245. (c) $C = (A \cap B') \cup (A' \cap B)$
Let us draw venn diagram and compare it with options.



This also represents $(A \cup B) - (A \cap B)$

246. (c) $x + \log_{15}(1 + 3^x) = x \log_{15}5 + \log_{15}12$

$\Rightarrow$ $x.\log_{15}15 + \log_{15}(1+3^x) = x\log_{15}5 + \log_{15}12$

$(\because \log_{15}15 = 1)$

$\Rightarrow$ $\log_{15}15^x + \log_{15}(1+3^x) = \log_{15}5^x + \log_{15}12$

$\Rightarrow$ $\log_{15}15^x(1 + 3^x) = \log_{15}(5^x \times 12)$

$(\because \log a + \log b = \log ab)$

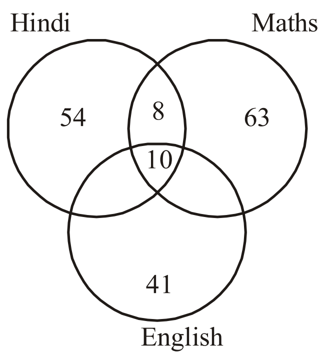
$\Rightarrow$ $15^x(1 + 3^x) = 5^x \times 12$

$\Rightarrow$ $3^x(1 + 3^x) = 12$

$3^x + 3^{2x} = 12.$

x = 1 satisfies the above equation.

247. (c) Let us represent the given data in Venn diagram as shown.



Number of students who are good in either Hindi or Maths but not in English = 54 + 18 + 63 = 125

248. (d) From the same Venn diagram,
Number of students who are good in Hindi and Maths but not English = 8

249. (d) Decimal number = 31.

2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	

So, binary form of 31 is 11111.

250. (a) $\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + ... + \frac{1}{\log_{100} N}$

$$= \frac{1}{\frac{\log N}{\log_2}} + \frac{1}{\frac{\log N}{\log_3}} + \frac{1}{\frac{\log N}{\log_4}} + ... + \frac{1}{\frac{\log N}{\log_{100}}}$$

$$\left(\because \log_a b = \frac{\log b}{\log a}\right)$$

$$= \frac{\log_2}{\log N} + \frac{\log_3}{\log N} + \frac{\log_4}{\log N} + ... + \frac{\log_{100}}{\log N}$$

$$= \frac{\log_2 + \log_3 + \log_4 + ... + \log_{100}}{\log N}$$

$$= \frac{\log(2.3.4....100)}{\log N}$$

$$= \frac{\log 100!}{\log N} = \frac{1}{\frac{\log N}{\log 100!}} = \frac{1}{\log_{100!} N}$$

251. (d) We know, $a^n + b^n$ is divisible by (a + b), if n is odd.
Here, n = 5 is odd
$\therefore$ $5^5 + 7^5$ is divisible by 5 + 7 = 12

252. (b) Number of students who like music, n(m) = 680
Number of students who like dance, n(d) = 215
Total number of students, $n(m \cup d) = 850$
$n(m \cup d) = n(m) + n(d) - n(m \cap d)$
$\Rightarrow 850 = 680 + 215 - n(m \cap d)$
$\Rightarrow n(m \cap d) = 895 - 850 = 45$

253. (b) $0 < a < 1$
Let $\log_{10}a = -x$
$\Rightarrow a = 10^{-x}$
10^{-x} can have values only between 0 and 1

254. (d) Time taken by train to cover first $5\text{km} = \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{5}{30} = \frac{1}{6}\text{hr}$$

Time taken by train to cover next $15\text{km} = \frac{15}{45} = \frac{1}{3}\text{hr}$.

Average speed $\frac{5 \quad 15}{\frac{1}{6} \quad \frac{1}{3}}$

$\frac{20}{\frac{1 \quad 2}{6}} \quad \frac{120}{3} \quad 40\text{km/hr}$

255. (c) Let $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} \quad x$

then $7^x = \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$

$7^x = \frac{1}{2}.\log_7 7\sqrt{7\sqrt{7}}$

$7^x = \frac{1}{2}\left[\log_7 7 \quad \frac{1}{2}\log_7 7\sqrt{7}\right]$

$= \frac{1}{2}\left[1 \quad \frac{1}{2}\log_7 7 \quad \frac{1}{2}\log_7 \sqrt{7}\right]$

$= \frac{1}{2}\left[1 \quad \frac{1}{2} \quad \frac{1}{4}\right]$

$7^x = \frac{7}{8}$

$x = \log_7\left(\frac{7}{8}\right)$

$x = \log_7 7 - \log_7 8$

$x = 1 - 3\log_7 2$ $\quad (\because \log_7 8 = \log_7 2^3)$

256. (c) Checking through option 'c' is incorrect.

257. (b) Number of numbers between 2999 and 8001

$= 8001 - 2999 - 1 = 5001$

3			

Number of numbers with all digit distinct and having 3 as starting digit

4			

$= 9 \times 8 \times 7 = 504$

Number of numbers with all digit distinct and having 4 as starting digit

$= 9 \times 8 \times 7 = 504$

Similarly number of numbers with starting digit 5, 6 and 7 respectively are 504, 504 and 504.

Total numbers = $5 \times 504 = 2520$

Hence, required number = $5001 - 2520 = 2481$

258. (a) $300 = 125 + 145 + 90 -$

$(|A \cap B| + |B \cap C| + |A \cap C|) + |A \cap B \cap C|$

$|A \cap B| + |B \cap C| + |A \cap C|$

$= 60 + |A \cap B \cap C|$ ...(i)

Again,

$|A \cap B| + |B \cap C| + |A \cap C| - 3|A \cap B \cap C| = 32$

$\Rightarrow |A \cap B| + |B \cap C| + |A \cap C|$

$= 32 + 3|A \cap B \cap C|$ ...(ii)

From (i) and (ii)

$|A \cap B \cap C| = 14$

259. (c) Exactly one $= |A| + |B| + |C| -$

$2[|A \cap B| + |B \cap C| + |A \cap C|] + 3|A \cap B \cap C|$

$= 125 + 145 + 90 - 2[32 + 3 \times 14] + 3 \times 14$

$= 360 - 106 = 254$

260. (b) $\log_9 27 + \log_8 32$

$= \log_9 3^3 + \log_8 2^5$

$= 3\log_9 3 + 5\log_8 2$

$3\log_{(3^2)} 3 + 5\log_{(2^3)} 2$

$\frac{3}{2}\log_3 3 + \frac{5}{3}\log_2 2$

$\frac{3}{2} \quad \frac{5}{3} \quad \frac{19}{6}$

261. (b) On substraction, we get

$$\begin{array}{r} 101101101 \\ -10110110 \\ \hline 10110111 \\ -11011 \\ \hline 10011100 \end{array}$$

$\Rightarrow x = 1, y = 0$

262. (c) $(0.2)^x = 2$

Taking log on both sides,

$x \log_{10}\frac{2}{10} = \log_{10} 2$

$x[\log_{10} 2 - \log_{10} 10] = \log_{10} 2$

$x[0.3010 - 1] = 0.3010$

$x = -\frac{0.3010}{0.6990} \approx -0.43$

263. (d) Given, x = {1, 2, 3, 4}

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

R is reflexive, if aRa for all $a \in x$

$(4, 4) \notin R$

$\therefore$ R is not reflexive ...(1)

R is transitive, if a R b, b R c $\Rightarrow$ aR C for all a, b, c $\in$ x (1, 2), (2, 3), $\in$ R, but (1, 3) $\notin$ R

$\therefore$ R is not transitive ...(2)

R is symmetric, if a R b $\Rightarrow$ b R a, for all a, b $\in$ x

$\therefore$ R is symmetric ...(3)

From (1), (2), (3) we can say, R, is neither reflexive nor transitive, but symmetric.

264. (d) Given, x R Y $\Rightarrow x^2 - 4xy + 3y^2 = 0$

$\Rightarrow x^2 - xy - 3xy + 3y^2 = 0$

$\Rightarrow x(x - y) - 3y(x - y) = 0$

$\Rightarrow (x - y)(x - 3y) = 0$

Reflexive property :

$x R x \Rightarrow (x - x)(-3x) = 0$

So, R is reflexive ...(1)

Symmetric property:

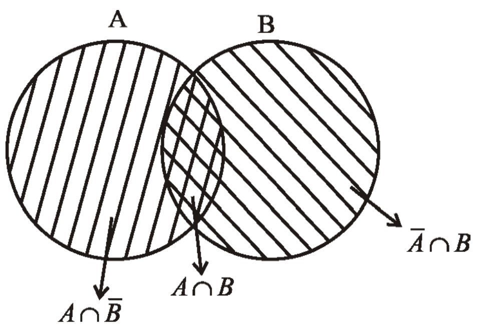
Let us check using an example (1, 2) and (2, 1)

for $(1, 2) \Rightarrow (1 - 2)(1 - 6) = (-1)(-5) = 10$

For $(2, 1) \Rightarrow (2 - 1)(2 - 3) = (1)(-1) = -1$

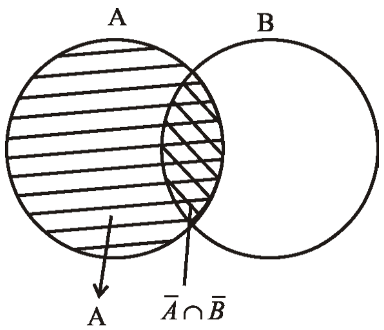
So, R is not symmetric ...(2)
Transitive property :
For $(9x, 3x) \Rightarrow (9x - 3x)(9x - 9x) = 0$
for $(3x, x) \Rightarrow (3x - 3x)(3x - 9x) = 0$
For $(9x, x) \Rightarrow (9x - x)(9x - 3x) \quad 0$
So, $(9x, 3x) \in R$, $(3x, x) \in R$ but $(9x, x) \notin R$
So, R is not transitive ...(3)
From (1), (2), (3), R is reflexive, but not symmetric and transitive.

265. (a) 1. $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup B$
Let us draw Venn diagram.



$\therefore$ 1 is correct.

2. $A \cup (\overline{A} \cap \overline{B}) = A$



$\therefore$ 2 is not correct.

266. (b) Statements (2) and (3) are correct.

267. (b) $A = \{\lambda, \{\lambda, \mu\}\}$
Power set $= \{\phi, \{\lambda\}, \{\{\lambda, \mu\}\}, \{\lambda, \{\lambda, \mu\}\}\}$

268. (b) Number of students who play chess, n (A) = 60
Number of students who play tennis, n (B) = 50
Number of students who play carrom, n (C) = 48
Given, $n(A \cap B) = 20$
$n(B \cap C) = 15$
$n(A \cap C) = 12$
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$= 60 + 50 + 48 - 20 - 15 - 12 + n(A \cap B \cap C)$
$= 111 + n(A \cap B \cap C)$
So, minimum number of students = 111

269. (b) $n(A \cup B \cup C) = 111 + n(A \cap B \cap C)$
Maximum number of students = 111 + 12 = 123

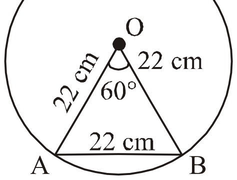
270. (d) $f(x) = \log_{10}(1 + x)$
$4 \cdot f(4) + 5 \cdot f(1) - \log_{10} 2$
$= 4 \cdot \log_{10}(1 + 4) + 5 \cdot \log_{10}(1 + 1) - \log_{10} 2$
$= 4 \log_{10} 5 + 5 \log_{10} 2 - \log_{10} 2$
$= 4 \log_{10} 5 + 4 \log_{10} 2$
$= 4 (\log_{10} 5 + \log_{10} 2)$
$= 4 (\log_{10} 10) = 4$

271. (c) f(r) is ratio of perimeter to area of circle of radius r.
Perimeter of circle $= 2\pi r$
Area of circle $= \pi r^2$

$$f(r) = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

So, $f(1) + f(2) = \frac{2}{1} + \frac{2}{2} = 2 + 1 = 3$

272. (a) Given, diameter of circle = 44 cm.
radius of circle = 22 cm
Chord of circle = 22 cm



In figure ΔOAB is equilateral triangle. Angle is 60°. So, arc is $\frac{1}{6}$ times circumstance.

Length of arc $= \frac{1}{6} \times 2\pi r = \frac{1}{6} \times 2 \times \frac{22}{7} \times 22$

$= \frac{484}{21}$ cm

Polynomial, Quadratic Equation & Inequalities

2

1. If the roots of the equation $4\beta^2 + \lambda\beta - 2 = 0$ are of the from $\frac{k}{k+1}$ and $\frac{k+1}{k+2}$, then what is the value of λ ?
(a) 2k (b) 7
(c) 2 (d) k + 1 *[2006-I]*

2. Given $4a - 2b + c = 0$ where a, b, c $\in$ R, which of the following statements is/are not true in general ?
1. (x + 2) will always be a factor of the expression $ax^2 + bx + c$.
2. (x – 2) will always be a factor of the expression $ax^2 + bx + c$.
3. There will be a factor of the expression $ax^2 + bx + c$ different from (x + 2).

Select the correct answer using the code given below :
(a) 1 and 2 only (b) 1, 2 and 3
(c) 2 only (d) 1 only *[2006-I]*

3. If the sum of the squares of the roots of $x^2 - (p - 2)x - (p + 1) = 0$ (p $\in$ R) is 5, then what is the value of p ? *[2006-I]*
(a) 0 (b) – 1
(c) 1 (d) $\frac{3}{2}$

4. What is the number of real solutions of $|x^2 - x - 6| = x + 2$? *[2006-I]*
(a) 4 (b) 3
(c) 2 (d) 1

5. If the roots of $x^2 - 2mx + m^2 - 1 = 0$ lie between – 2 and 4, then which one of the following is correct ?
(a) $-1 \le m \le 3$ (b) $-3 \le m \le 3$
(c) $-3 \le m \le 5$ (d) $-1 \le m \le 5$ *[2006-I]*

6. If $(\log_3 x)^2 + \log_3 x < 2$, then which one of the following is correct ?
(a) $0 < x < \frac{1}{9}$ (b) $\frac{1}{9} < x < 3$
(c) $3 < x < \infty$ (d) $\frac{1}{9} \le x \le 3$ *[2006-I]*

7. For what values of a does the equation $\cos 2x + a \sin x = 2a - 7$ possess a real solution ? *[2006-II]*
(a) a < 2 (b) $a \ge 8$
(c) a > 8 (d) a is any integer < – 2

8. If sin θ and cos θ are the roots of $ax^2 + bx + c = 0$, then constants a, b, c will satisfy which one of the following conditions? *[2006-II]*
(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 + b^2 - 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$ (d) $-a^2 + b^2 + 2ac = 0$

9. If $a^2 + b^2 + c^2 = 0$, then what is
$$\frac{(a^4 - b^4)^3 + (b^4 - c^4)^3 + (c^4 - a^4)^3}{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}$$
equal to ?
(a) $a^2b^2c^2$ (b) $-a^2b^2c^2$
(c) abc (d) $3a^2b^2c^2$ *[2006-II]*

10. If $0 < x < y < \pi$, then which one of the following is correct?
(a) x – cos x > y – cos y
(b) x – cos x < y – cos y
(c) x + cos x > y + cos y
(d) x + cos x < y + cos y *[2006-II]*

11. What is the $(m - 1)^{th}$ root of $\left[(a^m)^m - \left(\frac{1}{m}\right)\right]^{\frac{1}{m+1}}$?
(a) $a^{m+(1/m)}$ (b) $a^{m-(1/m)}$
(c) a (d) 1 *[2006-II]*

12. Let a, b $\in$ {1, 2, 3}. What is the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots ?
(a) 1 (b) 2
(c) 5 (d) 3 *[2006-II]*

13. If $px^2 + qx + r = p(x - \alpha)(x - \beta)$, and $p^3 + pq + r = 0$; p,q and r being real numbers, then which of the following is not possible ?
(a) $\alpha = \beta = p$ (b) $\alpha \ne \beta = p$
(c) $\alpha = \beta \ne p$ (d) $\beta \ne \alpha = p$ *[2006-II]*

14. If the equation $x^2 + k^2 = 2(k + 1)x$ has equal roots, then what is the value of k? *[2007-I]*
(a) $-\frac{1}{3}$ (b) $-\frac{1}{2}$
(c) 0 (d) 1

15. If $x = a^{1/3} - a^{-1/3}$, then what is $x^3 + 3x$ equal to? *[2007-I]*
(a) zero (b) $a + \left(\frac{1}{a}\right)$
(c) $a - \left(\frac{1}{a}\right)$ (d) $a^3 + \left(\frac{1}{a^3}\right)$

16. If $x^{1/3}+y^{1/3}+z^{1/3}=0$ then what is $(x+y+z)^3$ equal to?
(a) 1 (b) 3
(c) 3xyz (d) 27 xyz *[2007-I]*

17. If α, β are the roots of $ax^2+2bx+c=0$ and $\alpha+\delta, \beta+\delta$ are the roots of $Ax^2+2Bx+C=0$, then what is $(b^2-ac)/(B^2-AC)$ equal to? *[2007-I]*
(a) $(b/B)^2$ (b) $(a/A)^2$
(c) $(a^2b^2)/(A^2B^2)$ (d) $(ab)/(AB)$

18. If α, β are the roots of the equation $ax^2+bx+c=0$, then what is the value of $(a\alpha+b)^{-1}+(a\beta+b)^{-1}$? *[2007-I]*
(a) a/(bc) (b) b/ac
(c) –b/(ac) (d) –a/(bc)

19. If α, β are the roots of the equations $x^2-2x-1=0$, then what is the value of $\alpha^2\beta^{-2}+\alpha^{-2}\beta^2$ *[2007-I]*
(a) –2 (b) 0
(c) 30 (d) 34

20. Which one of the following values of x, y satisfies the in equation $2x+3y\le 6;\ x\ge 0,\ y\ge 0$? *[2007-I]*
(a) x = 0, y = 3 (b) x = 1, y = 2
(c) x = 1, y = 1 (d) x = 4, y = 0

21. What is the value of x at the intersection of $y=\dfrac{8}{(x^2+4)}$ and $x+y=2$? *[2007-I]*
(a) 0 (b) 1
(c) 2 (d) –1

22. If the roots of the equations $x^2-(a-1)x+(a+b)=0$ and $ax^2-2x+b=0$ are identical, then what are the values of a and b?
(a) a = 2, b = 4 (b) a = 2, b = –4
(c) $a=1, b=\frac{1}{2}$ (d) $a=-1, b=-\frac{1}{2}$
[2007-I]

23. How many real values of x satisfy the equation $|x|+|x-1|=1$?
(a) 1 (b) 2
(c) Infinite (d) No value of x *[2007-I]*

24. What is the number of digits in the numeral form of 8^{17}? (Given $\log_{10}2=0.3010$)
(a) 51 (b) 16
(c) 15 (d) 14 *[2007-I]*

25. If α and β are the roots of the equation $x^2+x+1=0$, then what is the equation whose roots are α^{19} and β^7?
(a) $x^2-x-1=0$ (b) $x^2-x+1=0$
(c) $x^2+x-1=0$ (d) $x^2+x+1=0$ *[2007-II]*

26. If α and β are the roots of the equation $x^2+6x+1=0$, then what is $|\alpha-\beta|$ equal to?
(a) 6 (b) $3\sqrt{2}$
(c) $4\sqrt{2}$ (d) 12 *[2007-II]*

27. If $r^{1/3}+\dfrac{1}{r^{1/3}}=3$ for a real number $r\neq 0$, then what is $r+\dfrac{1}{r}$ equal to? *[2007-II]*
(a) 27 (b) 36
(c) 9 (d) 18

28. The number of rows in a lecture hall equals the number of seats in a row. If the number of rows is doubled and the number of seats in every row is reduced by 10, the number of seats is increased by 300. If x denotes the number of rows in the lecture hall, then what is the value of x?
(a) 10 (b) 15
(c) 20 (d) 30 *[2007-II]*

29. If α, β are the roots of the equation $\ell x^2-mx+m=0$, $\ell\neq m, \ell\neq 0$, then which one of the following statements is correct? *[2007-II]*
(a) $\sqrt{\dfrac{\alpha}{\beta}}+\sqrt{\dfrac{\beta}{\alpha}}-\sqrt{\dfrac{m}{\ell}}=0$
(b) $\sqrt{\dfrac{\alpha}{\beta}}+\sqrt{\dfrac{\beta}{\alpha}}+\sqrt{\dfrac{m}{\ell}}=0$
(c) $\sqrt{\dfrac{\alpha+\beta}{\alpha\beta}}-\sqrt{\dfrac{m}{\ell}}=0$
(d) The arithmetic mean of α and β is the same as their geometric mean

30. For what value of k, are the roots of the quadratic equation $(k+1)x^2-2(k-1)x+1=0$ real and equal? *[2007-II]*
(a) k = 0 only (b) k = – 3 only
(c) k = 0 or k = 3 (d) k = 0 or k = – 3

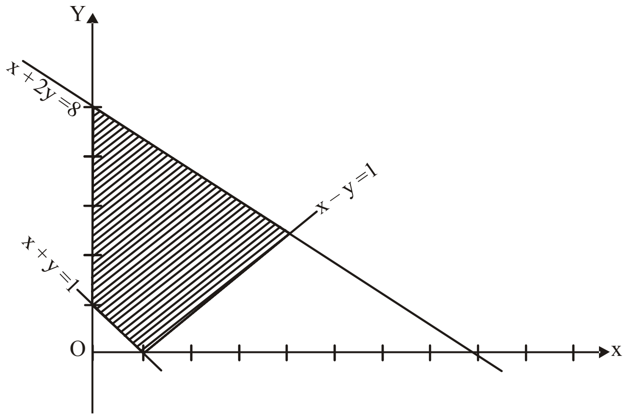
31. If roots of an equation $ax^2+bx+c=0$ are positive, then which one of the following is correct?
(a) Signs of a and c should be like
(b) Signs of b and c should be like
(c) Signs of a and b should be like
(d) None of the above *[2007-II]*

32. Which one of the following is correct? If $4<x^2<9$, then *[2007-II]*
(a) $2<x<3$ only (b) $-3<x<-2$ only
(c) $2<x<3, -3<x<-2$ (d) None of these

33. If α and β are the roots of the equation $ax^2+bx+c=0$, then what are the roots of the equation $cx^2+bx+a=0$?
(a) $\beta, \dfrac{1}{\alpha}$ (b) $\alpha, \dfrac{1}{\beta}$
(c) $-\alpha, -\beta$ (d) $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ *[2007-II]*

34. If x and y are real numbers such that $x>y$ and $|x|>|y|$, then which one of the following is correct?
(a) $x>0$ (b) $y>0$
(c) $y<0$ (d) $x<0$ *[2007-II]*

35. What are the linear constraints for which the shaded area in the above figure is the solution set ? *[2007-II]*



(a) $x - y \geq 1;\ x + 2y \leq 8;\ x + y \geq 1;\ x, y \geq 0$

(b) $x - y \leq 1;\ x + 2y \geq 8;\ x + y \leq 1;\ x, y \geq 0$

(c) $x - y \leq 1;\ x + 2y \leq 8;\ x + y \geq 1;\ x, y \geq 0$

(d) $x - y \leq 1;\ x + 2y \leq 8;\ x + y \leq 1;\ x, y \geq 0$

36. If x is real and $x^2 - 3x + 2 > 0$, $x^2 - 3x - 4 \leq 0$, then which one of the following is correct? *[2008-I]*
(a) $-1 \leq x \leq 4$ (b) $2 \leq x \leq 4$
(c) $-1 < x \leq 1$ (d) $-1 \leq x < 1$ or $2 < x \leq 4$

37. If $x = 2^{1/3} - 2^{-1/3}$, then what is the value of $2x^3 + 6x$?
(a) 1 (b) 2
(c) 3 (d) 4 *[2008-I]*

38. What is the value of

$\sqrt{5\sqrt{5\sqrt{5\sqrt{\ldots\infty}}}}$? *[2008-I]*

(a) 5 (b) $\sqrt{5}$
(c) 1 (d) $(5)^{1/4}$

39. For the real numbers p, q, r, x, y, let $p < x < q$ and $p < y < r$. Which one of the following is correct? *[2008-I]*
(a) $p < x < y < r$ (b) $p < x < q < r$
(c) $p < y < x < q$ (d) None of these

40. One root of the equation $x^2 = px + q$ is reciprocal of the other and $p \neq \pm 1$. What is the value of q? *[2008-I]*
(a) $q = -1$ (b) $q = 1$
(c) $q = 0$ (d) $q = \frac{1}{2}$

41. If the equation $x^2 + kx + 1 = 0$ has the roots α and β, then what is the value of $(\alpha + \beta) \times (\alpha^{-1} + \beta^{-1})$? *[2008-I]*
(a) k^2 (b) $\frac{1}{k^2}$
(c) $2k^2$ (d) $\frac{1}{(2k^2)}$

42. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then what is the value of $b^2 - 4c$? *[2008-II]*
(a) 1 (b) 2
(c) –2 (d) 3

43. If *r* and *s* are roots of $x^2 + px + q = 0$, then what is the value of $(1/r^2) + (1/s^2)$? *[2008-II]*
(a) $p^2 - 4q$ (b) $\frac{p^2 - 4q}{2}$
(c) $\frac{p^2 - 4q}{q^2}$ (d) $\frac{p^2 - 2q}{q^2}$

44. If *x* is an integer and satisfies $9 < 4x - 1 \leq 19$, then *x* is an element of which one of the following sets? *[2008-II]*
(a) {3, 4} (b) {2, 3, 4}
(c) {3, 4, 5} (d) {2, 3, 4, 5}

45. If $a = x + \sqrt{x^2 + 1}$, then what is *x* equal to? *[2008-II]*
(a) $(1/2)(a + a^{-1})$ (b) $(1/2)(a - a^{-1})$
(c) $a + a^{-1}$ (d) $a - a^{-1}$

46. A quadratic polynomial with two distinct roots has one real root. Then, the other root is *[2008-II]*
(a) not necessarily real, if the coefficients are real
(b) always imaginary
(c) always real
(d) real, if the coefficients are real

47. If sinα and cosα are the roots of the equation $px^2 + qx + r = 0$, then which one of the following is correct? *[2008-II]*
(a) $p^2 + q^2 - 2pr = 0$
(b) $p^2 - q^2 + 2pr = 0$
(c) $(p + r)^2 = 2(p^2 + r^2)$
(d) $(p - r)^2 = q^2 + r^2$

48. If α and β are the roots of $x^2 + 4x + 6 = 0$, then what is the value of $\alpha^3 + \beta^3$? *[2008-II]*
(a) –2/3 (b) 2/3
(c) 4 (d) 8

49. If sum of the roots of $3x^2 + (3p + 1)x - (p + 5) = 0$ is equal to their product, then what is the value of *p*? *[2008-II]*
(a) 2 (b) 3
(c) 4 (d) 9

50. If a polygon has 20 diagonals, then what is the number of sides? *[2008-II]*
(a) 6 (b) 10
(c) 12 (d) 8

51. Let α, γ be the roots of $Ax^2 - 4x + 1 = 0$ and β, δ be the roots of $Bx^2 - 6x + 1 = 0$. If α, γ, β, δ are in HP, then what are the values of *A* and *B* respectively? *[2009-I]*
(a) 3, 8 (b) –3, –8
(c) 3, –8 (d) –3, 8

52. If $2^x + 3^y = 17$ and $2^{x+2} - 3^{y+1} = 5$, then what is the value of *x*? *[2009-I]*
(a) 3 (b) 2
(c) 1 (d) 0

53. If $(x + a)$ is a factor of both the quadratic polynomials $x^2 + px + q$ and $x^2 + lx + m$, where *p*, *q*, *l* and *m* are constants, then which one of the following is correct? *[2009-I]*
(a) $a = (m - q)/(l - p)\ (l \neq p)$
(b) $a = (m + q)/(l + p)\ (l \neq -p)$
(c) $l = (m - q)/(a - p)\ (a \neq p)$
(d) $p = (m - q)/(a - l)\ (a \neq l)$

54. Which one of the following is one of the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$? *[2009-I]*
(a) $(c-a)/(b-c)$ (b) $(a-b)/(b-c)$
(c) $(b-c)/(a-b)$ (d) $(c-a)/(a-b)$

55. What is the value of x satisfying the equation
$16\left(\dfrac{a-x}{a+x}\right)^3=\dfrac{a+x}{a-x}$? *[2009-I]*
(a) $a/2$ (b) $a/3$
(c) $a/4$ (d) 0

56. If α, β are the roots of the equation $2x^2-2(1+n^2)x+(1+n^2+n^4)=0$, then what is the value of $\alpha^2+\beta^2$? *[2009-I]*
(a) $2n^2$ (b) $2n^4$
(c) 2 (d) n^2

57. The roots of $Ax^2+Bx+C=0$ are r and s. For the roots of $x^2+px+q=0$ to be r^2 and s^2, what must be the value of p? *[2009-I]*
(a) $(B^2-4AC)/A^2$ (b) $(B^2-2AC)/A^2$
(c) $(2AC-B^2)/A^2$ (d) B^2-2C

58. If α, β are the roots of $ax^2+bx+b=0$, then what is $\dfrac{\sqrt{\alpha}}{\sqrt{\beta}}+\dfrac{\sqrt{\beta}}{\sqrt{\alpha}}+\dfrac{\sqrt{b}}{\sqrt{a}}$ equal to? *[2009-II]*
(a) 0 (b) 1
(c) 2 (d) 3

59. If the roots of $ax^2+bx+c=0$ are $\sin\alpha$ and $\cos\alpha$ for some α, then which one of the following is correct? *[2009-II]*
(a) $a^2+b^2=2ac$ (b) $b^2-c^2=2ab$
(c) $b^2-a^2=2ac$ (d) $b^2+c^2=2ab$

60. If $x=2+2^{1/3}+2^{2/3}$, then what is the value of x^3-6x^2+6x? *[2009-II]*
(a) 1 (b) 2
(c) 3 (d) –2

61. The roots of the equation $(x-p)(x-q)=r^2$, where p, q, r are real, are *[2009-II]*
(a) always complex
(b) always real
(c) always purely imaginary
(d) None of these

62. The equation $x-2(x-1)^{-1}=1-2(x-1)^{-1}$ has *[2009-II]*
(a) no roots (b) one root
(c) two equal roots (d) infinite roots

63. If a, b and c are real numbers then the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are always *[2009-II]*
(a) real (b) imaginary
(c) positive (d) negative

64. For the two equations $x^2+mx+1=0$ and $x^2+x+m=0$, what is/are the value/ values of m for which these equations have at least one common root? *[2009-II]*
(a) –2 only (b) 1 only
(c) –2 and 1 (d) –2 and –1

65. Consider the equation $(x-p)(x-6)+1=0$ having integral coefficients. If the equation has integral roots, then what values can p have? *[2010-I]*
(a) 4 or 8 (b) 5 or 10
(c) 6 or 12 (d) 3 or 6

66. If $\dfrac{1}{2-\sqrt{-2}}$ is one of the roots of $ax^2+bx+c=0$, where a, b, c are real, then what are the values of a, b, c respectively?
(a) 6, – 4, 1 (b) 4, 6, – 1 *[2010-I]*
(c) 3, – 2, 1 (d) 6, 4, 1

67. If α, β are the roots of the quadratic equation $x^2-x+1=0$, then which one of the following is correct? *[2010-I]*
(a) $(\alpha^4-\beta^4)$ is real (b) $2(\alpha^6+\beta^5)=(\alpha\beta)^5$
(c) $(\alpha^6-\beta^6)=0$ (d) $(\alpha^8+\beta^8)=(\alpha\beta)^8$

68. If p and q are positive integers, then which one of the following equations has $p-\sqrt{q}$ as one of its roots? *[2010-I]*
(a) $x^2-2px-(p^2-q)=0$ (b) $x^2-2px+(p^2-q)=0$
(c) $x^2+2px-(p^2-q)=0$ (d) $x^2+2px+(p^2-q)=0$

69. If the product of the roots of the equation $x^2-5x+k=15$ is – 3, then what is the value of k? *[2010-I]*
(a) 12 (b) 15
(c) 16 (d) 18

70. If the equation $x^2-bx+1=0$ does not possess real roots, then which one of the following is correct? *[2010-I]*
(a) $-3<b<3$ (b) $-2<b<2$
(c) $b>2$ (d) $b<-2$

71. If p and q are the roots of the equation $x^2-px+q=0$, then what are the values of p and q respectively? *[2010-I]*
(a) 1 , 0 (b) 0, 1
(c) –2, 0 (d) –2, 1

72. If the equations $x^2+kx+64=0$ and $x^2-8x+k=0$ have real roots, then what is the value of k? *[2010-II]*
(a) 4 (b) 8
(c) 12 (d) 16

73. If the roots of the equation $(a^2+b^2)x^2-2b(a+c)x+(b^2+c^2)=0$ are equal, then which one of the following is correct? *[2010-II]*
(a) $2b=a+c$ (b) $b^2=ac$
(c) $b+c=2a$ (d) $b=ac$

74. If α and β are the roots of the equation $x^2-2x+4=0$, then what is the value of $\alpha^3+\beta^3$? *[2010-II]*
(a) 16 (b) – 16
(c) 8 (d) – 8

75. Which of the following are the two roots of the equation $(x^2+2)^2+8x^2=6x(x^2+2)$? *[2010-II]*
(a) $1\pm i$ (b) $2\pm i$
(c) $1\pm\sqrt{2}$ (d) $2\pm i\sqrt{2}$

76. If α and β are the roots of the equation $x^2+x+1=0$, then which of the following are the roots of the equation $x^2-x+1=0$? *[2010-II]*
(a) α^7 and β^{13} (b) α^{13} and β^7
(c) α^{20} and β^{20} (d) None of these

77. What is the solution set for the equation *[2011-I]*
$x^4-26x^2+25=0$
(a) $\{-5,-1,1,5\}$ (b) $\{-5,-1\}$
(c) $\{1,5\}$ (d) $\{-5,0,1,5\}$

78. If α and β are the roots of the equation $4x^2 + 3x + 7 = 0$, then what is the value of $(\alpha^{-2} + \beta^{-2})$? [2011-I]
(a) 47 / 49 (b) 49 / 47
(c) – 47 / 49 (d) – 49 / 47

79. What is the set of points (x, y) satisfying the equations $x^2 + y^2 = 4$ and $x + y = 2$? [2011-I]
(a) {(2, 0), (– 2, 0), (0, 2)}
(b) {(0, 2), (0, – 2)}
(c) {(0, 2), (2, 0)}
(d) {(2, 0), (– 2, 0), (0, 2), (0, – 2)}

80. If p, q and r are rational numbers, then the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are [2011-I]
(a) complex (b) pure imaginary
(c) irrational (d) rational

81. What is the sum of the roots of the equation
$\left(2-\sqrt{3}\right)x^2 - \left(7-4\sqrt{3}\right)x + \left(2+\sqrt{3}\right) = 0$? [2011-I]
(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $7-4\sqrt{3}$ (d) 4

82. One of the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is positive and the other root is negative. The condition for this to happen is [2011-I]
(a) $a > 0, b > 0, c > 0$ (b) $a > 0, b < 0, c > 0$
(c) $a < 0, b > 0, c < 0$ (d) $a < 0, c > 0$

83. What is the condition that one root of the equation $ax^2 + bx + c = 0$, $a \neq 0$ should be double the other? [2011-I]
(a) $2a^2 = 9bc$ (b) $2b^2 = 9ac$
(c) $2c^2 = 9ab$ (d) None of these

84. If $x + y \leq 4$, then the how many non-zero positive integer ordered pair (x, y)? [2011-I]
(a) 4 (b) 5
(c) 6 (d) 8

85. If 3 is the root of the equation $x^2 - 8x + k = 0$, then what is the value of k? [2011-I]
(a) – 15 (b) 9
(c) 15 (d) 24

86. If sum of squares of the roots of the equation $x^2 + kx - b = 0$ is $2b$, what is k equal to? [2011-I]
(a) 1 (b) b
(c) $-b$ (d) 0

87. If one root of the equation $ax^2 + bx + c = 0$, $a \neq 0$ is reciprocal of the other root, then which one of the following is correct? [2011-I]
(a) $a = c$ (b) $b = c$
(c) $a = -c$ (d) $b = 0$

88. The equation $x^2 - 4x + 29 = 0$ has one root $2 + 5i$. What is the other root? [2011-II]
$\left(i = \sqrt{-1}\right)$
(a) 2 (b) 5
(c) $2 + 5i$ (d) $2 - 5i$

89. Let α, β be the roots of the equation $(x - a)(x - b) = c$, $c \neq 0$. Then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are [2011-II]
(a) a, c (b) b, c
(c) a, b (d) $a + b, a + c$

90. If the equations $x^2 - px + q = 0$ and $x^2 - ax + b = 0$ have a common root and the roots of the second equation are equal, then which one of the following is correct? [2011-II]
(a) $aq = 2(b + p)$ (b) $aq = (b + p)$
(c) $ap = 2(b + q)$ (d) $ap = b + q$

91. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are α^{19} and β^7 is [2011-II]
(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$

92. What is the value of [2011-II]
$\sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8+ \ldots \infty}}}}$?
(a) 10 (b) 8
(c) 6 (d) 4

93. If $\sin\theta = x + \frac{a}{x}$ for all $x \in R - \{0\}$, then which one of the following is correct? [2011-II]
(a) $a \geq 4$ (b) $a \geq \frac{1}{2}$
(c) $a \leq \frac{1}{4}$ (d) $a \leq \frac{1}{2}$

94. The equation $\tan^4 x - 2\sec^2 x + a^2 = 0$ will have at least one real solution if [2011-II]
(a) $|a| \leq 4$ (b) $|a| \leq 2$
(c) $|a| \leq \sqrt{3}$ (d) None of the above

95. If the roots of the equation $x^2 - 4x - \log_3 N = 0$ are real, then what is the minimum value of N? [2011-II]
(a) 1/256 (b) 1/27
(c) 1/64 (d) 1/81

96. If one of the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ is 1, what is the second root? [2011-II]
(a) $-\frac{b(c-a)}{a(b-c)}$ (b) $\frac{b(c-a)}{a(b-c)}$
(c) $\frac{c(a-b)}{a(b-c)}$ (d) $-\frac{c(a-b)}{a(b-c)}$

97. What are the roots of the equation $2(y + 2)^2 - 5(y + 2) = 12$? [2011-II]
(a) – 7/2, 2 (b) – 3/2, 4
(c) – 5/3, 3 (d) 3/2, 4

98. If the roots of the equation $3x^2 - 5x + q = 0$ are equal, then what is the value of q? [2011-II]
(a) 2 (b) 5/12
(c) 12/25 (d) 25/12

99. If the difference between the roots of $ax^2 + bx + c = 0$ is 1, then which one of the following is correct? [2012-I]
(a) $b^2 = a(a + 4c)$ (b) $a^2 = b(b + 4c)$
(c) $a^2 = c(a + 4c)$ (d) $b^2 = a(b + 4c)$

100. If one of the roots of the equation $x^2 + ax - 6 = 0$ is 1, then what is $(a - 6)$ equal to? [2012-I]
(a) – 1 (b) 1
(c) 2 (d) – 2

101. If α and β are the roots of the equation $x^2 - q(1 + x) - r = 0$, then what is $(1 + \alpha)(1 + \beta)$ equal to? [2012-I]
(a) $1 - r$ (b) $q - r$
(c) $1 + r$ (d) $q + r$

DIRECTIONS (Qs. 102-103): *For the next [02] questions that follow:*

The equation formed by multiplying each root of $ax^2 + bx + c = 0$ by 2 is $x^2 + 36x + 24 = 0$

102. What is b:c equal to? [2012-I]
(a) 3 : 1 (b) 1 : 2
(c) 1 : 3 (d) 3 : 2

103. Which one of the following is correct? [2012-I]
(a) $bc = a^2$ (b) $bc = 36a^2$
(c) $bc = 72a^2$ (d) $bc = 108\,a^2$

104. What is the sum of the squares of the roots of the equation $x^2 + 2x - 143 = 0$ [2012-I]
(a) 170 (b) 180
(c) 190 (d) 290

105. The solution of the simultaneous linear equations $2x + y = 6$ and $3y = 8 + 4x$ will also be satisfied by which one of the following linear equations? [2012-I]
(a) $x + y = 5$ (b) $2x + y = 5$
(c) $2x - 3y = 10$ (d) $2x + 3y = 6$

106. If the roots of a quadratic equation are $m + n$ and $m - n$, then the quadratic equation will be : [2012-II]
(a) $x^2 + 2\,mx + m^2 - mn + n^2 = 0$
(b) $x^2 + 2\,mx + (m - n)^2 = 0$
(c) $x^2 - 2\,mx + m^2 - n^2 = 0$
(d) $x^2 + 2\,mx + m^2 - n^2 = 0$

107. If α, β are the roots of $x^2 + px - q = 0$ and γ, δ are the roots of $x^2 - px + r = 0$ then what is $(\beta + \gamma)(\beta + \delta)$ equal to ?
(a) $p + r$ (b) $p + q$ [2012-II]
(c) $q + r$ (d) $p - q$

108. If the roots of the quadratic equation $3x^2 - 5x + p = 0$ are real and unequal, then which one of the following is correct?
(a) $p = 25/12$ (b) $p < 25/12$ [2012-II]
(c) $p > 25/12$ (d) $p \le 25/12$

109. If $4^x - 6.2^x + 8 = 0$, then the values of x are [2013-I]
(a) 1, 2 (b) 1, 1
(c) 1, 0 (d) 2, 2

110. If the roots of a quadratic equation $ax^2 + bx + c = 0$ are α and β, then the quadratic equation having roots α^2 and β^2 is
(a) $x^2 - (b^2 - 2ac)\,x + c = 0$ [2013-I]
(b) $a^2x^2 - (b^2 - 2ac)\,x + c = 0$
(c) $ax^2 - (b^2 - 2ac)\,x + c^2 = 0$
(d) $a^2x^2 - (b^2 - 2ac)\,x + c^2 = 0$

111. If the roots of the equation $3ax^2 + 2bx + c = 0$ are in the ratio 2 : 3, then which one of the following is correct?
(a) $8ac = 25b$ (b) $8ac = 9b^2$ [2013-I]
(c) $8b^2 = 9ac$ (d) $8b^2 = 25ac$

112. If the sum of the roots of a quadratic equation is 3 and the product is 2, then the equation is [2013-I]
(a) $2x^2 - x + 3 = 0$ (b) $x^2 - 3x + 2 = 0$
(c) $x^2 + 3x + 2 = 0$ (d) $x^2 - 3x - 2 = 0$

113. If α and β are the roots of the equation $x^2 + bx + c = 0$, then what is the value of $\alpha^{-1} + \beta^{-1}$? [2013-I]
(a) $-\frac{b}{c}$ (b) $\frac{b}{c}$
(c) $\frac{c}{b}$ (d) $-\frac{c}{b}$

114. The area of a rectangle whose length is five more than twice its width is 75 square unit. The length is [2013-I]
(a) 5 unit (b) 10 unit
(c) 15 unit (d) 20 unit

115. $(x + 1)^2 - 1 = 0$ has [2013-I]
(a) one real root (b) two real roots
(c) two imaginary roots (d) four real roots

116. What is the positive square root of $7 + 4\sqrt{3}$? [2013-II]
(a) $\sqrt{3} - 1$ (b) $\sqrt{3} + 1$
(c) $\sqrt{3} - 2$ (d) $\sqrt{3} + 2$

117. If α, β are the roots of the equation $x^2 + x + 2 = 0$, then what is $\frac{\alpha^{10} + \beta^{10}}{\alpha^{-10} + \beta^{-10}}$ equal to? [2013-II]
(a) 4096 (b) 2048
(c) 1024 (d) 512

118. If *a* and *b* are rational and *b* is not perfect square, then the quadratic equation with rational coefficients whose one root is $3a + \sqrt{b}$ is [2013-II]
(a) $x^2 - 6ax + 9a^2 - b = 0$ (b) $3ax^2 + x - \sqrt{b} = 0$
(c) $x^2 + 3ax + \sqrt{b} = 0$ (d) $\sqrt{b}x^2 + x - 3a = 0$

119. How many real roots does the quadratic equation $f(x) = x^2 + 3|x| + 2 = 0$ have ? [2013-II]
(a) One (b) Two
(c) Fore (d) No real root

120. If α, β are the roots of the equation $ax^2 + bx + b = 0$, then what is the value of $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}}$? [2013-II]
(a) –1 (b) 0
(c) 1 (d) 2

121. The roots of the equation [2013-II]
$x^2 - 8x + 16 = 0$
(a) are imaginary (b) are distinct and real
(c) are equal and real (d) cannot be ascertained

122. What is the difference in the roots of the equation $x^2 - 10x + 9 = 0$? [2013-II]
(a) 2 (b) 3
(c) 5 (d) 8

123. If $8x - 9y = 20$ and $7x - 10y = 9$, then what is $2x - y$ equal to ? [2013-II]
(a) 10 (b) 11
(c) 12 (d) 13

124. The quadratic equation $x^2 + bx + 4 = 0$ will have real roots if *[2013-II]*
(a) $b \le -4$ only (b) $b \ge 4$ only
(c) $-4 < b < 4$ (d) $b \le -4, b \ge 4$

125. If α and β are the roots of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, then $(a\alpha + b)(a\beta + b)$ is equal to : *[2014-I]*
(a) ab (b) bc
(c) ca (d) abc

126. The roots of the equation $2a^2x^2 - 2abx + b^2 = 0$ when $a < 0$ and $b > 0$ are : *[2014-I]*
(a) Sometimes complex (b) Always irrational
(c) Always complex (d) Always real

127. Every quadratic equation $ax^2 + bx + c = 0$ where $a, b, c, \in R$, $a \neq 0$ has *[2014-II]*
(a) exactly one real root. (b) at least one real root.
(c) at least two real roots. (d) at most two real roots.

128. If α, β are the roots of $ax^2 + bx + c = 0$ and $a + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then what is h equal to ? *[2014-II]*
(a) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (b) $\frac{1}{2}\left(-\frac{b}{a} + \frac{q}{p}\right)$
(c) $\frac{1}{2}\left(\frac{b}{p} + \frac{q}{a}\right)$ (d) $\frac{1}{2}\left(-\frac{b}{p} + \frac{q}{a}\right)$

129. Consider the following statements in respect of the given equation : *[2015-I]*
$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$
1. All the roots of the equation are complex.
2. The sum of all the roots of the equation is 6.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

130. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtains 8 and 2 for roots. Another student makes a mistake only in the coefficient of first-degree term and finds –9 and –1 for roots. The correct equation is *[2015-I]*
(a) $x^2 - 10x + 9 = 0$ (b) $x^2 - 10x + 9 = 0$
(c) $x^2 - 10x + 16 = 0$ (d) $x^2 - 8x - 9 = 0$

131. If m and n are the roots of the equation $(x + p)(x + q) - k = 0$, then the roots of the equation $(x - m)(x - n) + k = 0$ are *[2015-I]*
(a) P and q (b) $\frac{1}{p}$ and $\frac{1}{q}$
(c) $-p$ and $-q$ (d) $p + q$ and $p - q$

132. If $2p + 3q = 18$ and $4p^2 + 4pq - 3q^2 - 36 = 0$, then what is $(2p + q)$ equal to? *[2015-I]*
(a) 6 (b) 7
(c) 10 (d) 20

133. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ is *[2015-II]*
(a) 4 (b) 3
(c) 2 (d) 1

134. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then *[2015-II]*
(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = a + b$
(c) $ab + b^2 = 2ac$ (d) $ab - b^2 = 2ac$

135. If the roots of the equation $x^2 - nx + m = 0$ differ by 1, then *[2015-II]*
(a) $n^2 - 4m - 1 = 0$ (b) $n^2 + 4m - 1 = 0$
(c) $m^2 + 4n + 1 = 0$ (d) $m^2 - 4n - 1 = 0$

136. If $x^2 + px + 4 > 0$ for all real values of x, then which one of the following is correct? *[2016-I]*
(a) $|p| < 4$ (b) $|p| \le 4$
(c) $|p| > 4$ (d) $|p| \ge 4$

DIRECTIONS (Qs. 137-138): *For the next two (2) items that follow:*

Consider the function $f(x) = \frac{27(x^{2/3} - x)}{4}$ *[2016-I]*

137. How many solutions does the function f(x) = 1 have?
(a) One (b) Two
(c) Three (d) Four

138. How many solutions does the function f(x) = –1 have?
(a) One (b) Two
(c) Three (d) Four

DIRECTIONS (Qs. 139-140): *For the next two (2) items that follow:*

Let α and β $(\alpha < \beta)$ be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$. *[2016-I]*

139. Consider the following:
1. $\beta < -\alpha$ 2. $\beta < |\alpha|$
Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

140. Consider the following:
1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0$
Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

141. If one root of the equation $(1 - m)x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m? *[2016-I]*
(a) $-\frac{9}{8}$ (b) $\frac{9}{8}$
(c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

142. If $c > 0$ and $4a + c < 2b$, then $ax^2 - bx + c = 0$ has a root in which one of the following intervals? *[2016-II]*
(a) (0, 2) (b) (2, 3)
(c) (3, 4) (d) (–2, 0)

143. If both the roots of the equation $x^2 - 2kx + k^2 - 4 = 0$ lie between –3 and 5, then which one of the following is correct? *[2016-II]*
(a) $-2 < k < 2$ (b) $-5 < k < 3$
(c) $-3 < k < 5$ (d) $-1 < k < 3$

DIRECTIONS (Qs. 144-145): *Consider the following for the next two (02) items that follow:*

Let α and β be the roots of the equation

$x^2 - (1 - 2a^2)x + (1 - 2a^2) = 0$

144. Under what condition does the above equation have real roots ? *[2016-II]*

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$

(c) $a^2 \le \frac{1}{2}$ (d) $a^2 \ge \frac{1}{2}$

145. Under what condition is $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$? *[2016-II]*

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$

(c) $a^2 > 1$ (d) $a^2 \in \left(\frac{1}{3}, \frac{1}{2}\right)$ only

146. What is the greatest value of the positive integer n satisfying the condition *[2016-II]*

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$?

(a) 8 (b) 9

(c) 10 (d) 11

DIRECTIONS (Qs. 147-148): *Consider the following for the next two (02) items that follow:*

$2x^2 + 3x - \alpha - 0$ has roots -2 and β while the equation $x^2 - 3mx + 2m^2 = 0$ has both roots positive, where $\alpha > 0$ and $\beta > 0$.

147. What is the value of α ? *[2016-II]*

(a) 1/2 (b) 1

(c) 2 (d) 4

148. If β, 2, 2m are in GP, then what is the value of $\beta\sqrt{m}$? *[2016-II]*

(a) 1 (b) 2

(c) 4 (d) 6

149. If the point (a, a) lies between the lines $|x + y| = 2$, then which one of the following is correct? *[2016-II]*

(a) $|a| < 2$ (b) $|a| < \sqrt{2}$

(c) $|a| < 1$ (d) $|a| < \frac{1}{\sqrt{2}}$

150. If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, then which one of the following is correct? *[2017-I]*

(a) $p^2m = l^2q$ (b) $m^2p = l^2p$

(c) $m^2p = q^2l$ (d) $m^2p^2 = l^2q$

151. If 1, ω, ω^2 are the cube roots of unity, then $(1 + \omega)(1 + \omega^2)(1 + \omega^3)(1 + \omega + \omega^2)$ is equal to *[2017-I]*

(a) –2 (b) –1

(c) 0 (d) 2

152. If the graph of a quadratic polynomial lies entirely above x-axis, then which one of the following is correct? *[2017-I]*

(a) Both the roots are real

(b) One root is real and the other is complex

(c) Both the roots are complex

(d) Cannot say

153. If cot α and cot β are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$, then the value of $\cot(\alpha + \beta)$ is *[2017-I]*

(a) $\frac{c-1}{b}$ (b) $\frac{1-c}{b}$

(c) $\frac{b}{c-1}$ (d) $\frac{b}{1-c}$

154. The roots of the equation *[2017-II]*

$(q - r)x^2 + (r - p)x + (p - q) = 0$ are

(a) $(r - p)/(q - r)$, 1/2 (b) $(p - q)/(q - r)$, 1

(c) $(q - r)/(p - q)$, 1 (d) $(r - p)/(p - q)$, 1/2

155. If α and β are the roots of the equation $1 + x + x^2 = 0$, then the matrix product $\begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix}$ is equal to *[2017-II]*

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$

156. If $|a|$ denotes the absolute value of an integer, then which of the following are correct? *[2017-II]*

1. $|ab| = |a||b|$ 2. $|a + b| \le |a| + |b|$
3. $|a - b| \ge ||a| - |b||$

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

157. The sum of all real roots of the equation $|x - 3|^2 + |x - 3| - 2 = 0$ is *[2017-II]*

(a) 2 (b) 3

(c) 4 (d) 6

158. It is given that the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real. For this, the minimum value of P is *[2017-II]*

(a) $\frac{1}{27}$ (b) $\frac{1}{64}$

(c) $\frac{1}{81}$ (d) 1

159. If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is *[2017-II]*

(a) $3x^2 + 8x + 16 = 0$ (b) $3x^2 - 8x - 16 = 0$

(c) $3x^2 + 8x - 16 = 0$ (d) $x^2 + 8x + 16 = 0$

160. In ΔPQR, $\angle R = \frac{\pi}{2}$. If than $\left(\frac{P}{2}\right)$ and tan $\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct? *[2017-II]*

(a) a = b + c (b) b = c + a

(c) c = a + b (d) b = c

161. The equation $|1 - x| + x^2 = 5$ has *[2018-I]*

(a) a rational root and an irrational root

(b) two rational roots

(c) two irrational roots

(d) no real roots

162. Let [x] denote the greatest integer function. What is the number of solutions of the equation $x^2 - 4x + [x] = 0$ in the interval [0, 2]? *[2018-I]*
(a) Zero (No solution) (b) One
(c) Two (d) Three

163. Consider the following expressions: *[2018-II]*
1. $x + x^2 - \frac{1}{x}$
2. $\sqrt{ax^2 + bx + x - c + \frac{d}{c} - \frac{e}{x^2}}$
3. $3x^2 - 5x + ab$
4. $\frac{2}{x^2 - ax + b^3}$
5. $\frac{1}{x} - \frac{2}{x+5}$

Which of the above are rational expressions?
(a) 1, 4 and 5 only (b) 1, 3, 4 and 5 only
(c) 2, 4 and 5 only (d) 1 and 2 only

164. If α and $\beta (\neq 0)$ are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$ where $x \in R$ has *[2018-II]*
(a) Least value $-\frac{1}{4}$
(b) Least value $-\frac{9}{4}$
(c) Greatest value $\frac{1}{4}$
(d) Greatest value $\frac{9}{4}$

165. Suppose f (x) is such a quadrant expression that it is positive for all real x.
If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x. Then for any real x. *[2018-II]*
(a) $g(x) < 0$ (b) $g(x) > 0$
(c) $g(x) = 0$ (d) $g(x) \geq 0$

166. The ration of roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are equal. If D_1 and D_2 are respective discriminates, then what is $\frac{D_1}{D_2}$ equal to? *[2018-II]*
(a) $\frac{a^2}{p^2}$ (b) $\frac{b^2}{q^2}$
(c) $\frac{c^2}{r^2}$ (d) None of these

167. What are the roots of the equation $|x^2 - x - 6| = x + 2$? *[2019-I]*
(a) –2, 1, 4 (b) 0, 2, 4
(c) 0, 1, 4 (d) –2, 2, 4

168. The equation $px^2 + qx + r = 0$ (where p, q, r, all are positive) has distinct real roots a and b.
Which one of the following is correct ? *[2019-I]*
(a) $a > 0, b > 0$ (b) $a < 0, b < 0$
(c) $a > 0, b < 0$ (d) $a < 0, b > 0$

169. If the roots of the equation $x^2 + px + q = 0$ are tan 19° and tan 26°, then which one of the following is correct ? *[2019-I]*
(a) $q - p = 1$ (b) $p - q = 1$
(c) $p + q = 2$ (d) $p + q = 3$

170. The number of real roots for the equation $x^2 + 9|x| + 20 = 0$ is *[2019-I]*
(a) Zero (b) One
(c) Two (d) Three

ANSWER KEY

1	(b)	18	(b)	35	(c)	52	(a)	69	(a)	86	(d)	103	(d)	120	(b)	137	(b)	154	(b)
2	(d)	19	(d)	36	(d)	53	(a)	70	(b)	87	(a)	104	(d)	121	(c)	138	(a)	155	(b)
3	(c)	20	(c)	37	(c)	54	(b)	71	(a)	88	(d)	105	(a)	122	(d)	139	(c)	156	(d)
4	(b)	21	(a)	38	(a)	55	(b)	72	(d)	89	(c)	106	(c)	123	(a)	140	(b)	157	(d)
5	(b)	22	(b)	39	(b)	56	(d)	73	(b)	90	(c)	107	(c)	124	(d)	141	(b)	158	(c)
6	(b)	23	(c)	40	(a)	57	(c)	74	(b)	91	(b)	108	(b)	125	(c)	142	(a)	159	(a)
7	(b)	24	(b)	41	(a)	58	(a)	75	(a)	92	(d)	109	(a)	126	(c)	143	(d)	160	(c)
8	(c)	25	(d)	42	(a)	59	(c)	76	(d)	93	(c)	110	(d)	127	(d)	144	(d)	161	(a)
9	(b)	26	(c)	43	(d)	60	(b)	77	(a)	94	(c)	111	(d)	128	(a)	145	(a)	162	(b)
10	(b)	27	(d)	44	(c)	61	(b)	78	(c)	95	(d)	112	(b)	129	(b)	146	(c)	163	(b)
11	(c)	28	(d)	45	(b)	62	(a)	79	(c)	96	(c)	113	(a)	130	(a)	147	(c)	164	(d)
12	(d)	29	(a)	46	(c)	63	(a)	80	(d)	97	(a)	114	(c)	131	(c)	148	(a)	165	(b)
13	(a)	30	(c)	47	(b)	64	(c)	81	(a)	98	(d)	115	(b)	132	(c)	149	(c)	166	(b)
14	(b)	31	(a)	48	(d)	65	(a)	82	(b)	99	(a)	116	(d)	133	(a)	150	(a)	167	(d)
15	(c)	32	(c)	49	(a)	66	(a)	83	(b)	100	(a)	117	(c)	134	(c)	151	(c)	168	(b)
16	(d)	33	(d)	50	(d)	67	(c)	84	(c)	101	(a)	118	(a)	135	(a)	152	(c)	169	(a)
17	(b)	34	(a)	51	(d)	68	(b)	85	(c)	102	(a)	119	(d)	136	(b)	153	(b)	170	(a)

HINTS & SOLUTIONS

1. (b) Let $\frac{k}{k+1}$ and $\frac{k+1}{k+2}$ are the roots of the equation $4\beta^2+\lambda\beta-2=0$, then

Sum of the roots $=\frac{k}{k+1}+\frac{k+1}{k+2}=-\frac{\lambda}{4}$ ...(i)

and product of the roots, $\frac{k}{k+1}\times\frac{k+1}{k+2}=-\frac{2}{4}$

$\Rightarrow \frac{k}{k+2}=-\frac{1}{2} \Rightarrow 2k=-k-2 \Rightarrow k=-\frac{2}{3}$

Putting the value of k in (i), we get

$$\frac{-\frac{2}{3}}{-\frac{2}{3}+1}+\frac{-\frac{2}{3}+1}{-\frac{2}{3}+2}=-\frac{\lambda}{4}$$

$$\Rightarrow \frac{-\frac{2}{3}}{\frac{1}{3}}+\frac{\frac{1}{3}}{\frac{4}{3}}=-\frac{\lambda}{4} \Rightarrow -2+\frac{1}{4}=-\frac{\lambda}{4}$$

$\Rightarrow \lambda=7$

2. (d) Given $4a-2b+c=0$

Substitute 2 in the equation, $ax^2+bx+c=0$

$\Rightarrow a(2)^2+b(2)+c=0$

$\Rightarrow 4a+2b+c=0$

So, $(x-2)$ is not the factor.

Substitute -2 in the equation, $ax^2+bx+c=0$

$\Rightarrow a(-2)^2+b(-2)+c=0$

$\Rightarrow 4a-2b+c=0$

So, $(x+2)$ is the factor.

$\therefore$ Only statement 1 is true.

3. (c) Let α and β be the roots of

$x^2-(p-2)x-(p+1)=0$,

Then $\alpha+\beta=(p-2)$ and $\alpha\beta=-(p+1)$

$\because \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow (\alpha+\beta)^2-2\alpha\beta=5$

$\Rightarrow (p-2)^2+2(p+1)=5$

$\Rightarrow p^2-4p+4+2p+2=5$

$\Rightarrow p^2-2p+1=0$

$\Rightarrow (p-1)^2=0$

$\Rightarrow p=1$

4. (b) The equation is $|x^2-x-6|=x+2$

for $x\geq 0$

$x^2-x-6=x+2$

$\Rightarrow x^2-2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$\Rightarrow x=4,-2$

and for $x<0$

$-x^2+x+6=x+2$

$\Rightarrow x^2=4 \Rightarrow x=\pm 2$

Thus, the number of real solutions are, -2, 2 and 4.

So, numbers of real solution of the equation $|x^2-x-6|=x+2$ are 3.

5. (b) Since, the roots of $x^2-2mx+m^2-1=0$ lie between -2 and 4 i.e., $b2-4ac\geq 0$ and $-2<\frac{-b}{2a}<4$

$\therefore (2m)^2-4(m^2-1)\geq 0$ ...(1)

and $-2<\frac{2m}{2}<4$

$\Rightarrow -2<m<4$

From (i)

$4m^2-4m^2+4\geq 0$

$\Rightarrow m\in R$

and $f(-2)>0$, also $f(4)>0$

$4+4m+m^2-1>0,\ 16-8m+m^2-1>0$

$\Rightarrow m^2+4m+3>0,\ m^2-8m+15>0$

$\Rightarrow (m+1)(m+3)>0,\ (m-3)(m-5)>0$

$\Rightarrow -3<m<-1$ and $3<m<5$.

Thus, the inteval in which it lies is $-1\leq m\leq 5$

6. (b) Given equation is $(\log_3 x)^2+\log_3 x<2$

$\Rightarrow (\log_3 x)^2+(\log_3 x)-2<0$

$\Rightarrow (\log_3 x+2)(\log_3 x-1)<0$

$\Rightarrow -2<\log_3 x<1$

$\Rightarrow \log_3 3^{-2}<\log_3 x<\log_3 3$

$\Rightarrow \frac{1}{9}<x<3$

7. (b) Given equation $\cos 2x+a\sin x=2a-7$ can be written as

$\cos^2 x-\sin^2 x+a\sin x=2a-7$

$\Rightarrow 1-\sin^2 x-\sin^2 x+a\sin x=2a-7$

$\Rightarrow 2\sin^2 x-a\sin x+(2a-7-1)=0$

$\Rightarrow 2\sin^2 x-a\sin x+2a-8=0$

This is a quadratic equation in $\sin x$ and its discriminant ≥ 0

Here, $a=2$, $b=-a$, $c=2a-8$

$\Rightarrow a^2-4.2.(2a-8)\geq 0$

$\Rightarrow a^2-16a+64\geq 0$

$\Rightarrow (a-8)^2\geq 0 \Rightarrow a\geq 8$

8. (c) As given $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2+bx+c=0$.

So, sum of roots $=\sin\theta+\cos\theta=-\frac{b}{a}$ ...(1)

and product of roots $=\sin\theta\cos\theta=\frac{c}{a}$ ...(2)

On squaring both sides in Eq. (1), we get

$$(\sin\theta+\cos\theta)^2=\frac{b^2}{a^2}$$

$\Rightarrow \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{b^2}{a^2}$

$\Rightarrow 1+\frac{2c}{a}=\frac{b^2}{a^2}$ using Eq. (2)

$\Rightarrow a^2+2ca=b^2$

$\Rightarrow a^2-b^2+2ac=0$

9. (b) We know that if $x + y + z = 0$
$x^3 + y^3 + z^3 = 3xyz$
Here both in nominator and denominator
$a^4 - b^4 + b^4 - c^4 + c^4 - a^4 = 0$
and $a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

Hence, $\dfrac{(a^4-b^4)^3+(b^4-c^4)^3+(c^4-a^4)^3}{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}$

$= \dfrac{3(a^4-b^4)(b^4-c^4)(c^4-a^4)}{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}$
$= (a^2 + b^2)(b^2 + c^2)(c^2 + a^2)$
$= (-c^2)(-a^2)(-b^2)$ $\quad (\because a^2 + b^2 + c^2 = 0)$
$= -a^2b^2c^2$

10. (b) Given that $0 < x < y < \pi$
We have $x < y \Rightarrow \cos y < \cos x$
So, $x + \cos y < y + \cos x$
$\Rightarrow x - \cos x < y - \cos y$

11. (c) The given expression $\left[(a^m)^{m-\frac{1}{m}}\right]^{\frac{1}{m+1}}$

$= \left[(a^m)^{\frac{m^2-1}{m}}\right]^{\frac{1}{m+1}} = \left[(a^m)^{\frac{(m-1)(m+1)}{m}}\right]^{\frac{1}{m+1}}$

$= [a^m]^{\frac{m-1}{m}} = a^{m-1}$
Its $(m - 1)^{th}$ root $= (a^{m-1})^{1/m-1} = a$.

Hence, $(m - 1)^{th}$ root of $\left[(a^m)^{m-\left(\frac{1}{m}\right)}\right]^{\frac{1}{(m+1)}}$

$= (m - 1)$th root of $a^{m-1} = a$

12. (d) The given equation is $ax^2 + bx + 1 = 0$
This equation has real roots.
When discriminant ≥ 0
$\therefore \ b^2 - 4a \geq 0$
$\Rightarrow b^2 \geq 4a$
a, b has to be selected from, three numbers, so total 3 selections are possible when (a, b) are (1,2), (1, 3) and (2, 3).
Thus, the number of equations of the form $ax^2 + bx + 1 = 0$ having real root is 3.

13. (a) Given equation is
$px^2 + qx + r = p(x - \alpha)(x - \beta)$
$\Rightarrow px^2 + qx + r = px^2 - p(\alpha + \beta)x + \alpha\beta p$
$\Rightarrow \alpha\beta p = r$ and $q = -(\alpha + \beta)p$ ...(1)
Also given that
$p^3 + pq + r = 0$
Putting value of q and r from (1)
$\Rightarrow p^3 - p^2(\alpha + \beta) + \alpha\beta p = 0$
$\Rightarrow p^2 - p(\alpha + \beta) + \alpha\beta = 0$
$\Rightarrow (p - \alpha)(p - \beta) = 0 \Rightarrow \alpha = \beta = p$

14. (b) $x^2 + k^2 = 2(k + 1)x$
$\Rightarrow x^2 - 2(k + 1)x + k^2 = 0$
For roots to be equal discriminant = 0
So, $\{-2(k + 1)\}^2 - 4k^2 = 0$
or, $4(k + 1)^2 - 4k^2 = 0$
or, $(k + 1)^2 - k^2 = 0$
or, $2k + 1 = 0$
$k = -\dfrac{1}{2}$

15. (c) In the given equation,
Given formula, $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$x = a^{1/3} - a^{-1/3}$
Raising both the sides to the power of cube
$x^3 = a - 3a^{2/3}.a^{-1/3} + 3a^{1/3} \times a^{-2/3} - a^{-1}$
$= a - 3a^{1/3} + 3a^{-1/3} - a^{-1} = a - 3(a^{1/3} - a^{-1/3}) - a^{-1}$
$\Rightarrow x^3 = a - 3x - a^{-1}$ or $x^3 + 3x = a - \dfrac{1}{a}$

16. (d) $x^{1/3} + y^{1/3} + z^{1/3} = 0$
Given formula, $(a + b)^3 = a^3 + b^3 + 3ab(A + b)$
so, $x^{1/3} = -(y^{1/3} + z^{1/3})$
Raising both the sides to the power of cube
$x = -\{y + z + 3y^{1/3}z^{1/3}(y^{1/3} + z^{1/3})\}$
$= -\{y + z + 3y^{1/3}z^{1/3}(-x^{1/3})\} = -\{y + z - 3x^{1/3}y^{1/3}z^{1/3}\}$
$= -y - z + 3x^{1/3}y^{1/3}z^{1/3} \Rightarrow x + y + z = 3x^{1/3}y^{1/3}z^{1/3}$
or $x + y + z = 3(xyz)^{1/3}$
$\Rightarrow (x + y + z)^3 = 27xyz$

17. (b) Since, α and β are the roots of $ax^2 + 2bx + c = 0$
$\therefore$ so, $\alpha + \beta = -\dfrac{2b}{a}$ and $\alpha\beta = \dfrac{c}{a}$
Also $\alpha + \delta$ and $\beta + \delta$ are the roots of $Ax^2 + 2Bx + C = 0$
so, sum of the roots $= \alpha + \beta + 2\delta = -\dfrac{2B}{A}$ and product of the roots $(\alpha + \delta)(\beta + \delta) = \dfrac{C}{A}$
$\Rightarrow -\dfrac{2b}{a} + 2\delta = -\dfrac{2B}{A}$
$\Rightarrow \delta = \dfrac{b}{a} - \dfrac{B}{A}$ ...(i)
and $(\alpha + \delta)(\beta + \delta) = \dfrac{C}{A}$
$\Rightarrow \alpha\beta + (\alpha + \beta)\delta + \delta^2 = \dfrac{C}{A}$ ...(ii)
Putting value of δ from equation (i) in equation (ii),
$\dfrac{c}{a} - \dfrac{2b}{a}\left(\dfrac{b}{a} - \dfrac{B}{A}\right) + \left(\dfrac{b}{a} - \dfrac{B}{A}\right)^2 = \dfrac{C}{A}$
$\Rightarrow \dfrac{c}{a} - \dfrac{2b^2}{a^2} + \dfrac{2bB}{aA} + \left(\dfrac{b}{a}\right)^2 + \left(\dfrac{B}{A}\right)^2 - \dfrac{2bB}{aA} = \dfrac{C}{A}$

$$\Rightarrow \frac{c}{a}-\left(\frac{b}{a}\right)^2+\left(\frac{B}{A}\right)^2=\frac{C}{A}$$

$$\Rightarrow \frac{B^2}{A^2}-\frac{C}{A}=\frac{b^2}{a^2}-\frac{c}{a} \Rightarrow \frac{B^2-AC}{A^2}=\frac{b^2-ac}{a^2}$$

$$\Rightarrow \frac{b^2-ac}{B^2-AC}=\left(\frac{a}{A}\right)^2$$

18. (b) Since, α and β are the roots of the equation $ax^2+bx+c=0$, then

Sum of the roots, $\alpha+\beta=-\frac{b}{a}$ and Product of the roots,

$\alpha\beta=\frac{c}{a}$

The expression, $(a\alpha+b)^{-1}+(a\beta+b)^{-1}$

$$=\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{\alpha\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$$

$$=\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$=\frac{a(-b/a)+2b}{a^2(c/a)+ab(-b/a)+b^2}=\frac{-b+2b}{ac-b^2+b^2}=\frac{b}{ac}$$

19. (d) Since, α and β are the roots of the equation $x^2-2x-1=0$, then
Sum of roots, $\alpha+\beta=2$ and
product of the roots $\alpha\beta=-1$
Since, $(\alpha+\beta)=\alpha^2+\beta^2+2\alpha\beta$
$\Rightarrow 4=\alpha^2+\beta^2-2$
$\Rightarrow \alpha^2+\beta^2=6$

Now, $\alpha^2\beta^{-2}+\alpha^{-2}\beta^2=\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\alpha^4+\beta^4}{(\alpha\beta)^2}$

$\Rightarrow (\alpha^2+\beta^2)^2=6^2$
$\Rightarrow \alpha^4+\beta^4+2\alpha^2\beta^2=36$
$\Rightarrow \alpha^4+\beta^4+2=36$ $(\because \alpha\beta=-1)$
$\Rightarrow \alpha^4+\beta^4=34$ ...(i)

$$\Rightarrow \frac{\alpha^4+\beta^4}{(\alpha\beta)^2}=\frac{34}{(-1)^2}=34$$

[Putting value of $\alpha^4+\beta^4=34$ from Equation (i)]

20. (c) There can be many values of x and y for this in equation. In the given options only x = 1, y = 1 satisfy the given equation.

21. (a) Given equations are

$y=\frac{8}{x^2+4}$ and $x+y=2$

Putting value of y from 1st equation into second equation.

$$x+\frac{8}{x^2+4}=2$$

$\Rightarrow x^3+4x+8=2x^2+8$
$\Rightarrow x^3-2x^2+4x=0$
$\Rightarrow x(x^2-2x+4)=0$
$\Rightarrow x=0$ [The other value of x is not real]

22. (b) Let α and β be the roots of both the equations
$x^2-(a-1)x+(a+b)=0$
$\Rightarrow \alpha+\beta=(a-1)$ and $\alpha\beta=(a+b)$
and $ax^2-2x+b=0$

$\Rightarrow \alpha+\beta=\frac{2}{a}$ and $\alpha\beta=\frac{b}{a}$

Equating the sums of roots

$\therefore a-1=\frac{2}{a}$

$\Rightarrow a^2-a-2=0 \Rightarrow a=-1, 2$

Equating the products of roots and $a+b=\frac{b}{a}$

If $a=-1$, $b=\frac{1}{2}$ and if $a=2$, $b=-4$

From the given option, a = 2, b = – 4 matches.

23. (c) The given equation is
$|x|+|x-1|=1$

We know that $|x|=\begin{cases}-x, \text{if } x<0\\ x, \text{if } x\ge 0\end{cases}$

$|x-1|=\begin{cases}-(x-1)=1-x, \text{if } x<1\\ x-1, \quad \text{if } x\ge 1\end{cases}$

Thus, three cases arise, case I; x < 0

Case I : if x < 0 then $|x|+|x-1|=1$ becomes $-x-(x-1)=1$ or $-x-x+1=1$

$\Rightarrow -2x=0$ or x = 0. So, equation is not valid for x < 0

Case II : $0\le x<1$.

For x = 0, equation is $0+|-1|=1$. and equation is satisfied.

For 0 < x < 1, equation is $x-(x-1)=1$.

Variable disappear, so, the equation is valid for all values of x in this integral.

Case III : $x\ge 1$

if x = 1 then equation becomes, | 1 | + | 0 | = 1 and equation is satisfied.

if x > 1 then $x+x-1=1 \Rightarrow 2x=2 \Rightarrow x=1$

So, equation is not valid for x > 1

So, this equation is defined for all values of x in the interval [0, 1]. So there are infinite number of real values of x.

24. (b) Let $x=8^{17}=(2^3)^{17}$
$\Rightarrow x=2^{51}$
Taking log on both sides of above equation, we get
$\log x=51\log 2$
$=51\times 0.3010=15.381$
$\therefore$ Number of digits in $8^{17}=15+1=16$

25. (d) If α and β are the roots of the equations
$x^2 + x + 1 = 0$
$\Rightarrow \alpha = \omega$ and $\beta = \omega^2$
or, $\alpha = \omega^2$ and $\beta = \omega$
$\therefore \alpha^{19} + \beta^7 = \omega^{19} + \omega^{14} = \omega + \omega^2 = -1$
or, $\alpha^{19} + \beta^7 = \omega^{38} + \omega^7 = \omega^2 + \omega = -1$
In either case $\alpha^{19} + \beta^7 = -1$
and $\alpha^{19} . \beta^7 = \omega^{19} . \omega^{14} = \omega^{33} = 1$
or $\omega^{38} . \omega^7 = \omega^{45} = 1$
$\therefore$ The required equation where roots are α^{19} and β^7 is
$x^2 - (\alpha^{19} + \beta^7)x + \alpha^{19}\beta^7 = 0 \Rightarrow x^2 + x + 1 = 0$

26. (c) α and β are the roots of the equation $x^2 + 6x + 1 = 0$
$\Rightarrow \alpha + \beta = -6$ and $\alpha\beta = 1$
Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= (-6)^2 - 4 = 36 - 4 = 32$
$\Rightarrow |\alpha - \beta| = \sqrt{32} = 4\sqrt{2}$

27. (d) Given equation is :
$$r^{1/3} + \frac{1}{r^{1/3}} = 3$$
Cubing both sides, we get
$$\left(r^{1/3} + \frac{1}{r^{1/3}}\right)^3 = 3^3 \quad [(a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$
$$\Rightarrow r + \frac{1}{r} + 3\left(r^{1/3} + \frac{1}{r^{1/3}}\right) = 27$$
$$\Rightarrow r + \frac{1}{r} + 3.3 = 27 \Rightarrow r + \frac{1}{r} + 27 - 9 = 18.$$

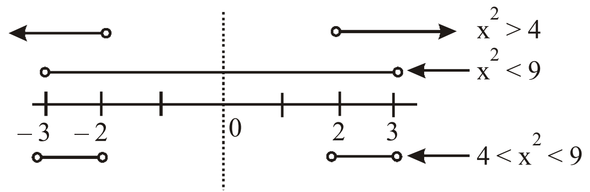
28. (d) As given
$\Rightarrow$ Number of rows = x
$\Rightarrow$ Number of seats in each row = x
Total number of seats in the hall = x^2
Revised number of rows = 2x
Revised number in each row = x – 10
Thus Revised number of seats = $2x(x - 10) = 2x^2 - 20x$
According to question,
$2x^2 - 20x = 300 + x^2$
$\Rightarrow x^2 - 20x - 300 = 0$
$\Rightarrow x^2 - 30x + 10x - 300 = 0$
$\Rightarrow (x - 30)(x + 10) = 0$
$\Rightarrow x = 30 \quad (\because x \neq -10)$

29. (a) As given : α and β are the roots of the quadratic equation
$\ell x^2 - mx + m = 0$
$\therefore$ So, sum of roots,
$\alpha + \beta = \frac{m}{\ell}$ and product of roots, $\alpha\beta = \frac{m}{\ell}$
$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{\alpha + \beta}{\sqrt{\alpha\beta}} = \frac{m/\ell}{\sqrt{m/\ell}}$$
$$\Rightarrow \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{m}{\ell}} = 0 \Rightarrow \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} - \sqrt{\frac{m}{\ell}} = 0$$

30. (c) As given :
roots of the quadratic equation
$(k + 1)x^2 - 2(k - 1)x + 1 = 0$ are real and equal,
Its discriminant
$\{-2(k - 1)\}^2 - 4(k + 1) = 0$
$\Rightarrow 4(k^2 - 2k + 1) - 4(k + 1) = 0$
$\Rightarrow k^2 - 2k + 1 - k - 1 = 0$
$\Rightarrow k^2 - 3k = 0 \quad \Rightarrow k = 0, k = 3$

31. (a) For roots of an equation $ax^2 + bx + c = 0$ to be positive sign of a and c should be like.

32. (c) In the given inequality
$\because 4 < x^2 < 9$
We consider
$x^2 > 4$ and $x^2 < 9$
We first consider ,
$x^2 > 4 \Rightarrow x^2 - 4 > 0$
$\Rightarrow x > -2$ and $x < 2$
Next; $x^2 < 9$
$\Rightarrow x^2 - 9 < 0 \quad \Rightarrow -3 < x < 3$
Combining both we get $-3 < x < -2, 2 < x < 3$
We represent this on number line



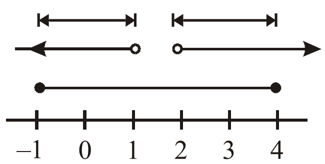
$\Rightarrow -3 < x < -2$ and $2 < x < 3$

33. (d) As given, α, and β are the roots of $ax^2 + bx + c = 0$, then the roots of $cx^2 + bx + a = 0$ will be reciprocal of α and β, i.e.,
$\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

34. (a) As given : x and y are real numbers such that $x > y$ and $|x| > |y|$, then x can not be negative or zero
$\Rightarrow x > 0.$

35. (c) The linear constraints for which the shaded region in the given figure is the solution set, are given by :
$x - y \le 1, x + 2y \le 8, x + y \ge 1, x, y \ge 0.$

36. (d) Consider first : $x^2 - 3x + 2 > 0$
$\Rightarrow (x - 1)(x - 2) > 0$
$\Rightarrow x < 1$ or $x > 2$ ...(1)
and $x^2 - 3x - 4 \le 0$
$\Rightarrow (x - 4)(x + 1) \le 0$
$\Rightarrow -1 \le x \le 4$ ...(2)
Combining (1) and (2)
$-1 \le x < 1$ or $2 < x \le 4$
Drawing on number line :



37. (c) Given equation is : $x = 2^{1/3} - 2^{-1/3}$
On cubing both sides, we get

$$x^3 = \left(2^{1/3} - 2^{-1/3}\right)^3$$

$$= 2 - 2^{-1} - 3.2^{1/3}.2^{-1/3}(2^{1/3} - 2^{-1/3})$$

$$\Rightarrow x^3 = -3x + 2 - \frac{1}{2} \Rightarrow x^3 = -3x + \frac{3}{2}$$

$$\Rightarrow 2x^3 + 6x = 3$$

38. (a) Let $x = \sqrt{5\sqrt{5\sqrt{5.......\infty}}}$
$\Rightarrow x = \sqrt{5x} \Rightarrow x^2 = 5x$
$\Rightarrow x^2 - 5x = 0 \Rightarrow x(x-5) = 0$
$\Rightarrow x = 0$ or 5
5 is given in the option.

39. (b) For the real number p, q, r, x and y
$p < x < q$ and $p < y < r$
$p < x < q < r$

40. (a) Let the roots of the equation $x^2 - px - q = 0$ be
α and $\frac{1}{\alpha}$

$\Rightarrow$ Product of roots $= \alpha . \frac{1}{\alpha} = -\frac{q}{1}$

$\Rightarrow 1 = -q \Rightarrow q = -1$

41. (a) As given : Roots of the equations $x^2 + kx + 1 = 0$ are α and β.
$\therefore \quad \alpha + \beta = -k$ and $\alpha\beta = 1$
Given expression

$$(\alpha + \beta)\left(\alpha^{-1} + \beta^{-1}\right) = (\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta)\left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(-k)^2}{1} = k^2$$

42. (a) Let α and $(\alpha + 1)$ be the roots of equation $x^2 - bx + c = 0$
$\therefore \alpha + (\alpha + 1) = b$
$\Rightarrow 2\alpha + 1 = b$
and $\alpha(\alpha + 1) = c$
Now, $b^2 - 4c = (2\alpha + 1)^2 - 4[\alpha(\alpha + 1)]$
$= 4\alpha^2 + 1 + 4\alpha - 4\alpha^2 - 4\alpha = 1$

43. (d) Since r and s are the roots of $x^2 + px + q = 0$.
Then, $r + s = -p$ and $rs = q$

$$\text{Now, } \frac{1}{r^2} + \frac{1}{s^2} = \frac{r^2 + s^2}{(rs)^2} = \frac{(r+s)^2 - 2rs}{(rs)^2}$$

$$= \frac{(-p)^2 - 2q}{q^2} = \frac{p^2 - 2q}{q^2}$$

44. (c) Given, $9 < 4x - 1 \le 19$
$\Rightarrow 9 < 4x - 1$ and $4x - 1 \le 19$
$\Rightarrow 9 + 1 < 4x$ and $4x \le 19 + 1$

$\Rightarrow x > \frac{5}{2}$ and $x \le 5$

$\therefore x \in \{3, 4, 5\}$

45. (b) $a = x + \sqrt{x^2 + 1} \Rightarrow a - x = \sqrt{x^2 + 1}$
$\Rightarrow x^2 + 1 = (a - x)^2 \Rightarrow x^2 + 1 = a^2 + x^2 - 2ax$

$\Rightarrow 2ax = a^2 - 1 \Rightarrow 2x = a - \frac{1}{a}$

$\Rightarrow x = \frac{1}{2}(a - a^{-1})$

46. (c) We know that, if a quadratic polynomial with two distinct root has one real root, then
$b^2 - 4ac > 0$
Then other root will always real.

47. (b) $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $px^2 + qx + r = 0$

$\therefore \quad \sin\alpha + \cos\alpha = \frac{-q}{p}$ ...(i)

and $\sin\alpha\cos\alpha = \frac{r}{p}$ ...(ii)

From equation (i)

$$(\sin\alpha + \cos\alpha)^2 = \frac{q^2}{p^2}$$

$$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{q^2}{p^2}$$

$$\Rightarrow 1 + 2.\frac{r}{p} = \frac{q^2}{p^2}$$

$\Rightarrow p^2 + 2pr = q^2$
$\Rightarrow p^2 - q^2 + 2pr = 0$

48. (d) α and β are the roots of $x^2 + 4x + 6 = 0$
$\therefore \alpha + \beta = -4$ and $\alpha\beta = 6$
Now, $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
$= (-4)^3 - 3 \times 6(-4) = -64 + 72 = 8$

49. (a) Let α and β are the roots of the given equation.

$\therefore \quad \alpha + \beta = \frac{-(3p+1)}{3}$

and $\quad \alpha\beta = \frac{-(p+5)}{3}$

Now, $\frac{-(3p+1)}{3} = \frac{-(p+5)}{3}$
$\Rightarrow 3p + 1 = p + 5$
$\therefore \quad 2p = 4$
$\therefore \quad p = 2$

50. (d) Number of diagonals $= \frac{n(n-3)}{2}$
where n is the number of sides of polygon
$\therefore \quad 20 = \frac{n(n-3)}{2}$
$\Rightarrow \quad 40 = n^2 - 3n$
$\Rightarrow \quad n^2 - 3n - 40 = 0$
$\Rightarrow \quad n^2 - 8n + 5n - 40 = 0$
$\Rightarrow \quad (n-8)(n+5) = 0$
$\Rightarrow \quad n - 8 = 0, n + 5 = 0$
$\therefore$ since, the number of diagonals, n cannot be negative.
$\Rightarrow \quad n = 8$

51. (d) Sum and product of roots of $Ax^2 - 4x + 1 = 0$ will be

$\alpha + \gamma = \frac{4}{A}$ and $\alpha\gamma = \frac{1}{A}$ respectively

Sum and product of roots of $Bx^2 - 6x + 1 = 0$ will be

$\beta + \delta = \frac{6}{B}$ and $\beta\delta = \frac{1}{B}$ respectively

Since, α, β, γ and δ are in HP.

Then, $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ and $\frac{1}{\delta}$ will be in AP.

$\Rightarrow \frac{1}{\beta} - \frac{1}{\alpha} = \frac{1}{\delta} - \frac{1}{\gamma} \Rightarrow \frac{1}{\beta} - \frac{1}{\delta} = \frac{1}{\alpha} - \frac{1}{\delta}$

$\Rightarrow \frac{\delta - \beta}{\beta\delta} = \frac{\gamma - \alpha}{\alpha\gamma}$

$\Rightarrow \frac{\sqrt{(\delta + \beta)^2 - 4\beta\delta}}{\beta\delta} = \frac{\sqrt{(\gamma + \alpha)^2 - 4\alpha\gamma}}{\alpha\gamma}$

$\Rightarrow 36 - 4B = 16 - 4A$

$\Rightarrow 4A + 4B = 20$

$\Rightarrow A + B = 5$

It is possible only, if

$A = -3$ and $B = 8$

52. (a) Given, $2^x + 3^y = 17$

and $2^{x+2} - 3^{y+1} = 5$

$\Rightarrow 4.2^x - 3.3^y = 5$

From equation (i) and (ii), we get

$2^x = 8$ and $3^y = 9$

$\Rightarrow x = 3$ and $y = 2$

53. (a) Given $(x + a)$ is a factor of quadratic polynomials

$x^2 + px + q$ and $x^2 + lx + m$

then $a^2 - ap + q = 0$

and $a^2 - la + m = 0$

(i) – (ii) $\Rightarrow -ap + q + la - m = 0$

$\Rightarrow (l - p)a = m - q$

$\Rightarrow a = \frac{m - q}{l - p}\ (l \neq p)$

54. (b) Given eqn is $(b - c)x^2 + (c - a)x + (a - b) = 0$

$\Rightarrow (b - c)x^2 - (b - c - b + a)x + (a - b) = 0$

$\Rightarrow (b - c)x(x - 1) - (a - b)(x - 1) = 0$

$\Rightarrow \{(b - c)x - (a - b)\}\{x - 1\} = 0$

$\Rightarrow x = \frac{a - b}{b - c}$ and $x = 1$

55. (b) Consider $16\left(\frac{a - x}{a + x}\right)^3 = \frac{a + x}{a - x}$

$\Rightarrow \left(\frac{a - x}{a + x}\right)^3 \times \left(\frac{a - x}{a + x}\right) = \frac{1}{16}$

$\Rightarrow \left(\frac{a - x}{a + x}\right)^4 = \left(\frac{1}{2}\right)^4$

$\Rightarrow \frac{a - x}{a + x} = \frac{1}{2}$

$\Rightarrow 2a - 2x = a + x$

$\Rightarrow a = 3x$

$\Rightarrow x = \frac{a}{3}$

56. (d) Since, α and β be the roots of

$2x^2 - 2(1 + n^2)x + (1 + n^2 + n^4) = 0$

$\therefore \alpha + \beta = -\left[\frac{-2(1 + n^2)}{2}\right] = (n^2 + 1)$

and $\alpha\beta = \frac{1 + n^2 + n^4}{2}$

Now, Consider $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (n^2 + 1)^2 - (1 + n^2 + n^4)$

$= n^4 + 1 + 2n^2 - 1 - n^2 - n^4 = n^2$

57. (c) Since, r and s are the roots of $Ax^2 + Bx + C = 0$, then

$r + s = -\frac{B}{A}$ and $rs = \frac{C}{A}$

Now, Given roots of $x^2 + px + q = 0$ be r^2 and s^2

$\therefore r^2 + s^2 = -p$ and $r^2s^2 = q$

$\Rightarrow (r + s)^2 - 2rs = -p$

$\Rightarrow \frac{B^2}{A^2} - \frac{2C}{A} = -p$

$\Rightarrow \frac{B^2 - 2AC}{A^2} = -p$

$\Rightarrow p = \frac{2AC - B^2}{A^2}$

58. (a) Let α and β are the roots of $ax^2 + bx + b = 0$

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{b}{a}$

Consider, $\frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \frac{\sqrt{b}}{\sqrt{a}} = \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \frac{\sqrt{b}}{\sqrt{a}}$

$= \frac{-b/a}{\sqrt{b/a}} + \sqrt{\frac{b}{a}} = -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}}$ (by rationalizing)

$= 0$

59. (c) Let $\sin\alpha$ and $\cos\alpha$ be the roots of $ax^2 + bx + c = 0$

Now, $\sin\alpha + \cos\alpha = \frac{-b}{a}$ and $\sin\alpha\cos\alpha = \frac{c}{a}$

Consider $\sin\alpha + \cos\alpha = \frac{-b}{a}$

Squaring both side, $(\sin\alpha + \cos\alpha)^2 = \frac{b^2}{a^2}$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$

$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow \frac{a + 2c}{a} = \frac{b^2}{a^2} \Rightarrow a + 2c = \frac{b^2}{a}$

$\Rightarrow a^2 + 2ac = b^2 \Rightarrow b^2 - a^2 = 2ac$

60. (b) Let $x = 2+2^{1/3}+2^{2/3}$

$= 2+2^{1/3}+(2^{1/3})^2 = 2+2^{1/3}(1+2^{1/3})$

$\Rightarrow x-2 = 2^{1/3}(1+2^{1/3})$

On cubing both sides, we get

$x^3 - 8 - 6x^2 + 12x = 2(1+2+3.2^{1/3}+3.2^{2/3})$

$\Rightarrow x^3 - 6x^2 + 6x = 14 + 6.2^{1/3} + 6.2^{2/3} - 6x$

$= 14 + 6.2^{1/3} + 6.2^{2/3} - 6(2+2^{1/3}+2^{2/3}) = 2$

61. (b) Given, equation is $(x-p)(x-q) = r^2$

$\Rightarrow x^2 - (p+q)x + pq - r^2 = 0$

Now, $D = \sqrt{(p+q)^2 - 4(pq - r^2)}$

$= \sqrt{(p-q)^2 + 4r^2} \geq 0$

Hence, roots are always real.

62. (a) Given, $x - 2(x-1)^{-1} = 1 - 2(x-1)^{-1}$

$$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$$

$$\Rightarrow \frac{x(x-1)-2}{x-1} = \frac{x-1-2}{x-1}$$

$\Rightarrow x^2 - x - 2 = x - 1 - 2$

$\Rightarrow x^2 - x - 2 - x + 3 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$

But $x = 1$ is not satisfied in the given equation.

Hence, no roots exist.

63. (a) Given equation is $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a) = 0$

$\Rightarrow 3x^2 - 2(b+a+c)x + ab + bc + ca = 0$

Now, here $A = 3$, $B = -2(a+b+c)$

$C = ab + bc + ca$

$\therefore D = \sqrt{B^2 - 4AC}$

$= \sqrt{(-2(a+b+c))^2 - 4(3)(ab+bc+ca)}$

$= \sqrt{4(a+b+c)^2 - 12(ab+bc+ca)}$

$= 2\sqrt{a^2+b^2+c^2-ab-bc-ca}$

$= 2\sqrt{\frac{1}{2}\{(a-b)^2-(b-c)^2+(c-a)^2\}}$

≥ 0

64. (c) Let the two equations be $x^2 + mx + 1 = 0$ and $x^2 + x + m = 0$.

Let given equations have common root α.

Therefore 'α' satisfies the both equations

Then, $\alpha^2 + m\alpha + 1 = 0$ and $\alpha^2 + \alpha + m = 0$

$$\Rightarrow \frac{\alpha^2}{m^2-1} = \frac{\alpha}{1-m} = \frac{1}{1-m}$$

By equating 2nd and 3rd,

$$\frac{\alpha}{1-m} = \frac{1}{1-m} \Rightarrow \alpha = 1$$

Also, by equating 1st and 3rd

$$\frac{\alpha^2}{m^2-1} = \frac{1}{1-m}$$

$\Rightarrow 1 - m = m^2 - 1$ ($\because \alpha = 1$)

$\Rightarrow m^2 + m - 2 = 0 \Rightarrow (m+2)(m-1) = 0$

$\Rightarrow m = 1$ and -2

65. (a) Given equation is $(x-p)(x-6)+1 = 0$

$\Rightarrow x^2 - 6x - px + 6p + 1 = 0$

$\Rightarrow x^2 - (p+6)x + (6p+1) = 0$

Now, $b^2 - 4ac = 0$

$a = 1$, $b = -(p+6)$, $c = 6p+1$

$\Rightarrow (p+6)^2 - 4(6p+1) = 0$

$\Rightarrow p^2 + 36 + 12p - 24p - 4 = 0$

$\Rightarrow p^2 - 12p + 32 = 0$

$\Rightarrow (p-4)(p-8) = 0$

$\Rightarrow p = 4, 8$

Hence, p can have 4 or 8.

66. (a) Given quadratic equation is $ax^2 + bx + c = 0$ whose one root is $\frac{1}{2-\sqrt{-2}}$

Consider $\frac{1}{2-\sqrt{-2}} = \frac{1}{2-\sqrt{2}i} \times \frac{2+\sqrt{2}i}{2+\sqrt{2}i}$

$= \frac{2+\sqrt{2}i}{4+2} = \frac{2+\sqrt{2}i}{6}$

$\therefore$ Another root will be $\frac{2-\sqrt{2}i}{6}$

($\because$ complex roots always occurs in pairs)

Thus, sum of roots $= \frac{2+\sqrt{2}i}{6} + \frac{2-2\sqrt{2}i}{6} = \frac{4}{6}$

and product of roots $= \left(\frac{2+\sqrt{2}i}{6}\right)\left(\frac{2-\sqrt{2}i}{6}\right)$

$= \frac{4+2}{36} = \frac{1}{6}$

$\therefore$ Required equation is

x^2 – (sum of roots) x + (product of roots) = 0

$$x^2 - \frac{4}{6}x + \frac{1}{6} = 0$$

$\Rightarrow 6x^2 - 4x + 1 = 0$

Thus, the values of a, b, c are 6, – 4, 1 respectively

67. (c) Let α and β be the roots of the equation $x^2 - x + 1 = 0$.

$\therefore \quad \alpha + \beta = -(-1) = 1$ ...(i)

$\alpha\beta = 1$

Now, $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{(1)^2 - 4(1)} = \sqrt{-3} = \sqrt{3}i$ ...(ii)

On solving (i), (ii) we get

$\alpha = \dfrac{1+i\sqrt{3}}{2}$ and $\beta = \dfrac{1-i\sqrt{3}}{2}$

$\Rightarrow \quad \alpha = \dfrac{1}{2} + i\cdot\dfrac{\sqrt{3}}{2}$ and $\beta = \dfrac{1}{2} - i\cdot\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \alpha = \cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}$ and $\beta = \cos\dfrac{\pi}{3} - i\sin\dfrac{\pi}{3}$

(a) $\alpha^4 - \beta^4 = \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3} - \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}$

$= 2i\sin\dfrac{4\pi}{3}$ (By Demoiver's thm)

$\Rightarrow \quad \alpha^4 - \beta^4$ is not real.

(b) $2\left(\alpha^5 + \beta^5\right)$

$= 2\left(\cos\dfrac{5\pi}{3} + i\sin\dfrac{5\pi}{3} + \cos\dfrac{5\pi}{3} - i\sin\dfrac{5\pi}{3}\right)$

$= 2.2\cos\dfrac{5\pi}{3} = 4.\dfrac{1}{2} = 2$

Now, $(\alpha\,\beta)^5 = 1$

$\Rightarrow \quad 2\left(\alpha^5 + \beta^5\right) \neq (\alpha\,\beta)^5$

(c) $\alpha^6 - \beta^6 = \cos\dfrac{6\pi}{3} + i\sin\dfrac{6\pi}{3} - \cos\dfrac{6\pi}{3} + i\sin\dfrac{6\pi}{3}$

$= 2i\sin 2\pi = 0$

Hence, option (c) is correct.

68. (b) If any equation has $p - \sqrt{q}$ as a root, then another root will be $p + \sqrt{q}$.

So, sum of roots $= p - \sqrt{q} + p + \sqrt{q} = 2p$

and product of roots $= \left(p - \sqrt{q}\right)\left(p + \sqrt{q}\right) = p^2 - q$

Now, required equation is

x^2 – (sum of roots) x + (product of roots) = 0

$\Rightarrow \quad x^2 - 2px + \left(p^2 - q\right) = 0$

69. (a) The given quadratic equation is

$x^2 - 5x + k = 15 \Rightarrow x^2 - 5x + k - 15 = 0$

Let α and β be the roots of the equation

$x^2 - 5x + k - 15 = 0$

Now, product of roots $= \alpha\beta = k - 15$

But $\alpha\beta = -3 \Rightarrow \quad -3 = k - 15$

$\Rightarrow \quad k = 15 - 3 = 12$

70. (b) Given quadratic equation is $x^2 - bx + 1 = 0$

It has no real roots. It means, equation has imaginary roots.

Which is possible when $B^2 - 4AC < 0$

Here, $B = -b, A = 1, C = 1$

$\Rightarrow \quad b^2 - 4 < 0 \;\Rightarrow\; b^2 < 4 \;\Rightarrow\; -2 < b < 2$

71. (a) Given quadratic equation is $x^2 - px + q = 0$

$\because$ p and q are the roots of $x^2 - px + q = 0$

$\therefore$ Sum of roots $= p + q = p \Rightarrow q = p - p = 0$

and product of roots $= pq = q \Rightarrow p = q/q = 1$

$\Rightarrow \quad q(p-1) = 0 \quad \Rightarrow \quad p = 1, q = 0$

72. (d) Given equations are $x^2 + kx + 64 = 0$ ...(i)

and $x^2 - 8x + k = 0$ ...(ii)

Since both the eqns have real roots, discriminant ≥ 0

$\Rightarrow \quad b^2 \geq 4ac$

from eq^n (i), we have

$k^2 \geq 4(64) \Rightarrow k^2 \geq 256$

$\Rightarrow \quad k \geq 16$ ...(A)

and from eq^n (ii), we have

$\Rightarrow \quad 64 \geq 4k \Rightarrow 4k \leq 64$

$\Rightarrow \quad k \leq 16$ ...(B)

Hence, from eq^n (A) and (B), we have

$k = 16$

73. (b) Since, the roots of the equation

$(a^2 + b^2)\,x^2 - 2b\,(a + c)\,x + (b^2 + c^2) = 0$ are equal.

therefore, discriminant = 0

$\Rightarrow \quad [2b\,(a + c)]^2 - 4\,(a^2 + b^2)\,(b^2 + c^2) = 0$

$\therefore \quad 4b^2\,(a + c)^2 = 4\,(a^2 + b^2)\,(b^2 + c^2)$

$\Rightarrow \quad b^2\,(a^2 + c^2 + 2ac) = a^2b^2 + b^2c^2 + a^2c^2 + b^4$

$\Rightarrow \quad a^2b^2 + b^2c^2 + 2acb^2 = a^2b^2 + b^2c^2 + a^2c^2 + b^4$

$\Rightarrow \quad b^4 + a^2c^2 - 2ac\,b^2 = 0$

$\Rightarrow \quad (b^2 - ac)^2 = 0 \Rightarrow b^2 = ac$

74. (b) Let α and β are the roots of $x^2 - 2x + 4 = 0$

$\therefore$ sum of roots $= \alpha + \beta = 2$, product $= \alpha\,\beta = 4$

Now, $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\,\beta\,(\alpha + \beta) = 2^3 - 3 \times 4 \times 2$

$= 8 - 24 = -16$

75. (a) Given eq^n is $(x^2 + 2)^2 + 8x^2 = 6x\,(x^2 + 2)$

$\Rightarrow \quad x^4 + 4x^2 + 4 + 8x^2 = 6x^3 + 12x$

$\Rightarrow \quad x^4 - 6x^3 + 12x^2 - 12\,x + 4 = 0$

$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

$\Rightarrow \quad x^4 + 4x^2 + 4 + 8x^2 = 6x^3 + 12x$

$\Rightarrow \quad x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

This can be factorised into $(x^2 - 4x + 2)(x^2 - 2x + 2) = 0$

Consider, $x^2 - 2x + 2 = 0$

Roots are $\dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(1)}}{2(1)} = \dfrac{2 \pm \sqrt{4-8}}{2}$

$= \dfrac{2 \pm \sqrt{-4}}{2} = \dfrac{2 \pm 2i}{2} = 1 \pm i$

Also, this equation is satisfied by $1 \pm i$. Hence, required roots are $1 \pm i$.

76. (d) Let α and β be the roots of the equation $x^2 + x + 1 = 0$

$\therefore \quad \alpha + \beta = -1$ and $\alpha\beta = 1$

$\Rightarrow \quad \alpha = \omega$ and $\beta = \omega^2$

$(\because \quad \omega^3 = 1, 1 + \omega + \omega^2 = 0)$

option a, b, c does not satisfies the eqn $x^2 - x + 1 = 0$

Hence, option (d) is correct.

77. (a) Consider, $x^4 - 26x^2 + 25 = 0$

$\Rightarrow \quad x^4 - 25x^2 - x^2 + 25 = 0$

$= x^2(x^2 - 25) - (x^2 - 25) = 0$

$\Rightarrow \quad (x^2 - 25)(x^2 - 1) = 0$

$\Rightarrow \quad (x - 5)(x + 5)(x - 1)(x + 1) = 0$

$\Rightarrow \quad x = 5, -5, 1, -1$

$\therefore$ Solution set for given equation is $\{5, -5, -1, 1\}$

78. (c) Let α and β be the roots of the equation.

$4x^2 + 3x + 7 = 0$

$\therefore \quad \text{Sum} = \alpha + \beta = -\frac{3}{4}$ and $\text{Product} = \alpha\beta = \frac{7}{4}$

Consider, $\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\frac{9}{16} - \frac{7}{2}}{\frac{49}{16}}$$

$$= \frac{\frac{9 - 56}{16}}{\frac{49}{16}} = \frac{-47}{16} \times \frac{16}{49} = -\frac{47}{49}$$

79. (c) The given equations are

$x^2 + y^2 = 4$ ... (i)

and $x + y = 2$ ... (ii)

$x^2 + (2 - x)^2 = 4$

$\Rightarrow \quad x^2 + 4 + x^2 - 4x = 4$

$\Rightarrow \quad 2x^2 = 4x$

$\Rightarrow \quad x = 2$

When $x = 2$ then $y = 2 - 2 = 0$

Similarly,

$y^2 + (2 - y)^2 = 4$

$\Rightarrow \quad y^2 + 4 + y^2 - 4y = 4$

$\Rightarrow \quad y = 2$ and $x = 0$

These equations are satisfied by only (2, 0) and (0, 2).

Hence, required set is $\{(0, 2), (2, 0)\}$

80. (d) The given equation is $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$

Now, discriminant

$= (-2p)^2 - 4(1)(p^2 - q^2 + 2qr - r^2)$

$= 4p^2 - 4p^2 + 4q^2 - 8qr + 4r^2$

$= (2q - 2r)^2 = 4(q - r)^2$

which is always greater than zero.

Therefore, the roots of given equation are rational.

81. (a) The given equation is

$(2 - \sqrt{3})x^2 - (7 - 4\sqrt{3})x + (2 + \sqrt{3}) = 0$

$\therefore \quad \text{Sum of roots} = \frac{(7 - 4\sqrt{3})}{2 - \sqrt{3}}$

$$= \frac{(7 - 4\sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{14 + 7\sqrt{3} - 8\sqrt{3} - 12}{4 - 3}$$

$= 2 - \sqrt{3}$

82. (b) Since one root of $ax^2 + bx + c = 0, a \neq 0$ is positive and another root is negative which is possible only if $a > 0$, $b < 0$, $c > 0$.

83. (b) Let the roots of the equation $ax^2 + bx + c = 0$ be α and 2α.

$\therefore \quad \text{Sum} = \alpha + 2\alpha = \frac{-b}{a}$ and $\text{product} = \alpha \cdot 2\alpha = \frac{c}{a}$

$\Rightarrow \alpha = \frac{-b}{3a}$ and $\alpha^2 = \frac{c}{2a}$

$\Rightarrow \left(\frac{-b}{3a}\right)^2 = \frac{c}{2a} \Rightarrow \frac{b^2}{9a^2} = \frac{c}{2a}$

$\Rightarrow 2b^2 = 9ac$

84. (c) Since x and y are non-zero positive integers therefore $x = 1, 2, 3, \ldots\ldots\ldots\ldots$ and $y = 1, 2, 3, \ldots\ldots\ldots\ldots$

Now, given $x + y \leq 4$

$\Rightarrow \quad (x, y)$ can be

(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)

Required number of ordered pairs = 6

85. (c) Since, 3 is the root of the equation $x^2 - 8x + k = 0$

$\therefore \quad 3$ satisfies the equation $x^2 - 8x + k = 0$

$\therefore \quad 9 - 24 + k = 0 \Rightarrow k = 15$

86. (d) Let the roots of the equation $x^2 + kx - b = 0$ be α and β.

$\therefore \quad \text{Sum} : \alpha + \beta = -k$ and $\text{Product} : \alpha\beta = -b$

According to the question, we have

$\alpha^2 + \beta^2 = 2b$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 2b$

$\Rightarrow k^2 + 2b = 2b \Rightarrow k = 0$

87. (a) Let the roots of $ax^2 + bx + c = 0, a \neq 0$ be α and $\frac{1}{\alpha}$.

$\therefore$ Product of roots $= \alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$\Rightarrow c = a$

88. (d) Since, complex roots occur in pairs therefore other root is $2 - 5i$.

89. (c) Given equation is

$(x - a)(x - b) = c, c \neq 0$

$\Rightarrow x^2 - (a + b)x + ab - c = 0$

Let α, β be the roots of this equation.

$\therefore \alpha + \beta = a + b, \alpha\beta = ab - c$

Consider $(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

Roots of this equation is

$$x = \frac{(\alpha + \beta) \pm \sqrt{(\alpha + \beta)^2 - 4(\alpha\beta + c)}}{2}$$

$$= \frac{(a+b) \pm \sqrt{(a+b)^2 - 4(ab - c + c)}}{2}$$

$$= \frac{(a+b) \pm \sqrt{a^2 + b^2 - 2ab}}{2}$$

$$= \frac{(a+b) \pm \sqrt{(a-b)^2}}{2} = \frac{(a+b) \pm (a-b)}{2}$$

$$= \frac{a+b+a-b}{2}, \frac{a+b-a+b}{2} = a, b.$$

Hence, roots of the equation
$(x-\alpha)(x-\beta) + c = 0$ are a and b.

90. (c) Given equations are $x^2 - px + q = 0$ ...(1)
and $x^2 - ax + b = 0$ ...(2)
Root of second equation is

$$x = \frac{a \pm \sqrt{a^2 - 4b}}{2}$$

Since, Roots of second equation are equal discriminant $= 0$

$\therefore \sqrt{a^2 - 4b} = 0 \Rightarrow a^2 = 4b$ ...(3)

$\therefore x = \frac{a}{2}$

Since, Equations (1) and (2) have common roots.

$\therefore x = \frac{a}{2}$ is the root of equation (1) also.

Thus, $x = \frac{a}{2}$ satisfies (1)

$$\Rightarrow \left(\frac{a}{2}\right)^2 - p\left(\frac{a}{2}\right) + q = 0$$

$\Rightarrow \frac{a^2}{4} = \frac{pa}{2} - q \Rightarrow 2b = pa - 2q$ (from (3)

$\Rightarrow ap = 2(b+q)$

91. (b) Given Equation is $x^2 + x + 1 = 0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{3}i}{2}$$

Thus, roots are ω and ω^2
$\therefore \alpha = \omega$ and $\beta = \omega^2$
So, $\alpha^{19} = (\omega)^{19} = (\omega^3)^6 . \omega = \omega$ $(\because \omega^3 = 1)$
and $\beta^7 = (\omega^2)^7 = \omega^{14} = (\omega^3)^4 . \omega^2 = \omega^2$
Now, $\alpha^{19} + \beta^7 = \omega + \omega^2 = -1$ $(\because 1 + \omega + \omega^2 = 0)$
and $(\alpha^{19})(\beta^7) = (\omega)(\omega^2) = 1$
$\therefore$ Required Quadratic equation whose roots are α^{19} and β^7 is
$x^2 + (\alpha^{19} + \beta^7)x + (\alpha^{19})(\beta^7) = 0$
$\Rightarrow x^2 + (-1)x + 1 = 0$
$\Rightarrow x^2 - x + 1 = 0$

92. (d) Let $y = \sqrt{8 + 2y}$

$\Rightarrow y^2 = 8 + 2y \Rightarrow y^2 - 2y - 8 = 0$
$\Rightarrow (y+2)(y-4) = 0$
$\Rightarrow y = 4, -2$
Hence, required value of given expression is 4.

93. (c) Given equation is

$\sin\theta = x + \frac{a}{x}, \; x \in R - \{0\}$

$\Rightarrow x^2 + a = x \sin\theta$
$\Rightarrow x^2 - x\sin\theta + a = 0$

Now, discriminant $= \sqrt{\sin^2\theta - 4a}$

For x to be real root,
discriminant ≥ 0

$\Rightarrow \sqrt{\sin^2\theta - 4a} \geq 0$
$\Rightarrow \sin^2\theta - 4a \geq 0 \Rightarrow \sin^2\theta \geq 4a$

$$\Rightarrow \frac{1}{\sin^2\theta} \leq \frac{1}{4a} \Rightarrow a \leq \frac{\sin^2\theta}{4}$$

$\Rightarrow a \leq \frac{1}{4}$ ($\because \sin^2\theta$ lies between 0 and 1)

94. (c) Given equation is
$\tan^4 x - 2\sec^2 x + a^2 = 0$
$\Rightarrow (\tan^2 x)^2 - 2[1 + \tan^2 x] + a^2 = 0$
$\Rightarrow (\tan^2 x)^2 - 2\tan^2 x + a^2 - 2 = 0$
This is the quadratic equation in $\tan x$.
So, roots of this will be real when discriminant ≥ 0
i.e., $b^2 - 4ac \geq 0$
$\Rightarrow (-2)^2 - 4(1)(a^2 - 2) \geq 0 \Rightarrow 4 - 4a^2 + 8 \geq 0$
$\Rightarrow 12 - 4a^2 \geq 0 \Rightarrow 4(3 - a^2) \geq 0$
$\Rightarrow 3 - a^2 \geq 0$ $(\because 4 \neq 0)$
$\Rightarrow a^2 \leq 3$
$\Rightarrow a \leq \pm\sqrt{3} \Rightarrow |a| \leq \sqrt{3}$

95. (d) Given equation is
$x^2 - 4x - \log_3 N = 0$
Since, roots are real
$\therefore b^2 - 4ac = 0 \Rightarrow (4)^2 - 4(-\log_3 N) \geq 0$
$\Rightarrow 16 \geq -4\log_3 N$
$\Rightarrow 4 \geq -\log_3 N$
$\Rightarrow 4 \geq \log_3 N^{-1}$
$\Rightarrow N^{-1} \geq 3^4 \geq 81$

$$\Rightarrow N \geq \frac{1}{81}$$

Hence, minimum value of N is $\frac{1}{81}$.

96. (c) Given equation is
$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$
Let α be the second root.
So, $(\alpha)(1) = \dfrac{c(a-b)}{a(b-c)}$
Hence, α = second root = $\dfrac{c(a-b)}{a(b-c)}$

97. (a) Given equation is $2(y+2)^2 - 5(y+2) = 12$
Let $y + 2 = a$
So, quadratic equation can be rewritten as
$2a^2 - 5a - 12 = 0$
$\Rightarrow 2a^2 - 8a + 3a - 12 = 0$
$\Rightarrow 2a(a-4) + 3(a-4) = 0$
$\Rightarrow (2a+3)(a-4) = 0$
$\Rightarrow 2a + 3 = 0$ or $a - 4 = 0$
$\Rightarrow a = \dfrac{-3}{2}$ or $a = 4$
$\Rightarrow y + 2 = \dfrac{-3}{2}$ or $y + 2 = 4$
$\Rightarrow y = \dfrac{-3}{2} - 2$ or $y = 2$
$\Rightarrow y = \dfrac{-7}{2}$ or 2 (Required roots)

98. (d) Given quadratic equation is
$3x^2 - 5x + q = 0$
Since, roots of this equation are equal therefore
$b^2 - 4ac = 0$
$\Rightarrow (-5)^2 - 4(3)(q) = 0$
$\Rightarrow 25 - 12q = 0$
$\Rightarrow q = \dfrac{25}{12}$

99. (a) Let α and β be the roots of $ax^2 + bx + c = 0$
$\alpha + \beta = -b/a$
$\alpha\beta = c/a$
Given
$\alpha - \beta = 1$
Consider $(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$
$\dfrac{b^2}{a^2} - 1 = \dfrac{4c}{a}$
$b^2 - a^2 = 4ac$
$b^2 = a(a + 4c)$

100. (a) Since 1 is the root of given equation
$\therefore$ it satisfies the equation.
$\therefore (1)^2 + a(1) - 6 = 0$
$\Rightarrow a - 6 = -1$

101. (a) Given equation is $x^2 - q(1+x) - r = 0$
$\Rightarrow x^2 - qx + (-q - r) = 0$
Now, $\alpha + \beta = q$
$\alpha\beta = -q - r$
Consider $(1+\alpha)(1+\beta)$
$= 1 + \alpha + \beta + \alpha\beta = 1 + q - q - r = 1 - r.$

102. (a) Let α, β be roots of equation $ax^2 + bx + c$
$\therefore \alpha + \beta = -b/a$, $\alpha\beta = c/a$
So, 2α and 2β are roots of equation $x^2 + 36x + 24$
Now, $2\alpha + 2\beta = -36$
$\Rightarrow \alpha + \beta = -18 \Rightarrow -\dfrac{b}{a} = -18 \Rightarrow b = 18a$
and $(2\alpha)(2\beta) = 24$
$\Rightarrow \alpha\beta = 6 \Rightarrow \dfrac{c}{a} = 6 \Rightarrow c = 6a$
$\therefore b : c = 3 : 1$

103. (d) $b = 18a$, $c = 6a$
$\therefore bc = (18a)(6a) = 108a^2$

104. (d) Let α, β be the roots of given equation
then $\alpha + \beta = -2$
and $\alpha\beta = -143$
Consider $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= (-2)^2 - 2(-143) = 4 + 286 = 290$

105. (a) Given equations are
$2x + y = 6 \Rightarrow 2x = 6 - y$ ...(1)
and $3y = 8 + 2(2x)$
$\Rightarrow 3y = 8 + 2(6 - y)$ (from (1))
$\Rightarrow 3y = 8 + 12 - 2y$
$\Rightarrow y = 4$ and $x = 1$
Now, equation given in option (a) is satisfied by $x = 1$ and $y = 4$

106. (c) Let $\alpha = m + n$, $\beta = m - n$
Now, $\alpha + \beta = m + n + m - n = 2m$
$\alpha \times \beta = (m+n)(m-n) = m^2 - n^2.$
Now, Required Quadratic Equation will be,
$x^2 - (\alpha+\beta)x + \alpha\beta = 0$
$x^2 - 2mx + m^2 - n^2 = 0$

107. (c) Since, α & β are the roots of $x^2 + px - q = 0$, then,
$\alpha + \beta = -p$ .... (1)
and $\alpha\beta = -q$ .... (2)
put the value of α from (2) in (1)
$-\dfrac{q}{\beta} + \beta = -p \Rightarrow -q + \beta^2 = -p\beta$
$\Rightarrow \beta^2 = q - p\beta$ .... (3)
Since γ & δ are the roots of $x^2 - px + r = 0$, then,
$\gamma + \delta = p$, $\gamma\delta = r$
Now, $(\beta+\gamma)(\beta+\delta) = \beta^2 + \beta\delta + \beta\gamma + \gamma\delta.$
$= \beta^2 + \beta[\gamma + \delta] + \gamma\delta. = q - p\beta + p\beta + r = q - r$

108. (b) The given equation is,
$3x^2 - 5x + p = 0$
We have, $a = 3$, $b = -5$, $c = p$
$D = b^2 - 4ac = 25 - 12p$
For Real and unequal, $D > 0$
$\therefore 25 - 12p > 0$
$\Rightarrow 25 > 12p \Rightarrow p < \dfrac{25}{12}$

109. (a) Given equation is.
$4^x - 6.2^x + 8 = 0$
$\Rightarrow (2^x)^2 - 6.2^x + 8 = 0$
Put $2^x = y$, we have
$y^2 - 6y + 8 = 0$
$\Rightarrow (y-4)(y-2) = 0$
$\Rightarrow y = 2, 4$
So, $2^x = 2 \Rightarrow x = 1$
and $2^x = 2^2 \Rightarrow x = 2$

110. (d) Quadratic equation is given by
$x^2 - (\text{sum of roots})\, x + (\text{product of roots}) = 0$
$\therefore$ Required equation is $x^2 - x(\alpha^2 + \beta^2) + (\alpha\beta)^2 = 0$
$\Rightarrow x^2 - x[(\alpha+\beta)^2 - 2\alpha\beta] + (\alpha\beta)^2 = 0$
$$\Rightarrow x^2 - x\left(\frac{b^2}{a^2} - \frac{2c}{a}\right) + \frac{c^2}{a^2} = 0$$
$\Rightarrow a^2x^2 - x(b^2 - 2ac) + c^2 = 0$

111. (d) Let roots of equation be 2α and 3α.
$$2\alpha + 3\alpha = -\frac{2b}{3a}$$
$$\Rightarrow \alpha = \frac{-2b}{15a} \quad \text{...(i)}$$
$$2\alpha \,.\, 3\alpha = \frac{c}{3a} \Rightarrow \alpha^2 = \frac{c}{18a} \quad \text{...(ii)}$$
Now, put value of α in equation (ii)
$$\therefore \left(\frac{-2b}{15a}\right)^2 = \frac{c}{18a} \Rightarrow \frac{4b^2}{225a^2} = \frac{c}{18a}$$
$\Rightarrow 8b^2 = 25ac$

112. (b) Quadratic equation can be given as
$x^2 - (\text{sum of roots})\, x + (\text{product of roots}) = 0$
Hence, Required quadratic equation is
$x^2 - 3x + 2 = 0$

113. (a) Consider $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$
Equation is $x^2 + bx + c = 0$
Now, sum of roots $= \alpha + \beta = -b$
and product of roots $= \alpha \,.\, \beta = c$
$$\therefore \frac{\alpha+\beta}{\alpha\beta} = -\frac{b}{c}$$
Hence, $\alpha^{-1} + \beta^{-1} = -\frac{b}{c}$

114. (c) Let width of the rectangle $= x$
so, length $= 2x + 5$
Given, Area of rectangle $= 75$
$\Rightarrow x(2x+5) = 75 \Rightarrow 2x^2 + 5x - 75 = 0$
$\Rightarrow 2x^2 + 15x - 10x - 75 = 0$
$\Rightarrow (x-5)(2x+15) = 0$
$$\Rightarrow x = 5, -\frac{15}{2}$$
Since, width can not be negative.
$\therefore x = 5$
$\therefore$ length $= 2x + 5 = 15$ unit.

115. (b) $(x+1)^2 - 1 = 0 \Rightarrow x + 1 = \pm 1$
$\Rightarrow x + 1 = 1$ or $x + 1 = -1$
$\Rightarrow x = 0$ or $x = -2$
Thus, $x = 0, -2$ two real roots.

116. (d) $7 + 4\sqrt{3} = (\sqrt{4})^2 + (\sqrt{3})^2 + 2\sqrt{4} \times \sqrt{3}$
$= (\sqrt{4} + \sqrt{3})^2$
$\sqrt{7 + 4\sqrt{3}} = 2 + \sqrt{3}$

117. (c) Here, α and β are the roots of the equation
$x^2 + x + 2 = 0$
$\alpha + \beta = -1$
$\alpha\beta = 2$
$$\frac{\alpha^{10} + \beta^{10}}{\alpha^{-10} + \beta^{-10}} = \frac{\alpha^{10} + \beta^{10}}{\frac{1}{\alpha^{10}} + \frac{1}{\beta^{10}}}$$
$$= \frac{(\alpha^{10} + \beta^{10})(\alpha\beta)^{10}}{(\alpha^{10} + \beta^{10})} = (\alpha\beta)^{10}$$
$\therefore (\alpha\beta)^{10} = 2^{10} = 1024$

118. (a) Since b is not a perfect square, therefore other root will be $3a - \sqrt{b}$
Required quadratic equation is
$x^2 - [(3a + \sqrt{b}) + (3a - \sqrt{b})]\, x + (3a + \sqrt{b})(3a - \sqrt{b}) = 0$
$\Rightarrow x^2 - 6ax + 9a^2 - b = 0$

119. (d) $f(x) = \begin{cases} x^2 + 3x + 2 = 0, & \text{for } x \ge 0 \\ x^2 - 3x + 2 = 0, & \text{for } x < 0 \end{cases}$
for $x \ge 0$
$x^2 + 3x + 2 = 0$
$$x = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2}$$
$x = -2, -1$
for $x < 0$
$x^2 - 3x + 2 = 0$
$$x = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2}$$
$x = 2, 1$
Since x as negative, therefore $x \ne 2, 1$
Hence the given equation has no real roots

120. (b) α and β are the roots of the given equation, then
$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{b}{a}$
$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}}$$
$$= \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\alpha\beta}$$
$$= \frac{\alpha + \beta + \alpha\beta}{\sqrt{\alpha\beta}} = \frac{-\frac{b}{a} + \frac{b}{a}}{\sqrt{\frac{b}{a}}} = 0$$

121. (c) Discriminant, $D = (-8)^2 - 4 \times 16 = 0$
$\therefore$ Roots are real and equal.

122. (d) $x^2 - 10x + 9 = 0$
$x^2 - x - 9x + 9 = 0$
$x(x-1) - 9(x-1) = 0$
$(x-1)(x-9) = 0$
$x = 1, 9$
Difference in roots $= 9 - 1 = 8$

123. (a) $8x - 9y = 20$ or $80x - 90y = 200$ ...(1)
$7x - 10y = 9$ or $63x - 90y = 81$ ...(2)
Subtracting (2) from (1), we get
$17x = 119$
$x = 7$
$8 \times 7 - 9y = 20$
$9y = 36$
$y = 4$
$2x - y = 2 \times 7 - 4 = 10$

124. (d) If root are real
$b^2 - 4 \times 4 \geq 0$
$b^2 \geq 16$
$b \leq -4,\ b \geq 4$

125. (c) Given equation $ax^2 + bx + c = 0$ (where $a \neq 0$)
α and β are roots of given equation.
$(a\alpha + b)(a\beta + b) = a^2\alpha\beta + ab\alpha + ab\beta + b^2$
$= a^2\alpha\beta + ab(\alpha + \beta) + b^2$
From the given quadratic equation

$$\alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$$

$$a^2 \times \frac{c}{a} + ab \times -\frac{b}{a} + b^2 = ac$$

126. (c) We have, $2a^2x^2 - 2abx + b^2 = 0$
Discriminent, $D = (-2ab)^2 - 4(2a^2)(b^2)$
$= 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$
Roots are always complex.

127. (d) Every quadratic equation
$ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$
has at most two real roots.

128. (a) $\because\ \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Also, $\alpha + h + \beta + h = -\frac{q}{p}$

$$\Rightarrow \alpha + \beta + 2h = -\frac{q}{p}$$

$$\Rightarrow 2h = -\frac{q}{p} + \frac{b}{a}$$

$$\left(\because \alpha + \beta = -\frac{b}{a}\right)$$

$$\Rightarrow h = \frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$$

129. (b) $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$
Let $x^2 + 2 = y$
$y^2 + 8x^2 = 6xy$
$y^2 - 6xy + 8x^2 = 0$

$$y = \frac{6x \pm \sqrt{36x^2 - 32x^2}}{2}$$

$$y = \frac{6x \pm 2x}{2} = 3x \pm x$$

$y = 4x, 2x$

At $y = 4x$,
$x^2 + 2 = 4x$
$x^2 - 4x + 2 = 0$
Discriminant, $D = 16 - 8 = 8 > 0$
Roots are real.
Sum of roots $= -(-4) = 4$
At $x = 2x$,
$x^2 + 2 = 2x$
$x^2 - 2x + 2 = 0$
$D = 4 - 8 = -4 < 0$
Roots are complex.
Sum of roots $= 2$
Sum of all roots $= 4 + 2 = 6$
only statement 2 is correct.
$\therefore$ Correct option is (b)

130. (a) Let correct equation is $ax^2 + bx + c = 0$
According to first student, equation is:
$ax^2 + bx + c_1 = 0$ and roots are 8 and 2

$$8 + 2 = \frac{-b}{a} \Rightarrow \frac{b}{a} = -10$$

Quadratic equation according to second student
$ax^2 + b_1x + c = 0$ and roots are -9 and -1

$$(-9) \times (-1) = \frac{c}{a} \Rightarrow \frac{c}{a} = 9$$

Putting value in original equation

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$x^2 - 10x + 9 = 0.$

131. (c) Here m and n are the roots of equation.
$(x + p)(x + q) - k = 0$
$x^2 + x(p + q) + pq - k = 0$ ... (i)
If m and n are the roots of equation, then
$(x - m)(x - n) = 0$
$\therefore\ x^2 - (m + n)x + mn = 0$ ... (ii)
Now equation (i) should be equal to equation (ii),
$(m + n) = -(p + q)$ and $mn = pq - k$
Now, we have to find roots of $(x - m)(x - n) + k = 0$
$x^2 - (m + n)x + mn + k = 0$
$x^2 + (p + q)x + (pq - k) + k = 0$
$x^2 + (p + q)x + pq = 0$
$x^2 + px + qx + pq = 0$
$x(x + p) + q(x + p) = 0$
$\therefore\ x + q = 0$ or $x + p = 0$
$\therefore\ x = -q$ and $x = -p$
$\therefore$ Option (c) is correct.

132. (c) $4p^2 + 4pq - 3q^2 - 36 = 0$

$\Rightarrow (4p^2 + 4pq + q^2) - 4q^2 - 36 = 0$

$(2p + q)^2 = 4q^2 + 36$

$2p + 3q = 18$

$p = \dfrac{18 - 3q}{2}$

$\Rightarrow \left[\dfrac{18 - 3q}{2} \times 2 + q\right]^2 = 4q^2 + 36$

$\Rightarrow (18 - 3q + q)^2$

$\Rightarrow (18 - 2q)^2 = 4q^2 + 36$

$\Rightarrow 324 + 4q^2 - 72q = 4q^2 + 36$

$72q = 324 - 36 = 288$

$q = \dfrac{288}{72} = 4$

Putting the value of q in $2p + 3q = 18$

$2p + 3 \times 4 = 18$

$2p = 18 - 12$

$p = \dfrac{6}{2} = 3$

$(2p + q) = 2 \times 3 + 4 = 10$

$\therefore$ Option (c) is correct.

133. (a) $x^2 - 3|x| + 2 = 0$

Case (i) when $x \geq 0$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1, 2$ (both roots satisfy the condition $x \geq 0$)

Case (ii) when $x < 0$

$x^2 + 3x + 2 = 0$

$(x + 1)(x + 2) = 0$

$x = -1, -2$ (both roots satisfy the condition $x < 0$)

So no. of real roots is 4.

134. (c) $ax^2 + bx + c = 0$

Let the root be α and β.

$\alpha + \beta = -\dfrac{b}{a}, \ \alpha.\beta = \dfrac{c}{a}$

$\Rightarrow \alpha + \beta = \alpha^2 + \beta^2$ … (given)

$\Rightarrow \alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow -\dfrac{b}{a} = \left(-\dfrac{b}{a}\right)^2 - \dfrac{2c}{a}$

$\boxed{\Rightarrow b^2 + ab = 2ac}$

135. (a) Let the root be α and β

$\because x^2 - nx + m = 0$

$\Rightarrow \alpha + \beta = n; \alpha.\beta = m$

$\Rightarrow \alpha - \beta = 1$

$\Rightarrow (\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$\Rightarrow n^2 = 1 + 4m$

$\Rightarrow n^2 - 4m - 1 = 0$

136. (b) $x^2 - px + 4 > 0 \ \forall$ real values of x.

If $b^2 - 4ac \leq 0$

$\Rightarrow p^2 - 4(1)(4) \leq 0$

$\Rightarrow p^2 \leq 16 \Rightarrow |p| \leq 4$

Sol. (137-138) : $f(x) = \dfrac{27}{4}(x^{2/3} - x)$

$= \dfrac{27}{4}((x^2)^{1/3} - x)$

137. (b) For $f(x) = 1$

$\dfrac{27}{4}((x^2)^{1/3} - x) = 1$

$(x^{1/3})^2 - x = \dfrac{4}{27}$

Put $x^{1/3} = y$

$x = y^3$

$y^2 - y^3 = \dfrac{4}{27}$

$\Rightarrow y^3 - y^2 + \dfrac{4}{27} = 0$

$\Rightarrow 27y^3 - 27y^2 + 4 = 0$

This is a cubic equation.

If we put $y = -\dfrac{1}{3}$ then $(3y + 1) = 0$ is a factor of cubic equation.

$3y + 1 \,)\, 27y^3 - 27y^2 + 4 \,(\, 9y^2 - 12y + 4$

$\underline{\overset{-}{27y^3} \overset{-}{+} 9y^2}$

$-36y^2 + 4$

$\underline{\overset{+}{-} 36y^2 \overset{+}{-} 12y}$

$12y + 4$

$\underline{\overset{-}{12y} \overset{-}{+} 4}$

$\times$

$(3y + 1)(9y^2 - 12y + 4) = 0$

$(3y + 1)(3y - 2)^2 = 0$

Hence $y = -\dfrac{1}{3}, \dfrac{2}{3}$

Thus $f(x) = 1$ has two solutions.

138. (a) Similarly for $f(x) = -1$ we will get $27y^3 - 27y^2 - 4 = 0$ and after solving it we will find that it has one real solution.

$y_0 = 1.1184,$

$x = (y_0)^3 = (1.1184)^3 = 1.4$

139. (c) Given quadratic equation,
$x^2 + bx + c = 0$ and roots are α and β where $\alpha < \beta$.
Hence roots of given quadratic equation are

$$\beta = \frac{-b+\sqrt{b^2-4c}}{2}$$

$$\alpha = \frac{-b-\sqrt{b^2-4c}}{2} \qquad (\because \alpha < \beta)$$

$$\Rightarrow -\alpha = \frac{b+\sqrt{b^2-4c}}{2} \text{ and } |\alpha| = \frac{b+\sqrt{b^2-4c}}{2}$$

$\therefore$ $\beta < -\alpha$ and $\beta < |\alpha|$ both are correct.

140. (b) Sum of roots = $\alpha + \beta = -b$
Multiplication of roots = $\alpha\beta = c$
Hence
$\alpha + \beta + \alpha\beta = -b + c$ $\qquad$ $\alpha^2\beta + \beta^2 = \alpha\beta(\alpha+\beta) = -bc$
$\because$ $b > 0$ & $c < 0$
$\therefore$ $-b + c < 0$ & $-bc > 0$

141. (b) Given equation is
$(\ell - m)x^2 + \ell x + 1 = 0$
Roots are α, β.
$\because$ One root is double the other.
$\beta = 2\alpha$
Sum of roots = $\alpha + \beta$

$$3\alpha = \frac{-\ell}{\ell - m} \qquad \alpha(2\alpha) = \frac{1}{(\ell - m)}$$

$$\Rightarrow \alpha^2 = \frac{\ell^2}{9(\ell-m)^2} \qquad 2\alpha^2 = \frac{1}{\ell - m}$$

$$\Rightarrow 2\frac{\ell^2}{9(\ell-m)^2} = \frac{1}{(\ell-m)}$$

$$\Rightarrow \frac{2\ell^2}{9(\ell-m)} = 1$$

$$\Rightarrow 2\ell^2 = 9(\ell - m) \Rightarrow 2\ell^2 - 9\ell + 9m = 0$$

For ℓ to be real discriminant should be $b^2 - 4ac \ge 0$

$81 - 4 \times 2 \times 9m \ge 0$

$m \le \frac{9}{8}$.

142. (a) Let $f(x) = ax^2 - bx + c$
$f(2) = 4a - 2b + c < 0$ (given)
$f(0) = c > 0$ (given)
So, we can see that sign of f(x) changes, when x changes from 0 to 2, so it has a root in the interval (0, 2).

143. (d) $x^2 - 2kx + k^2 - 4 = 0$
$\Rightarrow (x - k)^2 - 2^2 = 0$
$\Rightarrow (x - k - 2)(x - k + 2) = 0$
$\Rightarrow x = k + 2, k - 2.$
$\Rightarrow k + 2 < 5$ & $k - 2 > -3$
$\Rightarrow k < 3$ & $k > -1$
$\Rightarrow -1 < k < 3$

144. (d) Using $ax^2 + bx + c = 0$
$a = 1, b = -(1 - 2a^2)$ & $c = (1 - 2a^2)$
For roots to be real,
$b^2 - 4ac \ge 0$
$\Rightarrow [-(1 - 2a^2)]^2 - 4(1)(1 - 2a^2) \ge 0$
$\Rightarrow 4a^4 + 4a^2 - 3 \ge 0$
$\Rightarrow (2a^2 - 1)(2a^2 + 3) \ge 0$

$$\Rightarrow a^2 \ge \frac{1}{2} \text{ or } a^2 \le -\frac{3}{2}$$

145. (a) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1 \Rightarrow \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} < 1$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} < 1$$

$$\alpha + \beta = \frac{-b}{a} = (1 - 2a^2) \text{ \& } \alpha\beta = \frac{c}{a} = (1 - 2a^2)$$

On solving: $\frac{4a^4 - 1}{4a^4 - 4a^2 + 1} < 1$

$\Rightarrow 4a^4 - 1 < 4a^4 - 4a^2 + 1$

$4a^2 < 2 \Rightarrow a^2 < \frac{1}{2}$

146. (c) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\ldots + \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$
LHS of given inequality is in G.P.

$$\therefore \frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}} < 2 - \frac{1}{1000}$$

$$\Rightarrow 2 - \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$$

$\Rightarrow 2^{n-1} < 1000$
Now, $(2)^9 = 512$ & $(2)^{10} = 1024$
$\therefore$ $n - 1 = 9$
$\Rightarrow$ $n = 10$.

147. (c) $2x^2 + 3x - \alpha = 0$
Its roots are: -2 & β.

i.e., $\frac{-3}{2} = \beta - 2 \Rightarrow \beta = 2 - \frac{3}{2} = \frac{1}{2} \Rightarrow \beta = \frac{1}{2}$

$\frac{\alpha}{2} = 2\beta \Rightarrow \alpha = 4 \times \frac{1}{2} \Rightarrow \alpha = 2$

148. (a) $\beta = \frac{1}{2}$
β, 2, 2m are in GP.

$$\Rightarrow \frac{2}{\beta} = \frac{2m}{2}$$

$$\Rightarrow m = \frac{2}{\beta} = 2 \times \frac{2}{1}$$

$\Rightarrow m = 4$

$$\Rightarrow \beta\sqrt{m} = \frac{1}{2} \times \sqrt{4} = 1$$

149. (c) $|x+y| = 2 \Rightarrow x+y = \pm 2$

$\Rightarrow x+y+2=0$ and $x+y-2=0$

$\Rightarrow -2<2a<0$ and $-1<a<0$

$\Rightarrow |a|<1$

150. (a) Let a, b be the roots of
$x^2+p/x^2+q=0$
So, $a+b=-p/x^2$, $ab=q$ ...(i)
Let c, d be the roots of
$x^2+lx+m=0$
So, $c+d=-l$, $cd=m$ ...(ii)
Given that roots of both the equations are in the Same ratio.

So, $\frac{a}{b}=\frac{c}{d}$ ...(iii)

$\Rightarrow \frac{b}{a}=\frac{d}{c}$ ...(iv)

$(iii)+(iv) \Rightarrow \frac{a}{b}+\frac{b}{a}=\frac{c}{d}+\frac{d}{c}$

$\Rightarrow \frac{a^2+b^2}{ab}=\frac{c^2+d^2}{cd} \Rightarrow \frac{a^2+b^2}{ab}+2=\frac{c^2+d^2}{cd}+2$

$\Rightarrow \frac{a^2+b^2+2ab}{ab}=\frac{c^2+d^2+2cd}{cd}$

$\Rightarrow \frac{(a+b)^2}{ab}=\frac{(c+d)^2}{cd}$

$\Rightarrow \frac{(-P)^2}{q}=\frac{(-l)^2}{m}$ (from (i) and (ii))

$\Rightarrow P^2m=l^2q.$

151. (c) $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2)$
We know, $(1+\omega+\omega^2)=0$.
So, $(1+\omega)(1+\omega^2)(1+\omega^3)(0)=0$.

152. (c) Since the graph is not meeting the x - axis at all, roots are Complex numbers.

153. (b) Given equation, $x^2+bx+c=0$
Roots are $\cot\alpha$, $\cot\beta$.
Sum of roots = $\cot\alpha+\cot\beta=-b$
Product of roots = $\cot\alpha \cdot \cot\beta = c$

$\cot(\alpha+\beta)\ \frac{\cot\alpha\cdot\cot\beta-1}{\cot\beta+\cot\alpha}=\frac{c-1}{-b}=\frac{1-c}{b}$

154. (b) Given equation, $(q-r)x^2+(r-p)x+(p-q)=0$
On observing the equation, it is clear that 1 is root of equation.
If $x=1$, then $q-r+r-p+p-q=0$.
$\therefore$ 1 is one root of given equation.
Since, the given equation is quadratic equation, we know that product of roots is $\frac{c}{a}$.

Let the second root be α.

$\therefore\ (1)(\alpha)=\frac{p-q}{q-r} \Rightarrow \alpha=\frac{p-q}{q-r}.$

155. (b) α, β are roots of the equation $1+x+x^2=0$.
$1+x+x^2=0 \Rightarrow x^2+x+1=0$

Solving for x, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)}$

$=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm\sqrt{3}i}{2}$

$\therefore$ roots are $\frac{-1\pm\sqrt{3}i}{2}$ and $\frac{-1-\sqrt{3}i}{2}$

i.e., $\alpha=\omega$, $\beta=\omega^2$

$$\begin{bmatrix}1 & \beta\\ \alpha & \alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ 1 & \beta\end{bmatrix}=\begin{bmatrix}\alpha+\beta & \beta+\beta^2\\ \alpha^2+\alpha & \alpha\beta+\alpha\beta\end{bmatrix}$$

$$=\begin{bmatrix}\omega+\omega^2 & \omega^2+\omega^4\\ \omega^2+\omega & \omega^3+\omega^3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & \omega^2+\omega\\ -1 & 2\omega^3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1\\ -1 & 2\end{bmatrix}$$

156. (d) All the given three options are correct.

157. (d) $|x-3|^2+|x-3|-2=0$
Let $|x-3|=t$
$\therefore t^2+t-2=0 \Rightarrow t^2+2t-t-2=0$
$\Rightarrow t(t+2)-1(t+2)=0$
$\Rightarrow (t+2)(t-1)=0$
$\Rightarrow t=-2$ or $t=1$
Since t is modulus of a number, it cannot be negative.
$\therefore t=1 \Rightarrow |x-3|=1 \Rightarrow x-3=1$ or $x-3=-1$
$\Rightarrow x=4$ or 2
Sum of roots = 4 + 2 = 6.

158. (c) $x^2-4x-\log_3 P=0$
We know, roots are real if discriminant is greater than or equal to 0.
i.e., $b^2-4ac\ge 0 \Rightarrow b^2\ge 4ac$
In the given equation, $a=1$, $b=-4$, $c=-\log_3 P$.
$\therefore b^2\ge 4ac \Rightarrow (-4)^2\ge 4(1)(-\log_3 P)$
$\Rightarrow 16\ge -4\log_3 P$
$\Rightarrow 4\ge -\log_3 P$

$\Rightarrow 4\ge \log_3\left(\frac{1}{P}\right)$ $\left(\because \sin\theta-\log a=\log\frac{1}{a}\right)$

$\Rightarrow 3^4\ge\frac{1}{P}$

$\Rightarrow 81\ge\frac{1}{P} \Rightarrow P\ge\frac{1}{81}.$

$\therefore$ The minimum value of P is $\frac{1}{81}$.

159. (a) $3x^2 + 2x + 1 = 0$.

Sum of the roots $= \alpha + \beta = \frac{-2}{3}$ ....(1)

Product of the roots $= \alpha \cdot \beta = \frac{1}{3}$ ....(2)

We have to find the equation with the roots $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$.

Sum of the roots (S) $= \alpha + \beta^{-1} + \beta + \alpha^{-1}$

$$= \alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha}$$

$$= (\alpha + \beta) + \left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

$$= \frac{-2}{3} + \left(\frac{\frac{-2}{3}}{\frac{1}{3}}\right) \quad \text{(from (1), (2))}$$

$$= \frac{-2}{3} - 2 = \frac{-2-6}{3} = \frac{-8}{3}.$$

Product of the roots (P) $= (\alpha + \beta^{-1})(\beta + \alpha^{-1})$

$$= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = \alpha\beta + 2 + \frac{1}{\alpha\beta}$$

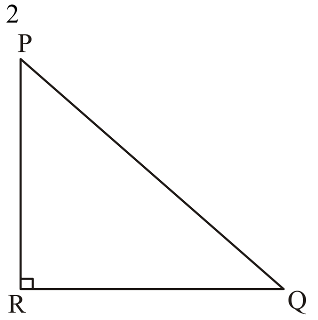
$$= \frac{1}{3} + 2 + \frac{1}{\frac{1}{3}}$$

$$= \frac{1}{3} + 2 + 3 = \frac{1}{3} + 5 = \frac{16}{3}.$$

So, the required equation is $x^2 - \text{s.x.} + P = 0$

$$= x^2 + \frac{8}{3}x + \frac{16}{3} = 0 \Rightarrow 3x^2 + 8x + 16 = 0$$

160. (c) $\angle R = \frac{\pi}{2}$



$$\therefore \angle P + \angle Q = \frac{\pi}{2}$$

$$\Rightarrow \frac{\angle P + \angle Q}{2} = \frac{\pi}{4} \quad ....(1)$$

$$\tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}\tan\frac{Q}{2}} \quad ....(2)$$

Given, $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are roots of $ax^2 + bx + c = 0$.

$$\therefore \tan\frac{P}{2} + \tan\frac{Q}{2} = \frac{-b}{a}; \tan\frac{P}{2} \cdot \tan\frac{Q}{2} = \frac{c}{a}.$$

$$\therefore (2) \Rightarrow \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \frac{\frac{-b}{a}}{1 - \frac{c}{a}}$$

$$\Rightarrow \tan\frac{\pi}{4} = \frac{\frac{-b}{a}}{1 - \frac{c}{a}} \quad \text{(from (1))}$$

$$\Rightarrow 1 = \frac{-b}{a-c} \Rightarrow -b = a - c \Rightarrow a + b = c.$$

161. (a) $|1 - x| + z^2 = 5$

$\Rightarrow |x - 1| + x^2 = 5$

First case: If $x < 1$, $|x - 1|$ is negative.

$\therefore \quad -(x - 1) + x^2 = 5$

$\Rightarrow \quad -x + 1 + x^2 = 5$

$\Rightarrow x^2 - x + 1 = 5$

$\Rightarrow x^2 - x - 4 = 0$

Roots are $\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$

$$= \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

Since, $x < 1$, root cannot be $\frac{1+\sqrt{17}}{2}$. So, the root is $\frac{1-\sqrt{17}}{2}$, which is irrational.

Second case: If $x > 1$, $|x - 1| = x - 1$

$\therefore \quad |x - 1| + 2^2 = 5$

$\Rightarrow \quad x - 1 + x^2 = 5$

$\Rightarrow \quad x^2 + x - 6 = 0$

$\Rightarrow \quad x^2 + 3x - 2x - 6 = 0$

$\Rightarrow \quad x(x - 3) - 2(x + 3) = 0$

$\Rightarrow \quad x = 2, -3$

Since $x > 1$, root cannot be –3. So, root is 2 which is rational.

$\therefore$ Given expression has one irrational root and One rational root.

162. (b) $x^2 - 4x + [x] = 0$

Given internal, [0, 2]

Case 1 : Let $0 \le x < 1$

$[x] = 0$

$\therefore x^2 - 4x + 0 = 0 \Rightarrow x(x - 4) = 0 \Rightarrow x = 0, x = 4$

$x = 4$ can't be taken in $0 \le x < 1$

$\therefore \boxed{x = 0}$

Case 2 : Let $1 \le x < 2$

$[x] = 1$

$\therefore x^2 - 4x + 1 = 0$

roots are $\frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

In internal $1 \le x < 2$, $2 \pm \sqrt{3}$ are not the roots.
Case 3 : Let $x = 2$
$[x] = 2$
$\therefore x^2 - 4x + 2 = 0$

roots are $\frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$

Since, $x = 2$, roots can't be $2 \pm \sqrt{2}$

$\therefore$ There is only one solution, $x = 0$

163. (b) A rational expression is nothing more than a fraction in which the numerator and denominator are polynomials. Here are some example of rational expressions are $\left(x + x^2 + \frac{1}{x}\right)$, $(3x^2 - 5x + ab)$, $\frac{2}{x^2 - ax + ab}$

164. (d) $\alpha + \beta = -\alpha$, $\alpha\beta = -\beta$
$\Rightarrow \alpha\beta + \beta = 0$
$\Rightarrow (\alpha + 1)\beta = 0$
$\Rightarrow \alpha = -1 \ (\beta \neq 0)$
$\Rightarrow (2\alpha + \beta) = 0$
$\Rightarrow \beta = 2$
$\therefore -x^2 + \alpha x + \beta = -x^2 - x + 2$

Greatest value $= -\frac{1+8}{-4} = \frac{9}{4}$

165. (b) Let $f(x) = ax^2 + bx + c$, $a > 0$, $b^2 < 4ac$ $\quad (\because f(x) > 0)$
Now, $g(x) = ax^2 + bx + c + 2ax + b + 2a$
$= ax^2 + (b + 2a)x + 2a + b + c$
Now, $(b + 2a)^2 - 4a(2a + b + c)$
$= b^2 + 4ab + 4a^2 - 8a^2 - 4ab - 4ac$
$= b^2 - 4ac - 8a^2 < 0$
$\Rightarrow g(x) > 0$

166. (b) $\frac{D_1}{D_2}$ = (ratio of coefficient of $x)^2 = \frac{b^2}{q^2}$

167. (d) Given, $|x^2 - x - 6| = x + 2$
$\therefore x^2 - x - 6 = x + 2$ and $x^2 - x - 6 = -(x + 2)$
$\Rightarrow x^2 - 2x - 8 = 0$
$\Rightarrow x^2 - x - 6 = -x - 2$
$\Rightarrow x^2 - 4x + 2x - 8 = 0$
$\Rightarrow x^2 = 4$
$\Rightarrow x(x - 4) + 2(x - 4) = 0$
$\Rightarrow x = +2, -2$
$\Rightarrow x = 4, -2$
$\therefore x = -2, 2, 4.$

168. (b) $px^2 + qx + r = 0$, (p, q, r are positive)
Whenever the coefficients and constant are positive in quadratic equation, its roots are always negative.
$\therefore a < 0, b < 0$

169. (a) Given, tan 19° and tan 26° are roots of $x^2 + px + q = 0$

$\therefore \tan 19° + \tan 26° = \frac{-p}{1} = -p$

$(\tan 19°)(\tan 26°) = \frac{q}{1} = q$

$\tan(19° + 26°) = \frac{\tan 19° + \tan 26°}{1 - \tan 19° \tan 26°}$

$\tan 45° = \frac{-p}{1-q} \Rightarrow 1 = \frac{-p}{1-q}$

$\Rightarrow 1 - q = -p$
$\Rightarrow q - p = 1$

170. (a) $x^2 + 9|x| + 20 = 0$
The sum of three positive quantities can never be zero. So, the equation has no solution.

Sequence and Series 3

1. It the sum of first 10 terms of an arithmetic progression with first term p and common difference q, is 4 times the sum of the first 5 terms, then what is the ratio $p : q$?
(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1 *[2006-I]*

2. One of the roots of a quadratic equation with real coefficients is $\frac{1}{(2-3i)}$. Which of the following implications is/are true?
1. The second root of the equation will be $\frac{1}{(3-2i)}$.
2. The equation has no real root.
3. The equation is $13x^2 - 4x + 1 = 0$.
Which of the above is/are correct ?
(a) 1 and 2 only (b) 3 only
(c) 2 and 3 only (d) 1, 2 and 3 *[2006-I]*

3. What is the sum of the first 50 terms of the series $(1 \times 3) + (3 \times 5) + (5 \times 7) + ...$?
(a) 1,71,650 (b) 26,600
(c) 26,650 (d) 26,900 *[2006-I]*

4. If $x = 1 \;\; \frac{y}{2} \;\; \left(\frac{y}{2}\right)^2 \;\; \left(\frac{y}{2}\right)^3 + ...$ where $| y | < 2$, what is 1y equal to ?
(a) $\frac{x-1}{x}$ (b) $\frac{x-1}{2x}$
(c) $\frac{2x-2}{x}$ (d) $\frac{2x+1}{2x}$ *[2006-I]*

5. What is the product of first 2n + 1 terms of a geometric progression ?
(a) The (n + 1)th power of the nth term of the GP
(b) The (2n + 1)th power of the nth term of the GP
(c) The (2n + 1)th power of the (n + 1)th term of the GP
(d) The nth power of the (n + 1)th terms of the GP
[2006-I]

6. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.
Assertion (A) : 1/8, $\log_{a^2} a \rightarrow$ exponent should be to the base, $\log^2$ are in GP but not in AP.
Reason: (R) : x, y, z are in AP as well as in GP if x = y =z.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation A
(c) A is true but R is false
(d) A is false but R is true *[2006-II]*

7. If x + 1, 4x + 1, and 8x + 1 are in geometric progression, then what is the non-trivial value of x ?
(a) – 1 (b) 1
(c) $\frac{1}{8}$ (d) $\frac{1}{4}$ *[2006-II]*

8. The equation $(a^2 + b^2) x^2 - 2b (a + c) x + (b^2 + c^2) = 0$ has equal roots. Which one of the following is correct about a, b, and c ?
(a) They are in AP
(b) They are in GP
(c) They are in HP
(d) They are neither in AP, nor in GP, nor in HP *[2006-II]*

9. If p^{th} term of an AP is q, and its q^{th} term is p, then what is the common difference ?
(a) – 1 (b) 0
(c) 2 (d) 1 *[2006-II]*

10. If a, b, c are in geometric progression and a, 2b, 3c are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$ *[2006-II]*

11. For an AP with first term u and common difference v, the p^{th} term is 15 uv more than the q^{th} term. Which one of the following is correct ?
(a) p = q + 15 v (b) p = q + 15 u
(c) p = q + 14 v (d) p = q + 14 u *[2006-II]*

12. If a, b and c are three positive numbers in an arithmetic progression, then: *[2006-II]*
(a) $ac > b^2$ (b) $b^2 > a + c$
(c) $ab + bc \leq 2ac$ (d) $ab + bc \geq 2ac$

13. If $|x| < \frac{1}{2}$, what is the value of

$$1+n\left[\frac{x}{1-x}\right]+\left[\frac{n(n+1)}{2!}\right]\left[\frac{x}{1-x}\right]^2+\ldots\ldots\ldots\ldots\ldots\infty\ ?$$

(a) $\left[\frac{1-x}{1-2x}\right]^n$ (b) $(1-x)^n$

(c) $\left[\frac{1-2x}{1-x}\right]^n$ (d) $\left(\frac{1}{1-x}\right)^n$ *[2006-II]*

14. The sum of the first (2p + 1) terms of an AP is {(p + 1) . (2p + 1)}. Which one of the following inferences can be drawn ?
(a) The $(p+1)^{th}$ term of the AP is (2p + 1)
(b) The $(2p+1)^{th}$ term of the AP is (2p + 1)
(c) The $(2p+1)^{th}$ term of the AP is (p + 1)
(d) The $(p+1)^{th}$ term of the AP is (p + 1) *[2006-II]*

15. a, b, c are in G.P. with $1 < a < b < n$, and $n > 1$ is an integer. $\log_a n$, $\log_b n$, $\log_c n$ form a sequence. This sequence is which one of the following ? *[2007-I]*
(a) Harmonic progression (b) Arithmetic progression
(c) Geometric progression (d) None of these

16. What is the sum of the series $1-\frac{1}{2}\ \ \frac{1}{4}-\frac{1}{8}\ \ .....?$

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$ *[2007-I]*

17. If b_1, b_2, b_3 are three consecutive terms of an arithmetic progression with common difference $d > 0$, then what is the value of d for which $b_3^2\ \ b_2b_3\ \ b_1d\ \ 2$?

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) 2 *[2007-I]*

18. If 1, x, y, z, 16 are in geometric progression, then what is the value of x + y + z ?
(a) 8 (b) 12
(c) 14 (d) 16 *[2007-II]*

19. If the nth term of an arithmetic progression is 3n + 7, then what is the sum of its first 50 terms?
(a) 3925 (b) 4100
(c) 4175 (d) 8200 *[2007-II]*

20. If, for positive real numbers x, y, z, the numbers x + y, 2y and y + z are in harmonic progression, then which one of the following is correct? *[2007-II]*
(a) x, y, z are in geometric progression
(b) x, y, z are in arithmetic progression
(c) x, y, z are in harmonic progression
(d) None of the above

21. What is the sum of the series

$$1+\frac{1}{8}+\frac{1.3}{8.16}+\frac{1.3.5}{8.16.24}+\ldots.\infty?$$

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$ *[2007-II]*

22. What is the geometric mean of the ratio of corresponding terms of two series where G_1 and G_2 are geometric means of the two series? *[2007-II]*
(a) $\log G_1 - \log G_2$ (b) $\log G_1 + \log G_2$
(c) $\frac{G_1}{G_2}$ (d) G_1G_2

23. If the points with the coordinates (a, ma), {b, (m + 1) b}, {c, (m + 2) c} are collinear, then which one of the following is correct? *[2007-II]*
(a) a, b, c are in arithmetic progression for all m
(b) a, b, c are in geometric progression for all m
(c) a, b, c are in harmonic progression for all m
(d) a, b, c are in arithmetic progression only for m = 1

24. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*

Assertion (A) : $0.3+0.03+0.003+.....=\frac{1}{3}$.

Reason (R) : For each (+)ve integer n, let $a_n = a + nd$, a and d are real numbers. Then, $a_1 + ... + a_n = \frac{n}{2}[2a + (n+1)d]$. *[2007-II]*
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

25. Which one of the following is correct ? If the positive numbers a, b, c, d are in AP, then bcd, cda, dab, abc
(a) are in AP
(b) are in GP
(c) are in HP
(d) are in none of the above progressions *[2007-II]*

26. What is the value of $9^{1/3}.\ 9^{1/19}.\ 9^{1/27}\ldots\ldots\ \infty$?
(a) 9 (b) 3
(c) $9^{1/3}$ (d) 1 *[2007-II]*

27. If a, b, c, d are in harmonical progression such that a > d, then which one of the following is correct?
(a) a + c = b + d (b) a + c > b + d
(c) ac = bd (d) ab = cd *[2007-II]*

28. After paying 30 out of 40 installments of a debt of Rs. 3600, one third of the debt is unpaid. If the installments are forming an arithmetic series, then what is the first instalment?
(a) Rs 50 (b) Rs 51
(c) Rs 105 (d) Rs 110 *[2008-I]*

29. The product of first nine terms of a GP is, in general, equal to which one of the following?
(a) The 9th power of the 4th term
(b) The 4th power of the 9th term
(c) The 5th power of the 9th term
(d) The 9th power of the 5th term [2008-I]

30. The difference between the nth term and (n– 1)th term of a sequence is independent of n. Then the sequence follows which one of the following?
(a) AP (b) GP
(c) HP (d) None of these [2008-I]

31. Which one of the following is correct?
If $\frac{1}{b-c}+\frac{1}{b-a}=\frac{1}{a}+\frac{1}{c}$, then a, b, c are in
(a) AP (b) HP
(c) GP (d) None of these [2008-I]

32. What is the 15th term of the series 3, 7, 13, 21, 31, 43, ...? [2008-II]
(a) 205 (b) 225
(c) 238 (d) 241

33. If the nth term of an arithmetic progression is $2n - 1$, then what is the sum upto n terms? [2008-II]
(a) n^2 (b) $n^2 - 1$
(c) $n^2 + 1$ (d) $\frac{1}{2}n(n+1)$

34. If the three observations are 3, – 6 and –6, then what is their harmonic mean? [2008-II]
(a) 0 (b) ∞
(c) $-1/2$ (d) -3

35. Sum of first n natural numbers is given by $\frac{n(n+1)}{2}$. What is the geometric mean of the series 1, 2, 4, 8, ..., 2^n? [2008-II]
(a) 2^n (b) $2^{\frac{n}{2}}$
(c) $2^{1/2}$ (d) 2^{n-1}

36. If the number of terms of an A.P. is $(2n + 1)$, then what is the ratio of the sum of the odd terms to the sum of even terms? [2008-II]
(a) $\frac{n}{n+1}$ (b) $\frac{n^2}{n+1}$
(c) $\frac{n+1}{n}$ (d) $\frac{n+1}{2n}$

37. If the sum of 'n' terms of an arithmetic progression is $n^2 - 2n$, then what is the n^{th} term? [2008-II]
(a) $3n - n^2$ (b) $2n - 3$
(c) $2n + 3$ (d) $2n - 5$

38. If a, $2a + 2$, $3a + 3$ are in GP, then what is the fourth term of the GP? [2008-II]
(a) – 13.5 (b) 13.5
(c) –27 (d) 27

39. What is sum to the 100 terms of the series $9 + 99 + 999 + ...$? [2008-II]
(a) $\frac{10}{9}(10^{100}-1)-100$ (b) $\frac{10}{9}(10^{99}-1)-100$
(c) $100\,(100^{10}-1)$ (d) $\frac{9}{100}(10^{100}-1)$

40. If the AM and GM of two numbers are 5 and 4 respectively, then what is the HM of those numbers? [2008-II]
(a) $\frac{5}{4}$ (b) $\frac{16}{5}$
(c) $\frac{9}{2}$ (d) 9

41. The harmonic mean of two numbers is 21.6.If one of the numbers is 27, then what is the other number? [2009-I]
(a) 16.2 (b) 17.3
(c) 18 (d) 20

42. If the sum of the first two terms and the sum of the first four terms of a geometric progression with positive common ratio are 8 and 80 respectively, then what is the 6th term? [2009-I]
(a) 88 (b) 243
(c) 486 (d) 1458

43. If $x > 1$ and $\log_2 x$, $\log_3 x$, $\log_x 16$ are in GP, then what is x equal to? [2009-I]
(a) 9 (b) 8
(c) 4 (d) 2

44. In a geometric progression with first term a and common ratio r, what is the arithmetic mean of first five terms? [2009-I]
(a) $a + 2r$ (b) ar^2
(c) $a(r^5 - 1)/(r - 1)$ (d) $a(r^5 - 1)/[5(r - 1)]$

45. If $(1 + 3 + 5 + ... + p) + (1 + 3 + 5 + ... + q)$ $= (1 + 3 + 5 + ... + r)$ [2009-II]
where each set of parentheses contains the sum of consecutive odd integers as shown, what is the smallest possible value of $(p + q + r)$ where $p > 6$?
(a) 12 (b) 21
(c) 45 (d) 54

46. If x^2, y^2, z^2 are in AP, then $y + z$, $z + x$, $x + y$ are in
(a) AP (b) HP [2009-II]
(c) GP (d) None of these

47. If x, $2x + 2$, $3x + 3$ are the first three terms of a GP, then what is its fourth term? [2009-II]
(a) –27/2 (b) 27/2
(c) –33/2 (d) 33/2

48. Which term of the sequence 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term? [2009-II]
(a) 27th (b) 28th
(c) 29th (d) No such term exists

49. In an AP, the m^{th} term $1/n$ and n^{th} term is $1/m$. What is its $(mn)^{th}$ term? [2009-II]
(a) $1/(mn)$ (b) m/n
(c) n/m (d) 1

50. The 59th term of an AP is 449 and the 449th term is 59. Which term is equal to 0 (zero)? [2010-I]
(a) 501st term (b) 502nd term
(c) 508th term (d) 509th term

51. If the AM and HM of two numbers are 27 and 12 respectively, then what is their GM equal to? [2010-I]
(a) 12 (b) 18
(c) 24 (d) 27

52. What is the sum of all natural numbers between 200 and 400 which are divisible by 7? *[2010-I]*
(a) 6729 (b) 8712
(c) 8729 (d) 9276

53. Let a, b, c be in AP. *[2010-I]*
Consider the following statements:
1. $\frac{1}{ab}, \frac{1}{ca}$ *and* $\frac{1}{bc}$ are in AP.
2. $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}$ *and* $\frac{1}{\sqrt{a}+\sqrt{b}}$ are in AP.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

54. If p times the pth term of an AP is q times the qth term, then what is the $(p+q)$th term equal to? *[2010-I]*
(a) $p+q$ (b) pq
(c) 1 (d) 0

55. The geometric mean of three numbers was computed as 6. It was subsequently found that, in this computation, a number 8 was wrongly read as 12. What is the correct geometric mean? *[2010-I]*
(a) 4 (b) $\sqrt[3]{5}$
(c) $2\sqrt[3]{18}$ (d) None of these

56. The arithmetic mean of two numbers exceeds their geometric mean by 2 and the geometric mean exceeds their harmonic mean by 1.6. What are the two numbers? *[2010-II]*
(a) 16, 4 (b) 81, 9
(c) 256, 16 (d) 625, 25

57. The sum of an infinite geometric progression is 6, If the sum of the first two terms is 9/2, then what is the first term? *[2010-II]*
(a) 1 (b) 5/2
(c) 3 or 3/2 (d) 9 or 3

58. If the AM and GM between two number are in the ratio $m : n$, then what is the ratio between the two numbers? *[2010-II]*
(a) $\frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$ (b) $\frac{m+n}{m-n}$
(c) $\frac{m^2-n^2}{m^2+n^2}$ (d) $\frac{m^2+n^2-mn}{m^2+n^2+mn}$

59. What is the geometric mean of the data 2, 4, 8, 16, 32? *[2011-I]*
(a) 2 (b) 4
(c) 8 (d) 16

60. If A, B and C are in AP and $b : c = \sqrt{3} : \sqrt{2}$, then what is the value of sin C? *[2011-I]*
(a) 1 (b) $\frac{1}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{2}}$

61. In a GP of positive terms, any term is equal to one-third of the sum of next two terms. What is the common ratio of the GP? *[2011-I]*
(a) $\frac{\sqrt{13}+1}{2}$ (b) $\frac{\sqrt{13}-1}{2}$
(c) $\frac{\sqrt{13}\quad 1}{3}$ (d) $\sqrt{13}$

62. Which term of a series $\frac{1}{4}, -\frac{1}{2}, 1, \ldots$ is -128? *[2011-I]*
(a) 9th (b) 10th
(c) 11th (d) 12th

63. If $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$, then a, b, c are in *[2011-I]*
(a) AP (b) GP
(c) HP (d) None of these

64. What is the sum of $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots$? *[2011-I]*
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{3\sqrt{3}}{2}$
(c) $\frac{2\sqrt{3}}{3}$ (d) $\sqrt{3}$

65. Which one of the following options is correct? *[2011-I]*
(a) $\sin^2 30°, \sin^2 45°, \sin^2 60°$ are in GP
(b) $\cos^2 30°, \cos^2 45°, \cos^2 60°$ are in GP
(c) $\cot^2 30°, \cot^2 45°, \cot^2 60°$ are in GP
(d) $\tan^2 30°, \tan^2 45°, \tan^2 60°$ are in GP

66. What is the 10th common term between the series $2+6+10+\ldots$ and $1+6+11+\ldots$? *[2011-II]*
(a) 180 (b) 186
(c) 196 (d) 206

67. If the 10th term of a GP is 9 and 4th term is 4, then what is its 7th term? *[2011-II]*
(a) 6 (b) 14
(c) 27/14 (d) 56/15

68. If $\log_{10} 2$, $\log_{10}(2^x-1)$, $\log_{10}(2^x+3)$ are three consecutive terms of an AP, then which one of the following is correct? *[2011-II]*
(a) $x=0$ (b) $x=1$
(c) $x=\log_2 5$ (d) $x=\log_5 2$

69. If n!, $3 \times (n!)$ and $(n+1)!$ are in GP, then the value of n will be *[2011-II]*
(a) 3 (b) 4
(c) 8 (d) 10

70. If a, b, c, d, e, f are in AP, then $(e-c)$ is equal to which one of the following? *[2011-II]*
(a) $2(c-a)$ (b) $2(d-c)$
(c) $2(f-d)$ (d) $(d-c)$

71. What is the geometric mean of 10, 40 and 60 (appox)? *[2011-II]*
(a) 10 (b) 28
(c) 29.6 (d) 70

72. If the arithmetic and geometric means of two numbers are 10, 8 respectively, then one number exceeds the other number by [2011-II]
    (a) 8 (b) 10
    (c) 12 (d) 16
73. If the sequence $\{S_n\}$ is a geometric progression and $S_2 S_{11} = S_p S_8$, then what is the value of p? [2012-I]
    (a) 1 (b) 3
    (c) 5 (d) cannot be determined
74. If 1/4, 1/x, 1/10 are in HP, then what is the value of x?
    (a) 5 (b) 6 [2012-I]
    (c) 7 (d) 8
75. If p, q, r are in AP as well as G.P., then which one of the following is correct? [2012-I]
    (a) $p = q \neq r$ (b) $p \neq q \neq r$
    (c) $p \neq q = r$ (d) $p = q = r$
76. The geometric mean and harmonic mean of two non negative observations are 10 and 8 respectively. Then what is the arithmetic mean of the observations equal to? [2012-I]
    (a) 4 (b) 9
    (c) 12, 5 (d) 2
77. What is the nth term of the sequence 1, 5, 9, 13, 17, ...?
    (a) $2n - 1$ (b) $2n + 1$ [2012-I]
    (c) $4n - 3$ (d) None of the above
78. What does the series

    $1+3^{-\frac{1}{2}}+3+\frac{1}{3\sqrt{3}}+\ldots$ represents? [2012-I]

    (a) AP (b) GP
    (c) HP (d) None of the above series
79. What is the sum of the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ equal to?
    (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ [2012-I]
    (c) 2 (d) $\frac{2}{3}$
80. Consider the following statements: [2012-II]
    1. The sum of cubes of first 20 natural numbers is 44400.
    2. The sum of squares of first 20 natural numbers is 2870.

    Which of the above statements is/are correct ?
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2
81. What is the sum of first eight terms of the series

    $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots..?$ [2012-II]

    (a) $\frac{89}{128}$ (b) $\frac{57}{384}$
    (c) $\frac{85}{128}$ (d) None of the above

DIRECTIONS (Qs. 82-83) : *For the next two (02) Questions that follow:*

The sum of first 10 terms and 20 terms of an AP are 120 and 440 respectively.

82. What is its first term? [2012-II]
    (a) 2 (b) 3
    (c) 4 (d) 5
83. What is the common difference? [2012-II]
    (a) 1 (b) 2
    (c) 3 (d) 4
84. What is the number of diagonals which can be drawn by joining the angular points of a polygon of 100 sides?
    (a) 4850 (b) 4950 [2012-II]
    (c) 5000 (d) 10000
85. The angles of a triangle are in AP and the least angle is 30°. What is the greatest angle (in radian) ? [2012-II]
    (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
    (c) $\frac{\pi}{4}$ (d) π
86. What is the geometric mean of the sequence 1, 2, 4, 8, ..., 2^n ? [2012-II]
    (a) $2^{n/2}$ (b) $2^{(n+1)/2}$
    (c) $2^{(n+1)} - 1$ (d) $2^{(n-1)}$
87. If the numbers $n-3$, $4n-2$, $5n+1$ are in AP, what is the value of n? [2013-I]
    (a) 1 (b) 2
    (c) 3 (d) 4
88. The harmonic mean H of two numbers is 4 and the arithmetic mean A and geometric mean G satisfy the equation $2A + G^2 = 27$. The two numbers are [2013-I]
    (a) 6, 3 (b) 9, 5
    (c) 12, 7 (d) 3, 1
89. If the positive integers *a, b, c, d* are in *AP,* then the numbers *abc, abd, acd, bcd* are in [2013-II]
    (a) *HP* (b) *AP*
    (c) *GP* (d) None of the above
90. What is 0.9 + 0.09 + 0.009 + ... equal to ? [2013-II]
    (a) 1 (b) 1.01
    (c) 1.001 (d) 1.1
91. The sum of the first five terms and the sum of the first ten terms of an *AP* are same. Which one of the following is the correct statement ? [2013-II]
    (a) The first term must be negative
    (b) The common difference must be negative
    (c) Either the first term or the common difference is negative but not both
    (d) Both the first term and the common difference are negative
92. What is the seventh term of the sequence 0, 3, 8, 15, 24,....? [2013-II]
    (a) 63 (b) 48
    (c) 35 (d) 33
93. The sum of an infinite *GP* is *x* and the common ratio *r* is such that $|r| < 1$. If the first term of the *GP* is 2, then which one of the following is correct ? [2014-I]
    (a) $-1 < x < 1$ (b) $-\infty < x < 1$
    (c) $1 < x < \infty$ (d) None of these

94. The sum of the series formed by the sequence 3, $\sqrt{3}$, 1...... upto infinity is : *[2014-I]*
(a) $\frac{3\sqrt{3}(\sqrt{3}\quad 1)}{2}$ (b) $\frac{3\sqrt{3}(\sqrt{3}-1)}{2}$
(c) $\frac{3(\sqrt{3}\quad 1)}{2}$ (d) $\frac{3(\sqrt{3}-1)}{2}$

DIRECTIONS (Qs. 95-96) : *For the next two (02) items that follow :*

Let S_n denote the sum of the n terms of an AP and $3S_n = S_{2n}$. *[2014-II]*

95. What is $S_{3n} = S_n$ equal to ?
(a) 4 : 1 (b) 6 : 1
(c) 8 : 1 (d) 10 : 1

96. What is $S_{3n} = S_{2n}$ equal to ?
(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 5 : 1

DIRECTIONS (Qs. 97-99) : *For the next three (03) items that follow :*

Let $f(x) = ax^2 + bx + c$ such that $f(1) = f(-1)$ and a, b, c are in Arithmetic Progression. *[2014-II]*

97. What is the value of b ?
(a) –1
(b) 0
(c) 1
(d) Cannot be determined deu to insufficient data

98. f'(a), f' (b), f' (c) are
(a) A.P.
(b) G.P.
(c) H.P.
(d) Arithmetico-geometric progression

99. f"(a), f" (b), f"(c) are
(a) in A.P. only (b) in G.P. only
(c) in both A.P. and G.P. (d) neither in A.P. nor in G.P.

100. What is the sum of the series 0.5 + 0.55 + 0.555 + ... to *n* terms? *[2015-I]*
(a) $\frac{5}{9}\left[n-\frac{2}{9}\left(1-\frac{1}{10^n}\right)\right]$ (b) $\frac{1}{9}\left[5-\frac{2}{9}\left(1-\frac{1}{10^n}\right)\right]$
(c) $\frac{1}{9}\left[n-\frac{5}{9}\left(1-\frac{1}{10^n}\right)\right]$ (d) $\frac{5}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

101. The value of the infinite product $6^{\frac{1}{2}}\times 6^{\frac{1}{2}}\times 6^{\frac{3}{8}}\times 6^{\frac{1}{4}}\times ...$ is *[2015-II]*
(a) 6 (b) 36
(c) 216 (d) ∞

102. The *n*th term of an AP. is $\frac{3\quad n}{4}$, then the sum of first 105 terms is *[2015-II]*
(a) 270 (b) 735
(c) 1409 (d) 1470

103. If p, q, r are in one geometric progression and a, b, c are in another geometric progression, then ap, bq, cr are in *[2015-II]*
(a) Arithmetic progression (b) Geometric progression
(c) Harmonic progression (d) None of the above

104. What is the sum of n terms of the series
$\sqrt{2}\quad \sqrt{8}\quad \sqrt{18}\quad \sqrt{32}$ .... ? *[2015-II]*
(a) $\frac{n(n-1)}{\sqrt{2}}$ (b) $\sqrt{2n}(n\quad 1)$
(c) $\frac{n(n\quad 1)}{\sqrt{2}}$ (d) $\frac{n(n-1)}{2}$

DIRECTIONS (Qs. 105-106) : *For the next two (2) items that follow*

Given that $a_n = \int_0^{\pi}\frac{\sin^2\{(n+1)x\}}{\sin 2x}dx$

105. Consider the following statements: *[2016-I]*
1. The sequence $\{a_{2n}\}$ is in AP with common difference zero.
2. The sequence $\{a_{2n+1}\}$ is in AP with common difference zero.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

106. What is $a_{n-1} - a_{n-4}$ equal to ?
(a) –1 (b) 0
(c) 1 (d) 2

DIRECTIONS (Qs. 107-108) : *For the next two (2) items that follow.*

Given that $\log_x y$, $\log_z x$, $\log_y z$ are in GP, xyz = 64 and x^3, y^3, z^3 are in A.P. *[2016-I]*

107. Which one of the following is correct ?
x, y and z are
(a) in AP only (b) in GP only
(c) in both AP and GP (d) neither in AP nor in GP

108. Which one of the following is correct?
xy, yz and zx are
(a) in AP only
(b) in GP only
(c) in both AP and GP
(d) neither in AP nor in GP

109. If m is the geometric mean of *[2016-I]*
$\left(\frac{y}{z}\right)^{\log(yz)}, \left(\frac{z}{x}\right)^{\log(zx)}$ and $\left(\frac{x}{y}\right)^{\log(xy)}$
then what is the value of m?
(a) 1 (b) 3
(c) 6 (d) 9

110. How many geometric progressions is/are possible containing 27, 8 and 12 as three of its/their terms? *[2016-II]*
(a) One (b) Two
(c) Four (d) Infinitely many

DIRECTIONS (Qs. 111-113) : *Consider the following for the next three (03) items that follow.*

Let a, x, y, z, b be in AP, where x + y + z = 15. Let a, p, q, r, b be in HP, where $p^{-1}+q^{-1}+r^{-1}=\frac{5}{3}$ *[2016-II]*

111. What is the value of ab?
(a) 10 (b) 9
(c) 8 (d) 6

112. What is the value of xyz?
(a) 120 (b) 105
(c) 90 (d) Cannot be determined

113. What is the value of pqr?
(a) 35/243 (b) 81/35
(c) 243/35 (d) Cannot be determined

DIRECTIONS (Qs. 114-115) : *Consider the following for the next two (02) items that follow*

The sixth term of an AP is 2 and its common difference is greater than 1. *[2016-II]*

114. What is the common difference of the AP so that the product of the first, fourth and fifth terms is greatest? *[2016-II]*
(a) 8/5 (b) 9/5
(c) 2 (d) 11/5

115. What is the first term of the AP so that the product of the first, fourth and fifth terms is greatest? *[2016-II]*
(a) – 4 (b) – 6
(c) – 8 (d) – 10

DIRECTIONS (Qs. 116-117) : *Consider the following for the next two (02) items that follow.*

The interior angles of a polygon of n sides are in AP. The smallest angle is 120° and the common difference is 5°. *[2016-II]*

116. How many possible values can n have?
(a) One (b) Two
(c) Three (d) Infinitely many

117. What is the largest interior angle of the polygon?
(a) 160° only (b) 195° only
(c) Either 160° or 195° (d) Nither 160 nor 195°

118. If $x^{\ell n\left(\frac{y}{z}\right)}.y^{\ell n(XZ)^2}.z^{\ell n\left(\frac{x}{y}\right)} = y^{4\ell n y}$ for any x >1, y >1 and z >1, then which one of the following is correct? *[2016-II]*
(a) ℓn y is the GM of ℓn x, ℓn x, ℓn x and ℓn z
(b) ℓn y is the AM of ℓn x, ℓn x, ℓn x and ℓn z
(c) ℓn y is the HM of ℓn x, ℓn x and ℓn z
(d) ℓn y is the AM of ℓn, In x, ℓn z and ℓn z

119. What is the sum of the series *[2017-I]*
0.3 + 0.33 + 0.333 + ... *n* terms?
(a) $\frac{1}{3}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$ (b) $\frac{1}{3}\left[n-\frac{2}{9}\left(1-\frac{1}{10^n}\right)\right]$
(c) $\frac{1}{3}\left[n-\frac{1}{3}\left(1-\frac{1}{10^n}\right)\right]$ (d) $\frac{1}{3}\left[n-\frac{1}{9}\left(1+\frac{1}{10^n}\right)\right]$

120. If the sum of m terms of an AP is n and the sum of n terms is m, then the sum of (m + n) terms is *[2017-I]*
(a) *mn* (b) *m* + *n*
(c) 2(*m* + *n*) (d) – (*m* + *n*)

121. The sum of the roots of the equation $x^2 + bx + c = 0$ (where b and c are non-zero) is equal to the sum of the reciprocals of their squares. Then $\frac{1}{c}, b, \frac{c}{b}$ are in *[2017-I]*
(a) AP (b) GP
(c) HP (d) None of the above

122. The sum of the roots of the equation $ax^2 + x + c = 0$ (where a and c are non-zero) is equal to the sum of the reciprocals of their squares. Then a, ca^2, c^2 are in *[2017-I]*
(a) AP (b) GP
(c) HP (d) None of the above

123. The fifth term of an AP of n terms, whose sum is $n^2 – 2n$, is *[2017-I]*
(a) 5 (b) 7
(c) 8 (d) 15

124. The sum of all the two-digit odd numbers is *[2017-I]*
(a) 2475 (b) 2530
(c) 4905 (d) 5049

125. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....$ is equal to *[2017-I]*
(a) $2^n – n – 1$ (b) $1 – 2^{–n}$
(c) $2^{–n} + n – 1$ (d) $2^n – 1$

126. Let x, y, z be positive real numbers such that x, y, z are in GP and $\tan^{–1}$ x, $\tan^{–1}$ y and $\tan^{–1}$ z are in AP. Then which one of the following is correct? *[2017-I]*
(a) x = y = z (b) xz = 1
(c) x ≠ y and y = z (d) x = y and y ≠ z

127. If $S_n = nP + \frac{n(n-1)Q}{2}$, where S_n denotes the sum of the first n terms of an AP, then the common difference is *[2017-II]*
(a) P + Q (b) 2P + 3Q
(c) 2Q (d) Q

128. The value of the product $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times ....$ up to infinite terms is *[2017-II]*
(a) 6 (b) 36
(c) 216 (d) 512

129. A person is to count 4500 notes. Let a_n denote the number of notes he counts in the nth minute. If $a_1 = a_2 = a_3 = ... = a_{10} = 150$, and a_{10}, a_{11}, a_{12}, ... are in AP with the common difference –2, then the time taken by him to count all the notes is *[2017-II]*
(a) 24 minutes (b) 34 minutes
(c) 125 minutes (d) 135 minutes

130. If $y = x + x^2 + x^3 + ...$ up to infinite terms where x < 1, then which one of the following is correct? *[2017-II]*
(a) $x = \frac{y}{1+y}$ (b) $x = \frac{y}{1-y}$
(c) $x = \frac{1+y}{y}$ (d) $x = \frac{1-y}{y}$

131. The value of $\frac{1}{\log_3 e} \quad \frac{1}{\log_3 e^2} \quad \frac{1}{\log_3 e^4}$ .... up to infinite terms is *[2017-II]*
(a) $\log_e 9$ (b) 0
(c) 1 (d) $\log_e 3$

132. If x_1 and x_2 are positive quantities, then the condition for the difference between the arithmetic mean and the geometric mean to be greater than 1 is *[2017-II]*
(a) $x_1 + x_2 > 2\sqrt{x_1x_2}$
(b) $\sqrt{x_1} + \sqrt{x_2} > \sqrt{2}$
(c) $|\sqrt{x_1} + \sqrt{x_2}| > \sqrt{2}$
(d) $x_1 + x_2 < 2\left(\sqrt{x_1x_2} + 1\right)$

133. If the ratio of AM to GM of two positive numbers a and b is 5 : 3, then a : b is equal to *[2018-I]*
(a) 3 : 5 (b) 2 : 9
(c) 9 : 1 (d) 5 : 3

134. If $x = 1 - y + y^2 - y^3 + ...$ up to infinite terms, where $|y| < 1$, then which one of the following is correct? *[2018-I]*
(a) $x \quad \frac{1}{1 \quad y}$ (b) $x \quad \frac{y}{1-y}$
(c) $x \quad \frac{y}{1 \quad y}$ (d) $x \quad \frac{y}{1-y}$

135. What is the sum of all two-digit numbers which when divided by 3 leave 2 as the remainder? *[2018-I]*
(a) 1565 (b) 1585
(c) 1635 (d) 1655

136. The third term of a GP is 3. What is the product of the first five terms? *[2018-I]*
(a) 216
(b) 226
(c) 243
(d) Cannot be determined due to insufficient data

137. If $x, \frac{3}{2}, z$ are in AP; x, 3, z are in GP; then which one of the following will be in HP? *[2018-I]*
(a) x, 6, z (b) x, 4, z
(c) x, 2, z (d) x, 1, z

138. If an infinite GP has the first term x and the sum 5, then which of the following is correct? *[2018-II]*
(a) $x < -10$ (b) $-10 < x < 0$
(c) $0 < x < 10$ (d) $x > 10$

139. The sum of the series $3 - 1 + \frac{1}{3} - \frac{1}{9}$ .... is equal to *[2018-II]*
(a) $\frac{20}{9}$ (b) $\frac{9}{20}$
(c) $\frac{9}{4}$ (d) $\frac{4}{9}$

140. Let T_r be the r^{th} term of an AP for r = 1, 2, 3,.... If for some distinct positive integers m and n we have $T_m = 1/n$ and $T_n = 1/m$, then what is T_{mn} equal to? *[2018-II]*
(a) $(mn)^{-1}$ (b) $m^{-1} + nn^{-1}$
(c) 1 (d) 0

141. If the second term of a GP is 2 and the sum of its infinite term is 8, then the GP is *[2018-II]*
(a) $8, 2, \frac{1}{2}, \frac{1}{8}, ...$ (b) $10, 2, \frac{2}{5}, \frac{2}{25}, ...$
(c) $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}, ...$ (d) $6, 3, \frac{3}{2}, \frac{3}{4}, ...$

142. If a, b, c are in AP or GP HP, then $\frac{a-b}{b-c}$ is equal to *[2018-II]*
(a) $\frac{b}{a}$ or 1 or $\frac{b}{c}$ (b) $\frac{c}{a}$ or $\frac{c}{b}$ or 1
(c) 1 or $\frac{a}{b}$ or $\frac{a}{c}$ (d) 1 or $\frac{a}{b}$ or $\frac{c}{a}$

143. If $\sin \beta$ is the harmonic mean of $\sin \alpha$ and $\cos \alpha$, and $\sin \theta$ is the arithmetic mean of $\sin \alpha$ and $\cos \alpha$, then which of the following is/are correct? *[2018-II]*
1. $\sqrt{2} \sin\left(\alpha + \frac{\pi}{4}\right) \sin \beta = \sin 2\alpha$
2. $\sqrt{2} \sin \theta = \cos\left(\alpha - \frac{\pi}{4}\right)$

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

144. If $x_i > 0, y_i > 0$ (i = 1, 2, 3, ... n) are the values of two variable X and Y with geometric mean P and Q respectively, then the geometric mean of $\frac{X}{Y}$ is *[2018-II]*
(a) $\frac{P}{Q}$ (b) antilog $\left(\frac{P}{Q}\right)$
(c) $n(\log P - \log Q)$ (d) $n(\log P + \log Q)$

145. What is the n^{th} term of the sequence 25, –125, 625, –3125, ...? *[2019-I]*
(a) $(-5)^{2n-1}$ (b) $(-1)^{2n} 5^{n+1}$
(c) $(-1)^{2n-1} 5^{n+1}$ (d) $(-1)^{n-1} 5^{n+1}$

146. The numbers 1, 5 and 25 can be three terms (not necessarily consecutive) of *[2019-I]*
(a) only one AP
(b) more than one but ûnite numbers of APs
(c) inûnite number of APs
(d) ûnite number of GPs

147. The sum of $(p + q)^{th}$ and $(p - q)^{th}$ terms of an AP is equal to *[2019-I]*
(a) $(2p)^{th}$ term (b) $(2q)^{th}$ term
(c) Twice the p^{th} term (d) Twice the q^{th} term

148. What is the Fourth term of an AP of n terms whose sum is n (n + 1) ? *[2019-I]*
(a) 6 (b) 8
(c) 12 (d) 20

ANSWER KEY

1	(a)	16	(d)	31	(b)	46	(a)	61	(b)	76	(c)	91	(c)	106	(b)	121	(c)	136	(c)
2	(c)	17	(c)	32	(d)	47	(a)	62	(b)	77	(c)	92	(b)	107	(c)	122	(a)	137	(a)
3	(a)	18	(c)	33	(a)	48	(b)	63	(c)	78	(d)	93	(c)	108	(c)	123	(b)	138	(c)
4	(c)	19	(c)	34	(b)	49	(d)	64	(b)	79	(d)	94	(a)	109	(a)	124	(a)	139	(c)
5	(c)	20	(a)	35	(b)	50	(c)	65	(d)	80	(b)	95	(b)	110	(d)	125	(c)	140	(c)
6	(a)	21	(a)	36	(c)	51	(b)	66	(b)	81	(c)	96	(a)	111	(b)	126	(a)	141	(c)
7	(c)	22	(c)	37	(b)	52	(c)	67	(a)	82	(b)	97	(b)	112	(b)	127	(d)	142	(c)
8	(b)	23	(c)	38	(a)	53	(c)	68	(c)	83	(b)	98	(a)	113	(c)	128	(a)	143	(c)
9	(a)	24	(b)	39	(a)	54	(d)	69	(c)	84	(a)	99	(c)	114	(a)	129	(b)	144	(b)
10	(a)	25	(c)	40	(b)	55	(c)	70	(b)	85	(a)	100	(d)	115	(b)	130	(a)	145	(d)
11	(b)	26	(b)	41	(c)	56	(a)	71	(b)	86	(a)	101	(b)	116	(a)	131	(a)	146	(c)
12	(d)	27	(b)	42	(c)	57	(d)	72	(c)	87	(a)	102	(d)	117	(a)	132	(c)	147	(c)
13	(a)	28	(b)	43	(a)	58	(a)	73	(c)	88	(a)	103	(b)	118	(b)	133	(c)	148	(b)
14	(d)	29	(d)	44	(d)	59	(c)	74	(c)	89	(a)	104	(c)	119	(a)	134	(a)		
15	(a)	30	(a)	45	(b)	60	(d)	75	(d)	90	(a)	105	(c)	120	(d)	135	(c)		

HINTS & SOLUTIONS

1. (a) Since first term = p and common difference = q.

Sum of first 10 terms = $\frac{10}{2}[2p+(10-1)q]$ and

Sum of first 5 terms = $\frac{5}{2}[2p+(5-1)q]$

According to question,

$$\frac{10}{2}[2p+9q] = 4\times\frac{5}{2}[2p+4q]$$

$\Rightarrow$ $2p+9q = 4p+8q$

$\Rightarrow$ $2p = q$

$\Rightarrow$ $p : q = 1 : 2$

2. (c) If one root is $\frac{1}{2-3i}$ i.e, $\frac{2+3i}{4+9} = \frac{2}{13}+\frac{3}{13}i$, then another root will be $\frac{2}{13}-\frac{3i}{13}$ i.e, $\frac{1}{2+3i}$. [Since complex roots are conjugate]

So, statement(1) is not correct.

Since, quadratic equation has two roots thus this equation has only imaginary roots. Statement (2) is correct.

$\therefore$ The equation is

x^2 – (Sum of roots) x + (Product of roots) = 0

Sum of roots = $\frac{1}{2-3i}+\frac{1}{2+3i} = \frac{4}{13}$

Product of roots = $\frac{1}{2-3i}\times\frac{1}{2+3i} = \frac{1}{13}$

$\Rightarrow$ $x^2 - \frac{4}{13}x + \frac{1}{13} = 0 \Rightarrow 13x^2 - 4x + 1 = 0$

$\Rightarrow$ So, statement 3 is correct.

Thus, (2) and (3) statements are correct.

3. (a) The given series is $(1\times 3)+(3\times 5)+(5\times 7)+$............

Its general term is given by

$T_n = (2n-1)(2n+1) = 4n^2 - 1$

Sum of series = $4\Sigma n^2 - \Sigma 1$

$$S_n = \frac{4n(n\quad 1)(2n\quad 1)}{6} - n$$

$$S_n = n\left[\frac{2(2n^2+3n+1)}{3} - 1\right]$$

$$S_n = n\left[\frac{4n^2+6n+2-3}{3}\right]$$

$$S_n = \left[\frac{n(4n^2+6n-1)}{3}\right]$$

For sum of first 50 terms of the series, n = 50,

$$S_{50} = \frac{50[4(50)^2+6(50)-1]}{3}$$

$$= \frac{50\times(10000+300-1)}{3} = \frac{50\times 10299}{3} = 171650$$

4. (c) $x = 1+\frac{y}{2}+\left(\frac{y}{2}\right)^2+\left(\frac{y}{2}\right)^3+.....$

Here, $\frac{y}{2}<1$ and this a G.P. with first term = 1 and common ratio = $\frac{y}{2}$ so,

$\Rightarrow$ $x = \frac{1}{1-\frac{y}{2}}$ $\Rightarrow$ $x = \frac{2}{2-y}$

$\Rightarrow$ $2x - xy = 2$ $\Rightarrow$ $y = \frac{2x-2}{x}$

5. (c) The GP is a, ar, ar^2,........ ar^{2n}
So, P = a . ar . ar^2 . ar^3........... ar^{2n}
$= a^{2n+1} . r^{1+2+......+2n}$

$= a^{(2n+1)} r^{\frac{2n(2n+1)}{2}} = a^{2n+1} r^{n(2n+1)} = (ar^n)^{(2n+1)}$

= (2n + 1)th power of the (n + 1)th term of G.P.

6. (a) If x, y, z are in GP then x . z = y^2

Here $x = \frac{1}{8}$, z = $\log_a a^2$, y = $(\log_{a^2} a)$

$\frac{1}{8} \times \log_a a^2 = \frac{1}{8}.2 = \frac{1}{4}$

and $(\log_{a^2} a)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Hence, $\frac{1}{8}, \log_{a^2} a, \log_a a^2$ are in GP.

Thus, both A and R are individually true and R is correct explanation of A.

7. (c) If (x + 1), (4x + 1) and (8x + 1) are in GP.
then $(4x + 1)^2 = (x + 1)(8x + 1)$
$\Rightarrow 16x^2 + 8x + 1 = 8x^2 + x + 8x + 1$
$\Rightarrow 8x^2 - x = 0 \Rightarrow x(8x - 1) = 0$

$\Rightarrow x = 0, \frac{1}{8}$, [$\frac{1}{8}$ is non–trivial value]

8. (b) The given equation
$(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$
has equal roots, so, discriminant = 0
Hence, $\{2b(a + c)\}^2 - 4(a^2 + b^2)(b^2 + c^2) = 0$
$\Rightarrow 4b^2(a^2 + c^2 + 2ca) - 4(a^2b^2 + a^2c^2 + b^4 + b^2c^2) = 0$
$\Rightarrow b^2a^2 + b^2c^2 + 2b^2ca - a^2b^2 - a^2c^2 - b^4 - b^2c^2 = 0$
$\Rightarrow 2b^2ca = b^4 + a^2c^2$
$\Rightarrow b^4 - 2b^2ca + a^2c^2 = 0$
$\Rightarrow (b^2)^2 - 2(b^2)(ac) + (ac)^2 = 0$
$\Rightarrow (b^2 - ac)^2 = 0$
$\Rightarrow b^2 = ac$
$\Rightarrow$ a, b, c are in GP.

9. (a) Let first term and common difference of an AP are a and d respectively.
Its P^{th} term = a + (p – 1) d = q ...(i)
and q^{th} term = a + (q – 1) d = p ...(ii)
Solving Eqs. (i) and (ii), we find
a = p + q –1, d = –1

10. (a) Given that a, b, c, are in GP.
Let r be common ratio of GP.
So, a = a, b = ar and c = ar^2 ...(i)
Also, given that a, 2b, 3c are in AP.

$\Rightarrow 2b = \frac{a + 3c}{2}$

$\Rightarrow 4b = a + 3c$
From Eq. (1)
$4ar = a + 3ar^2$
$\Rightarrow 3ar^2 - 4ar + a = 0$
$\Rightarrow 3r^2 - 4r + 1 = 0$
$\Rightarrow 3r^2 - 3r - r + 1 = 0$
$\Rightarrow 3r(r - 1) - 1(r - 1) = 0$
$\Rightarrow (r - 1)(3r - 1) = 0$

$\Rightarrow r = 1$ or $r = \frac{1}{3}$, $r = \frac{1}{3}$ is in the choice.

11. (b) Since, first term and common difference of an AP are u and v respectively.
p^{th} term, Tp = u + (p –1) v ...(i)
and q^{th} term, T_q = u + (q – 1) v ...(ii)
According to condition given in question,
$\Rightarrow T_p = T_q + 15uv$
$\Rightarrow T_p - T_q = 15uv$
$\Rightarrow u + (p - 1)v - u - (q - 1)v = 15uv$
$\Rightarrow v(p - 1 - q + 1) = 15uv$
$\Rightarrow v(p - q) = 15uv$
$\Rightarrow p - q = 15u \Rightarrow p = q + 15u$

12. (d) Since, a, b, c, are in AP

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in HP.

Since, AM $\geq$ HM

$\Rightarrow b \geq \frac{2ac}{a + c}$

[since AM= b and HM= $\frac{2ac}{a + c}$]

$\Rightarrow ab + bc \geq 2ac$

13. (a) Given that $1 + n\left[\frac{x}{1 - x}\right] + \frac{n(n + 1)}{2!}\left[\frac{x}{1 - x}\right]^2 + ...\infty$ is

expansion of $\left[1 - \frac{x}{1 - x}\right]^{-n}$.

So, it is $= \left[1 - \frac{x}{1 - x}\right]^{-n} = \left[\frac{1 - x - x}{1 - x}\right]^{-n} = \left[\frac{1 - x}{1 - 2x}\right]^n$

14. (d) Let the first term and common difference of an AP be a and d respectively.
Then, as given

$(p + 1)(2p + 1) = \left(\frac{2p + 1}{2}\right)\{2a + (2p + 1 - 1)d\}$

$\Rightarrow (p + 1) = \frac{1}{2}\{2a + 2pd\}$

$\Rightarrow (p + 1) = a + pd$
$\Rightarrow p + 1 = a + [(p + 1) - 1]d = t_{p+1}$
Hence, the inference is : the $(p + 1)^{th}$ term of the AP is (p + 1).

15. (a) If a, b, c are in G.P. then,
$b^2 = ac \Rightarrow b = (ac)^{1/2}$ ....(1)
Taking $\log_n$ on both the sides of eq. (1).

$$\log_n b = \frac{1}{2}\ (\log_n (ac)) \ = \frac{\log_n a \quad \log_n c}{2}$$

or, $\frac{\log_n a + \log_n c}{2} = \log_n b$

So, $\log_n a$, $\log_n b$ and $\log_n c$ are in AP.

Hence, $\frac{1}{\log_n a}, \frac{1}{\log_n b}, \frac{1}{\log_n c}$ are in H.P.

$$\log_a n = \frac{1}{\log_n a}$$

$$\log_b n = \frac{1}{\log_n b}$$

$$\log_c n = \frac{1}{\log_n c}$$

i.e. $\log_a n$, $\log_b n$, and $\log_c n$ are in HP.

16. (d) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ can be written as

$$1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \ldots$$

[$\because$ This is a GP with first term = 1 and common ratio $= -\frac{1}{2}$]

So, sum of the series

$$= \frac{1}{1 - \left(-\frac{1}{2}\right)} = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3}$$

17. (c) b_1, b_2, b_3 are in AP with common difference d,
so $b_2 = b_1 + d$ and $b_3 = b_1 + 2d$

As given, $b_3^2 = b_2 b_3 + b_1 d + 2$

$\Rightarrow \ (b_1 + 2d)^2 = (b_1 + d)\,(b_1 + 2d) + b_1 d + 2$

$\Rightarrow \ b_1^2 + 4d^2 + 4b_1 d = b_1^2 + 2b_1 d + b_1 d + 2d^2 + b_1 d + 2$

$\Rightarrow \ 2d^2 = 2$

$\Rightarrow \ d^2 = 1$

$\Rightarrow \ d = \pm 1$

i.e. d = 1 or – 1

Since, d > 0, –1 is discarded and d = 1

18. (c) As given 1, x, y, z 16 are in geometric progression.
Let common ratio be r,

$x = 1.\, r = r$

$y = 1 .\, r^2 = r^2$

$z = 1.\, r^3 = r^3$

and $16 = 1 .r^4 \Rightarrow 16 = r^4$

$\Rightarrow \ r = 2$

$\therefore \ x = 1.\, r = 2, \ y = 1.\, r^2 = 4,$

$z = 1 .\, r^3 = 8$

$\therefore \ x + y + z = 2 + 4 + 8 = 14$

19. (c) As given, n_{th} term

$T_n = 3n + 7$

Sum of n term, $S_n \quad \sum T_n$

$= \sum (3n + 7) = 3\sum n + 7\sum 1$

$$\frac{3n(n \quad 1)}{2} \quad 7n \quad n\left[\frac{3n \quad 3 \quad 14}{2}\right] \quad n\left[\frac{3n \quad 17}{2}\right]$$

Sum of 50 terms $S_{50} \quad 50\left[\frac{3 \times 50 \quad 17}{2}\right]$

$$50\left[\frac{167}{2}\right] \quad 25 \times 167 = 4175$$

20. (a) As given :
x +y, 2y and y +z are in harmonic progression.

$$2y = \frac{(x+y)(y+z)}{x+y+y+z}$$

$\Rightarrow \ \frac{1}{2y} \quad \frac{1}{x \quad y} \quad \frac{1}{y \quad z}$

$\Rightarrow \ y\,(x + 2y + z) = (xy + xz + y^2 + yz)$

$\Rightarrow \ xy + 2y^2 + yz = xy + xz + y^2 + yz$

$\Rightarrow \ y^2 = xz$

$\Rightarrow$ x, y, z are in geometric progession.

21. (a) As given the series is

$$S = 1 + \frac{1}{8} + \frac{1.3}{8.16} + \frac{1.3.5}{8.16.24} + \ldots.\infty$$

On comparing this series with

$S \quad (1 \quad x)^n \quad 1 \quad nx \quad \frac{n(n-1)}{2!}x^2 \quad \ldots\infty,$ we get

$nx \quad \frac{1}{8}$ ....(1)

and $\frac{n(n-1)}{2!}x^2 \quad \frac{1.3}{8.16}$ ....(2)

From Eqs. (1) and (2), we get

$$\frac{\frac{n(n-1)}{2!}x^2}{n^2x^2} \quad \frac{\frac{1.3}{8.16}}{\frac{1}{8}.\frac{1}{8}}$$

$\Rightarrow \ \frac{n-1}{2n} \quad \frac{3}{2}$

$\Rightarrow \ n - 1 = 3n$

$\Rightarrow \ n = -\frac{1}{2}$

On putting this value in Eq. (i)

$\Rightarrow \ \left(-\frac{1}{2}\right)x \quad \frac{1}{8}$

$\Rightarrow \ x = -\frac{1}{4}.$

But $S \quad (1 \quad x)^n \quad \left(1 - \frac{1}{4}\right)^{-1/2}$

$$\left(\frac{3}{4}\right)^{-1/2} \quad \frac{2}{\sqrt{3}}.$$

22. (c) Let series be a, G_1, b and a', G_2, b' so, $G_1 = \sqrt{ab}$ and $G_2 = \sqrt{a'b'}$

Series formed by ratio of the corresponding terms are :

$\frac{a}{a'}, \frac{G_1}{G_2}, \frac{b}{b'}$.

Geometric means of this series $= \sqrt{\frac{a}{a'}.\frac{b}{b'}}$

$= \sqrt{\frac{ab}{a'b'}} = \frac{\sqrt{ab}}{\sqrt{a'b'}} = \frac{G_1}{G_2}$

So, geometric mean of the ratio of corresponding term of two series where G_1 and G_2 are geometric means of two series is $\frac{G_1}{G_2}$.

23. (c) As given :

Points A (a,ma), B [b, (m +1)b]and C[c,(m + 2) c] are collinear.

$\Rightarrow$ $a\{(m+1)b-(m+2)c\}+b\{(m+2)c-ma\}$
$+c\{ma-(m+1)b\}=0$

$\Rightarrow$ $mab+ab-mac-2ac+mbc+2bc-mab+mac$
$-mbc-bc=0$

$\Rightarrow$ $ab-2ac+2bc-bc=0$

$\Rightarrow$ $ab+bc=2ac$

Dividing both the sides by abc, we get

$\frac{1}{c}+\frac{1}{a}=\frac{2}{b}$

$\Rightarrow$ $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P

$\Rightarrow$ a, b, c, are in Harmonic progression for all m.

24. (b) (A) : $0.3+0.03+0.003+... = \frac{1}{3}$

Let $S = 0.3+0.03+0.003+....$

This is geometric series with

first term, a = 0.3 and common ration, $r=\frac{1}{10}$.

So, $S=\frac{a}{1-r}=\frac{0.3}{1-\frac{1}{10}}=\frac{0.3\times10}{9}$

$=\frac{3}{9}=\frac{1}{3}$. So, (A) is true.

(R) : As given : $a_n = a+nd$

$\Rightarrow$ $a_1+a_2+...+a_n$
$=a+d+a+2d+...+a+nd$
$=na+(1+2+...+n)d$

$=na+\frac{n(n+1)d}{2} = \frac{n}{2}[2a+(n+1)d]$

So, (R) is also true

Hence, both (A) and (R) are true. but R is not the correct explanation of (A)

25. (c) As given : a, b, c, d are in AP

$\Rightarrow$ $\frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{1}{d}$ are in HP

Multiplying by abcd throughout

$\Rightarrow$ $\frac{abcd}{a},\frac{abcd}{b},\frac{abcd}{c},\frac{abcd}{d}$, are in HP

$\Rightarrow$ bcd, acd, abd, abc are in HP.

26. (b) The given expression $9^{1/3}, 9^{1/9}\, 9^{1/27} ... \infty$

Can be written as :

$9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...\infty} = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{\frac{1}{3}/\frac{2}{3}} = 9^{1/2} = 3$

27. (b) Since, a > d and a, b, c, d are in HP.

$\Rightarrow$ a > b > c > d.

$b=\frac{2ac}{a+c} \Rightarrow b^2d=\frac{2abcd}{a+c}$

and $c=\frac{2bd}{b+d} \Rightarrow c^2a=\frac{2abcd}{b+d}$

$\frac{c^2a}{b^2d}=\frac{a+c}{b+d}$

$\Rightarrow$ $\frac{a+c}{b+d}=\left(\frac{a}{b}\right).\left(\frac{c}{d}\right).\left(\frac{c}{b}\right)>1$

$\Rightarrow$ $a+c>b+d$

28. (b) Let first instalement be Rs. x and difference of consecutive instalments be Rs. d.

$\Rightarrow \frac{30}{2}[2x+29d]=\frac{3600\times2}{3}$

($\because$ $1/3^{rd}$ amount is unpaid, $2/3^{rd}$ amount is paid)

$\Rightarrow 2x+29d=\frac{2400}{15}$

$\Rightarrow 2x+29d=160$ ... (1)

Since total amount was 3600 and it was to be paid in 40 installment.

$\Rightarrow \frac{40}{2}[2x+39d]=3600$

$\Rightarrow 2x+39d=180$ ...(2)

On solving eqs. (1) and (2), we get

x = 51 and d = 2

First instalment = Rs. 51

29. (d) Let a be the first term and r, the common ratio

First nine terms of a GP are a, ar, ar^2, ... ar^8.

$\therefore$ $P = a.ar.\,ar^2 .... ar^8 = a^9.r^{1+2+...+8}$

$=a^9.r^{\frac{8.9}{2}}=a^9r^{36}=(ar^4)^9=(T_5)^9$

= 9th power of the 5th term

30. (a) If the sequence is in AP with first term, a and common difference, d.
$\Rightarrow T_n = a + (n-1)\,d$
Also $T_{n-1} = a + (n-2)\,d$
So, the sequence is in AP for which difference between the nth term and (n – 1)th term is independent of n.

31. (b) As given : $\frac{1}{b-c} + \frac{1}{b-a} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{1}{b-c} - \frac{1}{a} + \frac{1}{b-a} - \frac{1}{c} = 0$

$\Rightarrow \frac{a-b+c}{a(b-c)} + \frac{c-b+a}{c(b-a)} = 0$

$\Rightarrow (a-b+c)\left\{\frac{1}{a(b-c)} + \frac{1}{c(b-a)}\right\} = 0$

$\Rightarrow \frac{cb-ac+ab-ac}{ac(b-c)(b-a)} = 0$

$\Rightarrow cb \quad ab \quad 2ac$
Dividing both sides by abc

$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b} \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP. $\Rightarrow$ a, b, c are in HP.

32. (d) Let

$$S = 3 + 7 + 13 + 21 + 31 + \ldots\ldots\ldots + a_n$$
$$-S = \underline{\quad}\ 3 \pm 7 \pm 13 \pm 21 \pm 31 \pm \ldots \pm a_{n-1} \pm a_n$$
$$0 = 3 + 4 + 6 + 8 + 10 + 12 + \ldots - a_n$$

$\Rightarrow a_n = 3 + [4 + 6 + 8 + 10 + 12 + \ldots (n-1)]$

$= 3 + \frac{(n-1)}{2}[8 + \{(n-1)-1\} \times 2]$

$= 3 + \frac{(n-1)}{2}[8 + 2n - 4]$

$= 3 + \frac{(n-1)}{2}(2n+4)$

$= 3 + (n-1)(n+2)$

$\therefore$ 15th term $= a_{15} = 3 + (15-1)(15+2)$
$= 3 + 14 \times 17 = 241$

33. (a) Given $a_n = 2n - 1$

$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n}(2n-1)$

$= 2\sum_{k=1}^{n} n - n = 2.\frac{n(n\ \ 1)}{2} - n \quad n^2 \quad n - n \quad n^2$

34. (b) Harmonic mean of three number x1, x2, x3 is

$\frac{3}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$

$\therefore$ H. M $= \frac{3}{\frac{1}{3} + \left(\frac{-1}{6}\right) + \left(\frac{-1}{6}\right)} = \frac{3}{\frac{1}{3} - \frac{1}{3}} = \frac{3}{0} = \infty$

35. (b) Geometric mean = $\sqrt[n\ \ 1]{1.2.4.8\ldots 2^n}$

($\because$ there are $(n+1)$ multiples from 2° to 2^n)

$= \sqrt[n+1]{2^0.2^1.2^2.2^3\ldots 2^n}$

$= \sqrt[n+1]{2^{1+2+3+\ldots+n}} = \sqrt[n\ \ 1]{2^{\frac{n(n\ \ 1)}{2}}}$

$= \left[2^{\frac{n(n\ \ 1)}{2}}\right]^{\frac{1}{n\ \ 1}} \quad 2^{\frac{n}{2}}$

36. (c) Let the AP is
a, a + d, a + 2d, ..., a + (2n – 1) d, a + 2nd
Series of even terms.
a + d, a + 3d, ..., a + (2n –1)d, has n terms

Sum of even number $= \frac{n}{2}[(a+d) + \{a + (2n-1)d\}]$

$= \frac{n}{2}[2a + 2nd] = n[a + nd]$

Series of odd terms
a,a + 2d, a + 4d, ..., a + 2 nd, has (n + 1) terms.

Sum of odd numbers $= \frac{n+1}{2}[a + (a + 2\,nd)]$

$= \frac{n+1}{2}(2a + 2\,nd)$

$= (n+1)(a+nd)$

So, the required ratio $= \frac{n+1}{n}$

37. (b) Given,
$S_n = n^2 - 2n$
$\therefore\ a_n = S_n - S_{n-1}$
$= n^2 - 2n - [(n-1)^2 - 2(n-1)]$
$= n^2 - 2n - [n^2 + 1 - 2n - 2n + 2] = 2n - 3$

38. (a) Since, a, 2a + 2 and 3a + 3 are in GP
$\therefore\ (2a+2)^2 = a\,(3a+3)$
$\Rightarrow 4a^2 + 4 + 8a = 3a^2 + 3a \Rightarrow a^2 + 5a + 4 = 0$
$\Rightarrow a(a+4) + 1(a+4) = 0 \Rightarrow (a+4)(a+1) = 0$
$\Rightarrow a + 4 = 0$ or $a + 1 = 0$
$\Rightarrow a = -4$ or -1
Let the fourth term be x.

$\therefore\ \frac{a}{2a+2} = \frac{3a+3}{x}$

$\Rightarrow x = \frac{(3a+3)(2a+2)}{a}$

When a = – 4,
x = – 13.5
and a = – 1, x = 0
So, the fourth term is –13.5

39. (a) Let S = 9 + 99 + 999 + ...
$= (10^1 - 1) + (10^2 - 1) + (10^3 - 1) + ...$
$= (10 + 10^2 + 10^3 + ...) - (1 + 1 + 1 + ... \text{ 100 times})$

$= \frac{10(10^{100} - 1)}{10 - 1} - 100$ ($\because 10, 10^2, 10^3$ ....: is G.P. with $a = 10, r = 10, S_{100} = \frac{a(r^{100} - 1)}{r - 1}$)

$= \frac{10}{9}(10^{100} - 1) - 100$

40. (b) We know, $HM = \frac{(GM)^2}{AM}$

$\therefore HM = \frac{16}{5}$

41. (c) Harmonic mean = 21.6 and a = 27
We know that,

Harmonic mean $= \frac{2ab}{a \quad b} \Rightarrow 21.6 \quad \frac{2 \times 27 \times b}{27 \quad b}$

$\Rightarrow 583.2 = 54b - 21.6b$

$\Rightarrow b \quad \frac{583.2}{32.4} \quad 18$

42. (c) Let the Geometric progression be a, ar, ar^2, ar^3, … with common ratio r and first term 'a'.
According to the question, we have
$a + ar = 8 \Rightarrow a(1 + r) = 8$ ...(i)
and $a + ar + ar^2 + ar^3 = 80$
$\Rightarrow a(1 + r) + ar^2(1 + r) = 80$
$\Rightarrow a(1 + r)(1 + r^2) = 80$
$\Rightarrow 8(1 + r^2) = 80$ (from (i))

$\Rightarrow 1 \quad r^2 \quad \frac{80}{8} \quad 10$

$\Rightarrow r^2 = 10 - 1 = 9$
$\Rightarrow r = 3$ ($\because r > 0$)
From eq. (i), $a(1 + 3) = 8$
$\Rightarrow a = 2$
Now, 6^{th} term $= ar^5 = 2(3)^5 = 2 \times 243 = 486$

43. (a) $\log_2 x, \log_3 x, \log_x 16$ are in G.P.

$\therefore \frac{\log_3 x}{\log_2 x} = \frac{\log_x 16}{\log_3 x}$

$\Rightarrow (\log_3 x)^2 = \log_2 x \cdot \log x\, 16$
$\Rightarrow 2 \times \log_3 x = \log 2\, x \cdot \log x\, 2^4$
$\Rightarrow 2 \times \log_3 x = 4 \times \log_2 x \cdot \log_x 2$
$\Rightarrow \log_3 x = 2(\log_2 x \times \log_x 2)$

$\Rightarrow \log_3 x = 2\left[\frac{\log_2 x}{\log_2 x}\right]$ $\left(\because \log_b a = \frac{1}{\log_a b}\right)$

$\Rightarrow \log_3 x = 2 \Rightarrow x = 3^2 = 9$

44. (d) Let the Geometric progression be a, ar, ar^2, ar^3, ar^4,…
First five terms of a geometric progression are a, ar, ar^2, ar^3, ar^4.

$\therefore$ Mean $\frac{a \quad ar \quad ar^2 \quad ar^3 \quad ar^4}{5}$

$\frac{a(r^5 - 1)}{5(r - 1)}$ $\left(\because \text{Sum of G.P} \quad \frac{a(r^n - 1)}{r - 1}\right)$

45. (b) Since n^{th} term of A.P = a + (n – 1)d
$\therefore p = 1 + (n - 1)2$
($\because$ First term = a = 1 and common difference = d = 2)

$\Rightarrow n \quad \frac{p \quad 1}{2}$

$\therefore (1 + 3 + 5 + ... + p) + (1 + 3 + 5 + ... + q)$
$= (1 + 3 + 5 + ... + r)$

$\Rightarrow \frac{\frac{p \quad 1}{2}}{2}\left[2 \times 1 \quad \left(\frac{p+1}{2} - 1\right)2\right]$

$+ \frac{\left(\frac{q \quad 1}{2}\right)}{2}\left[2 \times 1 + \left(\frac{q+1}{2} - 1\right)2\right]$

$= \frac{r+1}{4}\left[2 \times 1 + \left(\frac{r+1}{2} - 1\right)2\right]$

$\Rightarrow \frac{p \quad 1}{4} \quad 2 + (p - 1) \quad \frac{q \quad 1}{4} \quad 2 \quad (q - 1)$

$\frac{r \quad 1}{4}[2 \quad r - 1]$

$\Rightarrow (p \quad 1)^2 \quad (q \quad 1)^2 \quad (r \quad 1)^2$

This is the possible only when p = 7, q = 5, r = 9
$\therefore p + q + r = 7 + 5 + 9 = 21$

46. (a) Let x^2, y^2, z^2 are in A.P
$\Rightarrow y^2 - x^2 = z^2 - y^2$
$2y^2 = x^2 + z^2$

(a) Suppose y + z, z + x and x + y are in A-P
$\therefore (z + x) - (y + z) = (x + y) - (z + x)$
$2(z + x) = (y + z) + (x + y)$
$\Rightarrow 2z + 2x = 2y + z + x \Rightarrow z + x = 2y$
$\Rightarrow$ x, y and z are in AP. Which is true

(b) Let y + z, z + x, x + y are in HP.

$\therefore z \quad x \quad \frac{2(y \quad z)(x \quad y)}{y \quad z \quad x \quad y}$

$\Rightarrow z \quad x \quad \frac{2(y \quad z)(x \quad y)}{2y \quad z \quad x}$

$\Rightarrow 2yz + z^2 + zx + 2xy + xz + x^2$
$= 2yx + 2y^2 + 2zx + 2yz$
$\Rightarrow z^2 + x^2 = 2y^2$
$\Rightarrow x^2, y^2$ and z^2 are in AP. Which is true.
Hence, y + z, z + x and x + y are in A.P.

47. (a) Since, x, 2x + 2, 3x + 3 are the terms of G..P

therefore $\frac{2x+2}{x}=\frac{3x+3}{2x+2}$

$\Rightarrow (2x+2)^2 = x(3x+3) \Rightarrow 4x^2+4+8x=3x^2+3x$

$\Rightarrow x^2+5x+4=0$

$\Rightarrow x^2+4x+x+4=0$

$\Rightarrow x(x+4)+1(x+4)=0$

$\Rightarrow x=-1,-4$

Now, first term a = x

Second term, $ar = 2(x+1) \Rightarrow r = \frac{2(x+1)}{x}$

$\therefore$ Fourth term $= ar^3 = x\left(\frac{2(x+1)}{x}\right)^3$

Put x = – 4, we get

Fourth term $= -4\left(\frac{2(-4+1)}{-4}\right)^3 = -4\times\left(\frac{3}{2}\right)^3 = -\frac{27}{2}$

48. (b) Given sequence is $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$

Which can be rewritten as $20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, ...$

This is an AP series.

Here, first term a = 20 and common difference $d = -\frac{3}{4}$

n^{th} term $= a + (n-1)d = 20+(n-1)\left(-\frac{3}{4}\right)$

$=\frac{83}{4}-\frac{3}{4}n$

For first negative term, n^{th} term < 0

$\Rightarrow \frac{83}{4}-\frac{3}{4}n<0 \Rightarrow 83<3n$

$\Rightarrow n>\frac{83}{3}=27.66$

So, n should be 28.

Hence, 28th term is first negative term.

49. (d) Let 'a' be the first term and 'd' be the common difference of an A.P

Now, Given m^{th} term $=\frac{1}{n}$

and n^{th} term $=\frac{1}{m}$

$\Rightarrow a+(m-1)d=\frac{1}{n}$ and

$a+(n-1)d=\frac{1}{m}$

By subtracting the above two eqns, we get

$(m-1-n+1)d=\frac{1}{n}-\frac{1}{m}$

$\Rightarrow (m-n)d=\frac{m-n}{mn}$

$\Rightarrow d=\frac{1}{mn}$

Now, $(mn)^{th}$ term = a + (mn – 1)d

$= a+(mn-1)\frac{1}{mn} = a+1-\frac{1}{mn}$

Now, $a=\frac{1}{m}-(n-1)d$

$=\frac{1}{m}-(n-1)\frac{1}{mn} = \frac{1}{m}-\frac{n}{mn}+\frac{1}{mn} = \frac{1}{mn}$

$\therefore (mn)^{th}$ term $=\frac{1}{mn}+1-\frac{1}{mn}=1$

50. (c) Let a and d be the first term and common difference of an AP respectively

$\therefore \quad a+58d=449$ and $a+448d=59$

On solving Eqs. (i) and (ii), we get

$a=507$ and $d=-1$

n^{th} term of AP is a + (n – 1) d.

Let us assume that n^{th} term will be zero.

$\therefore \quad a+(n-1)d=0$

$\Rightarrow \quad 0=507+(n-1)(-1)$

$\Rightarrow \quad 507=n-1 \quad \Rightarrow \quad n=508$

Hence, 508^{th} term will be zero.

51. (b) Given AM = 27 and HM = 12

and we know that

$(GM)^2 = (AM)(HM) = 27\times 12$

$\Rightarrow \quad GM=\sqrt{27\times12}=\sqrt{3\times3\times3\times3\times2\times2}$

$\Rightarrow \quad GM=3\times3\times2=18$

52. (c) The numbers between 200 and 400 which are divisible by 7, are

203, 210, 217, ..., 399

This is an A.P with first term = a = 203 and common difference = d = 7

Now, let number of terms be n.

Therefore from the n^{th} term of A.P = a + (n – 1) d we have

399 = 203 + (n – 1) 7

$\Rightarrow \quad \frac{196}{7}=(n-1)\Rightarrow n=29$

Required sum $=\frac{n}{2}[a+\ell]$ where ℓ = last term

Thus, required sum $=\frac{29}{2}[203+399]$

$=\frac{29\times602}{2}=8729$

53. (c) Let $\frac{1}{ab}, \frac{1}{ca}, \frac{1}{bc}$ are in AP.

$\Rightarrow \frac{1}{ca} - \frac{1}{ab} = \frac{1}{bc} - \frac{1}{ca}$

$\Rightarrow \frac{1}{a}\left(\frac{1}{c} - \frac{1}{b}\right) = \frac{1}{c}\left(\frac{1}{b} - \frac{1}{a}\right)$

$\Rightarrow \frac{b-c}{abc} = \frac{a-b}{abc}$

$\Rightarrow b - c = a - b \Rightarrow 2b = a + c$

$\Rightarrow$ a, b, c are in AP. Which is true

Now, $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

$\therefore \quad \frac{2}{\sqrt{c}+\sqrt{a}} = \frac{1}{\sqrt{b}+\sqrt{c}} + \frac{1}{\sqrt{a}+\sqrt{b}}$

$\Rightarrow \quad 2 \ \sqrt{b} \quad \sqrt{c} \quad \sqrt{a} \quad \sqrt{b}$

$\sqrt{c} \quad \sqrt{a} \quad \sqrt{a} \quad 2\sqrt{b} \quad \sqrt{c}$

$\Rightarrow \quad 2\left(\sqrt{ab} + b + \sqrt{ac} + \sqrt{bc}\right) = \sqrt{ac} + 2\sqrt{bc} + c + a$

$+2\sqrt{ab} + \sqrt{ac}$

$\Rightarrow \quad 2\sqrt{ab} \quad 2b \quad 2\sqrt{ac} \quad 2\sqrt{bc}$

$2\sqrt{ac} \quad 2\sqrt{bc} \quad 2\sqrt{ab} \quad c \quad a$

$\Rightarrow \quad 2b = a + c$

$\Rightarrow$ a, b, c are in A.P. Which is true.

Hence, both the statements are correct.

54. (d) Let a and d be the first term and common difference of an AP respectively

p^{th} term = a + (p – 1)d

and q^{th} term = a + (q – 1)d

According to question.

$p\left[a + (p-1)d\right] = q\left[a + (q-1)d\right]$

$\Rightarrow \quad pa + \left(p^2 - p\right)d = qa + \left(q^2 - q\right)d$

$\Rightarrow \quad (p-q)a = \left(q^2 - p^2 + p - q\right)d$

$\Rightarrow \quad (p-q)a = (p-q)(-p-q+1)d$

$\Rightarrow \quad a = -(p+q-1)d$

Now, $(p+q)^{th}$ term = $a + (p+q-1)d$

$= -(p+q-1)d + (p+q-1)d \quad = 0$

55. (c) We know geometric mean of 3 numbers x_1, x_2, x_3 is $\sqrt[3]{x_1 \cdot x_2 \cdot x_3}$

Given, if observations are x_1, x_2, 12, G.M. is 6

$\Rightarrow \quad \sqrt[3]{x_1 \cdot x_2 \cdot 12} = 16$

$\Rightarrow \quad x_1 \times x_2 \times 12 = 6^3 = 216$

$\Rightarrow \quad x_1 \times x_2 = \frac{216}{12} = 18$ ...(i)

Also, given that actual number is 8.

$\therefore$ Actual G.M. = $\sqrt[3]{x_1 \cdot x_2 \cdot 8} = \sqrt[3]{18 \times 8}$ (from (i))

$= \sqrt[3]{18 \times 2 \times 2 \times 2} = 2.\sqrt[3]{18}$

56. (a) Let A, G and H be the arithmetic mean, geometric mean and Harmonic mean of two numbers a and b respectively.

According to the Question

G = H + 1.6

and A = H + 1.6 + 2 = H + 3. 6

We have AH = G^2

(H + 3.6) H = $(H + 1.6)^2$

$\Rightarrow \quad H^2 + 3.6H = H^2 + 2.56 + 3.2\,H$

$\Rightarrow \quad H = \frac{2.56}{0.4} = 6.4$

$\therefore \quad$ A = 6. 4 + 3.6 = 10

and G = 6. 4 + 1.6 = 8

Now, $A = \frac{a+b}{2} \Rightarrow a + b = 2A$

$\Rightarrow \quad$ a + b = 20 ...(i)

and ab = G^2 = 64 ...(ii)

We know that, $(a - b)^2 = (a + b)^2 - 4ab$

= 400 – 256 = 144

$\Rightarrow \quad$ a – b = 12 ...(iii)

On solving Eqs. (i) and (iii), we get

a = 16 and b = 4

57. (d) Let 'a' be the first term and 'ar' be the second term of G.P with common ratio 'r'.

Given: $S_\infty = 6$ and $a + ar = \frac{9}{2} \Rightarrow \frac{a}{1-r} = 6$

$\Rightarrow \quad$ a = 6 (1 – r) ...(i)

and $a + ar = \frac{9}{2}$

$\Rightarrow \quad 6(1-r) + 6r(1-r) = \frac{9}{2}$ [from (i)]

$\Rightarrow \quad 12 - 12r + 12r - 12r^2 = 9$

$\Rightarrow \quad r^2 = \frac{3}{12} = \frac{1}{4} \Rightarrow r = \frac{1}{2}$ or $\frac{-1}{2} \Rightarrow a = 3$ or 9

58. (a) Let 'a' and 'b' two numbers.

A.M = $\frac{a \quad b}{2}$ and G.M = $\sqrt{ab}$

According to the question,

A : G = m : n

$\Rightarrow \quad \frac{a \quad b}{2\sqrt{ab}} = \frac{m}{n} \Rightarrow \frac{a \quad b^{\ 2}}{4ab} \quad \frac{m^2}{n^2}$ ...(i)

and $\frac{a + b^{\ 2} - 4ab}{4ab} \quad \frac{m^2 - n^2}{n^2}$

$$\Rightarrow \quad \frac{a-b^{\ 2}}{4ab} \quad \frac{m^2-n^2}{n^2} \qquad ...(ii)$$

Since, on dividing Equation (i) and (ii), we get

$$\frac{a \quad b^{\ 2}}{a-b^{\ 2}} = \frac{m^2}{m^2-n^2} \Rightarrow \frac{a \quad b}{a-b} \quad \frac{m}{\sqrt{m^2-n^2}}$$

$$\Rightarrow \quad \frac{a \quad b \quad a-b}{a+b \ - \ a-b} \quad \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

(Using componendo dividendo rule)

$$\Rightarrow \quad \frac{2a}{2b} \quad \frac{a}{b} \quad \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

59. (c) Required geometric mean = $\sqrt[5]{2.4.8.16.32}$

$= (32^3)^{1/5} = (2^{15})^{1/5} = 2^3 = 8$

60. (d) Let a – d, a and a + d be three numbers which are in A.P. since A, B and C are in A.P.

$\therefore$ A = a – d, B = a, C = a + d

$\Rightarrow$ a – d + a + a + d = 180°

($\because$ A, B and C are angles of a triangle)

$\Rightarrow$ a = 60°

$\Rightarrow$ A = 60° – d, B = 60°, C = 60° + d

Now by sine rule,

$$\frac{b}{c} \quad \frac{\sin B}{\sin C} \Rightarrow \frac{\sqrt{3}}{\sqrt{2}} \quad \frac{\sin 60}{\sin C}$$

$$\Rightarrow \quad \sin C = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} \quad \frac{1}{\sqrt{2}}$$

61. (b) Let a, ar and ar^2 be three positive terms of G.P. According to question,

$$a = \frac{1}{3}(ar + ar^2)$$

$\Rightarrow 3 = r + r^2$

$\Rightarrow r^2 + r - 3 = 0$

$$\Rightarrow \quad r = \frac{-1 \quad \sqrt{1 \quad 4\times 3}}{2}$$

$$\Rightarrow \quad r = \frac{-1 \pm \sqrt{13}}{2} = \frac{\sqrt{13}-1}{2}, -\left(\frac{1 \quad \sqrt{13}}{2}\right)$$

Since, r can not be negative.

$$\therefore \quad r = \frac{\sqrt{13}-1}{2}$$

62. (b) Let the given series be

$$\frac{1}{4}, \frac{-1}{2}, 1, ...$$

Let $a = \frac{1}{4}$ and $r = -\frac{1}{2} \times \frac{4}{1} = -2$

$\therefore$ General term = $T_n = ar^{n-1}$

$\Rightarrow \ -128 = \frac{1}{4}(-2)^{n-1}$

$\Rightarrow \ -512 = (-2)^{n-1}$

$\Rightarrow \ (-2)^9 = (-2)^{n-1}$

$\Rightarrow \ 9 = n - 1 \Rightarrow n = 10$

63. (c) Suppose a, b and c are in HP.

$$\therefore \quad b \quad \frac{2ac}{a \quad c}$$

Now, consider

$$\frac{1}{b-a} \quad \frac{1}{b-c} \quad \frac{1}{\frac{2ac}{a \quad c}-a} \quad \frac{1}{\frac{2ac}{a \quad c}-c}$$

$$= \frac{a \quad c}{a(c-a)} \quad \frac{a \quad c}{c(a-c)} = \left(\frac{a \quad c}{c-a}\right)\left(\frac{1}{a}-\frac{1}{c}\right)$$

$$= \frac{a+c}{c-a} \times \frac{c-a}{ca} = \frac{a \quad c}{ca} \quad \frac{1}{a} \quad \frac{1}{c}$$

Thus, our supposition is correct.

Hence, a, b and c are in HP.

64. (b) Given series is

$$\sqrt{3} \quad \frac{1}{\sqrt{3}} \quad \frac{1}{3\sqrt{3}} \quad ....$$

Since, $\left(\frac{1}{\sqrt{3}}\right)^2 = \sqrt{3} \times \frac{1}{3\sqrt{3}}$

$\Rightarrow$ Given series is a G.P.

$\therefore$ Sum upto $\infty \quad \frac{a}{1-r}$

where $a = \sqrt{3}$ = first term and $r = \frac{1}{3}$ = common ratio

$$\therefore \quad S_\infty \quad \frac{\sqrt{3}}{1-\frac{1}{3}} \quad \frac{3\sqrt{3}}{2}$$

65. (d) Three numbers a, b and c will be in G.P. if $b^2 = ac$. Only option (d) i.e. $\tan^2 30°$, $\tan^2 45°$ and $\tan^2 60°$ are in G.P.

$\because \quad \tan^2 30° = \frac{1}{3}$

$\tan^2 45° = 1$

and $\tan^2 60° = 3$

$\therefore \quad \tan^2 30°, \tan^2 45°$ and $\tan^2 60°$ are in G.P.

66. (b) Given two series are
$2 + 6 + 10 + 14 + 18 + 22 + 26 + 30 + \text{-----}$
and $1 + 6 + 11 + 16 + 21 + 26 + 31 + \text{-----}$
So, series which are made to be common terms of both series is 6, 26,-----
This is an A. P where $a = 6, d = 20$
$\therefore\ a_{10} = a + (n - 1)\, d$
$= 6 + (10 - 1)\, 20 = 6 + 9 \times 20$
$= 180 + 6 = 186$

67. (a) General term of G.P $= ar^{n-1}$.
Given : $a_{10} = 9$ and $a_4 = 4$
$\Rightarrow ar^9 = 9$ and $ar^3 = 4$
On dividing we get
$$\frac{ar^9}{ar^3} = \frac{9}{4} \Rightarrow r^6 = \frac{9}{4}$$
$ar^3 = 4 \Rightarrow (ar^3)^2 = 16$
$\Rightarrow a^2 r^6 = 16$
$$\Rightarrow a^2 \times \frac{9}{4} = 16 \Rightarrow a^2 = \frac{64}{9} \Rightarrow a = \frac{8}{3}$$
Thus, $a_7 = ar^6 = \frac{8}{3} \times \frac{9}{4} = 6$

68. (c) Let $\log_{10} 2$, $\log_{10}\left(2^x - 1\right)$ and $\log_{10}\left(2^x + 3\right)$ are in A.P
$$\therefore 2\log_{10}\ 2^x - 1\ = \log_{10} 2\quad \log_{10}\ 2^x\quad 3$$
$$\Rightarrow \log_{10}\left(2^x - 1\right)^2 = \log_{10} 2\left(2^x + 3\right)$$
$$\Rightarrow 2^{2x} + 1 - 2^{x+1} = 2.2^x + 6$$
$$\Rightarrow a^2 + 1 - 2a\quad 2a\quad 6 \text{ where } a = 2^x.$$
$\Rightarrow a^2 - 4a - 5 = 0$
$\Rightarrow a = 5$ or $a = -1$ $\qquad 2^x = 5 \Rightarrow x \log_2 = \log 5$
$$\Rightarrow x = \frac{\log 5}{\log 2} \Rightarrow x\quad \log_2 5$$

69. (c) Let $n!$, $3(n!)$ and $(n + 1)!$ are in G. P.
Then, $[3(n!)]^2 = (n!)\,(n + 1)!$
$$\Rightarrow 9 \times n! \times n!\quad n!\quad n\quad 1\ n!$$
$\Rightarrow 9 = (n + 1) \Rightarrow n = 8$

70. (b) Given,
a, b, c, d, e, f are in A.P
$\therefore\quad 2d = e + c$ ...(1)
Consider $e - c = 2d - c - c$ (from 1)
$= 2d - 2c = 2\,(d - c)$

71. (b) G.M$= (10 \times 40 \times 60)^{1/3} = 28.84 \simeq 28$

72. (c) Let a and b be two numbers such that
$$\text{A.M.} = \frac{a+b}{2} = 10$$
$\Rightarrow a + b = 20$ ...(1)
and G.M. $= \sqrt{ab} = 8 \Rightarrow ab = 64$ ...(2)
From (1) and (2), we have
$a^2 - 20a + 64 = 0 \Rightarrow (a - 4)\,(a - 16) = 0$
$\Rightarrow a = 4, 16$
Thus, when $a = 4, b = 16$ and when $a = 16, b = 4$
Hence, one number exceeds the other number by 12.

73. (c) Given $S_2S_{11} = S_pS_8$
$\Rightarrow\quad (ar)\,(ar^{10}) = ar^{p-1}\,(ar^7)$
where 'a' is the first term and 'r' is the common ratio of GP.
$\Rightarrow r^{11} = r^{7+p-1} \Rightarrow r^{11} = r^{6+p}$
$\Rightarrow 11 = 6 + p \Rightarrow p = 5$

74. (c) Since $\frac{1}{4}, \frac{1}{x}, \frac{1}{10}$ are in H.P.
$\therefore$ 4, x, 10 are in AP.
$$\Rightarrow 2x = 4 + 10 \Rightarrow x = \frac{14}{2} = 7$$

75. (d) Since p, q, r are in A.P.
$\therefore\ 2q = p + r$ ....(1)
Since p, q, r are in G.P.
$q^2 = pr \Rightarrow q = \sqrt{pr}$
$\therefore\ 2\sqrt{pr} = p + r$
$$\left(\sqrt{p}\right)^2 + \left(\sqrt{r}\right)^2 - 2\sqrt{p}.\sqrt{r} = 0$$
$$\Rightarrow \left(\sqrt{p} - \sqrt{r}\right)^2 = 0$$
$$\Rightarrow \sqrt{p} = \sqrt{r}$$
$\Rightarrow p = r$ ...(2)
$2q = 2p$
$\Rightarrow q = p$ ...(3)
from (2) and (3)
$p = q = r$

76. (c) Let 'a' and 'b' be two non-negative numbers.
G.M. $= \sqrt{ab} = 10$
$\Rightarrow\quad ab = 100$
$$\text{and H.M.} = \frac{2ab}{a+b} = 8$$
$$\Rightarrow\quad \frac{200}{a+b} = 8$$
$\Rightarrow\quad a + b = 25$
Consider $(a - b)^2 = (a + b)^2 - 4ab = 625 - 400 = 225$
$\Rightarrow a - b = 15$
and $a + b = 25$
$\Rightarrow 2a = 40 \Rightarrow a = 20$ and $b = 5$
$$\text{A.M.} = \frac{20+5}{2} = 12.5$$

77. (c) Given sequence is
1, 5, 9, 13, 17, ...
Which is an A.P.
Here a = 1, d = 4
$\therefore$ nth term = $a_n = a + (n-1)d = 1 + (n-1)4$
$= 1 + 4n - 4 = 4n - 3$.

78. (d) Given series is $1 + \frac{1}{\sqrt{3}} + 3 + \frac{1}{3\sqrt{3}} + ...$

Consider $\frac{a_2}{a_1} = \frac{1}{\sqrt{3}}, \frac{a_3}{a_2} = \frac{3}{1/\sqrt{3}}, \frac{a_4}{a_3} = \frac{1/3\sqrt{3}}{3}$

Also find $a_2 - a_1 = \frac{1}{\sqrt{3}} - 1 = \frac{1-\sqrt{3}}{\sqrt{3}}$

$a_3 - a_2 = 3 - \frac{1}{\sqrt{3}} = \frac{3\sqrt{3}-1}{\sqrt{3}}$

$a_4 - a_3 = \frac{1}{3\sqrt{3}} - 3 = \frac{1-9\sqrt{3}}{3\sqrt{3}}$

Thus, $\frac{a_2}{a_1} \neq \frac{a_3}{a_2} \neq \frac{a_4}{a_3}$

and $a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$
Hence, Given series is neither A.P., G.P. nor H.P.

79. (d) Given series is a G.P. with a = 1, $r = -\frac{1}{2}$

$\therefore S_\infty = \frac{a}{1-r} = \frac{1}{1-\left(\frac{-1}{2}\right)} = \frac{2}{3}$

80. (b) Consider the Ist statement $1^3 + 2^3 + ... + 20^3$.

$\left[\frac{n \quad n+1}{2}\right]^2$ Here n 20

$= \left[20 \times \frac{21}{2}\right]^2 = (210)^2 = 44100$
So, statement I is false.
Consider the IInd statement.

$1^2 \quad 2^2 \quad 3^2 \quad .... \quad 20^2 \quad \frac{n \ n \quad 1 \quad 2n \quad 1}{6}$

$= \frac{20 \times 21 \times 41}{6} = 2870$

So, statement II is true.

81. (c) a = 1

$r = -\frac{1}{2} \ (< 1)$

$\therefore$ Sum of Ist 8 terms is :-

$S_8 = \frac{1\left[1-\left(\frac{-1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $\left[\text{For G . P. } S_n \quad \frac{a \quad 1-r^n}{1-r}\right]$

$\frac{\frac{1}{1} - \frac{1}{256}}{\frac{3}{2}} \quad \frac{85}{128}$

82. (b) $S_{10} = \frac{10}{2}[2a + 9d]$
120 = 5 (2a + 9d)
2a + 9d = 24 ...(i)

$S_{20} = \frac{20}{2}[2a + 19d]$
440 = 10 [2a + 19d]
2a + 19d = 44 ...(ii)
Solving (i) & (ii), we get

$\boxed{a = 3}$

83. (b) put a = 3 in (i)
6 + 9 d = 24
$9d = 18 \Rightarrow \boxed{d = 2}$

84. (a) We have,
n = 100

Number of diagonals $= \frac{n^2 - 3n}{2}$

$= \frac{100^2 - 300}{2} = \frac{9700}{2} = 4850$

85. (a) We have,
a = 30°, n = 3,
$S_3 = 180°$

$S_3 = \frac{3}{2}\left[2 \times 30° + (3-1)d\right]$

$180 = \frac{3}{2}[60 + 2d]$

180 = 3 [30 + d]
30 + d = 60

$\boxed{d = 30}$

Now, largest angle = a + 2d = 30 + 60 = 90° = $\frac{\pi}{2}$

86. (a) Geometric mean = $\sqrt[n \ 1]{1.2.4.8...2^n}$
($\because$ there are $(n + 1)$ multiples from 2° to 2^n)

$= \sqrt[n+1]{2^0.2^1.2^2.2^3...2^n}$

$= \sqrt[n+1]{2^{1+2+3+...+n}} = \sqrt[n \ 1]{2^{\frac{n(n \ 1)}{2}}}$

$= \left[2^{\frac{n(n \ 1)}{2}}\right]^{\frac{1}{n \ 1}} \quad 2^{\frac{n}{2}}$

87. (a) $\because$ (n – 3), 4n – 2, 5n + 1 are in A.P.
$\Rightarrow$ (4n – 2) – (n – 3) = (5n + 1) – (4n – 2)
$\Rightarrow$ 3n + 1 = n + 3
$\Rightarrow$ n = 1

88. (a) Let the two numbers be a, b.

Given, H. M. = 4 $\Rightarrow \frac{2ab}{a+b} = 4 \Rightarrow ab = 2(a+b)$ ...(i)

Also, given $2A + G^2 = 27$

$\Rightarrow 2\left(\frac{a+b}{2}\right) + ab = 27$

$\Rightarrow a + b + ab = 27$

$\Rightarrow a + b + 2(a + b) = 27$ (from (i))

$\Rightarrow 3a + 3b = 27$

$\Rightarrow a + b = 9$ ...(ii)

From (i), ab = 2(9) = 18 ...(iii)

Solving (ii), (iii) we get

a = 3, b = 6 or a = 6, b = 3

89. (a) Given, a, b, c, d are in A.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ are in H.P.

$\Rightarrow \frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a}$ are also in H.P.

Now, multiply each term by abcd.

$\frac{abcd}{d}, \frac{abcd}{c}, \frac{abcd}{b}, \frac{abcd}{a}$

abc, abd, acd, bcd, are in H.P.

90. (a) $S = 0.9 + 0.09 + 0.009 + ...$

$= 9(0.1 + 0.01 + 0.001 + ...)$

$= 9\left[\frac{0.1}{1-0.1}\right] = 9 \times \frac{0.1}{0.9} = 1$

91. (c) $S_5 = S_{10}$

$\Rightarrow \frac{5}{2}[2a \quad 4d] = \frac{10}{2}[2a \quad 9d]$

$\Rightarrow 5a + 10d = 10a + 45d$

$\Rightarrow a = -7d$ or $d = \frac{-1}{7}a$

We see, If d is positive, then first term should be negative and common difference should be positive. If d is negative, then first term should be position and common difference should be negative.

92. (b) 0 + 3 = 3, 3 + 5 = 8, 8 + 7 = 15, 15 + 9 = 24, 24 + 11 = 35, 35 + 13 = 48

Sequence is 0, 3, 8, 15, 24, 35, 48

93. (c) GP = x

$\frac{a}{1-r} = x$ (where, a = Ist term and r = common ratio)

$\Rightarrow \frac{2}{1-r} = x$ ...(i) ($\because$ Given a = 2 and $|r| < 1$)

$\Rightarrow -1 < r < 1 \Rightarrow 1 > -r > -1$

$\Rightarrow 1 + 1 > 1 - r > 1 - 1$

$\Rightarrow 0 < 1 - r < 2$

$\Rightarrow \frac{1}{1-r} \quad \frac{1}{2}, \frac{2}{1-r} \quad 1$

from equation (i) x > 1

Hence, $1 < x < \infty$.

94. (a) $3, \sqrt{3}, 1, \frac{1}{\sqrt{3}}, ..., \infty$

This is a Geometric Progression with a = 3, $r = \frac{1}{\sqrt{3}}$

$S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{1}{\sqrt{3}}}$

$= \frac{3\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3} \quad 1}{\sqrt{3} \quad 1} = \frac{3\sqrt{3}(\sqrt{3} \quad 1)}{2}$

95. (b) Given, S_n = Sum of first n terms of an AP.

$S_n = \frac{h}{2}[2a + (n-1)d]$ or $S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$

Similarly, $S_{3n} = \frac{3n}{2}[3a + (3n-1)d]$

According to direction, $3S_n = 2S_{2n}$

Putting the value of S_n and S_{2n} in above equation.

$3\left(\frac{n}{2}\right)[2a + (n-1)d] = 2\left(\frac{n}{2}\right)[2a + (2n-1)d]$

$6a + 3(n-1)d = 4a + 2(n-1)d$

$2a = d(n+1)$

$\therefore S_n = \frac{n}{2}[d(n+1) + d(n-1)]$

$= \frac{n}{2}[dn + d + dn - d]$

$= \frac{n}{2}(2dn) = n^2 d$

Now, $S_{2n} = n[d(n+1) + (2n-1)d] = 3n^2 d$

$S_{3n} = \frac{3n}{2}[d(n + 1 + 3n - 1)] = 6n^2 d$

Hence, $\frac{S_{3n}}{S_n} \quad \frac{6n^2 d}{n^2 d} = \frac{6}{1} = 6 : 1$

96. (a) From explanation $\frac{S_{3n}}{S_{2n}} \quad \frac{6n^2 d}{3n^2 d} = \frac{2}{1} = 2 : 1$

97. (b) $f(x) = ax^2 + bx + c$

$\therefore f(1) = a + b + c$

and $f(-1) = a - b + c$

$\therefore f(1) = f(-1)$

$\Rightarrow a + b + c = a - b + c \Rightarrow b = 0$

98. (a) We have $f'(x) = 2ax$

$\therefore f'(a) = 2a^2, f'(b) = 2ab = 0$

and $f'(c) = 2ac$ ($\because b = 0$)

$\therefore f'(a) = 2a^2$

$f'(b) = 0$

and $f'(c) = -2a^2$ ($\because 2b = a + c \Rightarrow c = -a$)

Hence $f'(a)$, $f'(b)$ and $f'(c)$ are in AP.

99. (c) $f''(x) = 2a$

$\therefore f''(a) = f''(b) = f''(c)$

Hence, $f''(a)$, $f''(b)$ and $f''(c)$ are in both AP and GP.

100. (d) Given $0.5 + 0.55 + 0.555 + \ldots$ to n

$= 5\,[0.1 + 0.11 + 0.111 + \ldots \text{ to n terms}]$

$= \frac{5}{9}\,[0.9 + 0.99 + 0.999 + \ldots \text{ to n terms}]$

$= \frac{5}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots \text{ to n terms}\right]$

$= \frac{5}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots \text{ to n terms}\right]$

$= \frac{5}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \ldots + \left(1 - \frac{1}{10^n}\right)\right]$

$= \frac{5}{9}\left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \ldots \frac{1}{10^n}\right)\right]$

$= \frac{5}{9}\left[n - \frac{1}{10}\,\frac{\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{\left(1 - \frac{1}{10}\right)}\right]$

$= \frac{5}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

101. (b) $X = 6^{1/2} \times 6^{1/2} \times 6^{3/8} \times 6^{4/16} \times \ldots\ldots\infty.$

$6^{\left(\frac{1}{2}\ \frac{2}{4}\ \frac{3}{8}\ \frac{4}{16} \ldots\ldots\infty\right)}.$

Let $S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots\ldots \infty.$

$\frac{1}{2}S = \quad + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \ldots\ldots \infty.$

$\left(1 - \frac{1}{2}\right)S \quad \frac{1}{2} \quad \frac{2-1}{4} \quad \frac{3-2}{8} \quad \frac{4-3}{16} \quad \ldots\ldots\infty.$

$\frac{S}{2} \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \ldots\ldots\infty.$

$\frac{S}{2} \quad \frac{\frac{1}{2}}{1 - \frac{1}{2}} \quad 1.$

$S = 2 \quad \therefore x = 6^2 = 36.$

102. (d) $T_n = \frac{3+n}{4}$

$S_n = \sum_{n=0}^{\infty} T_n = \sum\left(\frac{3}{4} + \frac{n}{4}\right)$

$= \frac{3}{4}n + \frac{1}{4} \times \frac{n(n+1)}{2} = \frac{7}{8}n + \frac{n^2}{8}$

$S_{105} = \frac{7}{8} \times 105 + \frac{(105)^2}{8} = 1470$

103. (b) Let the common ratio be K_1 for p, q and r.

$\therefore$ $q = K_1 p$

& $r = (K_1)^2 p$

Let the common ratio be K_2 for a, b and c

$\therefore$ $b = K_2 a$

& $c = (K_2)^2 a$

$\therefore$ $bq = (K_1K_2)ap$

& $Cr = (K_1K_2)^2 ap$

So ap, bq, cr are in G.P.

104. (c) $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$

$= \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots$

$\therefore S_n \quad \sqrt{2}\ 1 \quad 2 \quad 3 \quad 4 \quad \ldots$ n terms

$= \sqrt{2}\,\frac{n(n+1)}{2} = \frac{n(n+1)}{\sqrt{2}}$

105. (c) $a_n = \int_0^{\pi} \frac{\sin^2\{(n+1)x\}}{\sin 2x}\,dx$

Since it is a definite integral will have a definite value.

The sequence $\{a_{2n}\}$ is in AP with common difference.

Statement (1) is correct.

The sequence $\{a_{2n+1}\}$ is also in AP with common difference.

Statement (2) is correct.

106. (b) $\because$ given sequence a_n also AP with no difference.

Thus $a_{n-1} - a_{n-4} = 0$

107. (c) Given $\log_x y$, $\log_z x$, $\log_y z$ are in G.P.

$\therefore (\log_z x)^2 = (\log_x y)(\log_y z)$

$\left(\frac{\log x}{\log z}\right)^2 = \left(\frac{\log y}{\log x}\right)\left(\frac{\log z}{\log y}\right) = \frac{\log z}{\log x}$

$\Rightarrow \left(\frac{\log x}{\log z}\right)^3 = 1$

$\Rightarrow \log x = \log z \Rightarrow x = z$

$\because xyz = 64$

$y = \frac{64}{x^2}$

Also given x^3, y^3 and z^3 are in A.P.

$\therefore y^3 = \frac{x^3+z^3}{2} = \frac{x^3+x^3}{2}$

$y^3 = x^3 \Rightarrow y = x$

$\Rightarrow x = y = z$

$x \cdot y \cdot z = 64$

$x = y = z = 4$

Thus x, y, z are in A.P. and G.P. both.

108. (c) Similarly xy, yz, zx are also in A.P. and G.P. both.

109. (a) Three terms are

$G_1 \quad \left(\frac{y}{z}\right)^{\log yz} \quad G_2 \quad \left(\frac{z}{x}\right)^{\log zx} \quad G_3 \quad \left(\frac{x}{y}\right)^{\log xy}$

Geometric mean of three terms is

$m = \sqrt[3]{G_1G_2G_3}$ ...(1)

$\therefore G_1G_2G_3 = \left(\frac{y}{z}\right)^{\log yz} \cdot \left(\frac{z}{x}\right)^{\log zx} \cdot \left(\frac{x}{y}\right)^{\log xy}$

$= \frac{y^{\log y} \cdot y^{\log z}}{z^{\log y} \cdot z^{\log z}} \times \frac{z^{\log z} \cdot z^{\log x}}{x^{\log z} \cdot x^{\log x}} \times \frac{x^{\log x} \cdot x^{\log y}}{y^{\log x} \cdot y^{\log y}}$

$= \left(\frac{y}{x}\right)^{\log z} \cdot \left(\frac{z}{y}\right)^{\log x} \cdot \left(\frac{x}{z}\right)^{\log y}$

Taking log both sides

$\log G_1G_2G_3 = \log\left[\left(\frac{y}{x}\right)^{\log z}\right] + \log\left[\left(\frac{z}{y}\right)^{\log x}\right]$

$+ \log\left[\left(\frac{x}{z}\right)^{\log y}\right]$

$= \log z \log y - \log z \log x + \log x \log z$

$- \log x \log y + \log y \log x - \log y \log z$

$\log G_1G_2G_3 = 0$

$G_1G_2G_3 = e^0 = 1$

Hence $m = \sqrt[3]{G_1G_2G_3} = (1)^{\frac{1}{3}}$

$\boxed{m = 1}$

110. (d) Let 'a' be the first term & 'x' be the common ratio. Also, suppose 27, 8 & 12 be the p^{th}, q^{th} & r^{th} term of the G.P.

$\therefore \quad ax^{p-1} = 27$

$ax^{q-1} = 8$

$\& \ ax^{r-1} = 12$

Now, $27 \times 8^2 = 12^3$

$\Rightarrow \quad ax^{p-1}\, x\, (ax^{q-1})^2 = (ax^{r-1})^3$

$\Rightarrow \quad x^{p-1} \cdot x^{2q-2} = x^{2r-3}$

$\Rightarrow \quad p - 1 + 2q - 2 = 3r - 3$

$\Rightarrow \quad p + 2q - 3r = 0$ ...(1)

There are infinitely many solutions for the eq. (1).

111. (b) $S_n = \frac{n}{2}(a \quad l)$

$\Rightarrow \quad a + x + y + z + b = \frac{5}{2}(a \quad b)$

$a + b + 15 = \frac{5}{2}(a \quad b)$

$\Rightarrow \quad a + b = 10$ ...(1)

$\& \ \frac{1}{a} \quad \frac{1}{p} \quad \frac{1}{q} \quad \frac{1}{r} \quad \frac{1}{b} \quad \frac{5}{2}\left(\frac{1}{a} \quad \frac{1}{b}\right)$

$\Rightarrow \quad \frac{1}{a} \quad \frac{1}{b} \quad \frac{5}{3} \quad \frac{5}{2}\left(\frac{1}{a} \quad \frac{1}{b}\right)$

$\Rightarrow \quad \frac{3(a \quad b)}{ab} \quad \frac{10}{3}.$ ...(2)

$\Rightarrow \quad \frac{3 \times 10}{ab} \quad \frac{10}{3}.$

$\Rightarrow \quad ab = 9$

112. (b) On solving eq (1) & (2), we get

(i) $a = 1 \ \& \ b = 9 \Rightarrow a + 4d = 9 \Rightarrow d = 2$

(ii) $a = 9 \ \& \ b = 1 \Rightarrow a + 4d = 1 \Rightarrow d = -2.$

For $a = 1 \ \& \ d = 2$,

$x = 3, y = 5 \ \& \ z = 7$

For $a = 9 \ \& \ d = -2$,

$x = 7, y = 5 \ \& \ z = 3$

$\Rightarrow \quad xyz = 7 \times 5 \times 3 = 105$

113. (c) Since a, p, q, r, b or 1, p, q, r, 9 are in H.P.

$\Rightarrow \quad \frac{1}{1 \quad 4d} \quad 9 \Rightarrow d = -\frac{2}{9}.$

$\frac{1}{p} = 1 - \frac{2}{9} \quad \frac{7}{9} \Rightarrow p \quad \frac{9}{7}$

$\frac{1}{q} = \frac{7}{9} - \frac{2}{9} \quad \frac{5}{9} \Rightarrow q \quad \frac{9}{5}$

$\& \ \frac{1}{r} = \frac{5}{9} - \frac{2}{9} \quad \frac{3}{9} \Rightarrow r \quad \frac{9}{3}$

$\Rightarrow \quad p \times q \times r = \frac{243}{35}$

114. (a) Let first term = a & common difference = x

$\therefore \quad a + 5x = 2 \Rightarrow a = 2 - 5x.$

Let $P = T_1 \times T_4 \times T_5$

$\Rightarrow \quad P = a(a + 3x)(a + 4x)$

$\Rightarrow \quad P = (2 - 5x)(2 - 5x + 3x)(2 - 5x + 4x)$

$\Rightarrow \quad P = -10x^3 + 34x^2 - 32x + 8.$

$\frac{dp}{dx} = 0 \Rightarrow 15x^2 - 34x + 16 = 0$

$\Rightarrow (5x - 8)(3x - 2) = 0$

$\Rightarrow \quad x = \frac{8}{5}, \left[\because x \quad \frac{2}{3} \quad 1\right]$

115. (b) Since, a = 2 – 5x

$\Rightarrow \quad a = 2 - 5\left(\frac{8}{5}\right)$

$\Rightarrow \quad a = -6$

116. (a) Here, $a = 120°$ and $d = 5$.
Sum of angles of polygon = $(n - 2)\ 180°$

$$\Rightarrow \frac{n}{2}[2a + (n-1)d] = (n-2)180$$

$$\Rightarrow \frac{n}{2}[2\times120 + (n-1)5] = (n-2)180$$

$$\Rightarrow n^2 - 25n + 144 = 0$$

$$\Rightarrow (n-9)(n-16) = 0$$

$$\therefore n = 9, 16$$

For $n = 9$, $T_9 = 120 + (9-1)\ 5 = 160$
For $n = 16$, $T_{16} = 120 + (16-1)\ 5 = 195$ [not possible]

117. (a) For $n = 9$
Largest angle = $T_9 = 120 + (9-1)\ 5 = 160$
For $n = 16$
Largest angle = $T_{16} = 120 + (16-1)\ 5 = 195$
(Not possible).

118. (b) $x^{\ln\left(\frac{y}{z}\right)} . y^{\ln\ xz} . z^{\ln\left(\frac{x}{y}\right)} \quad y^{4\ln y}$

$$\Rightarrow \ln\left[x^{\ln\left(\frac{y}{z}\right)}\right] \quad \ln\left[y^{\ln(xz)^2}\right] \quad \ln\left[z^{\ln\left(\frac{x}{y}\right)}\right] \quad \ln\left[y^{4\ln y}\right]$$

$$\Rightarrow \left[\ln\left(\frac{y}{z}\right)\ln x\right] + [2\ln(xz)\ln y] + \left[\ln\left(\frac{x}{y}\right)\ln z\right] = 4[\ln y]^2$$

$$\Rightarrow \ln x[\ln y - \ln z] + 2\ln y[\ln x + \ln z] + \ln z[\ln x - \ln y] = 4[\ln y]^2$$

$$\Rightarrow 3\ln x + \ln z = 4\ln y$$

$$\Rightarrow \frac{\ln x + \ln x + \ln x + \ln z}{4} = \ln y$$

∴ lny is the AM of lnx, lnx, lnx, lnx & lnz.

119. (a) 0.3 + 0.33 + 0.333 + ... n terms

$$= \frac{3}{10} + \frac{33}{100} + \frac{333}{1000} + ...$$

$$= 3\left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + ...\right)$$

$$= \frac{3}{9}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...\right)$$

$$= \frac{1}{3}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + ...\right]$$

$$= \frac{1}{3}\left[n - \left(\frac{1}{10} \quad \frac{1}{10^2} \quad \frac{1}{10^3} \quad ......\frac{1}{10^n}\right)\right]$$

$$= \frac{1}{3}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right]$$

$$\left(\text{Since, Sum of n terms of G.P} = \frac{a(1 - r^n)}{1 - r}\right]$$

$$= \frac{1}{3}\left[n - \frac{10^n - 1}{9(10^n)}\right] = \frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

120. (d) Given, $S_m = n$ and $S_n = m$
This is direct formula
$S_{m+n} = -(m+n)$

121. (c) Let α, β be the roots of $x^2 + bx + c = 0$

Given, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$
$\alpha + \beta = -b;\ \alpha.\beta = c$

$$\therefore \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2}{\alpha^2} + \frac{\beta^2}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow -b = \frac{b^2 - 2c}{c^2}$$

$\Rightarrow -bc^2 = b^2 - 2c \Rightarrow 2c = b^2 + bc^2$

$$\Rightarrow \frac{2c}{b} \quad b \quad c^2 \Rightarrow \frac{2}{b} = \frac{b}{c} + c$$

We know, if a, b, c are in A.P. 2b = a + c.

Similarly, we got $\frac{2}{b} = \frac{b}{c} + c$, which means $\frac{b}{c}, \frac{1}{b}, c$ are in A.P.

So, $\frac{c}{b}, b, \frac{1}{c}$ are in H.P.

122. (a) Let α, β be roots of $ax^2 + x + c = 0$.
Given, α + β

$$= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \frac{-1}{a} = \frac{\left(\frac{-1}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$\Rightarrow \frac{-1}{a} = \frac{\frac{1}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

$$\Rightarrow \frac{-1}{a} = \frac{1 - 2ac}{c^2} \Rightarrow -c^2 = a - 2a^2c$$

$\Rightarrow 2a^2c = a + c^2$
So, a, a^2c, c^2 are in A.P.

123. (b) Given, $S_n = n^2 - 2n$.
We know, $T_n = S_n - S_{n-1}$
So, $T_5 = S_5 - S_4$.
$S_5 = 5^2 - 2(5) = 25 - 10 = 15$
$S_4 = 4^2 - 2(4) = 16 - 8 = 8$
So, $T_5 = 15 - 8 = 7$.

124. (a) Sum of odd numbers $= n^2$
Two – digit odd numbers are from 11 – 99.
Number of odd numbers from 1 to 99 = 50
Sum of odd numbers from 1 to 99 $= 50^2 = 2500$
Number of odd numbers from 1 + 9 = 5
Sum of odd numbers from 1 to 9 $= 5^2 = 25$
So, Sum of all two digit odd numbers
$= 2500 - 25 = 2475$.

125. (c) $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....$

$$=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+......$$

$$=(1+1+1+....n)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....\right)$$

$$=n-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+.....\right)$$

$$=n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} \qquad \left(\because \text{G.P. } a=\frac{1}{2}, r=\frac{1}{2}\right)$$

$$=n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{\frac{1}{2}}$$

$= n - 1 + 2^{-n}$
$= 2^{-n} + n - 1$

126. (a) $2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$
($\because \tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in A.P)
and x, y, z are in G.P $\quad\therefore\quad y^2 = xz \qquad$ ...(i)

$$\Rightarrow \frac{2y}{1-y^2}=\frac{x+z}{1-xz}$$

$$\Rightarrow \frac{2y}{\cancel{1-y^2}}=\frac{x+z}{\cancel{1-y^2}} \qquad \text{(from (i))}$$

$\Rightarrow 2y = x + z$
x, y, z are in A.P.
Given x, y, z are also in G.P
So, x = y = z

127. (d) $S_n = np + \frac{n(n-1)Q}{2}$

We know, $T_1 = S_1$ and $T_2 = S_2 - S_1$
Common difference (d) $= T_2 - T_1$

$$\therefore S_1 = (1)P + \frac{1(1-1)Q}{2} = P + 0 = P$$

$$S_2 = (2)P + \frac{2(2-1)Q}{2} = 2P + \frac{2Q}{2} = 2P + Q.$$

$\therefore T_1 = P$; $T_2 = 2P + Q - P = P + Q$
$\therefore$ Common difference (d) $= T_2 - T_1$
$= P + Q - P = Q$.

128. (a) $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times ....$ upto infinite terms.

$$6^{\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ ....\infty} = 6$$

Sum of ∞ terms of G.P. is $S_\infty = \frac{a}{1-r}$. Here $a = \frac{1}{2}, r = \frac{1}{2}$

$$\therefore 6^{\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ ....\infty} \quad 6^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} \quad 6^{\frac{\frac{1}{2}}{\frac{1}{2}}} \quad 6^1 \quad 6.$$

129. (b) Given, $a_1 = a_2 = a_3 = ... = a_{10} = 150$
Also, $a_{10}, a_{11}, a_{12}, ...$ are in A.P. and $d = -2$
Since, $a_{10} = 150$, A.P. is 150, 148, 146, ...
For the first 10 minutes, he has counted $150 \times 10 = 1500$ notes.
Time taken to count remaining 3000 notes

$$S_n = \frac{n}{2}[2a + \ n-1\ \ d]$$

$$\Rightarrow 3000 = \frac{n}{2}[2\times 148 + \ n-1 \ \ -2\ ]$$

$$\Rightarrow 3000 = \frac{n}{2}\times 2\ \ 148 - n \ \ 1$$

$\Rightarrow 3000 = 148n - n^2 + n$
$\Rightarrow n^2 - 149n + 3000 = 0$
$\Rightarrow (n - 24)(n - 125)$
$\Rightarrow n = 24$, or $n = 125$.
Since he has taken 10 minutes to count 1500 notes, he will not take 125 min to count 3000 notes.
So, n = 24.
$\therefore$ Total time = 10 + 24 = 34 minutes.

130. (a) $y = x + x^2 + x^3 + ...$ upto infinite terms.
$x + x^2 + x^3 + ...$ is a G.P. with $a = x$ and $r = x$.

$$\therefore y \quad \frac{x}{1-x}$$

$\Rightarrow y - xy = x$
$\Rightarrow x + xy = y$
$\Rightarrow x(1 + y) = y$

$$\Rightarrow x = \frac{y}{1+y}.$$

131. (a) $\frac{1}{\log_3 e} \quad \frac{1}{\log_3 e^2} \quad \frac{1}{\log_3 e^4} \quad ....$

$$\frac{1}{\log_3 e} \quad \frac{1}{2\log_3 e} \quad \frac{1}{4\log_3 e} \quad .... \text{(Since, } \log_a b^m = m\log_a b)$$

$$\log_e 3 \quad \frac{\log_e 3}{2} \quad \frac{\log_e 3}{4} \quad .... \quad \left(\text{Since, } \log_a b \quad \frac{1}{\log_b a}\right)$$

$$\log_e 3\left(1 \quad \frac{1}{2} \quad \frac{1}{4} \quad ....\right)$$

$$\log_e 3\left(\frac{1}{1-\frac{1}{2}}\right)\left(\because 1, \frac{1}{2}, \frac{1}{4} .... \text{ is G.P. with a } \quad 1, r \quad \frac{1}{2}\right)$$

$= \log_e 3(2) = 2\log_e 3 = \log_e 3^2 = \log_e 9.$

132. (c) Arithmetic mean of $x_1, x_2 = \frac{x_1 \quad x_2}{2}$

Geometric mean of $x_1, x_2 = \sqrt{x_1 x_2}$

Given, $\frac{x_1 + x_2}{2} - \sqrt{x_1 x_2} > 1$

$\Rightarrow \frac{x_1 + x_2}{2} > \sqrt{x_1 x_2} + 1$

$\Rightarrow x_1 \quad x_2 \quad 2\sqrt{x_1 x_2} \quad 2$

$\Rightarrow x_1 + x_2 - 2\sqrt{x_1 x_2} \quad 2$

$\Rightarrow \sqrt{x_1}^{\,2} + \sqrt{x_2}^{\,2} - 2\sqrt{x_1}\sqrt{x_2} \quad 2$

$\Rightarrow \left(\sqrt{x_1} - \sqrt{x_2}\right)^2 > 2$

$\Rightarrow \left|\sqrt{x_1} - \sqrt{x_2}\right| \quad \sqrt{2}$

Correct option (c).

133. (c) A.M of two numbers $a, b \quad \frac{a \quad b}{2}$

G.M of two numbers $a, b \quad \sqrt{ab}$.

Given, $\frac{A.M}{G.M} \quad \frac{5}{3}$

$\Rightarrow \frac{\left(\frac{a \quad b}{2}\right)}{\sqrt{ab}} \quad \frac{5}{3}$

$\Rightarrow \frac{a \quad b}{\sqrt{ab}} \quad \frac{10}{3}$

$\Rightarrow \frac{a^2 \quad b^2 \quad 2ab}{ab} \quad \frac{100}{9}$ (Squaring on both sides)

$\Rightarrow \quad 9a^2 + 9b^2 + 18ab = 100ab$

$\Rightarrow \quad 9a^2 - 82ab + 9b^2 = 0$

$\Rightarrow \quad 9a^2 - 81ab - ab + 9b^2 = 0$

$\Rightarrow \quad 9a(a - 9b) - b(a - 9b) = 0$

$\Rightarrow \quad (9a - b)(a - 9b) = 0$

$\Rightarrow \quad 9a - b = 0; a - 9b = 0$

$\Rightarrow \quad b = 9a; a = 9b$

$\Rightarrow \quad \frac{a}{b} = \frac{1}{9}; \frac{a}{b} = \frac{9}{1}$

134. (a) $x = 1 - y + y^2 - y^3 + ...$ up to infinite terms.

We can see that the given series is geometric progression, with $a = 1$ and $r = -y$

$S_\infty \frac{a}{1-r} \quad \frac{1}{1- \; -y} \quad \frac{1}{1 \quad y}$

$\therefore x \quad \frac{1}{1 \quad y}.$

135. (c) The numbers which divided by 3, leaving remainder 2 will be of the form $3x + 2$

Given, $3x + 2$ is 2-digit number

So, x can be from 3 to 32

Sum of numbers $\sum_{x \; 3}^{32} 3x \quad 2$

$= 3(3 + 4 + 5 + ... + 32) + (2 + 2 + ...)$ ....(1)

3, 4, 5 .... 32 is an A. P with $a = 3, d = 1, T_n = 32$

$\therefore T_n = a + (n - 1)\,1$

$32 = 3 + (n - 1)\,1$

$\Rightarrow n - 1 = 29 \Rightarrow n = 30$ terms

$\therefore (1) \Rightarrow 3(3 + 4 + 5 + ... + 32) + (2 + 2 + ... 30 \text{ times})$

$3\left(\frac{30}{2} \; 3 \quad 32\right) \quad 2 \times 30 \quad \left(\because S_n = \frac{n}{2}(a + \ell)\right)$

$\frac{90}{2} \; 35 \quad 60$

$= (45 \times 35) + 60$

$= 1575 + 60 = 1635$

136. (c) Given, 3rd term of G.P. = 3

Let 'a' be the first term and 'r' be the common ratio.

$\therefore ar^2 = 3$

We know, $T_1 = a, T_2 = ar, T_3 = ar^2, T_4 = ar^3, T_5 = ar^4$

$T_1.T_2.T_3.T_4.T_5 = (a)(ar)(ar^2)(ar^3)(ar^4)$

$= a^5 r^{10}$

$= (ar^2)^5$

$= 3^5$

$= 243$

137. (a) $x, \frac{3}{2}, z$ are in A.P

If a, b, c are in A.P $2b = a + c$

$\therefore 2\left(\frac{3}{2}\right) \quad x \quad z$

$\Rightarrow 3 = x + z$ ....(1)

x, 3, z are in G.P.

If a, b, c are in G.P. $b^2 = ac$

$\therefore 3^2 = xz$

$\Rightarrow 9 = xz$ ....(2)

If x, 6, z are in H.P. $\frac{2}{6} \quad \frac{1}{x} \quad \frac{1}{z}$

$\left(\because \frac{2}{b} \quad \frac{1}{a} \quad \frac{1}{c}, \text{ if a, b, c are is H.P.}\right)$

$\Rightarrow \frac{1}{3} \quad \frac{z \quad x}{xz} \quad \frac{3}{9} \quad \frac{1}{3}$ from 1 2

L.H.S. = R.H.S.

138. (c) Sum of an infinite G.P. with first term x and common ratio $r(<1)$ is

$$s=\frac{x}{1-r}$$

From question $s=5$

then, $5=\frac{x}{1-r}$

$$1-r=\frac{x}{5}$$

$$r=1-\frac{x}{5}$$

For, $-1<1-\frac{x}{5}<1$

$$-2<-\frac{x}{5}<0$$

$$0<x<10$$

139. (c) The series is $\frac{3}{3^0}-\frac{3}{3^1}+\frac{3}{3^2}-\frac{3}{3^3}+....\infty$

This is a G.P. with first term $a=3$

and common ratio $r=-\frac{1}{3}$

$\therefore$ Sum $S=\frac{3}{1-\left(-\frac{1}{3}\right)}=\frac{3\times 3}{3+1}=\boxed{\frac{9}{4}}$

140. (c) $T_n=\frac{1}{m}, T_m=\frac{1}{n}$

$\Rightarrow$ 1^{st} term = c.d. $=\frac{1}{mn}$

$\Rightarrow T_{mn}=\frac{1}{mn}+\frac{mn-1}{mn}=1$

141. (c) Let the first term and common ratio of the G.P. is a and r respectively.

Then, $ar=2$, $\frac{a}{1-r}=8$

$\Rightarrow \frac{ar}{r-r^2}=8 \qquad \Rightarrow 2=8r-8r^2$

$\Rightarrow 4r^2-4r+1=0 \qquad \Rightarrow r=\frac{1}{2}$

$\therefore$ G.P. is $4, 2, 1, \frac{1}{2}, ...$

142. (c) From question,

$a, b, c \in$ A.P. $\Rightarrow a-b=b-c \Rightarrow \frac{a-b}{b-c}=1$

$a, b, c \in$ G.P. $\Rightarrow \frac{a}{b}=\frac{b}{c}=\frac{a-b}{b-c}$

$a, b, c \in$ H.P. $\Rightarrow b=\frac{2ac}{a+c} \Rightarrow ab+bc=2ac$

$\Rightarrow ab-ac=ac-bc$

$\Rightarrow \frac{a-b}{b-c}=\frac{a}{c}$

143. (c) From question,

$\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in H.P.

then, $\sin\beta=\frac{2\sin\alpha\cos\alpha}{\sin\alpha+\cos\alpha}$

and $\sin\alpha$, $\sin\theta$ and $\cos\alpha$ are in A.P.

then, $2\sin\theta=\sin\alpha+\cos\alpha$

Statement 1: $\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right).\sin\beta$

$=(\sin\alpha+\cos\alpha).\frac{2\sin\alpha\cos\alpha}{\sin\alpha+\cos\alpha}$

$=2\sin\alpha\cos\alpha=\sin 2\alpha$

Statement 2: $\cos\left(\alpha-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$

$=\frac{2\sin\theta}{\sqrt{2}}=\sqrt{2}\sin\theta$

144. (b) $(x_1.x_2 .... x_n)^{1/n}=P$

$(y_1.y_2 .... y_n)^{1/n}=Q$

$$\left(\frac{x_1.x_2 .... x_n}{y_1.y_2 .... y_n}\right)^{1/n}=\frac{(x_1.x_2 .... x_n)^{1/n}}{(y_1.y_2 .... y_n)^{1/n}}=\frac{P}{Q}$$

145. (d) Given series 25, –125, 625, –3125 ..... is geometric progression.

$a=t_1=25, t_2=-125$

$r=\frac{t_2}{t_1}=\frac{-125}{25}=-5$

n^{th} term $(t_n)=ar^{n-1}=(25)(-5)^{n-1}$

$=(5)^2(-1)^{n-1}(5)^{n-1}=(-1)^{n-1}(5)^{2+n-1}$

$=(-1)^{n-1}5^{n+1}$

146. (c) Given, numbers 1, 5, 25

Let p^{th} term $=1 \Rightarrow a+(p-1)d=1$ ...(1)

Let q^{th} term $=5 \Rightarrow a+(q-1)d=5$ ...(2)

Let r^{th} term $=25 \Rightarrow a+(r-1)d=25$ ...(3)

(3), (2) $\Rightarrow r-q=25-5=20$

(2), (1) $\Rightarrow q-p=5-1=4$

$\frac{r-q}{q-p}=\frac{20}{4}=5$ which is an integer.

So, the given series forms an AP.

Infinite AP's are possible.

147. (c) Let the first term of AP = a

Let the common difference of AP = d

$(p+q)^{th}$ term $=T_{p+q}=a+(p+q-1)d$

$(p-q)^{th}$ term $=T_{p-q}=a+(p-q-1)d$

$T_{p+q}+T_{p-q}=a+(p+q-1)d+a+(p-q-1)d$

$=2a+(p+q-1+p-q-1)d$

$=2a+(2p-2)d=2[a+(p-1)d]=2T_p$

148. (b) Given, sum of n terms $(S_n)=n(n+1)$

We know, $T_n=S_n-S_{n-1}$

$=[n(n+1)]-[(n-1)(n-1+1)]$

$=n(n+1)-(n-1)n$

$=n(n+1-n+1)$

$=2n$

$\therefore$ Fourth term, $T_4=2(4)=8$

Complex Numbers

4

1. If z_1, z_2 are any two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, which one of the following is correct ?
(a) $z_1 = \alpha z_2$ with $\alpha \in R$ (b) $z_1 \geq 0$ or $z_2 \geq 0$
(c) $z_1 = \alpha z_2$ with $\alpha > 0$ (d) $|z_1| = |z_2|$ *[2006-I]*

2. If α, β are real, what is $\left|\frac{\alpha + i\beta}{\beta + i\alpha}\right|$ equal to ?
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2 *[2006-I]*

3. If z = 1 + i, then what is the inverse of z^2 ?
(a) 2i (b) i
(c) $\frac{i}{2}$ (d) $-\frac{i}{2}$ *[2006-I]*

4. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*

Assertion (A) : If $Z_1 = 3 + \sqrt{-4}$, and $Z_2 = 3 + \sqrt{-25}$, Z_1/Z_2 is a complex number.
Reason (R) : If Z_1, Z_2 are complex numbers, then Z_1/Z_2 is always a complex number.
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2006-II]*

5. Let $z = i^3(1 + i)$ be a complex number. What is its argument?
(a) π (b) $\frac{\pi}{4}$
(c) $-\frac{\pi}{4}$ (d) $\frac{5\pi}{4}$ *[2006-II]*

6. Let z_1 and z_2 be two non-zero complex numbers such that
$$|z_1| = |z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right| = 2$$
What is the value of $|z_1 + z_2|$?
(a) 8 (b) 4
(c) 2 (d) 1 *[2006-II]*

7. Let z be a non-zero complex number. Then, what is z^{-1} (multiplicative inverse of z) equal to ?
(a) $\frac{\bar{z}}{|z|^2}$ (b) $\frac{z}{|z|^2}$
(c) $\frac{\bar{z}}{|z|}$ (d) $\frac{|z|}{z}$ *[2006-II]*

8. What is one of the values of $\sqrt{i} \quad \sqrt{-i}$?
(a) $\sqrt{2}$ (b) 0
(c) $\frac{1 \quad i}{\sqrt{2}}$ (d) $\frac{1-i}{\sqrt{2}}$ *[2007-I]*

9. What is the value of
$[-1 \quad i\sqrt{3})/2]^{10} \quad [-1 - i\sqrt{3})/2]^{10}$
(a) 1 (b) –1
(c) 2 (d) 0 *[2007-I]*

10. If ω denotes the cube root of unity, then what is the real root of the equation $x^3 - 27 = 0$?
(a) 3ω (b) $3\omega^2$
(c) -3ω (d) $3\omega^3$ *[2007-I]*

11. Let O be the origin and point A be represented by z. If OA is rotated through an angle $\pi/2$ in the anticlockwise direction keeping the length of OA same, then what represents the new point?
(a) –iz (b) $|z|i$
(c) iz (d) z *[2007-I]*

12. If $1, \omega, \omega^2$ are the three cube roots of unity, then what is $\frac{(a\omega^6 + b\omega^4 + c\omega^2)}{(b + c\omega^{10} + a\omega^8)}$ equal to?
(a) $\frac{a}{b}$ (b) b
(c) ω (d) ω^2 *[2007-II]*

13. What is the square root of the complex number – 5 + 12i ?
(a) 2 – 3i (b) 2 + 3i
(c) – 2 + 3i (d) $\sqrt{-5} + \sqrt{12i}$ *[2007-II]*

14. If $\alpha = \frac{1+i\sqrt{3}}{2}$, then what is the value of $1 + \alpha^8 + \alpha^{16} + \alpha^{24} + \alpha^{32}$?
(a) 0 (b) 1
(c) ω (d) $-\omega^2$ [2007-II]

15. A straight line is passing through the points represented by the complex numbers a + ib and $\frac{1}{-a+ib}$, where (a, b) ≠ (0,0).
Which one of the following is correct ?
(a) It passes through the origin
(b) It is parallel to the x-axis
(c) It is parallel to the y-axis
(d) It passes through (0, b) [2008-I]

16. Which one of the following is correct? If z and w are complex numbers and $\bar{w}$ denotes the conjugate of w, then $|z + w| = |z - w|$ holds only, if [2008-I]
(a) z = 0 or w = 0 (b) z = 0 and w = 0
(c) $z \cdot \bar{w}$ is purely real (d) $z \cdot \bar{w}$ is purely imaginary

17. What is the square root of $\frac{1}{2} - i\frac{\sqrt{3}}{2}$?
(a) $\pm\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$ (b) $\pm\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)$
(c) $\pm\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$ (d) $\pm\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$ [2008-I]

18. Let C be the set of complex number and z_1, z_2 are in C.
1. $\text{agr}(z_1) = \text{agr}(z_2) \Rightarrow z_1 = z_2$
2. $|z_1| = |z_2| \Rightarrow z_1 = z_2$
Which of the statements given above is/are correct? [2008-I]
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

19. What is arg (bi) where b > 0?
(a) 0 (b) $\frac{\pi}{2}$
(c) π (d) $\frac{3\pi}{2}$

20. If ω is a complex non-real cube root of unity, then ω satisfies which one of the following equations? [2008-I]
(a) $x^2 - x + 1 = 0$ (b) $x^2 + x + 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) $x^2 - x - 1 = 0$

21. For a positive integer n, what is the value of i^{4n+1}? [2008-II]
(a) 1 (b) –1
(c) i (d) $-i$

22. If ω is a complex cube root of unity, then what is the value of $1 - \frac{1}{(1+\omega)} - \frac{1}{(1+\omega^2)}$? [2008-II]
(a) 1 (b) 0
(c) ω (d) ω^2

23. Match List I with List II and select the correct answer using the code given below the lists [2008-II]

	List I		List II
A.	A cube root of unity	1.	$-2(1+i)$
B.	A square root of –1	2.	$2i$
C.	Cube of 1 – i	3.	$-i$
D.	Square of 1 + i	4.	$-\frac{1}{2}\left(1+i\sqrt{3}\right)$

Code :

	A	B	C	D
(a)	4	1	3	2
(b)	2	1	3	4
(c)	4	3	1	2
(d)	2	3	1	4

24. What is $\sqrt{3} \quad i \;/\; 1 \quad \sqrt{3}i$ equal to ? [2009-I]
(a) $1+i$ (b) $1-i$
(c) $\sqrt{3}(1-i)/2$ (d) $\left(\sqrt{3}-i\right)/2$

25. If $2x = 3 + 5i$, then what is the value of $2x^3 + 2x^2 - 7x + 72$? [2009-I]
(a) 4 (b) –4
(c) 8 (d) –8

26. **Assertion (A) :** $\left(\frac{-1+\sqrt{-3}}{2}\right)^{29} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{29} = -1$
Reason (R) : $\omega^2 = -1$ [2009-I]
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

27. If α is a complex number such that $\alpha^2 + \alpha + 1 = 0$, then what is α^{31} equal to? [2009-II]
(a) α (b) α^2
(c) 0 (d) 1

28. What is the modulus of $\frac{1 \quad 2i}{1-(1-i)^2}$ equal to? [2009-II]
(a) 5 (b) 4
(c) 3 (d) 1

29. What is the value of $-\sqrt{-1}^{\;4n \quad 3} \quad i^{41} \quad i^{-257} \;^{9}$, where $n \in N$? [2009-II]
(a) 0 (b) 1
(c) i (d) $-i$

30. If ω is the cube root of unity, then what is the conjugate of $2\omega^2 + 3i$? [2009-II]
(a) $2\omega - 3i$ (b) $3\omega + 2i$
(c) $2\omega + 3i$ (d) $3\omega - 2i$

31. If z is a complex number such that $z + z^{-1} = 1$, then what is the value of $z^{99} + z^{-99}$? [2009-II]
(a) 1 (b) –1
(c) 2 (d) –2

32. What is the value of $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{200} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{200} + 1$?
(a) –1 (b) 0 *[2010-I]*
(c) 1 (d) 2

33. If ω is a complex cube root of unity and $x = \omega^2 - \omega - 2$, then what is the value of $x^2 + 4x + 7$? *[2010-I]*
(a) – 2 (b) – 1
(c) 0 (d) 1

34. If $x^2 + y^2 = 1$, then what is $\frac{1+x+iy}{1+x-iy}$ equal to? *[2010-I]*
(a) $x - iy$ (b) $x + iy$
(c) $2x$ (d) $-2iy$

35. What is the modulus of $\left|\frac{1+2i}{1-(1-i)^2}\right|$? *[2010-I]*
(a) 1 (b) $\sqrt{5}$
(c) $\sqrt{3}$ (d) 5

36. What is the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$?
(a) 16 (b) 12 *[2010-I]*
(c) 8 (d) 4

37. What is the conjugate of $\left(\frac{1+2i}{2+i}\right)^2$? *[2010-II]*
(a) $\frac{7}{25} + i\frac{24}{25}$ (b) $-\frac{7}{25} - i\frac{24}{25}$
(c) $-\frac{7}{25} + i\frac{24}{25}$ (d) $\frac{7}{25} - i\frac{24}{25}$

38. What is $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6$ equal to ? *[2010-II]*
(a) – 1 (b) 0
(c) 1 (d) 2

39. If ω is a complex cube root of unity, then what is $\omega^{10} + \omega^{-10}$ equal to ? *[2010-II]*
(a) 2 (b) –1
(c) – 2 (d) 1

40. What is the value of $\left(-1 + i\sqrt{3}\right)^{48}$? *[2010-II]*
(a) 1 (b) 2
(c) 2^{24} (d) 2^{48}.

41. What is the value of
$$1 + i^2 + i^4 + i^6 + ... + i^{100},$$
where $i = \sqrt{-1}$? *[2010-II]*
(a) 0 (b) 1
(c) – 1 (d) None of these

42. If $z = 1 + \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$, then what is $|z|$ equal to ?
(a) $2\cos\frac{\pi}{5}$ (b) $2\sin\frac{\pi}{5}$ *[2011-I]*
(c) $2\cos\frac{\pi}{10}$ (d) $2\sin\frac{\pi}{10}$

43. What is modulus of $\frac{1}{1+3i} - \frac{1}{1-3i}$? *[2011-I]*
(a) $\frac{3}{5}$ (b) $\frac{9}{25}$
(c) $\frac{3}{25}$ (d) $\frac{5}{3}$

44. If ω is the imaginary cube root of unity, then what is $(2 - \omega + 2\omega^2)^{27}$ equal to? *[2011-I]*
(a) $3^{27}\omega$ (b) $-3^{27}\omega^2$
(c) 3^{27} (d) -3^{27}

45. What is the value of $(1 + i)^5 + (1 - i)^5$ where $i = \sqrt{-1}$? *[2011-II]*
(a) – 8 (b) 8
(c) $8i$ (d) $-8i$

46. What are the square roots of $-2i$? *[2011-II]*
$\left(i = \sqrt{-1}\right)$
(a) $\pm(1+i)$ (b) $\pm(1-i)$
(c) $\pm i$ (d) ± 1

47. If $z = 1 + i\tan\alpha$ where $\pi < \alpha < \frac{3\pi}{2}$, then what is $|z|$ equal to? *[2011-II]*
(a) sec α (b) – sec α
(c) $\sec^2\alpha$ (d) $-\sec^2\alpha$

48. The smallest positive integral value of n for which $\left(\frac{1-i}{1+i}\right)^n$ is purely imaginary with positive imaginary part is *[2011-II]*
(a) 1 (b) 3
(c) 4 (d) 5

49. If α and β are the complex cube roots of unity, then what is the value of $(1 + \alpha)(1 + \beta)(1 + \alpha^2)(1 + \beta^2)$? *[2011-II]*
(a) – 1 (b) 0
(c) 1 (d) 4

50. If p, q, r are positive integers and ω is the cube root of unity and $f(x) = x^{3p} + x^{3q+1} + x^{3r+2}$, then what is f(ω) equal to? *[2011-II]*
(a) ω (b) $-\omega^2$
(c) – ω (d) 0

51. If $z = \frac{1+2i}{2-i} - \frac{2-i}{1+2i}$, then what is the value of $z^2 + z\bar{z}$? $\left(i = \sqrt{-1}\right)$ [2011-II]
(a) 0 (b) – 1
(c) 1 (d) 8

52. What is the argument of $(1 - \sin\theta) + i\cos\theta$? $\left(i = \sqrt{-1}\right)$ [2011-II]
(a) $\frac{\pi}{2} - \frac{\theta}{2}$ (b) $\frac{\pi}{2} + \frac{\theta}{2}$
(c) $\frac{\pi}{4} - \frac{\theta}{2}$ (d) $\frac{\pi}{4} + \frac{\theta}{2}$

53. If $A + iB = \frac{4+2i}{1-2i}$ where $i = \sqrt{-1}$ then what is the value of A?
(a) – 8 (b) 0 [2012-I]
(c) 4 (d) 8

54. If $z = -\bar{z}$, then which one of the following is correct?
(a) real part of z is zero. [2012-I]
(b) The imaginary part of z is zero.
(c) The real part of z is equal to imaginary
(d) The sum of real and imaginary parts of z is z.

55. Consider the following statements : [2012-II]
1. $(\omega^{10} + 1)^7 + \omega = 0$
2. $(\omega^{105} + 1)^{10} = p^{10}$ for some prime number p
where $\omega \neq 1$ is a cube root of unity.
Which of the above statements is/are correct ?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

56. The value of the sum $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$ where $i = \sqrt{-1}$ is:
(a) i (b) –i [2012-II]
(c) 0 (d) i – 1

57. What is the modulus of $\frac{\sqrt{2}+i}{\sqrt{2}-i}$ where $i = \sqrt{-1}$? [2012-II]
(a) 3 (b) 1/2
(c) 1 (d) None of the above

58. What is $\sqrt{-i}$ where $i = \sqrt{-1}$ equal to? [2013-I]
(a) $\pm\frac{1-i}{\sqrt{2}}$ (b) $\pm\frac{1+i}{\sqrt{2}}$
(c) $\pm\frac{1-i}{2}$ (d) $\pm\frac{1+i}{2}$

59. What is the argument of the complex number (–1 –i) where $i = \sqrt{-1}$? [2013-I]
(a) $\frac{5\pi}{4}$ (b) $-\frac{5\pi}{4}$
(c) $\frac{3\pi}{4}$ (d) None of the above

60. What is one of the square roots of $3 + 4i$, where $i = \sqrt{-1}$? [2013-II]
(a) $2 + i$ (b) $2 - i$
(c) $-2 + i$ (d) $-3 - i$

61. If P and Q are two complex numbers, then the modulus of the quotient of P and Q is : [2014-I]
(a) Greater than the quotient of their moduli
(b) Less than the quotient of their moduli
(c) Less than or equal to the quotient of their moduli
(d) Equal to the quotient of their moduli

62. Let $z = x + iy$ Where x, y are real variables $i = \sqrt{-1}$. If $|2z - 1| = |z - 2|$, then the point z describes : [2014-I]
(a) A circle (b) An ellipse
(c) A hyperbola (d) A parabola

63. If $|z + \bar{z}| = |z - \bar{z}|$, then the locus of z is: [2014-I]
(a) A pair of straight lines
(b) A line
(c) A set of four straight lines
(d) A circle

64. What is the argument of the complex number $\frac{(1+i)(2+i)}{3-i}$ where $i = \sqrt{-1}$? [2014-I]
(a) 0 (b) $\frac{\pi}{4}$
(c) $-\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

65. Let z be a complex number such that $|z| = 4$ and arg $z = \frac{5\pi}{6}$. Where $i = \sqrt{-1}$. What is z equal to ? [2014-II]
(a) $2\sqrt{3} + 2i$ (b) $2\sqrt{3} - 2i$
(c) $-2\sqrt{3} + 2i$ (d) $-\sqrt{3} + i$

66. What is $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}}$ equal to, where n is a natural number and $i = \sqrt{-1}$? [2014-II]
(a) 2 (b) $2i$
(c) –2 (d) i

67. If $z = \frac{-2(1+2i)}{3+i}$ where $i = \sqrt{-1}$, then argument $\theta(-\pi < \theta \leq \pi)$ of z is [2015-I]
(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $\frac{5\pi}{6}$ (d) $-\frac{3\pi}{4}$

68. If 1, ω, ω^2 are the cube roots of unity, then the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ is [2015-I]
(a) –1 (b) 0
(c) 1 (d) 2

69. What is the square root of i, where $i=\sqrt{-1}$? *[2015-I]*
   (a) $\frac{1+i}{2}$ (b) $\frac{1-i}{2}$
   (c) $\frac{1+i}{\sqrt{2}}$ (d) None of these

70. (x^3-1) can be factorised as *[2015-I]*
   (a) $(x-1)(x-\omega)(x+\omega^2)$
   (b) $(x-1)(x-\omega)(x-\omega^2)$
   (c) $(x-1)(x+\omega)(x+\omega^2)$
   (d) $(x-1)(x+\omega)(x-\omega^2)$
   where ω is one of the cube roots of unity.

71. What is *[2015-I]*
$$\left[\frac{\sin\frac{\pi}{6}+i\left(1-\cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6}-i\left(1-\cos\frac{\pi}{6}\right)}\right]^3$$
   where $i=\sqrt{-1}$, equal to?
   (a) 1 (b) –1
   (c) i (d) $-i$

72. What is the real part of $(\sin x + i\cos x)^3$ where $i=\sqrt{-1}$? *[2015-I]*
   (a) $-\cos 3x$ (b) $-\sin 3x$
   (c) $\sin 3x$ (d) $\cos 3x$

73. If z_1 and z_2 are complex numbers with $|z_1|=|z_2|$, then which of the following is/are correct? *[2015-II]*
   1. $z_1=z_2$
   2. Real part of z_1 = Real part of z_2
   3. Imaginary part of z_1 = Imaginary part of z_2
   Select the correct answer using the code given below :
   (a) 1 only (b) 2 only
   (c) 3 only (d) None

74. If the point $z_1=1+i$ where $i=\sqrt{-1}$ is the reflection of a point $z_2=x+iy$ in the line $i\bar{z}-iz=5$, then the point z_2 is *[2015-II]*
   (a) $1+4i$ (b) $4+i$
   (c) $1-i$ (d) $-1-i$

75. $z\bar{z}+(3-i)z\ \ (3\ \ i)\bar{z}\ \ 1=0$ represents a circle with *[2015-II]*
   (a) centre (–3, –1) and radius 3
   (b) centre (–3, 1) and radius 3
   (c) centre (–3, –1) and radius 4
   (d) centre (–3, 1) and radius 4

76. Suppose ω is a cube root of unity with $\omega\neq1$. Suppose P and Q are the points on the complex plane defined by ω and ω^2. If O is the origin, then what is the angle between OP and OQ? *[2016-I]*
   (a) 60° (b) 90°
   (c) 120° (d) 150°

77. If $z=x+iy=\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{-25}$, where $i=\sqrt{-1}$, then what is the fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$? *[2016-I]*
   (a) π (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

DIRECTIONS (Qs. 78-79) : *For the next two (2) items that follow.*

Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2=i\bar{z}$, where $i=\sqrt{-1}$. *[2016-I]*

78. What is $z_1+z_2+z_3$ equal to?
   (a) i (b) –i
   (c) 0 (d) 1

79. Consider the following statements:
   1. $z_1z_2z_3$ is purely imaginary.
   2. $z_1z_2+z_2z_3+z_3z_1$ is purely real.
   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 80-81) : *For the next two (2) items that follow:*

Let z be a complex number satisfying *[2016-I]*
$$\left|\frac{z-4}{z-8}\right|=1 \text{ and } \left|\frac{z}{z-2}\right|=\frac{3}{2}$$

80. What is $|z|$ equal to?
   (a) 6 (b) 12
   (c) 18 (d) 36

81. What is $\left|\frac{z-6}{z+6}\right|$ equal to?
   (a) 3 (b) 2
   (c) 1 (d) 0

82. Suppose ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then what is $(\omega_1-\omega_2)^2$ equal to? *[2016-I]*
   (a) 3 (b) 1
   (c) –1 (d) –3

83. What is $\omega^{100}+\omega^{200}+\omega^{300}$ equal to, where ω is the cube root of unity? *[2016-II]*
   (a) 1 (b) 3ω
   (c) $3\omega^2$ (d) 0

84. If $\text{Re}\left(\frac{z-1}{z+1}\right)=0$, where $z=x+iy$ is a complex number, then which one of the following is correct? *[2016-II]*
   (a) $z=1+i$ (b) $|z|=2$
   (c) $z=1-i$ (d) $|z|=1$

85. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to? *[2016-II]*
   (a) 0 (b) $\frac{1}{2}$
   (c) $\frac{\sqrt{3}}{2}$ (d) 1

86. What is the number of distinct solutions of the equation $z^2 + |z| = 0$ (where z is a complex number)? [2016-II]
(a) One (b) Two
(c) Three (d) Five

87. What is $\sqrt{\frac{1+\omega^{2}}{1+\omega}}$ equal to, where ω is the cube root of unity? [2016-II]
(a) 1 (b) ω
(c) ω^2 (d) $i\omega$, where $i = \sqrt{-1}$

88. The value of $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$, where $i = \sqrt{-1}$, is [2017-I]
(a) 0 (b) 1
(c) i (d) $-i$

89. The value of [2017-I]
$$\left(\frac{-1+i\sqrt{3}}{2}\right)^{n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{n}$$
where n is not a multiple of 3 and $i = \sqrt{-1}$, is
(a) 1 (b) –1
(c) i (d) $-i$

90. The modulus and principal argument of the complex number $\frac{1 \quad 2i}{1- \quad 1-i \quad ^{2}}$ are respectively [2017-I]
(a) 1, 0 (b) 1, 1
(c) 2, 0 (d) 2, 1

91. If $|z+5| \le 3$, then the maximum value of $|z+1|$ is [2017-I]
(a) 0 (b) 4
(c) 6 (d) 10

92. The number of roots of the equation $z^2 = 2\bar{z}$ is [2017-I]
(a) 2 (b) 3
(c) 4 (d) zero

93. If $A = \begin{bmatrix} 4i-6 & 10i \\ 14i & 6+4i \end{bmatrix}$ and $k = \frac{1}{2i}$, where $i = \sqrt{-1}$, then kA is equal to [2017-II]
(a) $\begin{bmatrix} 2+3i & 5 \\ 7 & 2-3i \end{bmatrix}$ (b) $\begin{bmatrix} 2-3i & 5 \\ 7 & 2+3i \end{bmatrix}$
(c) $\begin{bmatrix} 2-3i & 7 \\ 5 & 2+3i \end{bmatrix}$ (d) $\begin{bmatrix} 2+3i & 5 \\ 7 & 2+3i \end{bmatrix}$

94. The smallest positive integer n for which $\left(\frac{1 \quad i}{1-i}\right)^{n} \quad 1$, is [2017-II]
(a) 1 (b) 4
(c) 8 (d) 16

95. Geometrically Re $(z^2 - i) = 2$, where $i = \sqrt{-1}$ and Re is the real part, represents. [2017-II]
(a) circle (b) ellipse
(c) rectangular hyperbola (d) parabola

96. What is the principal argument of (–1 – i), where $i = \sqrt{-1}$? [2018-I]
(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $-\frac{3\pi}{4}$ (d) $\frac{3\pi}{4}$

97. Let α and β be real numbers and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with Re(z) = 1, then it is necessary that [2018-I]
(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

98. The number of non-zero integral solutions of the equation $|1-2i|^x = 5^x$ is [2018-I]
(a) Zero (No solution) (b) One
(c) Two (d) Three

99. If α and β are different complex numbers with $|\alpha| = 1$, then what is $\left|\frac{\alpha-\beta}{1-\alpha\beta}\right|$ equal to? [2018-I]
(a) $|\beta|$ (b) 2
(c) 1 (d) 0

100. What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where $i = \sqrt{-1}$)? [2018-I]
(a) 0 (b) i
(c) –i (d) 1

101. The modulus-amplitude form of $\sqrt{3} \quad i$, where $i = \sqrt{-1}$ is [2018-I]
(a) $2\left(\cos\frac{\pi}{3} \quad i\sin\frac{\pi}{3}\right)$ (b) $2\left(\cos\frac{\pi}{6} \quad i\sin\frac{\pi}{6}\right)$
(c) $4\left(\cos\frac{\pi}{3} \quad i\sin\frac{\pi}{3}\right)$ (d) $4\left(\cos\frac{\pi}{6} \quad i\sin\frac{\pi}{6}\right)$

102. What is the value of the sum [2018-I]
$\sum_{n\ 2}^{11} i^{n} \quad i^{n\ 1}$, where $i = \sqrt{-1}$?
(a) i (b) 2i
(c) –2i (d) 1 + i

103. What is the value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} \quad \left(\frac{-1 \quad i\sqrt{3}}{2}\right)^{3n}$ where $i = \sqrt{-1}$? [2018-II]
(a) 3 (b) 2
(c) 1 (d) 0

104. Which one of the following is correct in respect of the cube roots of unity? [2018-II]
(a) They are collinear
(b) They lie on a circle of radius $\sqrt{3}$
(c) They form an equilateral triangle
(d) None of the above

105. If $A = \{x \in Z : x^2 - 1 = 0\}$ and $B = \{x \in Z : x^2 + x + 1 = 0\}$, where Z is set of complex numbers, then what is $A \cap B$ equal to ? *[2019-I]*
(a) Null set
(b) $\left\{\frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}\right\}$
(c) $\left\{\frac{-1+\sqrt{3}\,i}{4}, \frac{-1-\sqrt{3}\,i}{4}\right\}$
(d) $\left\{\frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}\,i}{2}\right\}$

106. If $\begin{bmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{bmatrix} = 6 + 11i$, then what are the values of x and y respectively ? *[2019-I]*
(a) –3, 4 (b) 3, 4
(c) 3, – 4 (d) –3, – 4

107. The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{2017} + z^{2018} + 1 = 0$ are *[2019-I]*
(a) $-1, \omega$ (b) $1, \omega^2$
(c) $-1, \omega^2$ (d) ω, ω^2

DIRECTIONS (Qs. 108-109): *Consider the following for the next 02 (two) items:*

A complex number is given by $z = \dfrac{1+2i}{1-(1-i)^2}$

108. What is the modulus of z? *[2019-I]*
(a) 4 (b) 2
(c) 1 (d) $\frac{1}{2}$

109. What is the principal argument of z? *[2019-I]*
(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

ANSWER KEY

1	(a)	12	(c)	23	(c)	34	(b)	45	(a)	56	(d)	67	(d)	78	(c)	89	(b)	100	(a)
2	(c)	13	(b)	24	(d)	35	(a)	46	(b)	57	(c)	68	(c)	79	(c)	90	(a)	101	(b)
3	(d)	14	(d)	25	(a)	36	(d)	47	(b)	58	(a)	69	(c)	80	(a)	91	(c)	102	(c)
4	(a)	15	(a)	26	(c)	37	(d)	48	(b)	59	(a)	70	(b)	81	(d)	92	(c)	103	(b)
5	(c)	16	(a)	27	(a)	38	(c)	49	(c)	60	(a)	71	(c)	82	(d)	93	(a)	104	(c)
6	(a)	17	(a)	28	(d)	39	(b)	50	(d)	61	(d)	72	(b)	83	(d)	94	(b)	105	(b)
7	(a)	18	(d)	29	(c)	40	(d)	51	(a)	62	(a)	73	(d)	84	(d)	95	(c)	106	(a)
8	(a)	19	(b)	30	(a)	41	(b)	52	(d)	63	(a)	74	(a)	85	(a)	96	(c)	107	(b)
9	(b)	20	(b)	31	(d)	42	(c)	53	(b)	64	(d)	75	(a)	86	(c)	97	(c)	108	(c)
10	(d)	21	(c)	32	(b)	43	(a)	54	(a)	65	(d)	76	(c)	87	(b)	98	(a)	109	(a)
11	(c)	22	(b)	33	(c)	44	(d)	55	(b)	66	(a)	77	(a)	88	(a)	99	(c)		

HINTS & SOLUTIONS

1. (a) Let $z_1 = r_1(\cos\theta + i\sin\theta)$
where = Argument of z_1 or $\theta = \text{Arg}(z_1)$
$|z_1| = r_1$ and $z_2 = r_2(\cos\phi + i\sin\phi)$
where $\phi = \text{Arg}(z_2)$ $|z_2| = r_2$
$z_1 + z_2 = r_1(\cos\theta + i\sin\theta) + r_2(\cos\phi + i\sin\phi)$
$= (r_1\cos\theta + r_2\cos\phi) + i(r_1\sin\theta + r_2\sin\phi)$

$$|z_1 + z_2| = \sqrt{(r_1\cos\theta + r_2\cos\phi)^2 + (r_1\sin\theta + r_2\sin\phi)^2}$$

$$= \sqrt{\begin{matrix} r_1^2\cos^2\theta + r_2^2\cos^2\phi + 2r_1r_2\cos\theta\cos\phi \\ + r_1^2\sin^2\theta + r_2^2\sin^2\phi + 2r_1r_2\sin\theta\sin\phi \end{matrix}}$$

$$= \sqrt{r_1^2 + r_2^2 + 2r_1r_2(\cos\theta\cos\phi + \sin\theta\sin\phi)}$$

As given : $|z_1 + z_2| = |z_1| + |z_2|$

So, $\sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta - \phi)} = r_1 + r_2$

Squaring both the sides
$r_1^2 + r_2^2 + 2r_1r_2\cos(\theta - \phi) = (r_1 + r_2)^2$
$= r_1^2 + r_2^2 + 2r_1r_2$
or, $2r_1r_2\cos(\theta - \phi) = 2r_1r_2$
or, $\cos(\theta - \phi) = 1$
$\theta - \phi = 0 \Rightarrow \theta = \phi$
$\text{Arg}(z_1) = \text{Arg}(z_2)$ so, $z_1 = \alpha z_2$
where $\alpha \in R$

2. (c) Let $z_1 = \alpha + i\beta$ and $z_2 = \beta + i\alpha$

Since, $\left|\frac{z_1}{z_2}\right| \quad \frac{|z_1|}{|z_2|}$

$$\therefore \left|\frac{\alpha + i\beta}{\beta + i\alpha}\right| \quad \frac{|\alpha + i\beta|}{|\beta + i\alpha|} = \frac{\sqrt{\alpha^2 + \beta^2}}{\sqrt{\alpha^2 + \beta^2}} = 1$$

3. (d) Given that $z = 1 + i$
$\Rightarrow z^2 = (1 + i)^2 = 1 + i^2 + 2i = 1 - 1 + 2i = 2i$

Inverse of $z^2 = \dfrac{1}{2i} = -\dfrac{i^2}{2i}$ [Since $i^2 = -1$; $-i^2 = 1$]

$= -\dfrac{i}{2}$

4. (a) $\frac{z_1}{z_2} = \frac{3+4i}{3+5i} = \frac{(3+4i)(3-5i)}{3^2-5^2} = \frac{9-3i+20}{9-25} = \frac{29-3i}{-16}$
which is complex number. Both A and R are individually true and R is correct explanation of A.

5. (c) The complex number is $z = i^3(1+i) = -i(1+i)$
$= -i - i^2$
$\Rightarrow z = 1 - i$

This gives, $\arg(z) = \tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

6. (a) As given $|z_1| = |z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right| = 2$

$\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = 2 \Rightarrow \frac{z_2 + z_1}{z_1 z_2} = 2$

$\Rightarrow \frac{|z_1 \quad z_2|}{|z_1||z_2|} \quad 2$

$\Rightarrow |z_1 + z_2| = 2|z_1||z_2|$

$\Rightarrow |z_1 + z_2| = 2.2.2$

$\Rightarrow |z_1 + z_2| = 8$

7. (a) Let z be a non-zero complex number, such that $z = x + iy$ where $x, y \in R$

Then $z^{-1} = \frac{1}{x + iy}$

So, $z^{-1} = \frac{x - iy}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{\bar{z}}{|z|^2}$

8. (a) $\sqrt{i} = \sqrt{\frac{2i}{2}} = \sqrt{\frac{1+2i-1}{2}}$

$= \sqrt{\frac{1 \quad 2i \quad i^2}{2}} \quad \sqrt{\frac{(1 \quad i)^2}{2}} = \frac{1+i}{\sqrt{2}}$

$\sqrt{-i} = \sqrt{-\frac{2i}{2}}$

$= \sqrt{\frac{1-2i-1}{2}} = \sqrt{\frac{1-2i \quad i^2}{2}} = \sqrt{\frac{(1-i)^2}{2}} = \frac{(1-i)}{\sqrt{2}}$

So, value of $\sqrt{i} + \sqrt{-i}$

$= \frac{(1+i)+(1-i)}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

9. (b) We know that $\frac{-1+i\sqrt{3}}{2} = \omega$,

so, $\frac{-1-i\sqrt{3}}{2} = \omega^2$

$\therefore \left[\frac{-1+i\sqrt{3}}{2}\right]^{10} + \left[\frac{-1-i\sqrt{3}}{2}\right]^{10}$

$= \omega^{10} + \omega^{20} = \omega^{3\times 3+1} + \omega^{3\times 6+2}$

$= \omega^3\omega^3\omega + \omega^3\omega^6\omega^2 = \omega + \omega^2$ (since $\omega^3 = 1$)

and $\omega + \omega^2 = -1$ $(1 + \omega + \omega^2 = 0)$

10. (d) The given equation is $x^3 - 27 = 0$

$\Rightarrow x^3 - 3^3 = 0 \Rightarrow x = 3, 3\omega, 3\omega^2$

Thus, real root of given equation is $3\omega^3$, since $\omega^3 = 1$

11. (c) Let $z = \cos\theta + i\sin\theta$

Now, on rotating through an angle $\frac{\pi}{2}$, z becomes

$Z = \cos\left(\frac{\pi}{2}+\theta\right) + i\sin\left(\frac{\pi}{2}+\theta\right)$

$= -\sin\theta + i\cos\theta = i^2\sin\theta + i\cos\theta$

$= i(\cos\theta + i\sin\theta) = iz$

12. (c) 1, ω and ω^2 are the three cube roots of unity.

$\Rightarrow 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$.

The given expression

$\frac{a\omega^6 + b\omega^4 + c\omega^2}{b + c\omega^{10} + a\omega^8} \quad \frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2}$

$[\omega^6 = 1, \omega^4 = \omega]$

$\frac{\omega(a + b\omega + c\omega^2)}{\omega(b + c\omega + a\omega^2)}$ [Multiplying N^r and D^r by ω.]

$\frac{\omega(a + b\omega + c\omega^2)}{(a\omega^3 + b\omega + c\omega^2)} \quad \frac{\omega(a + b\omega + c\omega^2)}{(a + b\omega + c\omega^2)} = \omega$

13. (b) Let $\sqrt{-5 \quad 12i} \quad x \quad iy$

$\Rightarrow (x + iy)^2 = -5 + 12i$

$\Rightarrow x^2 - y^2 + i2xy = -5 + 12i$

$\Rightarrow x^2 - y^2 = -5$ and $2xy = 12 \Rightarrow xy = 6$

$(x^2 - y^2)^2 + 4x^2y^2 = (x^2 + y^2)^2$

$\Rightarrow (-5)^2 + 4 \times (6)^2 = (x^2 + y^2)^2$

$\Rightarrow (x^2 + y^2)^2 = 25 + 144 = 169$

$\Rightarrow x^2 + y^2 = \quad 13$ and $x^2 - y^2 = -5$

Adding both $2x^2 = \pm 13 - 5 = 8$ or -18

$\Rightarrow x^2 = 4$ (–ve discarded)

$\Rightarrow x = \quad 2$ and $y^2 = \quad 13 - 4 = 9$ or -13

Discard $y^2 = -13$, $y^2 = 9$

$\Rightarrow y = \quad 3$

$\Rightarrow x + iy = \quad (2 + 3i)$

$\Rightarrow \sqrt{-5 \quad 12i} \quad (2 \quad 3i)$

14. (d) $\frac{-1 \quad i\sqrt{3}}{2}$ and $\frac{-1-i\sqrt{3}}{2}$

are complex cube roots of unity ω and ω^2

$\alpha \quad \frac{1 \quad i\sqrt{3}}{2} \quad -\omega^2$

$\therefore 1 + \alpha^8 + \alpha^{16} + \alpha^{24} + \alpha^{32}$

$= 1 + \omega^{16} + \omega^{32} + \omega^{48} + \omega^{64}$

$= 1 + \omega + \omega^2 + 1 + \omega = 0 + 1 + \omega = -\omega^2$

15. (a) Complex numbers are : (a + ib) and $\frac{1}{-a+ib}$

Second number is rationalized as,

$$\frac{-a-ib}{(-a+ib)(-a-ib)}=\frac{-a-ib}{a^2+b^2}$$

These two complex numbers represent two points,

(a, b) and $\left(\frac{-a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)$

Eq. of line passing through these point is

$$(y-b)=\frac{-\frac{b}{a^2+b^2}-b}{\frac{-a}{a^2+b^2}-a}(x-a)$$

$$\Rightarrow y-b=\frac{b}{a}(x-a)\Rightarrow ay-ab=bx-ab$$

$$\Rightarrow y=\frac{b}{a}x$$

So, line passes through the origin.

16. (a) Given that, $|z+w|=|z-w|\Rightarrow$ either z or, w is zero.

17. (a) Let $\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}=x-iy$

Squaring both the sides,

$$\Rightarrow\frac{1}{2}-\frac{i\sqrt{3}}{2}=(x-iy)^2\Rightarrow\frac{1}{2}-\frac{i\sqrt{3}}{2}=x^2-y^2-2ixy$$

Comparing real and imaginary parts, we get

$$x^2-y^2=\frac{1}{2} \quad ...(1)$$

and $2xy=\frac{\sqrt{3}}{2}$ ...(2)

and $(x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2=\frac{1}{4}+\frac{3}{4}=1$

$\Rightarrow x^2+y^2=1$ .... (3)

On solving eqs. (1) and (3) we get

$x^2=\frac{3}{4}$ and $y^2=\frac{1}{4}$

$$\Rightarrow x=\pm\frac{\sqrt{3}}{2},y=\pm\frac{1}{2}$$

$$\therefore\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}=\pm\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$$

18. (d) Both statements are not correct.

19. (b) arg (bi) $\tan^{-1}\left(\frac{b}{0}\right)=\tan^{-1}(\infty)=\frac{\pi}{2}(\because b>0)$

20. (b) If ω is cube root of unity then $\omega^2+\omega+1=0$

So, ω satisfies the equaion $x^2+x+1=0$

21. (c) $i^{4n+1}=(i^4)^n\times i=(1)^n\times i=1\times i=i$

22. (b) $1-\frac{1}{(1+\omega)}-\frac{1}{(1+\omega^2)}=1-\frac{1}{-\omega^2}-\frac{1}{-\omega}$

$$=\frac{\omega^2+1+\omega}{\omega^2}=\frac{0}{\omega^2}=0$$

23. (c)

A cube root of unity	$-\frac{1}{2}(1+i\sqrt{3})$
A square root of –1	$-i$
Cube of $1-i$	$-2(1+i)$
Square of $1+i$	$2i$

24. (d) $\frac{\sqrt{3}+i}{1+\sqrt{3}i}=\frac{(\sqrt{3}+i)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)}$

$$=\frac{\sqrt{3}-3i+i+\sqrt{3}}{1+3}=\frac{2\sqrt{3}-2i}{4}=\frac{(\sqrt{3}-i)}{2}$$

25. (a) Given $2x=3+5i$

$$\Rightarrow x=\frac{3+5i}{2}$$

Consider $x^3=\frac{27+125i^3+225i^2+135i}{8}$

$$=\frac{27-125i-225+135i}{8}\quad\left(\because\begin{array}{l}i^2=-1\\ i^3=-i\end{array}\right)$$

$$=\frac{-198+10i}{8}=\frac{-99+5i}{4}$$

and $x^2=\frac{9+25i^2+30i}{4}$

$$=\frac{9-25+30i}{4}=\frac{-8+15i}{2}$$

Now, Consider $2x^3+2x^2-7x+72$

$$=\left(\frac{-99+5i}{2}\right)+(-8+15i)-\frac{7(3+5i)}{2}+72$$

$$=-\frac{99}{2}+\frac{5i}{2}-8+15i-\frac{21}{2}-\frac{35}{2}i+72$$

$$=\left(-\frac{99}{2}-8-\frac{21}{2}+72\right)+\left(\frac{5}{2}+15-\frac{35}{2}\right)i$$

$$=\frac{-99-16-21+144}{2}=\frac{8}{2}=4$$

26. (c) (A) Consider $\left[\frac{-1+\sqrt{-3}}{2}\right]^{29} \quad \left[\frac{-1-\sqrt{-3}}{2}\right]^{29}$

$\left[\frac{-1+\sqrt{3}i}{2}\right]^{29} \quad \left[\frac{-1-\sqrt{3}i}{2}\right]^{29}$

$= (\omega)^{29} + (\omega^2)^{29} = \omega^{27}\cdot\omega^2 + (\omega^3)^{19}\cdot\omega$

$= (\omega^3)^9\,\omega^2 + (\omega^3)^{19}\cdot\omega$

$= 1.\omega^2 + 1.\omega \qquad (\because \omega^3 = 1)$

$= \omega^2 + \omega = -1$

(R) $\omega^2 \neq -1$

A is true but R is false.

27. (a) Since, α is a complex root therefore

$\alpha^2 + \alpha + 1 = 0 \Rightarrow \alpha = \omega$ or ω^2

consider $\alpha^{31} = (\omega)^{31}$

$= (\omega^3)^{10}.\ \omega$

$= \omega \qquad (\because \omega^3 \quad 1)$

$= \alpha$

28. (d) Let $z = \frac{1+2i}{1-(1-i)^2}$

$\Rightarrow |z| = \frac{|1+2i|}{|1-(1-i)^2|} = \frac{|1+2i|}{|1-1-i^2+2i|} = \frac{|1+2i|}{|1+2i|} = 1$

29. (c) Consider $(-\sqrt{-1})^{4n+3} + (i^{41} + i^{-257})^9$

$(-i)^{4n \quad 3} \quad \left[(i^4)^{10}.i^1 \quad (i^3)^{-85}.i^{-2}\right]^9$

$(-i)^{4n \quad 3} \quad \left[i \quad \frac{1}{(i^3)^{85}}.\frac{1}{i^2}\right]^9 \quad (-i)^{4n \quad 3} \quad \left(i \quad \frac{1}{i}\right)^9$

$= -(-1)^{4n+3}\,(i)^{4n}\,(i)^3 + (i-i)^9 = -(1)(-i) + 0 = i$

30. (a) Let $z = 2\omega^2 + 3i$

Since, ω is the cube root of unity

$\therefore \omega = \frac{-1+\sqrt{3}i}{2}$ and $\omega^2 = \frac{-1-\sqrt{3}i}{2}$

$\therefore z = 2\omega^2 + 3i$

$= 2\left[\frac{-1-\sqrt{3}i}{2}\right] + 3i$

$= -1-\sqrt{3}i + 3i = -1 + (3-\sqrt{3})i$

$\overline{z} = -1-(3-\sqrt{3})i = -1+\sqrt{3}i - 3i = 2\left(\frac{-1+\sqrt{3}i}{2}\right) - 3i$

$= 2\omega - 3i$

31. (d) Given, $z + z^{-1} = 1 \Rightarrow z + \frac{1}{z} = 1$

$\Rightarrow z^2 - z + 1 = 0 \Rightarrow z = -\omega$ and $-\omega^2$

when $z = -\omega$

$\therefore z^{99} + z^{-99} = (-\omega)^{99} + (-\omega)^{-99} = -1 - 1 = -2$

when $z = -\omega^2$

$\therefore z^{99} + z^{-99} = (-\omega^2)^{99} + (-\omega^2)^{-99}$

$= -1 - 1 = -2$

32. (b) Consider,

$\frac{i+\sqrt{3}}{-i+\sqrt{3}} = \frac{\left(i+\sqrt{3}\right)^2}{\left(\sqrt{3}-i\right)\left(\sqrt{3}+i\right)}$ (By Rationalizing)

$= \frac{i^2+3+2\sqrt{3}i}{3+1}$ $\left(\text{using}\,(a-b)(a+b) = a^2-b^2\right)$

$= \frac{-1+3+2\sqrt{3}i}{4} = \frac{1+\sqrt{3}i}{2}$

$= \frac{-(-1-\sqrt{3}i)}{2} = -\omega^2 \qquad (\because i^2 = -1)$

and consider

$\frac{i-\sqrt{3}}{i+\sqrt{3}} = \frac{\left(i-\sqrt{3}\right)^2}{i^2-\left(\sqrt{3}\right)^2}$ (By Rationalizing)

$= \frac{i^2+3-2i\sqrt{3}}{-4} = \frac{2-2i\sqrt{3}}{-4} \quad \left(\because i^2 = -1\right)$

$= \frac{-1+i\sqrt{3}}{2} = \omega$

$\therefore \left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{200} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{200} + 1$

$= \left(-\omega^2\right)^{200} + \omega^{200} + 1$

$= \omega^{400} + \omega^{200} + 1 = \omega^{3\times133+1} + \omega^{3\times66+2} + 1$

$= \left(\omega^3\right)^{133}\omega + \left(\omega^3\right)^{66}\omega^2 + 1$

$= \omega + \omega^2 + 1 \qquad \left(\because \omega^3 = 1\right)$

$= 0$

33. (c) Given $x = \omega^2 - \omega - 2$

$\Rightarrow x + 2 = \omega^2 - \omega$

On squaring both sides, we get

$(x+2)^2 = \left(\omega^2-\omega\right)^2$

$\Rightarrow x^2 + 4x + 4 = \omega^4 + \omega^2 - 2\,\omega^3$

Add 3 on both side

$\Rightarrow x^2 + 4x + 4 + 3 = \omega + \omega^2 - 2 + 3 \left(\because \omega^3 = 1\right)$

$\Rightarrow x^2 + 4x + 7 = 1 + \omega + \omega^2$

$\Rightarrow x^2 + 4x + 7 = 0 \left(\because 1 + \omega + \omega^2 = 0\right)$

34. (b) Consider $\frac{1+x+iy}{1+x-iy}=\frac{(1+x+iy)(1+x+iy)}{(1+x-iy)(1+x+iy)}$ (By Rationalizing)

$=\frac{(1+x)^2+iy(1+x)+iy(1+x)-y^2}{1+x^2+2x+y^2}\ (\because i^2=-1)$

$=\frac{1+x^2+2x-y^2+2iy(1+x)}{2(1+x)}\ (\because x^2+y^2=1)$

$=\frac{1-y^2+2x+x^2+2iy(1+x)}{2(1+x)}$

$=\frac{2x^2+2x+2iy(1+x)}{2(1+x)}=x+iy \quad (\because 1-y^2=x^2)$

35. (a) Consider $\frac{1+2i}{1-(1-i)^2}=\frac{1+2i}{1-(1-1-2i)}=\frac{1+2i}{1+2i}=1$

$\therefore \left|\frac{1\ \ 2i}{1-\ 1-i^{\ 2}}\right|\ \ 1$

36. (d) Consider $\frac{1+i}{1-i}=\frac{(1+i)(1+i)}{(1-i)(1+i)}=\frac{(1+i)^2}{1-i^2}=\frac{(1+i)^2}{1+1}$

$=\frac{1+i^2+2i}{2}=\frac{2i}{2}=i$

$\therefore \left(\frac{1\ \ i}{1-i}\right)^n\ \ i^n$

Now, $i^n=1$ is possible for $n=4$.

37. (d) Consider $\left(\frac{1+2i}{2+i}\right)^2=\left(\frac{(1+2i)(2-i)}{(2+i)(2-i)}\right)^2$

$=\left(\frac{2+4i-i-2i^2}{4-i^2}\right)^2\ \ \left(\frac{4\ \ 3i}{5}\right)^2$

$=\frac{1}{25}(16\ \ 9i^2\ \ 24i)\ \ \frac{7}{25}\ \ \frac{24i}{25}$

$\therefore$ Conjugate of $\left(\frac{1+2i}{2+i}\right)^2=\frac{7}{25}-\frac{24}{25}i$

($\because$ conjugate of $a+ib=a-ib$)

38. (c) Consider $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)=\frac{\sqrt{3}\ \ i\ \ \sqrt{3}\ \ i}{\sqrt{3}-i\ \ \sqrt{3}\ \ i}$

$=\frac{\sqrt{3}\ \ i^{\ 2}}{\sqrt{3}^{\ 2}-i^{\ 2}}=\frac{3+i^2+2\sqrt{3}i}{3-i^2}$

$=\frac{2+2\sqrt{3}i}{4}=\frac{1+\sqrt{3}i}{2}=-\omega^2 \quad (\because i^2=-1)$

Now, $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6=(-\omega^2)^6=\omega^{12}=1\ (\because \omega^3=1)$

39. (b) Consider $\omega^{10}+\omega^{-10}=\omega^{10}+\frac{1}{\omega^{10}}$

$=(\omega^3)^3.\omega\ \ \frac{1}{\omega^{3\ \ 3}.\omega}=\omega+\frac{1}{\omega}=-1$

40. (d) We know, $\omega=\frac{1+i\sqrt{3}}{2}$ and

$\omega^2=\frac{1-i\sqrt{3}}{2}\Rightarrow 1-i\sqrt{3}=2\omega^2$

Consider $(-1+i\sqrt{3})^{48}=\left[-(1-i\sqrt{3})\right]^{48}$

$=(1-i\sqrt{3})^{48}$

$\Rightarrow (2\omega^2)^{48}=2^{48}.\ \omega^{96}=2^{48}(\omega^3)^{32}=2^{48}$

41. (b) Consider $1+i^2+i^4+i^6+...+i^{100}$

$=1+[i^2+i^4+i^6+...+i^{100}]$

$=1+[(-1)+(1)-1+...+1]$

$=1-1+1-1+...+1=1$

42. (c) $|z|=\sqrt{1^2\ \ \cos^2\frac{\pi}{5}\ \ 2\cos\frac{\pi}{5}\ \ \sin^2\frac{\pi}{5}}$

$=\sqrt{1^2\ \ 1\ \ 2\cos\frac{\pi}{5}}=\sqrt{2\left(1\ \ \cos\frac{\pi}{5}\right)}$

$=\sqrt{2\left(1+2\cos^2\frac{\pi}{10}-1\right)}=\sqrt{2\left(2\cos^2\frac{\pi}{10}\right)}$

$=2\cos\frac{\pi}{10}$

43. (a) Let $z=\frac{1}{1+3i}-\frac{1}{1-3i}$

$=\frac{(1-3i)-(1+3i)}{(1+3i)(1-3i)}\ \ \frac{-6i}{(1)^2-(3i)^2}=-\frac{6i}{10}=-\frac{3}{5}i$

$\therefore |z|=\sqrt{(0)^2+\left(-\frac{3}{5}\right)^2}\ \ \sqrt{\frac{9}{25}}\ \ \frac{3}{5}$

44. (d) Consider,

$(2-\omega+2\omega^2)^{27}=[2(1+\omega^2)-\omega]^{27}$

$=(-2\omega-\omega)^{27}\ [\because 1+\omega+\omega^2=0]$

$=(-3\omega)^{27}=-3^{27}.\omega^{27}$

$=(-3)^{27}.(\omega^3)^9=(-3)^{27}$

45. (a) $(1+i)^5 = {}^5C_0(1)^{5-0}i^0 + {}^5C_1(1)^{5-1}(i)^1 + {}^5C_2(1)^{5-2}(i)^2$
$+ {}^5C_3(1)^{5-3}(i)^3 + {}^5C_4(1)^{5-4}(i)^4 + {}^5C_5(1)^{5-5}(i)^5$
$= 1 + 5i + 10i^2 + 10i^3 + 5i^4 + i^5$ .....(1)
$(1-i)^5 = {}^5C_0 i^0 - {}^5C_1 i + {}^5C_2 i^2 - {}^5C_3 i^3 + {}^5C_4 i^4 - {}^5C_5 i^5$
$= 1 - 5i + 10i^2 - 10i^3 + 5i^4 - i^5$ .....(2)
By adding (1) and (2), we get
$(1+i)^5 + (1-i)^5 = 1 + 5i + 10i^2 + 10i^3 + 5i^4 + i^5 + 1 - 5i + 10i^2 - 10i^3 + 5i^4 - i^5.$
$= 2 + 20i^2 + 10i^4 = 2 - 20 + 10 = -8$

46. (b) Let $Z = x + iy = -2i$
Let square root of z be $a + ib$.
Then $\sqrt{x+iy} = a + ib$

$$\Rightarrow x + iy = (a+ib)^2 = (a^2 - b^2) + (2ab)i$$

$$\Rightarrow -2i = (a^2 - b^2) + i(2ab)$$

Equating real and imaginary part, we get
$a^2 - b^2 = 0$ and $2ab = -2 \quad \Rightarrow ab = -1$
Since, $ab < 0$ therefore

$$\sqrt{x \quad iy} \quad \left[\sqrt{\frac{\sqrt{x^2 \quad y^2} \quad x}{2}} - i\sqrt{\frac{\sqrt{x^2 \quad y^2} - x}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{\sqrt{4}+0}{2}} - i\sqrt{\frac{\sqrt{4}-0}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{2}{2}} - i\sqrt{\frac{2}{2}}\right] = \pm[1 - i]$$

47. (b) Given : $z = 1 + i\tan\alpha,\ \pi < \alpha < \frac{3\pi}{2}$

$$|z| \quad \sqrt{1^{\ 2} + \ \tan\alpha^{\ 2}} \quad \sqrt{\sec^2\alpha}$$

$|z| = \sec\alpha.$

Since, $\pi < \alpha < \frac{3\pi}{2}$
$\therefore$ Sec α lies in IIIrd quadrant
In 3rd quadrant, sec α is negative.

48. (b) $\left(\frac{1-i}{1+i}\right)^n = \left[\frac{(1-i)^2}{(1+i)(1-i)}\right]^n$

$$\left[\frac{1+i^2-2i}{1-i^2}\right]^n \quad \left(\frac{-2i}{2}\right)^n \quad -i^{\ n}$$

For $n = 3$

$$(-i)^n = (-i)^3 = -i \times -i \times -i = -i^3 = -(-i) = i$$

Which is purely imaginary with positive imaginary part.
Hence, $n = 3$.

49. (c) Since, α and β are the complex cube roots of unity therefore $1 + \alpha + \alpha^2 = 0 = 1 + \beta + \beta^2$
and $\alpha^3 = 1 = \beta^3$.
Consider $(1+\alpha)(1+\beta)(1+\alpha^2)(1+\beta^2)$
$= (1+\alpha)(1+\alpha^2)(1+\beta)(1+\beta^2)$
$= (1+\alpha^2+\alpha+\alpha^3)(1+\beta^2+\beta+\beta^3)$
$= (0+\alpha^3)(0+\beta^3)$
$= (\alpha^3)(\beta^3) = (1)(1) = 1$

50. (d) Since ω is a cube root of unity
$\therefore\ \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Let $f(x) = (x^3)^p + (x^3)^q . x + (x^3)^r . x^2$
Now, put $x = \omega$
$f(\omega) = (\omega^3)^p + (\omega^3)^q.\omega + (\omega^3)^r.\omega^2$
$= 1^p + 1^q.\omega + 1^r.\omega^2 = 1 + \omega + \omega^2 = 0$

51. (a) $z = \frac{1+2i}{2-i} - \frac{2-i}{1+2i}$

$$z = \frac{(1+2i)^2 - (2-i)^2}{(2-i)(1+2i)} = \frac{1+4i^2+4i-4-i^2+4i}{2+4i-i-2i^2}$$

$$= \frac{-3-4+8i+1}{4+3i} = \frac{-6+8i}{4+3i} = \frac{(-6+8i)(4-3i)}{16+9}$$

$$= \frac{-24+18i+32i-24i^2}{25} = \frac{50i}{25} = 2i$$

Consider,

$$z^2 + z\bar{z} = (2i)^2 + (2i)(-2i) = 4i^2 - 4i^2 = 0$$

52. (d) Given complex number is
$(1 - \sin\theta) + i\cos\theta \equiv a + ib$

Argument $\equiv \tan\theta = \frac{b}{a}$

$$\Rightarrow \tan\theta = \frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}{\left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right)^2}$$

$$= \frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}} = \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{\theta}{2}}$$

$$\tan\theta = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

Hence, argument $= \frac{\pi}{4} + \frac{\theta}{2}$

53. (b) $A + iB = \frac{4+2i}{1-2i} \times \frac{1+2i}{1+2i}$ (By Rationalizing)

$= \frac{4+2i+8i-4}{(1)^2-(2i)^2} = \frac{0+10i}{5} = 2i = 0+2i$

$\Rightarrow A = 0, B = 2$

54. (a) Let $z = x + y$ then $\bar{z} = x - iy$

Now $z = -\bar{z}$

$\Rightarrow (x + iy) = -(x - iy) \Rightarrow x + iy = -x + iy$

$\Rightarrow 2x = 0 \Rightarrow Re(z) = 0$

55. (b) Statement – 1

$(\omega^{10} + 1)^7 + \omega = 0$

$L.H.S = \left[\ \omega^{3^{\ 3}}\ \ \omega\ \ \ 1\right]^7\ \ \ \omega$

$\Rightarrow\ (\omega + 1)^7 + \omega\ \ (\because\ \omega^3 = 1)$

$\Rightarrow\ -[\omega^2]^7 + \omega = -(\omega^3)^4 .\omega^2. + \omega. = -\omega^2 + \omega \neq 0$

Hence, LHS $\neq$ RHS

Statement – 2

$L.H.S = (\omega^{105} + 1)^{10} = \left[\left(\omega^3\right)^{35} + 1\right]^{10} = 2^{10}$

where 2 is a prime number.

Hence, statement – 2 is correct.

56. (d) $\sum_{n=1}^{13}\left[i^n + i^{n+1}\right] = \sum_{n=1}^{13} i^n\,[1+i]$

$= (1+i)\left[i + i^2 + i^3 \ldots .\, i^{13}\right] = \frac{1\ \ i}{1-i}\, i\left[1 - i^{13}\right]$

$= \frac{(-1+i)\left(1-i^{13}\right)}{(1-i)} = \frac{-1+i^{13}+i-i^{14}}{(1-i)}$

$= \frac{-1\ \ (i^2)^6.i\ \ i-(i^2)^7}{(1-i)} = \frac{2i+2i^2}{1-i^2} = (i-1)$

57. (c) Let $z = \frac{\sqrt{2}+i}{\sqrt{2}-i} = \frac{\sqrt{2}+i}{\sqrt{2}-i} \times \frac{\sqrt{2}+i}{\sqrt{2}+i}$

$= \frac{\left(\sqrt{2}+i\right)^2}{\left(\sqrt{2}\right)^2 - (i)^2} = \frac{2\sqrt{2}\ i\ \ \ 1}{3}$

$\Rightarrow z\ \ \frac{2\sqrt{2}}{3} i\ \ \frac{1}{3}$

Now, $|z| = \sqrt{\left(\frac{2\sqrt{2}}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = 1$

58. (a) Consider $\sqrt{-i} = \sqrt{e^{-i\pi/2}} = \pm\, e^{-i\pi/4} = \pm\left(\frac{1-i}{\sqrt{2}}\right)$

59. (a) Let $z = -1 - i$

Since Real part of $z < 0$ and Imaginary part of $z < 0$ therefore argument lies in III[rd] quadrant

$\arg(z) = \pi + \tan^{-1}\left|\frac{-1}{-1}\right| = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

60. (a) $3 + 4i = 2^2 + i^2 + 2 \times 2 \times i = (2 + i)^2$

$\sqrt{3\ \ 4i} = 2 + i$

61. (d) The two complex numbers are

$P = x + iy$ and $Q = \alpha + i\beta$

Quotient $= \frac{P}{Q} = \frac{x+iy}{\alpha+i\beta},\ \left|\frac{P}{Q}\right| = \left|\frac{x+iy}{\alpha+i\beta}\right|$

$\frac{\sqrt{x^2\ \ y^2}}{\sqrt{\alpha^2+\beta^2}}\ \ \sqrt{\frac{x^2\ \ y^2}{\alpha^2+\beta^2}}\ \ \frac{|Z_1|}{|Z_2|}\ \ \left|\frac{Z_1}{Z_2}\right|$

Hence, the quotient of their modulus is equal to the quotient of their moduli.

62. (a) $|2z - 1| = |z - 2|$

$|2\,(x + iy) - 1| = |x + iy - 2|$

$|(2x - 1) + 2yi| = |(x - 2) + iy|$

$\sqrt{(2x-1)^2\ \ y^2} = \sqrt{(x-2)^2\ \ y^2}$

Squaring both sides

$4x^2 + 1 - 4x + 4y^2 = x^2 + 4 - 4x + y^2$

$\Rightarrow 3x^2 + 3y^2 = 3$

$\Rightarrow\ x^2 + y^2 = 1$

It is the equation of a circle.

$\therefore$ The point z describes a circle.

63. (a) Let $z = x + iy,\ \bar{z} = x - iy$

$|z + \bar{z}| = |z - \bar{z}|$

$|(x + iy) + (x - iy)| = |(x + iy) - (x - iy)|$

$|2x| = |2y|$

$x = \pm y$

64. (d) $\frac{(1\ \ i)(2\ \ i)}{3-i} = \frac{1\ \ 3i}{3-i}$

$= \frac{1\ \ 3i}{3-i} \times \frac{3\ \ i}{3\ \ i} = \frac{10i}{10} = i$ or $0 + i$

argument, $\theta = \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}\left(\tan\frac{\pi}{2}\right) = \frac{\pi}{2}$

65. (d) Let $z = K(\cos\theta + i\sin\theta)$

$|z| = \sqrt{K^2(\cos^2\theta + \sin^2\theta)} = 4$

$\therefore\ K = 4$

Again $Arg(z) = \frac{5\pi}{6}$

So, $\theta = \frac{5\pi}{6}$

Now, $z = 4\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$

$= 4\left(\frac{-\sqrt{3}}{2} + i\frac{(1)}{2}\right) = -2\sqrt{3} + 2i$

66. (a) Given $\dfrac{(1\ \ i)^{4n\ \ 5}}{(1-i)^{4n\ \ 3}}$

$= \dfrac{(1\ \ i)^{4n\ \ 3}.(1\ \ i)^2}{(1-i)^{4n\ \ 3}} = \left(\dfrac{1\ \ i}{1-i}\right)^{4n\ \ 3}.(1\ \ i)^2$

$= \left[\dfrac{(1+i)(1+i)}{(1-i)(1+i)}\right]^{4n+3} \bullet\ (1+i^2+2i)$

$= \left[\dfrac{1\ \ i^2\ \ 2i}{1\ \ 1}\right]^{4n\ \ 3}.2i = (i)^{4n+3}.\ 2i = 2(i)^{4n+4}$

$= 2.\ (i^{4(n+1)}) = 2$

67. (d) $z = \dfrac{-2(1+2i)}{3+i}$

$= \dfrac{-2-4i}{3+i} = \dfrac{-2-4i}{3+i} \times \dfrac{3-i}{3-i}$

$= \dfrac{-6+2i-12i+4i^2}{10}$

$= \dfrac{-6-10i-4}{10} = \dfrac{-10-10i}{10} = -1-i$

$z = -1-i = r(\cos\theta + i\sin\theta)$

On comparing real and imaginary part on both sides, we get

$r\cos\theta = -1$ ...(i)

$r\sin\theta = -1$ ...(ii)

On dividing eq. (ii) by (i), we get

$\dfrac{r\sin\theta}{r\cos\theta} = \dfrac{-1}{-1}$

$\tan\theta = 1 = \tan\dfrac{\pi}{4}$

$\Rightarrow \quad \theta = \dfrac{\pi}{4}$

$\therefore \quad \theta = \dfrac{\pi}{4}$

$\therefore$ Option (b) is correct.

68. (c) Here cube root of unity is 1, ω, ω^2

Now as we know that $1+\omega+\omega^2 = 0$ and $\omega^3 = 1$

$\omega^8 = (\omega^3)^2.\ \omega^2 = \omega^2$

$\omega^4 = (\omega^3).\ \omega = \omega$

Now, $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (-\omega^2)(-\omega)(1+\omega)(1+\omega^2)$

$= \omega^3(1+\omega^2+\omega+\omega^3)$

$= \omega^3(\omega^3) = (1)(1) = 1$

$\therefore$ Option (c) is correct.

69. (c) Let $\sqrt{i} = x+iy$

$i = (x+iy)^2$

$x^2-y^2+2xyi = 0+i$

$x^2-y^2 = 0;\ 2xy = 1$

Now, $(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$(x^2+y^2)^2 = 0+1$

$x^2+y^2 = 1$

$x^2-y^2 = 0$ ... (i)

$x^2+y^2 = 1$ ...(ii)

$2x^2 = 1$

$x^2 = \dfrac{1}{2} \Rightarrow x = \pm\dfrac{1}{\sqrt{2}}$

$y^2 = \dfrac{1}{2} \Rightarrow y = \pm\dfrac{1}{\sqrt{2}}$

$\therefore\ \sqrt{i} = \pm\left(\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i\right)$

$= \dfrac{1}{\sqrt{2}}(1+i)$ or $\dfrac{-1}{\sqrt{2}}(1+i)$.

70. (b) As we know that cube root of unity is 1, ω and ω^2

$\therefore \quad x^3-1 = (x-1)(x-\omega)(x-\omega^2)$

$\therefore$ Option (b) is correct.

71. (c) $\left[\dfrac{\sin\frac{\pi}{6} + i\left(1-\cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6} - i\left(1-\cos\frac{\pi}{6}\right)}\right]^3$

$= \left[\dfrac{2\sin\frac{\pi}{12}\cos\frac{\pi}{12} + i\left(2\sin^2\frac{\pi}{12}\right)}{2\sin\frac{\pi}{12}\cos\frac{\pi}{12} - i\left(2\sin^2\frac{\pi}{12}\right)}\right]^3$

$= \left[\dfrac{\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}}{\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}}\right]^3 = \left(\dfrac{e^{i\frac{\pi}{12}}}{e^{-i\frac{\pi}{12}}}\right)^3$

$= \left(e^{i\frac{\pi}{6}}\right)^3 = e^{i\times 3\times\frac{\pi}{6}} = e^{i\frac{\pi}{2}} = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2} = i$

$\therefore$ Option (c) is correct.

72. (b) $(\sin x + i\cos x)^3$

$= \sin^3 x + (i)^3 \cos^3 x + 3i(\sin x)(\cos x)(\sin x + i\cos x)$

$= \sin^3 x - i\cos^3 x + 3i\sin^2 x\cos x - 3\sin x\cos^2 x$

$= \sin^3 x - 3\sin x\cos^2 x - i\cos x(\cos^2 x + \sin^2 x)$

$= \sin x(\sin^2 x - 3\cos^2 x) - i\cos x(\cos^2 x + 3\sin^2 x)$

Real part of $(\sin x + i\cos x)^3$

$= \sin x(\sin^2 x - 3\cos^2 x)$

$= \sin x[\sin^2 x - 3(1 - \sin^2 x)]$

$= \sin x[4\sin^2 x - 3]$

$= 4\sin^3 x - 3\sin x$

$= -(3\sin x - 4\sin^3 x) = -\sin 3x$

$\therefore$ Option (b) is correct.

73. (d) Let $Z_1 = a_1 + ib_1$

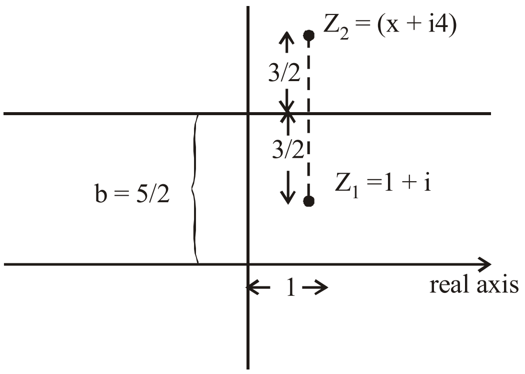
$Z_2 = a_2 + ib_2$

$|Z_1| = |Z_2|$

$$\sqrt{(a_1)^2 + (b_1)^2} = \sqrt{(a_2)^2 + (b_2)^2}$$

It is true for many values of a_1, a_2 & b_1, b_2. So a_1 must not equal to a_2, and b_1 must not equal to b_2.

74. (a) Let $z = a + bi$



$\Rightarrow \bar{z} = a - bi$

$\therefore i\bar{z} - iz = i[(a - bi) - (a + bi)] = 5$

$\Rightarrow i[-2bi] = 5$

$\Rightarrow b = \dfrac{5}{2}$

So from figure it is clear that

$x = 1,\ y = \dfrac{5}{2} + \dfrac{3}{2} = 4$

$z_2 = 1 + 4i$

75. (a) Let $z = x + iy$

$\bar{z} = x - iy$

$\Rightarrow z\bar{z} + (3 - i)z + (3 + i)\bar{z} + 1 = 0$

$\Rightarrow (x + iy)(x - iy) + (3 - i)(x + iy) + (3 + i)(x - iy) + 1 = 0$

$\Rightarrow x^2 + y^2 + 6x + 2y + 1 = 0$

$\Rightarrow (x + 3)^2 - 9 + (y + 1)^2 - 1 + 1 = 0$

$\Rightarrow (x + 3)^2 + (y + 1)^2 = (3)^2$

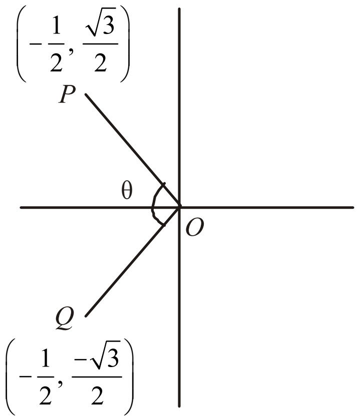
Centre $(-3, -1)$

radius $= 3$

76. (c) $\because \omega = \dfrac{-1}{2} + i\dfrac{\sqrt{3}}{2}$

$\&\ \omega^2 = \dfrac{-1}{2} - i\dfrac{\sqrt{3}}{2}$

$-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$ $\qquad (\because \omega \neq 1)$



P and Q are points on complex plane.

Angle between OP and OQ is

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

m_1 for line OP, $\qquad m_2$ for line OQ

$$m_1 = \frac{\frac{\sqrt{3}}{2} - 0}{\frac{-1}{2} - 0} \qquad m_2 = \frac{\frac{-\sqrt{3}}{2} - 0}{\frac{-1}{2} - 0}$$

$\Rightarrow m_1 = -\sqrt{3} \qquad \Rightarrow m_2 = \sqrt{3}$

$$\theta = \tan^{-1}\left[\frac{-\sqrt{3} - \sqrt{3}}{1 + (-\sqrt{3})(\sqrt{3})}\right]$$

$$= \tan^{-1}\left[\frac{-2\sqrt{3}}{-2}\right] = \pi - \tan^{-1}\sqrt{3} = \pi - \tan^{-1}\tan\frac{\pi}{3}$$

$$\theta = \pi - \frac{\pi}{3}$$

$\boxed{\theta = 120°}$

77. (a) $z = x + iy$

$$= \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$$

$$= \left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]^{-25}$$

$\because$ (Cosπ – i Sinπ)^n = Cos nπ – i Sin nπ

$\begin{bmatrix} \cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta \end{bmatrix}$

$$z = \left[\cos\left(\frac{25\pi}{4}\right) + i\sin\left(\frac{25\pi}{4}\right)\right]$$

$$= \left[\cos\left(6\pi \quad \frac{\pi}{4}\right) \quad i\sin\left(6\pi \quad \frac{\pi}{4}\right)\right]$$

$$= \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$$

$$= \frac{1+i}{\sqrt{2}}$$

$$\frac{z-\sqrt{2}}{z-i\sqrt{2}} \quad \frac{(1+i-2)\sqrt{2}}{\sqrt{2}(1+i-2i)}$$

$$= \frac{-1+i}{1-i} = -1$$

Amplitude $= \tan^{-1}\frac{b}{a} = \tan^{-1}\left(\frac{0}{-1}\right) = \tan^{-1}0$

$$= \tan^{-1}(\tan\pi) = \pi$$

Hence fundamental amplitude of $\left(\frac{z-\sqrt{2}}{z-i\sqrt{2}}\right)$ is π.

78. (c) Given $z^2 \quad i\bar{z}$

Let us suppose that $z = x+iy$

$\Rightarrow (x+iy)^2 = i(x-iy)$

$\Rightarrow x^2 - y^2 + 2xyi = ix + y$

Comparing real and imaginary part of both sides

$x^2 - y^2 = y$ and $2xy = x$.

Taking $2xy = x$

$(2y-1)x = 0$

$\Rightarrow x = 0$ and $y = \frac{1}{2}$

if $x = 0$

$\therefore y + y^2 = 0 \qquad (\because x^2 - y^2 = y)$

$y(y+1) = 0$

$y = 0$ and -1

$x = 0 \qquad y = 0$ and -1

If $y = \frac{1}{2}$

$$x^2 - \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow x^2 = \frac{3}{4}$$

$$\Rightarrow x = \pm\frac{\sqrt{3}}{2}$$

Since given numbers are non zero complex numbers.

So, $z_1 = 0 + (-1)i = -i$

$$z_2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i \qquad z_3 = \frac{-\sqrt{3}}{2} + \frac{1}{2}i$$

$$z_1 \quad z_2 \quad z_3 \quad (-i) \quad \left(\frac{\sqrt{3}}{2} \quad \frac{1}{2}i\right) \quad \left(-\frac{\sqrt{3}}{2} \quad \frac{1}{2}i\right) \quad 0$$

Hence $z_1 + z_2 + z_3 = 0$

79. (c) $z_1z_2z_3 = (-i)\left(\frac{\sqrt{3}+i}{2}\right)\left(\frac{-\sqrt{3}+i}{2}\right)$

$$= \frac{-i}{4}(i^2 - (\sqrt{3})^2)$$

$$= \frac{-i}{4}(-3-1) = i$$

Hence $z_1z_2z_3$ is purely imaginary.

$$z_1z_2 = -i\left(\frac{\sqrt{3}+i}{2}\right) = \frac{-\sqrt{3}i+1}{2}$$

$$z_2z_3 = \frac{(\sqrt{3}+i)(-\sqrt{3}+i)}{4}$$

$$= \left(\frac{-3-\sqrt{3}i+\sqrt{3}i+i^2}{4}\right) = -1$$

$$z_3z_1 = \frac{(-\sqrt{3}+i)(-i)}{2} = \frac{+\sqrt{3}i+1}{2}$$

$$z_1z_2 + z_2z_3 + z_3z_1 = \left(\frac{-\sqrt{3}i+1}{2}\right) + (-1) + \left(\frac{\sqrt{3}i+1}{2}\right)$$

$$= \left(\frac{-\sqrt{3}i+1+\sqrt{3}i+1}{2}\right) - 1$$

$$= 0 \in R$$

Hence $z_1z_2 + z_2z_3 + z_3z_1 = 0$ is purely real.

Hence both statements are correct.

80. (a) $\left|\frac{z-4}{z-8}\right| = 1$ and $\left|\frac{z}{z-2}\right| = \frac{3}{2}$

$\Rightarrow |z-4| = |z-8|$

Let $z = x + iy$

$|x + iy - 4| = |x + iy - 8|$

Squaring both sides, we get

$[(x-4)^2 + y^2] = [(x-8)^2 + y^2]$

$(x-4)^2 = (x-8)^2$

$\Rightarrow x^2 + 16 - 8x = x^2 + 64 - 16x$

$\Rightarrow 8x = 48 \Rightarrow x = 6$

when $\left|\frac{z}{z-2}\right|=\frac{3}{2}$

$\Rightarrow 2|z|=3|z-2|$

Squaring both sides, we get

$4(x^2+y^2)=9[(x-2)^2+y^2]$

$\Rightarrow 4x^2+4y^2=9x^2+36-36x+9y^2$

$\Rightarrow 5x^2+5y^2-36x+36=0$

as we know $x=6$

$5(6)^2+5y^2-36\times6+36=0$

$\Rightarrow 5y^2=0\Rightarrow y=0$

Hence $x=6$ and $y=0$.

$\Rightarrow z=6$

$|z|=6$

81. (d) $\left|\frac{z-6}{z+6}\right|=\left|\frac{6-6}{6+6}\right|=0$

82. (d) Cube root of unity are 1, $-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

ω_1 and ω_2 are two distinct cube roots of unity different from 1.

$\omega_1=-\frac{1}{2}\quad\frac{\sqrt{3}}{2}i$, $\omega_2=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$(\omega_1-\omega_2)^2=\left(-\frac{1}{2}\quad\frac{\sqrt{3}}{2}i\quad\frac{1}{2}\quad\frac{\sqrt{3}}{2}i\right)^2$

$=(\sqrt{3}i)^2=3i^2$

$(\omega_1-\omega_2)^2=-3$

83. (d) $\omega^{100}+\omega^{200}+\omega^{300}=(\omega^{99}\cdot\omega)+(\omega^{100})^2+(\omega^3)^{100}$

$=(\omega^{99}\cdot\omega)+(\omega^{99}\cdot\omega)^2+(\omega^3)^{100}$

$\because\ \omega^3\quad 1\qquad \omega^{99}=(\omega^3)^{33}\quad 1^{33}\quad 1$

$\Rightarrow\ \omega^{100}+\omega^{200}+\omega^{300}\quad(1\ \omega)\quad(1\ \omega)^2\quad 1^{100}$

$=\omega+\omega^2\quad 1\quad 1\quad \omega+\omega^2\quad 0$

84. (d) $\frac{z-1}{z+1}=\frac{x+iy-1}{x+iy+1}$

$\frac{z-1}{z+1}=\frac{x^2+y^2-1+2iy}{x^2+y^2+2x+1}$

$\Rightarrow \text{Re}\left(\frac{z-1}{z+1}\right)=\frac{x^2+y^2-1}{x^2+y^2+2x+1}=0$

$\Rightarrow x^2+y^2-1\quad 0$

$\Rightarrow x^2\quad y^2\quad 1$

Also, $z\bar{z}=x^2+y^2=1$

and $z\bar{z}\quad |z|^2$

$\Rightarrow |z|^2\quad 1$

$\Rightarrow |z|=1$

85. (a) $z\quad\left[\frac{\sqrt{3}}{2}\quad\frac{i}{2}\right]^{107}\quad\left[\frac{\sqrt{3}}{2}-\frac{i}{2}\right]^{107}$.

$\because\ \cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ & $\sin\frac{\pi}{6}=\frac{1}{2}$

$\Rightarrow\quad z=\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]^{107}+\left[\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right]^{107}$

Also, $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

$\Rightarrow\quad z\quad\cos\frac{107\pi}{6}\quad i\sin\frac{107\pi}{6}\quad\cos\frac{107\pi}{6}-i\sin\frac{107\pi}{6}$.

$\Rightarrow\quad \text{Im}(z)=0.$

86. (c) Let $z=x+iy$

$z^2+|z|=0$

$\Rightarrow x^2-y^2+2ixy+\sqrt{x^2+y^2}=0$

$\Rightarrow x^2-y^2+\sqrt{x^2+y^2}+i2xy=0+i0$

$\Rightarrow x^2-y^2+\sqrt{x^2+y^2}=0$ ...(1)

and $2xy=0\Rightarrow xy=0\Rightarrow x=0$ or $y=0$

Now : For $y=0$ in eq. (1) we get :

$x^2+\sqrt{x^2}=0$

$\Rightarrow x^2+|x|=0$

Clearly $x^2+|x|$ will always be greater than 0 for all $x>0$.

Let, $x\le 0$

$x^2+|x|=0$

$\Rightarrow x^2-x=0\Rightarrow x(x-1)=0$

$\Rightarrow x=0$ or $(x-1)=0$

$\Rightarrow x=0$ $(\because x\le 0)$

$\therefore$ **z = 0**

For $x=0$ in eq. (1) we get,

$-y^2+\sqrt{y^2}=0$

$-y^2+|y|=0$

If $y>0$, then

$-y^2+|y|=0$

$\Rightarrow -y^2+y=0$

$\Rightarrow y=0, y=1$

$\Rightarrow y=1$ $(\because y>0)$

$\therefore$ **z = i**

If $y<0$, then

$-y^2+|y|=0$

$\Rightarrow -y^2-y=0$

$\Rightarrow y=0, y=-1$

$\Rightarrow y=-1$ $(\because y<0)$

$\therefore$ **z = –i**

$\therefore$ There are only 3 distinct solutions.

87. (b) $X \quad \sqrt{\frac{1+\omega^2}{1+\omega}}$ $\quad (\because 1+\omega+\omega^2=0 \text{ and } \omega^3 \quad 1)$

$\Rightarrow X \quad \sqrt{\frac{-\omega}{-\omega^2}} \quad \sqrt{\frac{1}{\omega}} \quad \sqrt{\frac{\omega^3}{\omega}} \quad \sqrt{\omega^2}=\omega$

88. (a) $i^{2n}+i^{2n}.^{+1}+i^{2n}.^{+2}+i^{2n}.^{+3}$
$= i^{2n}+i^{2n}.i+i^{2n}.i^2+i^{2n}.i^3$
$= i^{2n}(1+i+i^2+i^3)$ [since, $i^2=-1$, $i^3=i^2.i=-i$]
$= i^{2n}(1+i-1-i)$
$= i^{2n}(0)$
$= 0$

89. (b) $\left(\frac{-1+i\sqrt{3}}{2}\right)^n+\left(\frac{-1-i\sqrt{3}}{2}\right)^n$

we know that cube roots of a unit use 1, ω, ω^2

$\omega=\frac{-1+i\sqrt{3}}{2}, \omega^2=\frac{-1-i\sqrt{3}}{2}$

So, $\left(\frac{-1+i\sqrt{3}}{2}\right)^n \quad \left(\frac{-1-i\sqrt{3}}{2}\right)^n \quad \omega^n+\omega^{2n}$

Given, n is not multiple of 3. So, n = 1, 2, 4, 5... In any case, $\omega^n+\omega^{2n}=\omega+\omega^2=-1$

90. (a) We know, if $Z=x+iy$, $|z|=\sqrt{x^2+y^2}$

Given, $\frac{1+2i}{1-(1-i)^2}=\frac{1+2i}{1-(1-2i+i^2)}=\frac{1+2i}{1+2i}=1$

Since, it is purely real number,
modulus = 1 and principal argument = 0

91. (c) $|z+4|\le 3$
$|z+1|=|z+4+(-3)|$
We know, $|z_1+z_2|\le|z_1|+|z_2|$
So, $|z+4+(-3)|\le|z+4|+|-3|$
$\le|3|+|-3|$
≤ 6
So, maximum value = 6.

92. (c) $z^2=2\bar{z}$
Let $z=x+iy \Rightarrow z^2=(x+iy)^2=x^2-y^2+2xyi$
$\therefore \quad z^2=2\bar{z}\Rightarrow x^2-y^2+2xyi=2(x-iy)$
$\Rightarrow \quad x^2-y^2=2x$; $2xy=-2y\Rightarrow 2(x+1)y=0.$
$x=-1$ and $y=0$
$\Rightarrow \quad (-1)^2-y^2=2(-1)$
$\Rightarrow \quad 1-y^2=-2$
$\Rightarrow \quad y^2=3\Rightarrow y= \quad \sqrt{3}$
$\therefore$ Roots are $-1 \quad \sqrt{3}\,i$ and $-1-\sqrt{3}\,i$.
and for $y=0$, $x^2-0=2x$
$x(x-2)=0$
$x=0$ and 2.
Hence, roots are 0, 2

93. (a) $A=\begin{bmatrix}4i-6 & 10i\\ 14i & 6+4i\end{bmatrix}$ and $K=\frac{1}{2i}$

$K=\frac{1}{2i}=\frac{i}{2i(i)}=\frac{i}{2i^2}=\frac{-i}{2}.$

$\therefore KA=\frac{-i}{2}\begin{bmatrix}4i-6 & 10i\\ 14i & 6+4i\end{bmatrix}$

$=\begin{bmatrix}(4i-6)\left(\frac{-i}{2}\right) & 10i\left(\frac{-i}{2}\right)\\ 14i\left(\frac{-i}{2}\right) & (6+4i)\left(\frac{-i}{2}\right)\end{bmatrix}$

$\begin{bmatrix}-2i^2+3i & -5i^2\\ -7i^2 & -3i-2i^2\end{bmatrix} \quad \begin{bmatrix}2 \quad 3i & 5\\ 7 & 2-3i\end{bmatrix}$

94. (b) $\left(\frac{1+i}{1-i}\right)^n=1$

We know, $i^2=-1$ and $i^4=1$.
Now, rationalise the denominator.

$\left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^n=\left(\frac{1+2i+i^2}{1-i^2}\right)^n=\left(\frac{1+2i-i}{1-(-1)}\right)^n=\left(\frac{2i}{2}\right)^n=-i^n$

The smallest positive integer for which $i^n=1$ is 4

95. (c) $\text{Re}(z^2-i)=2$
Let $z=x+iy$
Now, $z^2=(x+iy)^2=x^2+2ixy-y^2$
$z^2-i=x^2+2ixy-y^2-i$
$=x^2-y^2+i(2xy-1)$
But, $\text{Re}(z^2-i)=2$
i.e., $x^2-y^2=2$, which represents rectangular hyperbola.

96. (c) $z=x+iy=(-1-i)$
$\therefore x=-1, y=-1$
This lies in 3rd Quadrant.
$\therefore \arg(z)=\theta-\pi$

$=\tan^{-1}\left(\frac{y}{x}\right)-\pi$

$=\tan^{-1}\left(\frac{-1}{-1}\right)-\pi$

$=\tan^{-1}(1)-\pi$

$=\frac{\pi}{4}-\pi$

$\frac{-3\pi}{4}$

97. (c) Let $z = x + iy$

$\therefore z^2 + \alpha z + \beta = (x + iy)^2 + \alpha(x + iy) + \beta$

$= x^2 - y^2 + 2ixy + \alpha x + i\alpha y + \beta$

Given, $z^2 + \alpha z + \beta = 0$

$\therefore x^2 - y^2 + 2ixy + \alpha x + i\alpha y + \beta = 0$

$\Rightarrow x^2 - y^2 + \alpha x + \beta + i(2xy + \alpha y) = 0 + i.0$

Comparing real and imaginary parts, we get

$x^2 - y^2 + \alpha x + \beta = 0;$ ....(1)

$y(2x + \alpha) = 0$ ....(2)

$(2) \Rightarrow 2x + \alpha = 0$ $(\because y \neq 0)$

$\Rightarrow 2(1) + \alpha = 0$ $(\because$ Given $\text{Re}(z) = 1)$

$\Rightarrow \alpha = -2$

Now, (1)

$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0$

$\Rightarrow (1)^2 - y^2 + (-2)(1) + \beta = 0$

$\Rightarrow 1 - y^2 - 2 + \beta = 0$

$\Rightarrow -1 - y^2 + \beta = 0$

$\Rightarrow \beta = 1 + y^2$

Since, $y \in R$ and $y \neq 0$,

β is always greater than 1.

So, $\beta \in (1, \infty)$

98. (a) $|1 - 2i|^x = 5^x$

$$\Rightarrow \left(1^2 + -2^2 \right)^{\frac{1}{2} x} \quad 5^x \quad \because |x \quad iy| \quad \sqrt{x^2 \quad y^2}$$

$$\Rightarrow (5)^{\frac{x}{2}} = 5^x$$

$$\Rightarrow \frac{x}{2} = x \Rightarrow x = 2x$$

$\Rightarrow x = 0.$

There is no non zero integral solution.

99. (c) We know, it $|\alpha| = 1$

$\Rightarrow |\alpha|^2 = 1$

$\Rightarrow \alpha.\bar{\alpha} \quad 1$ ....(1)

$$\therefore \left|\frac{\alpha - \beta}{1 - \alpha\bar{\beta}}\right| \quad \left|\frac{\alpha - \beta}{\alpha.\bar{\alpha} - \alpha\bar{\beta}}\right| \quad \text{(from (1))}$$

$$\left|\frac{\alpha - \beta}{\alpha \ \bar{\alpha} - \bar{\beta}}\right|$$

$$\frac{\cancel{|\alpha - \beta|}}{|\alpha|\,\cancel{|\alpha - \beta|}} \qquad \because \text{since } |\bar{z}| \quad |z|$$

$$\frac{1}{|\alpha|} \quad \frac{1}{1} \quad 1$$

100. (a) $i^{1000} + i^{1001} + i^{1002} + i^{1003}$

$= i^{1000}(1 + i + i^2 + i^3)$

$i = \sqrt{-1}$

$\Rightarrow i^2 = -1$

$i^{1000} \ \cancel{1} \quad \cancel{i} - \cancel{1} - \cancel{i}$

$i^3 \quad i^2.i$

$= -i$

$= 0$

101. (b) Let $x \quad iy \quad \sqrt{3} \quad i$

Comparing real and imaginary parts, $x \quad \sqrt{3}, y \quad 1$

modulus-amplitude of $x + iy$ is $r(\cos\theta + \sin\theta)$, where

$$r \quad \sqrt{x^2 \quad y^2} \text{ and } \theta \quad \tan^{-1}\left(\frac{y}{x}\right)$$

$$r \quad \sqrt{\sqrt{3}^{\,2} \quad 1^{\,2}} \quad \sqrt{4} \quad 2$$

$$\theta \quad \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \frac{\pi}{6}$$

$\therefore$ modulus-amplitude is $2\left(\cos\frac{\pi}{6} \quad i\sin\frac{\pi}{6}\right)$.

102. (c) $\sum_{n \ 2}^{11} i^n \quad i^{n \ 1}$

We know, $i + i^2 + i^3 + i^4 = i - 1 - i + 1 = 0.$

Also, $i^3 + i^4 + i^5 + i^6 = 0$

The sum of 4 consecutive powers of i is always 0.

$$\therefore \sum_{n \ 2}^{11} i^n \quad i^{n \ 1} \quad i^2 \quad i^3 \quad i^3 \quad i^4 \quad i^4 \quad i^5 \quad i^5$$
$$i^6 \quad i^6 \quad i^7 \quad i^7 \quad i^8 \quad i^8 \quad i^9 \quad i^9 \quad i^{10} \quad i^{10}$$
$$i^{11} \quad i^{11} \quad i^{12}$$

$$i^2 \quad i^3 \quad i^4 \quad i^5 \quad i^6 \quad i^7 \quad i^8 \quad i^9 \quad i^{10} \quad i^{11}$$

$$i^3 \quad i^4 \quad i^5 \quad i^6 \quad i^7 \quad i^8 \quad i^9 \quad i^{10} \quad i^{11} \quad i^{12}$$

$= (i^2 + i^3 + 0) + (i^3 + i^4 + 0)$

$= i^2 + i^3 + i^3 + i^4$

$= i^2 + 2i^3 + i^4$

$= -1 + 2(-i) + 1$

$= -2i$

103. (b) From cube root of unity

$$\omega = \frac{-1 \quad \sqrt{3}i}{2} \text{ and}$$

$$\omega^2 = \frac{-1 - \sqrt{3}i}{2}$$

$$\omega^3 = \left(\frac{-1 + \sqrt{3}i}{2}\right)\left(\frac{-1 - \sqrt{3}i}{2}\right)$$

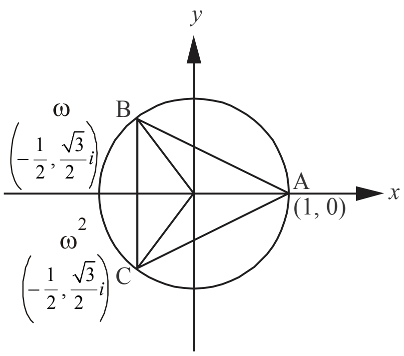
$$= \frac{(-1)^2 - (\sqrt{3}i)^2}{4} = \frac{4}{4} = 1$$

$\therefore \quad \omega^3 = 1$

Now, $(\omega)^{3n} + (\omega^2)^{3n}$

$= 1 + 1 = 2$

104. (c) Δ is equilateral.



105. (b) Given, $A = \{x \in z : x^3 - 1 = 0\}$
$B = \{x \in z : x^2 + x + 1 = 0\}$
The roots of $x^3 - 1 = 0$ are $1, \omega, \omega^2$
The roots of $x^2 + x + 1 = 0$ are ω, ω^2
$\therefore \quad A \cap B = \{1, \omega, \omega^2\} \cap \{\omega, \omega^2\} = \{\omega, \omega^2\}$

$$= \left\{\frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}\right\}$$

106. (a) $\begin{bmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{bmatrix} = 6 + 11i$

$\Rightarrow x[-i - (2i)(i)] - y[(-3i)(-i) - (2i)(1)] + 0 = 6 + 11i$
$\Rightarrow x(-i + 2) - y(-3 - 2i) = 6 + 11i$
$\Rightarrow -xi + 2x + 3y + 2yi = 6 + 11i$
$\Rightarrow (2x + 3y) + (-x + 2y)i = 6 + 11i$
$\therefore 2x + 3y = 6$ and $-x + 2y = 11$

Solving the equations,

$$\begin{aligned} 2\cancel{x} + 3y &= 6 \\ -2\cancel{x} + 4y &= 22 \\ \hline 7y &= 28 \Rightarrow y = 4 \end{aligned}$$

$2x + 3y = 6 \Rightarrow 2x + 12 = 6$
$\Rightarrow 2x = -6 \Rightarrow x = -3$
$\therefore x = -3, y = 4$

107. (b) Given equation, $z^3 + 2z^2 + 2z + 1 = 0$
$\Rightarrow z^3 - z^2 + z + z^2 - z + 1 + 2z^2 + 2z = 0$
$\Rightarrow (z + 1)(z^2 - z + 1) + 2z(z + 1) = 0$
$\Rightarrow (z + 1)(z^2 - z + 1 + 2z) = 0$
$\Rightarrow (z + 1)(z^2 + z + 1) = 0$
$\Rightarrow z = -1, \omega, \omega^2.$
$z^{2017} + z^{2018} + 1 = \omega + \omega^2 + 1 = 0$
$\therefore$ Common roots are ω, ω^2.

108. (c) $z = \dfrac{1+2i}{1-(1-i)^2} = \dfrac{1+2i}{1-(1-1-2i)}$

$= \dfrac{1+2i}{1+2i} = 1$

$\therefore |z| = 1$

109. (a) $z = \dfrac{1+2i}{1-(1-i)^2} = 1 = 1 + 0.i$

$\therefore$ Principal argument of $z = \tan\theta$

$= \dfrac{0}{1} = 0$

$\therefore \ \theta = 0°$

Binomial Theorem, Mathematical Induction

5

1. What is the coefficient of x^3 in $\frac{(3-2x)}{(1+3x)^3}$? *[2006-I]*
   (a) -272 (b) -540
   (c) -870 (d) -918
2. What are the last two digits of the number 9^{200} ? *[2006-II]*
   (a) 19 (b) 21 (c) 41 (d) 01
3. For any positive integer n, if $4^n - 3n$ is divided by 9, then what is the remainder ? *[2006-II]*
   (a) 8 (b) 6 (c) 4 (d) 1
4. What is the coefficient of x^5 in the expansion $(1-2x+3x^2-4x^3+.......\infty)^{-5}$? *[2007-I]*
   (a) $(10!)/(5!)^2$ (b) 5^{-5}
   (c) 5^5 (d) $10!/\{6!)(4!)\}$
5. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$?
   (a) $C(12, 7)\, x^3 y^{-3}$ (b) $C(12, 6)\, x^{-3} y^3$
   (c) $C(12, 7)\, x^{-3} y^3$ (d) $C(12, 6)\, x^3 y^{-3}$ *[2007-I]*
6. If x^4 occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then what is the value of r ? *[2007-II]*
   (a) 4 (b) 8 (c) 9 (d) 10
7. After simplification, what is the number of terms in the expansion of $[(3x+y)^5]^4-[(3x-y)^4]^5$? *[2007-II]*
   (a) 4 (b) 5 (c) 10 (d) 11
8. What is the coefficient of $x^3 y^4$ in $(2x+3y^2)^5$? *[2008-I]*
   (a) 240 (b) 360 (c) 720 (d) 1080
9. What is the approximate value of $(1.02)^8$?
   (a) 1.171 (b) 1.175
   (c) 1.177 (d) 1.179 *[2008-I]*
10. What is the last digit of $3^{3^{4n}}+1$, where n is a natural number? *[2008-I]*
    (a) 2 (b) 7
    (c) 8 (d) None of these
11. If t_r is the rth term in the expansion of $(1+x)^{101}$, then what is the ratio $\frac{t_{20}}{t_{19}}$ equal to ? *[2008-I]*
    (a) $\frac{20x}{19}$ (b) $83x$
    (c) $19x$ (d) $\frac{83x}{19}$
12. What is the value of *[2008-II]*
    $^8C_0 - {^8C_1} + {^8C_2} - {^8C_3} + {^8C_4} - {^8C_5} + {^8C_6} - {^8C_7} + {^8C_8}$
    (a) 0 (b) 1
    (c) 2 (d) 2^8
13. What is the term independent of x in the expansion of $\left(1+x+2x^3\right)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$? *[2009-I]*
    (a) 1/3
    (b) 17/54
    (c) 1/4
    (d) No such term exists in the expansion
14. What is the coefficient of x^4 in the expansion of $(1+2x+3x^2+4x^3+....)^{1/2}$? *[2009-II]*
    (a) 1/4 (b) 1/16
    (c) 1 (d) 1/128
15. Consider the following statements
    I. The coefficient of the middle term in the expansion of $(1+x)^8$ is equal to the middle term of $\left(x+\frac{1}{x}\right)^8$.
    II. The coefficient of the middle term in the expansion of $(1+x)^8$ is less than the coefficient of the fifth term in the expansion of $(1+x)^7$.

    Which of the above statements is/ are correct? *[2009-II]*
    (a) I only (b) II only
    (c) Both I and II (d) Neither I nor II
16. What is the sum of the coefficients of all the terms in the expansion of $(45x-49)^4$? *[2010-I]*
    (a) –256 (b) –100
    (c) 100 (d) 256
17. What is the coefficient of x^{17} in the expansion of $\left(3x-\frac{x^3}{6}\right)^9$? *[2010-II]*
    (a) $\frac{189}{8}$ (b) $\frac{567}{2}$
    (c) $\frac{21}{16}$ (d) None of these

18. What is the number of terms in the expansion of $(a+b+c)^n$, $n \in N$? [2010-II]
(a) $n+1$ (b) $n+2$
(c) $n(n+1)$ (d) $\frac{(n+1)(n+2)}{2}$

19. What is the sum of all the coefficients in the expansion of $(1+x)^n$? [2010-II]
(a) 2^n (b) $2^n - 1$
(c) 2^{n-1} (d) $2(n-1)$

20. What is the coefficient of x^4 in the expansion of $\left(\frac{1-x}{1+x}\right)^2$? [2010-II]
(a) -16 (b) 16
(c) 8 (d) -8

21. What is the middle term in the expansion of $\left(1-\frac{x}{2}\right)^8$? [2011-I]
(a) $\frac{35x^4}{8}$ (b) $\frac{17x^5}{8}$
(c) $\frac{35x^5}{8}$ (d) None of these

22. What is the ratio of coefficient of x^{15} to the term independent of x in $\left(x^2+\frac{2}{x}\right)^{15}$? [2011-II]
(a) 1/64 (b) 1/32
(c) 1/16 (d) 1/4

23. For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by [2011-II]
(a) 125 (b) 225
(c) 450 (d) None of the above

24. In the expansion of $[1+x]^n$, what is the sum of even binomial coefficients? [2012-I]
(a) 2^n (b) 2^{n-1}
(c) 2^{n+1} (d) None of the above

25. The value of the term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ is: [2012-I]
(a) 9 (b) 18
(c) 48 (d) 84

26. What is the sum of the coefficients in the expansion of $(1+x)^n$? [2013-I]
(a) 2^n (b) $2^n - 1$
(c) $2^n + 1$ (d) $n+1$

27. What is $\sum_{r\ 0}^{n} C(n,r)$ equal to ? [2013-II]
(a) $2^n - 1$ (b) n
(c) nl (d) 2^n

28. If $C(28, 2r) = C(28, 2r-4)$, then what is r equal to ? [2013-II]
(a) 7 (b) 8
(c) 12 (d) 16

29. Let n be a positive integer and
$(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$
What is $a_0 + a_1 + a_2 + \ldots + a_n$ equal to ? [2013-II]
(a) 1 (b) 2^n
(c) 2^{n-1} (d) 2^{n+1}

30 How many terms are there in the expansion of $(1+2x+x^2)^{10}$? [2013-II]
(a) 11 (b) 20
(c) 21 (d) 30

DIRECTIONS (Qs. 31-33): *For the next three (03) items that follow*

In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$ where n is a positive integer, the sum of the coefficients of x^5 and x^{10} is 0. [2014-I]

31. What is n equal to ?
(a) 5 (b) 10
(c) 15 (d) None of these

32. What is the value of the independent term ?
(a) 5005 (b) 7200
(c) –5005 (d) –7200

33. What is the sum of the coefficients of the two middle terms ?
(a) 0 (b) 1
(c) –1 (d) None of these

DIRECTIONS (Qs. 34-36) : *For the next three (03) items that follow*

Given that $C(n, r) : C(n, r+1) = 1 : 2$ and $C(n, r+1) : C(n, r+2) = 2 : 3$. [2014-I]

34. What is n equal to ?
(a) 11 (b) 12 (c) 13 (d) 14

35. What is r equal to ?
(a) 2 (b) 3 (c) 4 (d) 5

36. What is $P(n, r) : C(n, r)$ equal to ?
(a) 6 (b) 24 (c) 120 (d) 720

37. What is $\left(\frac{\sqrt{3}\ \ i}{\sqrt{3}-i}\right)^6$ equal to, where $i=\sqrt{-1}$? [2014-I]
(a) 1 (b) 1/6 (c) 6 (d) 2

DIRECTIONS (Qs. 38-42): *For the next five (05) items that follow*

Consider the expansion $\left(x^2\ \ \frac{1}{x}\right)^{15}$. [2014-II]

38. What is the independent term in the given expansion ?
(a) 2103 (b) 3003
(c) 4503 (d) None of these

39. What is the ratio of coefficient of x^{15} to the term independent of x in the given expansion ?
(a) 1 (b) 1/2
(c) 2/3 (d) 3/4

40. Consider the following statements :
1. There are 15 terms in the given expansion.
2. The coefficient of x^{12} is equal to that of x^3.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

41. Consider the following statements :
1. The term containing x^2 does not exist in the given expansion.
2. The sum of the coefficients of all the terms in the given expansion is 2^{15}.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

42. What is the sum of the coefficients of the middle terms in the given expansion ?
(a) C (15, 9) (b) C (16, 9)
(c) C (16, 8) (d) None of these

43. What is $\sum_{r=0}^{1} {}^{n+r}C_n$ equal to? *[2015-I]*
(a) ${}^{n+2}C_1$ (b) ${}^{n+2}C_n$
(c) ${}^{n+3}C_n$ (d) ${}^{n+2}C_{n+1}$

44. In the expansion of $\left(\sqrt{x} \quad \frac{1}{3x^2}\right)^{10}$ the value of constant term (independent of x) is *[2015-II]*
(a) 5 (b) 8
(c) 45 (d) 90

DIRECTIONS (Qs. 45-47): *For the next three (03) items that follow*

Consider the expansion of $(1 + x)^{2n+1}$

45. If the coefficients of x^r and x^{r+1} are equal in the expansion, then r is equal to *[2015-II]*
(a) n (b) $\frac{2n-1}{2}$
(c) $\frac{2n \quad 1}{2}$ (d) n + 1

46. The average of the coefficients of the two middle terms in the expansion is
(a) ${}^{2n+1}C_{n+2}$ (b) ${}^{2n+1}C_n$
(c) ${}^{2n+1}C_{n-1}$ (d) ${}^{2n}C_{n+1}$

47. The sum of the coefficients of all the terms in the expansion is
(a) 2^{2n-1} (b) 4^{n-1}
(c) 2×4^n (d) None of the above

48. The coefficient of x^{99} in the expansion of (x–1)(x–2)(x–3)..... (x – 100) is *[2015-II]*
(a) 5050 (b) 5000
(c) –5050 (d) –5000

49. What is ${}^{47}C_4 + {}^{51}C_3 + \sum_{j=2}^{5} {}^{52-j}C_3$ equal to? *[2016-II]*
(a) ${}^{52}C_4$ (b) ${}^{51}C_5$
(c) ${}^{53}C_4$ (d) ${}^{52}C_5$

50. The value of [C (7, 0) + C (7, 1)] + [C (7, 1) + C (7, 2)] + ... + [C (7, 6) + C (7, 7)] is *[2017-I]*
(a) 254 (b) 255
(c) 256 (d) 257

51. The expansion of $(x – y)^n$, $n \geq 5$ is done in the descending powers of x. If the sum of the fifth and sixth terms is zero, then $\frac{x}{y}$ is equal to *[2017-I]*
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$
(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

52. The number of terms in the expansion of $(x + a)^{100} + (x – a)^{100}$ after simplification is *[2017-II]*
(a) 202 (b) 101 (c) 51 (d) 50

53. In the expansion of $(1 + x)^{50}$, the sum of the coefficients of odd powers of x is *[2017-II]*
(a) 2^{26} (b) 2^{49}
(c) 2^{50} (d) 2^{51}

54. If $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^a + b}{3}$ then a and b are respectively *[2017-II]*
(a) n, 2 (b) n, 3
(c) n + 1, 2 (d) n + 1, 3

55. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of | z | is equal to *[2017-II]*
(a) $1+\sqrt{3}$ (b) $1+\sqrt{5}$
(c) $1-\sqrt{5}$ (d) $\sqrt{5}-1$

56. If $n \in N$, then $121^n – 25^n + 1900^n – (–4)^n$ is divisible by which one of the following? *[2018-I]*
(a) 1904 (b) 2000 (c) 2002 (d) 2006

57. In the expansion of $(1 + x)^{43}$, if the coefficients of $(2r + 1)^{th}$ and $(r + 2)^{th}$ terms are equal, then what is the value of $r(r \neq 1)$? *[2018-I]*
(a) 5 (b) 14 (c) 21 (d) 22

58. If the coefficients of a^m and a^n in the expansion of $(1 + a)^{m+n}$ are α and β, then which one of the following is correct? *[2018-I]*
(a) $\alpha = 2\beta$ (b) $\alpha = \beta$
(c) $2\alpha = \beta$ (d) $\alpha = (m+n)\beta$

59. What is the number of non-zero terms in the expansion of
$1 \quad 2\sqrt{3}\ x^{\ 11} \quad 1-2\sqrt{3}\ x^{\ 11}$ (after simplification)? *[2018-I]*
(a) 4 (b) 5 (c) 6 (d) 11

60. What is C(n, r) + 2C(n, r – 1) + C(n, r – 2) equal to? *[2018-I]*
(a) C(n + 1, r) (b) C(n – 1, r + 1)
(c) C(n, r + 1) (d) C(n + 2, r)

61. What is the coefficient of the middle term in the binomial expansion of $(2 + 3x)^4$? *[2018-II]*
(a) 6 (b) 12 (c) 108 (d) 216

62. Let the coefficient of the middle term of the binomial expansion of $(1 + x)^{2n}$ be α and those of two middle terms of the binomial expansion of $(1 + x)^{2n-1}$ be β and γ. Which one of the following relations is correct? *[2018-II]*
(a) $\alpha > \beta + \gamma$ (b) $\alpha < \beta + \gamma$
(c) $\alpha = \beta + \gamma$ (d) $\alpha = \beta\gamma$

63. If C(20, n + 2) = C(20, n – 2), then what is n equal to ? *[2019-I]*
(a) 8 (b) 10 (c) 12 (d) 16

64. What is the number of terms in the expansion of $[(2x – 3y)^2 (2x + 3y)^2]^2$? *[2019-I]*
(a) 4 (b) 5 (c) 8 (d) 16

65. In the expansion of $(1 + ax)^n$, the ûrst three terms are respectively 1, 12x and $64x^2$. What is n equal to? *[2019-I]*
(a) 6 (b) 9 (c) 10 (d) 12

ANSWER KEY

1	(d)	8	(c)	15	(a)	22	(b)	29	(b)	36	(b)	43	(a,d)	50	(a)	57	(b)	64	(b)
2	(d)	9	(a)	16	(d)	23	(b)	30	(c)	37	(a)	44	(a)	51	(b)	58	(b)	65	(b)
3	(d)	10	(d)	17	(a)	24	(b)	31	(c)	38	(b)	45	(a)	52	(c)	59	(c)		
4	(a)	11	(d)	18	(d)	25	(d)	32	(c)	39	(a)	46	(b)	53	(b)	60	(d)		
5	(d)	12	(a)	19	(a)	26	(a)	33	(a)	40	(b)	47	(c)	55	(d)	61	(d)		
6	(c)	13	(b)	20	(b)	27	(d)	34	(d)	41	(c)	48	(c)	55	(b)	62	(c)		
7	(c)	14	(c)	21	(a)	28	(b)	35	(c)	42	(c)	49	(a)	56	(b)	63	(b)		

HINTS & SOLUTIONS

1. (d) $\frac{(3-2x)}{(1+3x)^3} = (3-2x)(1+3x)^{-3}$

$= (3-2x)\left(1-9x+\frac{(-3)(-4)}{2!}.9x^2 + \frac{(-3)(-4)(-5)}{3!}.27x^3+....\right)$

[Expanding $(1+3x)^{-3}$]

$= (3-2x)(1-9x+54x^2-270x^3+.........)$

$\therefore$ Coefficient of $x^3 = -270\times 3 - 2\times 54$

$= -810-108 = -918$

2. (d) Using binomial theorem $9^{200} = (1+8)^{200}$

$= 1 + 8.200 + \frac{200\times 199}{2!}\times 8^2 + ...$

$= 1+1600+1273600+...$

From above, it is clear that the last two digits of the number 9^{200} are 01.

3. (d) Using binomial theorem. $4^n - 3n = (1+3)^n - 3n$

$= 1 + n.3 + \frac{n(n-1)}{2!}3^2 + ... - 3n$

$= 1 + \frac{n(n-1)}{2!}.3^2 + \frac{n(n-1)(n-2)}{3!}.3^3 + ...$

$\Rightarrow 4^n - 3n = 9\left\{\frac{n(n-1)}{2!}+...\right\}+1$

Thus, when $4^n - 3n$ is divided by 9, the remainder is 1.

4. (a) $1-2x+3x^2-4x^3+.....$

$= (1+x)^{-2}$, so, $(1-2x+3x^2-4x^3+...\infty)^{-5}$

$= ((1+x)^{-2})^{-5} = (1+x)^{10} \Rightarrow T_{r+1} = {}^{10}C_r x^r$

Putting r = 5, coefficient of $x^5 = {}^{10}C_5 = \frac{10!}{5!5!} = \frac{10!}{(5!)^2}$

5. (d) In the expansion of $\left(\frac{x\sqrt{y}}{3} - \frac{3}{y\sqrt{x}}\right)^{12}$, then middle term is $\frac{12}{2}+1 = 7^{th}$ term. $(r+1)_{th}$ term,

$T_{r+1} = {}^{12}C_r\left[\frac{x\sqrt{y}}{3}\right]^{12-r}.\left(-\frac{3}{y\sqrt{x}}\right)^r$

$\therefore \quad T_7 = T_{6+1} = {}^{12}C_6\left(\frac{x\sqrt{y}}{3}\right)^6\left(-\frac{3}{y\sqrt{x}}\right)^6$

$= {}^{12}C_6\frac{x^6y^3}{y^6x^3} = {}^{12}C_6\, x^3y^{-3} = C(12,6)x^3y^{-3}$

6. (c) In the expansion of $\left(x^4 \quad \frac{1}{x^3}\right)^{15}$, let T_r is the n_{th} term

$T_r = 15_{C_{r-1}}(x^4)^{15-r\ 1}\left(\frac{1}{x^3}\right)^{r-1}$

$= 15_{C_{r-1}}\, x^{64-4r-3r\ 3} = 15_{C_{r-1}}\, x^{67-7r}$

x^4 occurs in this term

$\Rightarrow \quad 4 = 67-7r$

$\Rightarrow \quad 7r = 63$

$\Rightarrow \quad r = 9.$

7. (c) Given expression is :

$[(3x+y)^5]^4 - [(3x-y)^4]^5 = [(3x+y)]^{20} - [(3x-y)]^{20}$

First and second expansion will have 21 terms each but odd terms in second expansion be Ist, 3rd, 5th.....21st will be equal and opposite to those of first expansion.

Thus, the number of terms in the expansion of above expression is 10.

8. (c) $T_r = {}^nC_{r-1}\,(2x)^{r-1}(3y^2)^{n-r+1}$

$T_4 = = {}^5C_3\,(2x)^3\,(3y^2)^2$

$= \frac{5!}{3!2!}2^3.x^3.9y^4 = \frac{5.4}{2.1}\times 8\times 9\times x^3y^4 = 720\,x^3\,y^4$

$\therefore$ Coefficient of $x^3y^4 = 720$

9. (a) $(1.02)^8 = (1+0.02)^8$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$

$n = 8, x = 0.02$

$(1+0.02)^8$

$= 1 + 8\times 0.02 + \frac{8\times 7}{2!}.(0.02)^2 + \frac{8.7.6}{3!}(.02)^3$

Neglecting higher terms

$= 1 + 0.16 + 28\times 0.0004 + 56\times 0.000008$

$\simeq 1 + 0.16 + 0.0112 = 1.171$

10. (d) In 3^n, last digit is 3, if n = 1, 9 if n = 2, 7 if n = 3 and 1 if n = 4 and it is repeated after than

Given expression is $3^{3^{4n}} + 1$

Let $x = 3^{3^{4n}} + 1 = 3^{81n} + 1$

$\Rightarrow x = 3^{80n}.3^n + 1$

Last digit of x will be decided by 3^n since 3^{80n} has power multiple of 4.

If n = 1 last digit is 3 + 1 = 4

n = 2 last digit is $3^2 + 1 = 9 + 1 = 10$

So, last digit is zero.

n = 3 last digit is $3^3 + 1 = 27 + 1 = 28$

last digit is 8.

If n = 4 last digit is $3^4 + 1 = 81 + 1 = 82$

last digit is 2.

So, there is no definite value of last digit.

11. (d) We find r_n term :

t_r is the rth term in the expansion of $(1+x)^{101}$.

$t_r = {}^{101}C_{r-1}.\,(x)^{(r-1)}$

$\therefore \frac{t_{20}}{t_{19}} \quad \frac{{}^{101}C_{19}}{{}^{101}C_{18}}.\frac{x^{19}}{x^{18}} \quad \frac{{}^{101}C_{19}x}{{}^{101}C_{18}} \quad = \frac{\frac{101!}{19!82!}}{\frac{101!}{18!83!}}x = \frac{83x}{19}$

12. (a) $(1-x)^n = {}^nC_0 - {}^nC_1(x) + {}^nC_2x^2 - {}^nC_3x^3 + ... + (-1)^n\,{}^nC_n$

Put $x = 1$ and $n = 8$

$\therefore (1-1)^8 = {}^8C_0 - {}^8C_1 + {}^8C_2 - {}^8C_3 + ... + {}^8C_8$

$\Rightarrow ({}^8C_0 - {}^8C_1 + {}^8C_2 - {}^8C_3 + ... + {}^8C_8) = 0$

13. (b) Given expansion is

$(1 \quad x \quad 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$(1 \quad x \quad 2x^3)\left[\left(\frac{3}{2}x^2\right)^9 - {}^9C_1\left(\frac{3}{2}x^2\right)^8.\frac{1}{3x}...........\right.$

$\left. + {}^9C_6\left(\frac{3}{2}x^2\right)^3\left(\frac{1}{3x}\right)^6 - {}^9C_7\left(\frac{3}{2}x^2\right)^2\left(\frac{1}{3x}\right)^7 ....\right]$

In the second bracket we have to search out terms of x^o and $\frac{1}{x^3}$ which when multiplied with the terms 1 and $2x^3$ in the first bracket will give a term independent of x. The term containing $\frac{1}{x}$ will not occur in the 2nd bracket.

$\therefore$ Term independent of x

$= 1 - \left[{}^9C_6\frac{3^3}{2^3}.\frac{1}{3^6}\right] - 2x^3\left[{}^9C_7\frac{3^2}{2^2}.\frac{1}{3^7}.\frac{1}{x^3}\right]$

$= \left[\frac{9.8.7}{1.2.3}.\frac{1}{8.27}\right] - 2\left[\frac{9.8}{1.2}.\frac{1}{4.243}\right]$

$= \frac{7}{18} - \frac{2}{27} \quad \frac{17}{54}$

14. (c) Consider $(1 + 2x + 3x^2 + 4x^3 + ...)^{1/2} = (1-x^{-2})^{1/2}$

As we know that

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + ...$

$\Rightarrow (1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + ...$

$\therefore$ Required coefficient of x^4 is 1.

15. (a) **Statement I :** Given expansion is $(1+x)^8$

Since, $n = 8$ is even

$\therefore \left(\frac{n}{2}+1\right)$ th term is the middle term.

ie- $\left(\frac{8}{4}+1\right)^{th} = 5^{th}$ term = middle term

Now, 5^{th} term = ${}^8C_4x^41^4 \quad {}^8C_4x^4$

Coeff of 5^{th} term (middle term) = ${}^8C_4$.

Now, consider the expansion $\left(x+\frac{1}{x}\right)^8$

It's middle term = 5^{th} term

and 5^{th} term = ${}^8C_4x^4\left(\frac{1}{x}\right)^4 = {}^8C_4$

Hence, statement I is correct.

Statement-II : Coeff. of middle term in $(x+1)^8$ is

$${}^8C_4 = \frac{8!}{4!4!} = 70$$

Coeff of 5^{th} term in $(1+x)^7 = {}^7C_4 = \frac{7!}{4!3!} = 35$

Hence, statement II is incorrect.

16. (d) Given expansion is $(45x-49)^4$.
To find the sum of the coefficients of all the terms in the expansion, we have to put $x = 1$ in the expansion.
Thus, required sum of coefficients $= (45-49)^4$
$= (-4)^4 = 256$

17. (a) Given expansion is

$$\left(3x - \frac{x^3}{6}\right)^9 \text{ where } a = 3x,\ b = \frac{-x^3}{6},\ n = 9$$

Now, General Term $= T_{r+1} = {}^nC_r\,(a)^{n-r}.\ b^r$

$$= {}^9C_r\,(3x)^{9-r}\left(\frac{-x^3}{6}\right)^r = {}^9C_r\,.\,3^{9-r}\,x^{9-r}\,.\,\frac{(-1)^r x^{3r}}{6^r}$$

$$= {}^9C_r\,3^{9-r}(-1)^r\,\frac{x^{9+2r}}{6^r}$$

We can get coeff of x^{17} when

$9 + 2r = 17$

$\Rightarrow\ 2r = 17 - 9$

$\Rightarrow\ r = \frac{8}{2} = 4$

Hence, required coefficient

$$= {}^9C_4\,\frac{3^5}{6^4} = \frac{126 \times 3}{16} = \frac{189}{8}$$

18. (d) Required number of terms in $(a+b+c)^n$

$$= {}^{n+2}C_2\ \frac{(n+2)!}{2!n!} = \frac{(n+1)(n+2)}{2}$$

19. (a) Given expansion is $(1+x)^n$.
Put $x = 1$, we get
Required sum $= (1+1)^n = 2^n$

20. (b) Consider $\left(\frac{1-x}{1+x}\right)^2 = (1-x)^2(1+x)^{-2}$

$= (1-2x+x^2)(1+x)^{-2}$

$= (1-2x+x^2)(1-2x+3x^2-4x^3+5x^4 - .....)$

$\therefore$ Coefficient of x^4 in $\left(\frac{1-x}{1+x}\right)^2 = 5+8+3 = 16$

21. (a) Since $n = 8$ is even number therefore middle term

$= \left(\frac{n}{2}+1\right)^{th}$ term $= (4+1) = 5^{th}$ term

Hence, $T_5 = {}^8C_4(1)^4\left(-\frac{x}{2}\right)^4$

$$= \frac{8!}{4!4!} \times \frac{x^4}{16} = \frac{8\times7\times6\times5}{4\times3\times2\times1}.\frac{x^4}{16} = \frac{70x^4}{16} = \frac{35x^4}{8}$$

22. (b) Given expansion is $\left(x^2 + \frac{2}{x}\right)^{15}$

$$T_{r+1} = {}^{15}C_r\,(x^2)^{15-r}\left(\frac{2}{x}\right)^r$$

$= {}^{15}C_r x^{30-2r}\,2^r.x^{-r} = {}^{15}C_r.\ x^{30-3r}.2^r$

Now, Above term will be independent of x when $30 - 3r = 0 \Rightarrow r = 10$

$\therefore$ Term independent of x $= {}^{15}C_{10}2^{10}$

Now , coeff of x^{15}

When $30 - 3r = 15 \Rightarrow r = 5$

$\therefore$ Required coeff $= {}^{15}C_5 2^5$

Thus, Required Ratio $= \frac{{}^{15}C_5.2^5}{{}^{15}C_{10}.2^{10}}$

$$= \frac{\frac{15!}{5!(10!)}}{\frac{15!}{10!5!}\times 2^5} = \frac{1}{2^5} = \frac{1}{32}$$

23. (b) Let P(n) : $2^{4n} - 15n - 1$
Put $n = 2$
P(2) $= 2^8 - 30 - 1 = 225$ which is divisible by 225.
Let us assume,
P(n) is true for $n = k$ is P(k) : $2^{4k} - 15k - 1$ is divisible by 225.

$\Rightarrow 2^{4k} - 15k - 1 = 225\lambda,\ \lambda \in R, k \in N$ ...(i)

To prove for $n = k + 1$
Consider

$2^{4k+4} - 15k - 15 - 1 = 2^{4k}.2^4 - 15k - 16$

$= 2^4\,[225\lambda + 1 + 15k] - 15k - 16$ (from (i))

$= 2^4 \cdot 225\lambda \cdot 2^4 + 15.2^4 \cdot k - 15k - 16$

$= 24 \cdot 225\lambda + 225k$

$= 225[2^4\lambda + k]$

$= 225\,r$ where

$r = 2^4\lambda + k$ is a constant

Hence, $2^{4n} - 15n - 1$ is divisible by 225.

24. (b) Sum of all binomial coefficients $= (1+1)^n = 2^n$

$\therefore$ Sum of even binomial coefficient $= \dfrac{2^n}{2} = 2^{n-1}$

25. (d) $\left(x^2 - \dfrac{1}{x}\right)^9$

$t_{r+1} = {}^9C_r \left(x^2\right)^{9-r} \left(\dfrac{-1}{x}\right)^r$

${}^9C_r x^{18-2r} \cdot (-1)^r \cdot x^{-r}$

$= \;{}^9C_r \;\; x^{\;18-3r} \;\; -1^{\;r}$ ...(1)

Term will be independent of x when

$18 - 3r = 0$

$r = 6$

Put r = 6, in [1]

$t_7 = {}^9C_6 (-1)^6 = \dfrac{9!}{6!3!} = 84$

26. (a) Given expansion is $(1 + x)^n$.
Put x = 1, we get
Sum of coefficient $= 2^n$.

27. (d) We know that, $(1+x)^n = {}^nC_0 + {}^nC_1^x + {}^nC_2^{x^2} + ... + {}^nC_n^{x^n}$
For x = 1, $(1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$\therefore \sum_{r\;\;0}^{n} c(n, r) = 2^n$

28. (b) $C(28, 2r) = C(28, 2r-4)$
${}^{28}C_{2r} = {}^{28}C_{2r-4}$
$\Rightarrow 2r + 2r - 4 = 28$
$\Rightarrow 4r = 32$
$\Rightarrow r = 8$

29. (b) $(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + ... + {}^nC_n\, x^n$
Putting x = 1
$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$
$\Rightarrow a_0 + a_1 + ... a_n = 2^n$
[here ${}^nC_0 = a_0,\ {}^nC_1 = a_1, ..., {}^nC_n\, a_n$]

30. (c) $(1 + 2x + x^2)^{10} = [(x+1)^2]^{10} = (x+1)^{20}$
Number of terms in the expansion of $(x+1)^{20}$
$= 20 + 1 = 21$

Sol. (31-33) $\left(x^3 - \dfrac{1}{x^2}\right)^n$

General term, $T_{r+1} = {}^nC_r (x^3)^{n-r} \cdot \left(-\dfrac{1}{x^2}\right)^r$

$= {}^nC_r . x^{(3n-3r)} . (-1)^r . x^{-2r}$

$= {}^nC_r . (-1)^r . x^{(3n-5r)}$ ...(i)

For the coefficient x^5
Put $3n - 5r = 5$
$5r = 3n - 5$

$\therefore\ r = \dfrac{3n}{5} - 1$

$\therefore$ Coefficient of $x^5 = {}^nC_{\left(\frac{3n}{5}-1\right)} (-1)^{\left(\frac{3n}{5}-1\right)}$

For the coefficient of x^{10}
Put $3n - 5r = 10$
$5r = 3n - 10$

$\therefore\ r = \dfrac{3n}{5} - 2$

$\therefore$ Coefficient of $x^{10} = {}^nC_{\left(\frac{3n}{5}-2\right)} (-1)^{\left(\frac{3n}{5}-2\right)}$

The sum of the coefficient of x^5 and $x^{10} = 0$

$\Rightarrow {}^nC_{\left(\frac{3n}{5}-1\right)} (-1)^{\left(\frac{3n}{5}-1\right)} + {}^nC_{\left(\frac{3n}{5}-2\right)} (-1)^{\left(\frac{3n}{5}-2\right)} = 0$

$\Rightarrow$

$(-1)^{\frac{3n}{5}} \left[{}^nC_{\left(\frac{3n}{5}-1\right)} . (-1)^{-1} + {}^nC_{\left(\frac{3n}{5}-2\right)} . (-1)^{(-2)} \right] = 0$

$\Rightarrow -{}^nC_{\left(\frac{3n}{5}-1\right)} + {}^nC_{\left(\frac{3n}{5}-2\right)} = 0$ ...(ii)

31. (c) From equation (ii)

${}^nC_{\left(\frac{3n}{5}-2\right)} \quad {}^nC_{\left(\frac{3n}{5}-1\right)}$

$\Rightarrow n = \left(\dfrac{3n}{5} - 2\right) + \left(\dfrac{3n}{5} - 1\right)$

$\left[\because {}^nC_x = {}^nC_y \Rightarrow n = x + y\right]$

$\Rightarrow n = \dfrac{6n}{5} - 3 \Rightarrow \dfrac{6n}{5} - n = 3$

$\Rightarrow \dfrac{n}{5} = 3 \qquad \therefore n = 15$

32. (c) For the independent term,
put $3n - 5r = 0$ [from eq. (i)]
$\Rightarrow 5r = 3n = 3 \times 15$
$5r = 3 \times 3 \times 5$
$r = 9$
Putting the value of r in eq. (i), we get
$T_{9+1} = {}^{15}C_9 . (-1)^9 . x^{(3 \times 15 - 5 \times 9)}$
$\Rightarrow T_{10} = -{}^{15}C_9 . x^0 = -{}^{15}C_9$
$\Rightarrow T_{10} = -{}^{15}C_6 \quad \left[\because {}^nC_r \quad {}^nC_{n-r}\right]$

$= \frac{-15!}{6!9!}$ $\left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$

$= -5005$

33. (a) $n = 15$

Total term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$ is 16.

$\therefore$ middle term = 8^{th} term and 9^{th} term

$T_8 = T_{(7+1)} = {}^{15}C_7 \cdot (-1)^7 \cdot x^{(3\times15-5\times7)}$

$= -{}^{15}C_7 \cdot x^{10}$ (from eq. (i))

$T_9 = T_{(8+1)} = {}^{15}C_8 \cdot (-1)^8 \cdot x^{(3\times15-5\times8)}$

$= -{}^{15}C_8 \cdot x^5$ (from eq. (ii))

The sum of the coefficients of the two middle terms

$= -{}^{15}C_7 + {}^{15}C_8 = -{}^{15}C_7 + {}^{15}C_7$. $\left[\because {}^nC_r = {}^nC_{n-r}\right]$

$= 0$

Sol. (34-36)

$\frac{{}^nC_r}{{}^nC_{r\ 1}} = \frac{1}{2}$

$\frac{\lfloor n \lfloor r+1 \lfloor n-r-1}{\lfloor r \lfloor n-r . \lfloor n} = \frac{1}{2}$

$\frac{r\ \ 1}{n-r} \quad \frac{1}{2} \Rightarrow 3r - n + 2 = 0$ ...(i)

$\frac{{}^nC_{r\ 1}}{{}^nC_{r\ 2}} = \frac{2}{3}$

$\frac{\lfloor n \lfloor r+2 \lfloor n-r-2}{\lfloor r+1 \lfloor n-r-1 \lfloor n} = \frac{2}{3}$

$\frac{r\ \ 2}{n-r-1} \quad \frac{2}{3} \Rightarrow 5r - 2n + 8 = 0$ ...(ii)

Solving equations (i) and (ii), we get

$n = 14, r = 4$

34. (d)

35. (c)

36. (b) $P(n, r) : C(n, r) = \lfloor r = 24$

37. (a) $\left(\frac{\sqrt{3}\ \ i}{\sqrt{3}-i}\right) = \frac{\sqrt{3}\ \ i}{\sqrt{3}-i} \times \frac{\sqrt{3}\ \ i}{\sqrt{3}\ \ i}$

$= \frac{3 + i^2 + 2\sqrt{3}i}{3 - i^2} = \frac{3 - 1 + 2\sqrt{3}i}{3+1}$

$= \frac{2(1\ \ \sqrt{3}i)}{4} \quad \frac{1}{2} \quad i\frac{\sqrt{3}}{2}$

$= \left(\cos\frac{\pi}{3} \quad i\sin\frac{\pi}{3}\right) \quad e^{i\frac{\pi}{3}}$

$\therefore \left(\frac{\sqrt{3}\ \ i}{\sqrt{3}-i}\right)^6 = (e^{i\frac{\pi}{3}})^6 = e^{i2\pi} \ \cos 2\pi + i \sin 2\pi$

$= 1 + 0.i = 1$

38. (b) $\left(x^2 \quad \frac{1}{x}\right)^{15}$

$T_{r+1} = {}^{15}C_r (x^2)^{15-r} \left(\frac{1}{x}\right)^r$

$= {}^{15}C_r x^{30-2r-r} = {}^{15}C_r x^{30-3r}$

For independent term,

$30 - 3r = 0 \Rightarrow r = 10$

Put $r = 10$, we get

$T_{10+1} = {}^{15}C_{10} = \frac{15!}{10!5!}$

$= \frac{15\times14\times13\times12\times11\times10!}{10!\times1\times2\times3\times4\times5} = 3003$

39. (a) For coefficient of x^{15},

$30 - 3r = 15$

$\Rightarrow r = 5$

$\therefore$ the coefficient of x^{15} is ${}^{15}C_5$.

and coefficient of independent of x is

$30 - 3r = 0$

$\Rightarrow r = 10$

So, coefficient of independent of x is ${}^{15}C_{10}$.

$\therefore$ Required ratio $= \frac{{}^{15}C_5}{{}^{15}C_{10}} = \frac{{}^{15}C_5}{{}^{15}C_5} = 1$

$\because {}^nC_r \quad {}^nC_{n-r}$

40. (b) 1. We know that, $(a + b)^n$ have total $(n + 1)$ number of terms

So, $\left(x^2 \quad \frac{1}{x}\right)^{15}$ haver 16 terms.

Hence, Statement 1 is false.

2. For coefficient of x^{12}

$30 - 3r = 12 \Rightarrow r = 6 \Rightarrow {}^{15}C_6$

and for coefficient of x^3,

$30 - 3r = 3 \Rightarrow r = 9 \Rightarrow {}^{15}C_9$

${}^{15}C_6 = {}^{15}C_9$

Hence, statement 2 is correct.

41. (c) 1. For coefficient of x^2,

$30 - 3r = 2 \Rightarrow r = \frac{28}{3}, r \notin N$

So, x^2 does not exist in the expansion

Hence, Statement 1 is correct.

2. Now,

$$\left(x^2 \quad \frac{1}{4}\right)^{15} = {}^{15}C_0(x^2)^{15} + {}^{15}C_1(x^2)^{14}\left(\frac{1}{x}\right)$$

$$+ \ldots + {}^{15}C_{15}\left(\frac{1}{x}\right)^{15}$$

Put x = 1 both sides, we get

$(1+1)^{15} = {}^{15}C_0 + {}^{15}C_1 + \ldots + {}^{15}C_{15}$

$\Rightarrow 2^{15} = {}^{15}C_0 + {}^{15}C_1 + \ldots + {}^{15}C_{15}$

Hence, Statement 2 is correct

42. (c) Given $\left(x^2 \quad \frac{1}{x}\right)^{15}$

Since, n is odd.

So, it has two middle terms T_8 and T_9.

$\therefore T_8 + T_9 = {}^{15}C_7 + {}^{15}C_8 = {}^{16}C_8$

$(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r)$

43. (a, d) $\sum_{r=0}^{1} {}^{n+r}C_n = {}^nC_n + {}^{n+1}C_n$

$$= 1 + \frac{(n+1)!}{(n+1-n)!\, n!} = 1 + \frac{(n+1)(n!)}{n!}$$

$= 1 + n + 1 = n + 2$

$${}^{n+2}C_{n+1} = \frac{(n+2)!}{(n+2-n-1)!\,(n+1)!}$$

$$= \frac{(n+2)\,(n+1)!}{(n+1)!} = n+2$$

OR

$$\sum_{r=0}^{1} {}^{n+r}C_n = {}^nC_n + {}^{n+1}C_n$$

$= 1 + (n+1) = n + 2$

Now,

$${}^{n+2}C_1 = \frac{(n+2)!}{1!(n+2-1)!} = \frac{(n+2)(n+1)!}{(n+1)!} = (n+2)$$

$\therefore$ Option (a and d) is correct.

44. (a) Let r^{th} term is independent of x.

$T_r = {}^nC_r x^r y^{n-r}$

$$= {}^{10}C_r \quad \sqrt{x}^{\ r}\left(\frac{1}{3x^2}\right)^{10-r}$$

$$= {}^{10}C_r\left(\frac{1}{3}\right)^{10-r} . \sqrt{x}^{\ r}\left(\frac{1}{x^2}\right)^{10-r}$$

Equating the coefficient of x to zero.

$\Rightarrow x^{r/2}.x^{-2(10-r)} = x^0$

$$\Rightarrow \frac{r}{2} - 20 + 2r = 0$$

$$\Rightarrow \frac{5}{2}r = 20 \Rightarrow r = 8$$

$$\text{Coefficient} = {}^{10}C_r\left(\frac{1}{3}\right)^{10-r}$$

$$= {}^{10}C_8\left(\frac{1}{3}\right)^{10-8} = \frac{10\times 9}{2}\times\frac{1}{9} = 5$$

45. (a) $(1+x)^{2n+1} = {}^{(2n+1)}C_0x^0 + {}^{(2n+1)}C_1x^1 + \ldots + {}^{(2n+1)}C_{2n+1}(x)^{2n+1}$

Coefficient of $x^r = {}^{(2n+1)}Cr$

Coefficient of $x^{r+1} = {}^{(2n+1)}Cr+1$

${}^{(2n+1)}C_r = {}^{(2n+1)}Cr+1$

$$\Rightarrow \frac{(2n+1)!}{r!(2n+1-r)!} = \frac{(2n+1)!}{(r+1)!(2n-r)!}$$

$$\Rightarrow \frac{2n-r\ !}{2n+1-r \quad 2n-r\ !} \quad \frac{r!}{r \quad 1\ r!}$$

$\Rightarrow (r+1) = 2n + 1 - r$

$\Rightarrow r = n$

46. (b) Total no. of terms in the expansion is 2n + 2. The middle two terms will be n^{th}, $(n+1)^{th}$ term. So.

$$\text{Average} = \frac{{}^{(2n+1)}C_n + {}^{(2n+1)}C_{n+1}}{2}$$

$$= \left[\frac{(2n+1)!}{n!(n+1)!} + \frac{(2n+1)!}{(n+1)!n!}\right]/2$$

$$= \frac{(2n+1)!}{n!(n+1)!} = {}^{(2n+1)}C_n$$

47. (c) Sum of all coefficient

$= {}^{2n\ \ 1}C_0 \quad {}^{2n\ \ 1}C_1 \ \ldots\ldots\ \ {}^{2n\ \ 1}C_{2n\ \ 1}$

$= (1+1)^{2n+1} = 2^{(2n+1)} = 2.2^{2n} = 2.4^n$

48. (c) Coefficient of x^1 in $[(x-1)(x-2)$ or $(x^2 - 3x + 2)]$

$= -3 = -1 - 2 = -(1+2)$

Coefficient of x^2 in $[(x-1)(x-2)(x-3)$ or $(x^3 - 6x^2 + 5x - 6)]$

$= -6 = -[1 + 2 + 3]$.

Coefficient of x^3 in $[(x-1)(x-2)(x-3)(x-4)$ or $(x^4 - 10x^3 - 29x^2 - 11x + 24)]$

$= -10 = -[1 + 2 + 3 + 4]$

$\therefore$ Coefficient of x^{99} in $[(x-1)(x-2)\ldots\ldots\ldots(x-100)]$

$$= -[1+2+3+\ldots\ldots+100] = \frac{-100(100 \quad 1)}{2} = -5050.$$

49. (a) ${}^{47}C_4 \quad {}^{51}C_3 \quad {}^{50}C_3 \quad {}^{49}C_3 \quad {}^{48}C_3 \quad {}^{47}C_3$

$\underline{{}^{47}C_3 \quad {}^{47}C_4} \quad {}^{48}C_3 \quad {}^{49}C_3 \quad {}^{50}C_3 \quad {}^{51}C_3$

$\underline{{}^{48}C_4 \quad {}^{48}C_3} \quad {}^{49}C_3 \quad {}^{50}C_3 \quad {}^{51}C_3$

$\left({}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}\right)$

$\underline{{}^{49}C_4 \quad {}^{49}C_3} \quad {}^{50}C_3 \quad {}^{51}C_3$

$\underline{{}^{50}C_4 \quad {}^{50}C_3} \quad {}^{51}C_3$

${}^{51}C_4 \quad {}^{51}C_3$

${}^{52}C_4$

50. (a) $\left(7_{c_0}+7_{c_1}\right)+\left(7_{c_1}+7_{c_2}\right)+....+\left(7_{c_6}+7_{c_7}\right)$

We know, $n_{c_r}+n_{c_{r-1}} = {}^{n+1}C_r$

$= 8_{c_1}+8_{c_2}+.....+8_{c_7}$

$8_{c_0} \quad 8_{c_1} \quad 8_{c_2} \quad ..... \quad 8_{c_7} \quad 8_{c_8} \; - \; 8_{c_0} \quad 8_{c_8}$

$= 2^8-(1+1)$

$\left[\text{Since, } n_{c_0}+n_{c_1}+n_{c_2}+....+n_{c_n} = 2^n\right]$

$= 256-2$

$= 254$

51. (b) $(x-y)^n$, $n \geq 5$

General term, $T_{r+1} = n_{c_r} x^{n-r}(-y)^r$.

$T_5+T_6 = 0$

$\Rightarrow \left[n_{c_4} x^{n-4}(-y)^4\right]+\left[n_{c_5} x^{n-5}(-y)^5\right] = 0$

$\Rightarrow n_{c_4} x^{n-4} y^4 - n_{c_5} x^{n-5} y^5 = 0$

$\Rightarrow n_{c_4} x^{n-4} y^4 = n_{c_5} x^{n-5} y^5$

$\Rightarrow \dfrac{x^{n-4-n+5}}{y} = \dfrac{n_{c_5}}{n_{c_4}} \Rightarrow \dfrac{x}{y} \quad \dfrac{\cancel{n!}}{5!\ n-5\ !} \times \dfrac{4!\ n-4\ !}{\cancel{n!}}$

$= \dfrac{\cancel{4!}(n-4)\cancel{(n-5)!}}{5\times \cancel{4!}\cancel{(n-5)!}} = \dfrac{n-4}{5}$

52. (c) $(x+a)^{100}+(x-a)^{100}$

Simple logic is we get ${}^{n}c_0, {}^{n}c_2, {}^{n}c_4 .... {}^{n}C_{100}$ in this expansion.

The number of terms from ${}^{n}c_0$ to ${}^{n}c_{100}$ are 51

53. (b) Sum of odd terms of expansion $(a+b)^n$ is $\frac{1}{2}.2^n$.

$\therefore$ Sum of odd terms of expansion $(1+x)^{50}$ is $\frac{1}{2}.2^{50}$.

$= 2^{-1}.2^{50} = 2^{49}$.

55. (d) $1.3+2.3^2+3.3^3+....+n.3^n = \dfrac{2n-1\ 3^a \quad b}{4}$

Let us put $3 = x$.

L.H.S: $S = x+2x^2+3x^3+....+n.x^n$ ....(1)

$xs = x^2+2x^3+3x^4+....+n.x^{n+1}$ ....(2)

$(1)-(2) \Rightarrow S-xS = (x+2x^2+3x^3+....+n.x^n)-(x^2+2x^3+3x^4+....+n.x^{n+1})$

$\Rightarrow S(1-x) = x+x^2+x^3+....+x^n-nx^{n+1}$

$\Rightarrow S(1-x) = \dfrac{x\left(1-x^n\right)}{1-x}-nx^{n+1}$

$\Rightarrow S \quad \left(\dfrac{1}{x-1}\right)\left(\dfrac{-x\ \ x^n-1\ +nx^{n\ \ 1}\ \ x-1}{x-1}\right)$

Put $x = 3$,

$\Rightarrow S = \dfrac{1}{2}\left(\dfrac{-3^{n+1}+3+2n.3^{n+1}}{2}\right) = \left(\dfrac{3^{n+1}(2n-1)+3}{4}\right)$

55. (b) $\left|z-\dfrac{4}{z}\right| = 2.$.

We know $|a-b| \geq |a|-|b|$

$\therefore \left|z-\dfrac{4}{z}\right| \geq |z|-\left|\dfrac{4}{z}\right|$

$\Rightarrow 2 \geq |z|-\dfrac{4}{|z|} \Rightarrow |z|^2-2|z|-4 \leq 0$

$\Rightarrow |z| \quad \dfrac{2\pm\sqrt{4-4\ 1\ \ -4}}{2\ 1} \quad \dfrac{2\quad\sqrt{20}}{2} \quad \dfrac{2\quad 2\sqrt{5}}{2} \quad 1 \quad \sqrt{5}$

So, $z \quad 1 \quad \sqrt{5}$.

56. (b) $n \in N$,

$121^n - 25^n + 1900^n - (-4^n)$

Let us substitute n = 1

We get, $(121)^1-(25)^1+(1900)^1-(-4^1)$

$= 121-25+1900+4$

$= 2025-25$

$= 2000$

So, given expression is divisible by 2000

57. (b) Given, in the expansion g $(1+x)^{43}$, coefficients of $(2r+1)^{th}$ term and $(r+2)^{th}$ term are equal.

Coefficient of $(2r+1)^{th}$ term $= n_{C_{2r}}$

Coefficient of $(r+2)^{th}$ term $= n_{C_{r\ \ 1}}$

$n_{C_{2r}} = n_{C_{r\ \ 1}}$

$\Rightarrow 43_{C_{2r}} \quad 43_{C_{r\ \ 1}} \qquad \because n \quad 43$

$\Rightarrow 2r + r + 1 = 43$
$\Rightarrow 3r + 1 = 43$
$\Rightarrow 3r + 42 \Rightarrow r = 14$

58. (b) $(1 + a)^{m+n}$

α = coefficient of $a^m = {}^{m\ \ n}C_m$

β = coefficient of $a^n = {}^{m\ \ n}C_n$

We know, ${}^nC_r \quad {}^nC_{n-r}$

$\therefore \beta \quad {}^{m\ \ n}C_n \quad {}^{m\ \ n}C_{m+n-n} \quad {}^{m\ \ n}C_m \quad \alpha$

$\therefore \alpha = \beta$

59. (c) We know, in the expansion of $(x + y)^n + (x - y)^n$, of

n = even, then number of non zero terms is $\frac{n}{2} + 1$

n = odd, then number of non zero terms in $\frac{n\ \ 1}{2}$.

Here, n = 11 which is odd.

$\therefore$ number of non zero terms $\quad \frac{11\ \ 1}{2} \quad 6$.

60. (d) $C(n, r) + 2C(n, r - 1) + C(n, r - 2)$

${}^nC_r \quad 2\ {}^nC_{r-1} \quad {}^nC_{r-2}$

${}^nC_r \quad {}^n\ C_{r-1} \quad {}^nC_{r-1} \quad {}^n\ C_{r-2}$

$\because {}^nC_r \quad {}^nC_{r-1} \quad {}^{n\ \ 1}C_r$

${}^{n\ \ 1}C_r \quad {}^{n\ \ 1}C_{r-1}$

${}^{n\ \ 2}C_r$

$= C(n + 2, r)$

61. (d) Middle term in the expansion of $(x + y)^n$

$= \left(\frac{n\ \ 1}{2}\right)^{th}$ term, if n is odd

$= \left(\frac{n}{2}\ \ 1\right)^{th}$ term, if n is even

Here $n = 4$

$\therefore$ Middle term is $\left(\frac{4}{2}\ \ 1\right)^{th} = 3^{rd}$ term

$4c_2 \times 2^2 \times 3^2 = 6 \times 4 \times 9$
$= 216$

62. (c) $\alpha = {}^{2n}C_n$
$\beta = {}^{2n-1}C_n$
$\gamma = {}^{2n-1}C_{n-1}$
$\beta + \gamma = {}^{2n-1}C_n + {}^{2n-1}C_{n-1} = {}^{2n}C_n = \alpha$

63. (b) Given, C (20, n + 2) = C (20, n – 2)
$\Rightarrow {}^{20}C_{n+2} = {}^{20}C_{n-2}$
$\Rightarrow 20 = n + 2 + n - 2 \qquad (\because {}^nc_r = {}^nc_s \Rightarrow n = r + s)$
$\Rightarrow 20 = 2n$
$\Rightarrow n = 10$

64. (b) $[(2x - 3y)^2 (2x + 3y)^2]^2$
$= [(4x^2 - 9y^2)^2]^2 = (4x^2 - 9y^2)^4$
$\therefore$ Number of terms = 4 + 1 = 5

65. (b) The first three terms in expansion of $(1 + ax)^n$ are
${}^nC_0, {}^nC_1ax, {}^nC_2a^2x^2$
Given, ${}^nC_0 = 1; {}^nC_1 ax = 12x; {}^nC_2 a^2x^2 = 64x^2$

$\Rightarrow nax = 12x; \ \frac{n(n-1)}{2}a^2 = 64$

$\Rightarrow na = 12 \Rightarrow a = \frac{12}{n}$

$\therefore \frac{n(n-1)}{2}a^2 = 64 \Rightarrow \frac{n(n-1)}{2} \times \frac{144}{n^2} = 64$

$\Rightarrow \frac{n-1}{n} = \frac{64 \times 2}{144} = \frac{8}{9}$

$\therefore n = 9$

Permutation and Combination

6

1. How many 3-digit numbers, each less than 600, can be formed from $\{1, 2, 3, 4, 7, 9\}$ if repetition of digits is allowed?
   (a) 216 (b) 180
   (c) 144 (d) 120 *[2006-I]*
2. There are four chairs with two chairs in each row. In how many ways can four persons be seated on the chairs, so that no chair remains unoccupied ?
   (a) 6 (b) 12
   (c) 24 (d) 48 *[2006-I]*
3. In how many ways can the letters of the word CORPORATION be arranged so that vowels always occupy even places ?
   (a) 120 (b) 2700
   (c) 720 (d) 7200 *[2006-I]*
4. If all permutations of the letters of the word 'LAGAN' are arranged as in dictionary, then what is the rank of 'NAAGL'?
   (a) 48th word (b) 49th word
   (c) 50th word (d) 51st word *[2006-I]*
5. If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected ?
   (a) 110 (b) 55
   (c) 22 (d) 11 *[2006-I]*

DIRECTIONS (Qs. 6 to 7) : *The following questions consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers.*

(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2006-II]*

6. **Assertion (A) :** The number of triangles that can be formed by joining the mid-points of any three adjacent faces of a cube is 20.
   Reason (R) : If there are n points on a plane and none of them are collinear, then the number of triangles that can be formed is C (n, 3).
7. **Assertion (A) :** The number of selections of 20 distinct things taken 8 at a time is same as that taken 12 at a time.
   Reason (R): $C(n, r) = C(n, s)$, if $n = r + s$
8. If the letters of the word BAZAR are arranged in dictionary order, then what is the 50th word? *[2006-II]*
   (a) ZAABR (b) ZBAAR
   (c) ZBRAA (d) ZAARB
9. In how many ways can 7 persons stand in the form of a ring? *[2006-II]*
   (a) $P(7, 2)$ (b) $7!$
   (c) $6!$ (d) $\frac{7!}{2}$
10. In how many ways can be letters of the word 'CABLE' be arranged so that the vowels should always occupy odd positions? *[2007-I]*
    (a) 12 (b) 18
    (c) 24 (d) 36
11. What is $\frac{(n+2)!+(n+1)\,(n-1)!}{(n+1)\,(n-1)!}$ equal to ? *[2007-II]*
    (a) 1
    (b) Always an odd integer
    (c) A perfect square
    (d) None of the above
12. A meeting is to be addressed by 5 speakers A, B, C, D, E. In how many ways can the speakers be ordered, if B must not precede A (immediately or otherwise) ? *[2007-II]*
    (a) 120 (b) 24
    (c) 60 (d) $5^4 \times 4$
13. On a railway route there are 20 stations. What is the number of different tickets required in order that it may be possible to travel from every station to every other station? *[2007-II]*
    (a) 40 (b) 380
    (c) 400 (d) 420
14. What is the number of five-digit numbers formed with 0, 1, 2, 3, 4 without any repetition of digits? *[2008-I]*
    (a) 24 (b) 48
    (c) 96 (d) 120

15. A group consists of 5 men and 5 women. If the number of different five-person committees containing k men and (5–k) women is 100, what is the value of k ? *[2008-I]*
(a) 2 only (b) 3 only
(c) 2 or 3 (d) 4

16. If 7 points out of 12 are in the same straight line, then what is the number of triangles formed ? *[2008-I]*
(a) 84 (b) 175
(c) 185 (d) 201

17. In how many ways can 3 books on Hindi and 3 books on English be arranged in a row on a shelf, so that not all the Hindi books are together? *[2008-II]*
(a) 144 (b) 360
(c) 576 (d) 720

18. How many words, with or without meaning can be formed by using all the letters of the word 'MACHINE', so that the vowels occurs only the odd positions? *[2008-II]*
(a) 1440 (b) 720
(c) 640 (d) 576

19. From 7 men and 4 women a committee of 6 is to be formed such that the committee contains at least two women. What is the nubmer of ways to do this? *[2008-II]*
(a) 210 (b) 371
(c) 462 (d) 5544

20. If $P(32, 6) = kC(32, 6)$, then what is the value of k? *[2009-I]*
(a) 6 (b) 32
(c) 120 (d) 720

21. What is the smallest natural number n such that $n!$ is divisible by 990? *[2009-I]*
(a) 9 (b) 11
(c) 33 (d) 99

22. What is the value of r, if $P(5, r) = P(6, r - 1)$? *[2009-I]*
(a) 9 (b) 5
(c) 4 (d) 2

23. What is the number of words formed from the letters of the word 'JOKE' so that the vowels and consonants alternate? *[2009-I]*
(a) 4 (b) 8
(c) 12 (d) None of these

24. If $C(n, 12) = C(n, 8)$, then what is the value of $C(22, n)$? *[2009-II]*
(a) 131 (b) 231
(c) 256 (d) 292

25. In a football championship 153 matches were played. Every team played one match with each other team. How many teams participated in the championship? *[2009-II]*
(a) 21 (b) 18
(c) 17 (d) 15

26. How many times does the digit 3 appear while writing the integers from 1 to 1000? *[2009-II]*
(a) 269 (b) 308
(c) 300 (d) None of these

27. What is the number of ways of arranging the letters of the word 'BANANA' so that no two N's appear together? *[2010-I]*
(a) 40 (b) 60
(c) 80 (d) 100

28. What is the number of three-digit odd numbers formed by using the digits 1, 2, 3, 4, 5, 6 if repetition of digits is allowed? *[2010-I]*
(a) 60 (b) 108
(c) 120 (d) 216

29. A team of 8 players is to be chosen from a group of 12 players. Out of the eight players one is to be elected as captain and another vice-captain. In how many ways can this be done? *[2010-I]*
(a) 27720 (b) 13860
(c) 6930 (d) 495

30. What is the number of words that can be formed from the letters of the word 'UNIVERSAL', the vowels remaining always together? *[2010-II]*
(a) 720 (b) 1440
(c) 17280 (d) 21540

31. What is the number of signals that can be sent by 6 flags of different colours taking one or more at a time? *[2010-II]*
(a) 21 (b) 63
(c) 720 (d) 1956

32. In how many ways can a committee consisting of 3 men and 2 women be formed from 7 men and 5 women? *[2010-II]*
(a) 45 (b) 350
(c) 700 (d) 4200

33. What is the total number of combination of n different things taken 1, 2, 3, ..., n at a time? *[2011-I]*
(a) 2^{n+1} (b) 2^{2n+1}
(c) 2^{n-1} (d) $2^n - 1$

34. 5 books are to be chosen from a lot of 10 books. If m is the number of ways of choice when one specified book is always included and n is the number of ways of choice when a specified book is always excluded, then which one of the following is correct? *[2011-I]*
(a) $m > n$ (b) $m = n$
(c) $m = n - 1$ (d) $m = n - 2$

35. In how many ways 6 girls can be seated in two chairs? *[2011-I]*
(a) 10 (b) 15
(c) 24 (d) 30

36. What is the value of n, if $P(15, n - 1) : P(16, n - 2) = 3 : 4$? *[2011-I]*
(a) 10 (b) 12
(c) 14 (d) 15

37. Using the digits 1, 2, 3, 4 and 5 only once, how many numbers greater than 41000 can be formed? *[2011-I]*
(a) 41 (b) 48
(c) 50 (d) 55

38. A, B, C, D and E are coplanar points and three of them lie in a straight line. What is the maximum number of triangles that can be drawn with these points as their vertices? [2011-I]
(a) 5 (b) 9
(c) 10 (d) 12

39. There are 4 candidates for the post of a lecturer in Mathematics and one is to be selected by votes of 5 men. What is the number of ways in which the votes can be given? [2011-II]
(a) 1048 (b) 1072
(c) 1024 (d) 625

40. What is the value of $\sum_{r=1}^{n} \frac{P(n,r)}{r!}$? [2011-II]
(a) $2^n - 1$ (b) 2^n
(c) 2^{n-1} (d) $2^n + 1$

41. What is the number of ways that 4 boys and 3 girls can be seated so that boys and girls alternate? [2012-I]
(a) 12 (b) 72
(c) 120 (d) 144

42. The number of permutations that can be formed from all the letters of the word 'BASEBALL' is: [2012-II]
(a) 540 (b) 1260
(c) 3780 (d) 5040

43. If P(77, 31) = x and C (77, 31) = y, then which one of the following is correct? [2013-I]
(a) x = y (b) 2x = y
(c) 77x = 31 y (d) x > y

44. In how many ways can the letters of the word 'GLOOMY' be arranged so that the two O's should not be together? [2013-I]
(a) 240 (b) 480
(c) 600 (d) 720

45. Out of 7 consonants and 4 vowels, words are to be formed by involving 3 consonants and 2 vowels. The number of such words formed is : [2014-I]
(a) 25200 (b) 22500
(c) 10080 (d) 5040

46. How many different words can be formed by taking four letters out of the letters of the word 'AGAIN' if each word has to start with *A* ? [2014-I]
(a) 6 (b) 12
(c) 24 (d) None of the above

47. What is the number of ways in which one can post 5 letters in 7 letters boxes ? [2014-II]
(a) 7^5 (b) 3^5
(c) 5^7 (d) 2520

48. What is the number of ways that a cricket team of 11 players can be made out of 15 players ? [2014-II]
(a) 364 (b) 1001
(c) 1365 (d) 32760

49. How many words can be formed using all the letters of the word 'NATION' so that all the three vowels should never come together? [2015-I]
(a) 354 (b) 348
(c) 288 (d) None of these

50. Let $A = \{x, y, z\}$ and $B = \{p, q, r, s\}$. What is the number of distinct relations from B to A? [2015-I]
(a) 4096 (b) 4094
(c) 128 (d) 126

51. If different words are formed with all the letters of the word 'AGAIN' and are arranged alphabetically among themselves as in a dictionary, the word at the 50th place will be [2015-II]
(a) NAAGI (b) NAAIG
(c) IAAGN (d) IAANG

52. The number of ways in which a cricket team of 11 players be chosen out of a batch of 15 players so that the captain of the team is always included, is [2015-II]
(a) 165 (b) 364
(c) 1001 (d) 1365

53. A polygon has 44 diagonals. The number of its sides is [2015-II]
(a) 11 (b) 10
(c) 8 (d) 7

54. The number of ways in which 3 holiday tickets can be given to 20 employees of an organization if each employee is eligible for any one or more of the tickets, is [2015-II]
(a) 1140 (b) 3420
(c) 6840 (d) 8000

55. The number of 3-digit even numbers that can be formed from the digits 0, 1, 2, 3, 4 and 5, repetition of digits being not allowed, is [2015-II]
(a) 60 (b) 56
(c) 52 (d) 48

56. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organization, if each employee is eligible for any one or more of the tickets? [2016-I]
(a) 60 (b) 120
(c) 500 (d) 1000

57. What is the number of four-digit decimal numbers (<1) in which no digit is repeated? [2016-I]
(a) 3024 (b) 4536
(c) 5040 (d) None of the above

58. What is the number of different messages that can be represented by three 0's and two 1's? [2016-I]
(a) 10 (b) 9
(c) 8 (d) 7

59. Out of 15 points in a plane, n points are in the same straight line. 445 triangles can be formed by joining these points. What is the value of n? [2016-II]
(a) 3 (b) 4
(c) 5 (d) 6

60. A five-digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3 and 4 without repetition of digits. What is the number of ways this can be done? [2016-II]
(a) 96 (b) 48
(c) 32 (d) No number can be formed

61. What is the number of odd integers between 1000 and 9999 with no digit repeated? [2016-II]
(a) 2100 (b) 2120
(c) 2240 (d) 3331

62. The number of different words (eight-letter words) ending and beginning with a consonant which can be made out of the letters of the word 'EQUATION' is *[2017-I]*
(a) 5200 (b) 4320
(c) 3000 (d) 2160

63. How many different permutations can be made out of the letters of the word 'PERMUTATION'? *[2017-II]*
(a) 19958400 (b) 19954800
(c) 19952400 (d) 39916800

64. A tea party is arranged for 16 people along two sides of a long table with eight chairs on each side. Four particular men wish to sit on one particular side and two particular men on the other side. The number of ways they can be seated is *[2017-II]*
(a) $24 \times 8! \times 8!$ (b) $(81)^3$
(c) $210 \times 8! \times 8!$ (d) 16!

65. How many numbers between 100 an 1000 can be formed with the digits 5, 6, 7, 8, 9, if the repetition of digits is not allowed? *[2018-I]*
(a) 3^5 (b) 5^3
(c) 120 (d) 60

66. How many four-digit numbers divisible by 10 can be formed using 1, 5, 0, 6, 7 without repetition of digits? *[2018-I]*
(a) 24 (b) 36
(c) 44 (d) 64

67. What is the number of triangles that can be formed by choosing the vertices from a set of 12 points in a plane, seven of which lie on the same straight line? *[2018-I]*
(a) 185 (b) 175
(c) 115 (d) 105

68. There are 17 cricket players, out of which 5 players can bowl. In how many ways can a team of 11 players be selected so as to include 3 bowlers? *[2018-II]*
(a) C (17, 11) (b) C (12, 8)
(c) $C(17, 5) \times (5, 3)$ (d) $C(5, 3) \times C(12, 8)$

69. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is *[2018-II]*
(a) 45360 (b) 30240
(c) 27216 (d) 15120

70. What is the sum of all three-digit numbers that can be formed using all the digits 3, 4 and 5, when repetition of digits is not allowed? *[2018-II]*
(a) 2664 (b) 3882
(c) 4044 (d) 4444

71. Three dice having digits 1, 2, 3, 4, 5 and 6 on their faces are marked I, II and III and rolled. Let x, y and z represent the number on die-I die-II and die -III respectively. What is the number of possible outcomes such that $x > y > z$? *[2018-II]*
(a) 14 (b) 16
(c) 18 (d) 20

72. There are 10 points in a plane. No three of these points are in a straight line. What is the total number of straight lines which can be formed by joining the points? *[2019-I]*
(a) 90 (b) 45
(c) 40 (d) 30

73. From 6 programmers and 4 typists, an ofûce wants to recruit 5 people. What is the number of ways this can be done so as to recruit at least one typist ? *[2019-I]*
(a) 209 (b) 210
(c) 246 (d) 242

74. How many three-digit even numbers can be formed using the digits 1, 2, 3, 4 and 5 when repetition of digits is **not** allowed ? *[2019-I]*
(a) 36 (b) 30
(c) 24 (d) 12

ANSWER KEY

1	(c)	9	(c)	17	(c)	25	(b)	33	(d)	41	(d)	49	(c)	57	(b)	65	(d)	73	(c)
2	(c)	10	(d)	18	(d)	26	(c)	34	(b)	42	(d)	50	(a)	58	(a)	66	(a)	74	(d)
3	(d)	11	(c)	19	(b)	27	(a)	35	(d)	43	(d)	51	(b)	59	(c)	67	(a)		
4	(b)	12	(b)	20	(d)	28	(b)	36	(c)	44	(a)	52	(c)	60	(d)	68	(d)		
5	(b)	13	(b)	21	(b)	29	(a)	37	(b)	45	(a)	53	(a)	61	(c)	69	(c)		
6	(a)	14	(c)	22	(c)	30	(c)	38	(b)	46	(c)	54	(d)	62	(b)	70	(a)		
7	(a)	15	(c)	23	(b)	31	(b)	39	(d)	47	(a)	55	(c)	63	(a)	71	(d)		
8	(d)	16	(c)	24	(b)	32	(b)	40	(a)	48	(c)	56	(d)	64	(c)	72	(b)		

HINTS & SOLUTIONS

1. (c) Three digit number less then 600 will have first element 100, and last element 599. First place will not have digit more than 6, hence, 7 and 9 can not be taken : So, first digit can be selected in 4 ways. Second digit can be selected in 6 ways and since repetition of digits are allowed, third digit can also be selected in 6 ways :
So, number of ways are $4 \times 6 \times 6 = 144$.

2. (c) First chair can be occupied in 4 ways and second chair can be occupied in 3 ways, third chair can be occupied in 2 ways and last chair can be occupied in one ways only. So total number of ways $= 4 \times 3 \times 2 \times 1 = 24$

3. (d) CORPORATION is 11 letter word.
It has 5 vowels (O, O, O, A, I) and 6 consonants (C, R, P, R, T, N)
In 11 letters, there are 5 even places (2^{nd}, 4^{th}, 6^{th}, 8^{th} and 10^{th} positions)

5 vowels can take 5 even places in $\frac{5!}{3!}$ ways
($\because$ Since O is repeated thrice)

Similarly, 6 consonasts can take 6 odd places in $\frac{6!}{2!}$ ways.
($\because$ R is repeated twice)

$\therefore$ Total number of ways $= \frac{5!}{3!} \times \frac{6!}{2!} = 20 \times 360 = 7200$

4. (b) Starting with the letter A and arranging the other four letters, there are 24 words. There are the first 24 words. Then starting with G that comes next in dictionary order and arranging A, A, L, N in different ways, there are $\frac{4!}{2!} = 12$ words. Next the 37th word starts with L, that comes next in dictionary order there are 12 words starting with L. This accounts up to the 48 words. The 49th word is 'NAAGL'

5. (b) Selection of 2 members out of 11 has ${}^{11}C_2$ number of ways
${}^{11}C_2 = 55$

6. (a) Number of faces in a cube = 6
Number of triangles formed by joining mid points of faces is selection of three points from 6 points = ${}^{6}C_3$
$= \frac{6!}{3!3!} = 20$
Hence , both A and R are individually true and R is correct explanation of A.

7. (a) Number of selection of 20 distinct things taken 8 at a time is given by
$${}^{20}C_8 = \frac{20!}{12!8!}$$
and selecting 12 out of 20 is
$${}^{20}C_{12} = \frac{20!}{12!8!}$$
Thus, both ${}^{20}C_8$ and ${}^{20}C_{12}$ are same.
$\Rightarrow$ Both A and R are individually true and R is correct explanation of A.

8. (d) With A at first place, rest 4 places will be arranged in 4! ways so, Number of words begin with A = 4! = 24 Similarly with B at first place,
Number of words begin with B $= \frac{4!}{2!} = 12$
[As there are two A_s]
Number of words begin with R $= \frac{4!}{2!} = 12$
Thus, 48 words have starting letter A, B and R.
So, 49th word will be ZAABR and 50th word will be ZAARB.

9. (c) Number of ways in which 7 persons can stand in the form of a ring $= (7-1)! = 6!$

10. (d) There are two vowels A and E. There are total 5 places out of which two places are to be occupied by vowels. So, 3 places can be occupied by 2 vowels in ${}^{3}P_2$ ways and after two vowels occupy two places, 3 consonants will occupy 3 places in ${}^{3}P_3 = 3!$ way, hence,
Required number of ways $= {}^{3}P_2 \times 3! = 6 \times 6 = 36$

11. (c) Given expression is :
$$\frac{(n \quad 2)! \quad (n \quad 1)!(n-1)!}{(n+1)!(n-1)!} \quad x \text{ (let)}$$
$$\Rightarrow x \quad \frac{(n \quad 2)(n \quad 1)n(n-1)! \quad (n \quad 1)(n-1)!}{(n+1)(n-1)!}$$
$= (n+2)n + 1 = n^2 + 2n + 1 = (n+1)^2$
Which is a perfect square.

12. (b) According to given restriction:
B must not precede A (immediately or otherwise),
$\Rightarrow$ A must follow B, i.e., B should addressed the meeting at first place
So, rest of the four speakers can address in 4! ways.
$\therefore$ Required number of ways = 4! = 24

13. (b) From each railway station, there are 19 different tickets to be issued. There are 20 railway station
So, total number of tickets $= 20 \times 19 = 380$.

14. (c) To make a 5 digit number, 0 can not come in the bagining. So, it can be filled in 4 ways. Rest of the places can be filled in 4! ways. So total number of digit formed = 4 × 4! = 4 × 24 = 96

15. (c) K men selected out of 5 and 5 – k women out of 5. These are ^5C_k and $^5C_{5-k}$
According to problem :
$^5C_k \times {}^5C_{5-k} = 100$
$$\Rightarrow \frac{5!}{k!(5-k)!} \times \frac{5!}{(5-k)!5!} = 100$$
$$\Rightarrow \left(\frac{5}{k!(5-k)!}\right)^2 = 100$$
$$\Rightarrow \frac{5!}{k!(5-k)!} = 10$$
This is true for k = 2 or 3.

16. (c) Number of triangles formed from 12 point $= {}^{12}C_3$
Since 7 parts are collinear, then $^7C_3$ triangles will not be formed so.
$= {}^{12}C_3 - {}^7C_3$
$$= \frac{12!}{3!9!} - \frac{7!}{3!4!} = \frac{12.11.10}{3.2.1} - \frac{7.6.5}{3.2.1}$$
$= 220 - 35 = 185$

17. (c) Total number of arrangement = 6! = 720
Total number of arrangement while all the Hindi books are together = 4! × 3! = 24 × 6 = 144
∴ The number of ways, in which books are arranged, while all the Hindi books are not together
= 720 – 144 = 576

18. (d) There are three vowels and they have four odd places to arrange. Other letters are four and has four places to arrange.
∴ The number of words = $^4P_3 \times 4!$
$$= \frac{4!}{(4-3)!} \times 4! \quad 576$$

19. (b) The required number of ways
$= {}^{11}C_6 - ({}^7C_6 \times {}^4C_0 + {}^7C_5 \times {}^4C_1)$
$$= \frac{11\times10\times9\times8\times7}{5\times4\times3\times2} - \left(7 + \frac{7\times6}{2}\times4\right)$$
$= 462 - (7 + 84) = 371$

20. (d) Since $^{32}P_6 = k\ {}^{32}C_6$
$$\Rightarrow \frac{32!}{(32-6)!} = k.\frac{32!}{6!(32-6)!}$$
$\Rightarrow k = 6! = 720$

21. (b) Consider option 'a'
Let us take $n = 9$
Since, 9! = 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 362880
which is not divisible by 990.
Now assume, $n = 11$
Since 11! = 39916800
which is divisible by 990.
Thus, required smallest natural number 11

22. (c) Given $P(5, r) = P(6, r - 1)$
$$\Rightarrow {}^5P_r \quad {}^6P_{r-1}$$
$$\Rightarrow \frac{5!}{(5-r)!} \quad \frac{6!}{(6-r \quad 1)!}$$
$$\Rightarrow \frac{5!}{(5-r)!} \quad \frac{6!}{(7-r)!}$$
$$\Rightarrow \frac{5!}{(5-r)!} \quad \frac{6\times5!}{(7-r)(6-r)(5-r)!}$$
$\Rightarrow (7-r)(6-r) = 6$
$\Rightarrow 42 - 13r + r^2 = 6$
$\Rightarrow r^2 - 13r + 36 = 0$
$\Rightarrow r^2 - 9r - 4r + 36 = 0$
$\Rightarrow (r-9)(r-4) = 0$
$\Rightarrow r = 4 \quad (\because r \neq 9)$

23. (b) Total number of letters = 4
No. of vowels = 2
No. of consonants = 2
Possibilities of words formed from the letters of word "JOKE" are
JOKE, KOJE, KEJO, JEKO, EJOK, EKOJ, OKEJ, OJEK
Thus, required number of words = 8

24. (b) Given $C(n, 12) = C(n, 8)$
$\Rightarrow {}^nC_{12} = {}^nC_8$
$$\Rightarrow \frac{n!}{(n-12)!12!} = \frac{n!}{(n-8)!8!}$$
$$\Rightarrow \frac{1}{(n-12)!(12\times11\times10\times9\times8!)}$$
$$= \frac{1}{(n-8)(n-9)(n-10)(n-11)(n-12)!8!}$$
$$\Rightarrow \frac{1}{12\times11\times10\times9} = \frac{1}{(n-8)(n-9)(n-10)(n-11)}$$
$\Rightarrow (n-8)(n-9)(n-10)(n-11)$
$= 12 \times 11 \times 10 \times 9$
$\Rightarrow n - 8 = 12, n - 9 = 11, n - 10 = 10$ and $n - 11 = 9$
$\Rightarrow n = 20$
$\Rightarrow C(22, n) = {}^{22}C_{20}$
$$= \frac{22!}{2!20!} = \frac{22\times21}{2} = 231$$

25. (b) Let total no. of team participated in a championship be n. Since, every team played one match with each other team.
$$\therefore {}^nC_2 = 153 \Rightarrow \frac{n!}{2!(n-2)!} = 153$$

$\Rightarrow \frac{n(n-1)(n-2)!}{2!(n-2)!} = 153 \Rightarrow \frac{n(n-1)}{2} = 153$

$\Rightarrow n(n-1) = 306$

$\Rightarrow n^2 - n - 306 = 0$

$\Rightarrow n^2 - 18n + 17n - 306 = 0$

$\Rightarrow n(n-18) + 17(n-18) = 0$

$\Rightarrow n = 18, -17$

n cannot be negative

$\therefore n \neq -17$

$\Rightarrow n = 18$

26. (c) Before 1000 there are one digit, two digits and three digits numbers.
Numbers of times 3 appear in one digit number = 20×9
Number of times 3 appear in two digit numbers = 11×9
Number of times 3 appear in three digit numbers = 21
Hence total number of times the digit 3 appear while writing the integers from 1 to 1000
$= 180 + 99 + 21 = 300$

27. (a) Total no. of letters in BANANA = 6
No. of repeated letter N = 2
No. of repeated letter A = 2
Therefore
Number of ways that can be formed by using the words
'BANANA' $= \frac{6!}{3!2!} = \frac{6\times5\times4\times3!}{3!\times2!} = 60$
Number of ways in which two N comes together
$= \frac{5!}{3!} = 20$
$\therefore$ Required number of ways $= 60 - 20 = 40$

28. (b) Total no. of digits = 6
To form a odd numbers we have only 3 choice for the unit digits.
Now, Extreme left place can be filled in 6 ways the middle place can be filled in 6 ways.
$\therefore$ Required number of numbers $= 6 \times 6 \times 3 = 108$

29. (a) Total no. of players = 12
No. of chosen players = 8
Number of ways to choose 8 players from 12 players
$= {}^{12}C_8 = \frac{12!}{8!4!} = \frac{12\times11\times10\times9\times8!}{8!4!} = 495$
Since, out of the 8 players 1 is to be elected as captain and another vice-captain
Therefore number of ways to choose a captain and a vice-captain
$= {}^8C_1 \times {}^7C_1 = 8\times7 = 56 \quad \left(\because {}^nC_1 = n\right)$
Hence, required number of ways $= 495 \times 56 = 27720$

30. (c) Consider the word UNIVERSAL
Total no. of vowels = U, I, E, A = 4
Let us consider these as a single letter.
UIEA N V R S L
Then, total no. of letters = 6
Then, number of ways to arrange them = 6! = 720
But vowels can also arranged in 4! or 24 ways.
Hence, total number of ways = 720 × 24 = 17280

31. (b) Required number of ways
${}^6C_0 \quad {}^6C_1 \quad {}^6C_2 \quad ...... \quad {}^6C_6 - 1 = 2^6 - 1 = 64 - 1 = 63$

32. (b) Total no. of Men = 7
Total no. of women = 5
Required number of ways $= {}^7C_3 \times {}^5C_2$
$= \frac{7!}{3!4!}\times\frac{5!}{2!3!} = \frac{7\times6\times5}{3\times2}\times\frac{5\times4}{2}$
$= 7\times5\times10 = 35\times10 = 350$

33. (d) Since, combinations of taking 1, 2, 3, ... , n things at a times are ${}^nC_1, {}^nC_2, ..., {}^nC_n$.
$\therefore$ Total number of combinations
$= {}^nC_1 + {}^nC_2 + ... + {}^nC_n$
$= 1 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n - 1$
$= 2^n - 1$

34. (b) Number of ways when one specified book is always included. So, 4 books can be chosen from remaining 9 ways:
$= m = {}^9C_4$
$\Rightarrow m = 126$
and number of ways when one specified book is always excluded $= n = {}^9C_5$
$\Rightarrow n = 126$
$\Rightarrow m = n$

35. (d) Required number of ways = 6 × 5 = 30

36. (c) Let P (15, $n-1$) : P (16, $n-2$) = 3 : 4

$\Rightarrow \frac{{}^{15}P_{n-1}}{{}^{16}P_{n-2}} \quad \frac{3}{4}$

$\Rightarrow \frac{15!}{(15-n \quad 1)!}\times\frac{(16-n \quad 2)!}{16!} \quad \frac{3}{4}$

$\Rightarrow \frac{15!}{(16-n)!}\times\frac{(18-n)!}{16!} \quad \frac{3}{4}$

$\Rightarrow \frac{(18-n)!}{16(16-n)!} \quad \frac{3}{4}$

$\Rightarrow \frac{(18-n)(17-n)(16-n)!}{16(16-n)!} \quad \frac{3}{4}$

$\Rightarrow (18-n)(17-n) = 12$

$\Rightarrow 306 - 17n - 18n + n^2 = 12$

$\Rightarrow n^2 - 35n + 294 = 0$

$\Rightarrow (n-14)(n-21) = 0$

$\Rightarrow n = 14 \qquad (\because n \neq 21)$

37. (b) We have to construct 5 digit numbers which are greater than 41000.
So, we have only 2 ways to choose 5th digit.

5^{th}	4^{th}	3^{rd}	2^{nd}	1^{st}

[$\because$ Only 4 or 5 can come at 5^{th} place]

Thus, for 4^{th} place we have 4 ways to choose digits.
For 3^{rd} place we have 3 ways.
For 2^{nd} place we have 2 ways.
and for unit place we have only 1 way.
Required number of ways $= 2 \times 4 \times 3 \times 2 \times 1 = 48$

38. (b) Number of triangles using 5 points out of three are on a straight line $= {}^5C_3 - {}^3C_3 = \frac{5!}{3!2!} - 1 = \frac{5 \times 4}{2} - 1$
$= 10 - 1 = 9$

39. (d) There are 4 candidates.
This means there are 4 blank spaces for 1 post.
Now, that 1 post is to be selected by votes of 5 men.
So, All 4 places can be fill by each man's votes.
$\therefore$ There is 5 ways for 1 place..
Hence, Required ways
$= 5 \times 5 \times 5 \times 5 = 625$ ($\because$ we have 4 places).

40. (a) Consider $\sum_{r\ 1}^{n} \frac{P\ n, r}{r!} \equiv \sum_{r\ 1}^{n} C\ n, r$

Now, consider
$(1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$
$\Rightarrow 2^n = 1 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$
$\Rightarrow 2^n - 1 = {}^nC_1 + {}^nC_2 + ... + {}^nC_n$
$\Rightarrow 2^n - 1 = \sum_{r=1}^{n} {}^nC_r \equiv \sum_{r=1}^{n} \frac{P(n, r)}{r!}$

41. (d) B G B G B G B
Required no. of ways $= 4! \times 3! = 144$

42. (d) There are total 8 letters in the word BASEBALL, in which we have 2 B's, 2A's and 2 L's.
$\therefore$ Required No. of permutations $= \frac{8!}{2! \times 2! \times 2!}$
$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{8} = 5040$

43. (d) As we know
$P(n, r) = r!\, C(n, r)$
$\therefore$ From the question, we have
$x = r!\,(y)$
Here $r = 31$
$\therefore$ $x = (31)!.\, y \rightarrow x > y$

44. (a) $\frac{6!}{2!} - 5! = 240$

45. (a) Number of words $= 5! \times {}^7C_3 \times {}^4C_2$
$= 120 \times \frac{7!}{4!3!} \times \frac{4!}{2!2!} = 25200$

46. (c) As 'A' must be first letter of each word.
Total number of words $= 4! = 24$

47. (a) First letter can be put any 7 letters boxes = 7 ways
Similarly, 2nd, 3rd, 4th and 5th letters be put in 7 ways each, respectively
$\therefore 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

48. (c) Number of ways that a cricket team of 11 players can be made out of 15 players $= {}^{15}C_{11} = \frac{15!}{11!\,4!}$
$= \frac{15 \times 14 \times 13 \times 12 \times 11!}{11! \times 1 \times 2 \times 3 \times 4} = 1365$

49. (c) The given word is 'NATION'.
Total number of words that can be formed from given word 'NATION'
$= \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$
Now numbers of words that can be formed from given word 'NATION', so that all vowels never come together.
$= 360 - \left[4! \times \frac{3!}{2!}\right] = 360 - [24 \times 3]$
$= 360 - 72 = 288$
$\therefore$ Option (c) is correct.

50. (a) $A = \{x, y, z\}$
$B = (p, q, r, s)$
$n(A) = 3$
$n(B) = 4$
$\therefore$ Number of distinct relations
$= 2^{n(A) \times n(B)} = 2^{3 \times 4} = 2^{12} = 4096$
$\therefore$ option (a) is correct.

51. (b)

First two words (according to dictionary)	no. of words form
AA – – –	$3! = 6$
AG – – –	$3! = 6$
AI – – –	$3! = 6$
AN – – –	$3! = 6$
GA – – –	$3! = 6$
GI – – –	$3!/2! = 3$
GN – – –	$3!/2! = 3$
IA – – –	$3! = 6$
IG – – –	$3!/2! = 3$
IN – – –	$3!/2! = 3$
NA – – –	$3! = 6$

total = 54

it means 50^{th} word will be starting with 'NA'.

NA —
- N A A G I (49^{th} place)
- N A A I G (50^{th} place)
- N A G A I (51^{th} place)
- N A G I A (52^{th} place)
- N A I A G (53^{th} place)
- N A I G A (54^{th} place)

52. (c) If captain is always included then we can choose 10 more players out of the remaining 14 players. So

$$^{14}C_{10} = \frac{14!}{10!4!} = 1001$$

53. (a) No. of diagonals in a polygen $= {}^nC_2 - n$

$\Rightarrow \quad 44 = {}^nC_2 - n$

$\Rightarrow \quad 44 = \dfrac{n!}{2!(n-2)!} - n$

$\Rightarrow \quad 44 = \dfrac{n(n-1)}{2} - n$

$\Rightarrow \quad 44 = \dfrac{n(n-3)}{2}$

$\Rightarrow \quad n^2 - 3n - 88 = 0$

$\Rightarrow \quad (n-11)(n+8) = 0$

$n \neq -8$

$n = 11$

54. (d) $\because$ Each employee is eligible for 1 or more of the tickets.

$\therefore$ No. of ways $= 20 \times 20 \times 20 = 8000$.

55. (c) No. of digits to be filled at one's place = 3

No. of digits to be filled at 10's place = 5

No. of digits to be filled at 100's place = 4

$\therefore$ Total no. of digits formed $= 3 \times 5 \times 4 = 60$

If zero is at 100's place;

Then; no. of digits to be filled at one's place = 2

& no. of digits to be filled at 10's place = 4

$\therefore$ No. of digits formed with zero at 100's place

$= 1 \times 2 \times 4 = 8$

$\therefore$ Required no. of digits formed $= 60 - 8 = 52$.

56. (d) No. of ways in which 3 holiday travel tickets are to be given to 10 employees $= 10^3 = 1000$

57. (b) Let the given 4 digit decimal number is • | | | | |

Places after decimal can be filled in the following ways:

• | 7 | 8 | 9 | 9 |

Total number of ways $= 7 \times 8 \times 9 \times 9 = 4536$

58. (a) Number of different messages that can be represented by three 0's and two 1's is 10.

Option (a) is correct.

59. (c) Here, $^{15}C_3 - {}^nC_3 = 445$

$\Rightarrow \quad {}^nC_3 = \dfrac{15!}{3!12!} - 445$

$\Rightarrow \quad {}^nC_3 = 10$

$^3C_3 = 1$; $^4C_3 = 4$; $^5C_3 = 10$

$\Rightarrow \quad \boxed{n = 5}$

60. (d) Since sum of digits = 10 (which is not divisible by 3)

$\therefore$ No numbers can be formed.

61. (c) **Case I**

When unit digit can be 1, 3, 5 or 7 & digit at thousand's place can be 1, 2, 3, 4, 5, 6, 7 or 8.

No. of ways digits can be filled are:

| 7 | 8 | 7 | 4 |

Total no's $= 7 \times 8 \times 7 \times 4 = 1568$.

Case II

When unit digit can be 9 & digit at thousand's place can be 1, 2, 3, 4, 5, 6, 7 or 8.

No. of ways digits can be filled are:

| 8 | 8 | 7 | 1 |

Total no's $= 8 \times 8 \times 7 \times 1 = 448$.

Case III

When unit digit can be 1, 3, 5 or 7 & digit at thousand's place can be 9.

No. of ways digits can be filled are:

| 1 | 8 | 7 | 4 |

Total no's $= 1 \times 8 \times 7 \times 4 = 224$.

$\therefore$ Number of odd digits between 1000 & 9999 with no digit repeated $= 1568 + 448 + 224$

$= 2240$.

62. (b) EQUATION – 8 letters.

Consonants – Q, T, N – 3 letters.

first letter of 8 – letter word can be any of 3 consonants

Last letter of 8 – letter word can be remaining 2 consonants.

The middle 6 – letters can be arranged in 6! ways.

So, number of different words $= 3 \times 2 \times 6!$

$= 6 \times 720 = 4320$.

63. (a) P E R M U T A T I O N

11 letters and T is repeated 2 times.

$\therefore$ Different permutations $\dfrac{11!}{2!}$

$= 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3$

$= 19958400$

64. (c) Number of ways $= \dfrac{8!8!}{4!6!} \times 10!$

$= (8!)^2 \times \dfrac{10!}{4!6!}$

$= 8!^2 \times \dfrac{10 \times 9 \times 8 \times 7}{4!}$

$= (8!)^2 \times \dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$

$= (210) \times (8!)^2$

65. (d) Number between 100 and 1000 are 3-digit numbers.
It is given that the digits should not be repeated.
Number of given digits = 5.
In a 3-digit number, first number can be arranged in 5 ways.
Second number in 4 ways.
Third number in 3 ways.
$\therefore$ Numbers that can be formed = $5 \times 4 \times 3 = 60$

66. (a) A number divisible by 10 means the last digit is 0.
So, the remaining 3 digits can be arranged in
$4 \times 3 \times 2$ ways = 24 ways.

67. (a) To form a triangle, we need 3 points. 12 points are given.
So, ${}^{12}C_3$ triangles can be formed.
But, given that 7 points are on a straight line. selecting 3 points from this set will not form a triangle.

So, number of triangles formed ${}^{12}C_3 - {}^{7}C_3$

$$= \frac{12!}{3!9!} - \frac{7!}{3!4!}$$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} - \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 220 - 35 = 185$$

68. (d) 3 bowlers are selected among 5 bowlers in ${}^5C_3$ ways.
Remaining 8 player's are selected from 12 player's in ${}^{12}C_8$ ways.
$\therefore$ total number of ways = ${}^{12}C_8 \times {}^5C_3$.

69. (c) Number of 5 digits numbers with all distinct digit is same as filling of 5 vacent placed out of 10 boxes.
First digit of any number can be choosen in 9 ways.
Remaining 4 digits can be choosen in remaining 9 digits in ${}^9P_4$ ways.
Total number of such number
$= 9 \times 9 \times 8 \times 7 \times 6 = \boxed{27216}$

70. (a) Number of 3 digit number made from digit 3, 4 or 5 and having all distinct digit = 3! = 6
and sum of such numbers are
$543 + 534 + 345 + 354 + 435 + 453 = 2664$

71. (d)

Cases	Dice III (z)	Dice II (y)	Dice I (x)	Total No. of Ways
I	1	2	3, 4, 5, 6	4 + 3 + 2 + 1 = 10
		3	4, 5, 6	
		4	5, 6	
		5	6	
II	2	3	4, 5, 6	3 + 2 + 1 = 6
		4	5, 6	
		5	6	
III	3	4	5, 6	2 + 1 = 3
		5	6	
IV	4	5	6	1

Total number of ways = 10 + 6 + 3 + 1 = 20

72. (b) A straight line can be formed by joining 2 points.
$\therefore$ Total number of straight lines = ${}^{10}C_2$

$$= \frac{10 \times 9}{2 \times 1} = 45$$

73. (c) Number of ways ${}^4C_1\,{}^6C_4 + {}^4C_2\,{}^6C_3 + {}^4C_3\,{}^6C_2 + {}^4C_4\,{}^6C_1$
$= (4)(15) + (6)(20) + (4)(15) + (1)(6)$
$= 60 + 120 + 60 + 6 = 246$

74. (d) Given digits are 1, 2, 3, 4 and 5.
Total number of 3-digit even numbers = ${}^4C_2 \times {}^2C_1$.

$$= \frac{3 \times 4}{2} \times 2 = 12.$$

Cartesian Coordinate System and Straight Line 7

1. The lines (p + 2q) x + (p – 3q) y = p – q for different values of p and q pass through the fixed point given by which one of the following ? *[2006-I]*
   (a) $\left(\frac{3}{2},\frac{5}{2}\right)$ (b) $\left(\frac{2}{5},\frac{2}{5}\right)$
   (c) $\left(\frac{3}{5},\frac{3}{5}\right)$ (d) $\left(\frac{2}{5},\frac{3}{5}\right)$

2. What is the angle between the two straight lines
   $y=(2-\sqrt{3})x+5$ and $y=(2+\sqrt{3})x-7$? *[2006-I]*
   (a) 60° (b) 45°
   (c) 30° (d) 15°

3. What is the image of the point (2, 3) in the line y = – x ?
   (a) (– 3, – 2) (b) (– 3, 2)
   (c) (– 2, – 3) (d) (3, 2) *[2006-II]*

4. The middle point of A (1, 2) and B (x, y) is C (2, 4). If BD is perpendicular to AB such that CD = 3 unit, then what is the length BD ? *[2006-II]*
   (a) $2\sqrt{2}$ unit (b) 2 unit
   (c) 3 unit (d) $3\sqrt{2}$ unit

5. If the points A(1, 2), B(2, 4) and C(3, a) are collinear, what is the length BC ? *[2006-II]*
   (a) $\sqrt{2}$ unit (b) $\sqrt{3}$ unit
   (c) $\sqrt{5}$ unit (d) 5 unit

6. What is the acute angle between the lines Ax + By = A + B and A (x – y) + B (x + y) = 2B? *[2007-I]*
   (a) 45° (b) $\tan^{-1}\left(\frac{A}{\sqrt{A^2+B^2}}\right)$
   (c) (d) 60°

7. If p be the length of the perpendicular from the orgin on the straight line x + 2by + 2p = 0, then what is the value of b?
   (a) $\frac{1}{p}$ (b) p
   (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$ *[2007-I]*

8. In what ratio does the line y – x + 2 = 0 cut the line joining (3, –1) and (8, 9)? *[2007-I]*
   (a) 2 : 3 (b) 3 : 2
   (c) 3 : –2 (d) 1 : 2

9. The points (2, –2), (8, 4), (4, 6) and (–1, 1) in order are the vertices of which one of the following quadrilaterals?
   (a) Square
   (b) Rhombus
   (c) Rectangle(but not square)
   (d) Trapezium *[2007-I]*

10. If p be the length of the perpendicular from the origin on the straight line ax + by = p and $b=\frac{\sqrt{3}}{2}$, then what is the angle between the perpendicular and the positive direction of x-axis? *[2007-I]*
    (a) 30° (b) 45° (c) 60° (d) 90°

11. The straight line ax + by + c = 0 and the coordinate axes form an isosceles triangle under which one of the following conditions?
    (a) |a| = |b| (b) |a| = |c|
    (c) |b| = |c| (d) none of these *[2007-I]*

12. The coordinates of P and Q are (–3, 4) and (2, 1), respectively. If PQ is extended to R such that PR = 2QR, then what are the coordinates of R ? *[2007-II]*
    (a) (3, 7) (b) (2, 4)
    (c) $\left(-\frac{1}{2},\frac{5}{2}\right)$ (d) (7, – 2)

13. Which one of the following points on the line 2x – 3y = 5 is equidistant from (1, 2) and (3, 4) ? *[2007-II]*
    (a) (7, 3) (b) (4, 1)
    (c) (1, – 1) (d) (– 2, – 3)

14. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.
    Assertion (A) : If two triangles with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3) satisfy the relation
    $$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$
    , then the triangles are congruent.

Reason (R) : For the given triangles satisfying the above relation implies that the triangles have equal area.
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2007-II]*

15. If A (2, 3), B (1, 4), C (0 – 2) and D (x, y) are the vertices of a parallelogram, then what is the value of (x, y) ?
(a) $(1, -3)$ (b) $(2, 4)$
(c) $(1, 1)$ (d) $(0, 0)$ *[2008-I]*

16. If O be the origin and A (x_1, y_1), B (x_2, y_2) are two points, then what is (OA) (OB) cos $\angle$ AOB ?
(a) $x_1^2 + x_2^2$ (b) $y_1^2 + y_2^2$
(c) $x_1 x_2 + y_1 y_2$ (d) $x_1 y_1 + x_2 y_2$ *[2008-I]*

17. The numerical value of the perimeter of a square exceeds that of its area by 4. What is the side of the square?
(a) 1 unit (b) 2 unit
(c) 3 unit (d) 4 unit *[2008-I]*

18. If (a, b), (c, d) and (a – c, b – d) are collinear, then which one of the following is correct ?
(a) $bc - ad = 0$ (b) $ab - cd = 0$
(c) $bc + ad = 0$ (d) $ab + cd = 0$ *[2008-I]*

19. The point of intersection of the two lines $2x + 3y + 4 = 0$ and $4x + 3y + 2 = 0$ is at a distance d from origin. What is the value of d ? *[2008-II]*
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{7}$

20. The line through the points (4, 3) and (2, 5) cuts off intercepts of lengths λ and μ on the axes. Which one of the following is correct? *[2008-II]*
(a) $\lambda > \mu$ (b) $\lambda < \mu$
(c) $\lambda > -\mu$ (d) $\lambda = \mu$

21. What is the locus of a point which is equidistant from the points $(a + b, a - b)$ and $(b - a, a + b)$? *[2008-II]*
(a) $bx - ay = 0$ (b) $bx + ay = 0$
(c) $-ax + by = 0$ (d) $ax + by = 0$

22. What is the area of the triangle formed by the lines $y - x = 0, y + x = 0, x = c$? *[2009-I]*
(a) $c/2$ (b) c^2
(c) $2c^2$ (d) $c^2/2$

23. What is the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$? *[2009-I]*
(a) $(1, 10)$ (b) $(5, 6)$
(c) $(6, 5)$ (d) $(7, 4)$

24. Consider the following statements : *[2009-I]*
1. The equation to a straight line parallel to the axis of x is $y = d$, where d is a constant.
2. The equation to the axis of x is $x = 0$.
Which of the statement (s) given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

25. What is the product of the perpendiculars from the two points $\left(\pm\sqrt{b^2 - a^2}, 0\right)$ to the line ax cos ϕ + by sin$\phi = ab$? *[2009-II]*
(a) a^2 (b) b^2
(c) ab (d) a/b

26. The middle point of the segment of the straight line joining the points (p, q) and $(q, -p)$ is $(r/2, s/2)$. What is the length of the segment? *[2009-II]*
(a) $[(s^2 + r^2)^{1/2}]/2$ (b) $[(s^2 + r^2)^{1/2}]/4$
(c) $(s^2 + r^2)^{1/2}$ (d) $s + r$

27. What is the locus of a point which is equidistant from the point $(m + n, n - m)$ and the point $(m - n, n + m)$? *[2009-II]*
(a) $mx = ny$ (b) $nx = -my$
(c) $nx = my$ (d) $mx = -ny$

28. Let O (0, 0, 0), P (3, 4, 5), Q (m, n, r) and R (1, 1, 1) be the vertices of a parallelogram taken in order. What is the value of $m + n + r$? *[2010-I]*
(a) 6 (b) 12
(c) 15 (d) More than 15

29. What is the image of the point (1, 2) on the line $3x + 4y - 1 = 0$? *[2010-I]*
(a) $\left(-\frac{7}{5}, -\frac{6}{5}\right)$ (b) $\left(\frac{7}{8}, \frac{1}{2}\right)$
(c) $\left(\frac{7}{8}, -\frac{1}{2}\right)$ (d) $\left(-\frac{7}{5}, \frac{1}{2}\right)$

30. If (–5, 4) divides the line segment between the coordinate axes in the ratio 1: 2, then what is its equation? *[2010-I]*
(a) $8x + 5y + 20 = 0$ (b) $5x + 8y - 7 = 0$
(c) $8x - 5y + 60 = 0$ (d) $5x - 8y + 57 = 0$

31. What is the equation to the straight line joining the origin to the point of intersection of the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$? *[2010-II]*
(a) $x + y = 0$ (b) $x + y + 1 = 0$
(c) $x - y = 0$ (d) $x + y + 2 = 0$

32. If the straight lines $x - 2y = 0$ and $kx + y = 1$ intersect at the point $\left(1, \frac{1}{2}\right)$, then what is the value of k? *[2010-II]*
(a) 1 (b) 2 (c) 1/2 (d) –1/2

33. What is the maximum number of straight lines that can be drawn with any four points in a plane such that each line contains at least two of these points? *[2010-II]*
(a) 2 (b) 4 (c) 6 (d) 12

34. A square is drawn by joining mid points of the sides of a square. Another square is drawn inside the second square in the same way and the process is continued in definitely. If the side of the first square is 16 cm, then what is the sum of the areas of all the squares? *[2010-II]*
(a) 256 sq cm (b) 512 sq cm
(c) 1024 sq cm (d) 512 / 3 sq cm

35. What is the slope of the line perpendicular to the line $\frac{x}{4}+\frac{y}{3}=1$? [2010-II]
(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$
(c) $-\frac{4}{3}$ (d) $\frac{4}{3}$

36. If the area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq unit, then what is the value of k? [2010-II]
(a) 3 (b) 6
(c) 9 (d) 12

37. What is the locus of a point which moves equidistant from the coordinate axes? [2011-I]
(a) $x \pm y = 0$ (b) $x + 2y = 0$
(c) $2x + y = 0$ (d) None of these

38. What is the equation of the line joining the origin with the point of intersection of the lines $4x + 3y = 12$ and $3x + 4y = 12$? [2011-I]
(a) $x + y = 1$ (b) $x - y = 1$
(c) $3y = 4x$ (d) $x = y$

39. If the sum of the squares of the distances of the point (x, y) from the points (a, 0) and (– a, 0) is $2b^2$, then which one of the following is correct ? [2011-I]
(a) $x^2 + a^2 = b^2 + y^2$ (b) $x^2 + a^2 = 2b^2 - y^2$
(c) $x^2 - a^2 = b^2 + y^2$ (d) $x^2 + a^2 = b^2 - y^2$

40. The line $mx + ny = 1$ passes through the points (1, 2) and (2, 1). What is the value of m? [2011-I]
(a) 1 (b) 3
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

41. What is the equation of the line passing through (2, – 3) and parallel to Y-axis? [2011-I]
(a) $Y = -3$ (b) $Y = 2$
(c) $X = 2$ (d) $X = -3$

42. What is the locus of the point which is at a distance 8 units to the left of Y-axis? [2011-I]
(a) $X = 8$ (b) $Y = 8$
(c) $X = -8$ (d) $Y = -8$

43. Two straight lines $x - 3y - 2 = 0$ and $2x - 6y - 6 = 0$ [2011-I]
(a) never intersect
(b) intersect at a single point
(c) intersect at infinite number of points
(d) intersect at more than one point (but finite number of points)

44. If (a, 0), (0, b) and (1, 1) are collinear, what is ($a + b - ab$) equal to? [2011-I]
(a) 2 (b) 1
(c) 0 (d) – 1

45. What are the co-ordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$? [2011-II]
(a) (2, 9) (b) (5, 6)
(c) (– 5, 6) (d) (6, 5)

46. How many diagonal will be there in an n-sided regular polygon? [2011-II]
(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n-3)}{2}$
(c) $n^2 - n$ (d) $\frac{n(n+1)}{2}$

47. If (p, q) is the point on the x-axis equidistant from the points (1, 2) and (2, 3), then which one of the following is correct? [2011-II]
(a) $p = 0, q = 4$ (b) $p = 4, q = 0$
(c) $p = 3/2, q = 0$ (d) $p = 1, q = 0$

48. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then which one of the following is correct? [2011-II]
(a) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ (b) $\frac{1}{p^2}=\frac{1}{a^2}-\frac{1}{b^2}$
(c) $\frac{1}{p}=\frac{1}{a}+\frac{1}{b}$ (d) $\frac{1}{p}=\frac{1}{a}-\frac{1}{b}$

49. For what value of k, are the lines $x + 2y - 9 = 0$ and $kx + 4y + 5 = 0$ parallel? [2011-II]
(a) 2 (b) – 1
(c) 1 (d) 0

50. What is the equation of a line parallel to x-axis at a distance of 5 units below x-axis? [2011-II]
(a) $x = 5$ (b) $x = -5$
(c) $y = 5$ (d) $y = -5$

51. What is the equation of line passing through (0, 1) and making an angle with the y-axis equal to the inclination of the line $x - y = 4$ with x-axis? [2012-I]
(a) $y = x + 1$ (b) $x = y + 1$
(c) $2x = y + 2$ (d) None of the above

52. What is the perimeter of the triangle with vertices A(–4, 2), B(0, –1) and C(3, 3)? [2012-I]
(a) $7+3\sqrt{2}$ (b) $10+5\sqrt{2}$
(c) $11+6\sqrt{2}$ (d) $5+\sqrt{2}$

53. If the mid point between the points (a + b, a – b) and (– a, b) lies on the line ax + by = k, what is k equal to? [2012-I]
(a) a/b (b) a + b
(c) ab (d) a – b

54. The acute angle which the perpendicular from origin on the line $7x - 3y = 4$ makes with the x-axis is [2012-I]
(a) zero (b) positive but not $\pi/4$
(c) negative (d) $\pi/4$

55. What is the distance between the lines 3x + 4y = 9 and 6x + 8y = 18? [2012-I]
(a) 0 (b) 3 units
(c) 9 units (d) 18 units

56. What is the perpendicular distance of the point (x, y) from x-axis? [2012-I]
(a) x (b) y
(c) $|x|$ (d) $|y|$

57. The line making an angle (– 120°) with x-axis is situated in the: *[2012-II]*
(a) first quadrant (b) second quadrant
(c) third quadrant (d) fourth quadrant

58. The locus of a point equidistant from three collinear points is: *[2012-II]*
(a) a straight line (b) a pair of points
(c) a point (d) the null set

59. The equation to the locus of a point which is always equidistant from the points (1, 0) and (0, –2) is : *[2012-II]*
(a) $2x + 4y + 3 = 0$
(b) $4x + 2y + 3 = 0$
(c) $2x + 4y - 3 = 0$
(d) $4x + 2y - 3 = 0$

60. The points (5, 1), (1, –1) and (11, 4) are : *[2012-II]*
(a) collinear
(b) vertices of right angled triangle
(c) vertices of equilateral triangle
(d) vertices of an isosceles triangle

61. What is the perpendicular distance between the parallel lines $3x + 4y = 9$ and $9x + 12y + 28 = 0$? *[2012-II]*
(a) $\frac{7}{3}$ units (b) $\frac{8}{3}$ units
(c) $\frac{10}{3}$ units (d) $\frac{11}{3}$ units

62. Let p, q, r, s be the distances from origin of the points (2, 6), (3, 4), (4, 5) and (–2, 5) respectively. Which one of the following is a whole number? *[2012-II]*
(a) p (b) q
(c) r (d) s

63. From the point (4, 3) a perpendicular is dropped on the x-axis as well as on the y-axis. If the lengths of perpendiculars are p, q respectively, then which one of the following is correct? *[2012-II]*
(a) $p = q$ (b) $3p = 4q$
(c) $4p = 3q$ (d) $p + q = 5$

64. The line $y = 0$ divides the line joining the points (3, –5) and (–4, 7) in the ratio : *[2012-II]*
(a) 3 : 4 (b) 4 : 5
(c) 5 : 7 (d) 7 : 9

65. The equation of a straight line which makes an angle 45° with the x-axis with y-intercept 101 units is : *[2012-II]*
(a) $10x + 101y = 1$ (b) $101x + y = 1$
(c) $x + y - 101 = 0$ (d) $x - y + 101 = 0$

66. If the points (2, 4), (2, 6) and $\left(2+\sqrt{3}, k\right)$ are the vertices of an equilateral triangle, then what is the value of k ?
(a) 6 (b) 5 *[2012-II]*
(c) – 3 (d) 1

67. What is the equation of a straight line which passes through (3, 4) and sum of whose x and y intercepts is 14 ?
(a) $4x + 3y = 24$ (b) $x + y = 14$ *[2013-I]*
(c) $4x - 3y = 0$ (d) $3x + 4y = 25$

68. The point whose abscissa is equal to its ordinate and which is equidistant from A (–1, 0) and B(0, 5) is *[2013-I]*
(a) (1, 1) (b) (2, 2)
(c) (–2, –2) (d) (3, 3)

69. What is the area of the triangle whose vertices are (3, 0), (0, 4) and (3, 4) ? *[2013-I]*
(a) 6 sq. unit (b) 7.5sq. unit
(c) 9 sq. unit (d) 12 sq. unit

70. A straight line passes through the points (5, 0) and (0, 3). The length of the perpendicular from the point (4, 4) on the line is
(a) $\frac{\sqrt{17}}{2}$ (b) $\sqrt{\frac{17}{2}}$ *[2013-I]*
(c) $\frac{15}{\sqrt{34}}$ (d) $\frac{17}{2}$

71. What is the inclination of the line $\sqrt{3}x - y - 1 = 0$?
(a) 30° (b) 60° *[2013-I]*
(c) 135° (d) 150°

72. Two straight line paths are represented by the equation $2x - y = 2$ and $-4x + 2y = 6$. Then the paths will
(a) cross each other at one point *[2013-I]*
(b) not cross each other
(c) cross each other at two points
(d) cross each other at infinitely many points

73. For what value of k, the equations $3x - y = 8$ and $9x - ky = 24$ will have infinitely many solutions ? *[2013-I]*
(a) 6 (b) 5 (c) 3 (d) 1

74. What is the area of the triangle bounded by the side $x = 0$, $y = 0$, and $x + y = 2$? *[2013-I]*
(a) 1 square unit (b) 2 square unit
(c) 4 square unit (d) 8 square unit

75. If the three vertices of the parallelogram *ABCD* are $A(1, a)$, $B(3, a)$, $C(2, b)$, then *D* is equal to *[2013-II]*
(a) $(3, b)$ (b) $(6, b)$
(c) $(4, b)$ (d) $(5, b)$

76. What is the equation of the line which passes through (4, –5) and is perpendicular to $3x + 4y + 5 = 0$? *[2013-II]*
(a) $4x - 3y - 31 = 0$ (b) $3x - 4y - 41 = 0$
(c) $4x + 3y - 1 = 0$ (d) $3x + 4y + 8 = 0$

77. For what value of *k* are the two straight lines $3x + 4y = 1$ and $4x + 3y + 2k = 0$ equidistant from the point (1, 1) ? *[2013-II]*
(a) $\frac{1}{2}$ (b) 2
(c) –2 (d) $-\frac{1}{2}$

78. A point *P* moves such that its distances from (1, 2) and (–2, 3) are equal. Then the locus of *P* is *[2013-II]*
(a) straight line (b) Parabola
(c) ellipse (d) hyperbola

79. The equation of the locus of a point which is equidistant from the axes is *[2013-II]*
(a) $y = 2x$ (b) $x = 2y$
(c) $y = \pm x$ (d) $2y + x = 0$

80. What angle does the line segment joining (5, 2) and (6, –15) subtend at (0, 0) ? [2013-II]
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

81. The length of latus rectum of the ellipse $4x^2 + 9y^2 = 36$ is [2013-II]
(a) $\frac{4}{3}$ (b) $\frac{8}{3}$
(c) 6 (d) 12

82. What is the equation to the straight line passing through (5, –2) and (–4, 7)? [2013-II]
(a) $5x-2y=4$ (b) $-4x+7y=9$
(c) $x+y=3$ (d) $x-y=-1$

83. What is the angle between the lines $x + y = 1$ and $x - y = 1$? [2013-II]
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

84. The centroid of the triangle with vertices (2, 3), (–2, –5) and (3, 5) is at [2013-II]
(a) (1, 1) (b) (2, –1)
(c) (1, –1) (d) (1, 2)

85. The equation of the line, the reciprocals of whose intercepts on the axes are m and n, is given by [2013-II]
(a) $nx+my=mn$ (b) $mx+ny=1$
(c) $mx+ny=mn$ (d) $mx-ny=1$

86. Consider the following points : [2014-I]
1. (0, 5) 2. (2, –1)
3. (3, –4)
Which of the above lie on the line $3x + y = 5$ and at a distance $\sqrt{10}$ from (1, 2) ?
(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 1, 2 and 3

87. What is the equation of the line through (1, 2) so that the segment of the line intercepted between the axes is bisected at this point ? [2014-I]
(a) $2x-y=4$ (b) $2x-y+4=0$
(c) $2x+y=4$ (d) $2x+y+4=0$

88. What is the equation of straight line passing through the point (4, 3) and making equal intercepts on the coordinate axes ? [2014-I]
(a) $x+y=7$ (b) $3x+4y=7$
(c) $x-y=1$ (d) None of these

89. A (3, 4) and B(5, –2) are two points and P is a point such that PA = PB. If the area of triangle PAB is 10 square unit, what are the coordinates of P ? [2014-II]
(a) (1, 0) only (b) (7, 2) only
(c) (1, 0) or (7, 2) (d) Neither (1, 0) nor (7, 2)

90. Which of the following is correc in respect of the equations $\frac{x-1}{2}=\frac{y-2}{3}$ and $2x+3y=5$? [2014-II]
(a) They represent two lines which are parallel.
(b) They represent two lines which are perpendicular.
(c) They represent two lines which are neither parallel nor perpendicular.
(d) The first equation does not represent a line.

DIRECTIONS (Qs. 91-93): *For the next three (3) items that follow:*

Consider the triangle ABC with vertices A (–2, 3), B (2, 1) and $C(1, 2)$.

91. What is the circumcentre of the triangle ABC ? [2015-I]
(a) (–2, –2) (b) (2, 2)
(c) (–2, 2) (d) (2, –2)

92. What is the centroid of the tirnalge ABC ? [2015-I]
(a) $\left(\frac{1}{3},1\right)$ (b) $\left(\frac{1}{3},2\right)$
(c) $\left(1,\frac{2}{3}\right)$ (d) $\left(\frac{1}{2},3\right)$

93. What is the foot of the altitude from the vertex A of the triangle ABC? [2015-I]
(a) (1, 4) (b) (–1, 3)
(c) (–2, 4) (d) (–1, 4)

94. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is [2015-I]
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{1}{3}$ (d) 3

95. The Perpendicular distance between the straight lines $6x + 8y + 15 = 0$ and $3x + 4y + 9 = 0$ is [2015-I]
(a) $\frac{3}{2}$ units (b) $\frac{3}{10}$ unit
(c) $\frac{3}{4}$ unit (d) $\frac{2}{7}$ unit

96. The length of perpendicular from the origin to a line is 5 units and the line makes an angle 120° with the positive direction of x-axis. The equation of the line is [2015-I]
(a) $x+\sqrt{3}y=5$ (b) $\sqrt{3}x+y=10$
(c) $\sqrt{3}x-y=10$ (d) None of these

97. The equation of the line joining the origin to the point of interesection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ is [2015-I]
(a) $x-y=0$ (b) $x+y=0$
(c) $x=0$ (d) $y=0$

98. If a line is perpendicular to the line $5x - y = 0$ and forms a triangle of area 5 square units with co-ordinate axes, then its equation is [2015-II]
(a) $x+5y\pm5\sqrt{2}=0$ (b) $x-5y\pm5\sqrt{2}=0$
(c) $5x+y\pm5\sqrt{2}=0$ (d) $5x-y\pm5\sqrt{2}=0$

99. The three lines $4x + 4y = 1$, $8x - 3y = 2$, $y = 0$ are *[2015-II]*
    (a) the sides of an isosceles triangle
    (b) concurrent
    (c) mutually perpendicular
    (d) the sides of an equilateral triangle
100. The line $3x + 4y - 24 = 0$ intersects the x-axis at A and y-axis at B. Then the circumcentre of the triangle OAB where O is the origin is *[2015-II]*
    (a) (2, 3) (b) (3, 3)
    (c) (4, 3) (d) None of the above
101. The product of y the perpendiculars from the two points $(\pm 4, 0)$ to the line $3x \cos\phi + 5y \sin\phi = 15$ is *[2015-II]*
    (a) 25 (b) 16
    (c) 9 (d) 8
102. The lines $2x = 3y = -z$ and $6x = -y = -4z$ *[2015-II]*
    (a) are perpendicular (b) are parallel
    (c) intersect at an angle 45° (d) intersect at an angle 60°
103. Two straight lines passing through the point A(3, 2) cut the line $2y = x + 3$ and x-axis perpendicularly at P and Q respectively. The equation of the line PQ is *[2015-II]*
    (a) $7x + y - 21 = 0$ (b) $x + 7y + 21 = 0$
    (c) $2x + y - 8 = 0$ (d) $x + 2y + 8 = 0$
104. A straight line intersects x and y axes at P and Q respectively. If (3, 5) is the middle point of PQ, then what is the area of the triangle OPQ? *[2016-I]*
    (a) 12 square units (b) 15 square units
    (c) 20 square units (d) 30 square units

DIRECTIONS (Qs. 105-106): *For the next two (2) items that follow:*

Consider the lines $y = 3x$, $y = 6x$ and $y = 9$ *[2016-I]*

105. What is the area of the triangle formed by these lines?
    (a) $\frac{27}{4}$ square units (b) $\frac{27}{2}$ square units
    (c) $\frac{19}{4}$ square units (d) $\frac{19}{2}$ square units
106. The centroid of the triangle is at which one of the following points?
    (a) (3, 6) (b) $\left(\frac{3}{2}, 6\right)$
    (c) (3, 3) (d) $\left(\frac{3}{2}, 9\right)$

DIRECTIONS (Qs. 107-108): *For the next two (2) items that follow:*

Consider the curves $y = |x - 1|$ and $|x| = 2$ *[2016-I]*

107. What is/are the point(s) of intersection of the curves ?
    (a) (–2, 3) only (b) (2, 1) only
    (c) (–2, 3) and (2, 1) (d) Neither (–2, 3) nor (2, 1)
108. What is the area of the region bounded by the curves and x-axis ?
    (a) 3 square units (b) 4 square units
    (c) 5 square units (d) 6 square units

DIRECTIONS (Qs. 109-110): *For the next two (2) items that follow:*

Consider the two lines $x + y + 1 = 0$ and $3x + 2y + 1 = 0$

109. What is the equation of the line passing through the point of intersection of the given lines and parallel to x-axis? *[2016-I]*
    (a) $y + 1 = 0$ (b) $y - 1 = 0$
    (c) $y - 2 = 0$ (d) $y + 2 = 0$
110. What is the equation of the line passing through the point of intersection of the given lines and parallel to y-axis? *[2016-I]*
    (a) $x + 1 = 0$ (b) $x - 1 = 0$
    (c) $x - 2 = 0$ (d) $x + 2 = 0$

DIRECTIONS (Qs. 111-113): *For the next three (3) items that follow:*

Consider a parallelogram whose vertices are A (1, 2), B (4, y), C (x, 6) and D (3, 5) taken in order. *[2016-I]*

111. What is the value of $AC^2 - BD^2$?
    (a) 25 (b) 30
    (c) 36 (d) 40
112. What is the point of intersection of the diagonals?
    (a) $\left(\frac{7}{2}, 4\right)$ (b) (3, 4)
    (c) $\left(\frac{7}{2}, 5\right)$ (d) (3, 5)
113. What is the area of the parallelogram?
    (a) $\frac{7}{2}$ square units (b) 4 square units
    (c) $\frac{11}{2}$ square units (d) 7 square units
114. (a, 2b) is the mid-point of the line segment joining the points (10, –6) and (k, 4). If $a - 2b = 7$, then what is the value of k? *[2016-I]*
    (a) 2 (b) 3
    (c) 4 (d) 5
115. An equilateral triangle has one vertex at (0,0) and another at $(3, \sqrt{3})$. What are the coordinates of the third vertex? *[2016-II]*
    (a) $(0, 2\sqrt{3})$ only
    (b) $(3, -\sqrt{3})$ only
    (c) $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$
    (d) Neither $(0, 2\sqrt{3})$ nor $(3, -\sqrt{3})$
116. What is the equation of the straight line which passes through the point of intersection of the straight lines $x + 2y = 5$ and $3x + 7y = 17$ and is perpendicular to the straight line $3x + 4y = 10$? *[2016-II]*
    (a) $4x + 3y + 2 = 0$ (b) $4x - y + 2 = 0$
    (c) $4x - 3y - 2 = 0$ (d) $4x - 3y + 2 = 0$
117. If (a, b) is at unit distance from the line $8x + 6y + 1 = 0$, then which of the following conditions are correct? *[2016-II]*
    1. $3a - 4b - 4 = 0$
    2. $8a + 6b + 11 = 0$
    3. $8a + 6b - 9 = 0$

    Select the correct answer using the code given below:
    (a) 1 and 2 only (b) 2 and 3 only
    (c) 1 and 3 only (d) 1, 2 and 3

118. A straight line cuts off an intercept of 2 units on the positive direction of x-axis and passes through the point (–3, 5). What is the foot of the perpendicular drawn from the point (3, 3) on this line? *[2016-II]*
(a) (1, 3) (b) (2, 0)
(c) (0, 2) (d) (1, 1)

119. What is the curve which passes through the point (1,1) and whose slope is $\frac{2y}{x}$? *[2016-II]*
(a) Circle (b) Parabola
(c) Ellipse (d) Hyperbola

120. If a vertex of a triangle is (1, 1) and the midpoints of two sides of the triangle through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is *[2017-I]*
(a) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ (b) $\left(-1, \frac{7}{3}\right)$
(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{3}\right)$

121. The incentre of the triangle with vertices $A(1, \sqrt{3}), B(0, 0)$ and C(2, 0) is *[2017-I]*
(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

122. If the three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), then what are the coordinates of the fourth vertex? *[2017-I]*
(a) (1, 2) (b) (1, 0)
(c) (0, 0) (d) (1, –1)

123. What is the ratio in which the point $C\left(-\frac{2}{7}, -\frac{20}{7}\right)$ divides the line joining the points A(–2, –2) and B(2, –4)? *[2017-I]*
(a) 1 : 3 (b) 3 : 4
(c) 1 : 2 (d) 2 : 3

124. What is the equation of the straight line parallel to 2x + 3y + 1 = 0 and passes through the point (–1, 2)? *[2017-I]*
(a) $2x + 3y - 4 = 0$ (b) $2x + 3y - 5 = 0$
(c) $x + y - 1 = 0$ (d) $3x - 2y + 7 = 0$

125. If the centroid of a triangle formed by (7, x), (y, –6) and (9, 10) is (6, 3), then the values of x and y are respectively *[2017-I]*
(a) 5, 2 (b) 2, 5
(c) 1, 0 (d) 0, 0

126. The points (a, b), (0, 0), (*–a*, *–b*) and (*ab*, b^2) are *[2017-II]*
(a) the vertices of a parallelogram
(b) the vertices of a rectangle
(c) the vertices of a square
(d) collinear

127. The distance of the point (1, 3) from the line 2x + 3y = 6, measured parallel to the line 4x + y = 4, is *[2017-II]*
(a) $\frac{5}{\sqrt{13}}$ units (b) $\frac{3}{\sqrt{17}}$ unit
(c) $\sqrt{17}$ units (d) $\frac{\sqrt{17}}{2}$ units

128. The equation of a straight line which cuts off an intercept of 5 units on negative direction of y-axis and makes an angle 120° with positive direction of x-axis is *[2017-II]*
(a) $y + \sqrt{3}x + 5 = 0$ (b) $y - \sqrt{3}x + 5 = 0$
(c) $y + \sqrt{3}x - 5 = 0$ (d) $y - \sqrt{3}x - 5 = 0$

129. The equation of the line passing through the point (2, 3) and the point of intersection of lines 2x – 3y + 7 = 0 and 7x + 4y + 2 = 0 is *[2017-II]*
(a) $21x + 46y - 180 = 0$ (b) $21x - 46y + 96 = 0$
(c) $46x + 21y + 155 = 0$ (d) $46x - 21y - 29 = 0$

130. What is the distance between the points which divide the line segment joining (4, 3) and (5, 7) internally and externally in the ratio 2 : 3? *[2018-I]*
(a) $\frac{12\sqrt{17}}{5}$ (b) $\frac{13\sqrt{17}}{5}$
(c) $\frac{\sqrt{17}}{5}$ (d) $\frac{6\sqrt{17}}{5}$

131. What is the equation of the straight line cutting off an intercept 2 from the negative direction of y-axis and inclined at 30° with the positive direction of x-axis? *[2018-I]*
(a) $x - 2\sqrt{3}y - 3\sqrt{2} = 0$
(b) $x + 2\sqrt{3}y - 3\sqrt{2} = 0$
(c) $x + \sqrt{3}y - 2\sqrt{3} = 0$
(d) $x - \sqrt{3}y - 2\sqrt{3} = 0$

132. What is the equation of the line passing through the point of intersection of the lines x + 2y – 3 = 0 and 2x – y + 5 = 0 and parallel to the line y – x + 10 = 0? *[2018-I]*
(a) $7x - 7y + 18 = 0$ (b) $5x - 7y + 18 = 0$
(c) $5x - 5y + 18 = 0$ (d) $x - y + 5 = 0$

133. Consider the following statements: *[2018-I]*
1. The length p of the perpendicular from the origin to the line ax + by = c satisfies the relation $p^2 = \frac{c^2}{a^2 + b^2}$.
2. The length p of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$ satisfies the relation $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.
3. The length p of the perpendicular from the origin to the line y = mx + c satisfies the relation $\frac{1}{p^2} = \frac{1 + m^2 + c^2}{c^2}$.

Which of the above is/are correct?
(a) 1, 2 and 3 (b) 1 only
(c) 1 and 2 only (d) 2 only

134. What is the equation of the straight line passing through the point (2, 3) and making an intercept on the positive y-axis equal to twice its intercept on the positive x-axis? *[2018-I]*
(a) $2x + y = 5$ (b) $2x + y = 7$
(c) $x + 2y = 7$ (d) $2x - y = 1$

135. The perpendiculars that fall from any point of the straight line $2x + 11y = 5$ upon the two straight lines $24x + 7y = 20$ and $4x - 3y = 2$ are *[2018-I]*
(a) 12 and 4 respectively
(b) 11 and 5 respectively
(c) Equal to each other
(d) Not equal to each other

136. The equation of the line, when the portion of it intercepted between the axes is divided by the point (2, 3) in the ratio of 3 : 2, is *[2018-I]*
(a) Either $x + y = 4$ or $9x + y = 12$
(b) Either $x + y = 5$ or $4x + 9y = 30$
(c) Either $x + y = 4$ or $x + 9y = 12$
(d) Either $x + y = 5$ or $9x + 4y = 30$

137. What is the distance between the straight lines $3x + 4y = 9$ and $6x + 8y = 15$? *[2018-I]*
(a) $\frac{3}{2}$ (b) $\frac{3}{10}$
(c) 6 (d) 5

138. The second degree equation *[2018-II]*
$x^2 + 4y - 2x - 4y + 2 = 0$ represents
(a) A point
(b) An ellipse of semi-major axis 1
(c) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$
(d) None of the above

139. The angle between the two lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ is given by $\tan^{-1}\theta$. What θ equal to? *[2018-II]*
(a) $\left|\frac{\ell m' - \ell' m}{\ell\ell' - mm'}\right|$ (b) $\left|\frac{\ell m' + \ell' m}{\ell\ell' + mm'}\right|$
(c) $\left|\frac{\ell m' - \ell' m}{\ell\ell' + mm'}\right|$ (d) $\left|\frac{\ell m' + \ell' m}{\ell\ell' - mm'}\right|$

140. Consider the following statements: *[2018-II]*
1. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_1 - c_2|}{\sqrt{1 - m^2}}$.
2. The distance between the lines $ax + by + c_1$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.
3. The distance between the lines $x = c$ and $x = c_2$ is $|c_1 - c_2|$.

Which of the above statements are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

141. What is equation of straight line pass through the point of intersection of the line $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$, and parallel the $4x + 5y - 6 = 0$? *[2018-II]*
(a) $20x + 25y - 54 = 0$
(b) $25x + 20y - 54 = 0$
(c) $4x + 5y - 54 = 0$
(d) $4x + 5y - 45 = 0$

142. Consider the following statements: *[2018-II]*
Statement I : If the line segment joining the points P(m, n) and Q(r, s) subtends an angle α at the origin, then $\cos\alpha = \frac{ms - nr}{\sqrt{(m^2 + n^2)(r^2 + s^2)}}$.
Statements II : In any triangle ABC, it is true that $a^2 = b^2 + c^2 - 2bc \cos A$.
What of the following is correct in respect of the above two statements?
(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I
(b) Both Statement I and Statement II are true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

143. Consider the following statements : *[2019-I]*
1. For an equation of a line, $x \cos q + y \sin q = p$, in normal form, the length of the perpendicular from the point (a, b) to the line is $|a \cos q + b \sin q + p|$.
2. The length of the perpendicular from the point (a, b) to the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\left|\frac{a\alpha + b\beta - ab}{\sqrt{a^2 + b^2}}\right|$.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

144. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. What is the value of c? *[2019-I]*
(a) 2 (b) –2
(c) 4 (d) –4

145. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b? *[2019-I]*
(a) 1 (b) 3
(c) 6 (d) $\frac{1}{2}$

146. What is the equation of the straight line which is perpendicular to $y = x$ and passes through (3, 2)? *[2019-I]*
(a) $x - y = 5$ (b) $x + y = 5$
(c) $x + y = 1$ (d) $x - y = 1$

147. The straight lines $x + y - 4 = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle, which is *[2019-I]*
(a) isosceles (b) right-angled
(c) equilateral (d) scalene

148. The centroid of the triangle with vertices A(2, –3, 3), B (5, –3, –4) and C (2, –3, –2) is the point *[2019-I]*
(a) (–3, 3, –1) (b) (3, –3, –1)
(c) (3, 1, –3) (d) (–3, –1, –3)

149. The minimum distance from the point (4, 2) to $y^2 = 8x$ is equal to *[2019-I]*
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) 2 (d) $3\sqrt{2}$

150. What is the minimum value of $a^2x + b^2y$ where $xy = c^2$? *[2019-I]*
(a) abc (b) 2abc
(c) 3abc (d) 4abc

ANSWER KEY

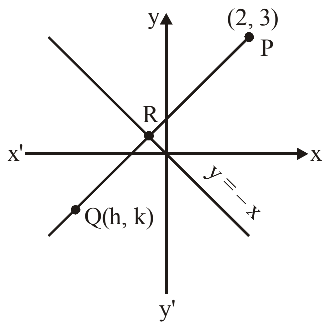
1	(d)	16	(c)	31	(c)	46	(b)	61	(d)	76	(a)	91	(a)	106	(b)	121	(d)	136	(d)
2	(a)	17	(b)	32	(c)	47	(b)	62	(b)	77	(d)	92	(b)	107	(c)	122	(a)	137	(b)
4	(b)	18	(a)	33	(c)	48	(a)	63	(c)	78	(a)	93	(d)	108	(c)	123	(b)	138	(a)
3	(a)	19	(c)	34	(b)	49	(a)	64	(c)	79	(c)	94	(b)	109	(d)	124	(a)	139	(c)
5	(c)	20	(d)	35	(d)	50	(d)	65	(d)	80	(c)	95	(b)	110	(b)	125	(a)	140	(b)
6	(a)	21	(c)	36	(a)	51	(a)	66	(b)	81	(b)	96	(b)	111	(c)	126	(b)	141	(a)
7	(d)	22	(b)	37	(a)	52	(b)	67	(a)	82	(c)	97	(a)	112	(a)	127	(d)	142	(d)
8	(a)	23	(b)	38	(d)	53	(c)	68	(b)	83	(d)	98	(a)	113	(d)	128	(a)	143	(d)
9	(d)	24	(a)	39	(d)	54	(c)	69	(a)	84	(a)	99	(b)	114	(a)	129	(b)	144	(d)
10	(c)	25	(a)	40	(d)	55	(a)	70	(b)	85	(b)	100	(c)	115	(c)	130	(a)	145	(c)
11	(a)	26	(c)	41	(c)	56	(d)	71	(b)	86	(c)	101	(c)	116	(d)	131	(d)	146	(b)
12	(d)	27	(c)	42	(c)	57	(c)	72	(b)	87	(c)	102	(a)	117	(b)	132	(c)	147	(a)
13	(b)	28	(c)	43	(a)	58	(d)	73	(c)	88	(a)	103	(a)	118	(d)	133	(c)	148	(b)
14	(a)	29	(a)	44	(c)	59	(a)	74	(b)	89	(c)	104	(d)	119	(b)	134	(b)	149	(b)
15	(a)	30	(c)	45	(b)	60	(a)	75	(c)	90	(b)	105	(a)	120	(d)	135	(c)	150	(b)

HINTS & SOLUTIONS

1. (d) As given, $(p + 2q)\,x + (p - 3q)\,y = p - q$
$\Rightarrow\ px + 2qx + py - 3qy = p - q$
$\Rightarrow\ p\,(x + y) - q\,(3y - 2x) = p - q$
Equation co-efficient of p and q
$\Rightarrow\ x + y = 1$ and $3y - 2x = 1$
Solving these, we get
$$x = \frac{2}{5},\ y = \frac{3}{5}.$$
So, line passes through $\left(\frac{2}{5}, \frac{3}{5}\right)$.

2. (a) The given lines are
$$y = (2-\sqrt{3})\,x + 5 \text{ and } y = (2+\sqrt{3})\,x - 7$$
Therefore, slope of first line $m_1 = 2-\sqrt{3}$ and slope of second line $m_2 = 2+\sqrt{3}$
$$\therefore\quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$$
$$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \ \Rightarrow\ \theta = \frac{\pi}{3} = 60^\circ$$

3. (a) Let there be a point P(2,3) on cartesian plane. Image of this point in the line $y = -x$ will lie on a line which is perpendicular to this line and distance of this point from $y = -x$ will be equal to distance of the image from this line.



Let Q be the image of p and let the co-ordinate of Q be (h, k)
Slope of line $y = -x$ is -1
Line joining P, Q will be perpendicular to $y = -x$ so, its slope = 1.
Let the equation of the line be $y = x + c$ since this passes through point (2, 3)
$3 = 2 + c \Rightarrow c = 1$
and the equation $y = x + 1$
The point of intersection R lies in the middle of P & Q.
Point of intersection of line $y = -x$ and $y = x + 1$ is
$2y = 1, \Rightarrow y = \frac{1}{2}$ and $x = -\frac{1}{2}$
Hence, $\frac{h+2}{2} = -\frac{1}{2}$ and $\frac{k+3}{2} = \frac{1}{2}$
$\Rightarrow\quad h = -3$ and $k = -2$
So, the image of the point (2, 3) in the line $y = -x$ is $(-3, -2)$.

4. (b) Given that mid point of A (1, 2) and B (x, y) is C (2, 4),

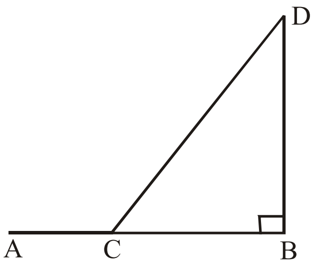
$$\frac{1+x}{2} = 2 \text{ and } \frac{2+y}{2} = 4$$

$\Rightarrow$ x = 3 and y = 6

So, coordinates of B are (3, 6).

Given that

BD $\perp$ AB and CD = 3 unit



$$BC = \sqrt{(2-3)^2 + (4-6)^2} = \sqrt{1+4} = \sqrt{5}$$

In right angled Δ BCD, $CD^2 = BC^2 + BD^2$

$\Rightarrow 9 = 5 + BD^2 \Rightarrow BD^2 = 4 \Rightarrow BD = 2$ unit

5. (c) Since the points are collinear.

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & a & 1 \end{vmatrix} = 0$$

Expanding the determinant

$$\Rightarrow 1\begin{vmatrix} 4 & 1 \\ a & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 3 & a \end{vmatrix} = 0$$

$\Rightarrow$ (4 – a) – 2(2 – 3) + 1(2a – 12) = 0

$\Rightarrow$ 4 – a + 2 + 2a – 12 = 0

$\Rightarrow$ a – 6 = 0

$\Rightarrow$ a = 6

Thus, Coordinates of C are (3, 6).

$$\text{Thus, } BC = \sqrt{(3-2)^2 + (6-4)^2}$$

$$= \sqrt{1+4} = \sqrt{5} \text{ unit}$$

6. (a) Lines are $L_1 \equiv Ax + By = A + B$ and $L_2 \equiv A(x - y) + B(x + y) = 2B$

Slope of L_1 is $-\frac{A}{B}$

$m_1 = -\frac{A}{B}$ [m_1 is the side of line L_1]

For line L_2:

Ax – Ay + Bx + By = 2B

(A + B)x – (A – B)y = 2B.

Slope of line L_2 in $\frac{(A+B)}{A-B}$

$m_2 = \frac{(A+B)}{(A-B)}$ [m_2 is the slope of line L_2]

If angle between line L_1 and L_2 is θ then

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{-\frac{A}{B} - \frac{A+B}{A-B}}{1 + \left(-\frac{A}{B}\right) \times \left(\frac{A+B}{A-B}\right)} = \frac{\frac{-A(A-B) - B(A+B)}{B(A-B)}}{\frac{B(A-B) - A(A+B)}{B(A-B)}}$$

$$= \frac{-A^2 + AB - AB - B^2}{AB - B^2 - A^2 - AB} = \frac{-B^2 - A^2}{-B^2 - A^2} = 1$$

$$\text{so, } \theta = \frac{\pi}{4}$$

7. (d) Length of perpendicular from the origin on the straight line x + 2by – 2p = 0 is

$$\left| \frac{0 + 2b \times 0 - 2p}{\sqrt{1^2 + (2b)^2}} \right| = p$$

$$\text{or } p = \left| \frac{-2p}{\sqrt{1^2 + 4b^2}} \right|$$

$$\text{or } p^2 = \frac{4p^2}{1 + 4b^2}$$

$$\frac{4}{1 + 4b^2} = 1$$

$\Rightarrow 1 + 4b^2 = 4$ or $4b^2 = 3$

$\Rightarrow b^2 = \frac{3}{4} \Rightarrow b = \pm\frac{\sqrt{3}}{2} \Rightarrow b = \frac{\sqrt{3}}{2}$ matches with the given option.

8. (a) Let the point of intersection divide the line segment joining points, (3, – 1) and (8, 9) in k : 1 ratio then:

The point is $\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$

Since this point lies on the line y – x + 2 = 0

We have, $\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$

$$= \frac{9k - 1 - 8k - 3}{k+1} + 2 = 0 = \frac{k-4}{k+1} + 2 = 0$$

= k – 4 + 2k + 2 = 0 = 3k – 2 = 0

$k = \frac{2}{3} : 1$ i.e. 2 : 3

9. (d) Let points be A(2, – 2), B(8, 4), C(4, 6) and D(– 1, 1) in order and are vertices of a quadrilateral.

$AB^2 = (8 - 2)^2 + (4 + 2)^2 = 36 + 36 = 72$

$BC^2 = (4 - 8)^2 + (6 - 4)^2 = 16 + 4 = 20$

$CD^2 = (4 + 1)^2 + (6 - 1)^2 = 25 + 25 = 50$

$AD^2 = (2 + 1)^2 + (-2 - 1)^2 = 9 + 9 = 18$

Thus AB $\neq$ BC $\neq$ CD $\neq$ AD

Hence, quadrilateral is a trapezium.

10. (c) Equation of line is $ax + by - p = 0$, then length of perpendicular, from the origin.

$$p = \left| \frac{a \times 0 + b \times 0 - p}{\sqrt{a^2 + b^2}} \right| \text{ or } p = \left| \frac{-p}{\sqrt{a^2+b^2}} \right|$$

$$\text{or } \left| \frac{1}{\sqrt{a^2+b^2}} \right| = 1 \text{ or } a^2 + b^2 = 1$$

$$b = \frac{\sqrt{3}}{2} \text{ or } b^2 = \frac{3}{4}$$

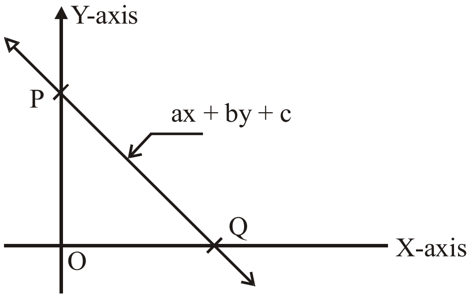
$$a^2 + \frac{3}{4} = 1$$

$$a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$$ $\left[a = -\frac{1}{2}\right.$ not taken since angle is with + ve direction to x-axis.]

Equation is $\frac{1}{2}x + \frac{\sqrt{3}}{2}y = p$ or $x \cos 60° + y \sin 60° = p$

Angle = 60°

11. (a) Co-ordinate axes and straight line $ax + by + c = 0$ form an isosceles triangle. This is possible when, the line makes equal intercept on both the axes.



Expressing $ax + by + c = 0$ in intercept form:

$$ax + by = -c \text{ or } \frac{x}{-\frac{c}{a}} + \frac{y}{-\frac{c}{b}} = 1$$

So, x-inntercept = $-\frac{c}{a}$ and y-intercept = $-\frac{c}{b}$

Since, $-\frac{c}{a} = -\frac{c}{b}$

Hence, $a = b$

Intercepts can be on both the sides of axis.

So, $|a| = |b|$

12. (d) As given :

Coordinates of P and Q are (– 3, 4) and (2, 1) respectively.

Let coordinates of R be (x, y).

As given : PR = 2 QR

$\Rightarrow$ PR – QR = QR $\Rightarrow$ PQ = QR.

So, Q is the mid point of P and R

$$\Rightarrow 2 = \frac{-3+x}{2} \text{ and } 1 = \frac{4+y}{2}$$

$\Rightarrow$ x = 7 and y = – 2

$\therefore$ Coordinates of R $\equiv$ (7, – 2)

13. (b) Let point P (x_1, y_1) be equidistant from point A (1, 2) and B (3, 4).

$\therefore$ PA = PB

$\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(1 - x_1)^2 + (2 - y_1)^2 = (3 - x_1)^2 + (4 - y_1)^2$

$\Rightarrow$ $1 + x_1^2 - 2x_1 + 4 + y_1^2 - 4y_1$

$= 9 + x_1^2 - 6x_1 + 16 + y_1^2 - 8y_1$

$\Rightarrow$ $4x_1 + 4y_1 = 20$

$\Rightarrow$ $x_1 + y_1 = 5$ ....(1)

As P (x_1, y_1) lies on $2x - 3y = 5$

$\therefore$ $2x_1 - 3y_1 = 5$ ....(2)

On solving Eqs. (1) and (2), we get

$x_1 = 4$ and $y_1 = 1$

$\therefore$ Coordinates of P are (4, 1).

14. (a) (A) and (R) are true and (R) is correct explanation of A.

15. (a) As given : A (2, 3), B (1, 4), C (0, – 2) and D (x, y) are the vertices of a parallelogram. Diagonals of a parallelogram bisect each other

So, mid-point are same for both diagonals AC and BD.

$$\frac{2+0}{2} = \frac{1+x}{2} \text{ and } \frac{3-2}{2} = \frac{4+y}{2}$$

$\Rightarrow$ x = 1 and y = – 3

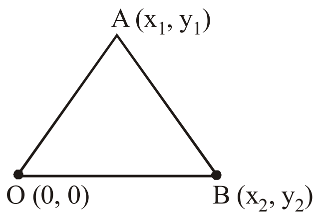
$\Rightarrow$ D(x, y) = (1, – 3)

16. (c) Let O (0, 0), A (x_1, y_1) and B (x_2, y_2) be three points

$$OA = \sqrt{x_1^2 + y_1^2}, \; OB = \sqrt{x_2^2 + y_2^2}$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In ΔAOB,



$$\cos \angle AOB = \frac{OA^2 + OB^2 - AB^2}{2.OA.OB}$$

$$\Rightarrow OA.OB \cos \angle AOB = \frac{OA^2 + OB^2 - AB^2}{2}$$

$$= \frac{x_1^2 + y_1^2 + x_2^2 + y_2^2 - \left\{(x_2 - x_1)^2 + (y_2 - y_1)^2\right\}}{2}$$

$\Rightarrow$ OA.OB.cos $\angle$AOB

$$= \frac{x_1^2 + y_1^2 + x_2^2 + y_2^2 - \left\{x_2^2 + x_1^2 - 2x_1x_2 + y_2^2 + y_1^2 - 2y_1y_2\right\}}{2}$$

$$= \frac{2(x_1x_2 + y_1y_2)}{2} = x_1x_2 + y_1y_2$$

17. (b) Let the side of the square = x units
Area of square = x^2 unit
and perimeter of square = 4x unit
According to question,
$x^2 + 4 = 4x$
$\Rightarrow x^2 - 4x + 4 = 0$
$\Rightarrow (x-2)^2 = 0$
$\Rightarrow x = 2$
$\therefore$ Side of square = 2 unit

18. (a) Let A, B and C having co-ordinates (a, b), (c, d) and {(a – c), (b – d)} respectively be the points
If these poins are collinear then

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$ gives

$$\begin{vmatrix} a & b & 1 \\ c-a & d-b & 0 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$R_3 \to R_2 + R_3$ gives

$$\begin{vmatrix} a & b & 1 \\ c-a & d-b & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1. \{a(d-b) - b(c-a)\} = 0$
$\Rightarrow ad - ab - bc + ab = 0$
$\Rightarrow bc - ad = 0$

19. (c) Given equations are
$2x + 3y + 4 = 0$ ...(i)
and $4x + 3y + 2 = 0$ ...(ii)
On solving (i) and (ii), the coordinates of the intersecting point are (1, – 2)

Now, $\sqrt{(0-1)^2 + \{0-(-2)\}^2} = d$

$\Rightarrow d = \sqrt{1+4} = \sqrt{5}$

20. (d) Let the equation of line be

$\frac{x}{\lambda} + \frac{y}{\mu} = 1$

The line passes through (4, 3) and (2, 5).

$\therefore \frac{4}{\lambda} + \frac{3}{\mu} = 1$, is possible when $\lambda = \mu = 7$

and $\frac{2}{\lambda} + \frac{3}{\mu} = 1$, is possible when $\lambda = \mu = 5$

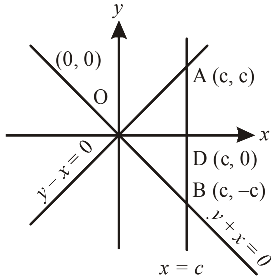
So, $\mu = \lambda$

21. (c) Let the coordinates of the point are (x, y)

$\therefore \sqrt{\{x-(a+b)\}^2 + \{y-(a-b)\}^2}$
$= \sqrt{\{x-(b-a)\}^2 + \{y-(a+b)\}^2}$

Squaring both sides, we get
$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (a-b)^2 - 2y(a-b)$
$= x^2 + (b-a)^2 - 2x(b-a) + y^2 + (a+b)^2 - 2y(a+b)$
$\Rightarrow 2x(a+b) + 2y(a-b) = 2x(b-a) + 2y(a+b)$
$\Rightarrow x\{(a+b) - (b-a)\} + y\{(a-b) - (a+b)\} = 0$
$\Rightarrow 2ax + (-2by) = 0$
$\Rightarrow ax - by = 0$
$\Rightarrow -ax + by = 0$

22. (b) The shaded portion shows the required area.
Since the area is symmetric about the x-axis therefore
Required area = 2 area (ΔAOD)



$= 2 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 2 \times \frac{1}{2} \times OD \times AD$

$= c \times c$
$= c^2$

23. (b) We know a line perpendicular to a given line
$ax - by + c = 0$ is $bx + ay + k = 0$
$\therefore$ The equation of line perpendicular to given line
$x + y - 11 = 0$ ...(i)
is $-x + y + \lambda = 0$ ...(ii)
Since, this equation passes through (2, 3).
Therefore (2, 3) satisfies the equation of line
$\therefore -2 + 3 + \lambda = 0$
$\Rightarrow \lambda = -1$
$\therefore$ From Eq. (ii),
$-x + y - 1 = 0$
$\Rightarrow y = x + 1$
And from eq. (i),
$x + x + 1 - 11 = 0$
$\Rightarrow 2x = 10$
$\Rightarrow x = 5$
Hence, coordinates of foot of perpendicular from (2, 3) to given line is (5, 6)

24. (a) We know that, the equation of x-axis is $y = 0$
Statement 1 says that the equation to a straight line parallel to the axis of x is $y = d$.
Since, d is constant therefore it can be zero.
Thus, only statement 1 is correct.

25. (a) Given equation of line is $ax \cos\phi + by \sin\phi - ab = 0$
Let d_1 be the perpendicular distance from $(\sqrt{b^2 - a^2}, 0)$ to the line $ax \cos\phi + by \sin\phi - ab = 0$

and d_2 from $(-\sqrt{b^2 - a^2}, 0)$ to the line $ax \cos\phi + by \sin\phi - ab = 0$

At point $(\sqrt{b^2 - a^2}, 0)$

$$d_1 = \frac{a\sqrt{b^2 - a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

At point $(-\sqrt{b^2 - a^2}, 0)$

$$d_2 = \frac{-a\sqrt{b^2 - a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

$$\therefore\ d_1 d_2 = -\frac{[a^2(b^2 - a^2)\cos^2\phi - a^2b^2]}{a^2\cos^2\phi + b^2\sin^2\phi}$$

$$= -\frac{a^2(-b^2\sin^2\phi - a^2\cos^2\phi)}{a^2\cos^2\phi + b^2\sin^2\phi} = a^2$$

26. (c) Two joining points are (p, q) and $(q, -p)$

Mid point of (p, q) and $(q, -p)$ is $\left(\frac{p+q}{2}, \frac{q-p}{2}\right)$

But it is given that the mid-point is $\left(\frac{r}{2}, \frac{s}{2}\right)$.

$\therefore\ \frac{p+q}{2} = \frac{r}{2}$ and $\frac{q-p}{2} = \frac{s}{2}$

$\Rightarrow p + q = r$ and $q - p = s$

Now, length of segment $= \sqrt{(p-q)^2 + (q+p)^2}$ (by distance formula)

$= \sqrt{s^2 + r^2} = (s^2 + r^2)^{1/2}$

27. (c) Let the locus of a point be (h, k)

Let the given points be

$P(m+n, n-m)$ and $Q(m-n, n+m)$

$\therefore$ By distance formula, we have

$\sqrt{[h-(m+n)]^2 + [k-(n-m)]^2}$

$= \sqrt{[h-(m-n)]^2 + [k-(n+m)]^2}$

$\Rightarrow h^2 + (m+n)^2 - 2h(m+n) + k^2 + (n-m)^2 - 2k(n-m)$

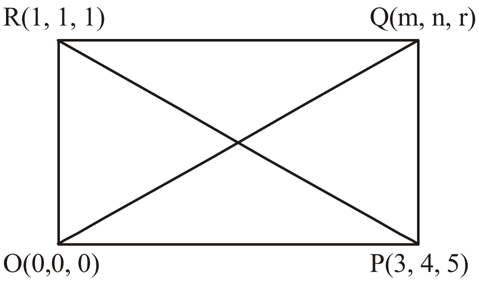
$= h^2 + (m-n)^2 - 2h(m-n) + k^2 + (n+m)^2 - 2k(n+m)$

$\Rightarrow 2h(m-n-m-n) + 2k(n+m-n+m) = 0$

$\Rightarrow -4hn + 4km = 0 \Rightarrow mk = nh$

Hence, locus of a point is $nx = my$.

28. (c)



OPQR is a parallelogram and OQ, PR are the diagonals of parallelogram.

We know that in a parallelogram, diagonals bisect each other.

$$\therefore\ \left(\frac{0+m}{2}, \frac{0+n}{2}, \frac{0+r}{2}\right) \equiv \left(\frac{1+3}{2}, \frac{1+4}{2}, \frac{1+5}{2}\right)$$

$$\therefore\ \frac{m}{2} = \frac{4}{2}, \frac{n}{2} = \frac{5}{2}, \frac{r}{2} = \frac{6}{2}$$

$\Rightarrow\ m = 4, n = 5, r = 6$

Hence, $m + n + r = 4 + 5 + 6 = 15$

29. (a) Let (a, b) be the image of point (1, 2) w.r.t. line $3x + 4y - 1 = 0$

$\therefore\ \left(\frac{a+1}{2}, \frac{b+2}{2}\right)$ will be on the line $3x + 4y - 1 = 0$

$\Rightarrow\ 3\left(\frac{a+1}{2}\right) + 4\left(\frac{b+2}{2}\right) - 1 = 0$

$\Rightarrow\ 3a + 3 + 4b + 8 - 2 = 0$

$\Rightarrow\ 3a + 4b + 9 = 0$

Now, co-ordinates given in option 'a' satisfies the equation $3a + 4b + 9 = 0$

Thus, the image of point $(1, 2)$ is $\left(-\frac{7}{5}, -\frac{6}{5}\right)$.

30. (c) Let $A(a, 0)$ and $B(0, b)$ be two points on x-axis and y-axis respectively

1 2

$A(a, 0)$ $(-5, 4)$ $B(0, b)$

Given $(-5, 4)$ divides line AB in the ratio 1 : 2.

By section formula we have

$$-5 = \frac{1\times 0 + 2\times a}{3}$$

$\Rightarrow\ a = \frac{-15}{2}$ and $4 = \frac{1\times b + 2\times 0}{3}$

$\Rightarrow\ b = 12$

Thus, $A = \left(\frac{-15}{2}, 0\right)$ and $B = (0, 12)$

Hence, equation of line joining $\left(\frac{-15}{2}, 0\right)$ and (0, 12) is

$$(y - 0) = \frac{12 - 0}{0 + \frac{15}{2}}.\left(x + \frac{15}{2}\right)$$

$\Rightarrow\ y = \frac{4}{5}(2x + 15)$

$\Rightarrow\ 5y = (8x + 60) \Rightarrow\ 8x - 5y + 60 = 0$

31. (c) We know that the equation of straight line passing through the intersection point of two lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, is

$$\left(\frac{x}{a} + \frac{y}{b} - 1\right) + \lambda\left(\frac{x}{b} + \frac{y}{a} - 1\right) = 0 \quad ...(i)$$

This line passes through the origin.

$\therefore \quad (0-1)+\lambda(0-1)=0 \Rightarrow \lambda=-1$

On putting the value of λ in Eq. (i), we get

$$\left(\frac{x}{a}+\frac{y}{b}-1\right)-1\left(\frac{x}{b}+\frac{y}{a}-1\right)=0$$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b}-1-\frac{x}{b}-\frac{y}{a}+1=0$$

$$\Rightarrow \quad x\left(\frac{1}{a}-\frac{1}{b}\right)-y\left(\frac{1}{a}-\frac{1}{b}\right)=0$$

$$\Rightarrow \quad x-y=0$$

32. (c) Since. the straight lines $x-2y=0$ and $kx+y=1$ intersect at the point $\left(1,\frac{1}{2}\right)$.

$\therefore$ The point $\left(1,\frac{1}{2}\right)$ satisfies the equation $kx+y=1$

$\therefore$ Put $x=1$, and $y=\frac{1}{2}$ in eq^n $kx+y=1$,

we get $k.1+\frac{1}{2}=1 \Rightarrow k=\frac{1}{2}$

33. (c) Required number of lines $= {}^4C_2=\frac{4!}{2!2!}=6$

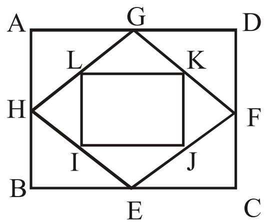
34. (b) Let ABCD, EFGH and IJKL be squares.

Let side of square ABCD = 16

Now, Area of ABCD $=(16)^2$

Area of EFGH $=\frac{(16)^2}{2}$,

Area of IJKL $=\frac{(16)^2}{4}$ So on.



Required sum,

$$=16^2+\frac{1}{2}(16)^2+\frac{1}{4}(16)^2+....\infty$$

$$=(16)^2\left\{1+\frac{1}{2}+\frac{1}{4}+....\infty\right\}$$

Now, $1+\frac{1}{2}+\frac{1}{4}+......\infty$ is a G.P

$\therefore$ Sum $=\frac{a}{1-r}$ where $a=1$ and $r=\frac{1}{2}$

$$\therefore \quad 1+\frac{1}{2}+\frac{1}{4}+......+\infty=\frac{1}{1-\frac{1}{2}}=2$$

$=256\times2=512$ sq. cm

35. (d) Given equation of the line $\frac{x}{4}+\frac{y}{3}=1$

can be written as

$3x+4y=12 \Rightarrow 4y=-3x+12$

$$\Rightarrow \quad y=\frac{-3}{4}x+\frac{12}{4}$$

The slope of the line $\frac{x}{y}+\frac{y}{3}=1$ is $\frac{-3}{4}$

$\therefore$ Slope of the line perpendicular to this line

$$=-\left(\frac{-1}{3/4}\right)=\frac{4}{3}$$

36. (a) Let the vertices of the Δ ABC be A (–3,0), B (3,0) and C (0,k).

$$\therefore \quad \text{Area of } \Delta \text{ ABC}=\frac{1}{2}\begin{vmatrix}-3 & 0 & 1\\ 3 & 0 & 1\\ 0 & k & 1\end{vmatrix}$$

Given, area is 9

$$\Rightarrow \quad 9=\frac{1}{2}\{-3(-k)+1(3k)\}$$

$$\Rightarrow \quad 18=3k+3k$$

$$\Rightarrow \quad k=\frac{18}{6}=3$$

37. (a) We have,

$y=x$ ... (i)

$y=-x$ ... (ii)

$-x=-y$ ... (iii)

$x=-y$ ... (iv)

Thus, the required locus of a point which moves equidistant from the co-ordinate axes is

$x\pm y=0$

38. (d) The equation of given lines are

$4x+3y=12$ ... (i)

and $3x+4y=12$ ... (ii)

On simplifying (i) and (ii), we get

$x=\frac{12}{7}$ and $y=\frac{12}{7}$

$\therefore$ Point of intersection of given line is $\left(\frac{12}{7},\frac{12}{7}\right)$.

Hence, the equation of line passing through (0, 0) and $\left(\frac{12}{7},\frac{12}{7}\right)$ is

$$\frac{y-0}{x-0}=\frac{\frac{12}{7}-0}{\frac{12}{7}-0} \Rightarrow y=x$$

39. (d) Let P (x, y) be a point and A = $(a, 0)$, B = $(-a, 0)$.

Now, $PA^2=(x-a)^2+y^2$

$PB^2=(x+a)^2+y^2$

Since the sum of the distances of the point P (x, y) from the points A $(a, 0)$ and B $(-a, 0)$ is $2b^2$.

$\therefore \quad PA^2+PB^2=2b^2$

$(x-a)^2+(y)^2+(x+a)^2+(y)^2=2b^2$

$\Rightarrow x^2+a^2-2ax+y^2+x^2+a^2+2ax+y^2=2b^2$

$\Rightarrow x^2+a^2+y^2=b^2$

$\Rightarrow x^2+a^2=b^2-y^2$

40. (d) Since line $mx + ny = 1$ passes through (1, 2) and (2, 1) therefore they satisfied the equation.

$\Rightarrow \quad m + 2n = 1 \qquad$ ... (i)

and $2m + n = 1 \qquad$ ... (ii)

From eqs. (i) and (ii), we get $m = n = \frac{1}{3}$

41. (c) Since required line is parallel to y-axis therefore its equation is $x = 2$. ($\because$ Required line passes through $(2, -3)$)

42. (c) Required locus is $X = -8$ which is at a distance of 8 units to the left of Y-axis.

43. (a) Given equation of straight lines are $x - 3y - 2 = 0$ and $2x - 6y - 6 = 0$

Here, $a_1 = 1, a_2 = 2, b_1 = -3, b_2 = -6,$
$c_1 = -2, c_2 = -6$

Now, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{1}{3}$

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ Both straight lines never intersect.

44. (c) Since, $(a, 0), (0, b)$ and $(1, 1)$ are collinear.

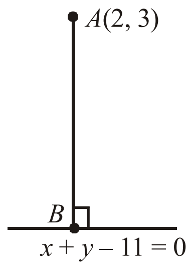
$$\therefore \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad a(b-1) + 1(0-b) = 0$

$\Rightarrow \quad ab - a - b = 0$

$\Rightarrow \quad a + b - ab = 0$

45. (b) Let B be the foot of perpendicular AB.



Now, $x + y - 11 = 0$

$\Rightarrow y = -x + 11 \qquad$ ...(1)

$\Rightarrow$ Slope $= -1 \qquad$ ...(2)

Since, AB is perpendicular to $x + y - 11 = 0$

$\therefore$ product of their slopes $= -1$

$\Rightarrow -1 \times$ Slope of $AB = -1$

$\Rightarrow$ Slope of AB = 1

Now, equation of AB is given as

$y - 3 = 1(x - 2)$ (using slope point form)

$\Rightarrow y - x = 1 \qquad$ ...(3)

Now, foot of perpendicular

= Point of intersection of line AB and $x + y - 11 = 0$

So, on solving equation (1) and (2) we get $x = 5, y = 6$.

Hence, $B = (5, 6)$.

46. (b) A n-sided regular polygon have $\frac{n(n-3)}{2}$ diagonals.

47. (b) Since (p, q) is the point on the x-axis

$\therefore q = 0$

Let $P = (p, 0)$

$A = (1, 2)$ and $B = (2, 3)$

Given: $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (1-p)^2 + 4 = (2-p)^2 + 9$

$\Rightarrow 1 + p^2 - 2p - 4 - p^2 + 4p = 5$

$\Rightarrow 2p = 8$

$\Rightarrow p = 4$

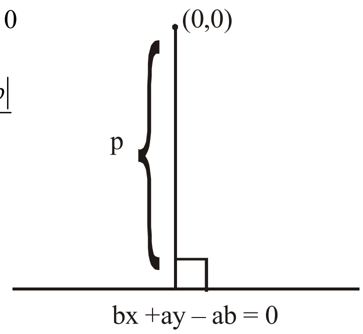
Hence, $p = 4, q = 0$

48. (a) Given equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow bx + ay - ab = 0$

$$p = \frac{|b.0 + a.0 - ab|}{\sqrt{b^2 + a^2}}$$

$$p = \frac{ab}{\sqrt{b^2 + a^2}}$$



on squaring both side, we get

$$p^2 = \frac{a^2b^2}{a^2 + b^2} \Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

49. (a) Given equation of lines are

$x + 2y - 9 = 0 \Rightarrow 2y = -x + 9$

$\Rightarrow y = -\frac{1}{2}x + \frac{9}{2} \qquad$ ...(1)

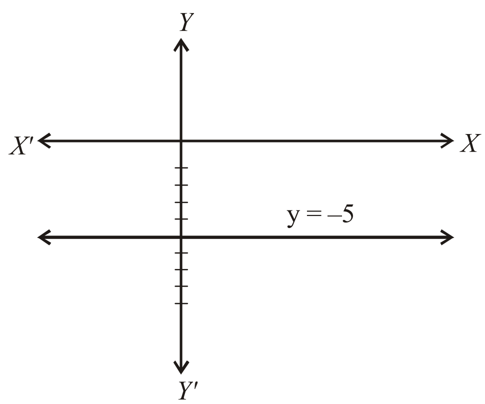
and $kx + 4y + 5 = 0 \Rightarrow 4y = -kx - 5$

$\Rightarrow y = \frac{-k}{4}x - \frac{5}{4} \qquad$ ...(2)

Since line (1) and line (2) are parallel therefore their slopes are equal.

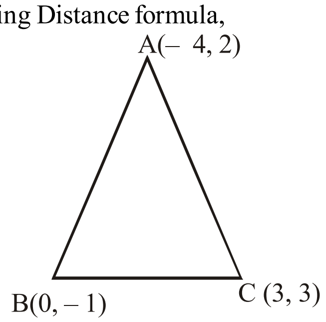
$\frac{-1}{2} = \frac{-k}{4} \Rightarrow k = 2$

50. (d) Equation of a line parallel to x-axis at a distance of 5 units below x-axis is $y = -5$



51. (a) Given line is $x - y = 4$
slope = 1 i.e. $m = 1$
Since required line passes through (0, 1)
$\therefore y - 1 = m(x - 0)$
$\Rightarrow y - 1 = 1(x) \Rightarrow y = x + 1$

52. (b) By using Distance formula,



We have, $AB = \sqrt{(0+4)^2 + (-1-2)^2} = \sqrt{16+9} = 5$

$BC = \sqrt{9+16} = 5$

$CA = \sqrt{49+(1)^2} = \sqrt{50} = 5\sqrt{2}$

Hence, Required Perimeter = AB + BC + CA
$= 10 + 5\sqrt{2}$

53. (c) Given points are $(a + b, a - b)$ and $(-a, b)$

Mid point is $\left(\frac{a+b-a}{2}, \frac{a-b+b}{2}\right) = \left(\frac{b}{2}, \frac{a}{2}\right)$

Since, it lies on $ax + by = k$

$\therefore \quad a\left(\frac{b}{2}\right) + b\left(\frac{a}{2}\right) = k \Rightarrow k = ab$

54. (c) Given line, $7x - 3y = 4$
$\Rightarrow 3y = 7x - 4$

$\Rightarrow y = \left(\frac{7}{3}\right)x - \left(\frac{4}{3}\right)$

$\therefore$ Slope $= \frac{7}{3}$

The slope of the line perpendicular to $7x - 3y = 4$ is $\left(\frac{-3}{7}\right)$.

If 'θ' is the angle between the perpendicular line with slope $\frac{-3}{7}$ and x-axis, then $\frac{-3}{7} = \tan\theta \Rightarrow \theta$ is negative.

55. (a) Given lines are
$3x + 4y = 9$ ...(1)
$2(3x + 4y = 9)$ ...(2)
Which are parallel to each other.

Distance between them $= \frac{|9-9|}{\sqrt{9+16}} = \frac{0}{5} = 0$

56. (d) $p = \frac{|y|}{\sqrt{1}} = |y|$

57. (c) The line making an angle –120° with x-axis is situated in the third quadrant.
$\therefore$ –120° means 120° clockwise which goes in 3rd quadrant.

58. (d) There can not be any point which is equidistant from three collinear points.
$\therefore$ Locus = null set.

59. (a) Let P (x, y) be the point.
Let $A = (1, 0)$ and $B = (0, -2)$ then
PA = PB
$\Rightarrow (PA)^2 = (PB)^2$.
$\Rightarrow (x-1)^2 + y^2 = x^2 + (y+2)^2$.
$\Rightarrow 1 - 2x = 4y + 4$
$\Rightarrow 2x + 4y + 3 = 0$

60. (a) Let A (5, 1); B (1, –1) and C (11, 4) are the given points.

Slope of $AB = \frac{-1-1}{-4} = \frac{-2}{-4} = \frac{1}{2}$

Slope of $BC = \frac{4-(-1)}{11-1} = \frac{5}{10} = \frac{1}{2}$

Slope of AB = Slope of BC
$\Rightarrow$ AB || BC and B is a common point
$\Rightarrow$ Points, A, B, C lie on a same straight line.
$\Rightarrow$ A (5, 1); B (1, –1) and C(11, 4) are collinear.

61. (d) The given lines are :-
$3x + 4y = 9$

$\Rightarrow y = \frac{9}{4} - \frac{3}{4}x$ .... (i)

and $9x + 12y + 28 = 0 \Rightarrow y = \frac{-7}{3} - \frac{3x}{4}$ .... (ii)

We have,

$m = \frac{-3}{4}; C_1 = \frac{9}{4}; C_2 = \frac{-7}{3}$

Now, Distance $= \frac{|C_1 - C_2|}{\sqrt{1+m^2}} = \frac{\left|\frac{9}{4} - \left(-\frac{7}{3}\right)\right|}{\sqrt{1+\frac{9}{16}}} = \frac{11}{3}$ units

62. (b) Let A (2, 6); B (3, 4); C (4, 5) and D (–2, 5) are the given points. Let O be the origin, i.e. O (0, 0)

$OA = \sqrt{(2-0)^2 + (6-0)^2} = \sqrt{40} = 2\sqrt{10}$ units

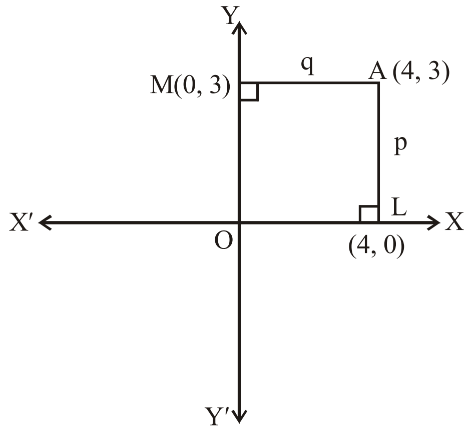
$OB = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9+16} = 5$ units

$OC = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41}$ units

$OD = \sqrt{(-2-0)^2 + (5-0)^2} = \sqrt{4+25} = \sqrt{29}$ units

So, q = OB = 5 units is the correct answer.

63. (c)



$p = \sqrt{(4-4)^2 + (3-0)^2} = 3$

$q = \sqrt{(4-0)^2 + (3-3)^2} = 4$

Now, $4p = 4 \times 3 = 12$

$3q = 3 \times 4 = 12$

$\therefore$ $4p = 3q$

64. (c) Let P(x, y) be the point of division that divides the line joining (3, –5) and (–4, 7) in the ratio of k : 1

Now, $y = \frac{7k-5}{k+1}$ .... (i)

Since, P lies on y = 0 or x -axis then, from eq. (i)

$0 = \frac{7k-5}{k+1} \Rightarrow 7k = 5 \Rightarrow k = \frac{5}{7}$

65. (d) The equation of the required line is,

$y = mx + c$ ..... (i)

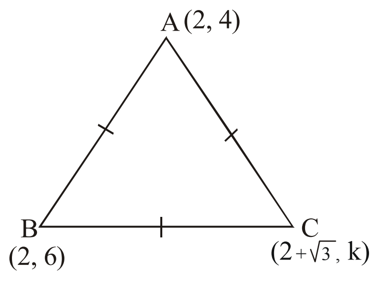
where $m = \tan 45° = 1$

and c = y – intercept = 101 units

$\therefore$ from (i)

$y = x + 101 \Rightarrow x - y + 101 = 0$

66. (b)



Since, ABC is an equilateral Δ,

$\therefore$ AB = BC = CA

Consider BC = AC

$\Rightarrow \sqrt{3+(k-6)^2} = \sqrt{3+(k-4)^2}$

$\Rightarrow \sqrt{39+k^2-12k} = \sqrt{k^2-8k+19}$

$\Rightarrow k^2 - 12k + 39 = k^2 - 8k + 19$

$\Rightarrow -4k = -20$

$\boxed{k = 5}$

67. (a) Equation of a straight line in intercept form is

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{3}{a} + \frac{4}{b} = 1$ .... (i)

Given, $a + b = 14$ .... (ii)

On solving (i) and (ii) we get

$\frac{3}{a} + \frac{4}{14-a} = 1 \Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow (a-7)(a-6) = 0$

$\Rightarrow a = 6$ and $b = 8$

or $a = 7$ and $b = 7$.

$\therefore$ Required eqns are $4x + 3y = 24$ or $x + y = 1$.

68. (b) Let the required point be P (x, x)

Since PA = PB

$\Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (x+1)^2 + x^2 = x^2 + (x-5)^2$

$\Rightarrow x^2 + 1 + 2x = x^2 + 25 - 10x$

$\Rightarrow 12x = 24 \Rightarrow x = 2$

Hence, Required point is (2, 2).

69. (a) Required Area

$$= \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 0 & 4 & 1 \\ 3 & 4 & 1 \end{vmatrix} = \frac{1}{2}[3(4-4)+1(0-12)] = 6$$

70. (b) Suppose equation of line is

$\frac{x}{5} + \frac{y}{3} = 1$

$\Rightarrow 3x + 5y - 15 = 0$

Now, length of perpendicular from (4, 4) on 3x + 5y – 15 = 0 is

$$P = \left|\frac{3.4+5.4-15}{\sqrt{34}}\right| = \left|\frac{17}{\sqrt{34}}\right| = \frac{\sqrt{17}\cdot\sqrt{17}}{\sqrt{17}\cdot\sqrt{2}}$$

$$P = \sqrt{\frac{17}{2}}$$

71. (b) Given equation can be written as

$y = \sqrt{3}x - 1$ on comparing with $y = mx + c$

We get $\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

72. (b) Given lines are

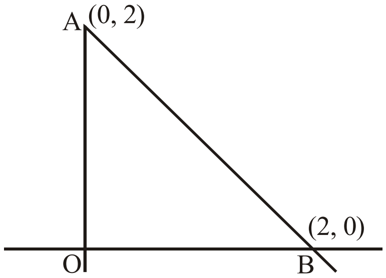
$2x - y = 2$ and $2x - y = -3$ since,

Both lines are parallel so they will never meet.

73. (c) For infinite solution $\frac{3}{9} = \frac{-1}{-k} = \frac{8}{24}$

$\Rightarrow \frac{1}{3} = \frac{1}{k} \Rightarrow k = 3$

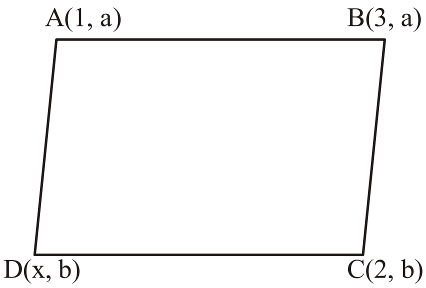
74. (b)



Area of $\Delta OAB = \frac{1}{2} \times OB \times OA$

$= \frac{1}{2} \times 2 \times 2 = 2$ square unit.

75. (c)



ABCD is a parallelogram.

AB || DC, then slope of line AB = slope of line DC $\frac{a-a}{3-1}$

$= \frac{b-y}{2-x} \Rightarrow y = b$

Also, AD || BC, $\frac{a-y}{1-x} = \frac{a-b}{3-2}$

$\frac{a-b}{1-x} = \frac{a-b}{3-2} \Rightarrow \frac{a-b}{1-x} \times \frac{1}{a-b} = 1$

$1 - x = 1$

$x = 0$

Then points D are (0, b)

76. (a) $3x + 4y + 5 = 0$ or $y = \frac{-3}{4}x + \frac{-5}{4}$

Slope $= \frac{-3}{4}$

Slope of required line, $m = \frac{-1}{-\frac{3}{4}} = \frac{4}{3}$

Also line passes through (4, –5)

Equation of line, $y + 5 = \frac{4}{3}(x - 4)$

$\Rightarrow 3y + 15 = 4x - 16$

$\Rightarrow 4x - 3y - 31 = 0$

77. (d) $d_1 = \left|\frac{3\times1+4\times1-1}{\sqrt{3^2+4^2}}\right| = \frac{6}{5}$

$d_2 = \left|\frac{4\times1+3\times1+2K}{\sqrt{3^2+4^2}}\right| = \frac{7+2K}{5}$

$d_1 = d_2$

$7 + 2K = 6$

$k = -\frac{1}{2}$

78. (a) Let moving point be P (x, y)

$\sqrt{(y-2)^2+(x-1)^2} = \sqrt{(y-3)^2+(x+2)^2}$

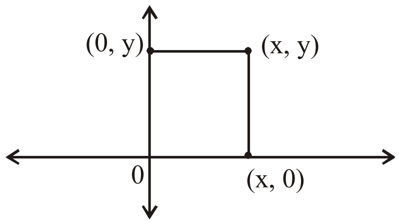
$\Rightarrow (y+2)^2 + (x-1)^2 = (y-3)^2 + (x+2)^2$

$\Rightarrow y^2 + 4 + 4y + x^2 + 1 - 2x = y^2 + 9 - 6y + x^2 + 4x + 4$

$\Rightarrow 10y - 6x - 8 = 0$

$\therefore$ Locus of P is a straight line.

79. (c)



$\sqrt{(x-0)^2+(y+y)^2} = \sqrt{(x-x)^2+(y-0)^2}$

$\Rightarrow \sqrt{(x-0)^2} = \sqrt{(y-0)^2}$

$\Rightarrow \sqrt{x^2} = \sqrt{y^2}$

$\Rightarrow y = \pm x$

80. (c) Slope of line joining (5, 2) and (0, 0)

$\tan A = m_1 = \frac{2-0}{5-0} = \frac{2}{5}$

Slope of line joining (6, –15) and (0, 0)

$\tan B = m_2 = \frac{-15}{6} = \frac{-5}{2}$

Now, $m_1.m_2 = \frac{2}{5}\left(-\frac{5}{2}\right) = -1$

Hence, both lines are perpendicular. and than angle between them $= \frac{\pi}{2}$

81. (b) $4x^2 + 9y^2 = 36$

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$

Length of latus rectum $= 2 \times \frac{2^2}{3} = \frac{8}{3}$

82. (c) Equation of line

$y + 2 = \frac{7+2}{-4-5}(x - 5)$

$\Rightarrow y + 2 = -x + 5$

$\Rightarrow x + y = 3$

83. (d) Slope of $x + y = 1$ is -1
Slope of $x - y = 1$ is 1
Let $\tan A = -1$, $\tan B = 1$

$$A = \frac{3\pi}{4}, B = \frac{\pi}{4}$$

$$A - B = \frac{\pi}{2}$$

84. (a) Centroid $= \left(\frac{2-2+3}{3}, \frac{3-5+5}{3}\right)$
$= (1, 1)$

85. (b) Let line be $\frac{x}{a} + \frac{y}{b} = 1$

given that $\frac{1}{a} = m$ and $\frac{1}{b} = n$

$$a = \frac{1}{m}, b = \frac{1}{n}$$

equation of line, $mx + ny = 1$

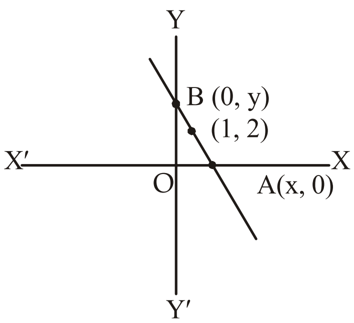
86. (c) All three points $(0, 5)$, $(2, -1)$ and $(3, -4)$ lie on $3x + y = 5$

$$\sqrt{(0-1)^2 + (5-2)^2} = \sqrt{10}$$

$$\sqrt{(2-1)^2 + (-1-2)^2} = \sqrt{10}$$

$$\sqrt{(3-1)^2 + (-4-2)^2} = \sqrt{40} = 2\sqrt{10}$$

87. (c)



$$\frac{0+x}{2} = 1; \frac{0+y}{2} = 2$$

$x = 2; y = 4$
Equation of line passing through $(2, 0)$ and $(0, 4)$

$$y - 0 = \frac{4-0}{0-2}(x-2)$$

$y = -2x + 4$
$2x + y = 4$

88. (a) Let equation of line be $\frac{x}{a} + \frac{y}{a} = 1$ or $x + y = a$
line passing through $(4, 3)$, then $a = 0$
Required equation, $x + y = 7$

89. (c) Given $A(3, 4)$ and $B(5, -2)$
Let, $P(x, y)$
Given that, $PA = PB$
$\Rightarrow PA^2 = PB^2$
$\Rightarrow (x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$
$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16$
$= x^2 - 10x + 25 + y^2 + 4y + 4$
$\Rightarrow 4x - 12y = 4$
$\Rightarrow x - 3y = 1$ ...(i)
$\because$ Area of $\Delta PAB = 10$

$$\therefore \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10$$

$\Rightarrow x(4+2) - y(3-5) + 1(-6-20) = \pm 20$
$\Rightarrow 6x + 2y - 26 = \pm 20$
$\Rightarrow 6x + 2y - 26 = 20$
or, $6x + 2y - 26 = -20$
$\Rightarrow 6x + 2y = 46$ ...(ii)
or $6x + 2y = 6$ ...(iii)
From eqs. (i) and (ii), we get
$x = 7, y = 2$
Similarly, from eqs. (i) and (iii), we get
$x = 1, y = 0$
Hence, coordinates of P are $(7, 2)$ or $(1, 0)$

90. (b) Given equation of line is

$$\frac{x-1}{2} = \frac{y-2}{3}$$

$\Rightarrow 3x - 3 = 2y - 4 \Rightarrow 3x - 2y + 1 = 0$

$$\Rightarrow y = \frac{3x}{2} + \frac{1}{2}$$

and equation of second line is $2x + 3y = 5$

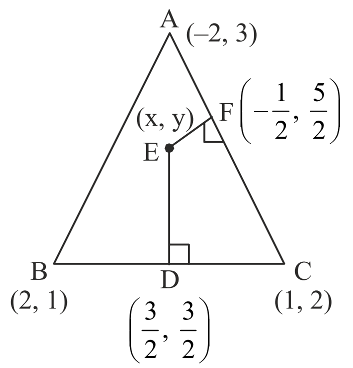
$$\Rightarrow y = \frac{-2}{3}x + \frac{5}{3}$$

$\therefore$ Slope of first line, $m_1 = \frac{3}{2}$

and slope of second line, $m_2 = -\frac{2}{3}$

$\therefore m_1 m_2 = -1$
Hence, two lines are perpendicular to each other.

91. (a) A circumcentre is a point at which perpendicular bisectors meet each other.
Here, 'E' represents circumcentre



Mid-point of BC $= \left(\frac{2+1}{2}, \frac{1+2}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

Slope of BC $= \frac{2-1}{1-2} = -1$

$\therefore$ Slope of DE = 1

Now, equation of $\overleftrightarrow{ED}$ is $\left(y - \frac{3}{2}\right) = 1\left(x - \frac{3}{2}\right)$

$\therefore$ $2y - 3 = 2x - 3$

$\therefore$ $x = y$ ... (i)

Now, mid-point of AC $= \left(\frac{-2+1}{2}, \frac{3+2}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$

Slope of AC $= \frac{3-2}{-2-1} = -\frac{1}{3}$

$\therefore$ Slope of EF = 3

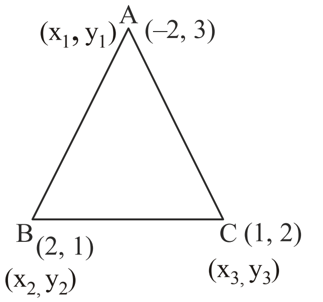
Now, equation of $\overleftrightarrow{EF}$ is $\left(y - \frac{5}{2}\right) = 3\left(x + \frac{1}{2}\right)$

$\therefore$ $2y - 5 = 6x + 3$ ... (ii)

From equations (i) and (ii),
$x = -2$ and $y = -2$
Hence, circumcentre of ΔABC is $(x, y) = (-2, -2)$
$\therefore$ Option (a) is correct.

92. (b) Centroid of the triangle

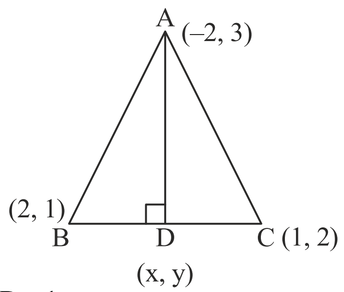
$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$



$= \left(\frac{-2+2+1}{3}, \frac{3+1+2}{3}\right) = \left(\frac{1}{3}, 2\right)$

$\therefore$ Option (b) is correct.

93. (d) Slope of BC $= \frac{2-1}{1-2} = -1$



Slope of AD = 1

Now, equation of $\overleftrightarrow{BC}$ is
$y - 2 = -1(x - 1)$

$\therefore$ $y - 2 = -x + 1$

$\therefore$ $x + y - 3 = 0$ ... (i)

and equation of $\overleftrightarrow{AD}$ is
$(y - 3) = 1(x + 2)$

$\therefore$ $x - y + 5 = 0$ ... (ii)

From equations (i) and (ii),
$x = -1$ and $y = 4$

$\therefore$ Foot of altitude from the vertex A of the triangle ABC is (–1, 4)

$\therefore$ Option (d) is correct.

94. (b) A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$
Slope of line $3x + y = 3$ is –3

Slope of line which passes through (2, 2) is $\frac{1}{3}$

$\therefore$ Equation of line passes through (2, 2) and having slope $\left(\frac{1}{3}\right)$ is

$(y - 2) = \frac{1}{3}(x - 2)$

$\therefore$ $3y - 6 = x - 2$

$\therefore$ $x - 3y + 4 = 0$

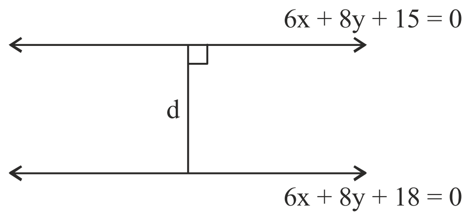
In order to find y-intercept of the line
Put x = 0 in $x - 3y + 4 = 0$

$\therefore$ $-3y = -4$

$\therefore$ $y = \frac{4}{3}$

$\therefore$ Option (b) is correct.

95. (b) $6x + 8y + 15 = 0$ ... (i)
and $3x + 4y + 9 = 0$ ... (ii)

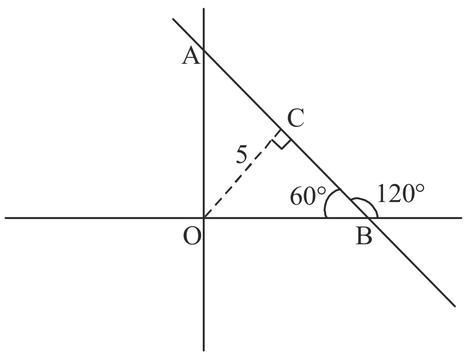


Multiply equation (ii) by 2, we get
$6x + 8y + 18 = 0$
Distance between the straight lines

$\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}} = \frac{18 - 15}{\sqrt{(6)^2 + (8)^2}} = \frac{3}{10}$ unit

$\therefore$ Option (b) is correct.

96. (b)



In ΔOCB,

$\sin 60° = \frac{5}{OB} \Rightarrow OB = \frac{5}{\sin 60°}$

$OB = \frac{5\times 2}{\sqrt{3}} = \frac{10}{\sqrt{3}}$

In ΔACO,

$\angle OAC = 30°$

$\sin 30° = \frac{5}{AO} \Rightarrow AO = \frac{5\times 2}{1} = 10$

Normal form of line AB

$\frac{X}{OB} + \frac{Y}{OA} = 1$

$\frac{\sqrt{3}X}{10} + \frac{Y}{10} = 1$

$\Rightarrow \sqrt{3}X + Y = 10.$

97. (a) $\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

and $\frac{x}{b} + \frac{y}{a} = 1$ ...(ii)

From solving equations (i) and (ii), we get the intersection point.

$bx + ay = ax + by$

$\Rightarrow (b-a)x = (b-a)y$

$\therefore x = y$ ...(iii)

$\Rightarrow \frac{x}{a} + \frac{x}{b} = 1$

$\therefore x(a+b) = ab$

$\therefore x = \frac{ab}{a+b}$ and $y = \frac{ab}{a+b}$ from equation (iii)

Now, equation of line joining (0, 0) and $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$

Here, slope of line = 1

$\therefore y = x$

$\therefore x - y = 0$

$\therefore$ Option (a) is correct.

98. (a) $5x - y = 0$

$y = 5x$ ...(1)

Slope = 5

Slope of perpendicular line will be $-\frac{1}{5}$.

Let equation of line is

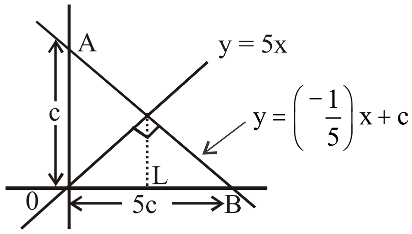
$y = \left(-\frac{1}{5}\right)x + c$ ...(2)

Putting y = 0

x = 5c

OB = 5c

Intersecting point A



Putting x = 0

$y = -\frac{1}{5}\times 0 + c$

$y = c$

area $\Delta AOB = \frac{1}{2}\times c \times 5c$

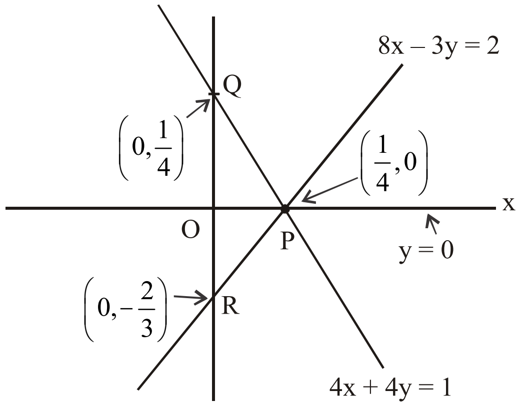
$5 = \frac{1}{2}\times 5c^2$

$c = \pm\sqrt{2}$

$y = \left(-\frac{1}{5}\right)x \pm \sqrt{2}$

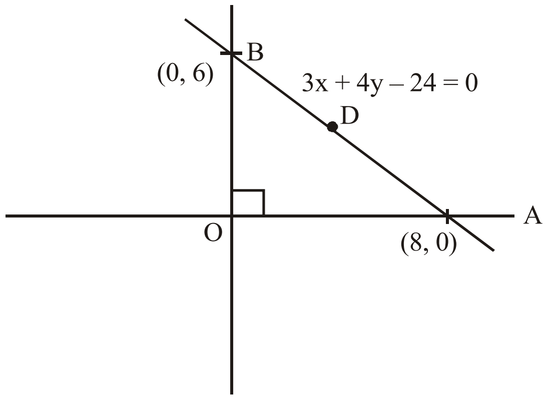
$5y + x \pm 5\sqrt{2} = 0$

99. (b)



So from the figure it is clear that all the three lines are concurrent at point P.

100. (c)



Since circumcentre of right angled triangle lies on the midpoint of hypotenuse.

So mid point of AB is $\left(\frac{0+8}{2}, \frac{6+0}{2}\right)$ or (4, 3)

101. (c) If length of perpendicular be p_1 from the point (4, 0)

$$p_1 = \left|\frac{12\cos\phi - 15}{\sqrt{(3\cos\phi)^2 + (5\cos\phi)^2}}\right|$$

$$= \frac{15 - 12\cos\phi}{\sqrt{(3\cos\phi)^2 + (5\cos\phi)^2}}$$

If length of perpendicular be p_2 from the point (– 4, 0)

$$p_2 = \left|\frac{-12\cos\phi - 15}{\sqrt{(3\cos\phi)^2 + (5\cos\phi)^2}}\right|$$

$$= \frac{(12\cos\phi + 15)}{\sqrt{(3\cos\phi)^2 + (5\sin\phi)^2}}$$

$$p_1 . p_2 = \frac{(15 - 12\cos\phi)(12\cos\phi + 15)}{(3\cos\phi)^2 + (5\sin\phi)^2}$$

$$= \frac{(225 - 144\cos^2\phi)}{9\cos^2\phi + 25\sin^2\phi} = \frac{9(25 - 16\cos^2\phi)}{25 - 16\cos^2\phi}$$

$= 9$

102. (a) $2x = 3y = -z$

or $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$

$6x = -y = -4z$

or $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$$\cos\theta = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_3^2}}$$

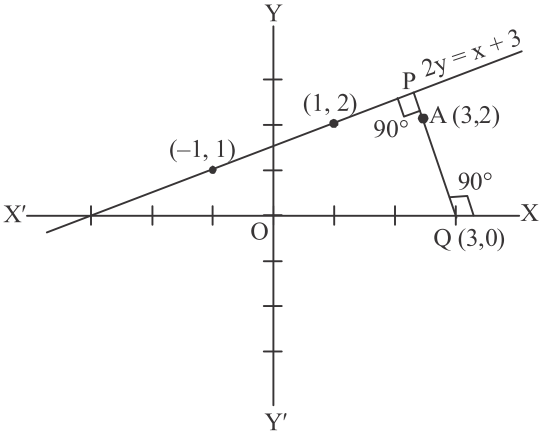
$$= \frac{(6 - 24 + 18)}{\sqrt{3^2 + 2^2 + (-6)^2} . \sqrt{2^2 + (-12)^2 + (-3)^2}}$$

$\cos\theta = 0$

$\theta = 90°$

So lines are perpendicular

103. (a)

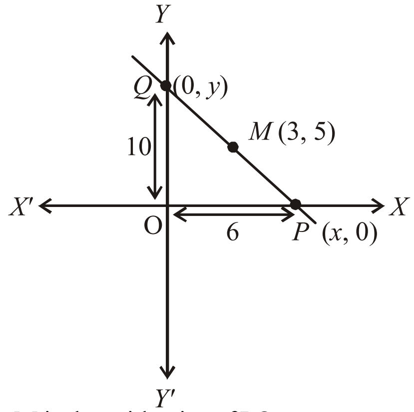


∵ Coordinates of Q are (3, 0) & it passes through PQ.

∴ Putting the values of (x = 3) & (y = 0) in options we get:

Equation of line PQ = 7x + y – 21 = 0

104. (d) As we know that line *PQ* intersects *x*-axis and *y*-axis at *P* and *Q*.



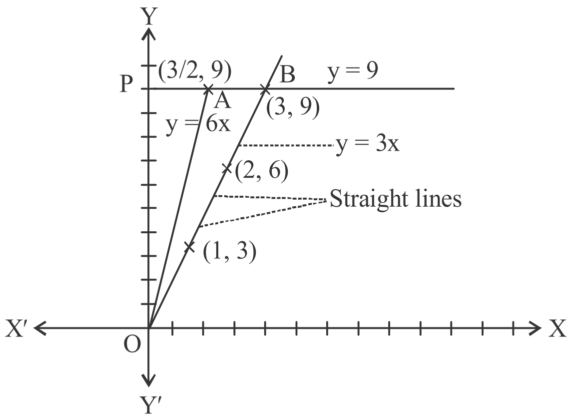
∵ M is the mid point of PQ

∴ $\frac{x+0}{2} = 3$ and $\frac{0+y}{2} = 5$

⇒ $x = 6$ and $y = 10$

Hence area of triangle *OPQ* = $\frac{1}{2} \times 6 \times 10 = 30$ sq. unit

105. (a) *OAB* is triangle.



Area of triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × AB × OP

$= \frac{1}{2} \times \frac{3}{2} \times 9 = \frac{27}{4}$

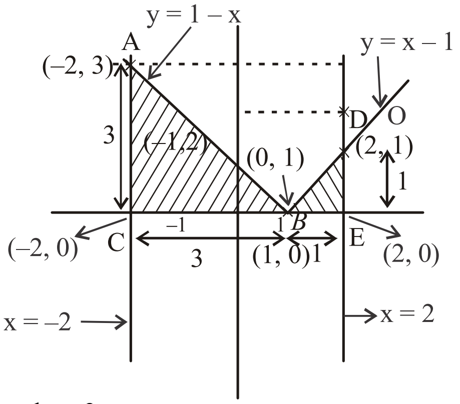
Area of triangle = $\frac{27}{4}$ square units

106. (b) Coordinates of *O, A, B* are (0, 0) $\left(\frac{3}{2}, 9\right)$, (3, 9) respectively.

∴ Centroid $C = \left[\left(\frac{0 + \frac{3}{2} + 3}{3}\right), \left(\frac{0+9+9}{3}\right)\right] = \left(\frac{3}{2}, 6\right)$

107. (c) $y = |x - 1|$ and $|x| = 2$

$$y = \begin{cases} x-1 & x \ge 1 \\ 1-x & x < 1 \end{cases}$$



and $x = 2$

$x = -2$

Hence curves intersect at (–2, 3) and (2, 1).

108. (c) Bounded region is shaded.

So area of bounded region has two triangles ACB and BDE.

$$\text{Area of } \Delta ACB = \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$$

$$\text{Area of } \Delta BDE = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

Area of region bounded by curves and x-axis is

$$\text{Area} = \Delta ACB + \Delta BDE = \frac{9}{2} + \frac{1}{2} = 5 \text{ square units}$$

Sol. (109 -110) :

Equations of lines

$x + y + 1 = 0$

$3x + 2y + 1 = 0$

$$\begin{array}{r} 3x + 3y + 3 = 0 \\ 3x + 2y + 1 = 0 \\ - \quad - \quad - \\ \hline y = -2 \end{array}$$

$x = 1$

Points of intersection (1, –2).

109. (d) Equation of x-axis

$y = 0$

Line parallel to x axis is

$y = k$

If this line passes through (1, –2) then

$k = -2$

$\Rightarrow y = -2$

$\Rightarrow y + 2 = 0$

Equation of line passing through (1, –2) and parallel to x-axis is

$y + 2 = 0$

110. (b) Equation of y-axis

$x = 0$

Equation of line parallel to y-axis is

$x = k$

If this line passes through (1, –2 then)

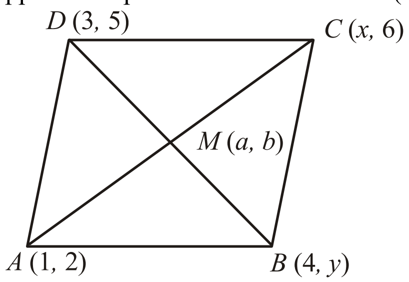
$x = 1$

Hence equation of line which passes through point of intersection of given line (1, –2) and parallel to y-axis

$x = 1$

$\Rightarrow x - 1 = 0$

111. (c) Suppose Mid point of AC and BD is $M(a, b)$.



$$a = \frac{1+x}{2} = \frac{3+4}{2}$$

$\Rightarrow x = 6$

$$b = \frac{5+y}{2} = \frac{2+6}{2}$$

$\Rightarrow y = 3$

$$a = \frac{7}{2}, b = 4$$

$$AC^2 = (1-x)^2 + (2-6)^2 = (1-6)^2 + (-4)^2 = 41$$

$$BD^2 = (3-4)^2 + (5-3)^2 = 1 + 4 = 5$$

$$AC^2 - BD^2 = 41 - 5$$

$$\boxed{AC^2 - BD^2 = 36}$$

112. (a) Point of intersection (a, b) is $\left(\frac{7}{2}, 4\right)$.

113. (d) Area of parallelogram = 2 area of Δ ADB

$\vec{a} = \overrightarrow{AB} = (4-1)\hat{i} + (3-2)\hat{j}$

$\vec{b} = \overrightarrow{AD} = (3-1)\hat{i} + (5-2)\hat{j}$

$$\therefore \text{Area of parallelogram} = 2\left[\frac{1}{2}\left|\vec{a} \times \vec{b}\right|\right] = \left|\vec{a} \times \vec{b}\right|$$

$$\text{Now; } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 2 & 3 & 0 \end{vmatrix} = 7\hat{k}.$$

$$\therefore \text{Area} = \left|\vec{a} \times \vec{b}\right| = \left|7\hat{k}\right| = \sqrt{49} = 7 \text{ square unit}$$

114. (a) $M \equiv$ mid point of line segment PQ

P	M	Q
(K, 4)	(a, 2b)	(10, –6)

$$\frac{K+10}{2} = a \Rightarrow K = (2a - 10)$$

$$2b = \frac{4-6}{2} = -1$$

given $a - 2b = 7$ ...(1)

Put the values of a & b in eq (1), we get

$$\frac{K+10}{2} + 1 = 7$$

$$\frac{K+10}{2} = 6 \Rightarrow K = 12 - 10$$

$$\boxed{K = 2}$$

115. (c) Let ABC is equilateral triangle with $A(0, 0)$ and $B\ (3, \sqrt{3})$ and C to be known.

$\therefore AB = \sqrt{(3-0)^2 + (\sqrt{3}-0)^2} = \sqrt{9+3} = \sqrt{12}$

Take option (a) i.e. $C\ (0, 2\sqrt{3})$

$CA = \sqrt{0^2 + (2\sqrt{3})^2} = \sqrt{12}$

$CB = \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{12}$

Take option (b) i.e. $C\ (3, -\sqrt{3})$

$CA = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{12}$

$CB = \sqrt{(0)^2 + (2\sqrt{3})^2} = \sqrt{12}$

$\therefore$ Both option (a) and (b) are correct.

116. (d) Intersecting lines are : $x+2y = 5$ & $3x + 7y = 17$

On solving these we get : $x = 1$ & $y = 2$

Equation of perpendicular line is

$3x + 4y = 10$ or $y = \dfrac{-3}{4}x + 10$

So, slope $= \dfrac{-3}{4}$

$\Rightarrow$ Slope of required line $= \dfrac{4}{3}$

$\therefore$ Equation of given line is

$(y - 2) = \dfrac{4}{3}(x - 1)$ or $4x - 3y + 2 = 0$

117. (b) Here $\dfrac{|8a+6b+1|}{\sqrt{8^2+6^2}} = 1 \Rightarrow |8a+6b+1| = 10$

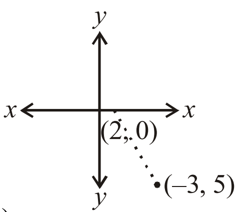
$\Rightarrow$ $8a + 6b + 1 = \pm 10$

$\Rightarrow$ $8a + 6b + 1 = 10$ & $8a + 6b + 1 = -10$

$\Rightarrow$ $8a + 6b - 9 = 0$ & $8a + 6b + 11 = 0$

118. (d) The given line passes through (–3, 5) and (2, 0). Its equation is

$y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$



$\Rightarrow (y-5) = \left(\dfrac{0-5}{2+3}\right)(x+3)$

$\Rightarrow y = -x + 2$ ...(1)

Slope $= m = -1$

and slope of perpendicular line $= -\dfrac{1}{m} = 1$

Equation of this line passing through (3, 3) is :

$(y - 3) = 1\ (x - 3)$

$\Rightarrow$ $y = x$.

From eq. (1) we get,

$x = -x + 2$

$\Rightarrow$ $\boldsymbol{x = 1}$ and $\boldsymbol{y = 1}$.

119. (b) $\dfrac{dy}{dx} = \dfrac{2y}{x}$

$\Rightarrow \dfrac{dy}{y} = 2\dfrac{dx}{x}$

On integration

$\int \dfrac{dy}{y} = 2\int \dfrac{dx}{x}$

$\Rightarrow \log y = 2\log x + \log a$

$\Rightarrow \log y = \log x^2 + \log a$

$\Rightarrow \log y = \log(x^2 \cdot a)$

$\Rightarrow y = x^2 a$

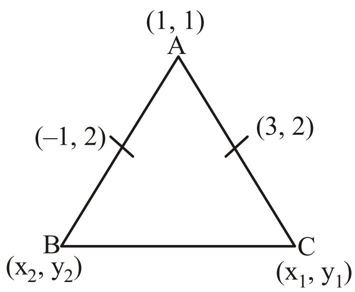
at (1, 1); $a = 1$

$\Rightarrow x^2 = y = 4\left(\dfrac{1}{4}\right)y$

$\Rightarrow$ the curve is parabola.

120. (d) Midpoint of AB = (– 1, 2)

$\Rightarrow \left(\dfrac{x_2+1}{2}, \dfrac{y_2+1}{2}\right) = (-1, 2)$



$\Rightarrow \dfrac{x_2+1}{2} = -1;\ \dfrac{y_2+1}{2} = 2$

$\Rightarrow x_2 + 1 = -2\ ;\ y_2 + 1 = 4$

$\Rightarrow x_2 = -3,\ y_2 = 3$

Midpoint of AC = (3, 2)

$\Rightarrow \left(\dfrac{x_1+1}{2}, \dfrac{y_1+1}{2}\right) = (3, 2)$

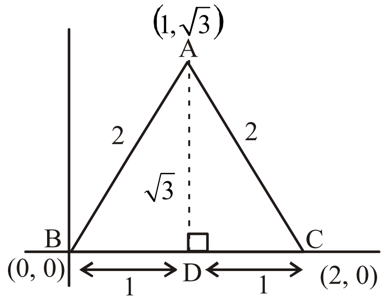
$\Rightarrow x_1 + 1 = 6\ ;\ y_1 + 1 = 4$

$\Rightarrow x_1 = 5,\ y_1 = 3$

So, vertices of triangle ABC are (1, 1), (–3, 3), (5, 3)

So, centroid $= \left(\dfrac{1-3+5}{3}, \dfrac{1+3+3}{3}\right) = \left(\dfrac{3}{3}, \dfrac{7}{3}\right) = \left(1, \dfrac{7}{3}\right)$

121. (d) Vertices of triangle, $A(1,\sqrt{3}), B(0,0), C(2,0)$
observe the figure,
$BD = 1, DC = 1, AD = \sqrt{3}$



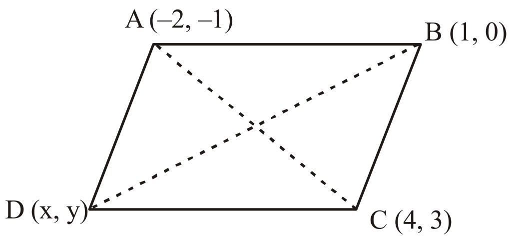
So, AB = 2. AC = 2 (Using Pythagoras theorem)
So, given triangle is equilateral triangle
In equilateral triangle, Incentre is controid only.

$$\therefore \text{Incentre} = \left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{\sqrt{3}}{3}\right)$$

$$= \left(1, \frac{1}{\sqrt{3}}\right)$$

122. (a) Given vertices of parallelogram are
(– 2, – 1), (1, 0), (4, 3)



Let the fourth vertex be (x, y)
We know, in a parallelogram diagonals bisect each other.
i.e, Midpoint of AC = Midpoint of BD

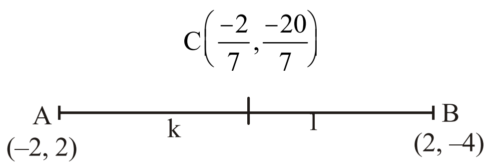
$$\Rightarrow \left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = \left(\frac{X+1}{2}, \frac{Y+0}{2}\right)$$

$$\Rightarrow (1,1) = \left(\frac{X+1}{2}, \frac{Y+0}{2}\right)$$

$\Rightarrow X + 1 = 2 ; Y + 0 = 2$
$\Rightarrow X = 1, Y = 2$

123. (b) $C = \left(\frac{2K-2}{K+1}, \frac{-4K+2}{K+1}\right)$

So, $\frac{2K-2}{K+1} = \frac{-2}{7}$



$$\Rightarrow \frac{\cancel{2}(K-1)}{K+1} = \frac{-\cancel{2}}{7}$$

$\Rightarrow 7K - 7 = -K - 1$

$\Rightarrow 8K = 6 \Rightarrow K = \frac{3}{4}$

$\therefore K : 1 = \frac{3}{4} : 1 = 3 : 4$

124. (a) The equation of line parallel to
$2x + 3y + 1 = 0$ is $2x + 3y + K = 0$
It is passing through point (–1, 2)
$\therefore 2(-1) + 3(2) + K = 0$
$\Rightarrow -2 + 6 + K = 0 \Rightarrow K = -4$
$\therefore$ Eqn. is $2x + 3y - 4 = 0$

125. (a) Centroid of given triangle

$$= \left(\frac{7+y+9}{3}, \frac{x-6+10}{3}\right)$$

$$\Rightarrow \left(\frac{16+y}{3}, \frac{x+4}{3}\right) = (6, 3)$$

$\Rightarrow 16 + y = 18 ; x + 4 = 9$
$\Rightarrow y = 2 ; x = 5$

126. (b) Given points, A(a, b), B(0, 0), C(–a, –b), D(ab, b^2).

Slope of $AB = \frac{b-0}{a-0} = \frac{b}{a}$

Slope of $BC = \frac{b}{a}$

Slope of $AC = \frac{b}{a}$

Slope of $BD = \frac{b}{a}$.

So, the points are collinear.

127. (d) The line $4x + y = 4$ can be written as $y = -4x + 4$.
So, slope is –4.
The line parallel to $4x + y = 4$ will have slope –4 only.
Given point = (1, 3)
Equation of line passing through (1, 3) and slope –4 is
$y - 3 = -4(x - 1)$
$\Rightarrow y - 3 = -4x + 4 \Rightarrow 4x + y = 7$.
Solving the two equations, we get

$$2x + 3y = 6 \Rightarrow \begin{array}{l} 4x + 6y = 12 \\ 4x + y = 7 \\ \cancel{(-)}\ \ (-)\ \ (-) \\ \hline 5y = 5 \Rightarrow y = 1 \end{array}$$

$2x + 3y = 6 \Rightarrow 2x + 3(1) = 6$

$\Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$.

Distance between the points (1, 3) and $\left(\frac{3}{2}, 1\right)$ is

$$\sqrt{\left(\frac{3}{2}-1\right)^2 + (1-3)^2} = \sqrt{\left(\frac{1}{2}\right)^2 + (-2)^2} = \sqrt{\frac{1}{4}+4} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}.$$

128. (a) Slope of the required line, m = tan 120° $= -\sqrt{3}$ (since it is making angle with positive direction of x-axis). Given, line cuts off an intercept of 5 units on negative direction of y-axis. This means line is intersecting y-axis at (0, –5)

P = (0, –5) and $m = -\sqrt{3}$.

$\therefore$ Equation of line is $y - (-5) = -\sqrt{3}(x - 0)$

$\Rightarrow y + 5 = -\sqrt{3}x$

$\Rightarrow y + \sqrt{3}x + 5 = 0.$

129. (b) Required line is $(2x - 3y + 7) + k(7x + 4y + 2) = 0$.
Given that this line passes through (2, 3)
$\therefore (2(2) - 3(3) + 7) + k(7(2) + 4(3) + 2) = 0.$
$\Rightarrow 4 - 9 + 7 + k(14 + 12 + 2) = 0$
$\Rightarrow 2 + k(28) = 0$

$\Rightarrow 28k = -2 \Rightarrow k = \frac{-1}{14}.$

$\therefore$ Required line

$\Rightarrow (2x - 3y + 7) + \left(\frac{-1}{14}\right)(7x + 4y + 2) = 0$

$\Rightarrow 28x - 42y + 98 - 7x - 4y - 2 = 0$
$\Rightarrow 21x - 46y + 96 = 0.$

130. (a) Let A(x, y) be the point that divides (4, 3) and (5, 7) internally is ratio 2:3

$$A = (x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

$$= \left(\frac{10 + 12}{2 + 3}, \frac{14 + 9}{2 + 3}\right) = \left(\frac{22}{5}, \frac{23}{5}\right).$$

Let B(x′, y′) be point that divides (4, 3) and (5, 7) externally in ratio 2 : 3

$$B = (x', y') = \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$$

$$= \left(\frac{10 - 12}{2 - 3}, \frac{14 - 9}{2 - 3}\right) = (2, -5)$$

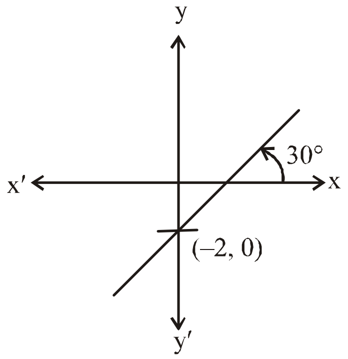
Distance between A and B $= \sqrt{(x' - x)^2 + (y' - y)^2}$

$$= \sqrt{\left(2 - \frac{22}{5}\right)^2 + \left(-5 - \frac{23}{5}\right)^2}$$

$$= \sqrt{\frac{(-12)^2}{25} + \frac{(-48)^2}{25}}$$

$$= \frac{12}{5}\sqrt{1 + 4^2} = \frac{12}{5}\sqrt{17}.$$

131. (d) Given, angle (θ) = 30°



$\therefore m = \tan 30° = \frac{1}{\sqrt{3}}.$

Point = (–2, 0) i.e., C = –2.
Slope-intercept form is y = mn + c

$\Rightarrow y = \frac{1}{\sqrt{3}}x - 2$

$\Rightarrow \sqrt{3}y = x - 2\sqrt{3}$

$\Rightarrow x - \sqrt{3}y - 2\sqrt{3} = 0.$

132. (c) Given lines,
$x + 2y - 3 = 0$ ....(1)
$2x - y + 5 = 0$ ....(2)

$(1) \times 2 \Rightarrow \quad 2x + 4y - 6 = 0$
$(2) \Rightarrow \quad 2x - y + 5 = 0$
(–) (+) (–)

$5y - 11 = 0 \Rightarrow y = \frac{11}{5}.$

$(1) \Rightarrow x + 2\left(\frac{11}{5}\right) - 3 = 0$

$\Rightarrow x + \frac{22}{5} - 3 = 0 \quad \Rightarrow x + \frac{7}{5} = 0 \Rightarrow x = \frac{-7}{5}.$

Given, the required line is parallel to y – x + 10 = 0.
$\Rightarrow y = x - 10$
$\Rightarrow y = (1)x + (-10)$
$\therefore$ slope (m) = 1
$\therefore$ Required line with slope 1 and passing through $\left(\frac{-7}{5}, \frac{11}{5}\right)$ is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - \frac{11}{5} = 1\left(x + \frac{7}{5}\right)$

$\Rightarrow 5y - 11 = 5x + 7 \Rightarrow 5x - 5y + 18 = 0.$

133. (c) 1: We know, the perpendicular distance from

(x_1, y_1) to line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Here, $(x_1, y_1) = (0, 0)$ and distance = P.

$$\therefore P = \frac{|a(0) + b(0) + (c)|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow P^2 = \frac{c^2}{a^2 + b^2}$$

$\therefore$ 1 is correct.

2: Line is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \left(\frac{1}{a}\right)x + \left(\frac{1}{b}\right)y + (-1) = 0$

$$P = \frac{\left|\frac{1}{a}(0) + \frac{1}{b}(0) - 1\right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$= P^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{a^2b^2}{b^2 + a^2}$$

$$\Rightarrow \frac{1}{P^2} = \frac{a^2 + b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

2 is correct.

3: Line is $y = mx + c \Rightarrow mx - y + c = 0$.

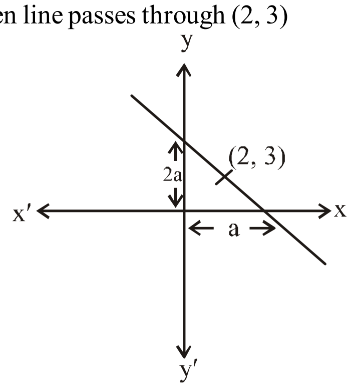
$$\therefore P = \frac{|m(0) - 0 + c|}{\sqrt{m^2 + 1}}$$

$$\Rightarrow P^2 = \frac{c^2}{m^2 + 1} \Rightarrow \frac{1}{P^2} = \frac{m^2 + 1}{c^2}$$

3 is wrong.

$\therefore$ Only 1 and 2 are correct.

134. (b) Given line passes through (2, 3)



Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{a} + \frac{y}{2a} = 1$$

$\Rightarrow 2x + y = 2a$ ....(1)

But this passes through (2, 3)

$\therefore 2a = 2(2) + 3 = 7$

$$\Rightarrow a = \frac{7}{2}.$$

$\therefore$ Equation of line is $2x + y = 2\left(\frac{7}{2}\right)$

$\Rightarrow 2x + y = 7$.

135. (c) We know, the perpendicular distance (d) from point (x_1, y_1) to line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Let us find a point on line $2x + 11y - 5 = 0$.

for $x = 0$, $2(0) + 11y - 5 = 0$

$\Rightarrow 11y - 5 = 0$

$$\Rightarrow y = \frac{5}{11}.$$

So, $(x, y) = \left(0, \frac{5}{11}\right)$.

Let us find perpendicular distances of this point to the given lines.

$24x + 7y = 20$

$\Rightarrow 24x + 7y - 20 = 0$

$$d_1 = \frac{\left|24(0) + 7\left(\frac{5}{11}\right) - 20\right|}{\sqrt{24^2 + 7^2}} = \frac{\left|\frac{35}{11} - 20\right|}{\sqrt{625}} = \frac{\left|\frac{-185}{11}\right|}{25} = \frac{185}{11 \times 25} = \frac{37}{55}$$

$4x - 3y = 2$

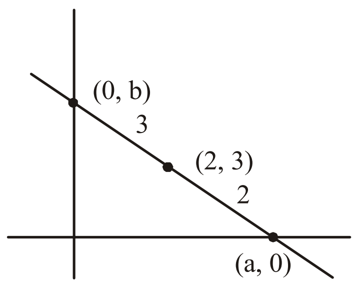
$\Rightarrow 4x - 3y - 2 = 0$

$$d_2 = \frac{\left|4(0) - 3\left(\frac{5}{11}\right) - 2\right|}{\sqrt{16 + 9}} = \frac{\left|\frac{-15}{11} - 2\right|}{\sqrt{25}} = \frac{37}{55}$$

$\therefore d_1 = d_2$

136. (d) Intercept form of line is $\frac{x}{a} + \frac{y}{b} = 1$.

We know, the point which divides a line joining two points (x_1, y_1) and (x_2, y_2) in the ratio m:n is



$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Case 1: m:n = 2:3

$\therefore (2,3) = \left(\frac{(2)(a)+3(0)}{2+3}, \frac{2(0)+3(b)}{2+3}\right)$

$\Rightarrow (2,3) = \left(\frac{2a}{5}, \frac{3b}{5}\right)$

$\Rightarrow \frac{2a}{5} = 2 \quad ; \frac{3b}{5} = 3$

$\Rightarrow a = 5; b = 5.$

$\therefore$ Equation of line is $\frac{x}{5}+\frac{y}{5} = 1 \Rightarrow x + y = 5$

Case 2: m:n = 3:2

$\therefore (2,3) = \left(\frac{3(a)+2(0)}{3+2}; \frac{3(0)+2(b)}{3+2}\right)$

$\Rightarrow (2,3) = \left(\frac{3a}{5}, \frac{2b}{5}\right)$

$\Rightarrow \frac{3a}{5} = 2; \frac{2b}{5} = 3$

$\Rightarrow a = \frac{10}{3}, b = \frac{15}{2}$

$\therefore$ Equation of line is $\frac{x}{\frac{10}{3}}+\frac{y}{\frac{15}{2}} = 1$

$\Rightarrow \frac{3x}{10}+\frac{2y}{15} = 1$

$\Rightarrow 9x + 4y = 30$

137. (b) Given lines, $L_1 = 3x + 4y - 9 = 0$

$L_2 = 6x + 8y - 15 = 0 \Rightarrow 3x + 4y - \frac{15}{2} = 0$.

Observe that the coefficients of x and y are same.

$\therefore L_1$ and L_2 are parallel lines.

Distance between parallel lines $= \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$

$= \frac{\left|-9+\frac{15}{2}\right|}{\sqrt{3^2+4^2}}$

$= \frac{\left|\frac{-18+15}{2}\right|}{\sqrt{25}} = \frac{3}{10}.$

138. (a) $(x^2 - 2x + 1) + (4y^2 - 4y + 1) = 0$

$\Rightarrow (x-1)^2 + (2y-1)^2 = 0$

$\Rightarrow x = 1, y = \frac{1}{2}$

It is a point.

139. (c) $\tan^{-1}(\theta) = \tan^{-1}\left|\frac{\ell m' - \ell' m}{\ell\ell' + mm'}\right|$

$\Rightarrow \theta = \left|\frac{\ell m' - \ell' m}{\ell\ell' + mm'}\right|$

140. (b) Using distance between two parallel lines formula.

141. (a) Given equatiosns: $\frac{x}{2}+\frac{y}{3} = 1$ and $\frac{x}{3}+\frac{y}{2} = 1$

Point of intersection $= \left(\frac{6}{5}, \frac{6}{5}\right)$

Let equation of line be $4x + 5y + k = 0$

Putting $\left(\frac{6}{5}, \frac{6}{5}\right)$, $k = -\frac{54}{5}$

$\therefore$ Equation of line is $20x + 25y - 54 = 0$

142. (d) $\cos\alpha = \frac{mr+ns}{\sqrt{m^2+n^2}\sqrt{r^2+s^2}}$

Statement 1 is false, statement 2 is true.

143. (d) The length of perpendicular from (α, β) to line

$x \cos\theta + y\sin\theta - p = 0$

$|\alpha\cos\theta + \beta\sin\theta - p|$

$\therefore$ Statement 1 is false.

The length of perpendicular from (α, β) to line

$\frac{x}{a}+\frac{y}{b} = 1$

$\frac{x}{a}+\frac{y}{b} = 1 \Rightarrow bx + ay - ab = 0$

So, perpendicular is $\left|\frac{b\alpha + a\beta - ab}{\sqrt{a^2+b^2}}\right|$

$\therefore$ Statement 2 is also false.

144. (d) Given, opposite vertices of rectangle are A (1, 3) and C (5, 1)

We know, diagonals of rectangle bisect each other.

So, midpoint of AC lies on line $y = 2x + c$.

Mid point of AC $= \left(\frac{1+5}{2}, \frac{3+1}{2}\right) = \left(\frac{6}{2}, \frac{4}{2}\right) = (3, 2)$

$y = 2x + c \Rightarrow 2 = 2(3) + c$

$\Rightarrow c = 2 - 6 = -4$

145. (c) Given lines, $3y + 4x = 1 \Rightarrow 4x + 3y - 1 = 0$

$y = x + 5 \Rightarrow x - y + 5 = 0$

$5y + bx = 3 \Rightarrow bx + 5y - 3 = 0$

Since, these lines are concurrent,

$$\begin{vmatrix} 4 & 3 & -1 \\ 1 & -1 & 5 \\ b & 5 & -3 \end{vmatrix} = 0$$

$\Rightarrow 4(3-25) - 3(-3-5b) - 1(5+b) = 0$
$\Rightarrow 4(-22) + 9 + 15b - 5 - b = 0$
$\Rightarrow -88 + 4 + 14b = 0$
$\Rightarrow -84 + 14b = 0$
$\Rightarrow b = 6$

146. (b) Given line, $y = x$
$\Rightarrow x - y = 0$

Slope of this line $= \frac{-1}{-1} = 1$

Slope of line perpendicular to this line $= -1$
The perpendicular line passes through (3, 2)
$\therefore$ Equation is $y - 2 = -1(x-3) \Rightarrow y - 2 = -x + 3$
$\Rightarrow x + y - 5 = 0 \Rightarrow x + y = 5$

147. (a) Given lines : $L_1 \equiv x + y - 4 = 0$
$L_2 \equiv 3x + y - 4 = 0$
$L_3 \equiv x + 3y - 4 = 0$

Slope of $L_1 = m_1 = \frac{-1}{1} = -1$

Slope of $L_2 = m_2 = \frac{-3}{1} = -3$

Slope of $L_3 = m_3 = \frac{-1}{3}$

Angle between L_1 and L_2

$$\Rightarrow \tan\theta_1 = \left|\frac{-1-(-3)}{1+(-1)(-3)}\right| = \frac{-1+3}{1+3} = \frac{1}{2}$$

Angle between L_2 and L_3

$$\Rightarrow \tan\theta_2 = \left|\frac{-3-\left(\frac{-1}{3}\right)}{1+(-3)\left(\frac{-1}{3}\right)}\right| = \left|\frac{-9+1}{3+3}\right| = \frac{4}{3}$$

Angle between L_1 and L_3

$$\Rightarrow \tan\theta_3 = \left|\frac{-1-\left(\frac{-1}{3}\right)}{1+(-1)\left(\frac{-1}{3}\right)}\right| = \left|\frac{-3+1}{3+1}\right| = \frac{1}{2}$$

$\therefore$ The triangle formed is an isosceles triangle.

148. (b) Given vertices of triangle are A (2, –3, 3), B (5, –3, –4) and C (2, –3, –2)

$$\text{Centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$= \left(\frac{2+5+2}{3}, \frac{-3-3-3}{3}, \frac{3-4-2}{3}\right)$$

$= (3, -3, -1)$

149. (b) Given equation is $y^2 - 8x$
Take an arbitrary point on this curve. If we take y as P, then point is $\left(\frac{P^2}{8}, P\right)$.

The distance between $\left(\frac{P^2}{8}, P\right)$ and (4, 2) is

$$d^2 = \left(\frac{P^2}{8} - 4\right)^2 + (P-2)^2 \quad ...(1)$$

$$= \frac{1}{64}(P^2 - 32)^2 + (P-2)^2$$

$$\Rightarrow 2d.\frac{dd}{dp} = \frac{1}{64} \times 2(P^2-32) \times 2P + 2(P-2)$$

$$= \frac{1}{16}(P^2-32)P + 2(P-2)$$

for minimum distance, $\frac{dd}{dp} = 0$

$\Rightarrow P^3 - 32P + 32P - 64 = 0$
$\Rightarrow P^3 = 64$
$\Rightarrow P = 4$

$$\therefore (1) \Rightarrow d^2 = \left(\frac{16}{8} - 4\right)^2 + (4-2)^2$$

$= (-2)^2 + (2)^2 = 8$
$\Rightarrow d = \sqrt{8} = 2\sqrt{2}$

150. (b) Let $p = a^2x + b^2y$ and $xy = c^2$

$$\Rightarrow y = \frac{c^2}{x} \quad ...(1)$$

$$\Rightarrow P = a^2x + b^2\left(\frac{c^2}{x}\right)$$

Now, $\frac{dP}{dx} = 0 \Rightarrow a^2 - \frac{b^2c^2}{x^2} = 0$

$$\therefore y = \frac{c^2}{\frac{bc}{a}} = \frac{ac^2}{bc} = \frac{ac}{b}$$

$$\Rightarrow a^2 = \frac{b^2c^2}{x^2}$$

$$\Rightarrow x = \frac{bc}{a} \quad \therefore P_{\min} = a^2\left(\frac{bc}{a}\right) + b^2\left(\frac{ac}{b}\right)$$

$= abc + abc = 2abc.$

Pair of Straight Lines

8

1. The bisector of the acute angle between the straight lines 3x – 4y – 3 = 0 and 12x + 5y + 6 = 0 passes through which one of the following points ? *[2006-II]*
   (a) (5, 3) (b) (– 3, 6)
   (c) (2, 7) (d) (– 1, 4)
2. What is the locus of the point of intersection of the straight lines (x/a) + (y/b) = m and (x/a) – (y/b) = 1/m ? *[2006-II]*
   (a) Circle (b) Parabola
   (c) Ellipse (d) Hyperbola
3. What does the equation $x^3y + xy^3 - xy = 0$ represent? *[2008-II]*
   (a) A pair of straight lines only
   (b) A pair of straight lines and a circle
   (c) A rectangular hyperbola only
   (d) A rectangular hyperbola and a circle
4. What is the value of λ if the straight line (2x + 3y + 4) + λ (6x – y + 12) = 0 is parallel to y-axis? *[2012-II]*
   (a) 3 (b) – 6
   (c) 4 (d) – 3
5. The value of k for which the lines $2x + 3y + a = 0$ and $5x + ky + a = 0$ represent family of parallel lines is *[2013-II]*
   (a) 3 (b) 4.5
   (c) 7.5 (d) 15
6. What is the equation of the line mid-way between the lines $3x - 4y + 12 = 0$ and $3x - 4y = 6$? *[2014-I]*
   (a) $3x - 4y - 9 = 0$ (b) $3x - 4y + 9 = 0$
   (c) $3x - 4y - 3 = 0$ (d) $3x - 4y + 3 = 0$
7. What is the product of the perpendiculars drawn from the points $(\pm\sqrt{a^2 - b^2}, 0)$ upon the line $bx \cos \alpha + ay \sin \alpha = ab$? *[2014-II]*
   (a) a^2 (b) b^2
   (c) $a^2 + b^2$ (d) $a + b$
8. What is the acute angle between the lines represented by the equations $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$? *[2016-I]*
   (a) 30° (b) 45°
   (c) 60° (d) 75°
9. What is the equation of the right bisector of the line segment joining (1, 1) and (2, 3)? *[2016-II]*
   (a) 2x + 4y –11= 0 (b) 2x – 4y – 5= 0
   (c) 2x – 4y –11= 0 (d) x– y + 1 = 0
10. What is the acute angle between the pair of straight lines $\sqrt{2}x + \sqrt{3}y = 1$ and $\sqrt{3}x + \sqrt{2}y = 2$? *[2017-I]*
   (a) $\tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
   (c) $\tan^{-1}(3)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
11. The angle between the lines x + y – 3 = 0 and x – y + 3 = 0 is α and the acute angle between the lines $x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$ is β. Which one of the following is correct? *[2017-II]*
   (a) α = β (b) α > β
   (c) α < β (d) α = 2β
12. What is the angle between the straight lines $(m^2 - mn)y = (mn + n^2)x + n^3$ and $(mn + m^2)y = (mn - n^2)x + m^3$, where m > n? *[2018-I]*
   (a) $\tan^{-1}\left(\frac{2mn}{m^2 + n^2}\right)$ (b) $\tan^{-1}\left(\frac{4m^2n^2}{m^4 - n^4}\right)$
   (c) $\tan^{-1}\left(\frac{4m^2n^2}{m^4 + n^4}\right)$ (d) 45°

ANSWER KEY

1	(c)	2	(d)	3	(b)	4	(a)	5	(c)	6	(d)	7	(b)	8	(a)	9	(a)	10	(a)
11	(*)	12	(b)																

HINTS & SOLUTIONS

1. (c) The equations of given straight lines are
$3x - 4y - 3 = 0$ ...(1)
and $12x + 5y + 6 = 0$ ...(2)
Re-writing equations, so that constant term in both have same sign. We write second equation so that its constant term is negative. Then, equation of bisector of the acute angle between the given straight lines is

$$\frac{3x-4y-3}{\sqrt{3^2+4^2}} = -\frac{(12x+5y+6)}{\sqrt{12^2+5^2}}$$

$$\Rightarrow \frac{3x-4y-3}{5} = -\frac{-12x-5y-6}{13}$$

$\Rightarrow 39x - 52y - 39 = -60x - 25y - 30$
$\Rightarrow -60x - 25y - 30 - 39x + 52y + 39 = 0$
$\Rightarrow -99x + 27y + 9 = 0$
$\Rightarrow -11x + 3y + 1 = 0$
Putting $x = 2$ and $y = 7$
This equation is satisfied by (2, 7).
Thus, the bisector of acute angle between the given straight lines passes through (2, 7)

2. (d) The given equation of straight lines are

$$\frac{x}{a} + \frac{y}{b} = m \quad ...(i)$$

and $$\frac{x}{a} - \frac{y}{b} = \frac{1}{m} \quad ...(ii)$$

From Eqs. (1) and (2), we get

$$\left(\frac{x}{a}+\frac{y}{b}\right)\left(\frac{x}{a}-\frac{y}{b}\right) = m \times \frac{1}{m} = 1$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

which represents a Hyperbola and is the locus of point of intersection the given straight lines.

3. (b) Given equation
$x^3y + xy^3 - xy = 0$
$\Rightarrow xy(x^2 + y^2) = xy$
$\Rightarrow xy(x^2 + y^2 - 1) = 0$
$\Rightarrow x^2 + y^2 = 1, xy = 0$
Above equations represent a pair of straight lines and a circle.

4. (a) The given line is,
$(2x + 3y + 4) + \lambda(6x - y + 12) = 0$
$\Rightarrow (2 + 6\lambda)x + (3 - \lambda)y + 4 + 12\lambda = 0$ .... (i)
Since line (i) is parallel to y-axis.
$\therefore$ Coefficient of $y = 0$
$3 - \lambda = 0 \Rightarrow \boxed{\lambda = 3}$

5. (c) $2x + 3y + a = 0$ or $y = \frac{-2}{3}x - \frac{a}{3}$

Slope $= \frac{-2}{3}$

$5x + ky + a = 0$ or $y = \frac{-5}{k}x \frac{-a}{5}$

Slope $= \frac{-5}{k}$
lines are parallel

$$\frac{-2}{3} = \frac{-5}{k}$$

$$k = \frac{15}{2} = 7.5$$

6. (d) $3x - 4y + 12 = 0$ or $y = \frac{3}{4}x + 3$

$3x - 4y = 6$ or $y = \frac{3}{4}x - \frac{3}{2}$

Equation of line mid-way between these two lines

$$y = \frac{3}{4}x + \left(\frac{3 - \frac{3}{2}}{2}\right)$$

$$y = \frac{3}{4}x + \frac{3}{4}$$

$4y = 3x + 3$
$3x - 4y + 3 = 0$

7. (b) Given equation of line is $bx\cos\alpha + ay\sin\alpha = ab$
Perpendicular distance from point $\left(\sqrt{a^2-b^2}, 0\right)$ is

$$d_1 = \left|\frac{b\cos\alpha\sqrt{a^2-b^2}+0-ab}{\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha}}\right|$$

($\because$ distance from (x_1, y_1) to $ax + by + c = 0$ is $= \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Similarly, perpendicular distance from point $(-\sqrt{a^2-b^2}, 0)$ is

$$d_2 = \left|\frac{-b\cos\alpha\sqrt{a^2-b^2}+0-ab}{\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha}}\right|$$

By product of d_1 and d_2, we get

$$= \frac{(b\cos\alpha\sqrt{a^2-b^2}-ab)(b\cos\alpha\sqrt{a^2-b^2}+ab)}{(\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha})(\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha})}$$

$$= \frac{b^2\cos^2\alpha(a^2-b^2)-a^2b^2}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$$= \frac{a^2b^2\cos^2\alpha-b^4\cos^2\alpha-a^2b^2}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$$= \frac{a^2b^2(\cos^2\alpha-1)-b^4\cos^2\alpha}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$$= \frac{-b^2[a^2\sin^2\alpha + b^2\cos^2\alpha]}{b^2\cos^2\alpha + a^2\sin^2\alpha}$$

Hence, product of the perpendiculars = $-b^2 = b^2$ (since, distance is positive)

8. (a) $y - \sqrt{3}x - 5 = 0$ (i)

$\sqrt{3}y - x + 6 = 0$ (ii)

$y = mx + c$

From (i) and from (ii)

$y = \sqrt{3}x + 5$ $\quad y = \frac{x}{\sqrt{3}} - \frac{6}{\sqrt{3}}$

$m_1 = \sqrt{3}$ $\quad m_2 = \frac{1}{\sqrt{3}}$

Angle between two lines,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\cdot\frac{1}{\sqrt{3}}}\right| = \frac{1}{\sqrt{3}}$$

$= \tan 30°$

$\boxed{\theta = 30°}$

9. (a) Equation of given line is

$$(y-3) = \left(\frac{3-1}{2-1}\right)(x-2)$$

$\Rightarrow y = 2x - 1$

Slope $m_1 = 2$

and slope of perpendicular $= -\frac{1}{2}$

The perpendicular is also bisector, therefore it will pass through its mid-point.

$\Rightarrow$ Coordinates of mid-point of given line are :

$\left(\frac{2+1}{2}, \frac{3+1}{2}\right)$ or $\left(\frac{3}{2}, 2\right)$.

So, equation of perpendicular bisector is :

$$(y-2) = -\frac{1}{2}\left(x - \frac{3}{2}\right)$$

$\Rightarrow 2x + 4y - 11 = 0$

10. (a) Given lines, $\sqrt{2}x + \sqrt{3}y = 1$ and $\sqrt{3}x + \sqrt{2}y = 2$

We know, Slope (m) $= \frac{-a}{b}$

$\therefore$ Slope of $\sqrt{2}x + \sqrt{3}y = 1$ is $m_1 = \frac{-\sqrt{2}}{\sqrt{3}}$

Slope of $\sqrt{3}x + \sqrt{2}y = 2$ is $m_2 = \frac{-\sqrt{3}}{\sqrt{2}}$

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{-\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}}}{1 + \left(\frac{-\sqrt{2}}{\sqrt{3}}\right)\left(\frac{-\sqrt{3}}{\sqrt{2}}\right)}\right|$$

$$= \left|\frac{\frac{-3+2}{\sqrt{6}}}{2}\right| = \left|\frac{-1}{2\sqrt{6}}\right| = \frac{1}{2\sqrt{6}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$$

11. (*) The slope of line $x + y - 3 = 0$ is -1

The slope of line $x - y + 3 = 0$ is 1

So, these are perpendicular lines and so angle between them is 90°.

$\therefore \alpha = 90°$

The slope of line $x - \sqrt{3}y + 2\sqrt{3} = 0$ is $m_1 = \frac{1}{\sqrt{3}}$

The slope of line $\sqrt{3}x - y + 1 = 0$ is $m_2 = \sqrt{3}$.

$$\therefore \tan\beta = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}\right| = \left|\frac{2}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}} \therefore \beta = 30°$$

$\therefore \alpha > \beta$

12. (b) Given straight lines are

$(m^2 - mn)\,y = (mn + n^2)x + n^3$ ...(1)

$(mn + m^2)\,y = (mn - n^2)x + m^3$ ...(2)

$$(1) \Rightarrow y = \frac{(mn + n^2)}{m^2 - mn}x + \frac{n^3}{m^2 - mn}$$

So, slope of (1), $m_1 = \frac{mn + n^2}{m^2 - mn}$

$$(2) \Rightarrow y = \frac{mn - n^2}{mn + m^2}x + \frac{m^3}{mn + m^2}$$

So, slope of (2), $m_2 = \frac{mn - n^2}{mn + m^2}$

If α is the angle between lines (1), (2), then

$$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\frac{mn + n^2}{m^2 - mn} - \frac{mn - n^2}{mn + m^2}}{1 + \left(\frac{mn + n^2}{m^2 - mn}\right)\left(\frac{mn - n^2}{mn + m^2}\right)}$$

$$= \frac{(mn + n^2)(mn + m^2) - (mn - n^2)(m^2 - mn)}{(m^2 - mn)(mn + m^2) + (mn + n^2)(mn - n^2)}$$

$$= \frac{m^2n^2 + m^3n + mn^3 + m^2n^2 - m^3n + m^2n^2 + m^2n^2 - mn^3}{m^3n + m^4 - m^2n^2 - m^3n + m^2n^2 - mn^3 + mn^3 - n^4}$$

$$\tan\alpha = \frac{4m^2n^2}{m^4 - n^4} \Rightarrow \alpha = \tan^{-1}\left(\frac{4m^2n^2}{m^4 - n^4}\right)$$

Circles

9

1. An equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$ with one of the vertices at (a, 0). What is the equation of the side opposite to this vertex ? *[2006-I]*
   (a) $2x - a = 0$ (b) $x + a = 0$
   (c) $2x + a = 0$ (d) $3x - 2a = 0$
2. What is the radius of the circle passing through the points (0, 0), (a, 0) and (0, b) ? *[2006-I]*
   (a) $\sqrt{a^2 - b^2}$ (b) $\sqrt{a^2 + b^2}$
   (c) $\frac{1}{2}\sqrt{a^2 + b^2}$ (d) $2\sqrt{a^2 + b^2}$
3. If two circles A, B of equal radii pass through the centres of each other, then what is the ratio of the length of the smaller arc to the circumference of the circle A cut off by the circle B? *[2006-II]*
   (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
   (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
4. If the extremities of a diameter of a circle are (0, 0) and $(a^3, 1/a^3)$, then the circle passes through which one of the following points ? *[2006-II]*
   (a) $(a^2, 1/a^2)$ (b) (a, 1/a)
   (c) (a, –a) (d) (1/a, a)
5. What is the length of the intercept made on the x-axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$? *[2006-II]*
   (a) $\frac{\sqrt{(g^2 - c)}}{2}$ (b) $\frac{\sqrt{(g^2 - 4c)}}{2}$
   (c) $2\sqrt{(g^2 - 4c)}$ (d) $2\sqrt{(g^2 - c)}$
6. Under which one of the following conditions does the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meet the x-axis in two points on opposite sides of the origin? *[2007-I]*
   (a) $c > 0$ (b) $c < 0$
   (c) $c = 0$ (d) $c \leq 0$
7. What is the equation of a circle, whose centre lies on the x-axis at a distance h from the origin and the circle passes through the origin? *[2007-I]*
   (a) $x^2 + y^2 - 2hx = 0$
   (b) $x^2 + y^2 - 2hx + h^2 = 0$
   (c) $x^2 + y^2 + 2hxy = 0$
   (d) $x^2 + y^2 - h^2 = 0$
8. Consider a circle of radius R. What is the length of a chord which subtends an angle θ at the centre? *[2007-II]*
   (a) $2R \sin\left(\frac{\theta}{2}\right)$ (b) $2R \sin \theta$
   (c) $2R \tan\left(\frac{\theta}{2}\right)$ (d) $2R \tan \theta$
9. What is the equation of circle which touches the lines x = 0, y = 0 and x = 2 ? *[2007-II]*
   (a) $x^2 + y^2 + 2x + 2y + 1 = 0$
   (b) $x^2 + y^2 - 4x - 4y + 1 = 0$
   (c) $x^2 + y^2 - 2x - 2y + 1 = 0$
   (d) None of these
10. Equation of a circle passing through origin is $x^2 + y^2 - 6x + 2y = 0$. What is the equation of one of its diameters? *[2008-I]*
    (a) $x + 3y = 0$ (b) $x + y = 0$
    (c) $x = y$ (d) $3x + y = 0$
11. Point (1, 2) relative to the circle $x^2 + y^2 + 4x - 2y - 4 = 0$ is a/an *[2008-I]*
    (a) exterior point
    (b) interior point, but not centre
    (c) boundary point
    (d) centre
12. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ $(c > 0)$ touches the y-axis, then which one of the following is correct? *[2008-I]*
    (a) $g = -\sqrt{c}$ only (b) $g = \pm\sqrt{c}$
    (c) $f = \sqrt{c}$ only (d) $f = \pm\sqrt{c}$

13. The equation of the circle which touches the axes at a distance 5 from the origin is $y^2 + x^2 - 2\alpha x - 2\alpha y + \alpha^2 = 0$. What is the value of α? [2008-II]
(a) 4 (b) 5
(c) 6 (d) 7

14. ABC is an equilateral triangle inscribed in a circle of centre O and radius 5 cm. Let the diameter through C meet the circle again at D. [2008-II]
Assertion (A) : $AD . BD < OB . OC$
Reason (R) : $2(AD^2 + BD^2) = CD^2 = 100$ sq cm
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

15. If x-axis is tangent to the circle $x^2 + y^2 + 2gx + 2fy + k = 0$, then which one of the following is correct? [2009-I]
(a) $g^2 = k$ (b) $g^2 = f$
(c) $f^2 = k$ (d) $f^2 = g$

16. The circle $x^2 + y^2 + 4x - 4y + 4 = 0$ touches [2009-II]
(a) Only the x-axis (b) Only the y-axis
(c) Both the axes (d) Neither of the axes

17. Consider the following statements in respect of circles $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 1$ [2010-I]
1. The radius of the first circle is twice that of the second circle.
2. Both the circles pass through the origin.

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

18. What is the equation to circle which touches both the axes and has centre on the line $x + y = 4$? [2010-II]
(a) $x^2 + y^2 - 4x + 4y + 4 = 0$
(b) $x^2 + y^2 - 4x - 4y + 4 = 0$
(c) $x^2 + y^2 + 4x - 4y - 4 = 0$
(d) $x^2 + y^2 + 4x + 4y - 4 = 0$

19. Under which of the following conditions does a general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ $(a \neq 0)$ represents a circle? [2010-II]
(a) $h = g, a = b$
(b) $h = g = f, a = b$
(c) $h = 0, a = b$
(d) $h = 0, g^2 + f^2 - c = a + b$

20. For the equation [2011-I]
$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$
where $a \neq 0$, to represent a circle, the condition will be
(a) $a = b$ and $c = 0$ (b) $f = g$ and $h = 0$
(c) $a = b$ and $h = 0$ (d) $f = g$ and $c = 0$

21. What is the radius of the circle touching x-axis at (3, 0) and y-axis at (0, 3)? [2011-II]
(a) 3 units (b) 4 units
(c) 5 units (d) 6 units

22. Which one of the following points lies inside a circle of radius 6 and centre at (3, 5) ? [2013-I]
(a) (–2, –1) (b) (0, 1)
(c) (–1, –2) (d) (2, –1)

23. The radius of the circle $x^2 + y^2 + x + c = 0$ passing through the origin is [2013-II]
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

DIRECTIONS (Qs. 24-25) : *For the next two (02) items that follow:*

Consider the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$. [2014-II]

24. What is the distance between the centres of the two circles?
(a) $\sqrt{a^2 + b^2}$ (b) $a^2 + b^2$
(c) $a + b$ (d) $2(a + b)$

25. The two circles touch each other if
(a) $c = \sqrt{a^2 + b^2}$ (b) $\frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$
(c) $c = \frac{1}{a^2} + \frac{1}{b^2}$ (d) $c = \frac{1}{a^2 + b^2}$

26. A straight line $x = y + 2$ touches the circle $4(x^2 + y^2) = r^2$. The value of r is [2015-II]
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) 2 (d) 1

27. If the centre of the circle passing through the origin is (3, 4), then the intercepts cut off by the circle on x-axis and y-axis respectively are [2015-II]
(a) 3 unit and 4 unit (b) 6 unit and 4 unit
(c) 3 unit and 8 unit (d) 6 unit and 8 unit

28. If a circle of radius b units with centre at (0, b) touches the line $y = x - \sqrt{2}$, then what is the value of b? [2016-I]
(a) $2 + \sqrt{2}$ (b) $2 - \sqrt{2}$
(c) $2\sqrt{2}$ (d) $\sqrt{2}$

DIRECTIONS (Qs. 29-30) : *For the next two (2) items that follow:*

Consider the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ [2016-I]

29. What is the distance between the centres of the two circles?

(a) 5 units (b) 6 units
(c) 8 units (d) 10 units

30. If the circles intersect at two distinct points, then which one of the following is correct?
(a) $r = 1$ (b) $1 < r < 2$
(c) $r = 2$ (d) $2 < r < 8$

DIRECTIONS (Qs. 31-32): *For the next two (2) items that follow:*

Consider a circle passing through the origin and the points (a, b) and (–b, –a). *[2016-I]*

31. On which line does the centre of the circle lie?
(a) $x + y = 0$ (b) $x - y = 0$
(c) $x + y = a + b$ (d) $x - y = a^2 - b^2$

32. What is the sum of the squares of the intercepts cut off by the circle on the axes?
(a) $\left(\frac{a^2+b^2}{a^2-b^2}\right)^2$ (b) $2\left(\frac{a^2+b^2}{a-b}\right)^2$
(c) $4\left(\frac{a^2+b^2}{a-b}\right)^2$ (d) None of the above

33. What is the radius of the circle passing through the point (2, 4) and having centre at the intersection of the lines $x - y = 4$ and $2x + 3y + 7 = 0$? *[2016-II]*
(a) 3 units (b) 5 units
(c) $3\sqrt{3}$ units (d) $5\sqrt{2}$ units

34. The two circles $x^2 + y^2 = r^2$ and $x^2 + y^2 - 10x + 16 = 0$ intersect at two distinct points. Then which one of the following is correct? *[2017-I]*
(a) $2 < r < 8$ (b) $r = 2$ or $r = 8$
(c) $r < 2$ (d) $r > 2$

35. What is the equation of the circle which passes through the points (3, –2) and (–2, 0) and having its centre on the line $2x - y - 3 = 0$? *[2017-I]*
(a) $x^2 + y^2 + 3x + 2 = 0$
(b) $x^2 + y^2 + 3x + 12y + 2 = 0$
(c) $x^2 + y^2 + 2x = 0$
(d) $x^2 + y^2 = 5$

36. The equation of the circle which passes through the points (1, 0), (0, –6) and (3, 4) is *[2017-II]*
(a) $4x^2 + 4y^2 + 142x + 47y + 140 = 0$
(b) $4x^2 + 4y^2 - 142x - 47y + 138 = 0$
(c) $4x^2 + 4y^2 - 142x + 47y + 138 = 0$
(d) $4x^2 + 4y^2 + 150x - 49y + 138 = 0$

37. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is *[2018-II]*
(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$
(b) $(x - x_1)^2 + (y - y_1)^2 = x_2y_2$
(c) $x^2 + y^2 + 2x_1x_2 + 2y_1y_2 = 0$
(d) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

38. If y-axis touches the circle $x^2 + y^2 + gx + fy + \frac{c}{4} = 0$, then the normal at this point intersects the circle at the point *[2018-II]*
(a) $\left(-\frac{g}{2}, -\frac{f}{2}\right)$ (b) $\left(-g, -\frac{f}{2}\right)$
(c) $\left(-\frac{g}{2}, f\right)$ (d) $(-g, -f)$

39. A circle is drawn on the chord of a circle $x^2 + y^2 = a^2$ as diameter. The chord lies on the line $x + y = a$. What is the equation of the circle? *[2019-I]*
(a) $x^2 + y^2 - ax - ay + a^2 = 0$
(b) $x^2 + y^2 - ax - ay = 0$
(c) $x^2 + y^2 + ax + ay = 0$
(d) $x^2 + y^2 + ax + ay - 2a^2 = 0$

40. The circle $x^2 + y^2 + 4x - 7y + 12 = 0$, cuts an intercept on y-axis equal to *[2019-I]*
(a) 1 (b) 3
(c) 4 (d) 7

ANSWER KEY

1	**(c)**	5	**(d)**	9	**(c)**	13	**(b)**	17	**(d)**	21	**(a)**	25	(b)	29	(a)	33	(d)	37	(d)
2	**(c)**	6	**(b)**	10	**(a)**	14	**(d)**	18	**(b)**	22	**(b)**	26	(b)	30	(d)	34	(a)	38	(b)
3	**(c)**	7	**(a)**	11	**(a)**	15	**(a)**	19	**(c)**	23	(b)	27	(d)	31	(a)	35	(b)	39	(b)
4	**(d)**	8	**(a)**	12	**(d)**	16	**(c)**	20	**(c)**	24	(a)	28	(a)	32	(b)	36	(c)	40	(a)

HINTS & SOLUTIONS

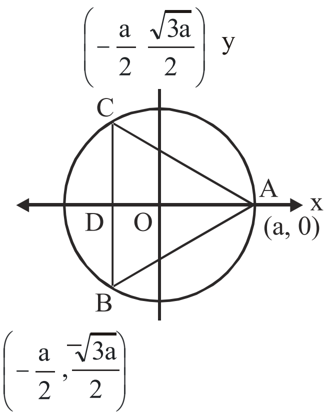
1. (c) Since the equilateral triangle is inscribed in the circle with centre at the origin, centroid lies on the origin.

So, $\frac{AO}{OD}=\frac{2}{1}$

and $OD=\frac{1}{2}AO=\frac{a}{2}$

So, other vertices of triangle have coordinates,

$\left(-\frac{a}{2},\frac{\sqrt{3}a}{2}\right)$ and $\left[-\frac{a}{2},-\frac{\sqrt{3}}{2}a\right]$



∴ Equation of line BC is :

$x=-\frac{a}{2}$

$\Rightarrow \ 2x+a=0$

2. (c) Let (h, k) be the centre of the circle. Since, circle is passing through (0, 0), (a, 0) and (0, b), distance between centre and these points would be same and equal to radius.

Hence, $h^2+k^2=(h-a)^2+k^2=h^2+(k-b)^2$

$\Rightarrow \ h^2+k^2=h^2+k^2+a^2-2ah=h^2+k^2+b^2-2bk$

$\Rightarrow \ h^2+k^2=h^2+k^2+a^2-2ah$

$\Rightarrow \ h=\frac{a}{2}$

Similarly, $k=\frac{b}{2}$

∴ Radius of circle $=\sqrt{h^2+k^2}=\frac{1}{2}\sqrt{a^2+b^2}$

3. (c) When two circles A and B of equal radii pass through the centres of each other, the angle made by arc of B at the centre of B is 90°.

So, length of small arc of B $=\frac{2\pi r 90°}{360°}=\frac{\pi r}{2}$

Hence, circumference of A cut off by the circle B

$=2\pi r-\frac{\pi r}{2}=\frac{3\pi r}{2}$

∴ Required ratio $=\frac{\pi r/2}{3\pi r/2}=\frac{1}{3}$

4. (d) Given the extremities of a diameter of a circle as (0, 0) and $\left(a^3,\frac{1}{a^3}\right)$, equation of circle is

$(x-0)\,(x-a^3)+(y-0)\left(y-\frac{1}{a^3}\right)=0$

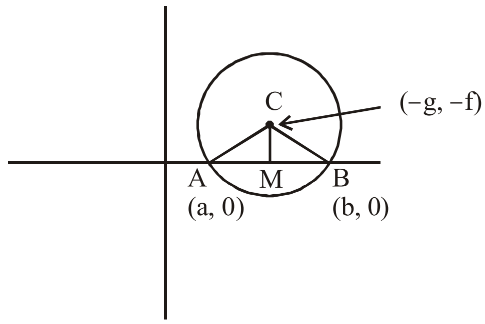
$\Rightarrow \ x^2-xa^3+y^2-\frac{y}{a^3}=0$

$\Rightarrow \ x^2+y^2-xa^3-\frac{y}{a^3}=0$

Putting $x=\frac{1}{a}$ and y = a, the equation is satisfied.

Thus, the circle passes through the point $\left(\frac{1}{a},a\right)$.

5. (d) Let the circle $S\equiv x^2+y^2+2gx+2fy+c=0$ cut x-axis at two points A, and B at x = a and x = b.
Let C be the centre and AB the chord.



Length of chord AB = b – a.
Perpendicular from centre c on chord bisect this chord at point M. Radius $r=\sqrt{g^2+f^2-c}$ also radius AC = $r=\sqrt{AM^2+CM^2}$

$AM=\frac{AB}{2}=\frac{b-a}{2}$

So, $r = \sqrt{\left(\frac{b-a}{2}\right)^2 + (-f)^2}$

Thus, $\sqrt{g^2+f^2-c} = \sqrt{\left(\frac{b-a}{2}\right)^2 + f^2}$

or $g^2 + f^2 - c = \left(\frac{b-a}{2}\right)^2 + f^2$

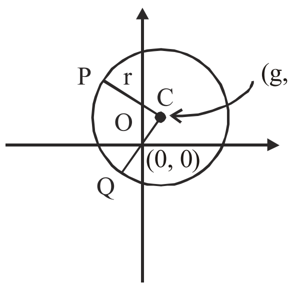
or $\left(\frac{b-a}{2}\right)^2 = g^2 - c$

$\frac{b-a}{2} = \sqrt{g^2 - c}$

Hence, $b - a = AB = 2\sqrt{g^2 - c}$

The length of the intercept made on the x-axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $2\sqrt{g^2 - c}$.

6. (b) For a circle to meet x-axis in two points on the opposite side of the origin its radius r, should be more than the distance of its centre from the origin.
Co-ordinate of centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(-g, -f)$:



In the figure shown,
OQ = OP = r, and distance of centre C, from origin, O is CO

$r > \sqrt{OC}$ i.e. $r > \sqrt{(-g)^2 + (-f)^2}$

or, $\sqrt{(-g)^2 + (-f)^2 - c} > \sqrt{(-g)^2 + (-f)^2}$

or, $g^2 + f^2 - c > g^2 + f^2$

or, $-c > 0$

or, $c < 0$

7. (a) Centre of the circle is (h, 0) and circle passes through the origin. In the general equation of circle:
$x^2 + y^2 + 2gx + 2fx + c = 0$
$g = -h$ and $f = 0$
so, $x^2 + y^2 - 2hx + 0 + c$
$= x^2 + y^2 - 2hx + c = 0$ ...(i)
since circle passes through origin (0, 0)
$0 + 0 - 0 + c = 0 \Rightarrow c = 0$
and equation (i) radius to $x^2 + y^2 - 2hx = 0$

8. (a) Let there be a circle of radius R and chord AB.
$OD \perp AB$ and $AD = DB$.
and $AD = 2AD$
$\angle AOB = \theta$

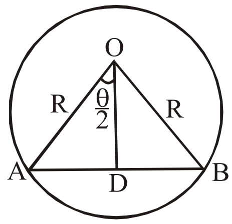
$\Rightarrow \angle AOD = \frac{\theta}{2}$

In ΔAOD,

$\sin\frac{\theta}{2} = \frac{AD}{OA}$

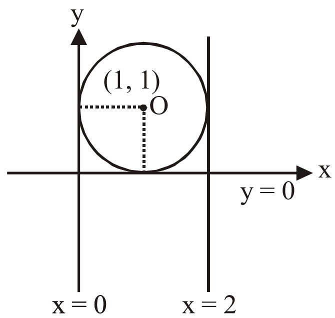
$\sin\frac{\theta}{2} = \frac{AD}{R}$

$AD = R\sin\frac{\theta}{2}$



$\therefore$ Length of chord $AB = 2AD = 2R\sin\frac{\theta}{2}$.

9. (c) Refer to the figure it is clear that coordinates of centre of circle are (1, 1) and diameter of circle = 2 and hence radius of circle is 1.



$\therefore$ Equation of circle with centre (1, 1) and radius = 1 is
$(x-1)^2 + (y-1)^2 = 1$
$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = 1$
$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

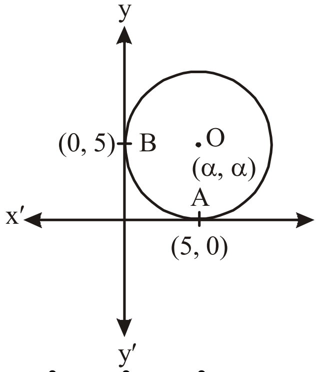
10. (a) Equation of the given circle is :
$x^2 + y^2 - 6x + 2y = 0$
$\Rightarrow x(x-6) + y(y+2) = 0$
or, $(x-0)(x-6) + (y-0)(y+2) = 0$
This is the equation of circle in diameter form.
Here, end points of diameter are (0, 0) and (6, – 2).
Hence, equation of diameter is a line which passes through the points (0, 0) and (6, – 2) which is

$(y - 0) = \frac{-2}{6}(x - 0) \Rightarrow x + 3y = 0$

11. (a) We put the co-ordinates of the given point in the given equation of circle
$x^2 + y^2 + 4x - 2y - 4 = 0$
At (1, 2)
$(1)^2 + (2)^2 + 4(1) - 2(2) - 4$
$= 1 + 4 + 4 - 4 - 4 = 1 > 0$
$\Rightarrow$ Point (1, 2) lies out side the circle i.e, an exterior point.

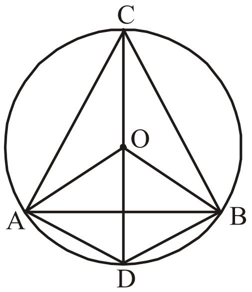
12. (d) As given, the circle $x^2+y^2+2gx+2fy+c=0$ touches y-axis.
then $r=\pm g$ and $g^2+f^2-c=g^2$
$\Rightarrow f^2-c=0 \Rightarrow f^2=c \Rightarrow f=\pm\sqrt{c}$

13. (b) Coordinates of the centre of given circle = (α, α) and
radius = $\sqrt{(\alpha)^2+(\alpha)^2-\alpha^2}=\sqrt{\alpha^2}=\alpha$



$\therefore (\alpha-5)^2+(\alpha)^2=(\alpha)^2$
$\Rightarrow \alpha^2+25-10\alpha=0$
$\Rightarrow (\alpha-5)^2=0$
$\Rightarrow \alpha=5$
Then, other root will always real.

14. (d)



(A) : Consider $AD\cdot BD < OB\cdot OC$
Now, OA = 5 cm (Radius), OB = 5 cm (Radius) and OC = 5 cm (Radius)
Since, ΔOAD and ΔOBD
are congruent by SAS therefore

$$AD = OA = 5 \text{ cm}$$

and $BD = OB = 5$ cm

Thus, $AD\cdot BD = 5\times 5 = 25$

and $OB\cdot OC = 5\times 5 = 25$

Thus, we have

$AD\cdot BD = 25 = OB\cdot OC$

Now (R) : $2\left(AD^2+BD^2\right)$

$$=2[25+25]=100$$

and $CD^2=(10)^2=100$

Thus, $2\left(AD^2+BD^2\right)=CD^2=100$ sq. cm.

Hence, (A) is false and (R) is true.

15. (a) Length of intercept on the x-axis made by the circle

$$x^2+y^2+2gx+2fy+k=0 \text{ is } 2\sqrt{g^2-k}$$

Since, circle touches the x-axis therefore intercept on x-axis = 0

$\therefore \sqrt{g^2-k}=0$

$\Rightarrow g^2=k$

16. (c) Given equation of circle is
$x^2+4x+4+y^2-4y=0$
Add 4 on both side,
$x^2+4x+4+y^2-4y+4=0+4$
$\Rightarrow (x+2)^2+(y-2)^2=2^2$
Here, we observe that the values of centre and radius are same.
Hence, it touches both the axes.

17. (d) The equation of first circle is $x^2+y^2-2x-2y=0$.

Radius $=\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$

and equation of second circle is $x^2+y^2=1$.
Radius = 1
From above it is clear that the radius of first circle is not twice that of second circle.
$\therefore$ Statement 1 is not correct.
Also, first circle passes through the origin while second circle does not pass through the origin.
Hence, neither 1 nor 2 statement is correct.

18. (b) The equation of circle, which touches both the axes, is given by

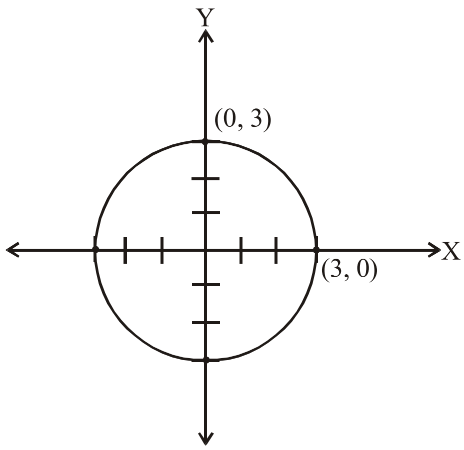
$$x^2+y^2-2rx-2ry+r^2=0 \quad \text{....... (i)}$$

Now, the centre (r, r) of this circle lies on the line
$x+y=4$
$r+r=4 \Rightarrow r=2$
$\therefore$ Put value of r in Eq. (i), we get
$x^2+y^2-4x-4y+4=0$
which is required equation of circle,

19. (c) The given equation represents a circle, if a = b, h = 0.

20. (c) The equation $ax^2+by^2+2hxy+2gx+2fy+c=0$ represents a circle, if $a=b$ and $h=0$.

21. (a) Since, circle is touching x-axis at (3, 0) and y-axis at (0, 3) therefore radius = 3 unit



22. (b) Radius of circle = 6, centre = (3, 5)

$\therefore$ Equation of circle is

$S \equiv (x-3)^2 + (y-5)^2 = (6)^2$

$\Rightarrow$ $S \equiv (x-3)^2 + (y-5)^2 - 36$

Now, consider all the four options.

(a) (–2, –1)

Put it in S

$S \equiv (-2-3)^2 + (-1-5)^2 - 36 = 25 + 36 - 36 = 25 > 0$

$\Rightarrow$ (–2, –1) is outside the circle.

(b) (0, 1)

$S \equiv (0-3)^2 + (1-5)^2 - 36$

$= 9 + 16 - 36 = 25 - 36 = -11 < 0$

Hence, (0, 1) lies inside a circle.

23. (b) Circle is passing through origin then C = 0

Now, $x^2 + y^2 + x = 0$

$$x^2 + x + \frac{1}{4} - \frac{1}{4} + y^2 = 0$$

$$\left(x + \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$$

$\therefore$ Radius of given circle is $\frac{1}{2}$ units

24. (a) Equations of circles are

$x^2 + y^2 + 2ax + c = 0$

and $x^2 + y^2 + 2by + c = 0$

Since, the centres of two circles are (–a, 0) and (0, –b)

$\therefore$ Distance between two centres = $\sqrt{a^2 + b^2}$

25. (b) Two circles touch each other, iff distance between two centres = Sum of radius of two circles

$$\sqrt{a^2 + b^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$$

On squaring both sides, we get

$$a^2 + b^2 = a^2 - c + b^2 - c + 2\sqrt{(a^2 - c)(b^2 - c)}$$

$$\Rightarrow c = \sqrt{(a^2 - c)(b^2 - c)}$$

Again, squaring both sides, we get

$$c^2 = a^2 b^2 - a^2 c - b^2 c + c^2$$

$$\Rightarrow a^2 b^2 = (a^2 + b^2)\, c \Rightarrow \frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$$

26. (b) $\because$ $4(x^2 + y^2) = r^2$

$$\Rightarrow x^2 + y^2 = \left(\frac{r}{2}\right)^2$$

Center (0, 0) and radius $\frac{r}{2}$

Eq. of line is ; $x - y - 2 = 0$

$\because$ Line touches the circle.

$$\therefore \frac{r}{2} = \frac{|0 - 0 - 2|}{\sqrt{(1)^2 + (-1)^2}}$$

$r = 2\sqrt{2}$

27. (d) Equation of circle having radius r and centre (3, 4) is

$= (x-3)^2 + (y-4)^2 = r^2$

if it is passing through (0, 0)

$\therefore$ $(0-3)^2 + (0-4)^2 = r^2$

$\Rightarrow$ $r^2 = 25$

equation of circle is

$(x-3)^2 + (y-4)^2 = 25$

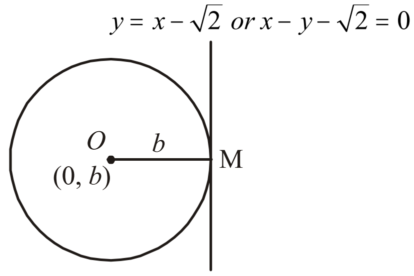
putting y = 0

$\therefore$ x = 6 unit = interception x-axis

intercept on y axis (putting x = 0) is

y = 8 unit

28. (a) Distance from the centre to the point of line which touches circle is OM = radius



$$r = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$\because$ $x_0 = 0$

& $Y_0 = b$

$$b = \frac{|1(0) + (-1)(b) - \sqrt{2}|}{\sqrt{(1)^2 + (-1)^2}}$$

$$b = \frac{b + \sqrt{2}}{\sqrt{2}}$$

$$(\sqrt{2} - 1)b = \sqrt{2}$$

$$\Rightarrow \boxed{b = 2 + \sqrt{2}}$$

Sol. (29–30) :

Given equation of circles

$(x-1)^2+(y-3)^2=r^2$

$(h_1, k_1) \equiv$ coordinates of centre $\equiv (1, 3)$

$\therefore\ x^2+y^2-8x+2y+8=0$

$\Rightarrow (x-4)^2+(y+1)^2=(3)^2$

$(h_2, k_2) \equiv$ coordinates of centre $\equiv (4, -1)$

29. (a) Distance between centres of two circles

$$d=\sqrt{(h_1-h_2)^2+(k_1-k_2)^2}$$

$$\Rightarrow d=\sqrt{(1-4)^2+(3+1)^2}$$

$$\Rightarrow d=\sqrt{25}$$

$d = 5$ units

30. (d) Radius of circle one $= r_1 = r$

Radius of circle two $= r_2 = 3$

$\because$ Circle intersects at two points so distance between circle is $d < r_1 + r_2$

$5 < r + 3$

$r > 2$ also $r \leq 5 + 2$. Hence, $2 < r < 8$

31. (a) Suppose; $x^2 + y^2 + 2gx + 2fy + c = 0$ is the eq. of the circle.

Since; it passes through

(0, 0); (a, b) & (–b, –a)

$\therefore C = 0$

$a^2 + b^2 + 2ga + 2fb = 0$ ...(1)

$\therefore\ a^2 + b^2 - 2gb - 2fb = 0$ ...(2)

on solving:

$g = -f$

$\therefore$ centre $\equiv (-g, -f)$ or $(+f, -f)$

$\therefore$ from options:

$x + y = 0$ is the line which passes through $(f, -f)$

32. (b) The two intercepts are : –2g & –2f

$\therefore$ from eq (1) & (2) we get;

$$g=\frac{-1}{2}\left(\frac{a^2+b^2}{a-b}\right)\ \&\ f=\frac{1}{2}\left(\frac{a^2+b^2}{a-b}\right)$$

is sum of squares of intercepts

$$=\left(\frac{a^2+b^2}{a-b}\right)^2+\left(\frac{a^2+b^2}{a-b}\right)^2$$

$$=2\left[\frac{a^2+b^2}{a-b}\right]^2$$

33. (d) We have

$x - y = 4$ & $2x + 3y + 7 = 0$

On solving, we get,

$x = 1$ & $y = -3$

(these are coordinates of centre of the circle)

$\Rightarrow$ radius $=\sqrt{(2-1)^2+(4+3)^2} = 5\sqrt{2}$

34. (a) For the circle, $x^2 + y^2 = r^2$, centre (0, 0) radius = r

If the circle $x^2 + y^2 - 10x + 16 = 0$ is compared with general form

$x^2 + y^2 + 2g + 2fy + c = 0$, we get

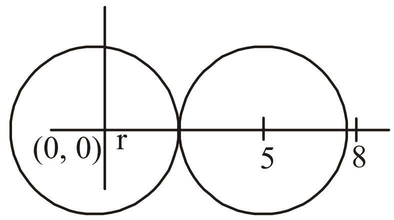
$2g = -10, 2f = 0, c = 16$

$\Rightarrow g = -5, f = 0$

$\therefore$ centre = (+ 5, 0)

radius $=\sqrt{g^2+f^2-c}=\sqrt{25+0-16}=\sqrt{9}=3$

Given, two circles intersect at two distinct points,



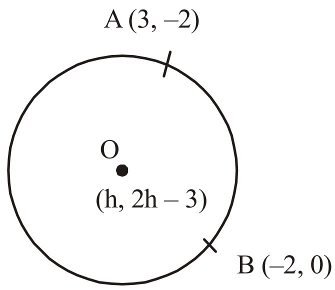
To interset at two points, r should be greater than 2

So, $2 < r < 8$

35. (b) Given, centre of circle lies on line $2x - y - 3 = 0$

Let $x = h$

$2h - y - 3 = 0 \Rightarrow y = 2n - 3$



$\therefore$ centre of circle = (h, 2h – 3)

We know, OA = OB

$\Rightarrow (h-3)^2 + (2h-3+2)^2 = (h+2)^2 + (2h-3)^2$

$\Rightarrow h^2 - 6h + 9 + (2h-1)^2 = h^2 + 4h + 4 + 4h^2 - 12h + 9$

$\Rightarrow \cancel{h^2} - 6h + 9 + \cancel{4h^2} - 4h + 1$

$= \cancel{h^2} + 4h + 4 + \cancel{4h^2} - 12h + 9$

$\Rightarrow -10h + 10 = -8h + 13$

$$\Rightarrow -2h = 3 \Rightarrow h=\frac{-3}{2}$$

$$\therefore \text{centre}=\left(\frac{-3}{2}, 2\left(\frac{-3}{2}\right)-3\right)=\left(\frac{-3}{2}, -6\right)$$

$$\therefore \text{radius}=(h+2)^2+(2h-3)^2$$

$$=\left(\frac{-3}{2}+2\right)^2+\left[2\left(\frac{-3}{2}\right)-3\right]^2=\frac{1}{4}+36$$

$\therefore$ Equation of circle

$$\Rightarrow \left(x+\frac{3}{2}\right)^2+(y+6)^2=\frac{1}{4}+36$$

$\Rightarrow x^2+y^2+3x+12y+2=0$

36. (c) Let A = (1, 0), B = (0, –6), C = (3, 4)

Equation of AB is L : $\frac{y-0}{-6-0}=\frac{x-1}{0-1}$

$$\Rightarrow \frac{y}{-6}=\frac{x-1}{-1}\Rightarrow y=6x-6$$

$\Rightarrow 6x - y - 6 = 0.$

Equation of circle (c) with AB as diameter is $(x-1)(x-0)+(y-0)(y+6)=0$

$\Rightarrow x^2 - x + y^2 + 6y = 0.$

The system of circle passing through the intersection of the circle C and the line L is given by C + kL = 0

$\Rightarrow x^2 - x + y^2 + 6y + k(6x - y - 6) = 0$

This circle is passing through (3, 4).

$\therefore (3)^2 - 3 + (4)^2 + 6(4) + k[6(3) - 4 - 6] = 0$

$\Rightarrow 9 - 3 + 16 + 24 + k(18 - 10) = 0$

$$\Rightarrow 46 + 8k = 0 \Rightarrow 8k = -46 \Rightarrow k=\frac{-46}{8}=\frac{-23}{4}$$

$\therefore$ Equation of circle is

$$x^2-x+y^2+6y+\left(\frac{-23}{4}\right)(6x-y-6)=0$$

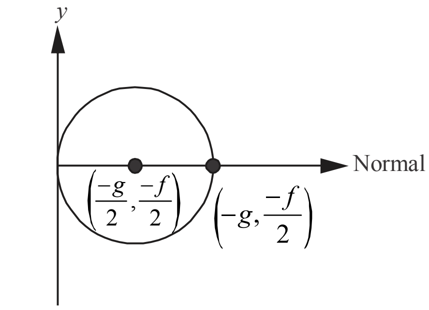
$\Rightarrow 4x^2 - 4x + 4y^2 + 24y - 138x + 23y + 138 = 0$

$\Rightarrow 4x^2 + 4y^2 - 142x + 47y + 138 = 0.$

37. (d) Equation of circle is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

38. (b)



39. (b) Given, equation of circle $\Rightarrow x^2 + y^2 = a^2$ ...(1)

Equation of chord $\Rightarrow x + y = a$ ...(2)

$(1) \Rightarrow x^2 + (a - x)^2 = a^2$

$\Rightarrow x^2 + a^2 + x^2 - 2ax = a^2$

$\Rightarrow 2x^2 = 2ax$

$\Rightarrow x = 0, a$

When, x = 0, y = a and when x = a, y = 0.

$\therefore$ Points of intersection are (0, a) and (a, 0)

$\therefore$ Equation of circle with chord as diameter is

$(x - 0)(x - a) + (y - a)(y - 0) = 0$

$\Rightarrow x(x - a) + y(y - a) = 0$

$\Rightarrow x^2 - ax + y^2 - ay = 0$

$\Rightarrow x^2 + y^2 - ax - ay = 0$

40. (a) Given circle, $x^2 + y^2 + 4x - 7y + 12 = 0$

Comparing with general form of circle,

$ax^2 + by^2 + 2gx + 2fy + c = 0,$

$f=\frac{-7}{2}$ and c = 12.

y - intercept = $2\sqrt{f^2-c}$

$$=2\sqrt{\left(\frac{-7}{2}\right)^2-12}=2\sqrt{\frac{49}{4}-12}$$

$$=2\sqrt{\frac{49-48}{4}}=2\left(\frac{1}{2}\right)=1$$

CONICS — Parabola, Ellipse & Hyperbola

10

1. If the latus rectum of an ellipse is equal to one half its minor axis, what is the eccentricity of the ellipse ?
   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
   (c) $\frac{3}{4}$ (d) $\frac{\sqrt{15}}{4}$ *[2006-I]*
2. P (2, 2) is a point on the parabola $y^2 = 2x$ and A is its vertex. Q is another point on the parabola such that PQ is perpendicular to AP. What is the length of PQ ?
   (a) $\sqrt{2}$ (b) $2\sqrt{2}$
   (c) $4\sqrt{2}$ (d) $6\sqrt{2}$ *[2006-I]*
3. The focal distance of a point on the parabola $y^2 = 12x$ is 4. What is the abscissa of the point ?
   (a) 1 (b) – 1
   (c) $2\sqrt{3}$ (d) – 2 *[2006-I]*
4. If (2, 0) is the vertex and the y-axis is the directrix of a parabola, then where is its focus ?
   (a) (0, 0) (b) (– 2, 0)
   (c) (4, 0) (d) (– 4, 0) *[2006-I]*
5. Which one of the following points lies outside the ellipse $(x^2 / a^2) + (y^2 / b^2) = 1$?
   (a) (a, 0) (b) (0, b)
   (c) (– a, 0) (d) (a, b) *[2006-II]*
6. What is the equation of the parabola, whose vertex and focus are on the x-axis at distance a and b from the origin respectively? (b > a > 0) *[2007-I]*
   (a) $y^2 = 8(b-a)(x-a)$ (b) $y^2 = 4(b+a)(x-a)$
   (c) $y^2 = 4(b-a)(x+a)$ (d) $y^2 = 4(b-a)(x-a)$
7. If the eccentricity and length of latus rectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis? *[2007-I]*
   (a) $\frac{7}{2}$ unit (b) 12 unit
   (c) $\frac{15}{2}$ unit (d) $\frac{15}{4}$ unit
8. In how many points do the ellipse $\frac{x^2}{4}+\frac{y^2}{8}=1$ and the circle $x^2 + y^2 = 9$ intersect ? *[2007-II]*
   (a) One (b) Two
   (c) Four (d) None of the above
9. If the foci of the conics $\frac{x^2}{a^2}+\frac{y^2}{7}=1$ and $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ were to coincide, then what is the value of a ? *[2007-II]*
   (a) 2 (b) 3
   (c) 4 (d) 16
10. Which one of the following is correct? The eccentricity of the conic
    $$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1, (\lambda \geq 0)$$ *[2008-I]*
    (a) increases with increase in λ
    (b) decreases with increase in λ
    (c) does not change with λ
    (d) None of the above
11. Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (b > a)$. Then, which one of the following is correct? *[2008-II]*
    (a) Real foci do not exist (b) Foci are $(\pm ae, 0)$
    (c) Foci are $(\pm be, 0)$ (d) Foci are $(0, \pm be)$
12. Consider the parabolas $S_1 \equiv y^2 - 4ax = 0$ and $S_2 \equiv y^2 - 4bx = 0$. S_2 will contain S_1, if *[2008-II]*
    (a) $a > b > 0$ (b) $b > a > 0$
    (c) $a > 0, b < 0$ but $|b| > a$ (d) $a < 0, b > 0$ but $b > |a|$
13. Equation of the hyperbola with eccentricity 3/2 and foci at (± 2, 0) is $5x^2 - 4y^2 = k^2$. What is the value of k? *[2008-II]*
    (a) 4/3 (b) 3/4
    (c) $(4/3)\sqrt{5}$ (d) $(3/4)\sqrt{5}$
14. What is the eccentricity of an ellipse, if its latusrectum is equal to one-half of its minor axis? *[2009-I]*
    (a) 1/4 (b) 1/2
    (c) $\sqrt{3}/4$ (d) $\sqrt{3}/2$

15. What is the sum of focal radii of any point on an ellipse equal to? [2009-I]
(a) Length of latusrectum
(b) Length of major-axis
(c) Length of minor-axis
(d) Length of semi-latusrectum

16. What does an equation of the first degree containing one arbitrary parameter passing through a fixed point represent? [2009-I]
(a) Circle (b) Straight line
(c) Parabola (d) Ellipse

17. The curve $y^2 = -4ax\ (a > 0)$ lies in [2009-II]
(a) First and fourth quadrants
(b) First and second quadrants
(c) Second and third quadrants
(d) Third and fourth quadrants

18. The ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ has the same eccentricity as the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. What is the ratio of a to b? [2009-II]
(a) 5/13 (b) 13/5
(c) 7/8 (d) 8/7

19. If (4,0) and (–4,0) are the foci of an ellipse and the semi-minor axis is 3, then the ellipse passes through which one of the following points? [2010-I]
(a) (2,0) (b) (0,5)
(c) (0,0) (d) (5,0)

20. What is the locus of points, the difference of whose distances from two points being constant? [2010-I]
(a) Pair of straight lines (b) An ellipse
(c) A hyperbola (d) A parabola

21. A circle is drawn with the two foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the end of the diameter. What is the equation of the circle? [2010-I]
(a) $x^2 + y^2 = a^2 + b^2$ (b) $x^2 + y^2 = a^2 - b^2$
(c) $x^2 + y^2 = 2(a^2 + b^2)$ (d) $x^2 + y^2 = 2(a^2 - b^2)$

22. What are the equations of the directrices of the ellipse $25x^2 + 16y^2 = 400$? [2010-I]
(a) $3x \pm 25 = 0$ (b) $3y \pm 25 = 0$
(c) $x \pm 15 = 0$ (d) $y \pm 25 = 0$

23. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. Let $P = (1, 2)$ and $Q = (2, 1)$. Which one of the following is correct? [2010-I]
(a) Q lies inside C but outside E
(b) Q lies outside both C and E
(c) P lies inside both C and E
(d) P lies inside C but outside E.

24. A point P moves such that the difference of its distances from two given points $(c, 0)$ and $(-c, 0)$ is constant. What is the locus of the point P? [2010-II]
(a) Circle (b) Ellipse
(c) Hyperbola (d) Parabola

25. If the latusrectum of an ellipse is equal to half its minor axis, then what is its eccentricity? [2010-II]
(a) $\frac{1}{2}$ (b) $\sqrt{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

26. What are the points of intersection of the curve $4x^2 - 9y^2 = 1$ with its conjugate axis? [2011-I]
(a) $(1/2, 0)$ and $(-1/2, 0)$ (b) $(0, 2)$ and $(0, -2)$
(c) $(0, 3)$ and $(0, -3)$ (d) No such point exists

27. What is the sum of the focal distances of a point of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$? [2011-I]
(a) a (b) b
(c) $2a$ (d) $2b$

28. What is the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of the latus rectum? [2011-II]
(a) 9 square units (b) 12 square units
(c) 14 square units (d) 18 square units

29. What is the focal distance of any point $P(x_1, y_1)$ on the parabola $y^2 = 4ax$? [2011-II]
(a) $x_1 + y_1$ (b) $x_1 y_1$
(c) ax_1 (d) $a + x_1$

30. If the latus rectum of an ellipse is equal to half of the minor axis, then what is its eccentricity? [2012-I]
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

31. What is the eccentricity of the conic $4x^2 + 9y^2 = 144$? [2012-I]
(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{\sqrt{5}}{4}$
(c) $\frac{3}{\sqrt{5}}$ (d) $\frac{2}{3}$

32. The sum of the focal distances of a point on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is: [2012-II]
(a) 4 units (b) 6 units
(c) 8 units (d) 10 units

33. The eccentricity e of an ellipse satisfies the condition: [2012-II]
(a) $e < 0$ (b) $0 < e < 1$
(c) $e = 1$ (d) $e > 1$

34. The equation of the ellipse whose vertices are (± 5, 0) and foci at (± 4, 0) is [2013-I]
(a) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{9}+\frac{y^2}{25}=1$
(c) $\frac{x^2}{16}+\frac{y^2}{25}=1$ (d) $\frac{x^2}{25}+\frac{y^2}{16}=1$

35. The difference of focal distances of any point on a hyperbola is equal to [2013-I]
(a) latus rectum (b) semi-transverse axis
(c) transverse axis (d) semi-latus rectum

36. The foci of the hyperbola $4x^2 - 9y^2 - 1 = 0$ are [2013-II]
(a) $(\pm\sqrt{13}, 0)$ (b) $\left(\pm\frac{\sqrt{13}}{6}, 0\right)$
(c) $\left(0, \pm\frac{\sqrt{13}}{6}\right)$ (d) None of these

37. The axis of the parabola $y^2 + 2x = 0$ is [2013-II]
(a) $x = 0$ (b) $y = 0$
(c) $x = 2$ (d) $y = 2$

38. What is the sum of the major and minor axes of the ellipse whose eccentricity is 4/5 and length of latus rectum is 14.4 unit ? [2014-I]
(a) 32 units (b) 48 units
(c) 64 units (d) None of these

DIRECTIONS (Qs. 39-40): *For the next two (02) items that follow:*

Consider an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. [2014-I]

39. What is the area of the greatest rectangle that can be inscribed in the ellipse ?
(a) ab (b) $2ab$
(c) $ab/2$ (d) $\sqrt{ab}$

40. What is the area included between the ellipse and the greatest rectangle inscribed in the ellipse ?
(a) $ab(\pi - 1)$ (b) $2ab(\pi - 1)$
(c) $ab(\pi - 2)$ (d) None of these

41. What is the equation of parabola whose verted is at (0, 0) and focus is at (0, –2) ? [2014-I]
(a) $y^2 + 8x = 0$ (b) $y^2 - 8x = 0$
(c) $x^2 + 8y = 0$ (d) $x^2 - 8y = 0$

42. What is the length of the latus rectum of the ellipse $25x^2 + 16y^2 = 400$? [2014-II]
(a) 25/2 (b) 25/4
(c) 16/5 (d) 32/5

DIRECTIONS (Qs. 43-45) : *For the next three (03) items that follow:*

The line $2y = 3x + 12$ cuts the parabola $4y = 3x^2$. [2014-II]

43. Where does the line cut the parabola ?
(a) At (–2, 3) only
(b) At (4, 12) only
(c) At both (–2, 3) and (4, 12)
(d) Neither at (–2, 3) nor (4, 12)

44. What is the area enclosed by the parabola and the line ?
(a) 27 square unit (b) 36 square unit
(c) 48 square unit (d) 54 square unit

45. What is the area enclosed by the parabola, the line and the *Y*-axis in the first quadrant ?
(a) 7 square unit (b) 14 square unit
(c) 20 square unit (d) 21 square unit

46. The point on the parabola $y^2 = 4ax$ nearest to the focus has its abscissa [2015-I]
(a) $x = 0$ (b) $x = a$
(c) $x = \frac{a}{2}$ (d) $x = 2a$

47. The hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through the point $(3\sqrt{5}, 1)$ and the length of its latus rectum is $\frac{4}{3}$ units. The length of the conjugate axis is [2015-I]
(a) 2 units (b) 3 units
(c) 4 units (d) 5 units

48. Consider any point P on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ in the first quadrant. Let r and s represent its distances from (4, 0) and (–4, 0) respectively, then (r + s) is equal to [2015-II]
(a) 10 unit (b) 9 unit
(c) 8 unit (d) 6 unit

49. The eccentricity of the hyperbola $16x^2 - 9y^2 = 1$ is [2015-II]
(a) $\frac{3}{5}$ (b) $\frac{5}{3}$
(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

50. What is the equation of the hyperbola having latus rectum and eccentrieity 8 and $\frac{3}{\sqrt{5}}$ respectively? [2016-II]
(a) $\frac{x^2}{25}-\frac{y^2}{20}=1$ (b) $\frac{x^2}{40}-\frac{y^2}{20}=1$
(c) $\frac{x^2}{40}-\frac{y^2}{30}=1$ (d) $\frac{x^2}{30}-\frac{y^2}{25}=1$

51. If the ellipse $9x^2 + 16y^2 = 144$ intercepts the line $3x + 4y = 12$, then what is the length of the chord so formed? [2016-II]
(a) 5 units (b) 6 units
(c) 8 units (d) 10 units

52. What is the eccentricity of rectangular hyperbola? *[2016-II]*
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\sqrt{5}$ (d) $\sqrt{6}$

DIRECTIONS (Qs. 53-54) : *Consider the following for the next two (02) items that follow.*

Consider the parabola $y = x^2 + 7x + 2$ and the straight line $y = 3x - 3$.

53. What are the coordinates of the point on the parabola which is closest to the straight line? *[2016-II]*
(a) (0, 2) (b) (–2, – 8)
(c) (–7, 2) (d) (1, 10)

54. What is the shortest distance from the above point on the parabola to the line? *[2016-II]*
(a) $\frac{\sqrt{10}}{2}$ (b) $\frac{\sqrt{10}}{5}$
(c) $\frac{1}{\sqrt{10}}$ (d) $\frac{\sqrt{5}}{4}$

55. What is the equation of the ellipse having foci (±2, 0) and the eccentricity $\frac{1}{4}$? *[2017-I]*
(a) $\frac{x^2}{64}+\frac{y^2}{60}=1$ (b) $\frac{x^2}{60}+\frac{y^2}{64}=1$
(c) $\frac{x^2}{20}+\frac{y^2}{24}=1$ (d) $\frac{x^2}{24}+\frac{y^2}{20}=1$

56. A man running round a racecourse notes that the sum of the distances of two flag-posts from him is always 10 m and the distance between the flag-posts is 8 m. The area of the path he encloses is *[2017-II]*
(a) 18π square metres (b) 15π square metres
(c) 12π square metres (d) 8π square metres

57. The position of the point (1, 2) relative to the ellipse $2x^2 + 7y^2 = 20$ is *[2017-II]*
(a) outside the ellipse
(b) inside the ellipse but not at the focus
(c) on the ellipse
(d) at the focus

58. The equation of the ellipse whose centre is at origin, major axis is along x-axis with eccentricity $\frac{3}{4}$ and latus rectum 4 units is *[2017-II]*
(a) $\frac{x^2}{1024}+\frac{7y^2}{64}=1$ (b) $\frac{49x^2}{1024}+\frac{7y^2}{64}=1$
(c) $\frac{7x^2}{1024}+\frac{49y^2}{64}=1$ (d) $\frac{x^2}{1024}+\frac{y^2}{64}=1$

59. What is the equation of the ellipse whose vertices are (± 5, 0) and foci are at (± 4, 0)? *[2018-I]*
(a) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{16}+\frac{y^2}{9}=1$
(c) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (d) $\frac{x^2}{9}+\frac{y^2}{25}=1$

60. The sum of the focal distances of a point on an ellipse is constant and equal to the *[2019-I]*
(a) length of minor axis
(b) length of major axis
(c) length of latus rectum
(d) sum of the lengths of semi-major and semi-minor axes

61. The equation $2x^2 - 3y^2 - 6 = 0$ represents *[2019-I]*
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

62. The two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ intersect *[2019-I]*
(a) at two points on the line y = x
(b) only at the origin
(c) at three points one of which lies on y + x = 0
(d) only at (4a, 4a)

ANSWER KEY

1	(b)	8	(d)	15	(b)	22	(b)	29	(d)	36	(b)	43	(c)	50	(a)	57	(a)
2	(d)	9	(c)	16	(b)	23	(d)	30	(c)	37	(b)	44	(a)	51	(a)	58	(b)
3	(a)	10	(b)	17	(c)	24	(c)	31	(a)	38	(c)	45	(c)	52	(a)	59	(a)
4	(c)	11	(d)	18	(b)	25	(c)	32	(a)	39	(b)	46	(a)	53	(b)	60	(b)
5	(d)	12	(b)	19	(d)	26	(d)	33	(b)	40	(c)	47	(c)	54	(c)	61	(d)
6	(d)	13	(c)	20	(c)	27	(c)	34	(a)	41	(c)	48	(a)	55	(a)	62	(a)
7	(c)	14	(d)	21	(b)	28	(d)	35	(c)	42	(d)	49	(b)	56	(b)		

HINTS & SOLUTIONS

1. (b) Length of latus rectum of an ellipse is $\frac{2b^2}{a}$ where b is semi minor axis and a is semi-major axis. As given,

$\frac{2b^2}{a} = b$

$\Rightarrow \quad 2b = a \Rightarrow \frac{b}{a} = \frac{1}{2}$

We know that eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

2. (d) Equation of parabola is $y^2 = 2x$, so vertex lies at origin
So, co-ordinates of vertex are A (0, 0).
Let (x_1, y_1) be the co-ordinates of the point Q

$\therefore \quad y_1^2 = 2x_1$ ...(i)

and slope of $PQ = \frac{y_1 - 2}{x_1 - 2}$

[co-ordinates of P is (2,2) as given]

Also, slope of $AP = \frac{2-0}{2-0} = 1$

Since, PQ and AP are perpendicular to each other, hence, slope of AP × Slope of PQ = –1

So, $1 \times \left(\frac{y_1 - 2}{x_1 - 2}\right) = -1$

$\Rightarrow \quad y_1 - 2 = -x_1 + 2$

$\Rightarrow \quad x_1 + y_1 = 4 \Rightarrow x_1 = 4 - y_1$

Putting value of x_1 in equation (i)

$y_1^2 = 8 - 2y_1$ or $y_1^2 + 2y_1 - 8 = 0$

$\Rightarrow \quad y_1 = -4$ and 2

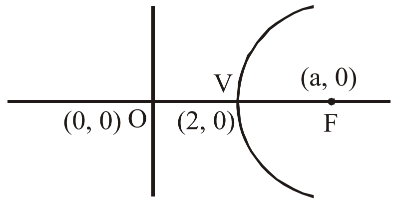
Hence, co-ordinates of point Q are (8, – 4).

So, required length $PQ = \sqrt{(8-2)^2 + (-4-2)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

3. (a) Focal distance of a point (x_1, y_1) on the parabola is $y^2 = 4ax$ is equal to its distance from directrix $x + a = 0$ is $x_1 + a$.
For $y^2 = 12x$; comparing with $y^2 = 4ax$.
$4a = 12 \Rightarrow a = 3$,
so, $\quad x_1 + 3 = 4$
$\Rightarrow \quad x_1 = 1$

4. (c) Vertex is (2, 0). Since, y-axis is the directrix of a parabola. Equation directrix is x = 0. So, axis of parabola is x-axis. Let the focus be (a, 0)



Distance of the vertex of a parabola from directrix
= its distance from focus
So, OV = VF $\Rightarrow (2-0)^2 = (a-2)^2$
$\Rightarrow \quad a^2 = 4a$ Þ $a = 4$
$\Rightarrow \quad$ Focus is (4, 0)

5. (d) The equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

The point for which $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 > 0$ is outside ellipse.

Since, at (a, 0), $1 + 0 - 1 = 0$
It lies on the ellipse.
At (0, b), $0 + 1 - 1 = 0$
It lies on the ellipse.
At (– a, 0), $1 + 0 - 1 = 0$
It lies on the ellipse.
At (a, b), $1 + 1 - 1 > 0$
So, the point (a, b) lies outside the ellipse.

6. (d) The parabola's vertex and focus lie on x-axis at points (a, 0) and (b, 0). Vertex and focus lie on the x-axis hence, the axis of parabola is x-axis. Equation of parabola Vertex whose is a point (x_1, y_1) then is
$(y - y_1)^2 = 4k(x - x_1)$
So, $y_1 = 0$ and $x_1 = a$ and k = distance between focus and vertex = (b – a) so the equation is
$(y - 0)^2 = 4(b - a) . (x - a)$
i.e., $\quad y^2 = 4(b - a)(x - a)$

7. (c) Length of latus rectum of a hyperbola is $\frac{2b^2}{a}$ where a is the half of the distance between two vertex of the hyperbola.

Latus rectum $= \frac{2b^2}{a} = \frac{10}{3}$

or, $b^2 = \frac{5a}{3}$ ...(1)

In case of hyperbola,
$b^2 = a^2(e^2 - 1)$ ...(2)

Putting value of b^2 from equation (1) and $e = \frac{\sqrt{13}}{3}$ in equation (2),

$$\frac{5a}{3} = a^2\left(\frac{13}{9} - 1\right)$$

or, $\frac{5a}{3} = \frac{4a^2}{9}$

$\Rightarrow \quad 4a^2 - 15a = 0$ or $a(4 - 15a) = 0$

$a \neq 0$, hence, $a = \frac{15}{4}$

Length of transverse axis $= 2a = 2 \times \frac{15}{4} = \frac{15}{2}$

8. (d) The given equation of circle is: $x^2 + y^2 = 9$ ....(1)

and ellipse is : $\frac{x^2}{4} + \frac{y^2}{8} = 1$ ....(2)

From eqn. (1) and (2), we get

$$\frac{x^2}{4} + \frac{9 - x^2}{8} = 1$$

$\Rightarrow \quad 2x^2 + 9 - x^2 = 8 \Rightarrow \quad x^2 = -1$

$\Rightarrow$ x is not real .

Hence, circle and ellipse do not intersect.

9. (c) The equation of ellipse is given as :

$$\frac{x^2}{a^2} + \frac{y^2}{7} = 1$$

Eccentricity is given by :

$$e = \sqrt{1 - \frac{7}{a^2}}$$

Therefore, foci of ellipse are $(\pm\, ae, 0)$ ie,

$$\left(\pm a\sqrt{1 - \frac{7}{a^2}}, 0\right)$$

Now, the equation of given hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25} \Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

So, $a = \frac{12}{5}$ and $b = \frac{9}{5}$

$$\therefore \quad e' = \sqrt{1 + \frac{81/25}{144/25}} = \sqrt{\frac{144 + 81}{144}} = \sqrt{\frac{225}{144}}$$

$$= \frac{15}{12}.$$

$\therefore$ Foci of hyperbola are $\left(\pm\frac{12}{5}.\frac{15}{12}, 0\right)$ ie, $(\pm 3, 0)$.

Since these foci coincides.

$$\Rightarrow \quad 3 = a\sqrt{1 - \frac{7}{a^2}}$$

$$\Rightarrow \quad \frac{3}{a} = \sqrt{1 - \frac{7}{a^2}}$$

$$\Rightarrow \quad \frac{9}{a^2} = 1 - \frac{7}{a^2}$$

$$\Rightarrow \quad \frac{16}{a^2} = 1 \quad \Rightarrow \quad a = 4$$

10. (b) Equation of the given conic is an equation of ellipse

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} (x \geq 0)$$

$\Rightarrow \quad A^2 = a^2 + \lambda$ and $B^2 = b^2 + \lambda$

Eccentricity, $e = \sqrt{1 - \frac{B^2}{A^2}} = \sqrt{1 - \frac{b^2 + \lambda}{a^2 + \lambda}}$

$$= \sqrt{\frac{a^2 + \lambda - b^2 - \lambda}{a^2 + \lambda}} = \sqrt{\frac{a^2 - b^2}{a^2 + \lambda}}$$

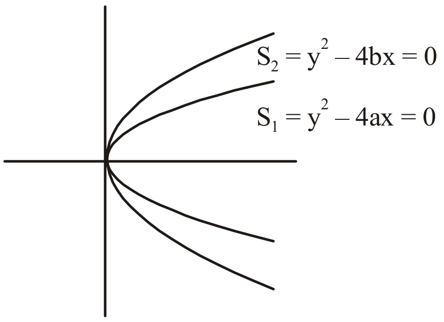
λ is in the denominator so, when λ increases, the eccentricity decreases.

11. (d) Given equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since, $b > a$

$\therefore$ Foci $= (0, \pm\, be)$

12. (b) If a and $b > 0$, then graphic representation would be as follows :



S_2 will contain S_1,

if latusrectum of $S_2 >$ latusrectum of S_1

$\Rightarrow 4b > 4a$

$\therefore b > a > 0$

13. (c) Given equation of hyperbola

$5x^2 - 4y^2 = k^2$

$$\Rightarrow \frac{x^2}{\frac{k^2}{5}} - \frac{y^2}{\frac{k^2}{4}} = 1$$

$\therefore\ a=\frac{k}{\sqrt{5}}$ and $b=\frac{k}{2}$

The eccentricity $\frac{3}{2}$ and foci at $(\pm 2,0)$ of

$5x^2-4y^2=k^2$

Then, $e=\frac{3}{2}$ and $\pm ae=2$

$\Rightarrow \frac{k}{\sqrt{5}}.\frac{3}{2}=2 \Rightarrow k=\frac{4}{3}\sqrt{5}$

14. (d) Since, Latusrectum of an ellipse $=\frac{2b^2}{a}$

and minor axis = 2b

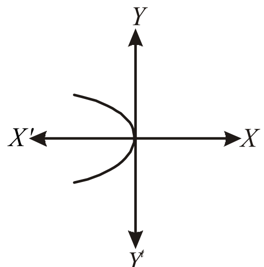
$\therefore\ b=\frac{2b^2}{a}\Rightarrow a=2b$

Also, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{b^2}{4b^2}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

15. (b) We know sum of the focal distances of a point on the ellipse is constant and is equal to the length of the major axis.
Thus, we know that, the sum of focal radii of any point on ellipse is equal to length of major axis.

16. (b) From the given information, we have an equation of the first degree which contains one arbitrary parameter.
Therefore the required equation represents a straight line.

17. (c) Given curve $y^2=-4ax$ which is one of the form of parabola



It is clear from the figure that curve lies in the second and third quadrants.

18. (b) Given ellipse is $\frac{x^2}{169}+\frac{y^2}{25}=1$

$\therefore\ e=\sqrt{1-\frac{25}{169}}=\frac{12}{13}$

Also, standard equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and eccentricity, $e=\sqrt{1-\frac{b^2}{a^2}}$

$\therefore\ \frac{12}{13}=\sqrt{1-\frac{b^2}{a^2}} \Rightarrow \frac{b^2}{a^2}=1-\frac{144}{169}=\frac{25}{169}$

$\Rightarrow \frac{b}{a}=\frac{5}{13}\Rightarrow \frac{a}{b}=\frac{13}{5}$

19. (d) Foci of an ellipse are (4, 0) and (– 4, 0). (Given)

$\therefore\ 2ae=8 \Rightarrow ae=4$

and semi minor axis is 3 $\therefore b=3$

We know that, $e=\sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow \left(\frac{4}{a}\right)^2=\left(1-\frac{9}{a^2}\right)\left(\because e=\frac{4}{a},\ b=3\right)$

$\Rightarrow \frac{16}{a^2}=\frac{a^2-9}{a^2}$

$\Rightarrow 16=a^2-9 \Rightarrow a^2=25\Rightarrow a=5$

Now, standard equation of an ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Thus, the equation of an ellipse is

$\frac{x^2}{25}+\frac{y^2}{9}=1$

which is satisfied by (5, 0). Hence the ellipse passes through (5, 0).

20. (c) We know that the locus of the difference of whose distances from two points being constant, is a hyperbola.

21. (b) Foci of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are given as $(ae, 0)$ and $(-ae, 0)$.
Since, two foci are at the end of the diameter
$\therefore$ Equation of circle, is

$(x-ae)(x+ae)+(y-0)(y-0)=0$

$\Rightarrow x^2-a^2e^2+y^2=0$

$\Rightarrow x^2+y^2-a^2\left(1-\frac{b^2}{a^2}\right)=0\ \left(\because e=\sqrt{1-\frac{b^2}{a^2}}\right)$

$\Rightarrow x^2+y^2-a^2+b^2=0$

$\Rightarrow x^2+y^2=a^2-b^2$

22. (b) Given equation of ellipse is $25x^2+16y^2=400$ which can be rewritten as

$\frac{x^2}{16}+\frac{y^2}{25}=1$

We know standard equation of ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

On comparing given equation with standard equation, we get $a = 4$ and $b = 5$ ($b > a$)

$\therefore$ Equations of the directrices are

$$y = \pm\frac{b}{e} = \pm\frac{5}{\sqrt{1-\frac{16}{25}}} = \pm\frac{25}{3} \qquad \left(\because e = \sqrt{1-\frac{a^2}{b^2}}\right)$$

$\Rightarrow \quad 3y \pm 25 = 0$

23. (d) Given equation of ellipse E is $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\Rightarrow \quad \frac{4x^2+9y^2}{36}=1 \Rightarrow \quad 4x^2+9y^2=36$

$\Rightarrow \quad 4x^2+9y^2-36=0 \qquad ...(1)$

and C : Eqn of circle is $x^2+y^2=9$

Which can be rewritten as

$x^2+y^2-9=0 \qquad ...(2)$

For a point $P(1, 2)$ we have

$4(1)^2+9(2)^2-36=40-36>0 \quad$ [from (1)]

and $1^2+2^2-9=5-9<0 \quad$ [from (2)]

$\therefore$ Point P lies outside of E and inside of C.

24. (c) When a point P moves such that the difference of its distances from two given points (c, 0) and (– c, 0) is constant, then the locus of the point P is hyperbola. It is the definition of hyperbola also.

25. (c) Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Length of minor axis = 2b

and length of latus rectum = $\frac{2b^2}{a}$

According to the question,

$$\frac{2b^2}{a} = b \Rightarrow 2b = a \Rightarrow 4b^2 = a^2$$

Now, eccentricity of ellipse

$$e = \frac{\sqrt{a^2-b^2}}{a}$$

$$e = \frac{\sqrt{4b^2-b^2}}{2b} = \frac{\sqrt{3}b}{2b} = \frac{\sqrt{3}}{2}$$

$\Rightarrow \quad e = \frac{\sqrt{3}}{2}$

26. (d) The given equation of curve is

$4x^2 - 9y^2 = 1$

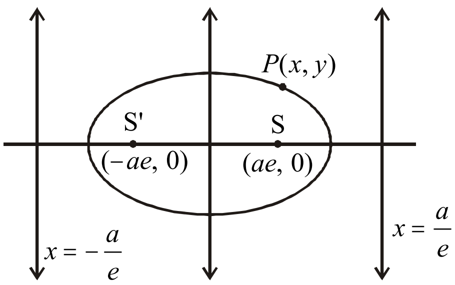
$\Rightarrow \quad \frac{x^2}{1/4}-\frac{y^2}{1/9}=1$

This is an equation of a hyperbola which does not intersect with conjugate axes.

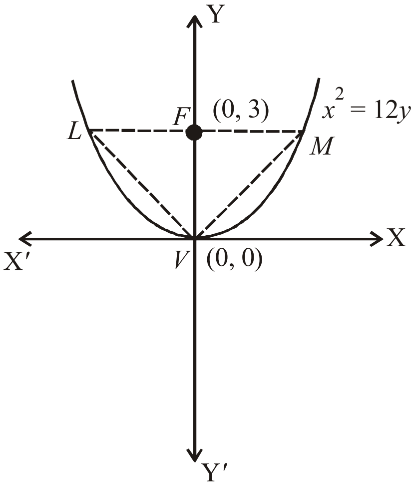
Hence, no point of intersection exists.

27. (c) By definition of ellipse, we have

PS + PS' = $2a$



28. (d)



Given parabola is $x^2 = 12y$ which is of the form $x^2 = 4ay$.

$\Rightarrow 4a = 12 \Rightarrow a = 3$

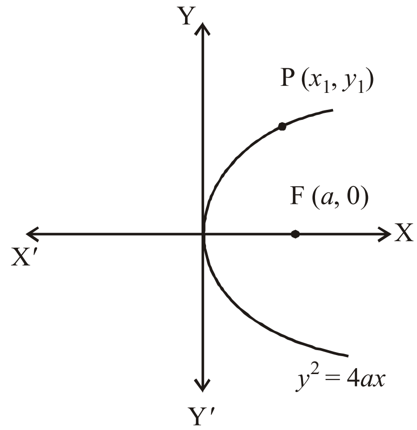
Now, LM is the latus rectum whose length = $4a = 4 \times 3 = 12$

So, area of $\Delta\, LMV = \frac{1}{2} \times LM \times VF$.

$= \left(\frac{1}{2}\times 12\times 3\right)$ sq. unit

= 18 square unit

29. (d)



Focal-Distance :
The distance between a point on a parabola and its focus is called its focal distance.
Let $F(a, 0)$ be a focus on parabola $y^2 = 4ax$.
Since, $P(x_1, y_1)$ on $y^2 = 4ax$

$\therefore \quad y_1^2 = 4ax_1 \quad ...(1)$

Now, Focal distance

$$PF = \sqrt{(a-x_1)^2 + y_1^2}$$
$$= \sqrt{a^2 + x_1^2 - 2ax_1 + y_1^2}$$
$$= \sqrt{a^2 + x_1^2 - 2ax_1 + 4ax_1}$$

(from 1)

$$= \sqrt{a^2 + x_1^2 + 2ax_1}$$
$$= \sqrt{(a+x_1)^2} = a + x_1$$

Hence, focal distance $= a + x_1$.

30. (c) Length of minor axis = 2b and latus rectum $= \frac{2b^2}{a}$

According to given condition $\frac{2b^2}{a} = b$

$\Rightarrow 2b = a$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{4b^2}} = \sqrt{\frac{3}{4}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

31. (a) Given equation can be written as

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

This is an ellipse.
$\Rightarrow a^2 = 36, b^2 = 16$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{36}} = \sqrt{\frac{20}{36}} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

32. (a) Given equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

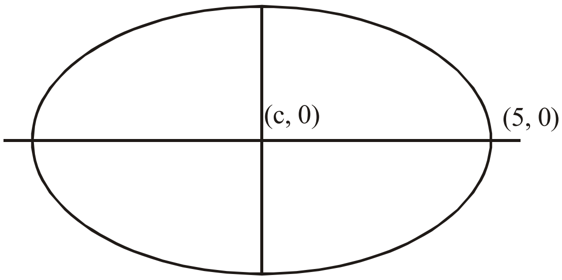
$$\Rightarrow \quad \frac{x^2}{(2)^2} + \frac{y^2}{(3)^2} = 1$$

$\Rightarrow \quad a = 2$ and $b = 3$
Length of major axis = 2a = 4
Since, we have
Sum of the focal distances of a point on ellipse = length of major axis.
$\therefore$ Required Ans = 4 units.

33. (b) The eccentricity e of an ellipse satisfies the condition : $0 < e < 1$.

34. (a) Since vertices of an ellipse are $(\pm a, 0)$ and foci are $(\pm ae, 0)$
$\therefore \quad a = 5$ and $ae = 4$

$\Rightarrow e = \frac{4}{5}$



We know, $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{16}{25} = 1 - \frac{b^2}{25}$

$\Rightarrow \frac{b^2}{25} = \frac{9}{25} \Rightarrow b^2 = 9$

Hence, Required equation is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

35. (c) Transverse axis
Proof: The focal distance of any point (x, y) on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$e|x| - a$ from the nearer focus
$e|x| + a$ from the farther focus.
Difference $= (e|x| + a) - (e|x| - a) = 2a$
= length of transverse axes.

36. (b) $4x^2 - 9y^2 = 1$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} - \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$$

eccentricity, $e = \sqrt{1 + \frac{\left(\frac{1}{3}\right)^2}{\left(\frac{1}{2}\right)^2}} = \frac{\sqrt{13}}{3}$

foci $= \left(\pm\frac{1}{2} \times \frac{\sqrt{13}}{3}, 0\right) = \left(\pm\frac{\sqrt{13}}{6}, 0\right)$

37. (b) $y^2 + 2x = 0 \Rightarrow y^2 = -2x$, which is in the form $y^2 = -4ax$. Therefore axis of parabola is x-axis and its equation is $y = 0$.

38. (c) Let 2a and 2b be the length of major and minor axis respectively.

$$\sqrt{1-\frac{b^2}{a^2}} = \frac{4}{5}$$

$$\frac{b^2}{a^2} = \frac{9}{25} \quad \text{...(i)}$$

Also, $\frac{2b^2}{a} = 14.4$

$\frac{b^2}{a} = 7.2, b^2 = 7.2\,a$

Putting value of $\frac{b^2}{a}$ in equation (i)

$\frac{7.2}{a} = \frac{9}{25} \Rightarrow a = 20$

$b^2 = 7.2 \times 20 = 144$

$b = 12$

the sum of the major and minor axes

$= 2a + 2b$

$= 2(a + b) = 2(20 + 12) = 64$ units

39. (b) Given equation of ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let A (a cos θ, b sin θ) be any point on ellipse (1st quadrant)

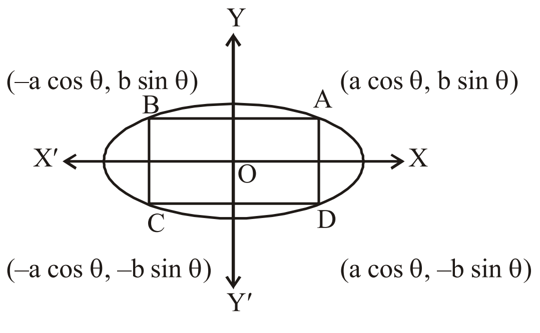
Coordinate of B = [a cos (π – θ), b sin (π – θ)]

= (– a cos θ, b sin θ) (2nd quadrant)

Coordinate of C = [a cos (π + θ), b sin (π + θ)] (3rd quadrant)

Coordinate of D = [a cos (2π – θ), b sin (2π – θ)]

= (a cos θ, – b sin θ) (4th quadrant)



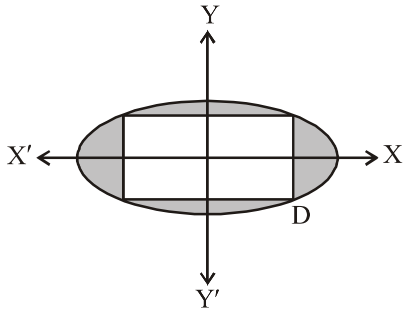
Area of the rectangle ABCD

= (a cos θ + a cos θ) (b sin θ + b sin θ)

= 2a cos θ × 2b sin θ = 2ab sin 2θ

= 2ab × 1 = 2ab

40. (c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Area of ellipse is π ab

Area of shaded region = Area of ellipse – Area of rectangle

= π ab – 2ab = ab (π – 2)

41. (c) Focus is (0, – 2)

a = – 2 and parabola is along y-axis downward

$x^2 = 4ay$

$x^2 = -8y$

or $x^2 + 8y = 0$

42. (d) Equation of ellipse is $25x^2 + 16y^2 = 400$

$\frac{x^2}{16} + \frac{y^2}{25} = 1$

Here, $a^2 = 16$ and $b^2 = 25$

∴ Length of latus rectum $= \frac{2a^2}{b} = \frac{2 \times 16}{5} = \frac{32}{5}$

43. (c) Equation of line

$2y = 3x + 12$ ...(i)

Equation of parabola

$4y = 3x^2$ ...(ii)

From eqs. (i) and (ii), we get

$2(3x + 12) = 3x^2$

$3x^2 - 6x - 24 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

∴ $x = 4$

and $x = -2$

Now putting the value of x in eqn (ii)

We get y = 12 and y = 3

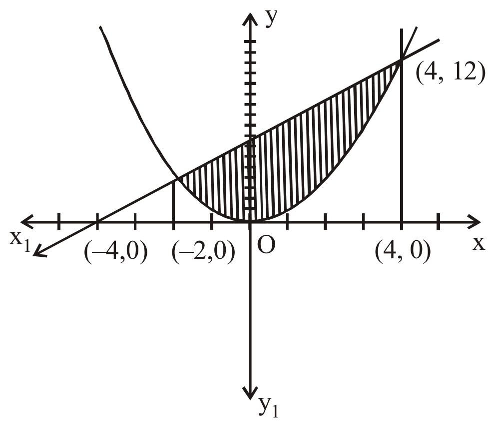
Thus, the points (–2, 3) and (4, 12)

44. (a) Equation of line $2y = 3x + 12$, $y = \frac{3x+12}{2}$

Equation of parabola $4y = 3x^2$, $y = \frac{3x^2}{4}$

$$= \int_{-2}^{4} \left[\frac{3x+12}{2} - \frac{3x^2}{4}\right] dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$$



$$= \frac{1}{2}\left[\left\{\frac{3(4)^2}{2}+12(4)\right\}-\left\{\frac{3(-2)^2}{2}+12(-2)\right\}\right]$$

$$-\frac{3}{4}\left[\frac{4^3}{3}-\frac{(-2)^3}{3}\right]$$

$$= \frac{1}{2}[24+48-6+24]-\frac{3}{4}\left[\frac{64+8}{3}\right]$$

$$= \frac{1}{2}\times 90-18 = 27 \text{ sq. units.}$$

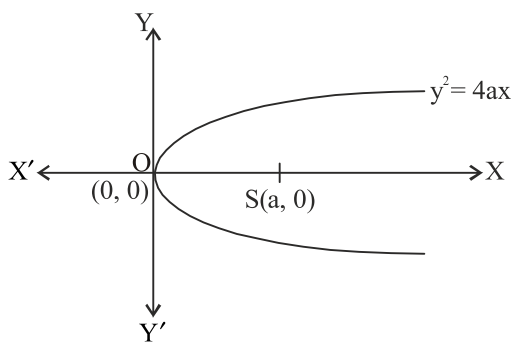
45. (c) Equation of line $2y = 3x + 12$ and equations of parabola $4y = 3x^2$

$$= \int_0^4 \left(\frac{3x+12}{2}-\frac{3x^2}{4}\right)dx = \left(\frac{3}{4}x^2+6x-\frac{x^3}{4}\right)_0^4$$

$= 3 \times 4 + 24 - 16 = 36 - 16 = 20$ sq. units.

$\therefore$ Area enclosed by the parabola, the line and the y axis in first quadrant = 20 sq. units

46. (a) Here, 'S' represents focus O(0, 0) is a point which is on parabola $y^2 = 4ax$ and nearest to focus $(a, 0)$



$\therefore$ abscissa of O (0, 0) is x = 0

$\therefore$ Option (a) is correct.

47. (c) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Hyperbola passes through $(3\sqrt{5}, 1)$

$$\therefore \quad \frac{(3\sqrt{5})^2}{a^2}-\frac{1}{b^2}=1$$

$$\frac{45}{a^2}-\frac{1}{b^2}=1 \qquad \text{...(i)}$$

Now length of latus rectum $= \frac{2b^2}{a}$

$$\Rightarrow \quad \frac{4}{3}=\frac{2b^2}{a}$$

$$\Rightarrow \quad \frac{2}{3}=\frac{b^2}{a} \Rightarrow a = \frac{3b^2}{2} \qquad \text{...(ii)}$$

Putting the value of '*a*' from equation (ii) in equation (i),

$$\Rightarrow \quad \frac{45\times 4}{9b^4}-\frac{1}{b^2}=1$$

$$\Rightarrow \quad \frac{20}{b^4}-\frac{1}{b^2}=1$$

$20 - b^2 = b^4$

$b^4 + b^2 - 20 = 0$

$b^4 + 5b^2 - 4b^2 - 20 = 0$

$b^2 (b^2 + 5) - 4(b^2 + 5) = 0$

$(b^2 - 4)(b^2 + 5) = 0$

$b^2 = 4, b^2 = -5$

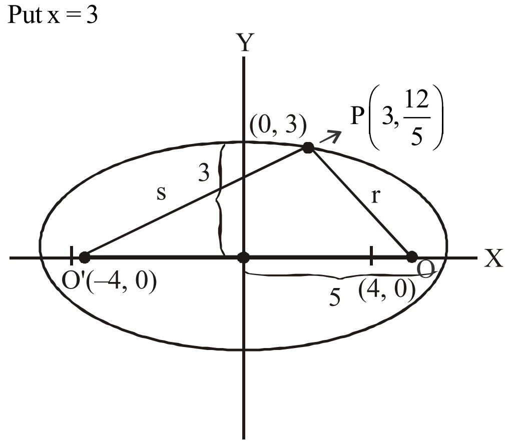
$\therefore \quad b^2 = 4 \Rightarrow b = 2$

Now length of conjugate axis = 2b = 2(2) = 4

$\therefore$ Option (c) is correct.

48. (a) $\frac{x^2}{25}+\frac{y^2}{9}=1$

Put x = 3



$$\frac{9}{25}+\frac{y^2}{9}=1$$

$$y=\frac{12}{5}$$

P = (3, 12/5)

$$r = PO = \sqrt{(4-3)^2+\left(0-\frac{12}{5}\right)^2}$$

= 13/5

$$S = PO' = \sqrt{(-4-3)^2+\left(0-\frac{12}{5}\right)^2}$$

= 37/5

$$r+s=\frac{13}{5}+\frac{37}{5}=\frac{50}{5}=10 \text{ unit}$$

49. (b) $\because 16x^2-9y^2=1$

or $\frac{x^2}{\left(\frac{1}{4}\right)^2}-\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$

Comparing with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore a^2=\frac{1}{16}$

& $b^2=\frac{1}{9}$

$\because e=\sqrt{1+\frac{b^2}{a^2}}$

$\therefore e=\sqrt{1+\frac{1/9}{1/16}}$

$\Rightarrow \quad e=\frac{5}{3}$

50. (a) Let the equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Latus rectum = 8 = $\frac{2b^2}{a}$ $\Rightarrow b^2 = 4a$ ...... (i)

Also, $b^2 = a^2(e^2-1)$

$\Rightarrow$ $4a = a^2(e^2-1)$ [From (i)]

$\Rightarrow$ $4a = a^2\left[\left(\frac{3}{\sqrt{5}}\right)^2-1\right]$

$\Rightarrow$ a = 5 & $b^2 = 20$

$\therefore$ Equation is $\frac{x^2}{25}-\frac{y^2}{20}=1$

51. (a) Here,

$9x^2+16y^2=144$ and $3x+4y=12$

$$\Rightarrow x=\frac{12-4y}{3}$$

So, $9\left(\frac{12-4y}{3}\right)^2+16y^2=144$

On solving we get, $y = 0, 3$

For $y = 0; x = 4$

For $y = 3; x = 0$

$\Rightarrow$ Length of chord $=\sqrt{(0-3)^2+(4-0)^2}=\sqrt{9+16}$

$=\sqrt{25}=5$ units

52. (a) Here, $b^2=a^2(e^2-1)$

For rectangular hyperbola : $a = b$

$\Rightarrow b^2=b^2(e^2-1)$

$\Rightarrow e^2-1=1$

$\Rightarrow e^2=2 \Rightarrow e=\pm\sqrt{2}$

For hyperbola, e > 1.

Hence, e = $\sqrt{2}$

53. (b) Parabola Eq : $y=x^2+7x+2$

Line eq. : $y = 3x-3$

Since all the points given in the options lie on the parabola.

Thus we will calculate the distance from the given line to these points :

for (0, 2) : distance = $=\frac{|3(0)-(2)-3|}{\sqrt{(3)^2+(-1)^2}}=\frac{5}{\sqrt{10}}$

for (–2, –8) : distance $=\frac{|3(-2)-(-8)-3|}{\sqrt{10}}=\frac{1}{\sqrt{10}}$

for (–7, 2) : distance $=\frac{|3(-7)-2-3|}{\sqrt{10}}=\frac{26}{\sqrt{10}}$

for (1, 10) : distance $=\frac{|3(1)-10-3|}{\sqrt{10}}=\frac{10}{\sqrt{10}}$

$\therefore$ (–2, –8) is the given point.

54. (c)

55. (a) foci: $(\pm 2, 0)$, $e=\frac{1}{4}$

$c = 2, e=\frac{1}{4}=\frac{c}{a}\Rightarrow\frac{2}{a}=\frac{1}{4}\Rightarrow a=8$

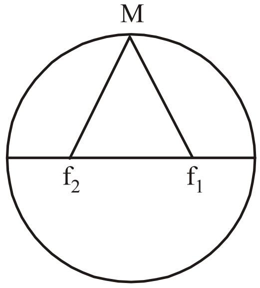
We know, $a^2-b^2=c^2$

$\Rightarrow b^2=a^2-c^2=8^2-2^2=64-4=60$

Eqn of ellipse $\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\Rightarrow \frac{x^2}{64}+\frac{y^2}{60}=1$

56. (b) Given that sum of the distances of two flag-posts from him is always 10m. So, the race course is in the shape of ellipse.
From the given figure, $Mf_1 + Mf_2 = 10$



Let 'a' be the length of semi major axis and 'b' be the length of semi minor axis.
$\therefore\ Mf_1 + Mf_2 = 10 \Rightarrow 2a = 10 \Rightarrow a = 5$
Also, $f_1f_2 = 8$
Let $f_1 = (C, 0)$ and $f_2 = (-C, 0)$.
$\therefore\ f_1f_2 = 8 \Rightarrow 2C = 8 \Rightarrow C = 4$
We know, $a^2 = b^2 + c^2 \Rightarrow 5^2 = b^2 + 4^2 \Rightarrow b^2 = 25 - 16 = 9 = 3^2$
$\therefore\ b = 3.$
Area of the racecourse $= \pi ab = \pi \times 5 \times 3 = 15\pi$ sq. m

57. (a) Given ellipse, $2x^2 + 7y^2 = 20$.
Given point, (1, 2)
$2(1)^2 + 7(2)^2 - 20 = 2 + 28 - 20 = 38 - 20 = 18 > 0.$
$\therefore$ Point is outside the ellipse.

58. (b) Given, $b^2 = 2a,\ c^2 = \left(\frac{3}{4}\right)^2 a^2 = \frac{9}{16}a^2$
We know, $a^2 = b^2 + c^2$
So,

$$a^2 = 2a + \frac{9}{16}a^2 \Rightarrow 16a^2 = 32a + 9a^2 \Rightarrow 7a^2 = 32a$$

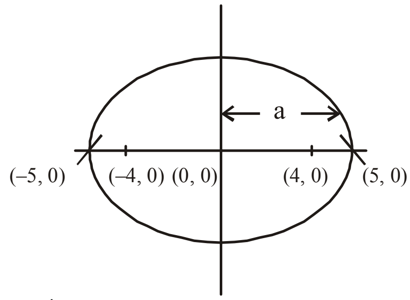
$$\Rightarrow a = \frac{32}{7}$$

$$\therefore b^2 = \frac{64}{7}$$

Equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\left(\frac{32}{7}\right)^2} + \frac{y^2}{\frac{64}{7}} = 1 \Rightarrow \frac{49x^2}{1024} + \frac{7y^2}{64} = 1.$$

59. (a) Given,
vertices, (5, 0) and (–5, 0)
foci (4,0) and (–4, 0)



i.e., $ae = 4$

$$\Rightarrow e = \frac{4}{a} = \frac{4}{5}.$$

we know, $b^2 = a^2(1 - e^2)$

$$= 5^2\left(1 - \left(\frac{4}{5}\right)^2\right)$$

$$= 25\left(\frac{25-16}{25}\right) = 9$$

$\therefore\ a^2 = 25, b^2 = 9$

Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1.$

61. (d) Given equation, $2x^2 - 3y^2 - 6 = 0$
$\Rightarrow 2x^2 - 3y^2 = 6$

$$\Rightarrow \frac{x^2}{3} - \frac{y^2}{2} = 1$$

This equation represents hyperbola.

62. (a) The parabolas $y^2 = 4ax$ and $x^2 = 4ay$
They intersect at (0, 0) and (4a, 4a)
These points lie on $y = x$

TRIGONOMETRY — Ratio & Identity, Trigonometric Equations

11

1. The difference of two angles is 1°; the circular measure of their sum is 1. What is the smaller angle in circular measure ?

(a) $\left[\frac{180}{\pi}-1\right]$ (b) $\left[1-\frac{\pi}{180}\right]$

(c) $\frac{1}{2}\left[1-\frac{\pi}{180}\right]$ (d) $\frac{1}{2}\left[\frac{180}{\pi}-1\right]$ *[2006-I]*

2. A positive acute angle is divided into two parts whose tangents are $\frac{1}{8}$ and $\frac{7}{9}$. What is the value of this angle ?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$ *[2006-I]*

3. If an angle B is complement of an angle A, what are the greatest and least values of cos A cos B respectively ?

(a) $0, -\frac{1}{2}$ (b) $\frac{1}{2}, -1$

(c) 1, 0 (d) $\frac{1}{2}, -\frac{1}{2}$ *[2006-I]*

4. Three expressions are given below :

$Q_1 = \sin(A+B) + \sin(B+C) + \sin(C+A)$

$Q_2 = \cos(A-B) + \cos(B-C) + \cos(C-A)$

$Q_3 = \sin A(\cos B + \cos C) + \sin B(\cos C + \cos A) + \sin C(\cos A + \cos B)$

Which one of the following is correct ?

(a) $Q_1 = Q_2$

(b) $Q_2 = Q_3$

(c) $Q_1 = Q_3$

(d) All the expressions are different *[2006-I]*

5. For what values of x is the equation $2\sin\theta = x + \frac{1}{x}$ valid ?

(a) $x = \pm 1$ (b) All real values of x

(c) $-1 < x < 1$ (d) $x > 1$ and $x < -1$ *[2006-I]*

6. If $\sin(\pi\cos x) = \cos(\pi\sin x)$, then what is one of the values of sin 2x ?

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$

(c) $-\frac{3}{4}$ (d) -1 *[2006-I]*

7. In a triangle ABC, if cos A = cos B cos C, what is the value of tan A – tan B – tan C ?

(a) 0 (b) – 1

(c) 1 + tan A tan B tan C (d) tan A tan B tan C – 1 *[2006-I]*

8. What is the value of $\sqrt{3}$ cosec 20° – sec 20° ?

(a) 4 (b) 2

(c) 1 (d) 0 *[2006-I]*

9. Let $45° \le \theta < 90°$. If $\tan\theta + \cot\theta = (\tan\theta)^i + (\cot\theta)^i$ for some $i \ge 2$, then what is the value of $\sin\theta + \cos\theta$?

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{(\sqrt{3}+1)}{2}$ (d) $\frac{2}{(\sqrt{3}+1)}$ *[2006-II]*

10. Given that $\tan\theta = m \ne 0$, $\tan 2\theta = n \ne 0$ and $\tan\theta + \tan 2\theta = \tan 3\theta$, then which one of the following is correct ?

(a) m = n (b) m + n = 1

(c) m + n = 0 (d) mn = – 1 *[2006-II]*

11. Let A and B be obtuse angles such that $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$. What is the value of sin (A + B) ?

(a) $-\frac{63}{65}$ (b) $-\frac{33}{65}$

(c) $\frac{33}{65}$ (d) $\frac{63}{65}$ *[2006-II]*

12. If $\tan^2 B = \frac{1-\sin A}{1 \quad \sin A}$ then what is the value of A + 2B ?
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ *[2006-II]*

13. Given that cos 20° – sin 20° = p, then what is the value of sin 40° ?
(a) $1 - p^2$ (b) $1 + p^2$
(c) p^2 (d) $p^2 - 1$ *[2006-II]*

14. Given that p = tan α + tan β, and q = cot α + cot β; then what is $\left(\frac{1}{p} - \frac{1}{q}\right)$ equal to ?
(a) cot (α – β) (b) tan (α – β)
(c) tan (α + β) (d) cot (α + β) *[2006-II]*

15. A is a certain positive acute angle which satisfies the following equation :
Number of degrees in A + Number of radians in A = (180 + π)/3
What is the angle A ?
(a) 20° (b) 40°
(c) 60° (d) 80° *[2006-II]*

16. If $\sin^3\theta + \cos^3\theta = 0$, then what is the value of θ ?
(a) $\frac{-\pi}{4}$ (b) 0
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$ *[2006-II]*

17. What is the value of

$$\frac{\text{cosec}(\pi+\theta)\cot\{(9\pi/2-\theta)\}\text{cosec}^2(2\pi-\theta)}{\cot(2\pi-\theta)\sec^2(\pi-\theta)\sec\{(3\pi/2)+\theta\}}$$

(a) 0 (b) 1
(c) –1 (d) ∞ *[2007-I]*

18. What is the value of
sin(A + B) sin (A – B) + sin (B + C) sin (B – C) + sin (C + A) sin (C – A)? *[2007-I]*
(a) 0 (b) sinA + sin B + sin C
(c) cosA + cos B + cos C (d) 1

19. Given that tan α = m/(m + 1), tan β = 1/(2m+ 1), then what is the value of α + β ?
(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$ *[2007-I]*

20. If x = r sin θ cos φ, y = r sin θ sin φ and z = r cos θ, then $x^2 + y^2 + z^2$ is independent of which of the following?
(a) r only (b) r, φ
(c) θ, φ (d) r, θ *[2007-I]*

21. What is the minimum value of cos θ + cos2θ?
(a) –2 (b) $-\frac{9}{8}$
(c) 0 (d) $-\frac{9}{16}$ *[2007-I]*

22. If 3 tan θ + 4 = 0, where (π/2) < θ < π, then what is the value of 2cotθ –5 cos θ + sin θ?
(a) $-\frac{53}{10}$ (b) $\frac{7}{10}$
(c) $\frac{23}{10}$ (d) $\frac{37}{10}$ *[2007-I]*

23. What is the value of cosec (13π/12)?
(a) $\sqrt{6} \quad \sqrt{2}$ (b) $-\sqrt{6} \quad \sqrt{2}$
(c) $\sqrt{6} - \sqrt{2}$ (d) $-\sqrt{6} - \sqrt{2}$ *[2007-I]*

24. What is the value of (secθ – cosθ) (cosecθ – sinθ) (cotθ + tanθ)?
(a) 1 (b) 2
(c) sin θ (d) cos θ *[2007-I]*

25. If $\alpha + \beta \quad \frac{\pi}{2}$ and β + γ = α ; then which one of the following is correct?
(a) 2 tan β + tan γ = tan α
(b) tan β + 2tan γ = tan α
(c) tan β + 2tan γ = tan α
(d) 2 (tan β + tan γ) = tan α *[2007-I]*

26. What is the value of $\frac{(\cos 10° + \sin 20°)}{(\cos 20° - \sin 10°)}$?
(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) $-\sqrt{3}$ *[2007-I]*

27. If α and β are such that tan α = 2 tan β, then what is sin (α + β) equal to ?
(a) 1 (b) 2 sin (α – β)
(c) sin (α – β) (d) 3 sin (α – β) *[2007-II]*

28. What is the value of
cos 306° + cos 234° + cos 162° + cos 18° ?
(a) 1 (b) –1
(c) 0 (d) 2 *[2007-II]*

29. Let ABCD be a square and let P be a point on AB such that AP : PB = 1 : 2. If ∠APD = θ, then what is the value of cos θ ?
(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{10}}$ (d) $\frac{2}{\sqrt{5}}$ *[2007-II]*

30. If $\cos 3A = \frac{1}{2}$, then how many values can sin A assume? $(0 < A < 360°)$
(a) 3 (b) 4
(c) 5 (d) 6 *[2007-II]*

31. Let $0° < \theta < 45°$. Which one of the following is correct?
(a) $\sin^2\theta + \cos^6\theta = \sin^6\theta + \cos^2\theta$
(b) $\operatorname{cosec}^2\theta + \cot^6\theta = \operatorname{cosec}^6\theta + \cot^2\theta$
(c) $\sin^2\theta - \cos^4\theta = \sin^4\theta + \cos^2\theta$
(d) $\operatorname{cosec}^2\theta + \cot^4\theta = \operatorname{cosec}^4\theta + \cot^2\theta$ *[2007-II]*

32. If sin A = sin B and cos A = cos B, then which one of the following is correct ?
(a) $B = n\pi + A$
(b) $A = 2n\pi - B$
(c) $A = 2n\pi + B$
(d) $B = n\pi - A$ (n is an integer) *[2007-II]*

33. If $\alpha = \frac{\pi}{8}$, what is the value of $\cos\alpha \cos 2\alpha \cos 4\alpha$?
(a) 0 (b) $\frac{1}{4}$
(c) 8 (d) 4 *[2007-II]*

34. What is the value of cot (– 870°)?
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$ *[2007-II]*

35. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*

Let $X = \{\theta \in [0, 2\pi] : \sin\theta = \cos\theta\}$

Assertion (A) : The number of elements in X is 2.
Reason (R) : sin θ and cos θ are both negative both in second and fourth quadrants.
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2008-I]*

36. What is the measure of the angle 114° 35′ 30″ in radian?
(a) 1 rad (b) 2 rad
(c) 3 rad (d) 4 rad *[2008-I]*

37. What is the value of $\left(\sin 22\frac{1°}{2} + \cos 22\frac{1°}{2}\right)^4$?
(a) $\frac{3+2\sqrt{2}}{2}$ (b) $\frac{1+2\sqrt{2}}{2}$
(c) $\frac{3\sqrt{2}+2}{2}$ (d) 1 *[2008-I]*

38. Which one of the following is correct?
$\left(1+\cos 67\frac{1°}{2}\right)\left(1+\cos 112\frac{1°}{2}\right)$ is
(a) an irrational number and is greater than 1
(b) a rational number but not an integer
(c) an integer
(d) an irrational number and is less than 1 *[2008-I]*

39. If $\sin 2A = \frac{4}{5}$, then what is the value of $\tan A \left(0 \le A \le \frac{\pi}{4}\right)$?
(a) 1 (b) – 1
(c) $\frac{1}{2}$ (d) 2 *[2008-I]*

40. What is the value of $\frac{\cos 10° - \sin 10°}{\cos 10° + \sin 10°}$?
(a) tan 35° (b) tan 10°
(c) $\frac{1}{\sqrt{2}}$ (d) 1 *[2008-I]*

41. For what value of x does the equation $4\sin x + 3\sin 2x - 2\sin 3x + \sin 4x = 2\sqrt{3}$ hold ?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ *[2008-I]*

42. Which one of the following pairs is not correctly matched?
(a) $\sin 2\pi$: $\sin(-2\pi)$
(b) tan 45° : tan (– 315°)
(c) $\cot(\tan^{-1} 0.5)$: $\tan(\cos^{-1} 0.5)$
(d) tan 420° : tan (– 60°) *[2008-I]*

43. What is the value of $\sin\left(\frac{5\pi}{12}\right)$?
(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{\sqrt{6}+\sqrt{2}}{4}$
(c) $\frac{\sqrt{3}+\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}+1}{2}$ *[2008-I]*

44. What is the correct sequence of the following values?
1. $\sin\left(\frac{\pi}{12}\right)$ 2. $\cos\left(\frac{\pi}{12}\right)$
3. $\cot\left(\frac{\pi}{12}\right)$
Select the correct answer using the code given below
(a) 3 > 2 > 1 (b) 1 > 2 > 3
(c) 1 > 3 > 2 (d) 3 > 1 > 2 *[2008-I]*

45. What is the value of cos 15°?
(a) $\frac{1}{2}\left(\sqrt{2-\sqrt{3}}\right)$ (b) $\frac{1}{2}\left(\sqrt{2+\sqrt{3}}\right)$
(c) $\sqrt{2}+\sqrt{3}$ (d) $\sqrt{2}-\sqrt{3}$ *[2008-II]*

46. How many values of θ between 0° and 360° satisfy $\tan\theta = k \neq 0$, where k is a given number? *[2008-II]*
(a) 1 (b) 2
(c) 4 (d) Many

47. If $\sin x + \sin y = a$, $\cos x + \cos y = b$, then what is the value of $\cos(x - y)$? *[2008-II]*
(a) $a^2 - 1$ (b) $b^2 - 1$
(c) $\frac{1}{2}\left(a^2 + b^2 - 2\right)$ (d) $\frac{1}{2}\left(a^2 + b^2\right)$

48. What is $\sqrt{2\ \ \sqrt{2\ \ \sqrt{2\ \ 2\cos 4A}}}$ equal to? *[2008-II]*
(a) $\cos A$ (b) $\cos(2A)$
(c) $2\cos(A/2)$ (d) $\sqrt{2\cos A}$

49. The equation $\tan^2\phi + \tan^6\phi = \tan^3\phi \cdot \sec^2\phi$ is
(a) identity for only one value of ϕ
(b) not an identity
(c) identity for all values of ϕ
(d) None of the above

50. If $\sec A + \tan A = p$, then what is the value of $\sin A$? *[2008-II]*
(a) $\frac{p^2 - 1}{p^2 + 1}$ (b) $\frac{p^2 + 1}{p^2 - 1}$
(c) 1 (d) None of these

51. What is the value of $\tan(-1575°)$? *[2009-I]*
(a) 1 (b) 1/2
(c) 0 (d) –1

52. For which acute angle θ, $\mathrm{cosec}^2\theta = 3\sqrt{3}\cot\theta - 5$? *[2009-I]*
(a) $\frac{5\pi}{12}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

53. If $\tan^2\theta = 2\tan^2\phi + 1$, then which one of the following is correct? *[2009-I]*
(a) $\cos(2\theta) = \cos(2\phi) - 1$
(b) $\cos(2\theta) = \cos(2\phi) + 1$
(c) $\cos(2\theta) = \left[\cos(2\phi) - 1\right]/2$
(d) $\cos(2\theta) = \left[\cos(2\phi) + 1\right]/2$

54. What is the value of $1 - \sin 10° \sin 50° \sin 70°$? *[2009-I]*
(a) 1/8 (b) 3/8
(c) 5/8 (d) 7/8

55. The sines of two angles of a triangle are equal to 5/13 and 99/101. What is the cosine of the third angle? *[2009-I]*
(a) 255/1313 (b) 265/1313
(c) 275/1313 (d) 770/1313

56. After subtending an angle of 1000° from its inital position, the revolving line will be situated in which one of the following quadrants? *[2009-I]*
(a) First quadrant (b) Second quadrant
(c) Third quadrant (d) Fourth quadrant

57. One radian is approximately equal to which one of the following? *[2009-I]*
(a) 90° (b) 180°
(c) 57° (d) 47°

58. If $\cot(x + y) = 1/\sqrt{3}$, $\cot(x - y) = \sqrt{3}$ then what are the smallest positive values of x and y respectively? *[2009-I]*
(a) 45°, 30° (b) 30°, 45°
(c) 15°, 60° (d) 45°, 15°

59. $x = \sin\theta\cos\theta$ and $y = \sin\theta + \cos\theta$ are satisfied by which one of the following equations? *[2009-I]*
(a) $y^2 - 2x = 1$ (b) $y^2 + 2x = 1$
(c) $y^2 - 2x = -1$ (d) $y^2 + 2x = -1$

60. If $\sin^4 x - \cos^4 x = p$, then which one of the following is correct? *[2009-I]*
(a) $p = 1$ (b) $p = 0$
(c) $|p|\ \ 1$ (d) $|p| \leq 1$

61. If $\cos\theta < \sin\theta$ and θ lies in the first quadrant, then which one of the following is correct? *[2009-I]*
(a) $0 < \theta < \pi/4$ (b) $\pi/4 < \theta < \pi/2$
(c) $0 < \theta < \pi/3$ (d) $\pi/3 < \theta < \pi/2$

62. If $\sin^2 x + \sin^2 y = 1$, then what is the value of $\cot(x + y)$? *[2009-I]*
(a) 1 (b) $\sqrt{3}$
(c) 0 (d) $1/\sqrt{3}$

63. What is the value of $\cos 10° + \cos 110° + \cos 130°$? *[2009-I]*
(a) –1 (b) 0
(c) 1 (d) 2

64. What is the length of arc of a circle of radius 5 cm subtending a central angle measuring 15°? *[2009-II]*
(a) $5\pi/12$cm (b) $7\pi/12$cm
(c) $\pi/12$cm (d) $\pi/5$cm

65. What is the maximum value of $\sin\theta\cos\theta$? *[2009-II]*
(a) 1 (b) 1/2
(c) $1/\sqrt{2}$ (d) $\sqrt{3}/2$

66. If $\sin x + \mathrm{cosec}\, x = 2$, then what is the value of $\sin^4 x + \mathrm{cosec}^4 x$? *[2009-II]*
(a) 2 (b) 4
(c) 8 (d) 16

67. What is the value of $\tan 15° + \cot 15°$? *[2009-II]*
(a) $\sqrt{3}$ (b) $2\sqrt{3}$
(c) 4 (d) 2

68. If $A + B + C = \pi/2$, then what is the value of $\tan A \tan B + \tan B \tan C + \tan C \tan A$? *[2009-II]*
(a) 0 (b) 1
(c) –1 (d) $\tan A \tan B \tan C$

69. If $(\sin x + \mathrm{cosec}\, x)^2 + (\cos x + \sec x)^2 = k + \tan^2 x + \cot^2 x$, then what is the value of k? *[2009-II]*
(a) 8 (b) 7
(c) 4 (d) 3

70. If $p = \sin(989°)\cos(991°)$, then which one of the following is correct? *[2009-II]*
(a) p is finite and positive
(b) p is finite and negative
(c) $p = 0$
(d) p is undefined

71. If $A = \frac{41\pi}{12}$, then what is the value of $\frac{1-3\tan^2 A}{3\tan A - \tan^3 A}$? *[2009-II]*
(a) –1 (b) 1
(c) 1/3 (d) 3

72. Consider the following statements *[2009-II]*
I. If $\theta = 1200°$, then $(\sec\theta + \tan\theta)^{-1}$ is positive.
II. If $\theta = 1200°$, then $(\text{cosec}\theta - \cot\theta)$ is negative.
Which of the statements given above is/are correct?
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

73. If $\cot\theta = 2\cos\theta$, where $(\pi/2) < \theta < \pi$, then what is the value of θ ? *[2009-II]*
(a) $5\pi/6$ (b) $2\pi/3$
(c) $3\pi/4$ (d) $11\pi/12$

74. If $\cot\theta = 5/12$ and θ lies in the third quadrant, then what is $(2\sin\theta + 3\cos\theta)$ equal to? *[2009-II]*
(a) – 4
(b) $-p^2$ for some odd prime p
(c) $(-q/p)$ where p is an odd prime and q a positive integer with (q/p) not an integer
(d) $-p$ for some odd prime p

75. What is the value of *[2009-II]*
$\cos(\pi/9) + \cos(\pi/3) + \cos(5\pi/9) + \cos(7\pi/9)$?
(a) 1 (b) –1
(c) –1/2 (d) 1/2

76. What is the value of $\sqrt{3}$ cosec 20° – sec 20° ? *[2009-II]*
(a) 4 (b) 3
(c) 2 (d) 1

77. Match List_I with List_II and select the correct answer using the code given below the lists *[2009-II]*

	List-I		List-II
A.	tan 15°	1.	$-2-\sqrt{3}$
B.	tan 75°	2.	$2+\sqrt{3}$
C.	tan 105°	3.	$-2+\sqrt{3}$
		4.	$2-\sqrt{3}$

Codes :

	A	B	C
(a)	4	1	2
(b)	4	2	1
(c)	3	2	1
(d)	2	1	4

78. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then what is the value of (A+B)? *[2010-I]*
(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

79. If $\cos x \neq -1$, then what is $\frac{\sin x}{1+\cos x}$ equal to? *[2010-I]*
(a) $-\cot\frac{x}{2}$ (b) $\cot\frac{x}{2}$
(c) $\tan\frac{x}{2}$ (d) $-\tan\frac{x}{2}$

80. What is the value of $\frac{1+\tan 15°}{1-\tan 15°}$? *[2010-I]*
(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

81. What is the value of $\sqrt{3}$ cosec 20º – sec 20º ? *[2010-I]*
(a) 1/4 (b) 4
(c) 2 (d) 1

82. What is $\tan\left(7\frac{1}{2}\right)^{o}$ equal to? *[2010-I]*
(a) $\sqrt{6}+\sqrt{3}-\sqrt{2}+2$ (b) $\sqrt{6}+\sqrt{3}+\sqrt{2}+2$
(c) $\sqrt{6}-\sqrt{3}+\sqrt{2}-2$ (d) $\sqrt{6}+\sqrt{3}+\sqrt{2}-2$

83. What is the value of $\frac{\cos 15° + \cos 45°}{\cos^3 15° + \cos^3 45°}$? *[2010-I]*
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) None of these

84. The angle A lies in the third quadrant and it satisfies the equation $4(\sin^2 x + \cos x) = 1$. What is the measure of the angle A? *[2010-I]*
(a) 225° (b) 240°
(c) 210° (d) None of these

85. What is $\frac{\sin\theta + 1}{\cos\theta}$ equal to ? *[2010-II]*
(a) $\frac{\sin\theta+\cos\theta-1}{\sin\theta+\cos\theta+1}$ (b) $\frac{\sin\theta+\cos\theta+1}{\sin\theta+\cos\theta-1}$
(c) $\frac{\sin\theta-\cos\theta-1}{\sin\theta+\cos\theta+1}$ (d) $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

86. One of the angles of a triangle is 1/2 radian and the other is 99°. What is the third angle in radian measure? *[2010-II]*
(a) $\frac{9\pi-10}{\pi}$ (b) $\frac{90\pi-100}{7\pi}$
(c) $\frac{90\pi-10}{\pi}$ (d) None of these

87. What is $\left(\frac{\sec 18}{\sec 144} \quad \frac{\operatorname{cosec} 18}{\operatorname{cosec} 144}\right)$ equal to? [2010-II]
(a) sec 18° (b) cosec 18°
(c) –sec 18° (d) –cosec 18°

88. If α and β are positive angles such that $\alpha + \beta = \frac{\pi}{4}$, then what is $(1 + \tan \alpha)(1 + \tan \beta)$ equal to? [2010-II]
(a) 0 (b) 1
(c) 2 (d) 3

89. What is the value of (sin 50° – sin 70° + sin 10°) ? [2010-II]
(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{2}$ (d) 0

90. If $\cos A + \cos B = m$ and $\sin A + \sin B = n$, where $m, n \neq 0$, then what is $\sin (A + B)$ equal to ? [2010-II]
(a) $\frac{mn}{m^2 + n^2}$ (b) $\frac{2mn}{m^2 + n^2}$
(c) $\frac{m^2 + n^2}{2mn}$ (d) $\frac{mn}{m + n}$

91. If $y = \sec^2 \theta + \cos^2 \theta$, where $0 < \theta < \frac{\pi}{2}$, then which one of the following is correct? [2010-II]
(a) $y = 0$ (b) $0 \leq y \leq 2$
(c) $y \geq 2$ (d) None of these

92. If $\tan A = 3/4$ and $\tan B = -12/5$, then how many values can $\cot (A - B)$ have depending on the actual values of A and B?
(a) 1 (b) 2 [2010-II]
(c) 3 (d) 4

93. What is the value of sin 15° sin 75°? [2010-II]
(a) 1/4 (b) 1/8
(c) 1/16 (d) 1

94. What is the value of $\frac{\sin \theta + \cos \theta - \tan \theta}{\sec \theta + \operatorname{cosec} - \cot \theta}$, when $\theta = \frac{3\pi}{4}$?
(a) 0 (b) 1 [2010-II]
(c) –1 (d) None of these

95. What is the value of $\sin 292 \frac{1}{2}°$? [2010-II]
(a) $\frac{1}{3}\sqrt{2 + \sqrt{3}}$ (b) $-\frac{1}{3}\sqrt{2 - \sqrt{3}}$
(c) $\frac{1}{2}\sqrt{2 + \sqrt{2}}$ (d) $-\frac{1}{2}\sqrt{2 + \sqrt{2}}$

96. Which one of the following is correct? [2010-II]
(a) sin 1° > sin 1 (b) sin 1° < sin 1
(c) sin 1° = sin 1 (d) $\sin 1° = \frac{\pi}{180} \sin 1$

97. If in general, the value of sin A is known, but the value of A is not known, then how many values of $\tan\left(\frac{A}{2}\right)$ can be calculated? [2011-I]
(a) 1 (b) 2
(c) 3 (d) 4

98. If $x = \sin\theta + \cos\theta$ and $y = \sin\theta.\cos\theta$, then what is the value of $x^4 - 4x^2y - 2x^2 + 4y^2 + 4y + 1$?
[2011-I]
(a) 0 (b) 1
(c) 2 (d) None of these

99. If $(1 + \tan\theta)(1 + \tan\phi) = 2$, then what is $(\theta + \phi)$ equal to?
(a) 30° (b) 45° [2011-I]
(c) 60° (d) 90°

100. If an angle α is divided into two parts A and B such that $A - B = x$ and $\tan A : \tan B = 2 : 1$, then what is $\sin x$ equal to?
(a) 3 sin α (b) (2 sin α) / 3 [2011-I]
(c) (sin α) / 3 (d) 2 sin α

101. What is the value of
tan 9° – tan 27° – tan 63° + tan 81°? [2011-II]
(a) 1 (b) 2
(c) 3 (d) 4

102. If $x = y \cos\left(\frac{2\pi}{3}\right) = z \cos\left(\frac{4\pi}{3}\right)$, then what is $xy + yz + zx$ equal to? [2011-II]
(a) – 1 (b) 0
(c) 1 (d) 2

103. If $\sin A + \sin B + \sin C = 3$ then what is $\cos A + \cos B + \cos C$ equal to? [2011-II]
(a) – 1 (b) 0
(c) 1 (d) 3

104. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then what is $\cot (A - B)$ equal to? [2011-II]
(a) $\frac{1}{y} - \frac{1}{x}$ (b) $\frac{1}{x} - \frac{1}{y}$
(c) $\frac{1}{x} + \frac{1}{y}$ (d) $-\frac{1}{x} - \frac{1}{y}$

105. If $\tan A = 1/2$ and $\tan B = 1/3$, then what is the value of $4A + 4B$? [2011-II]
(a) $\pi/4$ (b) $\pi/2$
(c) π (d) 2π

106. What is the maximum value of $3 \cos x + 4 \sin x + 5$?
(a) 5 (b) 7 [2011-II]
(c) 10 (d) 12

107. If $\sin\theta = \cos^2\theta$, then what is $\cos^2\theta(1 + \cos^2\theta)$ equal to? [2011-II]
(a) 1 (b) 0
(c) $\cos^2 \theta$ (d) $2 \sin \theta$

108. What is the value of tan 15°. tan195° ? [2011-II]
(a) $7-4\sqrt{3}$ (b) $7+4\sqrt{3}$
(c) $7+2\sqrt{3}$ (d) $7+6\sqrt{3}$

109. What is $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}$ equal to? [2011-II]
(a) 2 tan x (b) 2 cosec x
(c) 2 cos x (d) 2 sin x

110. If sin $3A$ = 1, then how many distinct values can sin A assume? [2011-II]
(a) 1 (b) 2
(c) 3 (d) 4

111. What is $\frac{\sin\theta}{\text{cosec}\theta}\ \frac{\cos\theta}{\sec\theta}$ equal to? [2012-I]
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) 2

112. If tanθ + secθ = 4, then what is the value of sinθ? [2012-I]
(a) 8/17 (b) 8/15
(c) 15/17 (d) 23/32

113. What is the angle subtended by 1 m pole at a distance 1 km on the ground in sexagesimal measure? [2012-I]
(a) $\frac{9}{50\pi}$ degree (b) $\frac{9}{5\pi}$ degree
(c) 3.4 minute (d) 3.5 minute

114. If cotA cotB = 2, then what is the value of cos(A + B) sec (A – B)? [2012-I]
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) 1 (d) –1

115. What is $\tan\left(\frac{\pi}{12}\right)$ equal to? [2012-I]
(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $\sqrt{2}-\sqrt{3}$ (d) $\sqrt{3}-\sqrt{2}$

116. If θ = 18°, then what is the value of $4\sin^2\theta + 2\sin\theta$? [2012-I]
(a) – 1 (b) 1
(c) 0 (d) 2

117. If $\text{cosec}\theta - \cot\theta = \frac{1}{\sqrt{3}}$ where $\theta \neq 0$, then what is the value of cos θ? [2012-I]
(a) 0 (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

118. What is the maximum value of sin3θcos2θ + cos3θsin2θ? [2012-I]
(a) 1 (b) 2
(c) 4 (d) 10

119. What is sinAcosAtanA + cosAsinAcotA equal to? [2012-I]
(a) sinA (b) cosA
(c) tanA (d) 1

120. Which one of the following is positive in the third quadrant? [2012-I]
(a) sinθ (b) cosθ
(c) tanθ (d) secθ

121. What is the value of sin (1920°)? [2012-I]
(a) 1/2 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{2}$ (d) 1/3

122. Let sin(A + B) = 1 and sin (A – B) = $\frac{1}{2}$ where $A, B \in \left[0, \frac{\pi}{2}\right]$. What is the value of A? [2012-I]
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

123. What is tan(A + 2B). tan(2A + B) equal to? [2012-I]
(a) – 1 (b) 0
(c) 1 (d) 2

124. What is $\sin^2A - \sin^2B$ equal to? [2012-I]
(a) 0 (b) 1/2
(c) 1 (d) 2

125. What is the value of sin420°.cos390° + cos(– 300°).sin(–330°)? [2012-I]
(a) 0 (b) 1
(c) 2 (d) – 1

126. Consider the following statements: [2012-I]
1. 1° in radian measure is less than 0.02 radians.
2. 1 radian in degree measure is greater than 45°.

Which of the above statements is/are correct? [2012-I]
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

127. What is maximum value of $\sin^2x$? [2012-I]
(a) – 1 (b) 0
(c) 1 (d) Infinity

128. If ABCD is a cyclic quadrilateral then what is sinA + sinB – sinC – sinD equal to? [2012-I]
(a) 0 (b) 1
(c) 2 (d) 2(sin A + sin B)

129. What is the value of sin 15° ? [2012-II]
(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

130. If $4\sin^2\theta = 1$, where $0 < \theta < 2\pi$, how many values does θ take? [2012-II]
(a) 1 (b) 2
(c) 4 (d) None of the above

131. What is the value of sin 18° cos 36° equal to? [2012-II]
(a) 4 (b) 2
(c) 1 (d) 1/4

132. If $\sec\alpha = \frac{13}{5}$ where $270° < \alpha < 360°$ then what is $\sin\alpha$ equal to ? [2012-II]
(a) $\frac{5}{13}$ (b) $\frac{12}{13}$
(c) $-\frac{12}{13}$ (d) $-\frac{13}{12}$

133. What is tan (– 585°) equal to ? [2012-II]
(a) 1 (b) – 1
(c) $-\sqrt{2}$ (d) $-\sqrt{3}$

134. Consider the following statements : [2012-II]
1. The value of cos 46° – sin 46° is positive.
2. The value of cos 44° – sin 44° is negative.
Which of the above statement is/are correct ? [2012-II]
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

135. The angle subtended at the centre of a circle of radius 3 cm by an arc of length 1 cm is : [2012-II]
(a) $\frac{30°}{\pi}$ (b) $\frac{60°}{\pi}$
(c) 60° (d) None of the above

136. If $\sin A = \frac{2}{\sqrt{5}}$ and $\cos B = \frac{1}{\sqrt{10}}$ where A and B are acute angles, then what is A + B equal to ? [2012-II]
(a) 135° (b) 90°
(c) 75° (d) 60°

137. If cosecθ + cotθ = c, then what is cosθ equal to ? [2013-I]
(a) $\frac{c}{c^2-1}$ (b) $\frac{c}{c^2+1}$
(c) $\frac{c^2-1}{c^2+1}$ (d) None of the above

138. If sinθ + 2 cosθ = 1, then what is 2sinθ – cosθ equal to ?
(a) 0 (b) 1 [2013-I]
(c) 2 (d) 4

139. If A + B = 90°, then what is $\sqrt{\sin A \sec B - \sin A \cos B}$ equal to ? [2013-I]
(a) sin A (b) cos A
(c) tan A (d) 0

140. What is $\tan^4 A - \sec^4 A + \tan^2 A + \sec^2 A$ equal to ?
(a) 0 (b) 1 [2013-I]
(c) 2 (d) –1

141. What is the value of tan 105° ? [2013-I]
(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}+1}{1-\sqrt{3}}$
(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}+2}{\sqrt{3}-1}$

142. If tan A = x + 1 and tan B = x – 1, then x^2 tan (A – B) has the value: [2013-I]
(a) 1 (b) x
(c) 0 (d) 2

143. What is the value of $(\sin^4\theta - \cos^4\theta + 1)\,\text{cosec}^2\theta$?
(a) – 2 (b) 0 [2013-I]
(c) 1 (d) 2

144. The expression $\frac{\cot x + \text{cosec}x - 1}{\cot x - \text{cosec}x + 1}$ is equal to : [2013-I]
(a) $\frac{\sin x}{1-\cos x}$ (b) $\frac{1-\cos x}{\sin x}$
(c) $\frac{1+\cos x}{\sin x}$ (d) $\frac{\sin x}{1+\cos x}$

145. What is $\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ equal to : [2013-I]
(a) sin x. cos x (b) tan x
(c) sin x (d) cos x

146. What is $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$ equal to ? [2013-II]
(a) 0 (b) 1
(c) 2 (d) 3

147. What is $\sin^2 20° + \sin^2 70°$ equal to ? [2013-II]
(a) 1 (b) 0
(c) –1 (d) $\frac{1}{2}$

148. What is $(1-\sin^2\theta)(1+\tan^2\theta)$ equal to ? [2013-II]
(a) $\sin^2\theta$ (b) $\cos^2\theta$
(c) $\tan^2\theta$ (d) 1

149. What is tan 15° equal to ? [2013-II]
(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $1-\sqrt{3}$ (d) $1+\sqrt{3}$

150. Consider the following: [2013-II]
1. $\tan\left(\frac{\pi}{6}\right)$ 2. $\tan\left(\frac{3\pi}{4}\right)$
3. $\tan\left(\frac{5\pi}{4}\right)$ 4. $\tan\left(\frac{2\pi}{3}\right)$
What is the correct order ?
(a) $1<4<2<3$ (b) $4<2<1<3$
(c) $4<2<3<1$ (d) $1<4<3<2$

151. If $\cos x = \frac{1}{3}$, then what is $\sin x \,.\, \cot x \,.\, \text{cosec}x \,.\, \tan x$ equal to ? [2013-II]
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) 2 (d) 1

152. The complete solution of $3\tan^2 x = 1$ is given by : *[2014-I]*
(a) $x = n\pi \pm \frac{\pi}{3}$ (b) $x = n\pi + \frac{\pi}{3}$ only
(c) $x = n\pi \pm \frac{\pi}{6}$ (d) $x = n\pi + \frac{\pi}{6}$ only
where $n \in Z$

153. What is the value of cos 36° ? *[2014-I]*
(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{4}$
(c) $\frac{\sqrt{10+2\sqrt{5}}}{4}$ (d) $\frac{\sqrt{10-2\sqrt{5}}}{4}$

154. Consider the following statements : *[2014-I]*
1. Value of sin θ oscillates between –1 and 1.
2. Value of cos θ oscillates between 0 and 1.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

155. Consider the following statements : *[2014-I]*
1. $n\left(\sin^2 67\frac{1°}{2} - \sin^2 22\frac{1°}{2}\right) > 1$ for all positive integers $n \geq 2$.
2. If x is any positive real number, then $nx > 1$ for all positive integers $n \geq 2$.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

156. Consider the following statements : *[2014-I]*
1. If 3θ is an acute angle such that sin 3θ = cos 2θ, then the mesurement of θ in radian equals to $\frac{\pi}{10}$.
2. One radian is the angle subtended at the centre of a circle by an arc of the same circle whose length is equal to the diameter of that circle.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

157. Consider the following statements : *[2014-I]*
1. $\sin|x| + \cos|x|$ is always positive.
2. $\sin(x^2) + \cos(x^2)$ is always positive.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

158. What is $\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A}$ equal to ? *[2014-I]*
(a) sec A – tan A (b) 2 sec A . tan A
(c) 4 sec A . tan A (d) 4 cosec A . cot A

159. What is $\frac{\cot 224° - \cot 134°}{\cot 226° + \cot 316°}$ equal to ? *[2014-I]*
(a) – cosec 88° (b) – cosec 2°
(c) – cosec 44° (d) – cosec 46°

160. What is cos 20° + cos 100° + cos 140° equal to ? *[2014-I]*
(a) 2 (b) 1
(c) 1/2 (d) 0

161. What is $\sin^2(3\pi) + \cos^2(4\pi) + \tan^2(5\pi)$ equal to ? *[2014-I]*
(a) 0 (b) 1
(c) 2 (d) 3

162. What is $\sqrt{1+\sin 2\theta}$ equal to ? *[2014-II]*
(a) $\cos\theta - \sin\theta$ (b) $\cos\theta + \sin\theta$
(c) $2\cos\theta + \sin\theta$ (d) $\cos\theta + 2\sin\theta$

163. If cot A = 2 and cot B = 3, then what is the value of A + B ? *[2014-II]*
(a) $\pi/6$ (b) π
(c) $\pi/2$ (d) $\pi/4$

164. What is $\sin^2 66\frac{1°}{2} - \sin^2 23\frac{1°}{2}$ equal to ? *[2014-II]*
(a) sin 47° (b) cos 47°
(c) 2 sin 47° (d) 2 cos 47°

165. What is $\frac{\cos 7x - \cos 3x}{\sin 7x - 2\sin 5x + \sin 3x}$ equal to ? *[2014-II]*
(a) tan x (b) cot x
(c) tan 2x (d) cot 2x

166. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then what is $\frac{\tan x}{\tan y}$ equal to ? *[2014-II]*
(a) $\frac{b}{a}$ (b) $\frac{a}{b}$
(c) ab (d) 1

167. If sin A sin (60° – A) sin (60° + A) = k sin 3A, then what is k equal to ? *[2014-II]*
(a) 1/4 (b) 1/2
(c) 1 (d) 4

168. The line $y = \sqrt{3}$ meets the graph y = tan x, where $x \in \left(0, \frac{\pi}{2}\right)$, in k points. What is k equal to? *[2014-II]*
(a) One (b) Two
(c) Three (d) Infinity

169. Which one of the following is one of the solutions of the equation of the equation $\tan 2\theta . \tan\theta = 1$? *[2014-II]*
(a) $\pi/12$ (b) $\pi/6$
(c) $\pi/4$ (d) $\pi/3$

DIRECTIONS (Qs. 170-172): *For the next three (03) items that follow.*

Given that $16\sin^5 x = p\sin 5x + q\sin 3x + r\sin x$.

170. What is the value of p ? *[2014-II]*
(a) 1 (b) 2
(c) –1 (d) –2

171. What is the value of q ? *[2014-II]*
 (a) 3 (b) 5
 (c) 10 (d) –5

172. What is the value of *r* ? *[2014-II]*
 (a) 5 (b) 8
 (c) 10 (d) –10

173. Let θ be a positive angle. If the number of degrees in θ is divided by the number of radians in θ, then an irrational number $\frac{180}{\pi}$ results. If the number of degrees in θ is multiplied by the number of radians in θ, then an irrational number $\frac{125\pi}{9}$ results. The angle θ must be equal to *[2015-I]*
 (a) 30° (b) 45°
 (c) 50° (d) 60°

DIRECTIONS (Qs. 174-175): *For the next two (2) items that follow.*

Let α be the root of the equation $25\cos^2\theta + 5\cos\theta - 12 = 0$, where $\frac{\pi}{2} < \alpha < \pi$.

174. What is $\tan\alpha$ equal to? *[2015-I]*
 (a) $\frac{-3}{4}$ (b) $\frac{3}{4}$
 (c) $\frac{-4}{3}$ (d) $\frac{-4}{5}$

175. What is $\sin 2\alpha$ equal to? *[2015-I]*
 (a) $\frac{24}{25}$ (b) $\frac{-24}{25}$
 (c) $\frac{-5}{12}$ (d) $\frac{-21}{25}$

176. $(1 - \sin A + \cos A)^2$ is equal to *[2015-I]*
 (a) $2(1 - \cos A)(1 + \sin A)$
 (b) $2(1 - \sin A)(1 + \cos A)$
 (c) $2(1 - \cos A)(1 - \sin A)$
 (d) None of the above

177. What is $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$ equal to? *[2015-II]*
 (a) $\sin\theta - \cos\theta$ (b) $\sin\theta + \cos\theta$
 (c) $2\sin\theta$ (d) $2\cos\theta$

178. The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \sin^2 20° + ... + \sin^2 90°$ is *[2015-II]*
 (a) 7 (b) 8
 (c) 9 (d) $\frac{19}{2}$

179. On simplifying $\frac{\sin^3 A + \sin 3A}{\sin A} \quad \frac{\cos^3 A - \cos 3A}{\cos A}$, we get *[2015-II]*
 (a) sin3A (b) cos3A
 (c) sin A + cos A (d) 3

180. If sin x + sin y = a and cos x + cos y = b, then $\tan^2\left(\frac{x+y}{2}\right) \quad \tan^2\left(\frac{x-y}{2}\right)$ is equal to *[2015-II]*
 (a) $\frac{a^4 \quad b^4 \quad 4b^2}{a^2b^2 \quad b^4}$ (b) $\frac{a^4 - b^4 \quad 4b^2}{a^2b^2 \quad b^4}$
 (c) $\frac{a^4 - b^4 \quad 4a^2}{a^2b^2 \quad a^4}$ (d) None of the above

DIRECTIONS (Qs. 181-182) : *For the next two (02) items that follow .*

Consider a triangle ABC satisfying

$$2a\sin^2\left(\frac{C}{2}\right) \quad 2c\sin^2\left(\frac{A}{2}\right) \quad 2a \quad 2c - 3b$$

181. The sides of the triangle are in *[2015-II]*
 (a) G.P.
 (b) A.P.
 (c) H.P.
 (d) Neither in G.P. nor in A.P nor in H.P.

182. sin A, sin B, sin C are in *[2015-II]*
 (a) G.P.
 (b) A.P.
 (c) H.P.
 (d) Neither in G.P. nor in A.P nor in H.P.

183. If $p = \tan\left(-\frac{11\pi}{6}\right)$, $q = \tan\left(\frac{21\pi}{4}\right)$ and $r = \cot\left(\frac{283\pi}{6}\right)$, then which of the following is/are correct ? *[2015-II]*
 1. The value of p × r is 2.
 2. p, q and r are in G.P.
 Select the correct answer using the code given below :
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 184-185) : *For the next two (2) items that follow.*

Given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$.

184. What is $\tan(\alpha+\beta)$ equal to? *[2016-I]*
 (a) $b(c-1)$ (b) $c(b-1)$
 (c) $c(b-1)^{-1}$ (d) $b(c-1)^{-1}$

185. What is $\sin(\alpha+\beta)\sec\alpha\sec\beta$ equal to? *[2016-I]*
 (a) b (b) – b
 (c) c (d) – c

186. If $A = (\cos 12° - \cos 36°)(\sin 96° + \sin 24°)$ *[2016-I]* and $B = (\sin 60° - \sin 12°)(\cos 48° - \cos 72°)$, then what is $\frac{A}{B}$ equal to?
 (a) –1 (b) 0
 (c) 1 (d) 2

187. sin A + 2 sin 2A + sin 3A is equal to which of the following? *[2016-II]*
 1. $4\sin 2A\cos^2\left(\frac{A}{2}\right)$
 2. $2\sin 2A\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2$
 3. $8\sin A\cos A\cos^2\left(\frac{A}{2}\right)$

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

188. If $x = \sin 70^0.\sin 50^0$ and $y = \cos 60^0.\cos 80^0$, then what is xy equal to ? [2016-II]
(a) 1/16 (b) 1/8
(c) 1/4 (d) 1/2

189. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 4$, then what is the value of $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 =$? [2016-II]
(a) 0 (b) 1
(c) 2 (d) 4

190. What is the value of [2016-II]
$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)?$$
(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{2\sqrt{2}}$
(c) $\frac{1}{2}-\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{8}$

191. If $x\cos\theta + y\sin\theta = z$, then what is the value of $(x\sin\theta - y\cos\theta)^2$? [2016-II]
(a) $x^2+y^2-z^2$ (b) $x^2-y^2-z^2$
(c) $x^2-y^2+z^2$ (d) $x^2+y^2+z^2$

192. If $\sin 18^0 = \frac{\sqrt{5}-1}{4}$, then what is the value of sin 81° ? [2016-II]
(a) $\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$ (b) $\frac{\sqrt{3+\sqrt{5}}+\sqrt{5+\sqrt{5}}}{4}$
(c) $\frac{\sqrt{3-\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$ (d) $\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{4}$

193. What is $\frac{1-\tan 2^0 \cot 62^0}{\tan 152^0 - \cot 88^0}$ equal to? [2016-II]
(a) $\sqrt{3}$ (b) $-\sqrt{3}$
(c) $\sqrt{2}-1$ (d) $1-\sqrt{2}$

194. If $\sin A = \frac{3}{5}$, where $450° < A < 540°$, then $\cos\frac{A}{2}$ is equal to [2017-I]
(a) $\frac{1}{\sqrt{10}}$ (b) $-\sqrt{\frac{3}{10}}$
(c) $\frac{\sqrt{3}}{\sqrt{10}}$ (d) None of the above

195. What is $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}$ equal to? [2017-I]
(a) 0 (b) 1
(c) 2 (d) 4

196. If $K=\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, then what is the value of K? [2017-I]
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

197. The expression $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$ is equal to [2017-I]
(a) $\tan\left(\frac{\alpha+\beta}{2}\right)$ (b) $\cot\left(\frac{\alpha+\beta}{2}\right)$
(c) $\sin\left(\frac{\alpha+\beta}{2}\right)$ (d) $\cos\left(\frac{\alpha+\beta}{2}\right)$

198. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan\alpha$ is equal to [2017-I]
(a) –1 (b) 0
(c) 1 (d) 2

199. What is the value of tan 18°? [2017-I]
(a) $\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ (b) $\frac{\sqrt{5}-1}{\sqrt{10+\sqrt{5}}}$
(c) $\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$ (d) $\frac{\sqrt{10+\sqrt{5}}}{\sqrt{5}-1}$

200 If $\tan(\alpha+\beta) = 2$ and $\tan(\alpha-\beta) = 1$, then $\tan(2\alpha)$ is equal to [2017-I]
(a) –3 (b) –2
(c) $-\frac{1}{3}$ (d) 1

201. If $\sec\theta - \operatorname{cosec}\theta = \frac{4}{3}$, then what is $(\sin\theta - \cos\theta)$ equal to? [2017-I]
(a) –2 only (b) $\frac{1}{2}$ only
(c) Both –2 and $\frac{1}{2}$ (d) Neither $\frac{1}{2}$ nor –2

202. The value of tan 9° – tan 27° – tan 63° + tan 81° is equal to [2017-II]
(a) –1 (b) 0
(c) 1 (d) 4

203. The value of $\sqrt{3}$ cosec 20° - sec 20° is equal to [2017-II]
(a) 4 (b) 2
(c) 1 (d) –4

204. Angle α is divided into two parts A and B such that A – B = x and tan A : tan B = p : q. The value of sin x is equal to [2017-II]
(a) $\frac{(p+q)\sin\alpha}{p-q}$ (b) $\frac{p\sin\alpha}{p+q}$
(c) $\frac{p\sin\alpha}{p-q}$ (d) $\frac{(p-q)\sin\alpha}{p+q}$

205. $\sqrt{1+\sin A} = -\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)$ is true if *[2017-II]*

(a) $\frac{3\pi}{2} < A < \frac{5\pi}{2}$ only (b) $\frac{\pi}{2} < A < \frac{3\pi}{2}$ only

(c) $\frac{3\pi}{2} < A < \frac{7\pi}{2}$ (d) $0 < A < \frac{3\pi}{2}$

206. If $\sin x \quad \frac{1}{\sqrt{5}}, \sin y \quad \frac{1}{\sqrt{10}}$, where $0 \quad x \quad \frac{\pi}{2}$, $0 \quad y \quad \frac{\pi}{2}$, then what is (x + y) equal to? *[2018-I]*

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) 0

207. What is $\frac{\sin 5x - \sin 3x}{\cos 5x \quad \cos 3x}$ equal to? *[2018-I]*

(a) sin x (b) cos x
(c) tan x (d) cot x

208. What is sin 105° + cos 105° equal to? *[2018-I]*

(a) sin 50° (b) cos 50°

(c) $\frac{1}{\sqrt{2}}$ (d) 0

209. If $\frac{\sin\ x \quad y}{\sin\ x - y} \quad \frac{a \quad b}{a - b}$, then what is $\frac{\tan x}{\tan y}$ equal to? *[2018-I]*

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$

(c) $\frac{a \quad b}{a - b}$ (d) $\frac{a - b}{a \quad b}$

210. If $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$, where $0 < \beta < \alpha < 2\pi$, then which one of the following is correct? *[2018-I]*

(a) $\alpha = \pi - \beta$ (b) $\alpha = \pi + \beta$
(c) $\alpha = 2\pi - \beta$ (d) $2\alpha = \pi + 2\beta$

211. Suppose cos A is given. If only one value of $\cos\left(\frac{A}{2}\right)$ is possible, then A must be *[2018-I]*

(a) An odd multiple of 90°
(b) A multiple of 90°
(c) An odd multiple of 180°
(d) A multiple of 180°

212. If cos α + cos β + cos γ = 0, where $0 < \alpha \le \frac{\pi}{2}$, $0 < \beta \le \frac{\pi}{2}$, $0 < \gamma \le \frac{\pi}{2}$, then what is the value of sin α + sin β + sin γ? *[2018-I]*

(a) 0 (b) 3

(c) $\frac{5\sqrt{2}}{2}$ (d) $\frac{3\sqrt{2}}{2}$

213. What is the period of the function f(x) = sin x? *[2018-I]*

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) 2π

214. What is $\frac{2\tan\theta}{1+\tan^2\theta}$ equal to? *[2018-II]*

(a) cos 2θ (b) tan 2θ
(c) sin 2θ (d) cosec 2θ

215. If sec (θ – α), sec θ and sec (θ + α) are in AP, where cos $\alpha \neq 1$, then what is the value of $\sin^2\theta + \cos\alpha$? *[2018-II]*

(a) 0 (b) 1
(c) – 1 (d) 1/2

216. A is an angle in the fourth quadrant. If satisfies the trigonometric equation $3(3-\tan^2 A - \cot A)^2 = 1$.
Which one of the following is a value of A? *[2018-II]*

(a) 300° (b) 315°
(c) 330° (d) 345°

217. What is/are the solutions of the trigonometric equation cosec x + cot x = $\sqrt{3}$, where $0 < x < 2\pi$? *[2018-II]*

(a) $\frac{5\pi}{3}$ only (b) $\frac{\pi}{3}$ only

(c) π only (d) $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

218. If $\theta \quad \frac{\pi}{8}$, then what is the value of
$(2\cos\theta + 1)^{10}\ (2\cos 2\theta - 1)^{10}\ (2\cos\theta - 1)^{10}\ (2\cos 4\theta - 1)^{10}$? *[2018-II]*

(a) 0 (b) 1
(c) 2 (d) 4

219. If cos α and cos β ($0 < \alpha < \beta < \pi$) are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of sec α × sec β ? *[2018-II]*

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

220. If A = $\sin^2\theta + \cos^4\theta$, the for all real θ, which one of the following is correct? *[2018-II]*

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$

(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

221. What is the least value of $25\,\text{cosec}^2 x + 36\sec^2 x$? *[2019-I]*

(a) 1 (b) 11
(c) 120 (d) 121

222. What is the value of [2019-I]

$\frac{\sin 34° \cos 236° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$?

(a) –2 (b) –1
(c) 2 (d) 1

223. tan 54° can be expressed as [2019-I]

(a) $\frac{\sin 9° + \cos 9°}{\sin 9° - \cos 9°}$ (b) $\frac{\sin 9° - \cos 9°}{\sin 9° + \cos 9°}$

(c) $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$ (d) $\frac{\sin 36°}{\cos 36°}$

DIRECTIONS (Qs. 224-226) : *Consider the following for the next 03 (three) items.*

If $p = X \cos\theta - Y \sin\theta$, $q = X \sin\theta + Y \cos\theta$ and $p^2 + 4pq + q^2 = AX^2 + BY^2$, $0 \le 0 \le \frac{\pi}{2}$.

224. What is the value of θ? [2019-I]

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

225. What is the value of A? [2019-I]
(a) 4 (b) 3
(c) 2 (d) 1

226. What is the value of B ? [2019-I]
(a) –1 (b) 0
(c) 1 (d) 2

DIRECTIONS (Qs. 227-228) : *Consider the following for the next 02 (two) items.*

It is given that $\cos(\theta - \alpha) = a$, $\cos(\theta - \beta) = b$

227. What is $\cos(\alpha - \beta)$ equal to ? [2019-I]

(a) $ab + \sqrt{1-a^2}\sqrt{1-b^2}$ (b) $ab - \sqrt{1-a^2}\sqrt{1-b^2}$

(c) $a\sqrt{1-b^2} - b\sqrt{1-a^2}$ (d) $a\sqrt{1-b^2} + b\sqrt{1-a^2}$

228. What is $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$ equal to ? [2019-I]

(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $b^2 - a^2$ (d) $-(a^2 + b^2)$

229. If $\sin\alpha + \cos\alpha = p$, then what is $\cos^2(2\alpha)$ equal to ? [2019-I]

(a) p^2 (b) $p^2 - 1$
(c) $p^2(2 - p^2)$ (d) $p^2 + 1$

230. If $\tan\theta = \frac{1}{2}$ and $\tan\varphi = \frac{1}{3}$, then what is the value of ? [2019-I]

(a) 0 (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

231. If $\cos A = \frac{3}{4}$, then what is the value of $\sin\left(\frac{A}{2}\right)\sin\left(\frac{3A}{2}\right)$? [2019-I]

(a) $\frac{5}{8}$ (b) $\frac{5}{16}$

(c) $\frac{5}{24}$ (d) $\frac{7}{32}$

232. What is the value of tan 75° + cot 75° ? [2019-I]
(a) 2 (b) 4
(c) $2\sqrt{3}$ (d) $4\sqrt{3}$

233. What is the value of cos 46° cos 47° cos 48° cos 49° cos 50°.... cos 135° ? [2019-I]
(a) –1 (b) 0
(c) 1 (d) Greater than 1

234. If $\sin 2\theta = \cos 3\theta$, where $0 < \theta < \frac{\pi}{2}$, then what is $\sin\theta$ equal to ? [2019-I]

(a) $\frac{\sqrt{5}+1}{4}$ (b) $\frac{\sqrt{5}-1}{4}$

(c) $\frac{\sqrt{5}+1}{16}$ (d) $\frac{\sqrt{5}-1}{16}$

235. What is $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha \sec^2\beta$ equal to [2019-I]
(a) 0 (b) 1
(c) 2 (d) 4

236. If $p = \operatorname{cosec}\theta - \cot\theta$ and $q = (\operatorname{cosec}\theta + \cot\theta)^{-1}$, then which one of the following is correct ? [2019-I]
(a) pq = 1 (b) p = q
(c) p + q = 1 (d) p + q = 0

237. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then what is $(\cos\theta - \sin\theta)$ equal to ? [2019-I]

(a) $-\sqrt{2}\cos\theta$ (b) $-\sqrt{2}\sin\theta$

(c) $\sqrt{2}\sin\theta$ (d) $2\sin\theta$

238. If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$, then in which quadrant does θ lie ? [2019-I]
(a) First (b) Second
(c) Third (d) Fourth

ANSWER KEY

1	(c)	25	(b)	49	(b)	73	(a)	97	(b)	121	(c)	145	(d)	169	(b)	193	(b)	217	(b)
2	(b)	26	(c)	50	(a)	74	(d)	98	(a)	122	(b)	146	(c)	170	(a)	194	(d)	218	(b)
3	(d)	27	(d)	51	(a)	75	(d)	99	(b)	123	(c)	147	(a)	171	(d)	195	(d)	219	(a)
4	(c)	28	(c)	52	(c)	76	(a)	100	(c)	124	(b)	148	(d)	172	(c)	196	(c)	220	(b)
5	(a)	29	(a)	53	(c)	77	(b)	101	(d)	125	(b)	149	(a)	173	(c)	197	(a)	221	(d)
6	(c)	30	(d)	54	(d)	78	(b)	102	(b)	126	(c)	150	(b)	174	(a)	198	(b)	222	(a)
7	(a)	31	(d)	55	(a)	79	(c)	103	(b)	127	(c)	151	(d)	175	(b)	199	(a)	223	(c)
8	(a)	32	(a)	56	(d)	80	(d)	104	(c)	128	(a)	152	(c)	176	(b)	200	(a)	224	(c)
9	(a)	33	(a)	57	(c)	81	(b)	105	(c)	129	(a)	153	(b)	177	(b)	201	(b)	225	(b)
10	(c)	34	(a)	58	(d)	82	(c)	106	(c)	130	(c)	154	(a)	178	(d)	202	(d)	226	(a)
11	(a)	35	(c)	59	(a)	83	(d)	107	(a)	131	(d)	155	(a)	179	(d)	203	(a)	227	(a)
12	(a)	36	(b)	60	(d)	84	(c)	108	(a)	132	(c)	156	(a)	180	(b)	204	(d)	228	(a)
13	(a)	37	(a)	61	(b)	85	(d)	109	(b)	133	(b)	157	(d)	181	(b)	205	(c)	229	(c)
14	(d)	38	(d)	62	(c)	86	(d)	110	(b)	134	(d)	158	(c)	182	(b)	206	(c)	230	(c)
15	(c)	39	(c)	63	(b)	87	(a)	111	(a)	135	(b)	159	(b)	183	(b)	207	(c)	231	(b)
16	(a)	40	(a)	64	(a)	88	(c)	112	(c)	136	(a)	160	(d)	184	(d)	208	(c)	232	(b)
17	(b)	41	(a)	65	(b)	89	(d)	113	(a)	137	(c)	161	(b)	185	(b)	209	(a)	233	(b)
18	(a)	42	(d)	66	(a)	90	(b)	114	(a)	138	(c)	162	(b)	186	(c)	210	(b)	234	(b)
19	(b)	43	(b)	67	(c)	91	(c)	115	(a)	139	(b)	163	(d)	187	(c)	211	(c)	235	(a)
20	(c)	44	(a)	68	(b)	92	(d)	116	(b)	140	(a)	164	(b)	188	(a)	212	(b)	236	(b)
21	(b)	45	(b)	69	(b)	93	(a)	117	(c)	141	(b)	165	(b)	189	(a)	213	(d)	237	(c)
22	(c)	46	(b)	70	(b)	94	(b)	118	(a)	142	(d)	166	(b)	190	(d)	214	(c)	238	(c)
23	(d)	47	(c)	71	(b)	95	(c)	119	(d)	143	(d)	167	(a)	191	(a)	215	(a)		
24	(a)	48	(c)	72	(d)	96	(b)	120	(c)	144	(c)	168	(a)	192	(a)	216	(a)		

HINTS & SOLUTIONS

1. (c) Let the angles are α and β, then $\alpha - \beta = 1°$

$$\Rightarrow \quad \alpha - \beta = \frac{\pi}{180°} \text{ is circular measure} \qquad \text{...(i)}$$

As given, $\quad \alpha + \beta = 1 \qquad$ ...(ii)

On solving Eqs. (i) and (ii), we get,

$$\alpha = \frac{1}{2}\left[1 + \frac{\pi}{180°}\right] \text{ and } \beta = \frac{1}{2}\left[1 - \frac{\pi}{180°}\right]$$

β is the smaller angle.

Hence, smaller angle $= \frac{1}{2}\left[1 - \frac{\pi}{180°}\right]$

2. (b) Let two parts of an angle θ are ϕ and ψ. So, $\theta = \phi + \psi$

So, $\tan\theta = \tan(\phi + \psi)$

$$= \frac{\tan\phi + \tan\psi}{1 - \tan\phi\tan\psi} = \frac{\frac{1}{8} + \frac{7}{9}}{1 - \frac{1}{8}\cdot\frac{7}{9}} = \frac{\frac{9+56}{72}}{\frac{72-7}{72}} = \frac{\frac{65}{72}}{\frac{65}{72}} = 1$$

$$= \tan\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$

3. (d) Since, A and B are complementary angles, then A + B = 90°

Now, $\cos A \cos B = \cos A \cos(90° - A)$

$$= \cos A \sin A = \frac{1}{2}\sin 2A$$

Since, $-1 \le \sin 2A \le 1$

Hence, $-\frac{1}{2} \le \frac{1}{2}\sin 2A \le \frac{1}{2}$.

Thus, greatest and least values of cos A cos B are $\frac{1}{2}$ and $-\frac{1}{2}$.

4. (c) We take Q_3 first,

$Q_3 = \sin A(\cos B + \cos C) + \sin B(\cos C + \cos A) + \sin C(\cos A + \cos B)$

$= \sin A\cos B + \sin A\cos C + \sin B\cos C + \sin B\cos A + \sin C\cos A + \sin C\cos B$

$= \sin(A+B) + \sin(B+C) + \sin(C+A) = Q_1$

$\Rightarrow \quad Q_3 = Q_1$

5. (a) Given : $2\sin\theta = x + \frac{1}{x}$

We know that $-1 \le \sin\theta < 1, -2 \le 2\sin\theta < 2$

So, $-2 \le x + \frac{1}{x} < 2$

Thus, the given equation is valid only if $x = \pm 1$

6. (c) Given that : $\sin(\pi\cos x) = \cos(\pi\sin x)$

So, $\cos\left(\frac{\pi}{2} - \pi\cos x\right) = \cos(\pi\sin x)$

$\Rightarrow \frac{\pi}{2} - \pi\cos x = \pi\sin x$

$\Rightarrow \sin x + \cos x = \frac{1}{2}$

Squaring both sides, we get

$\sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{4}$

$\Rightarrow \sin 2x = \frac{1}{4} - 1 = -\frac{3}{4}$

7. (a) As given, $\cos A = \cos B \cos C$ ...(1)

$\tan A - \tan B - \tan C$

$= \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} - \frac{\sin C}{\cos C}$

$= \frac{\sin A}{\cos A} - \frac{(\sin B\cos C + \cos B\sin C)}{\cos B\cos C}$

$= \frac{\sin A - \sin(B \quad C)}{\cos A}$ ....[using (1)]

$= \frac{\sin A - \sin(\pi - A)}{\cos A}$ [Since, $A + B + C = \pi$. So, $B + C = \pi - A$]

$= \frac{\sin A - \sin A}{\cos A} = 0$

8. (a) $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$

$= \frac{4\cdot\frac{1}{2}(\sqrt{3}\cos 20° - \sin 20°)}{2\sin 20°\cos 20°}$

$= \left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)\left(\frac{4}{2\sin 20°\cos 20°}\right)$

$= (\sin 60°\cos 20° - \cos 60°\sin 20°)\left(\frac{4}{\sin 40°}\right)$

$= \sin 40° \; \frac{4}{\sin 40°} = 4$

($\because \sin(A - B) = \sin A\cos B - \cos A\sin B$)

9. (a) As given, $\tan\theta + \cot\theta = (\tan\theta)^i + (\cot\theta)^i$

Also, $45° \leq \theta < 90°$ and $i \geq 2$

which is possible only when $\theta = 45°$

Since, $\tan 45° + \cot 45° = 1 + 1 = 2$

and $(\tan 45°)^i + (\cot 45°)^i = 1 + 1 = 2$

Thus, $\sin\theta + \cos\theta = \sin 45° + \cos 45°$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

10. (c) Given that $\tan\theta = m$ and $\tan 2\theta = n$

We know from fundamentals that

$\Rightarrow \tan 3\theta = \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta}$

Since, $\tan 3\theta = \tan\theta + \tan 2\theta$.....(as given)

$\Rightarrow \tan\theta + \tan 2\theta = \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta}$

$\Rightarrow (\tan\theta + \tan 2\theta)(1 - \tan\theta\tan 2\theta) - (\tan\theta + \tan 2\theta) = 0$

$\Rightarrow (\tan\theta + \tan 2\theta)\{1 - \tan\theta\tan 2\theta - 1\} = 0$

$\Rightarrow (\tan\theta + \tan 2\theta) - (\tan\theta\tan 2\theta) = 0$

$\Rightarrow (m + n) - mn = 0; \Rightarrow (m + n) = 0$

[since $m \neq 0$ and $n \neq 0$]

11. (a) $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

It is given that A and B are obtuse angle

$\Rightarrow \cos A = \pm\sqrt{1 - \sin^2 A} = \pm\sqrt{1 - \frac{16}{25}} = -\frac{3}{5}$

Negative sign is taken for cos A since A being obtuse lies in second quadrant.

$\sin B = \pm\sqrt{1 - \cos^2 B} = \pm\sqrt{1 - \left(\frac{-12}{13}\right)^2}$

$= \sqrt{\frac{169 - 144}{169}} = \frac{5}{13}$

Positive sign is taken since, sin B is positive in second quadrant.

$\Rightarrow \cos A = \frac{-3}{5}$ and $\sin B = \frac{3}{13}$

$\therefore \sin(A + B) = \sin A\cos B + \cos A\sin B$

$= \frac{4}{5}\times\left(\frac{-12}{13}\right) + \frac{-3}{5}\times\left(\frac{5}{13}\right) = -\frac{48}{65} - \frac{15}{65}$

$= \frac{-48 - 15}{65} = \frac{-63}{65}$

12. (a) Given equation is $\tan^2 B = \frac{1 - \sin A}{1 + \sin A}$

$\Rightarrow$ Applying componendo and dividendo

$\frac{1 + \tan^2 B}{1 - \tan^2 B} = \frac{2}{2\sin A}$

$\Rightarrow \sin A = \frac{1 - \tan^2 B}{1 + \tan^2 B} \Rightarrow \sin A = \cos 2B$

$\Rightarrow \sin A = \sin\left(\frac{\pi}{2} - 2B\right)$

$\Rightarrow A = \frac{\pi}{2} - 2B \Rightarrow A + 2B = \frac{\pi}{2}$

13. (a) As given, $\cos 20° - \sin 20° = p$
Squaring both sides, we get
$(\cos 20° - \sin 20°)^2 = p^2$
$\Rightarrow \cos^2 20° + \sin^2 20° - 2\sin 20° \cos 20° = p^2$
$\Rightarrow 1 - \sin 40° = p^2 \Rightarrow \sin 40° = 1 - p^2$

14. (d) Since, $p = \tan\alpha + \tan\beta$
and $q = \cot\alpha + \cot\beta$
$q = \cot\alpha + \cot\beta$

$$\Rightarrow q = \frac{1}{\tan\alpha} + \frac{1}{\tan\beta} = \frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}$$

$$q = \frac{p}{\tan\alpha\tan\beta} \Rightarrow \frac{1}{q} = \frac{\tan\alpha\tan\beta}{p}$$

Hence, $\frac{1}{p} - \frac{1}{q} = \frac{1}{p} - \frac{\tan\alpha\tan\beta}{p}$

$$= \frac{1 - \tan\alpha\tan\beta}{p} = \frac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta} = \frac{1}{\tan(\alpha + \beta)}$$

$= \cot(\alpha + \beta)$

15. (c) Given that number of degrees in A + Number of radians

in $A = \frac{180° + \pi}{3} = \frac{180°}{3} + \frac{\pi}{3} = 60° + \frac{\pi}{3}$

$\Rightarrow$ Angle $A = 60°$

16. (a) Since, $\sin^3\theta + \cos^3\theta = 0$
$\Rightarrow (\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta) = 0$
$(\because a^3 + b^3 = (a + b)(a^2 - ab + b^2))$
$\Rightarrow (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 0$

$$\Rightarrow (\sin\theta + \cos\theta)\left(1 - \frac{\sin 2\theta}{2}\right) = 0$$

$\Rightarrow \sin\theta + \cos\theta = 0$
or $\sin 2\theta = 2$
(discarded since $\sin^2\theta = 2$ is not possible)
$\Rightarrow \sin\theta + \cos\theta = 0 \Rightarrow \sin\theta = -\cos\theta$
$\Rightarrow \tan\theta = -1 \Rightarrow \theta = -\pi/4$

17. (b) The expression

$$\frac{\operatorname{cosec}(\pi + \theta)\cot\left\{\frac{9\pi}{2} - \theta\right\}\operatorname{cosec}^2(2\pi - \theta)}{\cot(2\pi - \theta)\sec^2(\pi - \theta)\sec\left\{\left(\frac{3\pi}{2}\right) + \theta\right\}}$$

$$= \frac{-\operatorname{cosec}\theta.\tan\theta.\operatorname{cosec}^2\theta}{-\cot\theta\sec^2\theta\operatorname{cosec}\theta}$$

$$= \frac{\tan^2\theta\operatorname{cosec}^2\theta}{\sec^2\theta} = \tan^2\theta \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1$$

18. (a) $\sin(A + B)\sin(A - B)$

$$= \frac{1}{2}\{2\sin(A + B).\sin(A + B)\}$$

$$= \frac{1}{2}\{\cos(A - B - A - B) - \cos(A - B + A + B)\}$$

[Since $2\sin X\sin Y = \cos(X - Y) - \cos(X + Y)$]

$$= \frac{1}{2}\{\cos 2B - \cos 2A\}$$

Also, $\sin(B + C)\sin(B - C) = \frac{1}{2}\{\cos 2C - \cos 2B\}$
and $\sin(C + A).\sin(C - A)$

$$= \frac{1}{2}\{\cos 2A - \cos 2C\}$$

$\therefore \sin(A + B)\sin(A - B) + \sin(B + C)\sin(B - c) + \sin(C + A).\sin(C - A)$

$$= \frac{1}{2}\{\cos 2C - \cos 2B + \cos 2A - \cos 2C + \cos 2B - \cos 2A\} = 0$$

19. (b) As given, $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta \quad \frac{1}{2m \quad 1}$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{\frac{m}{m \quad 1} \quad \frac{1}{2m \quad 1}}{1 - \frac{m}{m \quad 1} \times \frac{1}{2m \quad 1}} \quad \frac{m(2m \quad 1) \quad (m \quad 1)}{(m \quad 1)(2m \quad 1) - m}$$

$$= \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

So, $\alpha + \beta = \frac{\pi}{4}$

20. (c) As given, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$.
and $z = r\cos\theta$
Now, $x^2 + y^2 + z^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\phi\sin^2\theta + r^2\cos^2\theta$
$= r^2\sin^2\theta(\sin^2\phi + \cos^2\phi) + r^2\cos^2\theta$
$= r^2\sin^2\theta + r^2\cos^2\theta$
$= r^2(\sin^2\theta + \cos^2\theta)$
$= r^2$
Thus, $x^2 + y^2 + z^2$ is independent of θ and ϕ.

21. (b) Let $A = \cos\theta + \cos 2\theta$
$\therefore$ On differentiating w.r.t. to θ, we get

$$\frac{dA}{d\theta} = -\sin\theta - 2\sin 2\theta$$

Put $\frac{dA}{d\theta} = 0$ for maxima or minima.

$\sin\theta + 2\sin 2\theta = 0 \Rightarrow \sin\theta + 4\sin\theta\cos\theta$
$\Rightarrow \sin\theta(1 + 4\cos\theta) = 0$

$\Rightarrow$ $\sin\theta = 0$, or $4\cos\theta + 1 = 0$

$\Rightarrow$ $\cos\theta = 1$ or $\cos\theta = -\dfrac{1}{4}$

Now, $\dfrac{d^2A}{d\theta^2} = -\cos\theta - 4\cos 2\theta$

$= -\cos\theta - 4(2\cos^2\theta - 1)$

For $\cos\theta = 1$

$\dfrac{d^2A}{d\theta^2} = -\cos\theta - 4(2\cos^2\theta - 1)$

$= -1 - 4(2(1) - 1) = -1 - 4 = -5 < 0$

So, A is maximum at $\cos\theta = 1$

$\Rightarrow \left(\dfrac{d^2A}{d\theta^2}\right)_{\cos\theta = \frac{-1}{4}} = \dfrac{1}{4} - 4\left(2.\dfrac{1}{16} - 1\right) > 0$

[Since $\cos 2\theta = 2\cos^2\theta - 1$]

$\therefore$ A is minimum at $\theta = \cos^{-1}\left(\dfrac{-1}{4}\right)$.

Now minimum value of $\cos\theta + \cos 2\theta$

or of $\cos\theta + 2\cos^2\theta - 1$

$= \left(\dfrac{-1}{4}\right) + 2\left(\dfrac{1}{16}\right) - 1$

$= \dfrac{-1}{4} + \dfrac{1}{8} - 1 \quad \dfrac{-2+1-8}{8} \quad \dfrac{-9}{8}$

22. (c) As given, $3\tan\theta + 4 = 0 \Rightarrow \tan\theta = -\dfrac{4}{3}$

[θ lies in second quadrant i.e., $\dfrac{\pi}{2} < \theta < \pi$]

$\therefore$ $\cot\theta = -\dfrac{3}{4} \Rightarrow \cos\theta = -\dfrac{3}{5}$ and $\sin\theta = \dfrac{4}{5}$

Now, $2\cot\theta - 5\cos\theta + \sin\theta$

$= -\dfrac{6}{4} \quad \dfrac{15}{5} \quad \dfrac{4}{5} \quad \dfrac{-30 \quad 60 \quad 16}{20} \quad \dfrac{23}{10}$

23. (d) $\text{cosec}\left(\dfrac{13\pi}{12}\right) = \text{cosec}\left(\pi + \dfrac{\pi}{12}\right)$

$= -\text{cosec}\dfrac{\pi}{12} = -\text{cosec}\,15°$

$= -\sqrt{1+\cot^2 15°}$

$= -\sqrt{1+(2+\sqrt{3})^2}$ $\quad \left[\because \cot 15 \quad 2 \quad \sqrt{3}\right]$

$= -\sqrt{1+4+3+4\sqrt{3}}$

$= -\sqrt{8+4\sqrt{3}} = -\sqrt{6+2+2\sqrt{12}}$

$= -\sqrt{(\sqrt{6})^2 + (\sqrt{2})^2 + 2(\sqrt{6})(\sqrt{2})}$

$= -\sqrt{(\sqrt{6}+\sqrt{2})^2} = -\sqrt{6} - \sqrt{2}$

24. (a) The given expression is :

$(\sec\theta - \cos\theta)(\text{cosec}\,\theta - \sin\theta)(\cot\theta + \tan\theta)$

$= \left(\dfrac{1}{\cos\theta} - \cos\theta\right)\left(\dfrac{1}{\sin\theta} - \sin\theta\right)\left(\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}\right)$

$= \left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)\left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$

$= \dfrac{\sin^2\theta}{\cos\theta}.\dfrac{\cos^2\theta}{\sin\theta} \times \dfrac{1}{\sin\theta.\cos\theta} = \dfrac{\sin^2\theta.\cos^2\theta}{\cos^2\theta.\sin^2\theta} = 1$

25. (b) As given, $\alpha + \beta = \dfrac{\pi}{2}$ and $\beta + \gamma = \alpha$

$\Rightarrow \tan(\beta + \gamma) = \tan\alpha$

$\Rightarrow \dfrac{\tan\beta + \tan\gamma}{1 - \tan\beta\tan\gamma} = \tan\alpha$

$\Rightarrow \tan\beta + \tan\gamma = \tan\alpha - \tan\alpha\tan\beta\tan\gamma$

$\Rightarrow \tan\beta + \tan\gamma = \tan\alpha - \tan\alpha\cot\alpha\tan\gamma$

$\left(\because \beta + \alpha = \dfrac{\pi}{2} \Rightarrow \beta = \pi/2 - \alpha \Rightarrow \tan(\pi/2 - \alpha) = \cot\alpha\right)$

$\Rightarrow \tan\beta + \tan\gamma = \tan\alpha - \tan\gamma$

$\Rightarrow \tan\beta + 2\tan\gamma = \tan\alpha$

26. (c) The given expression is, $\dfrac{\cos 10° + \sin 20°}{\cos 20° - \sin 10°}$

$= \dfrac{\cos(90° - 80°) + \sin 20°}{\text{cso}(90° - 70°) - \sin 10°}$

$= \dfrac{\sin 80° + \sin 20°}{\sin 70° - \sin 10°} = \dfrac{2\sin\dfrac{80+20}{2}.\cos\dfrac{80-20}{2}}{2\cos\dfrac{70+10}{2}.\sin\dfrac{70-10}{2}}$

$= \dfrac{2\sin 50°\cos 30°}{2\cos 40°\sin 30°} = \dfrac{\sin(90° - 40°)\cot 30°}{\cos 40°}$

$= \dfrac{\cos 40°\cot 30°}{\cos 40°} = \cot 30° = \sqrt{3}$

27. (d) As given :

$\tan\alpha = 2\tan\beta$

$\Rightarrow \dfrac{\tan\alpha}{\tan\beta} \quad 2 \Rightarrow \dfrac{\sin\alpha/\cos\alpha}{\sin\beta/\cos\beta} \quad 2$

$\Rightarrow \dfrac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta} \quad 2$

Using componendo and dividendo we get

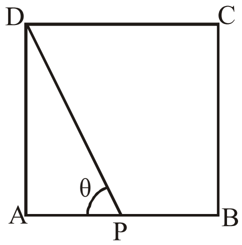
$\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \dfrac{2+1}{2-1} = 3$

$\Rightarrow \dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} \quad 3$

$\sin(\alpha+\beta) = 3\sin(\alpha-\beta)$

28. (c) Given expression is :
$\cos 306° + \cos 234° + \cos 162° + \cos 18° = x$ (Let)
$\Rightarrow x = \cos(360° - 54°) + \cos(180° + 54°) + \cos(180° - 18°) + \cos 18°$
$= \cos 54° - \cos 54° - \cos 18° + \cos 18° = 0$

29. (a) Let, ABCD be a square having length of side = x
$\therefore$ So, AD = AB = x



As given : $\frac{AP}{PB} = \frac{1}{2}$

$\Rightarrow \frac{AP}{PB + AP} = \frac{1}{2+1}$

$\Rightarrow AP = \frac{1}{1+2} x = \frac{x}{3}$

$\cos\theta = \frac{AP}{PD}$ $\quad (\because PB + AP = AB)$

Now, $PD^2 = AP^2 + AD^2 = \left(\frac{x}{3}\right)^2 + x^2$

$= \frac{x^2}{9} + x^2 = \frac{10x^2}{9}$

$\Rightarrow PD = \frac{\sqrt{10}x}{3}$

$\cos\theta = \frac{x/3}{\sqrt{10}x/3} = \frac{1}{\sqrt{10}}$

30. (b) Given, $\cos 3A = \frac{1}{2}$

$\Rightarrow \cos 3A = \cos\left(\frac{\pi}{3}\right)$ or $\left(\frac{5\pi}{3}\right)$

Since, $O < A < 360°$,

A can take the values, $\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}$ and $\frac{17\pi}{9}$.

So, sin A can assume 6 values.

31. (d) (a) In such a problem, we have to check option, one-by-one
$\sin^2\theta + \cos^6\theta = \sin^6\theta + \cos^2\theta$
$\Rightarrow \sin^6\theta - \cos^6\theta = \sin^2\theta - \cos^2\theta$
Taking L.H.S,
$\sin^6\theta - \cos^6\theta = (\sin^2\theta)^3 - (\cos^2\theta)^3$
$= (\sin^2\theta - \cos^2\theta)(\sin^4\theta + \sin^2\theta\cos^2\theta + \cos^4\theta)$
$(\because a^3 - b^3 = (a-b)(a^2 + ab + b^2))$
$= (\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta)$
$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta$
$= \sin^2\theta - \cos^2\theta - \sin^2\theta\cos^2\theta$
Which is not equal to R.H.S., $\sin^2\theta - \cos^2\theta$
Option (a) is not correct.

(b) $\because \operatorname{cosec}^6\theta - \cot^6\theta$
$(\operatorname{cosec}^2\theta - \cot^2\theta)[(\operatorname{cosec}^2\theta - \cot^2\theta)^2 + (\operatorname{cosec}\theta\cot\theta)]$
Which is not equal to $\operatorname{cosec}^2\theta - \cot^2\theta$.
$\therefore$ Option (b) is also not correct.

(c) $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$
$= 1 - 2\sin^2\theta\cos^2\theta$.
Which is not equal to $\sin^2\theta - \cos^2\theta$.
Hence, option (c) is also not correct.

(d) $(\operatorname{cosec}^2\theta + \cot^4\theta)$
$= \operatorname{cosec}^2\theta + (\operatorname{cosec}^2\theta - 1)^2$
$= \operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta + 1 - 2\operatorname{cosec}^2\theta$
$= \operatorname{cosec}^4\theta + 1 - \operatorname{cosec}^2\theta$
$= \operatorname{cosec}^4\theta + \cot^2\theta$
Thus option (d) is correct.

32. (a) As given :
$\sin A = \sin B$
and $\cos A = \cos B$
$\Rightarrow \frac{\sin A}{\cos A} = \frac{\sin B}{\cos B}$
$\Rightarrow \tan A = \tan B$ or $\tan B = \tan A = \tan(n\pi + A)$
$\Rightarrow B = n\pi + A$

33. (a) As given : $\alpha = \frac{\pi}{8}$

$\cos\alpha\cos 2\alpha\cos 4\alpha = \cos\frac{\pi}{8}.\cos\frac{\pi}{4}\cos\frac{\pi}{2} = 0$

$(\because \cos\frac{\pi}{2} = 0)$

34. (a) $\cot(-870°) = -\cot(2 \times 360° + 150°)$
$= -\cot 150° = -\cot(180° - 30°) = \cot 30° = \sqrt{3}$.

35. (c) (A) : $X = \{\theta \in [0, 2\pi] : \sin\theta = \cos\theta\}$
Number of elements in X is 2. Since, $\sin\theta = \cos\theta$ is possible at $\theta = 45°$ and $225°$
Since, $\cos\theta$ is negative in IInd quadrant but $\sin\theta$ is positive, R is false.

36. (b) From relation between minute 6 seconds measure : 60" = I'

$\Rightarrow 30'' = \frac{1'}{2}$

$\Rightarrow 35'30'' = \left(35 + \frac{1}{2}\right)' = \left(\frac{71}{2}\right)'$

Also, 60' = 1°

$\therefore 1' = \left(\frac{1}{60}\right)°$

$\Rightarrow \left(\frac{71}{2}\right)' = \left(\frac{71}{2} \times \frac{1}{60}\right)° = \left(\frac{71}{120}\right)°$

$\therefore \quad 114°35'30' = \left(114 + \frac{71}{120}\right)° = \left(\frac{13751}{120}\right)°$

We know that, 2π rad = 360°

$\Rightarrow 1° = \frac{2\pi}{360}$ rad

$\Rightarrow \left(\frac{13751}{120}\right)^{\circ} = \frac{2\pi}{360^{\circ}} \times \frac{13751}{120}$ rad

$= \frac{2 \times 22 \times 13751}{7 \times 360 \times 120}$ rad $= 2.0008069$ rad

$\Rightarrow 114° 35' 30'' = 2$ rad (approx.)

37. (a) Let $x = \left(\sin 22\frac{1°}{2} + \cos^2 22\frac{1°}{2}\right)^4$

$= \left\{\left(\sin 22\frac{1°}{2} + \cos 22\frac{1}{2}°\right)^2\right\}^2$

$= \left(\sin^2 22\frac{1°}{2} + \cos^2 22\frac{1°}{2} + 2\sin 22\frac{1}{2}^{\circ}\cos 22\frac{1°}{2}\right)^2$

$= (1 + \sin 45°)^2$ $\quad (\because 2\sin\theta\cos\theta = \sin 2\theta)$

$= \left(1 + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)^2$

$= \frac{2+1+2\sqrt{2}}{2} = \frac{3+2\sqrt{2}}{2}$

38. (d) The given expression

$\left(1 + \cos 67\frac{1°}{2}\right)\left(1 + \cos 112\frac{1°}{2}\right)$

Can also be writters as :

$\left(1 + \cos 67\frac{1°}{2}\right)\left\{1 + \cos\left(180° - 67\frac{1°}{2}\right)\right\}$

$= \left(1 + \cos 67\frac{1°}{2}\right)\left(1 - \cos 67\frac{1°}{2}\right)$

$= 1 - \cos^2 67°\frac{1°}{2} = \sin^2 67\frac{1°}{2}$

$= \frac{1 - \cos 135°}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$ $\quad \left(\because \sin^2 A = \frac{1 - \cos 2A}{2}\right)$

Which is an irrational number and is less than 1.

39. (c) As given : $\sin 2A = \frac{4}{5}$

$\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$

$\Rightarrow \frac{2\tan A}{1 + \tan^2 A} = \frac{4}{5}$

$\Rightarrow$ $10\tan A = 4 + 4\tan^2 A$

$\Rightarrow$ $5\tan A = 2 + 2\tan^2 A$

$\Rightarrow$ $2\tan^2 A - 5\tan A + 2 = 0$

$\Rightarrow$ $2\tan^2 A - 4\tan A - \tan A + 2 = 0$

$\Rightarrow$ $2\tan A(\tan A - 2) - 1(\tan A - 2) = 0$

$\Rightarrow$ $(2\tan A - 1)(\tan A - 2) = 0$

$\Rightarrow \tan A = \frac{1}{2}$ (since $A \le \frac{\pi}{4} \Rightarrow \tan A \ne 2$)

40. (a) Given expression,

$\frac{\cos 10° - \sin 10°}{\cos 10° + \sin 10°} = \frac{1 - \tan 10°}{1 + \tan 10°} = x$ (let)

$x = \frac{\tan 45 - \tan 10°}{1 + \tan 45.\tan 10°} = \tan(45 - 10) = \tan 35°.$

41. (a) Given expression

$4\sin x + 3\sin 2x - 2\sin 3x + \sin 4x = 2\sqrt{3}$

A quick way is to take from choices take choice (a) first, Let $x = \frac{\pi}{6}$

$\therefore 4\sin\frac{\pi}{6} + 3\sin\frac{\pi}{3} - 2\sin\frac{\pi}{2} + \sin\frac{2\pi}{3}$

$= 4\left(\frac{1}{2}\right) + \frac{3\sqrt{3}}{2} - 2 + \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$ Equation is satisfied

So, $x = \frac{\pi}{6}$ is true

42. (d) Fourth pair is not correct matched explained below

$\tan 420° = \tan(360 + 60) = \tan 60°$

$\tan 60° \ne \tan(-60°)$

43. (b) $\sin\frac{5\pi}{12} = \sin 75°$

$= \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

44. (a) We work out the given statements.

1. $\sin\frac{\pi}{12} = \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

2. $\cos\frac{\pi}{12} = \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

3. $\cot\frac{\pi}{12} = \cot 15° = 2 + \sqrt{3}$

So, correct sequence is 3 > 2 > 1.

45. (b) $\cos 2\theta = 2\cos^2\theta - 1$

Put $\theta = 15°$

$\therefore \cos 30° = 2\cos^2 15° - 1$

$\Rightarrow \frac{\sqrt{3}}{2} + 1 = 2\cos^2 15°$

$\Rightarrow \cos^2 15° = \frac{\sqrt{3}+2}{4}$

$\Rightarrow \cos 15° = \frac{1}{2}\left(\sqrt{2+\sqrt{3}}\right)$

46. (b) Given, equation is
$\tan\theta = k, k \neq 0$
$\Rightarrow \theta = \tan^{-1} k$
Now, we know,
If $\tan^{-1}x = \theta$ then $-\infty < x < \infty$ and $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$
Thus, θ will have 2 values between 0º and 360º.
or
The equation $\tan\theta = k, -\infty < k < \infty$ for any real values of k there are two values of the form α and π + α, in the interval $0 \leq \theta < 2\pi$, which satisfies the given equation.

47. (c) Given $\sin x + \sin y = a$
and $\cos x + \cos y = b$
$\therefore a^2 + b^2 = (\sin x + \sin y)^2 + (\cos x + \cos y)^2$
$= \sin^2 x + \sin^2 y + 2\sin x \sin y + \cos^2 x + \cos^2 y + 2\cos x \cos y$
$= (\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2(\cos x \cos y + \sin x \sin y)$
$= 1 + 1 + 2\cos(x - y)$
$\Rightarrow 2\cos(x - y) = a^2 + b^2 - 2$
$\Rightarrow \cos(x - y) = \frac{1}{2}(a^2 + b^2 - 2)$

48. (c) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4A}}}$
$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 4A)}}}$
$= \sqrt{2+\sqrt{2+2\cos 2A}} \qquad \sqrt{2\quad\sqrt{2(1\quad \cos 2A)}}$
$(\because 1 + \cos 4A = 2\cos^2 2A)$
$= \sqrt{2+2\cos A} = \sqrt{2(1\quad \cos A)}$
$(\because 1 + \cos 2A = 2\cos^2 A)$
$= 2\cos\left(\frac{A}{2}\right) \qquad \left(\because 1+\cos A = 2\cos^2\left(\frac{A}{2}\right)\right)$

49. (b) Given equation is
$\tan^2\phi + \tan^6\phi = \tan^3\phi . \sec^2\phi$
$\Rightarrow \tan^2\phi(1 + \tan^4\phi) = \tan^3\phi . \sec^2\phi$
$\Rightarrow (1 + \tan^4\phi) = \tan\phi . \sec^2\phi$
Now, $\sec^2\phi = 1 + \tan^2\phi$
$\therefore (1 + \tan^4\phi) = \tan\phi(1 + \tan^2\phi)$
$\Rightarrow 1 + \tan^4\phi = \tan\phi + \tan^3\phi$
It is not an identity

50. (a) $\sec A + \tan A = P$
$\Rightarrow \frac{1}{\cos A} + \frac{\sin A}{\cos A} = p$
$\Rightarrow \frac{1+\sin A}{\cos A} = p$
$\Rightarrow \frac{(1+\sin A)^2}{\cos^2 A} = p^2$
$\Rightarrow \frac{(1+\sin A)^2}{1-\sin^2 A} = p^2$
$\Rightarrow \frac{(1+\sin A)^2}{(1+\sin A)(1-\sin A)} = p^2$
$\Rightarrow \frac{1+\sin A}{1-\sin A} = p^2$
$\Rightarrow \frac{(1+\sin A)+(1-\sin A)}{(1+\sin A)-(1-\sin A)} = \frac{p^2+1}{p^2-1}$
(Using componendo and dividendo)
$\Rightarrow \frac{2}{2\sin A} = \frac{p^2+1}{p^2-1}$
$\Rightarrow \sin A = \frac{p^2-1}{p^2+1}$

51. (a) $\tan(-1575°) = -\tan(4 \times 360° + 135°)$
$= -\tan 135° = -\tan(90° + 45°) = \cot 45° = 1$

52. (c) Given, $\operatorname{cosec}^2\theta = 3\sqrt{3}\cot\theta - 5$
$\Rightarrow 1 + \cot^2\theta - 3\sqrt{3}\cot\theta + 5 = 0$
[Since, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$]
$\Rightarrow \cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$
Work with option, we find that
This equation is satisfied by $\theta = \frac{\pi}{6}$.
Thus, $\theta = \frac{\pi}{6}$

53. (c) Work with option,
$\cos(2\phi) - 1 = \frac{1-\tan^2\phi}{1+\tan^2\phi} - 1$
$= -\frac{2\tan^2\phi}{1+\tan^2\phi} = \frac{-(\tan^2\theta - 1)}{1+\frac{\tan^2\theta - 1}{2}}$
$= \frac{1-\tan^2\theta}{1+\tan^2\theta} \times 2 = \cos(2\theta)2$
Thus, $\cos 2\theta = \frac{\cos(2\phi)-1}{2}$

54. (d) $1 - \sin 10° \sin 50° \sin 70°$
$= 1 - \frac{1}{2}[2\sin 70° \sin 10° \sin 50°]$
$= 1 - \frac{1}{2}[(\cos 60° - \cos 80°)\sin 50°]$
$[\because 2\sin A \sin B = \cos(A - B) - \cos(A + B)]$
$= 1 - \frac{1}{2}\left[\frac{1}{2}\sin 50° - \frac{1}{2} 2\cos 80° \sin 50°\right]$
$= 1 - \frac{1}{4}[\sin 50° - \sin 130° + \sin 30°]$
$(\because 2\cos A \sin B = \sin(A + B) - \sin(A - B))$
$= 1 - \frac{1}{8} = \frac{7}{8} \qquad (\because \sin 130° = \sin(180° - 50°) = \sin 50°)$

55. (a) Let $\sin\theta = \frac{5}{13}$ and $\sin\ \phi = \frac{99}{101}$

$\therefore \cos\{\pi - (\theta + \phi)\}$

$= -\cos(\theta + \phi)$

$= -\{\cos\theta\cos\phi - \sin\theta\sin\phi\}$

$= -\left\{\sqrt{1 - \frac{25}{169}}\sqrt{1 - \left(\frac{99}{101}\right)^2} - \frac{5}{13} \times \frac{99}{101}\right\}$

$= -\left\{\frac{12}{13} \times \frac{20}{101} - \frac{5}{13} \times \frac{99}{101}\right\}$

$= -\left\{\frac{240}{1313} - \frac{495}{1313}\right\} = \frac{255}{1313}$

56. (d) $1000° = 2 \times 360° + 280°$
It is clear that the revolving live will be in fourth quadrant.

57. (c) 1 radian is approximately equal to 57°

58. (d) Since $\cot(x \quad y) \quad \frac{1}{\sqrt{3}} \quad \cot 60$ $[\cot 60° = \frac{1}{\sqrt{3}}]$

$\Rightarrow x + y = 60°$ ...(i)

and $\cot(x - y) = \sqrt{3} = \cot 30°$

$\Rightarrow x - y = 30°$ ...(ii)

From equations (i) and (ii), we get

$x = 45°$ and $y = 15°$

59. (a) Given, $x = \sin\theta\cos\theta$ and $y = \sin\theta + \cos\theta$

$\therefore \quad y^2 - 2x = (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 2\sin\theta\cos\theta = 1$

60. (d) Consider $\sin^4 x - \cos^4 x = p$

$\Rightarrow (\sin^2 x)^2 - (\cos^2 x)^2 = p$

$\Rightarrow (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = p$

$\Rightarrow \sin^2 x - \cos^2 x = p \ (\because \sin^2 x + \cos^2 x = 1)$

$\Rightarrow -\cos 2x = p \quad (\because \cos^2 x - \sin^2 x = \cos 2x)$

$\Rightarrow \cos 2x = -p$

$\therefore \quad |p| \le 1$

61. (b) We know that, for $\frac{\pi}{4} < \theta \quad \frac{\pi}{2}$

$\cos\theta < \sin\theta$

62. (c) Given $\sin^2 x + \sin^2 y = 1$

$\Rightarrow \sin^2 x = 1 - \sin^2 y$

$\Rightarrow \sin^2 x = \cos^2 y$

$\Rightarrow \sin x = \cos y$

Similarly By considering

$\sin^2 y = 1 - \sin^2 x$, we have

$\cos x = \sin y$

Now, Consider $\cot(x + y) = \frac{\cos(x \quad y)}{\sin(x \quad y)}$

$= \frac{\cos x\cos y - \sin x\sin y}{\sin(x \quad y)}$

$= \frac{\cos x\cos y - \cos x\ \cos y}{\sin(x \quad y)} \quad 0$

63. (b) Consider $\cos 10° + \cos 110° + \cos 130°$

$= \cos 130° + \cos 10° + \cos 110°$

$= 2\cos\left(\frac{130+10}{2}\right)\cos\left(\frac{130-10}{2}\right) + \cos 110°$

$= 2\cos\left(\frac{140}{2}\right)\cos\left(\frac{120}{2}\right) + \cos 110°$

$= 2\cos 60°\cos 70° + \cos 110°$

$= \cos 70° + \cos 110° \quad \left(\because \cos 60 \quad \frac{1}{2}\right)$

$= \cos(180° - 110°) + \cos 110°$

$= -\cos 110° + \cos 110° = 0 \quad (\because \cos(180° - \theta) = -\cos\theta)$

64. (a) We know length of arc of a circle $= 2\pi r\frac{\theta}{360}$

where 'r' is the radius and θ is the central angle.

So, $r = 5$, $\theta = 15°$

$\therefore$ Length $= 2\pi \times 5 \times \frac{15°}{360°} \quad = \frac{5\pi}{12}$ cm

65. (b) Let $P = \sin\theta\cos\theta$

Multiply and divide by 2, we get

$P = \frac{2\sin\theta\cos\theta}{2} = \frac{\sin 2\theta}{2}$

The maximum value of $\sin 2\theta$ is 1.

$\therefore$ Maximum value of $P = \frac{1}{2}$

66. (a) Given $\sin x + \operatorname{cosec} x = 2$

Consider $\sin^4 x + \operatorname{cosec}^4 x = (\sin^2 x + \operatorname{cosec}^2 x)^2 - 2(\sin^2 x \cos^2 x)$

$= [(\sin x + \operatorname{cosec} x)^2 - 2)]^2 - 2$

$= (4 - 2)^2 - 2 = 2$

67. (c) Consider $\tan 15° + \cot 15° = \frac{\sin 15°}{\cos 15°} + \frac{\cos 15°}{\sin 15°}$

$= \frac{\sin^2 15° + \cos^2 15°}{\cos 15° \sin 15°} = \frac{2 \times 1}{2\cos 15° \sin 15°}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{2}{\sin 30°} \quad 4\left(\because \sin 30° \quad \frac{1}{2}\right)$

68. (b) Given $A + B + C = \frac{\pi}{2}$

Take tan on both sides,

$\Rightarrow \tan(A + B + C) = \tan\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A} = \frac{1}{0}$

$\left(\because \frac{1}{0} = \infty\right)$

$\Rightarrow \tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

69. (b) Given,

$(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2$
$= k + \tan^2 x + \cot^2 x$

$\Rightarrow \sin^2 x + \operatorname{cosec}^2 x + 2 \sin x \operatorname{cosec} x + \cos^2 x + \sec^2 x + 2 \sec x \cos x = k + \tan^2 x + \cot^2 x$

$\Rightarrow \sin^2 x + \operatorname{cosec}^2 x + 2 + \cos^2 x + \sec^2 x + 2 = k + \tan^2 x + \cot^2 x$

($\because \sin x \operatorname{cosec} x = 1$ and $\sec x \cos x = 1$)

$\Rightarrow 1 + \operatorname{cosec}^2 x - \cot^2 x + \sec^2 x - \tan^2 x + 4 = k$

$\Rightarrow 1 + 1 + 1 + 1 + 4 = k \Rightarrow k = 7$

70. (b) Given, $p = \sin(989^\circ)\cos(991^\circ)$

Which can be written as

$= \sin(1080^\circ - 91^\circ)\cos(1080^\circ - 89^\circ)$

$= -\sin 91^\circ \cos 89^\circ$

$= -\sin(90^\circ + 1^\circ)\cos 89^\circ$

$= -\cos 1^\circ \cos 89^\circ$

As $\cos 1^\circ$ and $\cos 89^\circ$ are positive.

therefore their product is also + ve

Hence, p is finite and negative.

71. (b) Consider $\dfrac{1 - 3\tan^2 A}{3\tan A - \tan^3 A} = \dfrac{1}{\tan 3A}$

$\left(\because \tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right)$

$= \dfrac{1}{\tan\dfrac{41\pi}{4}} \quad \left(\because A \quad \dfrac{4\pi}{12}\right)$

$= \dfrac{1}{\tan\left(10\pi + \dfrac{\pi}{4}\right)} = \dfrac{1}{\tan\dfrac{\pi}{4}} = 1$

72. (d) **Statement-I :** Let $\theta = 1200°$

Consider $(\sec\theta + \tan\theta)^{-1} \quad \dfrac{1}{\sec\theta + \tan\theta}$

$\dfrac{\sec\theta - \tan\theta}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$

$\dfrac{\sec\theta - \tan\theta}{(\sec^2\theta - \tan^2\theta)} = \sec\theta - \tan\theta$

When $1200 \div 360$

we get remainder as 120°

Now, put $\theta = 120°$

$\Rightarrow (\sec\theta + \tan\theta)^{-1} \quad \sec 120 \; - \tan 120$

$= \dfrac{1}{\cos 120} - \tan(90 \quad 30\ )$

$= \dfrac{1}{\cos(90 \quad 30)} - \cot 30$

$\dfrac{1}{-\sin 30} \quad \sqrt{3}$

$= -2 \quad \sqrt{3}$ which is negative

Now,

Statement-II :

Consider $\operatorname{cosec}\theta - \cot\theta$

$\operatorname{cosec} 120 \; - \cot 120$

$= \dfrac{1}{\sin(90 \quad 30)} - \cot(90 \quad 30)$

$\dfrac{1}{\cos 30} \quad \tan 30$

$\dfrac{2}{\sqrt{3}} \quad \dfrac{1}{\sqrt{3}} \quad \dfrac{3}{\sqrt{3}}$

which is positive.

Hence, both statements are incorrect.

73. (a) Given $\cot\theta = 2\cos\theta,\ \dfrac{\pi}{2} < \theta < \pi$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta} = 2\cos\theta \Rightarrow \dfrac{\cos\theta}{2\cos\theta} = \sin\theta$

$\Rightarrow \dfrac{1}{2} = \sin\theta \Rightarrow \sin\dfrac{\pi}{6} = \sin\theta \Rightarrow \theta = \dfrac{\pi}{6}$

But $\dfrac{\pi}{2} < \theta < \pi \ \therefore \theta = \pi - \dfrac{\pi}{6} \quad \dfrac{5\pi}{6}$

74. (d) Let $\cot\theta = \dfrac{5}{12}$

$\Rightarrow \tan\theta = \dfrac{12}{5} = \dfrac{\text{perpendicular (p)}}{\text{base (b)}}$

$\therefore$ Hypotenuse (H) $= \sqrt{p^2 + b^2}$

$= \sqrt{(12)^2 + (5)^2} \;= \sqrt{144 + 25} \quad \sqrt{169} \quad 13$

Consider $2\sin\theta + 3\cos\theta$

$= 2\left(\dfrac{p}{H}\right) + 3\left(\dfrac{b}{H}\right)$ (H – Height)

But θ lies in 3rd quadrant and $\sin\theta$, $\cos\theta$ both are negative in 3rd quadrant

$\therefore 2\sin\theta + 3\cos\theta = 2\left(\dfrac{-p}{H}\right) + 3\left(\dfrac{-b}{H}\right)$

$= 2\left(\dfrac{-12}{13}\right) + 3\left(\dfrac{-5}{13}\right)$

$= \dfrac{-24 - 15}{13} = \dfrac{-39}{13} = -3$

which is an odd prime.

75. (d) Consider $\cos\dfrac{\pi}{9} + \cos\dfrac{\pi}{3} + \cos\dfrac{5\pi}{9} + \cos\dfrac{7\pi}{9}$

$\cos\dfrac{\pi}{9} \quad \dfrac{1}{2} \quad \cos\dfrac{5\pi}{9} \quad \cos\dfrac{7\pi}{9} \quad \left(\because \cos\dfrac{\pi}{3} \quad \dfrac{1}{2}\right)$

$= \dfrac{1}{2} + \left(\cos\dfrac{\pi}{9} + \cos\dfrac{5\pi}{9}\right) + \cos\dfrac{7\pi}{9}$

$= \dfrac{1}{2} + \left[2\cos\dfrac{6\pi}{18}\cos\dfrac{4\pi}{18}\right] + \cos\dfrac{7\pi}{9}$

$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$=\frac{1}{2}+\left[2\cos\frac{\pi}{3}\cos\frac{2\pi}{9}\right]+\cos\frac{7\pi}{9}$

$=\frac{1}{2}+\left[2.\frac{1}{2}\cos\frac{2\pi}{9}\right]+\cos\frac{7\pi}{9}$

$=\frac{1}{2}+\cos\frac{2\pi}{9}+\cos\frac{7\pi}{9}=\frac{1}{2}+2\cos\left(\frac{9\pi}{18}\right)\cos\left(\frac{5\pi}{18}\right)$

$=\frac{1}{2}+2\cos\frac{\pi}{2}\cos\frac{5\pi}{18} \quad \frac{1}{2}\left(\because \cos\frac{\pi}{2} \quad 0\right)$

76. (a) Consider $\sqrt{3}$ cosec $20^\circ-\sec 20^\circ=\frac{\sqrt{3}}{\sin 20^\circ}-\frac{1}{\cos 20^\circ}$

$=\frac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}$

Multiply and divide by 2 in N^r.

$=2\left(\frac{\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}\right)$

$2\left(\frac{\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}\right)$

$(\because \sin 60^\circ=\frac{\sqrt{3}}{2}$ and $\cos 60^\circ=\frac{1}{2})$

$=\frac{2\times 2[\sin(60^\circ-20^\circ)]}{2\sin 20^\circ\cos 20^\circ}$

($\because \sin A\cos B-\cos A\sin B=\sin(A-B)$ and $\sin 2\theta=2\sin\theta\cos\theta$)

$=\frac{4\sin 40^\circ}{\sin 40^\circ}=4$

77. (b) (A) $\tan 15^\circ=\tan(45^\circ-30^\circ)$

$=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{3+1-2\sqrt{3}}{2}=2-\sqrt{3}$

(B) $\tan 75^\circ=\tan(45^\circ+30^\circ) \quad \frac{\tan 45^\circ \quad \tan 30^\circ}{1-\tan 45^\circ\tan 30^\circ}$

$\frac{1 \quad \frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} \quad \frac{\sqrt{3} \quad 1}{\sqrt{3}-1}\times\frac{\sqrt{3} \quad 1}{\sqrt{3} \quad 1}$ (By Rationaliziang)

$2 \quad \sqrt{3}$

(C) $\tan(105^\circ)=\tan(60^\circ+45^\circ)$

$=\frac{\sqrt{3} \quad 1}{1-\sqrt{3}}\times\frac{1 \quad \sqrt{3}}{1 \quad \sqrt{3}} \quad \frac{(\sqrt{3} \quad 1)^2}{1-3}$

$=\frac{4+2\sqrt{3}}{-2}=-2-\sqrt{3}$

Hence, option (b) is correct.

78. (b) Let $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$

We know, $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}=\frac{\frac{5}{6}}{\frac{5}{6}}=1=\tan\pi/4$

$\Rightarrow A+B=\pi/4$

79. (c) Consider $\frac{\sin x}{1+\cos x}=\frac{2\sin x/2\cos x/2}{1+2\cos^2(x/2)-1}$

($\because \sin 2x=2\sin x\cos x$ and $\cos 2x=2\cos^2 x-1$)

$=\frac{2\sin x/2\cos x/2}{2\cos^2 x/2}=\frac{\sin x/2}{\cos x/2}=\tan x/2$

80. (d) Consider $\frac{1+\tan 15^\circ}{1-\tan 15^\circ}=\frac{\tan 45^\circ+\tan 15^\circ}{1-\tan 45^\circ\tan 15^\circ}$ $(\because \tan 45^\circ=1)$

$=\tan(45^\circ+15^\circ) \quad \left(\because \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}\right)$

$=\tan 60^\circ=\sqrt{3}$

81. (b) Consider $\sqrt{3}$ cosec $20^\circ-\sec 20^\circ$

$=\frac{\sqrt{3}}{\sin 20^\circ}-\frac{1}{\cos 20^\circ}=\frac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}$

Multiply and divide by 4

$=\frac{4}{2\sin 20^\circ\cos 20^\circ}\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)$

$=\frac{4}{\sin 40^\circ}\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)$

$(\because \sin 2\theta=2\sin\theta\cos\theta)$

$=\frac{4}{\sin 40^\circ}(\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ)$

$=\frac{4}{\sin 40^\circ}\sin(60^\circ-20^\circ)$

$(\because \sin(A-B)=\sin A\cos B-\cos A\sin B)$

$=4$

82. (c) $\tan\left(7\frac{1}{2}\right)^\circ=\frac{\sin\left(7\frac{1}{2}\right)^\circ}{\cos\left(7\frac{1}{2}\right)^\circ}$

Multiply and divide by $2\sin\left(7\frac{1}{2}\right)$; we get

$$\frac{2\sin^2\left(7\frac{1}{2}\right)^\circ}{2\sin\left(7\frac{1}{2}\right)^\circ\cos\left(7\frac{1}{2}\right)^\circ} = \frac{2\sin^2\left(\frac{15}{2}\right)^\circ}{2\sin\left(\frac{15}{2}\right)^\circ\cos\left(\frac{15}{2}\right)^\circ}$$

$$= \frac{1-\cos\left(2\times\frac{15}{2}\right)^\circ}{\sin\left(2\times\frac{15}{2}\right)^\circ}$$

$\left(\because \cos 2\theta = 1-2\sin^2\theta \text{ and } \sin 2\theta = 2\sin\theta\cos\theta\right)$

$$= \frac{1-\cos 15^\circ}{\sin 15^\circ} = \frac{1-\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \sqrt{2}\left(\sqrt{3}+1\right)-\left(2+\sqrt{3}\right)$$

$$= \sqrt{6}+\sqrt{2}-2-\sqrt{3} = \sqrt{6}-\sqrt{3}+\sqrt{2}-2$$

83. (d) Consider $\frac{\cos 15^\circ + \cos 45^\circ}{\cos^3 15^\circ + \cos^3 45^\circ}$

$$= \frac{\cos 15^\circ + \cos 45^\circ}{(\cos 15^\circ + \cos 45^\circ)(\cos^2 45^\circ + \cos^2 15^\circ - \cos 45^\circ \cos 15^\circ)}$$

$\left(\because a^3+b^3 = (a+b)(a^2+b^2-ab)\right)$

$$= \frac{1}{(\cos^2 45^\circ + \cos^2 15^\circ - \cos 45^\circ \cos 15^\circ)}$$

$$= \frac{1}{\frac{1}{2}+(\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ)^2 - \frac{\cos 15^\circ}{\sqrt{2}}}$$

$[\because \cos 15^\circ = \cos(45^\circ - 30^\circ)]$

$$= \frac{1}{\frac{1}{2}+\left(\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)^2 - \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}$$

$$= \frac{1}{\frac{1}{2}+\frac{3+1+2\sqrt{3}}{8}-\frac{\sqrt{3}+1}{4}}$$

$$= \frac{1}{\frac{4+4+2\sqrt{3}-2\sqrt{3}-2}{8}} = \frac{8}{6} = \frac{4}{3}$$

84. (c) Given equation is $4\left(\sin^2 x + \cos x\right) = 1$

$\Rightarrow 4\sin^2 x + 4\cos x = 1$

$\Rightarrow 4\sin^2 x + 4\cos x - 1 = 0$

$\Rightarrow 4\left(1-\cos^2 x\right) + 4\cos x - 1 = 0$

$\Rightarrow 4 - 4\cos^2 x + 4\cos x - 1 = 0$

$\Rightarrow -4\cos^2 x + 4\cos x + 3 = 0$

$\Rightarrow 4\cos^2 x - 4\cos x - 3 = 0$

This is the quadratic in $\cos x$.

$\Rightarrow 4\cos^2 x - 6\cos x + 2\cos x - 3 = 0$

$\Rightarrow (2\cos x - 3)(2\cos x + 1) = 0$

$\Rightarrow \cos x = \frac{3}{2}$ and $\cos x = -\frac{1}{2}$

$\cos x = \frac{3}{2}$ is not possible therefore $\cos x = -\frac{1}{2}$

$\Rightarrow \cos A = -\frac{1}{2} = \cos 210^\circ$

$\Rightarrow A = 210^\circ$

85. (d) Consider $\frac{1+\sin\theta}{\cos\theta} = \frac{1+\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}{\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}$

$\left(\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} \text{ and } \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$$= \frac{\left(1+\tan\frac{\theta}{2}\right)^2}{\left(1-\tan\frac{\theta}{2}\right)(1+\tan\frac{\theta}{2})}$$

$$= \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

Multiplied and divide by $2\sin\frac{\theta}{2}$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}-2\sin^2\frac{\theta}{2}}$$

$$= \frac{\sin\theta + 1 - \cos\theta}{\sin\theta - 1 + \cos\theta}$$

$(\because \sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta)$

86. (d) Let $\angle A = \frac{1}{2}$ radian, $\angle B = 99^\circ = \frac{99^\circ \times \pi}{180^\circ} = \frac{11\pi}{20}$

We know that, $\angle A + \angle B + \angle C = \pi$

(by angle sum property of triangle)

$\Rightarrow \frac{1}{2} + \frac{11\pi}{20} + \angle C = \pi$

$\Rightarrow \angle C = \pi - \frac{11\pi}{20} - \frac{1}{2} = \frac{9\pi - 10}{20}$

Hence, the third angle in radian is $\frac{9\pi - 10}{20}$.

87. (a) Consider $\left(\dfrac{\sec 18°}{\sec 144°}+\dfrac{\cos ec 18°}{\cos ec 144°}\right)$

$=\dfrac{\sec 18°}{\sec(180°-36°)}+\dfrac{\cos ec 18°}{\cos ec\,(180°-36°)}$

$=-\dfrac{\sec 18°}{\sec 36°}+\dfrac{\cos ec 18°}{\cos ec 36°}$ ($\because$ sin, cosec are + ve in 2^{nd} quadrant and sec is –ve in 2^{nd} quadrant)

$=\dfrac{\sin 36°}{\sin 18°}-\dfrac{\cos 36°}{\cos 18°}=\dfrac{\sin 36°\cos 18°-\cos 36°\sin 18°}{\sin 18°\cos 18°}$

$=\dfrac{\sin(36°-18°)}{\sin 18°\cos 18°}=\dfrac{\sin 18°}{\sin 18°\cos 18°}=\sec 18°$

$\left(\because \dfrac{1}{\cos x}=\sec x\right)$

88. (c) Let $\alpha+\beta=\dfrac{\pi}{4}$

$\Rightarrow \tan(\alpha+\beta)=\tan\dfrac{\pi}{4}$

$\Rightarrow \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=1$

$\Rightarrow \tan\alpha+\tan\beta=1-\tan\alpha\ \tan\beta$

By adding 1 on both sides, we get

$1+\tan\alpha+\tan\beta+\tan\alpha\ \tan\beta=2$

$\Rightarrow (1+\tan\alpha)(1+\tan\beta)=2$

89. (d) Consider $\sin 50°-\sin 70°+\sin 10°$

$=2\cos\dfrac{70°+50°}{2}.\sin\dfrac{50°-70°}{2}+\sin 10°$

$\left[\because \text{SinA}-\text{SinB}=2\cos\left(\dfrac{A+B}{2}\right)\text{Sin}\left(\dfrac{A-B}{2}\right)\right]$

$=-2\cos 60°\ \sin 10°+\sin 10°=-\sin 10°+\sin 10°=0$

$\left(\because \cos 60°=\dfrac{1}{2}\right)$

90. (b) Let $\cos A+\cos B=m$ ...(i)

and $\sin A+\sin B=n$ ...(ii)

Consider $\sin(A+B)=\dfrac{(m^2+n^2)\sin(A+B)}{m^2+n^2}$

$=\dfrac{[2+2\cos(A–B)]\sin(A+B)}{2+2\cos(A–B)}$

(from i and ii)

$=\dfrac{2\sin(A\quad B)\quad 2\sin(A\quad B)\cos(A-B)}{1\quad 1\quad 2\cos(A-B)}$

$=\dfrac{2\sin(A+B)+\sin(A+B+A-B)+\sin(A+B-A+B)}{1+1+2\cos(A-B)}$

$=\dfrac{2\sin(A+B)+\sin 2A+\sin 2B}{1+1+2\cos(A-B)}$

$=\dfrac{2(\cos A\quad \cos B)(\sin A\quad \sin B)}{\sin^2 A\quad \cos^2 A\quad \sin^2 B\quad \cos^2 B\quad 2\cos A\cos B\quad 2\sin A\sin B}$

$=\dfrac{2(\cos A\quad \cos B)(\sin A\quad \sin B)}{(\sin A\quad \sin B)^2\quad (\cos A\quad \cos B)^2}$

$=\dfrac{2mn}{m^2\quad n^2}$ (from (i) and (ii))

Hence, $\sin(A+B)=\dfrac{2mn}{m^2\quad n^2}$

91. (c) Since, $\cos^2\theta$ lies between 0 and 1 therefore,

$\sec^2\theta+\cos^2\theta\geq 2,\ \forall 0<\theta\quad \dfrac{\pi}{2}$

$\therefore\quad y\geq 2$

92. (d) $\tan A=\dfrac{3}{4}$ and $\tan B=-\dfrac{12}{5}$

$\therefore\quad \cot(A–B)=\dfrac{1}{\tan(A–B)}=\dfrac{1+\tan A\tan B}{\tan A–\tan B}$

93. (a) Consider $\sin 15°\sin 75°$

$=\sin(45°-30°)\sin(45°+30°)$

$=(\sin 45°\cos 30°-\cos 45°\sin 30°)(\sin 45°\cos 30°+\cos 45°\sin 30°)$

(using $\sin(A+B)=\sin A\cos B+\cos A\sin B$ and $\sin(A–B)\ \sin A\cos B-\cos A\sin B$)

$=\left(\dfrac{1}{\sqrt{2}}.\dfrac{\sqrt{3}}{2}-\dfrac{1}{\sqrt{2}}.\dfrac{1}{2}\right)\left(\dfrac{1}{\sqrt{2}}.\dfrac{\sqrt{3}}{2}+\dfrac{1}{\sqrt{2}}.\dfrac{1}{2}\right)$

$=\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)=\dfrac{3-1}{8}=\dfrac{2}{8}=\dfrac{1}{4}$

94. (b) Consider

$\dfrac{\sin\theta+\cos\theta-\tan\theta}{\sec\theta+\cos ec\theta-\cot\theta}$

Now, put value of $\theta=\dfrac{3\pi}{4}$, we get

$\dfrac{\sin\frac{3\pi}{4}+\cos\frac{3\pi}{4}-\tan\frac{3\pi}{4}}{\sec\frac{3\pi}{4}+\cos ec\frac{3\pi}{4}-\cot\frac{3\pi}{4}}$

$=\dfrac{\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\quad \tan\frac{\pi}{4}}{-\frac{1}{\cos\frac{\pi}{4}}\quad \frac{1}{\sin\frac{\pi}{4}}\quad \frac{1}{\tan\frac{\pi}{4}}}=\dfrac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}+1}{-\sqrt{2}+\sqrt{2}+1}=1$

95. (c) $\sin\left(292\frac{1}{2}\right)° = \sin\frac{585°}{2}$

$= \sqrt{\frac{1-\cos 585°}{2}}$

$\left(\because \cos 2\theta = 1-2\sin^2\theta \Rightarrow \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}\right)$

$= \sqrt{\frac{1-\cos(360°+225°)}{2}} = \sqrt{\frac{1-\cos 225°}{2}}$

$= \sqrt{\frac{1-\cos(180°+45°)}{2}}$

$= \sqrt{\frac{1 \quad \cos 45}{2}} \quad \sqrt{\frac{1 \quad \frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

96. (b) We know that, $1° < 1$ radian $\left(\because 1 \quad \frac{\pi}{180}\text{radian}\right)$

$\Rightarrow \sin 1° < \sin 1$

97. (b) We know, $\sin A = \frac{2\tan\frac{A}{2}}{1 \quad \tan^2\frac{A}{2}}$ ... (1)

If $\sin A$ is known then equation (1) becomes quadratic equation in $\tan\left(\frac{A}{2}\right)$. This mean 2 values of $\tan\left(\frac{A}{2}\right)$ can be calculated.

98. (a) Let $x = \sin\theta + \cos\theta$ and $y = \sin\theta \,.\, \cos\theta$

Now, $x^4 - 4x^2y - 2x^2 + 4y^2 + 4y + 1$

$= (\sin\theta + \cos\theta)^4 - 4(\sin\theta + \cos\theta)^2 y -$
$2(\sin\theta + \cos\theta)^2 + 4y^2 + 4y + 1$

$= (\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta)^2 -$
$4(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta)y$
$-2(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) + 4y^2 + 4y + 1$

$= (1+2y)^2 - 4(1+2y)y - 2(1+2y) + 4y^2 + 4y + 1$

$= 1 + 4y^2 + 4y - 4y - 8y^2 - 2 - 4y + 4y^2 + 4y + 1 = 0$

99. (b) Given, $(1+\tan\theta)(1+\tan\phi) = 2$

$\Rightarrow 1 + \tan\theta + \tan\phi + \tan\theta\tan\phi = 2$

$\Rightarrow \tan\theta + \tan\phi = 1 - \tan\theta\tan\phi$

$\Rightarrow \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi} = 1$

$\Rightarrow \tan(\theta+\phi) = \tan 45°$

$\Rightarrow \theta + \phi = 45°$

100. (c) Since, angle α is divided into two parts A and B.

$\therefore \alpha = A + B$ ... (1)

and $x = A - B$ (given) ... (2)

On solving (1) and (2) we get,

$A = \frac{\alpha \quad x}{2}, B = \frac{\alpha - x}{2}$

Now, consider $\frac{\tan A}{\tan B} \quad \frac{2}{1}$

$\Rightarrow \frac{\tan\left(\frac{\alpha \quad x}{2}\right)}{\tan\left(\frac{\alpha - x}{2}\right)} \quad \frac{2}{1}$

$\Rightarrow \frac{\sin\left(\frac{\alpha+x}{2}\right)\cos\left(\frac{\alpha-x}{2}\right)}{\cos\left(\frac{\alpha+x}{2}\right)\sin\left(\frac{\alpha-x}{2}\right)} \quad \frac{2}{1}$

Multiply and divide by 2,

$\Rightarrow \frac{2\sin\left(\frac{\alpha+x}{2}\right)\cos\left(\frac{\alpha-x}{2}\right)}{2\cos\left(\frac{\alpha+x}{2}\right)\sin\left(\frac{\alpha-x}{2}\right)} \quad 2$

$\Rightarrow \frac{\sin\alpha \quad \sin x}{\sin\alpha - \sin x} \quad 2$

$\Rightarrow \sin\alpha \quad \sin x \quad 2\sin\alpha - 2\sin x$

$\Rightarrow 3\sin x = \sin\alpha$

$\Rightarrow \sin x = \frac{\sin\alpha}{3}$

101. (d) Given expression is

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= \tan 9° - \tan 27° - \tan(90° - 27°) + \tan(90° - 9°)$

$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$

$= (\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \frac{1}{\sin 9 \quad \cos 9} - \frac{1}{\sin 27 \quad \cos 27}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin(90° - 36°)}$

$= \frac{2}{\sin 18°} - \frac{2}{\cos 36°}$

$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right] = 8 \times \frac{2}{4} = 4$

(By putting value of sin 18° and cos 36°)

102. (b) Let $x = y\cos\left(\frac{2\pi}{3}\right) = z\cos\left(\frac{4\pi}{3}\right)$

$\Rightarrow x = y\cos\left(\pi - \frac{\pi}{3}\right) = -y\cos\frac{\pi}{3} \quad \frac{-y}{2}$ ...(1)

and $x = z\cos\left(\pi + \frac{\pi}{3}\right) = -z\cos\frac{\pi}{3} = \frac{-z}{2}$ ...(2)

from (1) and (2)

$\frac{-y}{2} = \frac{-z}{2} \Rightarrow y = z$

Thus, $xy + yz + zx = zx + z^2 + xz = 2xz + z^2$

$= -y.(y) + y^2 = -y^2 + y^2 = 0$

103. (b) Let $\sin A + \sin B + \sin C = 3$

$\Rightarrow \sin A = \sin B = \sin C = 1$ ($\because$ max value of sin is 1)

$\therefore \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1-1} = 0$

Similarly, $\cos B = 0 = \cos C$

Hence, $\cos A + \cos B + \cos C = 0 + 0 + 0 = 0$

104. (c) Let $\tan A - \tan B = x$ and $\cot B - \cot A = y$.

$\Rightarrow \frac{1}{\tan B} - \frac{1}{\tan A} = y$

$\Rightarrow \frac{\tan A - \tan B}{\tan A \tan B} = y \Rightarrow \frac{x}{\tan A \tan B} = y$

Consider $\cot(A-B) = \left(\frac{1}{\tan(A-B)}\right)$

$= \frac{1 + \tan A \tan B}{\tan A - \tan B} = \frac{1 + \frac{x}{y}}{x} = \frac{y+x}{xy} = \frac{1}{x} + \frac{1}{y}.$

105. (c) Let $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

We know,

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2}+\frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}} = \frac{5}{6}\times\frac{6}{5}$

$\tan(A+B) = 1$

$\Rightarrow A + B = \tan^{-1}(1) = \frac{\pi}{4}$

Multiply by 4 on both side,

$4(A+B) = \frac{\pi}{4} \times 4 \Rightarrow 4A + 4B = \pi$

106. (c) Maximum value of

$3\cos x + 4\sin x + 5 = \sqrt{(4)^2 + (3)^2} + 5 = \sqrt{16+9} + 5$

$= \sqrt{25} + 5 = 5 + 5 = 10$

107. (a) Let $\sin\theta = \cos^2\theta$

$\Rightarrow \sin^2\theta = \cos^4\theta$ ...(1)

Consider

$\cos^2\theta(1 + \cos^2\theta) = \cos^2\theta + \cos^4\theta$

$= \cos^2\theta + \sin^2\theta$ (using 1)

$= 1$

108. (a) Consider $\tan 15° \tan 195°$

$= \tan 15° \tan(180 + 15°)$

$= \tan 15° \tan 15°$ ($\because \tan(180+\theta) = \tan\theta$)

$= (\tan 15°)^2$

$= \left(2 - \sqrt{3}\right)^2$ $\left(\because \tan 15° = 2 - \sqrt{3}\right)$

$= 4 + 3 - 4\sqrt{3} = 7 - 4\sqrt{3}$

109. (b) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = \frac{\sin^2 x + 1 + \cos^2 x + 2\cos x}{(1+\cos x)(\sin x)}$

$= \frac{2 + 2\cos x}{(1+\cos x)(\sin x)} = \frac{2(1+\cos x)}{(1+\cos x)(\sin x)}$

$= \frac{2}{\sin x} = 2\text{cosec } x$

110. (b) Let $\sin 3A = 1$

$\Rightarrow 3\sin A - 4\sin^3 A = 1$

$\Rightarrow 4\sin^3 A - 3\sin A + 1 = 0$

$\Rightarrow (\sin A + 1)(4\sin^2 A - 4\sin A + 1) = 0$

$\Rightarrow (\sin A + 1)(2\sin A - 1)^2 = 0$

$\Rightarrow \sin A = -1$ or $\frac{1}{2}$

Hence, $\sin A$ can take two distinct values.

111. (a) $\frac{\sin\theta}{\text{cosec}\theta} + \frac{\cos\theta}{\sec\theta} = \frac{\sin\theta}{\frac{1}{\sin\theta}} + \frac{\cos\theta}{\frac{1}{\cos\theta}} = \sin^2\theta + \cos^2\theta = 1$

112. (c) $\tan\theta + \sec\theta = 4$

$\Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = 4$

$\Rightarrow 1 + \sin\theta = 4\cos\theta$

Squaring on both side,

$(1 + \sin\theta)^2 = 16\cos^2\theta = 16(1 - \sin^2\theta)$

$(1 + \sin\theta)^2 = 16(1 - \sin\theta)(1 + \sin\theta)$

$1 + \sin\theta = 16 - 16\sin\theta$

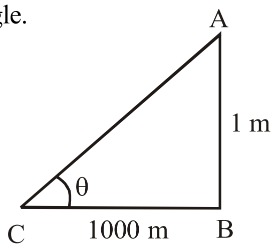
$17\sin\theta = 16 - 1$

$\sin\theta = \frac{15}{17}$

113. (a) Let AB be the pole of 1m.

BC = 1 km = 1000 m

Let 'θ' be the required angle.



Now, $\tan\theta = \left(\frac{1}{1000}\right)^\circ$

Since 'θ' is very small

$\therefore \tan\theta = \theta = \left(\frac{1}{1000}\right)^\circ$

Now, consider option (a)

$\left(\frac{9}{50\pi}\right)^\circ = \left(\frac{9}{50\times180}\right)^\circ = \left(\frac{9}{9000}\right)^\circ = \left(\frac{1}{1000}\right)^\circ$

Hence, required angle $= \left(\frac{9}{50\pi}\right)^\circ$

114. (a) Consider cos (A + B). sec (A – B)

$= \frac{\cos(A+B)}{\cos(A-B)} = \frac{\cos A\cos B - \sin A\sin B}{\cos A\cos B + \sin A\sin B}$

Divide Nr and Dr by sinA sinB,

$\frac{\cot A\cot B - 1}{\cot A\cot B + 1} = \frac{2-1}{2+1} = \frac{1}{3}$

115. (a) $\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\pi/3 - \tan\pi/4}{1 + \tan\pi/3\tan\pi/4}$

$= \frac{\sqrt{3}-1-3+\sqrt{3}}{(1)^2 - (\sqrt{3})^2} = \frac{2(\sqrt{3}-2)}{-2} = 2-\sqrt{3}$

116. (b) Consider $4\sin^2\theta + 2\sin\theta = 2\sin\theta\,(2\sin\theta + 1)$
Put θ = 18° in the above we get
Required expression = 2sin18° (2 sin 18° + 1)

As we know, $\sin 18° = \frac{\sqrt{5}-1}{4}$

$= 2\left(\frac{\sqrt{5}-1}{4}\right)\left[2\left(\frac{\sqrt{5}-1}{4}\right)+1\right]$

$= \left(\frac{\sqrt{5}-1}{2}\right)\left(\frac{\sqrt{5}-1}{2}+1\right)$

$= \frac{\sqrt{5}-1}{2}\left[\frac{\sqrt{5}+1}{2}\right] = \frac{5-1}{4} = 1$

117. (c) Consider, $\operatorname{cosec}\theta - \cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1}{\sqrt{3}} \Rightarrow \frac{1-\cos\theta}{\sin\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1-\left(1-2\sin^2\theta/2\right)}{2\sin\theta/2\cos\theta/2} = \frac{1}{\sqrt{3}} \Rightarrow \frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\sin\theta/2}{\cos\theta/2} = \frac{1}{\sqrt{3}} \Rightarrow \tan\frac{\theta}{2} = \tan 30°$

$\Rightarrow \theta = 60°$

$\therefore \cos\theta = \cos 60° = \frac{1}{2}$

118. (a) Consider $\sin 3\theta\cos 2\theta + \cos 3\theta\sin 2\theta$

$= \sin(3\theta + 2\theta)$ $\quad \left[\because \sin A\cos B + \cos A\sin B = \sin(A+B)\right]$

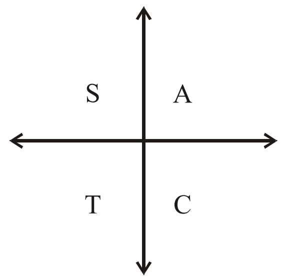
$= \sin(5\theta)$
We know, $-1 < \sin\theta < 1$
Hence, maximum value of given expression is 1.

119. (d) Given expression

$= \sin A\cos A.\frac{\sin A}{\cos A} + \cos A\sin A\frac{\cos A}{\sin A}$

$= \sin^2 A + \cos^2 A = 1$

120. (c) tanθ is positive in third quadrant



121. (c) $\sin(1920°) = \sin(360 \times 5 + 120°)$

$= \sin 120°$ $\quad (\because \sin(360° + \theta) = \sin\theta)$

$= \sin(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

122. (b) sin (A + B) = 1
⇒ sin (A + B) = sin 90°
⇒ A + B = 90° ...(1)

Given $\sin(A - B) = \frac{1}{2} = \sin 30°$

⇒ A – B = 30° ...(2)
On solving (1) and (2), we get
A = 60
B = 30

123. (c) tan(A + 2B). tan(2A + B)
Put A = 60 and B = 30 in above expression
We get tan(120°).tan(150°)
= tan(90° + 30°) tan(90° + 60°)

$= \cot 30°.\cot 60° = \sqrt{3}.\frac{1}{\sqrt{3}} = 1$

124. (b) $\sin^2 A - \sin^2 B = \sin^2 60° - \sin^2 30° = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

125. (b) Given expression
= sin (360° + 60°).cos(360° + 30°) + cos (360° – 60°) (–sin (360° – 30°)
= sin 60°.cos30° + cos60°sin30° = sin(60° + 30°)
= sin 90° = 1

126. (c) (1) $1° = \left(\frac{\pi}{180}\right)$ radian = 0.01746 radian which is less than 0.02 radian.

(2) 1 radian $= \left(\frac{180}{\pi}\right)^\circ = 57°16'22''$ approx. which is greater than 45°.
Hence, both statements are true.

127. (c) $-1 < \sin x < 1 \Rightarrow 1 < \sin^2 x < 1$
Hence, maximum value of $\sin^2 x = 1$

128. (a) We know in cyclic quadrilateral, ABCD
A + C = 180°, B + D = 180°
∴ A = 180° – C, B = 180° – D
sin A + sin B – sin C – sin D
= sin (180° – C) + sin (180° – D) – sin C – sin D
= sin C + sin D – sin C – sin D = 0

129. (a) $\sin 15° = \sin[45° - 30°]$
$= \sin 45°.\cos 30° - \cos 45° \sin 30°$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

130. (c) $4\sin^2\theta = 1$

$$\Rightarrow \sin^2\theta = \frac{1}{4} \Rightarrow \sin\theta \quad \frac{1}{2} \Rightarrow \theta \quad \frac{\pi}{6}$$

Hence θ take 4 values. (one value for each quadrant)

131. (d) We know that $\sin 18° = \frac{\sqrt{5}-1}{4}$

Now, $\cos 36° = 1 - 2\sin^2 18°$

$$= 1 - \frac{2}{16}\left(\sqrt{5}-1\right)^2 = 1 - \frac{\left(3-\sqrt{5}\right)}{4}$$

$$\cos 36° = \frac{\sqrt{5}+1}{4}$$

Now, $\sin 18° \cos 36° = \frac{\left(\sqrt{5}\right)^2 - (1)^2}{16} = \frac{4}{16} = \frac{1}{4}$

132. (c) We have $\sec \alpha = \frac{13}{5}$

Since $\frac{3\pi}{2} < \alpha < 2\pi$

$\therefore \sin\alpha < 0$

Now, $\sin\alpha = -\sqrt{1 - \frac{1}{\sec^2\alpha}}$

$$= -\sqrt{1 - \frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

133. (b) $\tan(-585°) = -\tan 585°$

$$= -\tan[540° + 45°] = -\tan\left[3\pi + \frac{\pi}{4}\right]$$

$$= -\left[\frac{\tan 3\pi + \tan\frac{\pi}{4}}{1 - \tan 3\pi \tan\frac{\pi}{4}}\right] = -\left[\frac{0+1}{1-0\times 1}\right] = -1$$

134. (d) Since $\cos\theta > \sin\theta$, in $\left[0, \frac{\pi}{4}\right]$

and $\cos\theta < \sin\theta$, in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$\therefore \quad \cos 46° - \sin 46° = -ve$
and $\cos 44° - \sin 44° = +ve$
So, both the above statements are incorrect.

135. (b) Let θ be the required angle

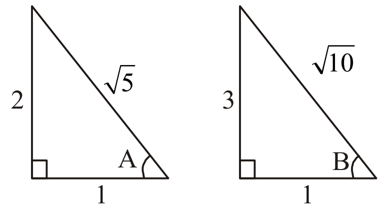
$$\therefore \quad \theta \quad \frac{\text{arc}}{\text{radius}} \quad \frac{1}{3} \text{ radians}$$

Now, 1 radian $= \frac{180°}{\pi}$

$$\therefore \quad \frac{1}{3} \text{ radian} = \frac{180}{\pi} \times \frac{1}{3} \quad \frac{60}{\pi}$$

Hence, Required angle $= \theta = \frac{60°}{\pi}$

136. (a) Given, $\sin A = \frac{2}{\sqrt{5}}$ and $\cos B = \frac{2}{\sqrt{10}}$



$$\therefore \quad \cos A = \frac{1}{\sqrt{5}} \qquad \sin B = \frac{3}{\sqrt{10}}$$

$\therefore \sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \left(\frac{2}{\sqrt{15}}\right)\left(\frac{1}{\sqrt{10}}\right) + \left(\frac{1}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right)$$

$$= \frac{2}{\sqrt{50}} + \frac{3}{\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{5}{\sqrt{25\times 2}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin(A+B) = \frac{1}{\sqrt{2}} \Rightarrow \sin(90+45) = \frac{1}{\sqrt{2}}$$

$(\because \sin(90+45) = \cos 45°)$

$\therefore \quad A = 90°$ and $B = 45°$
$\therefore \quad A + B = 90° + 45° = 135°$

137. (c) Let $\operatorname{cosec}\theta + \cot\theta = c$

$$\Rightarrow \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = c \Rightarrow \frac{1+\cos\theta}{\sin\theta} = c$$

$$\Rightarrow \frac{1 + \left(2\cos^2\frac{\theta}{2} - 1\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = c \Rightarrow \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = c$$

$$\Rightarrow \cot\frac{\theta}{2} = c \Rightarrow \cos\theta = \frac{1 - \frac{1}{c^2}}{1 + \frac{1}{c^2}} = \frac{c^2-1}{c^2+1}$$

$$\left(\because \cos\theta = \frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}\right)$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{c}$$

138. (c) Let $\sin\theta + 2\cos\theta = 1$ .... (i)
Consider $2\sin\theta - \cos\theta = \alpha$ (let) .... (ii)
squaring and adding
$\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta + 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta = 1 + \alpha^2$.
$\Rightarrow (\sin^2\theta + \cos^2\theta) + 4(\cos^2\theta + \sin^2\theta) = 1 + \alpha^2$.
$\Rightarrow 1 + 4 = 1 + \alpha^2 \Rightarrow \alpha^2 = 4 \Rightarrow \alpha = 2$.

139. (b) Let $A + B = 90°$
Consider $\sqrt{\sin A\sec B - \sin A\cos B}$
$= \sqrt{\sin A\sec(90° - A) - \sin A\cos(90° - A)}$
$= \sqrt{\sin A\,\text{cosec}A - \sin A\sin A} = \sqrt{1 - \sin^2 A} = \cos A$.

140. (a) Consider $\tan^4 A - \sec^4 A + \tan^2 A + \sec^2 A$
$= (\tan^2 A)^2 - (\sec^2 A)^2 + \tan^2 A + \sec^2 A$.
$= (\tan^2 A + \sec^2 A)(\tan^2 A - \sec^2 A) + \tan^2 A + \sec^2 A$
$= -(\tan^2 A + \sec^2 A) + \tan^2 A + \sec^2 A = 0$.

141. (b) Consider $\tan(105°) = \tan(60 + 45°)$
$$= \frac{\tan 60° + \tan 45°}{1 - \tan 60°\tan 45°}$$
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \qquad \left(\because \tan 60° = \sqrt{3} \text{ and } \tan 45° = 1\right)$$

142. (d) $x^2\tan(A - B)$
$$= x^2\left(\frac{\tan A - \tan B}{1 + \tan A\tan B}\right) = x^2\left(\frac{(x+1)-(x-1)}{1+\left(x^2-1\right)}\right)$$
$$= x^2\left(\frac{2}{x^2}\right) = 2$$

143. (d) Consider $(\sin^4\theta - \cos^4\theta + 1)\,\text{cosec}^2\theta$.
$$= \left[\left(\sin^2\theta\right)^2 - \left(\cos^2\theta\right)^2 + 1\right]\text{cosec}^2\theta$$
$$= \left[\left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta + \cos^2\theta\right) + 1\right]\text{cosec}^2\theta$$
$$= \left[1 - \cos^2\theta + \sin^2\theta\right]\text{cosec}^2\theta$$
$$= \left(\sin^2\theta + \sin^2\theta\right)\text{cosec}^2\theta$$
$$= 2\sin^2\theta \times \frac{1}{\sin^2\theta} = 2$$

144. (c) $$\frac{\cot x + \text{cosec}x - \left(\text{cosec}^2 x - \cot^2 x\right)}{\cot x - \text{cosec}x + 1}$$
$$= \frac{\cot x + \text{cosec}x - \left[(\text{cosec}\,x - \cot x)(\text{cosec}x + \cot x)\right]}{\cot x - \text{cosec}x + 1}$$
$$= \frac{(\cot x + \text{cosec}x)(1 + \cot x - \text{cosec}x)}{(\cot x - \text{cosec}x + 1)}$$
$= \cot x + \text{cosec}\,x$
$$= \frac{\cos x}{\sin x} + \frac{1}{\sin x} = \frac{1 + \cos x}{\sin x}$$

145. (d) $$\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2}}$$
$$= \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$$
$(\because \cos 2x = \cos^2 x - \sin^2 x)$

146. (c) $$\frac{\cot 54}{\tan 36}\quad\frac{\tan 20}{\cot 70} = \frac{\cot(90° - 36°)}{\tan 36°} + \frac{\tan(90° - 70°)}{\cot 70°}$$
$$= \frac{\tan 36}{\tan 36}\quad\frac{\cot 70}{\cot 70} = 1 + 1 = 2$$

147. (a) $\sin^2 20° + \sin^2 70° = \sin^2 20° + \sin^2(90° - 20°)$
$= \sin^2 20° + \cos^2 20° = 1$

148. (d) $(1 - \sin^2\theta)(1 + \tan^2\theta) = \cos^2\theta \,.\, \sec^2\theta$
$$= \cos^2\theta \,.\, \frac{1}{\cos^2\theta} = 1$$

149. (a) $$\tan 15° = \sqrt{\frac{1 - \cos 30}{1 \quad \cos 30}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 \quad \frac{\sqrt{3}}{2}}}\quad\sqrt{\frac{2-\sqrt{3}}{2 \quad \sqrt{3}}}$$
$$= \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}}\quad\sqrt{\frac{(2-\sqrt{3})^2}{1}} = 2 - \sqrt{3}$$

150. (b) 1. $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ 2. $\tan\left(\frac{3\pi}{4}\right) = -1$
3. $\tan\left(\frac{5\pi}{4}\right) = 1$ 4. $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$
$$-\sqrt{3} < -1 \quad \frac{1}{\sqrt{3}} \quad 1$$
Hence, $4 < 2 < 1 < 3$

151. (d) $\sin x.\cot x.\,\text{cosec}\,x.\tan x$
$= (\sin x.\,\text{cosec}\,x).(\cot x.\tan x)$
$= 1 \times 1 = 1$

152. (c) $3\tan^2 x = 1$
$$\tan x = \frac{1}{\sqrt{3}}$$
$$\tan x = \tan\left(\quad\frac{\pi}{6}\right)$$
$$x = n\pi \quad \frac{\pi}{6}$$

154. (a) $\sin\theta \in [-1, 1]$; $Q \in R$, the value of $\sin\theta$ lies between -1 to 1.
$\cos\theta \in [-1, 1]$; $Q \in R$, the value of $\sin\theta$ lies between -1 to 1.

155. (a) **Statement 1 :**

Given $n\left(\sin^2 67\frac{1^\circ}{2} - \sin^2 22\frac{1^\circ}{2}\right)$

or $n\left(\sin^2 \frac{135^\circ}{2} - \sin^2 \frac{45^\circ}{2}\right)$

$= n\left(\sin\frac{135^\circ}{2} + \sin\frac{45^\circ}{2}\right)\left(\sin\frac{135^\circ}{2} - \sin\frac{45^\circ}{2}\right)$

$= n\left[2\sin\frac{\left(\frac{135^\circ}{2}+\frac{45^\circ}{2}\right)}{2}.\cos\frac{\left(\frac{135^\circ}{2}-\frac{45^\circ}{2}\right)}{2}\right]$

$\left[2.\cos\frac{\left(\frac{135^\circ+45^\circ}{2}\right)}{2}.\sin\frac{\left(\frac{135^\circ-45^\circ}{2}\right)}{2}\right]$

$= n\left[2.\sin\left(\frac{90^\circ}{2}\right).\cos\left(\frac{45^\circ}{2}\right)\right]$

$\left[2.\cos\left(\frac{90^\circ}{2}\right).\sin\left(\frac{45^\circ}{2}\right)\right]$

$= 2n\left(2\sin\frac{45^\circ}{2}.\cos\frac{45^\circ}{2}\right)$ $(\sin 45^\circ . \cos 45^\circ)$

$= 2n . \sin\left(2\times\frac{45^\circ}{2}\right)\left(\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)$

$= 2n.\sin 45 \times \frac{1}{2} = n.\frac{1}{\sqrt{2}} = \frac{n}{\sqrt{2}}$

$\therefore \frac{n}{\sqrt{2}} > 1$ for all positive integers $n \geq 2$.

$\therefore$ Statement 1 is true

Statement 2

$nx > 1, \forall\ n \geq 2$

$\Rightarrow n > \frac{1}{x}, \forall n \geq 2$

$x \in (0, \infty)$, then we take $x = 1$

$n > 1$, but n is always greater or equal to 2 for all x positive real number.

$\therefore$ Statement 2 is false.

156. (a) **Statement : 1**

$\sin 3\theta = \cos 2\theta$

$\sin 3\theta = \sin\left(\frac{\pi}{2} - 2\theta\right)$

$3\theta = \frac{\pi}{2} - 2\theta$

$5\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{10}$

Statement : 2

One radian is the angle subtended at the centre of a circle by an arc of the same circle whose length is equal to radius of that circle.

Hence, statement 1 is correct.

157. (d) **Statement 1 :** $f_1(x) = \sin|x| + \cos|x|$, the value of $|\sin x|$ and $|\cos x|$ depends on its angles. $\sin|x| + \cos|x|$ is not always positive.

Statement 2 : $f_2(x) = \sin(x^2) + \cos(x^2)$, the value of x^2 between any value which lies in the interval $\left(\pi, \frac{3\pi}{2}\right)$, then value of $f_2(x) = \sin(x^2) + \cos(x^2)$ is always negative.

158. (c) $\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A}$

$= \frac{(1+\sin A)^2 - (1-\sin A)^2}{(1-\sin A)(1+\sin A)} = \frac{4\sin A}{\cos^2 A}$

$= \frac{4\sin A}{\cos A}.\frac{1}{\cos A} = 4\sec A.\tan A$

159. (b) $\frac{\cot 224^\circ - \cot 134^\circ}{\cot 226^\circ + \cot 316^\circ}$

$= \frac{\cot(180^\circ+44^\circ) - \cot(180^\circ - 46^\circ)}{\cot(180+46^\circ) + \cot(270^\circ+46^\circ)^\circ}$

$= \frac{\cot 44^\circ + \cot 46^\circ}{\cot 46^\circ - \tan 46^\circ} = \frac{\tan 46^\circ + \tan 44^\circ}{\tan 44^\circ - \tan 46^\circ}$

$= \frac{\sin(46^\circ+44^\circ)}{\sin(44^\circ-46^\circ)} = -\operatorname{cosec} 2^\circ$

160. (d) $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ$

$= (\cos 140^\circ + \cos 20^\circ) + \cos 100^\circ$

$= 2\cos\left(\frac{160^\circ}{2}\right).\cos\left(\frac{120^\circ}{2}\right) + \cos 100^\circ$

$= 2.\cos 80^\circ.\frac{1}{2} + \cos 100^\circ$

$= 2\cos\left(\frac{180^\circ}{2}\right).\cos\left(\frac{20^\circ}{2}\right)$

$= 2\cos 90^\circ . \cos 10^\circ$

$= 2 \times 0 \times \cos 10^\circ = 0$

161. (b) $\sin^2(3\pi) + \cos^2(4\pi) + \tan^2(5\pi)$

$= \sin^2(3\pi) + \cos^2(\pi + 3\pi) + \tan^2(5\pi)$

$= (\sin^2(3\pi) + \cos^2(3\pi)) + \tan^2(2\times 2\pi + \pi)$

$= 1 + \tan^2\pi = \sec^2\pi = 1$

162. (b) Consider, $\sqrt{1+\sin 2\theta}$

$= \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$

$= \sqrt{(\sin\theta + \cos\theta)^2} = \sin\theta + \cos\theta$

163. (d) $\cot A = 2$ and $\cot B = 3$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} = \frac{6-1}{2+3} = \frac{5}{5} = 1$$

$$\Rightarrow \cot(A+B) = \cot\left(\frac{\pi}{4}\right) \Rightarrow A+B = \frac{\pi}{4}$$

164. (b) $\sin^2 66\frac{1°}{2} - \sin^2 23\frac{1°}{2}$

$$= \left[\sin\left(90° - 23\frac{1°}{2}\right)\right]^2 - \sin^2 23\frac{1°}{2}$$

$$= \cos^2 23\frac{1°}{2} - \sin^2 23\frac{1°}{2}$$

$$= \cos 2\left(23\frac{1°}{2}\right) = \cos 47°$$

$(\because \cos 2A = \cos^2 A - \sin^2 A)$

$$= \cos\left[2 \times \left(\frac{47}{2}\right)\right] = \cos 47°$$

165. (b) $\dfrac{\cos 7x - \cos 3x}{\sin 7x - 2\sin 5x + \sin 3x}$

$$= \frac{-2\sin\frac{7x+3x}{2}.\sin\frac{7x-3x}{2}}{2\sin\frac{7x+3x}{2}.\cos\frac{7x-3x}{2} - 2\sin 5x}$$

$$\left(\begin{array}{l}\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right) \\ \text{and } \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\end{array}\right)$$

$$= \frac{-2\sin 5x.\sin 2x}{2\sin 5x\cos 2x - 2\sin 5x}$$

$$= \frac{-2\sin 5x.\sin 2x}{-2\sin 5x[1-\cos 2x]}$$

$$= \frac{\sin 2x}{1 - 1 + 2\sin^2 x} \qquad (\because \cos 2x = 1 - 2\sin^2 x)$$

$$= \frac{2\sin x\cos x}{2\sin^2 x} = \cot x$$

166. (b) $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

Applying componendo and dividendo, we get

$$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$$\Rightarrow \frac{2\sin x.\cos y}{2\cos x.\sin y} = \frac{2a}{2b} \Rightarrow \tan x \,.\, \cot y = \frac{a}{b}$$

$$\therefore \frac{\tan x}{\tan y} = \frac{a}{b}$$

167. (a) $\sin A \bullet \sin(60° - A)\sin(60° + A) = k\sin 3A$

$$\Rightarrow \sin A.\frac{\sin 3A}{4\sin A} = k.\sin 3A$$

$$\left[\because \sin(60° + A).\sin(60° - A) = \frac{\sin 3A}{4\sin A}\right]$$

$$\Rightarrow \frac{\sin 3A}{4} = k.\sin 3A$$

$$\therefore k = \frac{1}{4}$$

168. (a) Line $y = \sqrt{3}$ and graph $y = \tan x$

Now, we have $\sqrt{3} = \tan x$

$\Rightarrow \tan x = \tan 60°$

$\Rightarrow x = 60° \qquad \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

Hence, one intersecting point is possible in the given domain i.e., k = 1.

169. (b) $\tan 2\theta.\tan\theta = 1$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta}.\tan\theta = 1$$

$\Rightarrow 2\tan^2\theta = 1 - \tan^2\theta \Rightarrow 3\tan^2\theta = 1$

$$\Rightarrow \tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\Rightarrow \tan^2\theta = \tan^2(30°) = \tan^2\left(\frac{\pi}{6}\right) \quad \left[\because \theta = n\pi \pm \frac{\pi}{6}\right]$$

$$\therefore \theta = \frac{\pi}{6}$$

Sol. (Qs. 170–172)

$16\sin^5 x = 16(\sin^2 x)^2.\sin x$

$$= 16\left(\frac{1-\cos 2x}{2}\right)^2.\sin x$$

$= 4(1 + \cos^2 2x - 2\cos 2x).\sin x$

$$= 4\left(1 + \frac{1+\cos 4x}{2} - 2\cos 2x\right).\sin x$$

$$= \frac{4}{2}(3 + \cos 4x - 4\cos 2x).\sin x$$

$= (6 + 2\cos 4x - 8\cos 2x)\sin x$

$= 6\sin x + 2\sin x\cos 4x - 8\cos 2x.\sin x$

$= 6\sin x + \sin 5x - \sin 3x - 4(\sin 3x - \sin x)$

$[\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

$= 6\sin x + \sin 5x - \sin 3x - 4\sin 3x + 4\sin x$

$= \sin 5x - 5\sin 3x + 10\sin x.$

170. (a) Clearly, p = 1, hence option (a) is correct.

171. (d) Clearly, q = –5, hence option (d) is correct.

172. (c) Clearly, r = 10, hence option (c) is correct.

173. (c) From going by the options,
option (a), $\theta = 30°$, as we know that
$180° = \pi$ radian

$\therefore \quad 30° = \frac{30\pi}{180}$ radian

Now according to question,

$$\frac{30° \times 180°}{30° \pi} = \frac{180}{\pi}$$

Now number of degree in θ is multiplied by number of radians in θ.

$$\therefore \quad 30° \times \frac{30\pi}{180} = \frac{900\pi}{180} = \frac{10\pi}{2} = 5\pi \neq \frac{125\pi}{9}$$

From option (b),
$\theta = 45°$

$\therefore \quad 45° = \frac{45°\pi}{180}$ radian

Now according to question,

$$\frac{45° \times 180}{45° \pi} = \frac{180}{\pi}$$

Now number of degree in θ is multiplied by number of radian in θ.

$$\therefore \quad 45° \times \frac{45° \pi}{180°} = \frac{45\pi}{4} \neq \frac{125\pi}{9}$$

From option (c),
$\theta = 50°$
As we know that $180° = \pi$ radian

$\therefore \quad 50° = \frac{50\pi}{180}$ radian

Now according to question

$$\frac{50° \times 180°}{50° \pi} = \frac{180}{\pi}$$

Now number of degree in 'θ' is multiplied by number of radian in θ.

$$\therefore \quad 50° \times \frac{50\pi}{180} = \frac{2500\pi}{180} = \frac{125\pi}{9}$$

$\therefore$ Option (c) is correct.

Sol. (174-175):

174. (a) Here α is the root of equation
$25\cos^2\theta + 5\cos\theta - 12 = 0$
$\Rightarrow \quad 25\cos^2\alpha + 5\cos\alpha - 12 = 0$
$\Rightarrow \quad 25\cos^2\alpha + 20\cos\alpha - 15\cos\alpha - 12 = 0$
$\Rightarrow \quad 5\cos\alpha(5\cos\alpha + 4) - 3(5\cos\alpha + 4) = 0$
$(5\cos\alpha - 3)(5\cos\alpha + 4) = 0$

$\cos\alpha = \frac{3}{5}$ or $\cos\alpha = \frac{-4}{5}$

Here, $\frac{\pi}{2} < \alpha < \pi$

$\therefore \quad \cos\alpha = \frac{-4}{5}$

($\because$ In 2nd quadrant, $\cos\alpha$ value is negative)

Now, $\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1 - \frac{16}{25}}$

$\therefore \quad \sin\alpha = \frac{3}{5}$

$\therefore \quad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3}{5} \times \frac{-5}{4} = \frac{-3}{4}$

$\therefore$ Option (a) is correct.

175. (b) $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$

$= 2\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right)$

$= \frac{6}{5} \times \frac{-4}{5} = \frac{-24}{25}$

$\therefore$ Option (b) is correct.

176. (b) $(1 - \sin A + \cos A)^2$
$= 1 + \sin^2 A + \cos^2 A - 2\sin A - 2\sin A \cdot \cos A + 2\cos A$
$= 2 - 2\sin A - 2\sin A\cos A + 2\cos A$
$= 2(1 - \sin A) + 2\cos A(1 - \sin A)$
$= 2(1 + \cos A)(1 - \sin A)$
$\therefore$ Option (b) is correct.

177. (b) $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$

$$= \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta - \sin\theta}$$

$= \cos\theta + \sin\theta$
$\therefore$ Option (b) is correct.

178. (d) $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots\ldots + \ldots\ldots + \sin^2 75° + \sin^2 80° + \sin^2 85° + \sin^2 90°$
$\Rightarrow \quad \sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots\ldots \sin^2(90 - 15°) + \sin^2(90 - 10) + \sin^2(90 - 5) + 1$
$\Rightarrow \quad \sin^2 5 + \sin^2 10 + \sin^2 15° + \ldots\ldots + \cos^2 15° + \cos^2 10° + \cos^2 5° + 1$
$\Rightarrow \quad (1 + 1 + 1 + \ldots. 8 \text{ times}) + \sin^2 45° + 1$

$\Rightarrow \quad 8 + \frac{1}{2} + 1 = \frac{19}{2}$

179. (d) $\frac{\sin^3 A + \sin 3A}{\sin A} + \frac{\cos^3 A - \cos 3A}{\cos A}$

$$\Rightarrow \quad \frac{\sin^3 A + 3\sin A - 4\sin^3 A}{\sin A}$$

$$+\frac{\cos^3 A-\left[4\cos^3 A-3\cos A\right]}{\cos A}$$

$$\Rightarrow \quad \frac{3\sin A-3\sin^3 A}{\sin A}+\frac{\left(-3\cos^3 A+3\cos A\right)}{\cos A}$$

$= 3 - 3\sin^2 A - 3\cos^2 A + 3$

$= 6 - 3(\cos^2 A + \sin^2 A) = 6 - 3(1) = 3$

180. (b) $\sin x + \sin y = a$

$$\Rightarrow \quad 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=a \quad ...(1)$$

$\cos x + \cos y = b$

$$\Rightarrow \quad 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=b \quad ...(2)$$

dividing eq (1) & (2)

$$\tan\left(\frac{x+y}{2}\right)=\frac{a}{b}$$

Squaring eq (1) & (2) and adding -

$$4\cos^2\left(\frac{x-y}{2}\right)=a^2+b^2$$

$$\sec^2\left(\frac{x-y}{2}\right)=\frac{4}{a^2+b^2}$$

again, $\tan^2\left(\frac{x+y}{2}\right)+\tan^2\left(\frac{x-y}{2}\right)$

$$=\left(\frac{a}{b}\right)^2+\sec^2\left(\frac{x-y}{2}\right)-1$$

$$=\frac{a^2}{b^2}+\frac{4}{a^2 \quad b^2}-1=\frac{a^4-b^4+4b^2}{a^2b^2+b^4}$$

181. (b) $2a\sin^2\left(\frac{C}{2}\right) \quad 2c\sin^2\left(\frac{A}{2}\right) \quad 2a \quad 2c-3b$

$$\Rightarrow \quad 2a\times\frac{(s-a)(s-b)}{ab}+2c\times\frac{(s-b)(s-c)}{bc}=2a+2c-3b$$

$$\Rightarrow \quad \frac{2}{b}(s-b)[s-a+s-c]=2a+2c-3b$$

$$\Rightarrow \quad \frac{2}{b} \; s-b \; b \quad 2a \quad 2c-3b \qquad [\because 2s-a-c=b]$$

$\Rightarrow \quad a + c = 2b$

So a, b, c are in A.P.

182. (b) As we have already proven

$2b = a + c$

as-

$a = R\sin A$

$b = R\sin B$

$c = R\sin C$

$\Rightarrow 2(R\sin B) = R\sin A + R\sin C$

$\Rightarrow 2R(\sin B) = R(\sin A + \sin C)$

$2\sin B = \sin A + \sin C$

183. (b) $p=\tan\left(-\frac{11\pi}{6}\right)$

$$p=-\tan\left(2\pi-\frac{\pi}{6}\right)$$

$$p=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$$

$$q=\tan\left(\frac{21\pi}{4}\right)=\tan\left(6\pi-\frac{3\pi}{4}\right)$$

$$q=-\tan\frac{3\pi}{4}=+\tan\frac{\pi}{4}=1$$

$$r=\cot\left(\frac{283\pi}{6}\right)=\cot\left(46\pi+\frac{7\pi}{6}\right)=\cot\left(\pi+\frac{\pi}{6}\right)$$

$$r=\cot\frac{\pi}{6}=\sqrt{3}$$

$$p\times r=\frac{1}{\sqrt{3}}\times\sqrt{3}=1$$

$\therefore$ Statement (1) is incorrect.

also $\frac{p}{q}=\frac{q}{r}=\frac{1}{\sqrt{3}}$

So p, q, r are in G.P.

$\therefore$ Statement (2) is correct.

184. (d) $x^2 + bx + c = 0 \qquad b \neq 0$

$\because$ $\tan\alpha$ and $\tan\beta$ are roots of equation.

$$\therefore \quad \tan\alpha+\tan\beta=\frac{-b}{1}=-b \quad ...(1)$$

$$\therefore \quad \tan\alpha\tan\beta=\frac{c}{1}=c$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{-b}{1-c}$$

$= b(c-1)^{-1}$

185. (b) $\sin(\alpha+\beta)\sec\alpha\sec\beta$

$$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$=\tan\alpha+\tan\beta$

$=-b$

186. (c) Given $A = (\cos 12° - \cos 36°)(\sin 96° + \sin 24°)$

$B = (\sin 60° - \sin 12°)(\cos 48° - \cos 72°)$

$$\frac{A}{B} \quad \frac{[2\sin 24 \quad \sin 12 \quad][2\sin 60 \quad \cos 36 \quad]}{[2\cos 36 \quad \sin 24 \quad][2\sin 60 \quad \sin 12 \quad]}$$

$$\Rightarrow \frac{A}{B}=1$$

187. (c) Let $A = 30°$

$\Rightarrow$ $\sin A + 2\sin 2A + \sin 3A = \sin 30° + 2\sin 60° + \sin 90°$

$$=\frac{1}{2}+\frac{2\sqrt{3}}{2}+1=\frac{2\sqrt{3}+3}{2}$$

$(\because 2\cos^2 A = 1 + \cos 2A)$

Now, $4 \sin 2A \cos^2\left(\frac{A}{2}\right) = 2 \sin 2A\,[1 + \cos A]$

$= 2 \sin 60° [1 + \cos 30°] = \frac{2\sqrt{3}\quad 3}{2}$

Also, $\sin 2A = 2 \sin A \cos A$ & $\sin^2 A + \cos^2 A = 1$

$2 \sin 2A\left[\sin\frac{A}{2} + \cos\frac{A}{2}\right]^2$

$= 2\sin 2A\left[\sin^2\frac{A}{2} + \cos^2\frac{A}{2}\quad 2\sin\frac{A}{2}\cos\frac{A}{2}\right]$

$= 2\sin 2A\ \ 1 + \sin A\ \ = 2\sin 60°\ \ 1 + \sin 30°$

$= \frac{3\sqrt{3}}{2}$

& $8 \sin A \cos A \cos^2\left(\frac{A}{2}\right)$

$= 4 \sin A \cos A\,[1 + \cos A]$

$= 4 \sin 30° \cos 30° [1 + \cos 30°]$

$= \frac{2\sqrt{3}\quad 3}{2}$

188. (a) $x = \sin 70°.\sin 50°$ and $y = \cos 60°\,.\,\cos 80°$

$\Rightarrow xy = \cos 60°\,.\,\sin 70°\,.\,\sin 50°\,.\,\cos 80°$

$xy = \frac{1}{2}\cdot\sin(90-20)\cdot\sin(90-40)\cdot\cos 80$

$\Rightarrow xy = \frac{1}{2}\cdot\cos 20\cdot\cos 40\cdot\cos 80$

$(\because \sin(90-x) = \cos x)$

$\Rightarrow xy = \frac{1}{2}\cdot\cos 20°\cdot\cos(60-20)°\cdot\cos(60+20)°$

$\Rightarrow xy = \frac{1}{2}\left[\frac{1}{4}\cos 3(20°)\right] = \frac{1}{2}\times\frac{1}{4}\times\cos 60° = \frac{1}{16}.$

$\left[\because \cos\theta\cdot\cos(60-\theta)\cdot\cos(60+\theta) = \frac{1}{4}\cos 3\theta\right]$

189. (a) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 4$ ...(1)

Since max. value of $\sin\theta = 1$

We have four terms in LHS of eq. (1).

$\Rightarrow$ Every term should be $= 1$

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = \sin\theta_4 = 1$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \theta_4 = 90°$

Now,

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 = (\cos 90°)\times 4 = 0$.

190. (d) $\left[1\quad \cos\frac{\pi}{8}\right]\left[1\quad \cos\frac{3\pi}{8}\right]\left[1\quad \cos\frac{5\pi}{8}\right]\left[1\quad \frac{\cos 7\pi}{8}\right]$

We have,

$\cos\frac{7\pi}{8} = \cos\left[\pi - \frac{\pi}{8}\right] = -\cos\frac{\pi}{8}$

and $\cos\frac{5\pi}{8} = \cos\left[\pi - \frac{3\pi}{8}\right] = -\cos\frac{3\pi}{8}$

$\therefore\ \left[1\quad \cos\frac{\pi}{8}\right]\left[1\quad \cos\frac{3\pi}{8}\right]\left[1-\cos\frac{\pi}{8}\right]\left[1-\cos\frac{3\pi}{8}\right]$

$= \left[1-\cos^2\frac{\pi}{8}\right]\left[1-\cos^2\frac{3\pi}{8}\right]\quad \sin^2\frac{\pi}{8}\ \sin^2\frac{3\pi}{8}$

$\frac{1}{4}\left[2\sin^2\frac{\pi}{8}\ \ 2\sin^2\frac{3\pi}{8}\right]$

$= \frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)\right]$

$\left(\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right)$

$= \frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)\left(1\quad \frac{1}{\sqrt{2}}\right)\right]\quad \frac{1}{8}$

191. (a) Here, $z = x\cos\theta + y\sin\theta$

$z^2 = x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta$

$\Rightarrow 2xy\sin\theta\cos\theta = z^2 - x^2\cos^2\theta - y^2\sin^2\theta$

Let, $L = (x\sin\theta - y\cos\theta)^2$

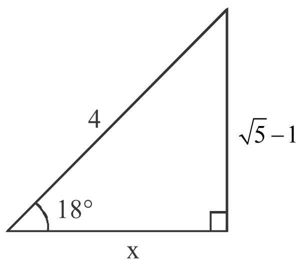
$\Rightarrow L = x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta$

$\Rightarrow L = x^2\sin^2\theta + y^2\cos^2\theta - [z^2 - x^2\cos^2\theta - y^2\sin^2\theta]$

$\Rightarrow L = x^2[\sin^2\theta + \cos^2\theta] + y^2[\sin^2\theta + \cos^2\theta] - z^2$

$\Rightarrow L = x^2 + y^2 - z^2$

192. (a)



$\because\ \ \sin 18\quad \frac{\sqrt{5}-1}{4}$

$x^2 = 4^2 - \left(\sqrt{5}-1\right)^2$

$\Rightarrow\ \ x = \sqrt{10+2\sqrt{5}}$

$\Rightarrow\ \ \cos 18\quad \frac{\sqrt{10\quad 2\sqrt{5}}}{4}$

$\Rightarrow\ \ 2\cos^2 9 - 1 = \frac{\sqrt{10+2\sqrt{5}}}{4}$

$\cos^2 9\quad \frac{\sqrt{10\quad 2\sqrt{5}}\quad 4}{8}$

$\Rightarrow\ \ \sin^2 81\quad \frac{4\quad \sqrt{10\quad 2\sqrt{5}}}{8}$

After squaring all the options available, we come to a conclusion that option (a) is correct.

193. (b) $L = \dfrac{1-\tan 2°\cot 62°}{\tan 152° - \cot 88°} = \dfrac{1-\tan 2°\cot(90-28)°}{\tan(180-28)° - \cot(90-2)°}$

$\Rightarrow L = \dfrac{1-\tan 2°\tan 28°}{-\tan 28° - \tan 2°} = -\left[\dfrac{1-\tan 2°\tan 28°}{\tan 2° + \tan 28°}\right]$

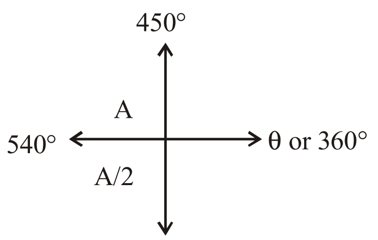
$\Rightarrow L = -\dfrac{1}{\tan(2+28)°} = -\dfrac{1}{\tan 30°} = -\sqrt{3}$

$\left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}\right]$

194. (d) Sin A $\dfrac{3}{5}$; 450 A 540

$\Rightarrow 225 \quad \dfrac{A}{2} \quad 270$

$\cos A = \dfrac{-4}{5}$ $(\because$ A lies in Q2$)$



$\therefore \cos^2\dfrac{A}{2} = \dfrac{1+\cos A}{2} = \dfrac{1}{10}$

$\Rightarrow \cos\dfrac{A}{2} \quad \dfrac{-1}{\sqrt{10}}$ $\left(\because \dfrac{A}{2}\text{ lies in Q3}\right)$

195. (d) $\dfrac{1}{\sin 10°} - \dfrac{\sqrt{3}}{\cos 10°} = \dfrac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10°\cos 10°}$

$= \dfrac{2\times 2\left[\dfrac{1}{2}\cos 10° - \dfrac{\sqrt{3}}{2}\sin 10°\right]}{2\sin 10°\cos 10°}$

$= \dfrac{4(\cos 60°\cos 10° - \sin 60°\sin 10°)}{\sin 20°}$

$= \dfrac{4.\cos(60°+10°)}{\sin 20°} = 4.\dfrac{\cos 70°}{\sin 20°} = 4.\dfrac{\sin 20°}{\sin 20°} = 4$

196. (c) $K = \sin\left(\dfrac{\pi}{18}\right)\sin\left(\dfrac{5\pi}{18}\right)\sin\left(\dfrac{7\pi}{18}\right)$

We know, 2 sin A sin B = cos (A – B) – cos (A + B)

$K = \dfrac{1}{2}.\sin\dfrac{\pi}{18}\left[2\sin\dfrac{5\pi}{18}\sin\dfrac{7\pi}{18}\right]$

$= \dfrac{1}{2}.\sin\dfrac{\pi}{18}\left[\cos\dfrac{2\pi}{18} - \cos\dfrac{\overset{2}{\cancel{12}}\pi}{\underset{3}{\cancel{18}}}\right]$

$= \dfrac{1}{2}.\dfrac{1}{2}\left[2\sin\dfrac{\pi}{18}\cos\dfrac{2\pi}{18} - 2\sin\dfrac{\pi}{18}\cos\dfrac{2\pi}{3}\right]$

$= \dfrac{1}{4}\left[\sin\left(\dfrac{3\pi}{18}\right) + \sin\left(\dfrac{-\pi}{18}\right) - 2\sin\dfrac{\pi}{18}\cos\left(\pi - \dfrac{\pi}{3}\right)\right]$

$= \dfrac{1}{4}\left[\sin\dfrac{\pi}{6} - \sin\dfrac{\pi}{18} + 2\sin\dfrac{\pi}{18}.\cos\dfrac{\pi}{3}\right]$

$= \dfrac{1}{4}\left[\sin\dfrac{\pi}{6} - \sin\dfrac{\cancel{\pi}}{\cancel{18}} + \cancel{2}\sin\dfrac{\cancel{\pi}}{\cancel{18}}.\dfrac{1}{\cancel{2}}\right]$

$= \dfrac{1}{4}\sin\dfrac{\pi}{6} = \dfrac{1}{4}\times\dfrac{1}{2} = \dfrac{1}{8}$

197. (a) $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \dfrac{\cancel{2}\sin\left(\dfrac{\alpha+\beta}{2}\right)\cancel{\cos\left(\dfrac{\alpha-\beta}{2}\right)}}{\cancel{2}\cos\left(\dfrac{\alpha+\beta}{2}\right)\cancel{\cos\left(\dfrac{\alpha-\beta}{2}\right)}}$

$= \tan\left(\dfrac{\alpha+\beta}{2}\right)$

198. (b) Given, sin θ = 3 sin (θ + 2α)

$\Rightarrow \dfrac{\sin\ \theta + 2\alpha}{\sin\theta} \quad \dfrac{1}{3}$

Apply componendo and divide do rule

$\Rightarrow \dfrac{\sin(\theta+2\alpha) + \sin\theta}{\sin(\theta+2\alpha) - \sin\theta} = \dfrac{1+3}{1-3}$

$\Rightarrow \dfrac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha} = \dfrac{4}{-2} = -2$

$\Rightarrow \dfrac{\tan(\theta+\alpha)}{\tan\alpha} = -2$

$\Rightarrow \tan(\theta+\alpha) = -2\tan\alpha \Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$

199. (a) $\tan 18 \quad \dfrac{\sin 18°}{\cos 18} \quad \dfrac{\dfrac{\sqrt{5}-1}{4}}{\dfrac{\sqrt{10 \quad 2\sqrt{5}}}{4}} \quad \dfrac{\sqrt{5}-1}{\sqrt{10 \quad 2\sqrt{5}}}$

200. (a) tan (α + β) = 2

tan (α – β) = 1

tan 2α = tan [(α + β) + (α – β)]

$= \dfrac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)}$

$= \dfrac{2+1}{1-2.1} = \dfrac{3}{-1} = -3$

201. (b) $\sec\theta - \text{cosec}\,\theta = \frac{4}{3} \Rightarrow \frac{1}{\cos\theta} - \frac{1}{\sin\theta} = \frac{4}{3}$

$\Rightarrow \frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = \frac{4}{3}$ ...(i)

$\Rightarrow \frac{(\sin\theta - \cos\theta)^2}{(\sin\theta\cos\theta)^2} = \frac{16}{9}$

$\Rightarrow \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta} = \frac{16}{9}$

$\Rightarrow \frac{1 - 2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta} = \frac{16}{9}$

Let $\sin\theta\cos\theta = x \Rightarrow \frac{1-2x}{x^2} = \frac{16}{9}$

$\Rightarrow 16x^2 + 18x - 9 = 0$

$\Rightarrow (8x-3)(2x+3) = 0$

$\Rightarrow x = \frac{3}{8}, x = \frac{-3}{2}$

$\therefore \sin\theta\cos\theta = \frac{3}{8}$

from (i), $\sin\theta - \cos\theta = \frac{4}{3} \times \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

202. (d) $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= \tan 9° - \tan 27° - \tan(90° - 27°) + \tan(90° - 9°)$

$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$

$= \tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

We know,

$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$

$\therefore \tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} \quad 2\left(\frac{\sin 54° - \sin 18}{\sin 18 \ \sin 54}\right)$

We also know, $\sin C - \sin D$

$= 2\cos\frac{(C+D)}{2}\sin\left(\frac{C-D}{2}\right).$

$\therefore 2\left(\frac{\sin 54° - \sin 18}{\sin 18 \ \sin 54}\right) \quad 2 \ 2 \ \frac{\cos 36 \ \sin 18}{\sin 18 \ \sin 54}$

$= 2 \cdot 2 \cdot \frac{\sin 54° \cdot \sin 18°}{\sin 18° \cdot \sin 54°} = 4$

203. (a) $\sqrt{3}\text{cosec } 20° - \sec 20° = \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$

$\frac{\sqrt{3}\cos 20° - \sin 20}{\sin 20 \ \cos 20}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20° \cos 20°}$

$= \frac{2(\cos 30° \cos 20° - \sin 30° \sin 20°)}{\left(\frac{2\sin 20° \cos 20°}{2}\right)}$

$\frac{4[\cos \ 30 \quad 20 \ ]}{\sin 40}$

$\frac{4\cos 50}{\sin 40} \quad \frac{4\cos \ 90° - 40}{\sin 40} \quad \frac{4\sin 40}{\sin 40} \quad 4.$

204. (d) $A - B = x$ and $\tan A : \tan B = p : q$

Also, given $\alpha = A + B$

$A - B = x$ and $\alpha = A + B$

$\Rightarrow A = \frac{x+\alpha}{2}$ and $B = \frac{\alpha - x}{2}$

Now, $\frac{\tan A}{\tan B} \quad \frac{\tan\left(\frac{x+\alpha}{2}\right)}{\tan\left(\frac{\alpha - x}{2}\right)} \quad \frac{p}{q}$ (Given)

$\Rightarrow \frac{2\sin\left(\frac{\alpha+x}{2}\right)\cos\left(\frac{\alpha-x}{2}\right)}{2\cos\left(\frac{\alpha+x}{2}\right)\sin\left(\frac{\alpha-x}{2}\right)} = \frac{p}{q}$

$\Rightarrow \frac{\sin\alpha \quad \sin x}{\sin\alpha - \sin x} \quad \frac{p}{q}$

$\Rightarrow \frac{\sin\alpha + \sin x + \sin\alpha - \sin x}{\sin\alpha + \sin x - \sin\alpha + \sin x} = \frac{p+q}{p-q}$

$\Rightarrow \frac{2\sin\alpha}{2\sin x} \quad \frac{p \quad q}{p-q}$

$\Rightarrow \frac{\sin\alpha}{\sin x} = \frac{p \quad q}{p-q} \Rightarrow \sin x \quad \frac{\sin\alpha \quad p-q}{p \quad q}$

205. (c) $\sqrt{1 \quad \sin A} \quad -\left(\sin\frac{A}{2} \quad \cos\frac{A}{2}\right)$

We know, $1 + \sin A = \left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)^2$

$\therefore \sqrt{1 \quad \sin A} \quad \left|\cos\frac{A}{2} \quad \sin\frac{A}{2}\right|$

We know, $|x| \quad \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x \quad 0 \end{cases}$

$\left|\sin\frac{A}{2} + \cos\frac{A}{2}\right| = \begin{cases} \sin\frac{A}{2} + \cos\frac{A}{2}, & \text{if } 2n\pi - \frac{\pi}{4} \leq \frac{A}{2} \leq 2n\pi + \frac{3\pi}{4} \\ -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right), & \text{if otherwise} \end{cases}$

$\therefore \sqrt{1 \quad \sin A} \quad -\left(\sin\frac{A}{2} \quad \cos\frac{A}{2}\right)$ when

$\frac{3\pi}{4} < \frac{A}{2} < \frac{7\pi}{4}$

$\Rightarrow \frac{3\pi}{2} < A < \frac{7\pi}{2}.$

206\. (c) $\sin x = \frac{1}{\sqrt{5}}, \sin y = \frac{1}{\sqrt{10}}, 0 < x < \frac{\pi}{2}, 0 < y < \frac{\pi}{2}$

$\cos x = \sqrt{1-\sin^2 x}$ $\quad$ $\cos y = \sqrt{1-\sin^2 y}$

$= \sqrt{1-\frac{1}{5}}$ $\quad$ $= \sqrt{1-\frac{1}{10}}$

$= \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$ $\quad$ $= \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$.

sin (x + y) = sin x cos y + cos x sin y

$= \frac{1}{\sqrt{5}}.\frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}}.\frac{1}{\sqrt{10}}$

$= \frac{5}{\sqrt{5}\cdot\sqrt{10}} = \frac{\sqrt{5}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{10}} = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$.

$\therefore \quad x + y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

207\. (c) $\frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$

$\sin c - \sin d = 2\cos\left(\frac{c+d}{2}\right)\sin\left(\frac{c-d}{2}\right)$

$\cos c - \cos d = 2\cos\left(\frac{c+d}{2}\right)\cos\left(\frac{c-d}{2}\right)$

$\therefore \frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x} = \frac{2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$

$= \frac{\sin\left(\frac{2x}{2}\right)}{\cos\left(\frac{2x}{2}\right)} = \frac{\sin x}{\cos x} = \tan x$.

208\. (c) sin 105° + cos 105°

= sin (60° + 45°) + cos (60° + 45°)

= (sin 60° . sin 45° + cos 60° . cos 45°) + (cos 60° cos 45° – sin 60° sin 45°)

$= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}$

$= \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

209\. (a) $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

Applying componendo and dividendo, we get

$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$

$\Rightarrow \frac{2\sin\left(\frac{x+y+x-y}{2}\right)\cos\left(\frac{x+y-x+y}{2}\right)}{2\cos\left(\frac{x+y+x-y}{2}\right)\sin\left(\frac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$

$\Rightarrow \frac{\sin x.\cos y}{\cos x.\sin y} = \frac{a}{b}$

$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$.

210\. (b) $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$

$\sin\alpha + \sin\beta = 0$

$\Rightarrow \sin\alpha = -\sin\beta$

$\Rightarrow \sin\alpha = \sin(\pi+\beta)$

$\Rightarrow \alpha = \pi + \beta$

211\. (c) Given, $\cos\frac{A}{2}$ has only one value.

We know, $\cos A = 2\cos^2\frac{A}{2} - 1$

$\Rightarrow 2\cos^2\frac{A}{2} = \cos A + 1 \Rightarrow \cos\frac{A}{2} = \sqrt{\frac{\cos A + 1}{2}}$

Since, $\cos\frac{A}{2}$ is single value, $\frac{\cos A + 1}{2} = 0$

$\Rightarrow \cos A = -1$

So, A is an odd multiple of 180°.

212\. (b) $\cos\alpha + \cos\beta + \cos\gamma = 0$ ....(1)

Given, $0 < \alpha \le \frac{\pi}{2}, 0 < \beta \le \frac{\pi}{2}, 0 < \gamma \le \frac{\pi}{2}$.

(1) is satisfied when $\alpha = \frac{\pi}{2}, \beta = \frac{\pi}{2}$ and $\gamma = \frac{\pi}{2}$.

$\therefore \sin\alpha + \sin\beta + \sin\gamma = \sin\frac{\pi}{2} + \sin\frac{\pi}{2} + \sin\frac{\pi}{2}$.

= 1 + 1 + 1 = 3.

213\. (d) Period of the function, sin x is 2π.

214\. (c) $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta}$

$= 2\tan\theta.\cos^2\theta$

$= 2\sin\theta.\cos\theta = \sin 2\theta$

215\. (a) $\frac{2}{\cos\theta} = \frac{\cos(\theta+\alpha)+\cos(\theta-\alpha)}{\cos(\theta+\alpha)\cos(\theta-\alpha)} = \frac{2\cos\theta.\cos\alpha}{\cos^2\theta - \sin^2\alpha}$

$\Rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$

$\Rightarrow \sin^2\alpha = \cos^2\theta(1-\cos\alpha)$

$\Rightarrow \cos^2\theta = \frac{\sin^2\alpha}{1-\cos\alpha} = 1 + \cos\alpha$

$\Rightarrow 1 - \sin^2\theta = 1 + \cos\alpha$

$\Rightarrow \sin^2\theta + \cos\alpha = 0$

216\. (a) Checking through options

300° = – 60°

So, $3[3 - \tan^2(-60°) - \cot(-60°)]^2$

$= 3\left[3 - 3 + \frac{1}{\sqrt{3}}\right]^2 = 3 \times \frac{1}{3} = 1$

217. (b) Given equation:

$\text{cosec } x + \cot x = \sqrt{3}$

$\text{cosec } x - \cot x = \frac{1}{\sqrt{3}}$

$\Rightarrow 2 \text{ cosec } x = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow \text{cosec } x = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$

Possible values of $x = \frac{\pi}{3}, \frac{2\pi}{3}$

∴ Option (b) is correct.

218. (b)

$[(2\cos\theta+1)(2\cos\theta-1)]^{10}[2\cos 2\theta-1]^{10}[2\cos 4\theta-1]^{10}$

when, $\theta = \frac{\pi}{8}$ (given)

$= \left(4\cos^2\frac{\pi}{8}-1\right)^{10}\left(2\times\frac{1}{\sqrt{2}}-1\right)^{10}(-1)^{10}$

$= \left(4\times\frac{2+\sqrt{2}}{4}-1\right)^{10}\left(\sqrt{2}-1\right)^{10}$

$= [(\sqrt{2}+1)(\sqrt{2}-1)]^{10} = 1^{10} = 1$

219. (a) Product of roots $= \frac{c}{a}$

$\Rightarrow \cos\alpha \, . \, \cos\beta = -\frac{3}{4}$

$\Rightarrow \frac{1}{\cos\alpha . \cos\beta} = \sec\alpha \, . \, \sec\beta = -\frac{4}{3}$

220. (b) $A = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + (1-\sin^2\theta)^2$

$= 1 + \sin^4\theta - \sin^2\theta$

$= 1 - \sin^2\theta(1-\sin^2\theta)$

$= 1 - \sin^2\theta \, . \, \cos^2\theta$

$= \frac{4-4\sin^2\theta.\cos^2\theta}{4} = \frac{4-\sin^2(2\theta)}{4}$

As, we know, $0 \le \sin^2 2\theta \le 1$

$\therefore \quad A = \frac{4-0}{4}$ or $\frac{4-1}{4} \Rightarrow \frac{3}{4} \le A \le 1$

221. (d) $25 \text{ cosec}^2 x + 36 \sec^2 x$.

Minimum value $= (\sqrt{25}+\sqrt{36})^2$

$= (5+6)^2 = (11)^2 = 121$

222. (a) $\frac{\sin 34°\cos 236° - \sin 56°\sin 124°}{\cos 28°\cos 88° + \cos 178°\sin 208°}$

$= \frac{\sin 34°(-\cos 56°) - \sin 56°\cos 34°}{\cos 28°\sin 2° + \cos 2°\sin 28°}$

$= \frac{-\sin 34°\cos 56° - \sin 56°\cos 34°}{\sin(28°+2°)}$

$= -\frac{\sin(34°+56°)}{\sin 30°} = \frac{-\sin 90°}{\sin 30°} = \frac{-1}{\frac{1}{2}} = -2$

223. (c) $\tan 54° = \tan(45°+9°)$

$= \frac{\tan 45° + \tan 9°}{1-\tan 45° \cdot \tan 9°} = \frac{1+\tan 9°}{1-\tan 9°}$

$= \frac{1+\frac{\sin 9°}{\cos 9°}}{1-\frac{\sin 9°}{\cos 9°}} = \frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$

Sol. (224-226)

224. (c) $p = x\cos\theta - y\sin\theta$

$q = x\sin\theta + y\cos\theta$

Given, $p^2 + 4pq + q^2 = Ax^2 + By^2$

Let us take $\theta = \frac{\pi}{4}$.

$p = x\cos\frac{\pi}{4} - y\cos\frac{\pi}{4} = \frac{x-y}{\sqrt{2}}$

$q = x\sin\frac{\pi}{4} + y\cos\frac{\pi}{4} = \frac{x+y}{\sqrt{2}}$

$pq = \frac{x^2-y^2}{2} \Rightarrow 2pq = x^2 - y^2$

$\Rightarrow 4pq = 2x^2 - 2y^2$ ...(1)

Now, $p^2 + q^2 = x^2\cos^2\theta + y^2\sin^2\theta - 2xy\cos\theta\sin\theta + x^2\sin^2\theta + y^2\cos^2\theta + 2xy\sin\theta\cos\theta = x^2 + y^2$ ...(2)

From (1), (2), $p^2 + q^2 + 4pq = x^2 + y^2 + 2x^2 - 2y^2$

$= 3x^2 - y^2$

Comparing this with the given form, we get

$\theta = \frac{\pi}{4}, A = 3, B = -1$

225. (b) 226. (a)

227. (a) Given, $\cos(\theta-\alpha) = a \Rightarrow \sin(\theta-\alpha) = \sqrt{1-a^2}$

$\cos(\theta-\beta) = b \Rightarrow \sin(\theta-\beta) = \sqrt{1-b^2}$

$\therefore \cos(\alpha-\beta) = \cos(\theta-\beta-(\theta-\alpha))$

$= \cos(\theta-\beta)\cos(\theta-\alpha) + \sin(\theta-\beta)\sin(\theta-\alpha)$

$= (b)(a) + \sqrt{1-b^2}\sqrt{1-a^2}$

$= ab + \sqrt{1-a^2}\sqrt{1-b^2}$

228. (a) $\sin^2(\alpha-\beta)+2ab\cos(\alpha-\beta)=1-\cos^2(\alpha-\beta)+2ab\cos(\alpha-\beta)=1-\cos(\alpha-\beta)\,[\cos(\alpha-\beta)-2ab]$

$$=1-(ab+\sqrt{1-a^2}\sqrt{1-b^2})\left[ab+\sqrt{1-a^2}\sqrt{1-b^2}-2ab\right]$$

$$=1-(\sqrt{1-a^2}\sqrt{1-b^2}+ab)(\sqrt{1-a^2}\sqrt{1-b^2}-ab)$$

$$=1-\left[(\sqrt{1-a^2}\sqrt{1-b^2})^2-(ab)^2\right]$$

$= 1 - [(1-a^2)(1-b^2) - a^2b^2]$
$= 1 - (1 - b^2 - a^2 + a^2b^2 - a^2b^2)$
$= 1 - 1 + b^2 + a^2 = a^2 + b^2$

229. (c) $\sin\alpha + \cos\alpha = p$
$\Rightarrow (\sin\alpha + \cos\alpha)^2 = p^2$
$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = p^2$
$\Rightarrow 1 + \sin 2\alpha = p^2$
$\Rightarrow \sin 2\alpha = p^2 - 1$
$\cos^2 2\alpha = 1 - \sin^2 2\alpha = 1 - (p^2 - 1)^2$
$= 1 - (p^4 + 1 - 2p^2) = -p^4 + 2p^2$
$= p^2(2 - p^2)$

230. (c) $\tan\theta = \frac{1}{2}$

$\tan\phi = \frac{1}{3}$

$$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}=\frac{\frac{5}{6}}{\frac{5}{6}}=1$$

$$\therefore \theta+\phi=\tan^{-1}(1)=\frac{\pi}{4}$$

231. (b) $\cos A = \frac{3}{4}$

$$\sin\left(\frac{A}{2}\right).\sin\left(\frac{3A}{2}\right)=\frac{1}{2}\left[2\sin\frac{A}{2}\sin\frac{3A}{2}\right]$$

$$=\frac{1}{2}\left[\cos\left(\frac{3A}{2}-\frac{A}{2}\right)-\cos\left(\frac{3A}{2}+\frac{A}{2}\right)\right]$$

$$=\frac{1}{2}[\cos A-\cos 2A]=\frac{1}{2}[\cos A-(2\cos^2 A-1)]$$

$$=\frac{1}{2}[\cos A-2\cos^2 A+1]$$

$$=\frac{1}{2}\left[\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)^2+1\right]$$

$$=\frac{1}{2}\left[\frac{3}{4}-\frac{18}{16}+1\right]=\frac{1}{2}\left[\frac{12-18+16}{16}\right]$$

$$=\frac{1}{2}\left[\frac{10}{16}\right]=\frac{5}{16}$$

232. (b) $\tan 75^\circ + \cot 75^\circ = 2+\sqrt{3}+2-\sqrt{3} = 4$

233. (b) $\cos 46^\circ \cos 47^\circ \cos 48^\circ \cos 49^\circ \ldots\ldots \cos 135^\circ$
We know, $\cos 90^\circ = 0$
$\therefore$ Given expression has $\cos 90^\circ$ and so it's value is 0.

234. (b) $\sin 2\theta = \cos 3\theta$
$\Rightarrow \sin 2\theta = \sin(90^\circ - 3\theta)$

$2\theta = 90^\circ - 3\theta$ $\qquad \therefore \sin\theta = \sin 18^\circ = \frac{\sqrt{5}-1}{4}$
$\Rightarrow 5\theta = 90^\circ$
$\Rightarrow \theta = 18^\circ$

235. (a) $(1+\tan\alpha\tan\beta)^2+(\tan\alpha-\tan\beta)^2-\sec^2\alpha\sec^2\beta$

$$=1+\tan^2\alpha\tan^2\beta+2\tan\alpha\tan\beta+\tan^2\alpha+\tan^2\beta-2\tan\alpha\tan\beta-\sec^2\alpha\sec^2\beta$$

$$=1+\tan^2\alpha\tan^2\beta+\tan^2\alpha+\tan^2\beta-\sec^2\alpha\sec^2\beta$$

$$=(1+\tan^2\alpha)(1+\tan^2\beta)-\sec^2\alpha\sec^2\beta$$

$$=\sec^2\alpha\sec^2\beta-\sec^2\alpha\sec^2\beta$$

$=0$

236. (b) $p = \text{cosec}\,\theta - \cot\theta$
$q = (\text{cosec}\,\theta + \cot\theta)^{-1}$

$$\Rightarrow \frac{1}{q}=\text{cosec}\,\theta+\cot\theta$$

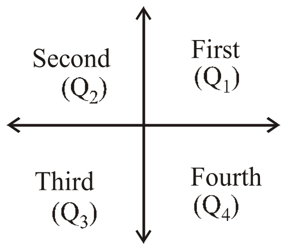
We know, $\text{cosec}^2\theta - \cot^2\theta = 1$
$\Rightarrow (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$$\Rightarrow \left(\frac{1}{q}\right)(p)=1$$

$\Rightarrow p = q$

237. (c) $\sin\theta+\cos\theta=\sqrt{2}\cos\theta$ ...(1)
Let $\cos\theta - \sin\theta = P$ ...(2)
$(1)^2 + (2)^2 \Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 2\cos^2\theta + p^2$
$\Rightarrow 2 = 2\cos^2\theta + p^2$
$\Rightarrow p^2 = 2(1 - \cos^2\theta) = 2\sin^2\theta$
$\Rightarrow p = \sqrt{2}\sin\theta$

238. (c) $\sin\theta = \frac{-1}{2}, \tan\theta = \frac{1}{\sqrt{3}}$



$\sin\theta$ is negative, $\tan\theta$ is positive
θ lies in third quadrant.

Properties of Triangle, Inverse Trigonometric Function

12

1. In a triangle ABC, $a = 2b$ and $\angle A = 3\angle B$. Which one of the following is correct ?
   (a) The triangle is isosceles
   (b) The triangle is equilateral
   (c) The triangle is right-angled
   (d) Such triangle does not exist *[2006-I]*

2. What is the value of
   $\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z) - \cot(\cot^{-1}x + \cot^{-1}y + \cot^{-1}z)$?
   (a) 0 (b) $2(x + y + z)$
   (c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{2} + x + y + z$ *[2006-I]*

3. What is the value of x that satisfies the equation $\cos^{-1}x = 2\sin^{-1}x$?
   (a) $\frac{1}{2}$ (b) -1
   (c) 1 (d) $-\frac{1}{2}$ *[2006-I]*

4. The median AD of a triangle ABC is bisected at F, and BF is produced to meet the side AC in P. If $AP = \lambda AC$, then what is the value of λ ?
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
   (c) $\frac{2}{3}$ (d) $\frac{1}{3}$ *[2006-I]*

5. What is the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?
   (a) $-\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
   (c) $-\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$ *[2006-I]*

6. What are the values of (x, y) satisfying the simultaneous equations $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ and $\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$?
   (a) (0, 1) (b) $(\frac{1}{2}, 1)$
   (c) $(1, \frac{1}{2})$ (d) $(\frac{\sqrt{3}}{2}, 1)$ *[2006-II]*

7. If the perimeter of a triangle ABC is 30 cm, then what is the value of $a\cos^2(C/2) + c\cos^2(A/2)$?
   (a) 15 cm (b) 10 cm
   (c) $\frac{15}{2}$ cm (d) 13 cm *[2006-II]*

8. In ΔABC, if $\angle A : \angle B : \angle C = 1 : 2 : 3$, then what is BC : CA : AB ?
   (a) 1 : 2 : 3 (b) $1 : \sqrt{3} : 2$
   (c) $2 : \sqrt{3} : 1$ (d) $\sqrt{3} : 1 : 2$ *[2006-II]*

9. The angles A, B, C of a triangle are in the ratio 2 : 5 : 5. What is the value of tan B tan C ?
   (a) $4 + \sqrt{3}$ (b) $4 + 2\sqrt{3}$
   (c) $7 + 4\sqrt{3}$ (d) $3 + 3\sqrt{3}$ *[2006-II]*

10. If A, B and C are angles of a triangle such that tan A = 1, tan B = 2, then what is the value of tan C?
    (a) 0 (b) 1
    (c) 2 (d) 3 *[2007-I]*

11. What is $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$ where $x > 0$, equal to?
    (a) $\sqrt{\frac{(x^2 \quad 1)}{(x^2 \quad 2)}}$ (b) $\sqrt{\frac{(x^2 \quad 2)}{(x^2 \quad 1)}}$
    (c) $\frac{(x^2+1)}{(x^2+2)}$ (d) $\frac{(x^2+2)}{(x^2+1)}$ *[2007-I]*

12. In a triangle ABC, if a = 2b and A = 3B then which one of the following is correct ?
    (a) The triangle is obtuse-angled
    (b) The triangle is acute-angled but not right-angled
    (c) The triangle is right-angled
    (d) The triangle is isosceles but not obtuse-angled *[2007-I]*

13. If $\sin^{-1} x = \tan^{-1} y$, what is the value of $\frac{1}{x^2} - \frac{1}{y^2}$?
   (a) 1 (b) –1
   (c) 0 (d) 2 *[2007-II]*

14. What is the value of :
$$\cos\left[\tan^{-1}\left\{\tan\left(\frac{15\pi}{4}\right)\right\}\right]?$$
   (a) $-\frac{1}{\sqrt{2}}$ (b) 0
   (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$ *[2007-II]*

15. Two angles of a triangle are $\tan^{-1}\frac{1}{2}$ and $\tan^{-1}\frac{1}{3}$. What is the third angle?
   (a) 30° (b) 45°
   (c) 90° (d) 135° *[2007-II]*

16. If median of the ΔABC through A is perpendicular to BC, then which one of the following is correct?
   (a) $\tan A + \tan B = 0$ (b) $\tan B - \tan C = 0$
   (c) $\tan C + 2\tan A = 0$ (d) $\tan B + \tan C = 0$
   [2007-II]

17. If $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \theta$, then what is the value of $\text{cosec}^{-1}(\sqrt{5})$?
   (a) $\left(\frac{\pi}{2}\right) + \theta$ (b) $\left(\frac{\pi}{2}\right) - \theta$
   (c) $\frac{\pi}{2}$ (d) $-\theta$ *[2007-II]*

18. What is the value of
$$\tan^{-1}\left(\frac{m}{n}\right) - \tan^{-1}\left(\frac{m-n}{m+n}\right)?$$
   (a) π (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$ *[2007-II]*

19. What is $\tan(\cos^{-1} x)$ equal to ?
   (a) $\frac{\sqrt{1-x^2}}{x}$ (b) $\frac{x}{1\ \ x^2}$
   (c) $\frac{\sqrt{1\ \ x^2}}{x}$ (d) $\sqrt{1-x^2}$ *[2008-I]*

20. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then what is the value of x?
   (a) $x = -\frac{1}{2}$ (b) $x = 1$
   (c) $x = \frac{1}{2}$ (d) $x = \frac{\sqrt{3}}{2}$ *[2008-I]*

21. In a triangle ABC, $b = \sqrt{3}$ cm, $c = 1$ cm, $\angle A = 30°$, what is the value of a?
   (a) $\sqrt{2}$ cm (b) 2 cm
   (c) 1 cm (d) $\frac{1}{2}$ cm *[2008-I]*

22. Let $-1 \le x \le 1$. If $\cos(\sin^{-1} x) = \frac{1}{2}$, then how many value does $\tan(\cos^{-1} x)$ assume?
   (a) One (b) Two
   (c) Four (d) Infinite *[2008-I]*

23. The equation $\sin^{-1}(3x - 4x^3) = 3\sin^{-1}(x)$ is true for all values of x lying in which one of the following intervals?
   (a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (b) $\left[\frac{1}{2}, 1\right]$
   (c) $\left[-1, -\frac{1}{2}\right]$ (d) $[-1, 1]$ *[2008-I]*

24. Which one of the following is not correct? *[2008-II]*
   (a) $\sin^{-1}\{\sin(5\pi/4)\} = -\pi/4$
   (b) $\sec^{-1}\{\sec(5\pi/4)\} = 3\pi/4$
   (c) $\tan^{-1}\{\tan(5\pi/4)\} = \pi/4$
   (d) $\text{cosec}^{-1}\{\text{cosec}(7\pi/4)\} = \pi/4$

25. If $\sin^{-1}x + \sin^{-1}y = \pi/2$ and $\cos^{-1}x - \cos^{-1}y = 0$, then values x and y are respectively *[2008-II]*
   (a) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}, \frac{1}{2}$
   (c) $\frac{1}{2}, -\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

26. ABC is a triangle in which $AB = 6$ cm, $BC = 8$ cm and $CA = 10$ cm. What is the value of $\cot(A/4)$? *[2008-II]*
   (a) $\sqrt{5} - 2$ (b) $\sqrt{5} + 2$
   (c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

27. If the sides of a triangle are 6cm, 10cm and 14 cm, then what is the largest angle included by the sides? *[2009-I]*
   (a) 90° (b) 120°
   (c) 135° (d) 150°

28. For finding the area of a triangle ABC, which of the following entities are required? *[2009-I]*
(a) Angles A, B and side a
(b) Angles A, B and side b
(c) Angles A, B and side c
(d) Either (a) or (b) or (c)

29. The formula $\sin^{-1}\{2x(1-x^2)\} = 2\sin^{-1} x$ is true for all values of x lying in the interval *[2009-I]*
(a) $[-1, 1]$ (b) $[0, 1]$
(c) $[-1, 0]$ (d) $\left[-1/\sqrt{2}, 1/\sqrt{2}\right]$

30. If $\sin A = 1/\sqrt{5}$, $\cos B = 3/\sqrt{10}$; A, B being positive acute angles, then what is $(A + B)$ equal to? *[2009-I]*
(a) $\pi/6$ (b) $\pi/4$
(c) $\pi/3$ (d) $\pi/2$

31. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then what is the value of x? *[2009-I]*
(a) a/b (b) ab
(c) b/a (d) $\frac{a-b}{1+ab}$

32. If in a ΔABC, $\cos B = (\sin A)/(2 \sin C)$, then the triangle is *[2009-II]*
(a) Isosceles triangle
(b) Equilateral triangle
(c) Right angled triangle
(d) Scalene triangle

33. If $\sin^{-1} x + \cot^{-1}(1/2) = \pi/2$, then what is the value of x? *[2009-II]*
(a) 0 (b) $1/\sqrt{5}$
(c) $2/\sqrt{5}$ (d) $\sqrt{3}/2$

34. In a ΔABC, $a + b = 3(1+\sqrt{3})$ cm and $a - b = 3(1-\sqrt{3})$ cm. If angle A is 30°, then what is the angle B? *[2009-II]*
(a) 120° (b) 90°
(c) 75° (d) 60°

35. What is the principle value of $\text{cosec}^{-1}(-\sqrt{2})$? *[2010-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $-\frac{\pi}{4}$ (d) 0

36. If $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$, then what is the value of x? *[2010-I]*
(a) 1 (b) 7
(c) 13 (d) 17

37. If angles A, B and C are in AP, then what is $\sin A + 2 \sin B + \sin C$ equal to? *[2010-I]*
(a) $4 \sin B \cos^2\left(\frac{A-C}{2}\right)$
(b) $4 \sin B \cos^2\left(\frac{A-C}{4}\right)$
(c) $4 \sin (2B) \cos^2\left(\frac{A-C}{2}\right)$
(d) $4 \sin (2B) \cos^2\left(\frac{A-C}{4}\right)$

38. **Statement I :** If $-1 \le x < 0$, then $\cos(\sin^{-1} x) = -\sqrt{1-x^2}$
Statement II : If $-1 \le x < 0$, then $\sin(\cos^{-1} x) = \sqrt{1-x^2}$
Which one of the following is correct in respect of the aobve statements? *[2010-I]*
(a) Both statements I and II are independently correct and statement II is the correct explanation of statement I
(b) Both statements I and II are independently correct but statement II is not the correct explanation of statement I
(c) Statement I is correct but statement II is false.
(d) Statement I is false but statement II is correct.

39. In a triangle ABC, $BC = \sqrt{39}$, $AC = 5$ and $AB = 7$. What is the measure of the angle A? *[2010-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

40. What is the value of $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}$? *[2010-II]*
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

41. ABC is a triangle in which BC = 10 cm, CA = 6 cm and AB = 8 cm. Which one of the following is correct? *[2010-II]*
(a) ABC is an acute angled triangle
(b) ABC is an obtuse angled triangle
(c) ABC is a right angled triangle
(d) None of these

42. In a ΔABC, if c = 2, A = 120°, a = $\sqrt{6}$, then what is C equal to ? *[2011-I]*
(a) 30° (b) 45°
(c) 60° (d) 75°

DIRECTIONS (Qs. 43-46) : *Read the following information carefully and give the answer.*

ABC is a triangle right-angled at *B*. The hypotenuse (*AC*) is four times the perpendicular (*BD*) drawn to it from the opposite vertex and AD < DC.

43. What is one of the acute angle of the triangle? *[2011-I]*
(a) 15° (b) 30°
(c) 45° (d) None of these

44. What is $\angle ABD$? *[2011-I]*
(a) 15° (b) 30°
(c) 45° (d) None of these

45. What is *AD*:*DC* equal to? *[2011-I]*
(a) $(7-2\sqrt{3}):1$ (b) $(7-4\sqrt{3}):1$
(c) 1 : 2 (d) None of these

46. What is tan (*A* – *C*) equal to? *[2011-I]*
(a) 0 (b) 1
(c) 2 (d) None of these

47. Consider the following *[2011-I]*

I. $\text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) = -\frac{\pi}{3}$

II. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

Which of the above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

48. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then what is *x* equal to?
(a) 0 (b) 1 *[2011-I]*
(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

49. What is the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$? *[2011-II]*
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

50. In any triangle *ABC*, the sides are 6 cm, 10 cm and 14 cm. Then the triangle is obtuse angled with the obtuse angle equal to *[2011-II]*
(a) 150° (b) 135°
(c) 120° (d) 105°

51. In a triangle *ABC*, if $A = \tan^{-1} 2$ and $B = \tan^{-1} 3$, then C is equal to *[2011-II]*
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

52. If the sides of a triangle are in the ratio $2:\sqrt{6}:1+\sqrt{3}$, then what is the smallest angle of the triangle? *[2011-II]*
(a) 75° (b) 60°
(c) 45° (d) 30°

53. In a triangle ABC, *a* = 8, *b* = 10 and *c* = 12. What is the angle C equal to? *[2011-II]*
(a) *A*/2 (b) 2A
(c) 3A (d) 3A/2

54. The sides *a*, *b*, *c* of a triangle ABC are in arithmetic progression and '*a*' is the smallest side. What is cos *A* equal to? *[2011-II]*
(a) $\frac{3c-4b}{2c}$ (b) $\frac{3c-4b}{2b}$
(c) $\frac{4c-3b}{2c}$ (d) $\frac{3b-4c}{2c}$

55. What is the value of $\cos\left\{\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}\right\}$? *[2012-I]*
(a) 63/65 (b) 33/65
(c) 22/65 (d) 11/65

56. In a triangle ABC if the angles A, B, C are in AP, then which one of the following is correct? *[2012-I]*
(a) c = a + b (b) $c^2 = a^2 + b^2 - ab$
(c) $a^2 = b^2 + c^2 - bc$ (d) $b^2 = a^2 + c^2 - ac$

57. If $\sin^{-1}1 + \sin^{-1}\frac{4}{5} = \sin^{-1}x$, then what is x equal to?
(a) 3/5 (b) 4/5 *[2012-I]*
(c) 1 (d) 0

58. If $\tan^{-1}2$, $\tan^{-1}3$ are two angles of a triangle, then what is the third angle? *[2012-I]*
(a) $\tan^{-1}2$ (b) $\tan^{-1}4$
(c) $\pi/4$ (d) $\pi/3$

59. What is the value of $\sec^2\tan^{-1}\left(\frac{5}{11}\right)$? *[2012-I]*
(a) 121/96 (b) 211/921
(c) 146/121 (d) 267/121

60. What is $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right]$ equal to ? *[2012-II]*
(a) 0 (b) 1/2
(c) 1 (d) 2

61. In any triangle ABC, a = 18, b = 24 and c = 30. Then what is sin C equal to : *[2013-I]*

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1

62. If $\sin^{-1}\left(\frac{2a}{1\quad a^2}\right)\quad \sin^{-1}\left(\frac{2b}{1\quad b^2}\right)\quad 2\tan^{-1}x$, then x is equal to *[2013-I]*

(a) $\frac{a-b}{1+ab}$ (b) $\frac{a-b}{1-ab}$

(c) $\frac{2ab}{1+ab}$ (d) $\frac{a+b}{1-ab}$

63. If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{3}\quad 1)$, then what is the area of the tringle ? *[2013-II]*

(a) $\frac{\sqrt{3}\quad 1}{2}$ (b) $2(\sqrt{3}\quad 1)$

(c) $\frac{\sqrt{3}\quad 1}{3}$ (d) $\frac{\sqrt{3}-1}{2}$

64. What is $\tan^{-1}\left(\frac{1}{2}\right)\quad \tan^{-1}\left(\frac{1}{3}\right)$ equal to ? *[2013-II]*

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

65. If x and y are positive and $xy > 1$, then what is $\tan^{-1}x + \tan^{-1}y$ equal to ? *[2014-I]*

(a) $\tan^{-1}\left(\frac{x\quad y}{1-xy}\right)$ (b) $\pi\quad \tan^{-1}\left(\frac{x\quad y}{1-xy}\right)$

(c) $\pi - \tan^{-1}\left(\frac{x\quad y}{1-xy}\right)$ (d) $\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

66. Consider the following statements : *[2014-I]*
1. There exists no triangle ABC for which $\sin A + \sin B = \sin C$.
2. If the angle of a triangle are in the ratio 1 : 2 : 3, then its sides will be in the ratio $1 : \sqrt{3} : 2$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

67. Consider the following statements : *[2014-I]*
1. $\tan^{-1}1 + \tan^{-1}(0.5) = \pi/2$
2. $\sin^{-1}(1/3) + \cos^{-1}(1/3) = \pi/2$

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

68. If $A + B + C = \pi$, then what is $\cos(A + B) + \cos C$ equal to ? *[2014-I]*

(a) 0 (b) $2\cos C$
(c) $\cos C - \sin C$ (d) $2\sin C$

69. What is $\sin^{-1}\sin\frac{3\pi}{5}$ equal to ? *[2014-I]*

(a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$

(c) $\frac{\pi}{5}$ (d) None of these

70. What is $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5}$ equal to ? *[2014-II]*

(a) $\pi/2$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/6$

71. In a triangle ABC, c = 2, A = 45°, a = $2\sqrt{2}$, than what is C equal to ? *[2014-II]*

(a) 30° (b) 15°
(c) 45° (d) None of these

72. In a triangle ABC, sin A – cos B = cos C, then what is B equal to ? *[2014-II]*

(a) π (b) $\pi/3$

(c) $\pi/2$ (d) $\pi/4$

73. In a triangle ABC, $a = (1+\sqrt{3})$ cm, b = 2 cm and angle C = 60°. Then the other two angles are *[2015-I]*

(a) 45° and 75° (b) 30° and 90°
(c) 105° and 15° (d) 100° and 20°

74. The equation $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ is satisfied by *[2015-I]*

(a) $x = 1$ (b) $x = -1$

(c) $x = 0$ (d) $x = \frac{1}{2}$

DIRECTIONS (Qs. 75-77) : *For the next three (3) items that follow.*

Consider $x = 4\tan^{-1}\left(\frac{1}{5}\right)$, $y = \tan^{-1}\left(\frac{1}{70}\right)$ and $z = \tan^{-1}\left(\frac{1}{99}\right)$. *[2015-I]*

75. What is x equal to?

(a) $\tan^{-1}\left(\frac{60}{119}\right)$ (b) $\tan^{-1}\left(\frac{120}{119}\right)$

(c) $\tan^{-1}\left(\frac{90}{169}\right)$ (d) $\tan^{-1}\left(\frac{170}{169}\right)$

76. What is $x - y$ equal to?

(a) $\tan^{-1}\left(\frac{828}{845}\right)$ (b) $\tan^{-1}\left(\frac{8287}{8450}\right)$

(c) $\tan^{-1}\left(\frac{8281}{8450}\right)$ (d) $\tan^{-1}\left(\frac{8287}{8471}\right)$

77. What is $x - y + z$ equal to? *[2015-I]*

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

78. The value of tan $\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$ is *[2015-II]*

(a) $-\frac{7}{17}$ (b) $\frac{5}{16}$

(c) $\frac{5}{4}$ (d) $\frac{7}{17}$

79. Consider the following : *[2015-II]*

1. $\sin^{-1}\frac{4}{5} \quad \sin^{-1}\frac{3}{5} \quad \frac{\pi}{2}$

2. $\tan^{-1}\sqrt{3} \quad \tan^{-1}1 \quad -\tan^{-1}(2 \quad \sqrt{3})$

which of the above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

80. If a, b, c are the sides of a triangle ABC, then $a^{\frac{1}{p}} + b^{\frac{1}{p}} - c^{\frac{1}{p}}$ where $p > 1$, is *[2015-II]*

(a) always negative

(b) always positive

(c) always zero

(d) positive if $1 < p < 2$ and negative if $p > 2$

DIRECTIONS (Qs. 81-82) : *For the next two (2) items that follow:*

Consider a tiangle ABC in which

$\cos A + \cos B + \cos C = \sqrt{3}\sin\frac{\pi}{3}$ *[2016-I]*

81. What is the value of $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

82. What is the value of

$$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)?$$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{16}$ (d) None of the above

83. Consider the following statements: *[2016-I]*

1. There exists $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which $\tan^{-1}(\tan\theta) \neq \theta$.

2. $\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{2\sqrt{2}(\sqrt{3}-1)}{15}\right)$

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

84. Consider the following statements: *[2016-I]*

1. $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \pi$

2. There exist $x, y \in [-1, 1]$, where $x \neq y$ such that $\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

85. Consider the following statements: *[2016-I]*

1. If ABC is an equilateral triangle, then $3\tan(A+B)\tan C = 1$.

2. If ABC is a triangle in which $A = 78°$, $B = 66°$, then $\tan\left(\frac{A}{2}+C\right) < \tan A$

3. If ABC is any triangle, then $\tan\left(\frac{A \quad B}{2}\right)\sin\left(\frac{C}{2}\right) \quad \cos\left(\frac{C}{2}\right)$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) 1 and 2 (d) 2 and 3

86. What is the value of $\cos(2\cos^{-1}(0.8))$? *[2016-II]*

(a) 0.81 (b) 0.56

(c) 0.48 (d) 0.28

87. Consider the following for triangle ABC : *[2017-I]*

1. $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$

2. $\tan\left(\frac{B+C}{2}\right) = \cot\left(\frac{A}{2}\right)$

3. $\sin(B+C) = \cos A$

4. $\tan(B+C) = -\cot A$

Which of the above are correct?

(a) 1 and 3 (b) 1 and 2

(c) 1 and 4 (d) 2 and 3

88. The value of $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is equal to *[2017-II]*

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

89. In a triangle ABC, $a - 2b + c = 0$. The value of $\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$ is *[2017-II]*

(a) $\frac{9}{2}$ (b) 3

(c) $\frac{3}{2}$ (d) 1

90. In triangle ABC, if $\frac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C}=2$ then the triangle is *[2017-II]*
(a) right-angled (b) equilateral
(c) isosceles (d) obtuse-angled

91. The principal value of $\sin^{-1} x$ lies in the interval *[2017-II]*
(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
(c) $\left[0,\frac{\pi}{2}\right]$ (d) $[0,\pi]$

92. In a triangle ABC if a = 2, b = 3 and $\sin A=\frac{2}{3}$, then what is angle B equal to? *[2018-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

93. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$? *[2018-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

94. If x, x – y and x + y are the angles of a triangle (not an equilateral triangle) such that tan (x – y), tan x and tan (x + y) are in GP, then what is x equal to? *[2018-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

95. ABC is a triangle inscribed in a circle with centre O. Let $\alpha = \angle BAC$, where $45° < \alpha < 90°$. Let $\beta = \angle BOC$. Which one of the following is correct? *[2018-I]*
(a) $\cos\beta=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ (b) $\cos\beta=\frac{1+\tan^2\alpha}{1-\tan^2\alpha}$
(c) $\cos\beta=\frac{2\tan\alpha}{1+\tan^2\alpha}$ (d) $\sin\beta=2\sin^2\alpha$

96. What is $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)$ equal to? *[2018-I]*
(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

97. If A + B + C = 180°, then what is sin 2A – sin 2B – sin 2C equal to? *[2018-II]*
(a) – 4 sin A sin B sin C (b) – 4 cos A sin B cos C
(c) – 4 cos A cos B sin C (d) – 4 sin A cos B cos C

98. Consider the following values of x: *[2018-II]*
1. 8 2. –4
3. $\frac{1}{6}$ 4. $-\frac{1}{4}$
Which of the above values of x is/are the solutions of the equation $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$
(a) 3 only (b) 2 and 3 only
(c) 1 and 4 only (d) 4 only

99. Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$: Then the value of θ in the interval $(0, \pi)$ is *[2018-II]*
(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

100. What is the value of $\sin^{-1}\frac{4}{5}+\sec^{-1}\frac{5}{4}-\frac{\pi}{2}$? *[2019-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) 0

101. If $\sin^{-1}\frac{2p}{1+p^2}-\cos^{-1}\frac{1-q^2}{1+q^2}=\tan^{-1}\frac{2x}{1-x^2}$, then what is x equal to ? *[2019-I]*
(a) $\frac{p+q}{1+pq}$ (b) $\frac{p-q}{1+pq}$
(c) $\frac{pq}{1+pq}$ (d) $\frac{p+q}{1-pq}$

102. If the angles of a triangle ABC are in the ratio 1 : 2 : 3. then the corresponding sides are in the ratio *[2019-I]*
(a) 1 : 2 : 3 (b) 3 : 2 : 1
(c) $1:\sqrt{3}:2$ (d) $1:\sqrt{3}:\sqrt{2}$

103. What is the derivative of $\sec^2(\tan^{-1}x)$ with respect to x? *[2019-I]*
(a) 2x (b) x^2+1
(c) x + 1 (d) x^2

ANSWER KEY

1	(c)	11	(a)	21	(c)	31	(d)	41	(c)	51	(b)	61	(d)	71	(a)	81	(c)	91	(b)	101	(b)
2	(a)	12	(c)	22	(b)	32	(a)	42	(b)	52	(c)	62	(d)	72	(c)	82	(d)	92	(b)	102	(c)
3	(a)	13	(a)	23	(d)	33	(b)	43	(a)	53	(b)	63	(a)	73	(a)	83	(b)	93	(c)	103	(a)
4	(d)	14	(c)	24	(d)	34	(d)	44	(a)	54	(c)	64	(c)	74	(c)	84	(d)	94	(b)		
5	(d)	15	(d)	25	(d)	35	(c)	45	(b)	55	(b)	65	(b)	75	(b)	85	(b)	95	(a)		
6	(b)	16	(b)	26	(b)	36	(c)	46	(d)	56	(d)	66	(c)	76	(c)	86	(d)	96	(b)		
7	(a)	17	(b)	27	(b)	37	(b)	47	(c)	57	(a)	67	(b)	77	(d)	87	(b)	97	(d)		
8	(b)	18	(c)	28	(c)	38	(d)	48	(d)	58	(c)	68	(a)	78	(a)	88	(b)	98	(a)		
9	(c)	19	(a)	29	(d)	39	(b)	49	(d)	59	(c)	69	(b)	79	(a)	89	(b)	99	(b)		
10	(c)	20	(d)	30	(b)	40	(b)	50	(c)	60	(c)	70	(a)	80	(b)	90	(a)	100	(d)		

HINTS & SOLUTIONS

1. (c) From properties of triangle we know that

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

Given that a = 2b and A = 3B.

We get $\frac{2b}{\sin 3B} \quad \frac{b}{\sin B}$.

$\Rightarrow \frac{\sin 3B}{\sin B}=2 \Rightarrow \frac{3\sin B-4\sin^3 B}{\sin B}=2$

$\Rightarrow 3-4\sin^2 B=2$

$\Rightarrow \sin^2 B=\left(\frac{1}{2}\right)^2 \Rightarrow \sin B=\frac{1}{2}=\sin\frac{\pi}{6}$

$\Rightarrow B=\frac{\pi}{6}$ since A = 3 B

So, $\angle A \quad \frac{\pi}{2}$

Thus, triangle is right angled triangle.

2. (a) $\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)$

$-\cot(\cot^{-1}x+\cot^{-1}y+\cot^{-1}z)$

$=\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)$

$-\cot\left(\frac{\pi}{2}-\tan^{-1}x+\frac{\pi}{2}-\tan^{-1}y+\frac{\pi}{2}-\tan^{-1}z\right)$

$\left(\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right)$

$=\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)$

$-\cot\{3\pi/2-(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)$

$=\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)$

$-\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)=0$

3. (a) Given that $\cos^{-1}x=2\sin^{-1}x$

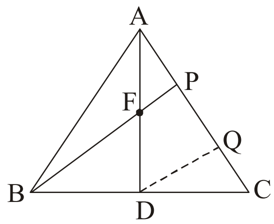
$\Rightarrow \frac{\pi}{2}-\sin^{-1}x=2\sin^{-1}x$

$\Rightarrow \frac{\pi}{2}=3\sin^{-1}x$

$\Rightarrow \sin^{-1}x=\frac{\pi}{6}$

So, $x=\sin\frac{\pi}{6}=\frac{1}{2}$

4. (d) Given, AD is the median of ΔABC. F is mid-point of AD and BF is produced to meet the side AC in P.



Draw DQ ∥ FP

In ΔADQ, F is mid-point of AD and FP ∥ DQ.

∴ P is mid-point of AQ (converse of mid-point theorem)

$\Rightarrow$ AP = PQ ...(1)

In ΔBCP, D is mid-point of BC and DQ ∥ BP.

∴ Q is midpoint of PC.

$\Rightarrow$ PQ = QC ...(2)

From (1), (2) we get, AP = PQ = QC

From figure, AP + PQ + QC = AC

$\Rightarrow$ AP + AP + AP = AC

$\Rightarrow 3\times AP = AC$

$\Rightarrow AP=\frac{1}{3}\times AC \quad \therefore \quad \lambda=\frac{1}{3}$

5. (d) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{3}\right)\right]=\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$=\frac{\pi}{3}$ $\quad\left[\text{Since, } \sin\frac{2\pi}{3}=\sin\left(\pi-\frac{\pi}{3}\right)\right]$

6. (b) Given that

$$\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3} \quad ...(i)$$

and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}$

$$\Rightarrow \frac{\pi}{2} - \sin^{-1} x - \frac{\pi}{2} + \sin^{-1} y = \frac{\pi}{3}$$

$$[\text{since } \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}]$$

$$\Rightarrow -\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3} \quad ...(ii)$$

On solving eq. (i) and (ii), we get

$$2\sin^{-1} y = \pi \text{ and } 2\sin^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow \sin^{-1} y = \frac{\pi}{2} \text{ and } \sin^{-1} x = \frac{\pi}{6}$$

Hence, $y = \sin\frac{\pi}{2}$ and $x = \sin\frac{\pi}{6}$

$$\Rightarrow x = \frac{1}{2} \text{ and } y = 1$$

7. (a) We know from properties of triangle that

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} \Rightarrow \cos^2\frac{A}{2} = \frac{s(s-a)}{bc}$$

and $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}} \Rightarrow \cos^2\frac{C}{2} = \frac{s(s-c)}{ab}$

So, $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2}$

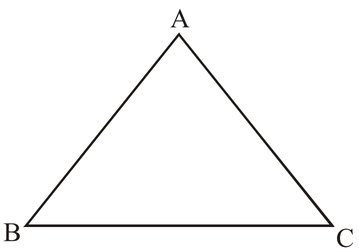
$$= a\left(\frac{s(s-c)}{ab}\right) + c\left(\frac{s(s-a)}{bc}\right) = \frac{s(s-c+s-a)}{b}$$

$$= \frac{s(2s-a-c)}{b} = \frac{s(a+b+c-a-c)}{b} = \frac{s.b}{b} = s$$

$$= \frac{30}{2} = 15\,\text{cm} \quad [\text{given that } 2s = 30]$$

8. (b) Ratio of angles is given by $\angle A : \angle B : \angle C = 1 : 2 : 3$

Let $\angle A = x$, $\angle B = 2x$ and $\angle C = 3x$



We know that in a triangle

$\angle A + \angle B + \angle C = 180°$

$$\Rightarrow x + 2x + 3x = 180°$$

$$\Rightarrow x = \frac{180°}{6} = 30°$$

So, $\angle A = 30°$, $\angle B = 60°$ and $\angle C = 90°$

From sin law

$$\frac{BC}{\sin A} = \frac{CA}{\sin B} = \frac{AB}{\sin C} = K$$

$$\frac{BC}{\sin 30°} = \frac{CA}{\sin 60°} = \frac{AB}{\sin 90°} = K$$

$BC = K \sin 30°$

$CA = K \sin 60°$

$AB = K \sin 90°$

$BC : CA : AB$

$$= \sin 30° : \sin 60° : \sin 90° = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$$

9. (c) Let the angles A, B and C of a triangle are 2x, 5x and 5x, respectively

So, $2x + 5x + 5x = 180°$

$$\Rightarrow x = \frac{180°}{12} = 15°$$

Angles are 30°, 75°, 75°

$\angle B = 75°$ and $\angle C = 75°$

$\therefore \tan B \tan C = (\tan 75°)^2 = (\tan(45° + 30°))^2$

$$= \left(\frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}\right)^2 = \left(\frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}\right)^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$$

$$= \left(\frac{(\sqrt{3}+1)^2}{3-1}\right)^2 = \frac{1}{4}[3+1+2\sqrt{3}]^2$$

$$= \frac{1}{4}[4+2\sqrt{3}]^2 = \frac{1}{4}[16+12+16\sqrt{3}]$$

$$= \frac{1}{4}[28+16\sqrt{3}] = 7 + 4\sqrt{3}$$

10. (c) In any triangle ABC, $A + B + C = \pi$

or, $A + B = \pi - C$

so, $\tan(A + B) = \tan(\pi - C)$

or, $-\tan C = \tan(A + B)$

so, $\tan C = -\tan(A + B)$

$$= -\frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

As given, $\tan A = 1$, and $\tan B = 2$

putting these values,

$$\tan C = -\frac{1+3}{1-1\times 3} = \frac{4}{2} = 2$$

11. (a) Let $\alpha = \tan^{-1} x \Rightarrow \tan\alpha = x$

then $\cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha}} \quad \frac{1}{\sqrt{1 \quad x^2}}$

$\Rightarrow \cos(\tan^{-1} x) = \left\{\frac{1}{\sqrt{1+x^2}}\right\}$

so, $\cot^{-1}\{\cos(\tan^{-1} x)\} = \cot^{-1}\left\{\frac{1}{\sqrt{1 \quad x^2}}\right\}$

Let $\cot^{-1}\left(\frac{1}{\sqrt{1 \quad x^2}}\right) = \beta$

$\Rightarrow \cot\beta = \frac{1}{\sqrt{1+x^2}}$

and $\sin\beta = \frac{1}{\sqrt{1+\cot^2\beta}} \quad \frac{\sqrt{1 \quad x^2}}{\sqrt{x^2 \quad 1 \quad 1}} \quad \sqrt{\frac{x^2 \quad 1}{x^2 \quad 2}}$

$\Rightarrow \sin[\cot^{-1}\{\cos(\tan^{-1})\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

12. (c) We know from the Sine law that

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\Rightarrow \frac{2b}{\sin 3B} = \frac{b}{\sin B}$

$\Rightarrow 2\sin B = \sin 3B$

$\Rightarrow 2\sin B = 3\sin B - 4\sin^3 B$

$\Rightarrow \sin B - 4\sin^3 B = 0$

$\Rightarrow \sin B(1 - 4\sin^2 B) = 0$

$\Rightarrow \sin B = 0$ or $1 - 4\sin^2 B = 0 \Rightarrow B = 0$ or $B = 30°$

$\Rightarrow B = 30°$ and $A = 3 \times 30° = 90°$

$\Rightarrow B = 0$ is not possible so, $B = 30°$ and $A = 3 \times 30° = 90°$

$\Rightarrow$ The triangle is right angled triangle.

13. (a) Let, $\sin^{-1} x = \tan^{-1} y = \theta$

$\Rightarrow x = \sin\theta$ and $y = \tan\theta$

$\frac{1}{x^2} \quad \frac{1}{\sin^2\theta} \quad \text{cosec}^2\theta$

and $\frac{1}{y^2} \quad \frac{1}{\tan^2\theta} \quad \cot^2\theta.$

$\Rightarrow \frac{1}{x^2} - \frac{1}{y^2} = \text{cosec}^2\theta - \cot^2\theta \quad 1$

14. (c) The given trigonometric expression is :

$\cos\left[\tan^{-1}\left\{\tan\left(\frac{15\pi}{4}\right)\right\}\right]$

$= \cos\left[\tan^{-1}\left\{\tan\left(4\pi - \frac{\pi}{4}\right)\right\}\right]$

$= \cos\left[\tan^{-1}\left\{-\tan\frac{\pi}{4}\right\}\right] \quad \cos\left[\tan^{-1}\tan\left(\frac{-\pi}{4}\right)\right]$

$\left(\text{Since } \tan^{-1}\theta \text{ is defined for } \frac{-\pi}{2} < \theta < \frac{-\pi}{2}\right)$

$= \cos\left(\frac{-\pi}{4}\right)$

$\cos\frac{\pi}{4} \quad \frac{1}{\sqrt{2}}$ [since $\cos(-\theta) = \cos\theta$]

15. (d) In any ΔABC

$\angle A + \angle B + \angle C = \pi$

$\Rightarrow$ let $A \quad \tan^{-1}\frac{1}{2}$ and $B \quad \tan^{-1}\frac{1}{3}$

$\Rightarrow \tan^{-1}\frac{1}{2} \quad \tan^{-1}\frac{1}{3} \quad \angle C = \pi$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{2} \quad \frac{1}{3}}{1-\frac{1}{6}}\right) + \angle C = \pi$

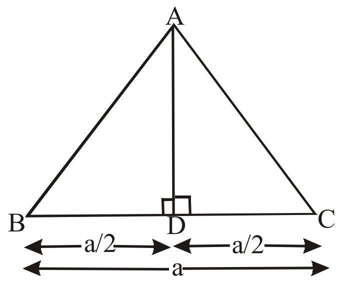
[$\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$]

$\Rightarrow \tan^{-1}\left(\frac{5/6}{5/6}\right) + \angle C = \pi \Rightarrow \frac{\pi}{4} + \angle C = \pi$

$\Rightarrow \angle C = \pi - \frac{\pi}{4} \quad \frac{3\pi}{4} \quad 135$

16. (b) In the given ΔABC

let $BC = a$



$\therefore \quad BD \quad CD \quad \frac{a}{2}$

In ΔADB,

$\tan B \quad \frac{AD}{BD} \quad \frac{AD}{a/2}$

$\Rightarrow \tan B \quad \frac{2AD}{a}$ ....(1)

In ΔADC,

$\tan C \quad \frac{AD}{CD} \quad \frac{AD}{a/2}$

$\tan C \quad \frac{2AD}{a}$ ....(2)

From eqs. (1) and (2), we get

$\tan B = \tan C$

$\Rightarrow \tan B - \tan C = 0$

17. (b) Let, $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)=\theta \Rightarrow \cos\theta=\frac{1}{\sqrt{5}}$

$\Rightarrow \sec\theta=\sqrt{5} \Rightarrow \sec^{-1}(\sqrt{5})=\theta$

$\Rightarrow \frac{\pi}{2}-\text{cosec}^{-1}(\sqrt{5})=\theta \quad (\because \sec^{-1}x+\text{cosec}^{-1}x=\frac{\pi}{2})$

$\Rightarrow \text{cosec}^{-1}(\sqrt{5})=\frac{\pi}{2}-\theta$

18. (c) The given expression is :

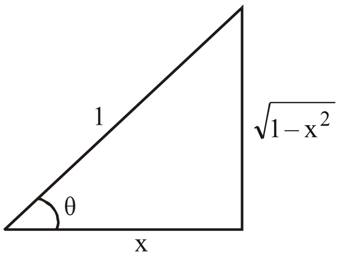
$\tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}\left(\frac{m-n}{m+n}\right)$

$=\tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}\left(\frac{1-\frac{n}{m}}{1+\frac{n}{m}}\right)$

$=\tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}(1)+\tan^{-1}\left(\frac{n}{m}\right)$

$=\tan^{-1}\left(\frac{m}{n}\right)+\cot^{-1}\left(\frac{m}{n}\right)-\frac{\pi}{4}=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

19. (a) Let $\cos^{-1}x=\theta$



$\Rightarrow \cos\theta=x \Rightarrow \sin\theta=\sqrt{1-x^2}$

$\Rightarrow \tan\theta=\frac{\sqrt{1-x^2}}{x}$ and $\theta=\cos^{-1}x$

This can be represented by a triangle with hypotenuous = 1 and sides x and $\sqrt{1-x^2}$.

$\Rightarrow \tan(\cos^{-1}x)=\frac{\sqrt{1-x^2}}{x}$

20. (d) As given :

$\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$ ...(1)

and we know that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ ...(2)

On adding Eqs. (1) and (2) we get

$2\sin^{-1}x=\frac{\pi}{6}+\frac{\pi}{2}=\frac{2\pi}{3}$

$\Rightarrow \sin^{-1}x=\frac{\pi}{3} \Rightarrow x=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

21. (c) As given : In a triangle ABC, AC = $b=\sqrt{3}$ cm, AB = c = 1m and $\angle A=30°$

From cosine formulae

$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{(\sqrt{3})^2+1^2-a^2}{2\sqrt{3}.1}$

$\Rightarrow \frac{\sqrt{3}}{2}=\frac{3+1-a^2}{2\sqrt{3}} \Rightarrow 3=4-a^2$

$\Rightarrow a^2=4-3=1 \Rightarrow a=1$ cm

22. (b) As given :

$\cos\left(\sin^{-1}x\right)=\frac{1}{2}$

$\Rightarrow \sin^{-1}x=\cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \sin^{-1}x=\frac{\pi}{3} \Rightarrow x=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\therefore \tan(\cos^{-1}x)=\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

$=\tan\left(\pm\frac{\pi}{6}\right)=\pm\frac{1}{\sqrt{3}}$

Hence, tan $(\cos^{-1}x)$ have two values.

23. (d) Let $\sin^{-1}x=\theta \Rightarrow x=\sin\theta$

$\sin^{-1}(3\sin\theta-4\sin^3\theta)=\sin^{-1}\sin3\theta=3\theta=3\sin^{-1}x$

Equaion $\sin^{-1}(3x-4x^3)=3\sin^{-1}x$ is true for all values of x lying in the interval [–1, 1].

24. (d) **Option** (a)

$\sin^{-1}\left(\sin\frac{5\pi}{4}\right)=\frac{-\pi}{4}$

$\Rightarrow \sin\frac{5\pi}{4}=\sin\left(\frac{-\pi}{4}\right)$

$\Rightarrow \sin\left(\pi+\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}$

$\Rightarrow -\sin\left(\frac{\pi}{4}\right)=-\sin\left(\frac{\pi}{4}\right)$

Hence it is correct

Option (b)

$\sec^{-1}\left(\sec\frac{5\pi}{4}\right)=\frac{3\pi}{4}$

$\Rightarrow \sec\frac{5\pi}{4}=\sec\frac{3\pi}{4}$

$\Rightarrow \sec\left(2\pi-\frac{3\pi}{4}\right)=\sec\frac{3\pi}{4}$

$\Rightarrow \sec\frac{3\pi}{4}=\sec\frac{3\pi}{4}$

Hence, it is correct

Option (c)

$$\tan^{-1}\left(\tan\frac{5\pi}{4}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan\frac{5\pi}{4}=\tan\frac{\pi}{4}$$

$$\Rightarrow \tan\left(\pi \quad \frac{\pi}{4}\right) \quad \tan\frac{\pi}{4}$$

$$\Rightarrow \tan\frac{\pi}{4}=\tan\frac{\pi}{4}$$

Hence it is correct Option (d).

$$\text{cosec}^{-1}\left(\text{cosec}\frac{7\pi}{4}\right)=\frac{\pi}{4}$$

$$\Rightarrow \text{cosec}\frac{7\pi}{4}=\text{cosec}\frac{\pi}{4}$$

$$\Rightarrow \text{cosec}\left(2\pi-\frac{\pi}{4}\right)=\text{cosec}\frac{\pi}{4}$$

$$\Rightarrow -\text{cosec}\frac{\pi}{4}=\text{cosec}\frac{\pi}{4}$$

Hence it is not correct.

25. (d) Given, $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$ ...(i)

and $\cos^{-1}x-\cos^{-1}y=0$

$$\Rightarrow \left(\frac{\pi}{2}-\sin^{-1}x\right)-\left(\frac{\pi}{2}-\sin^{-1}y\right)=0$$

$$\Rightarrow \sin^{-1}y-\sin^{-1}x \quad 0$$

$$\Rightarrow \sin^{-1}y \quad \sin^{-1}x \qquad \text{...(ii)}$$

From equations (i) and (ii), we get

$$2\sin^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow x=\frac{1}{\sqrt{2}}$$

From equation (ii)

$$y=\frac{1}{\sqrt{2}}$$

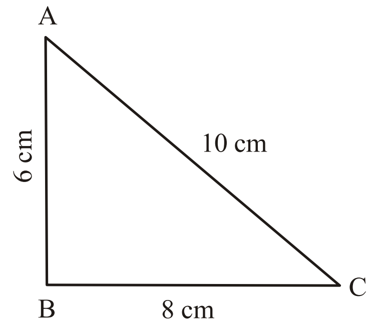
26. (b) Here, AB = 6 cm, BC = 8 cm and CA = 10 cm
So, c = 6 cm, a = 8 cm, b = 10 cm

$$S=\frac{a+b+c}{2}=\frac{24}{2}=12$$

$$\therefore \tan\frac{A}{2}=\sqrt{\frac{(12-10)(12-6)}{12(12-8)}}$$

$$\left(\because \tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right)$$

$$=\sqrt{\frac{1}{4}}=\frac{1}{2}$$



$$\therefore \cot\frac{A}{2}=2$$

$$\text{Now, } \cot\left(\frac{A}{4}+\frac{A}{4}\right)=\frac{\cot^2\frac{A}{4}-1}{2\cot\frac{A}{4}}$$

$$\cot\left(\frac{A}{2}\right)=\frac{\cot^2\frac{A}{4}-1}{2\cot\frac{A}{4}}$$

$$\text{Let } \cot\left(\frac{A}{4}\right)=x$$

$$\therefore \quad 2=\frac{x^2-1}{2x}$$

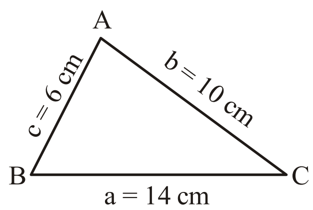
$$\Rightarrow x^2-4x-1=0$$

$$\Rightarrow x=\frac{4\pm\sqrt{16+4}}{2}$$

$$\Rightarrow x=\frac{4\pm2\sqrt{5}}{2}=2\pm\sqrt{5}$$

$$\text{So, } \cot\left(\frac{A}{4}\right)=\sqrt{5}+2 \text{ or } 2-\sqrt{5}$$

27. (b) We know that largest side has greatest angle opposite it.
$\therefore$ a = 14 cm, b = 10 cm and c = 6 cm



$$\therefore \cos A=\frac{c^2+b^2-a^2}{2bc}$$

$$=\frac{36+100-196}{2\times6\times10}=-\frac{1}{2}=\cos120°$$

$$\Rightarrow \angle A=120°$$

28. (c) For finding the area of a triangle ABC, $\angle A$, $\angle B$ and side c are required.

29. (d) $\sin^{-1}\{2x(1-x^2)\} = 2\sin^{-1} x$ is true

$\forall x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

30. (b) Given, $\sin A = \frac{1}{\sqrt{5}}$ and $\cos B = \frac{3}{\sqrt{10}}$

$\therefore \sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \sqrt{1-\frac{1}{5}} \times \sqrt{1-\frac{9}{10}}$

$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}} = \frac{3+2}{\sqrt{50}} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow A + B = \frac{\pi}{4}$

31. (d) Given,

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore\ 2\tan^{-1} a - 2\tan^{-1} b = 2\tan^{-1} x$

$\Rightarrow \tan^{-1} a - \tan^{-1} b = \tan^{-1} x$

$\Rightarrow \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1} x$

$\Rightarrow x = \frac{a-b}{1+ab}$

32. (a) Consider $\cos B = \frac{\sin A}{2\sin C}$

$\Rightarrow \frac{c^2+a^2-b^2}{2ac} = \frac{a}{2c}$

$\left(\because \frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{\sin A}{\sin C} = \frac{a}{c}\right)$

$\Rightarrow c^2 + a^2 - b^2 = \frac{2a^2c}{2c}$

$\Rightarrow c^2 + a^2 - b^2 = a^2$

$\Rightarrow c^2 - b^2 = 0$

$\Rightarrow c = b$

Hence, Δ ABC is isosceles triangle.

33. (b) Let $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

As we know $\cot^{-1} x = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$\therefore \sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x + \sin^{-1}\left(\frac{1}{\sqrt{1+\frac{1}{4}}}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x + \sin^{-1}\left(\frac{2}{\sqrt{5}}\right) = \frac{\pi}{2}$ ...(1)

Now, $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$

$\therefore \sin^{-1}\left(\frac{2}{\sqrt{5}}\right) = \cos^{-1}\sqrt{1-\frac{4}{5}} = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore$ From equation (1), we have

$\sin^{-1} x + \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{2}$

since, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\therefore x = \frac{1}{\sqrt{5}}$

34. (d) Given, $a + b = 3(1+\sqrt{3})$.......... (1)

and $a - b = 3(1-\sqrt{3})$ .... (2)

By adding (1) and (2) we get

$(a+b)+(a-b) = 3+3\sqrt{3}+3-3\sqrt{3}$

$\Rightarrow 2a = 6 \Rightarrow a = 3$

$\therefore b = 3-3+3\sqrt{3} = 3\sqrt{3}$

By using sine rule, $\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$

Given $\angle A = 30$ $\Rightarrow \frac{3}{\sin 30^o} = \frac{3\sqrt{3}}{\sin B}$

$\Rightarrow \sin B = \sqrt{3} \times \frac{1}{2} \Rightarrow \sin B = \sin 60^o$

$\Rightarrow B = 60^o$

35. (c) Let the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right) = \theta$

$\Rightarrow -\sqrt{2} = \text{cosec}\ \theta \Rightarrow -\sqrt{2} = \frac{1}{\sin\theta}$

$\Rightarrow \sin\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = -\frac{\pi}{4}$

Principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$

36. (c) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{5}{x} + \cos^{-1}\frac{\sqrt{x^2-144}}{x} = \frac{\pi}{2}$

$\left(\because \sin^{-1} x = \cos^{-1}\sqrt{1-x^2}\right)$

But we know $\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}$

$\therefore \quad \frac{5}{x} = \frac{\sqrt{x^2 - 144}}{x}$

$\Rightarrow \quad 5 = \sqrt{x^2 - 144}$

$\Rightarrow \quad 25 = x^2 - 144 \Rightarrow x^2 = 169$

$\Rightarrow \quad x = 13$

37. (b) Since, A, B, C are in AP.

$\therefore B - A = C - B$

$\Rightarrow \quad 2B = A + C$

But we know $A + B + C = 180°$

$\Rightarrow \quad 3B = 180° \Rightarrow B = 60°$

Consider $\sin A + 2\sin B + \sin C$

$= 2\sin\frac{A+C}{2}\cos\frac{A-C}{2} + 2\sin B$

$= 2\sin B\left[\cos\frac{A-C}{2} + 1\right] \qquad (\because A + C = 2B)$

$= 2\sin B\left[2\cos^2\left(\frac{A-C}{4}\right)\right]$

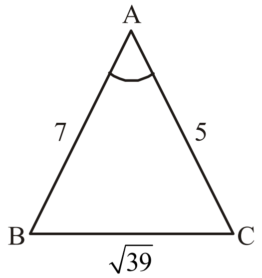
$= 4\sin B\cos^2\left(\frac{A-C}{4}\right)$

38. (d) $(\text{I})\cos\left(\sin^{-1}x\right) = \cos\left(\cos^{-1}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

$(\text{II})\sin\left(\cos^{-1}x\right) = \sin\left(\sin^{-1}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

Hence, statement I is false and II is true.

39. (b) Let a, b, c be the sides of ΔABC and $\angle A = \theta$



$\therefore \quad a = \sqrt{39},\ b = 5 \text{ and } c = 7$

and $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 49 - 39}{2 \times 5 \times 7}$

$= \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad A = \frac{\pi}{3}$

40. (b) Consider $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}$

$= \tan^{-1}\left(\frac{4/5}{\sqrt{1 - 16/25}}\right) + \tan^{-1}\left(\frac{2/3}{1 - 1/9}\right)$

$\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}} \text{ and } 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= \tan^{-1}\left(\frac{4/5}{3/5}\right) + \tan^{-1}\left(\frac{2/3}{8/9}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right) + \cot^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{2}$

$\left(\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right)$

41. (c) Given a ΔABC in which BC = 10 cm, CA = 6cm and AB = 8cm.

Since, $CA^2 + AB^2 = 36 + 64 = 100 = BC^2$

$\therefore$ ΔABC is a right angled triangle.

42. (b) Let $c = 2$, $\angle A = 120°$ and $a = \sqrt{6}$ in ΔABC,

$\therefore$ By Sine rule, we have

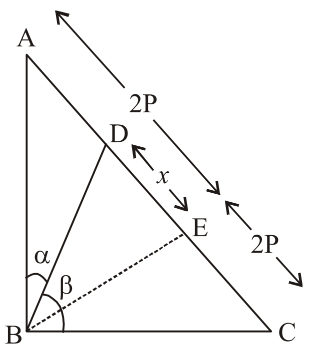
$\frac{a}{\sin A} = \frac{c}{\sin C} \quad \Rightarrow \quad \frac{\sqrt{6}}{\sin 120°} = \frac{2}{\sin C}$

$\Rightarrow \sin C = \frac{2 \times \sqrt{3}}{\sqrt{6} \times 2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin C = \sin 45° \quad \Rightarrow \quad \angle C = 45°$

43. (a) Let BD = P and DE = x

$\Rightarrow$ AC = 4P



Let E be mid-point of AC.

Then, AE = EC = BE = 2P

In ΔBDE, $(BE)^2 = (BD)^2 + (ED)^2$

$\Rightarrow \quad (2P)^2 = (P)^2 + x^2$

$\Rightarrow \quad 4P^2 = P^2 + x^2$

$\Rightarrow \quad 3P^2 = x^2 \Rightarrow x = \sqrt{3}P$

Now, $AD = 2P - x = 2P - \sqrt{3}P = P(2-\sqrt{3})$

$DC = 2P + x = 2P + \sqrt{3}P = P(2+\sqrt{3})$

In ΔBAD, $\tan A = \dfrac{BD}{AD} = \dfrac{P}{P(2-\sqrt{3})}$

$= \dfrac{1}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2-\sqrt{3}} = 2+\sqrt{3} = \tan 75°$

$\tan\alpha = \dfrac{AD}{BD} = \dfrac{P(2-\sqrt{3})}{P} = 2-\sqrt{3} = \tan 15°$

$\Rightarrow \quad \alpha = 15°$

As, ΔABC is right angled at B, from figure $\alpha + \beta = 90°$

$\Rightarrow \quad 15 + \beta = 90° \Rightarrow \beta = 75°$

In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 75° + 90° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 165° = 15°$

$\therefore$ One of the a cute angle is 15°

44. (a) $\angle ABD = \alpha = 15°$

45. (b) $AD : DC = \dfrac{AD}{DC} = \dfrac{P(2-\sqrt{3})}{P(2+\sqrt{3})} = \dfrac{2-\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{4+3-4\sqrt{3}}{1} = \dfrac{7-4\sqrt{3}}{1}$

$\therefore AD : DC = 7-4\sqrt{3} : 1$

46. (d) $\tan(A-C) = \tan(75° - 15°) = \tan 60° = \sqrt{3}$

47. (c) Consider (I) :

$$\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \frac{-\pi}{3}$$

Let $\text{cosec}^{-1}\left(\dfrac{-2}{\sqrt{3}}\right) = \theta$

$\Rightarrow \dfrac{-2}{\sqrt{3}} = \text{cosec}\,\theta \Rightarrow \dfrac{-\sqrt{3}}{2} = \dfrac{1}{\text{cosec}\,\theta}$

$\Rightarrow \dfrac{-\sqrt{3}}{2} = \sin\theta \Rightarrow \theta = \dfrac{-\pi}{3}$

Now, consider (II) :

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Let $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = \theta$

$\Rightarrow \dfrac{2}{\sqrt{3}} = \sec\theta \quad \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sec\theta}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \cos\theta \quad \Rightarrow \theta = \dfrac{\pi}{6}$

Hence, both statements I and II are correct.

48. (d) Let $\sin\left[\sin^{-1}\left(\dfrac{1}{5}\right) + \cos^{-1}x\right] = 1$

$\Rightarrow \sin^{-1}\left(\dfrac{1}{5}\right) + \cos^{-1}x = \sin^{-1}1$

$\Rightarrow \sin^{-1}\left(\dfrac{1}{5}\right) + \cos^{-1}x = \dfrac{\pi}{2}$

$\Rightarrow x = \dfrac{1}{5} \qquad \left(\because \sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}\right)$

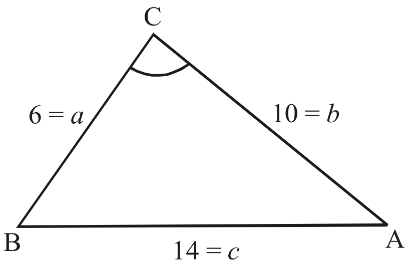
49. (d) Let $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = y$ where $0 < y \le \dfrac{\pi}{2}$

$\Rightarrow \sec y = \dfrac{2}{\sqrt{3}}$

$\Rightarrow \sec y = \sec\dfrac{\pi}{6} \Rightarrow y = \dfrac{\pi}{6}$

$\therefore$ The principal value of $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = \dfrac{\pi}{6}$

50. (c)



Since, c = 14 is the largest side

$\therefore$ Angle C will be obtuse

$\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{(6)^2+(10)^2-(14)^2}{2(6)(10)}$

$= \dfrac{36+100-196}{2\times 6\times 10} = \dfrac{-1}{2}$

$\Rightarrow C = \cos^{-1}\left(\dfrac{-1}{2}\right) = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3} = \dfrac{2\times 180}{3} = 120$

51. (b) We have $A = \tan^{-1}2 \Rightarrow \tan A = 2$

and $B = \tan^{-1}3 \Rightarrow \tan B = 3$.

Since, A, B, C are angles of a triangle

$\therefore A + B + C = \pi$

$\Rightarrow C = \pi - (A+B)$ ...(1)

Now, $A + B = \tan^{-1}2 + \tan^{-1}3$

$= \pi + \tan^{-1}\left(\frac{2+3}{1-2.3}\right)$

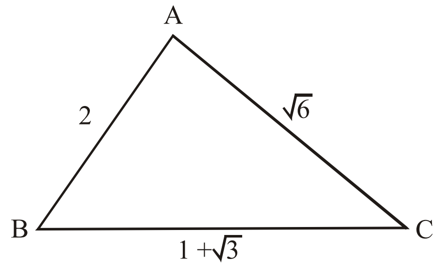
$\left[\because \tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}\right]$

$= \pi + \tan^{-1}(-1) = \pi - \tan^{-1}(1)$

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore$ from (1), $C = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

52. (c)



Let ABC be a triangle with sides a = $1+\sqrt{3}$, b = 2 and c = $\sqrt{6}$

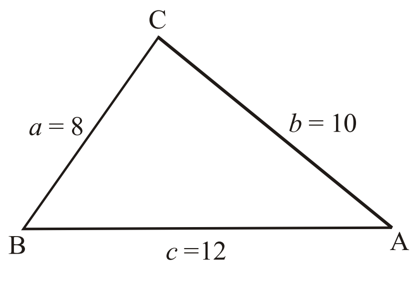
So, $\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{(1+\sqrt{3})^2+(\sqrt{6})^2-4}{2(1+\sqrt{3})(\sqrt{6})}$

$= \frac{2\sqrt{3}+6}{2\sqrt{6}+\sqrt{18}} = \frac{3+\sqrt{3}}{\sqrt{6}+3\sqrt{2}}$

$= \frac{\sqrt{3}\sqrt{3}+\sqrt{3}}{(\sqrt{3}+\sqrt{3}\sqrt{3})\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ B = 45° is the smallest angle.
($\because$ smallest side is $b = 2$)

53. (b)



$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{164-144}{2(8)(10)} = \frac{1}{8}$

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{3}{4}$

Sin $C = \frac{3\sqrt{7}}{8}$ and Sin A $= \frac{\sqrt{7}}{4}$

$\frac{\cos C}{\cos A} = \frac{1}{6} < 1 \Rightarrow \cos C < \cos A \Rightarrow C > A$

$\cos(C-A) = \cos C \cos A + \sin C \sin A$

$= \frac{1}{8}\times\frac{3}{4} + \frac{3\sqrt{7}}{8}\times\frac{\sqrt{7}}{4} = \frac{3}{4} = \cos A$

$\Rightarrow C - A = A \Rightarrow C = 2A$

54. (c) Given a, b, c are in arithmetic progression.
$\therefore 2b = a + c$ ...(1)
Now, we know

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{b^2+c^2-(2b-c)^2}{2bc}$ (from 1)

$= \frac{b^2+c^2-4b^2-c^2+4bc}{2bc}$

$= \frac{-3b^2+4bc}{2bc} = \frac{4c-3b}{2c}$

55. (b) Let $\cos^{-1}\frac{4}{5} = A \Rightarrow \cos A = \frac{4}{5} \Rightarrow \sin A = \frac{3}{5}$

Let $\cos^{-1}\frac{12}{13} = B \Rightarrow \cos B = \frac{12}{13} \Rightarrow \sin B = \frac{5}{13}$

Now, $\cos\left(\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}\right) = \cos(A+B)$

$= \cos A \cos B - \sin A \sin B$

$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{33}{65}$

56. (d) Since A, B, C are in A.P.
$\therefore 2B = A + C$
Also, $A + B + C = 180° \Rightarrow 2B + B = 180°$
$\Rightarrow 3B = 180° \Rightarrow B = 60°$
Now, we know

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\Rightarrow \cos 60° = \frac{a^2+c^2-b^2}{2ac}$

$\Rightarrow \frac{1}{2} = \frac{a^2+c^2-b^2}{2ac} \Rightarrow ac = a^2 + c^2 - b^2$

$\Rightarrow b^2 = a^2 + c^2 - ac$

57. (a) Let $\sin^{-1}(1) + \sin^{-1}\left(\frac{4}{5}\right) = \sin^{-1}x$

Let $\sin^{-1}(1) = \theta \Rightarrow \sin\theta = 1 \Rightarrow \cos\theta = 0$

and $\sin^{-1}\left(\frac{4}{5}\right) = \phi \Rightarrow \sin\phi = \left(\frac{4}{5}\right) \Rightarrow \cos\phi = \sqrt{1-\frac{16}{25}}$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$\therefore \quad \sin^{-1}x = \theta + \phi$

$\Rightarrow \quad x = \sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$$= 1 \times \frac{3}{5} + 0 \times \frac{4}{5}$$

$$\Rightarrow x = \frac{3}{5}$$

58. (c) Let A = $\tan^{-1}2$, B = $\tan^{-1}3$ and C be the angles of a triangle.
By angle sum property, we have

$$\tan^{-1}2 + \tan^{-1}3 + C = 180°$$

$$\Rightarrow \tan^{-1}\left(\frac{5}{-5}\right) = 180° - C$$

$$\Rightarrow \tan^{-1}(-1) = 180° - C$$

$$\Rightarrow \frac{3\pi}{4} = \pi - C \Rightarrow C = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$$

Hence, third angle is $\frac{\pi}{4}$.

59. (c) Let $\sec^2\left(\tan^{-1}\left(\frac{5}{11}\right)\right)$

$$= 1 + \tan^2\left(\tan^{-1}\left(\frac{5}{11}\right)\right) \qquad (\because \sec^2\theta - \tan^2\theta = 1)$$

$$= 1 + \left[\tan\left(\tan^{-1}\left(\frac{5}{11}\right)\right)\right]^2 = 1 + \left(\frac{5}{11}\right)^2$$

$$= 1 + \frac{25}{121} = \frac{146}{121}$$

60. (c) $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right]$

$$= \sin\left[\sin^{-1}\left\{\frac{3}{5}\sqrt{1 - \frac{16}{25}} + \frac{4}{5}\sqrt{1 - \frac{9}{25}}\right\}\right]$$

$$= \sin\left[\sin^{-1}\left\{\frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5}\right\}\right]$$

$$= \sin\left[\sin^{-1}\left\{\frac{9}{25} + \frac{16}{25}\right\}\right] = \sin\left[\sin^{-1}(1)\right]$$

$$= \sin\frac{\pi}{2} = 1$$

61. (d) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$$\Rightarrow \cos C = \frac{(18)^2 + (24)^2 - (30)^2}{2 \times 18 \times 24} = \frac{9 + 16 - 5^2}{2 \times 3 \times 4} = 0$$

Now, $\sin C = \sqrt{1 - \cos^2 C} = \sqrt{1 - 0} = 1$
Hence $\sin C = 1$

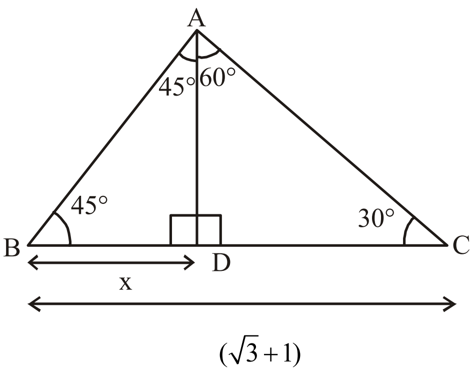
62. (d) Since a > 0, b > 0 and $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

$\therefore$ Given expression is $2\tan^{-1}a + 2\tan^{-1}b = 2\tan^{-1}x$

$$\Rightarrow 2\tan^{-1}\left(\frac{a + b}{1 - ab}\right) = 2\tan^{-1}x$$

$$\Rightarrow x = \frac{a + b}{1 - ab}$$

63. (a)



From ΔADB, AD = BD = x
In ΔADC,

$$\tan 30° = \frac{x}{\sqrt{3} + 1 - x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{\sqrt{3} + 1 - x} \Rightarrow \sqrt{3}x \quad \sqrt{3} \quad 1 - x$$

$$\Rightarrow (\sqrt{3} \quad 1)x = \sqrt{3} \quad 1$$

$$x = \frac{\sqrt{3} \quad 1}{\sqrt{3} \quad 1} = 1.$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times (\sqrt{3} + 1) \times 1 \quad \frac{\sqrt{3} \quad 1}{2}$$

64. (c) $\tan^{-1}\left(\frac{1}{2}\right) \quad \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}\right)$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) \quad \frac{\pi}{4}$$

65. (b) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left[\frac{x\quad y}{1-xy}\right]$, when $xy<1$.

And if $x<0,\ y<0$ and $xy>1$, then

$\tan^{-1}x+\tan^{-1}y=\pi\quad\tan^{-1}\left(\frac{x\quad y}{1-xy}\right)$

66. (c) 1. Given, $\sin A+\sin B=\sin C$

$a+b=c\qquad\left(\because \text{By sine law}, \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=K\right)$

Here, the sum of two sides of ΔABC is equal to the third side, but it is not possible (Because by triangle inequality, the sum of the length of two sides of a triangle is always greater than the length of the third side)

$\boxed{a\quad b\quad c}$

2. Ratio of angles of a triangle

$A:B:C=1:2:3$

$A+B+C=180°$

$\therefore A=30°$

$B=60°$

$C=90°$

the ratio in sides according to sine rule

$a:b:c=\sin A:\sin B:\sin C$

$=\sin 30°:\sin 60°:\sin 90°$

$=\frac{1}{2},\frac{\sqrt{3}}{2},1=\frac{1}{2}:\frac{\sqrt{3}}{2}:1\Rightarrow 1:\sqrt{3}:2$

67. (b) 1. L.H.S.

$\tan^{-1}1\quad\tan^{-1}\left(\frac{1}{2}\right)$

$=\tan^{-1}1\quad\cot^{-1}\left(\frac{1}{\frac{1}{2}}\right)=\tan^{-1}+\cot^{-1}2\neq\frac{\pi}{2}$

So, L.H.S. $\neq$ R.H.S.

2. $\sin^{-1}\frac{1}{3}+\cos^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{2}$

$\left\{\sin^{-1}x\quad\cos^{-1}x\quad\frac{\pi}{2}\right\}$

68. (a) $A+B+C=\pi$

$A+B=\pi-C$

$\cos(A+B)=\cos(\pi-C)$

$\cos(A+B)=-\cos C$

or $\cos(A+B)+\cos C=0$

69. (b) $\sin^{-1}\sin\frac{3\pi}{5}=\sin^{-1}\sin\left(\pi-\frac{2\pi}{5}\right)$

$=\sin^{-1}\sin\frac{2\pi}{5}\quad\frac{2\pi}{5}$

70. (a) $\sin^{-1}\frac{3}{5}\quad\sin^{-1}\frac{4}{5}=\sin^{-1}\frac{3}{5}\quad\cos^{-1}\frac{3}{5}$

$\left(\text{let}\sin^{-1}\frac{4}{5}=\theta\Rightarrow\sin\theta=\frac{4}{5}\Rightarrow\cos\theta\quad\frac{3}{5}\right)$

$=\frac{\pi}{2}$

71. (a) According to sine rule,

$\frac{a}{\sin A}\quad\frac{b}{\sin B}\quad\frac{c}{\sin C}$

$\therefore\ \frac{a}{\sin A}\quad\frac{c}{\sin C}$

$\Rightarrow\sin C=\frac{c.\sin A}{a}\quad\frac{2.\sin 45}{2\sqrt{2}}$

$=\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}\quad\frac{1}{2}=\sin 30°$

$\therefore C=30°$

72. (c) In a ΔABC, we have

$\sin A-\cos B=\cos C\Rightarrow\sin A=\cos B+\cos C$

$\Rightarrow 2\sin\frac{A}{2}.\cos\frac{A}{2}=2\cos\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)$

$[\because \sin 2A=2\sin A\cdot\cos A]$

and $\cos B\quad\cos C\quad 2\cos\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow 2\sin\frac{A}{2}.\cos\frac{A}{2}=2\cos\left(90°-\frac{A}{2}\right).\cos\left(\frac{B-C}{2}\right)$

$\left[\because A\quad B\quad C\quad 180\Rightarrow\left(\frac{B+C}{2}\right)\quad 90\ -\frac{A}{2}\right]$

$\Rightarrow 2\sin\frac{A}{2}.\cos\frac{A}{2}\quad 2\sin\frac{A}{2}.\cos\left(\frac{B-C}{2}\right)$

$[\because \cos(90°-\theta)=\sin\theta]$

$\Rightarrow\quad\cos\frac{A}{2}=\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow\quad\frac{A}{2}=\frac{B-C}{2}$

$\Rightarrow\quad A+C=B$ ...(i)

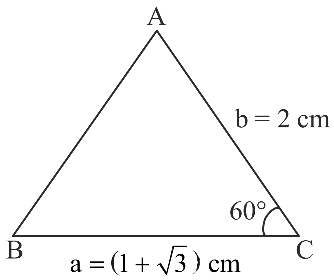
Also, $A+C=180°-B$ ...(ii)

So, $180°-B=B$

$\Rightarrow\quad 2B=180°$

$\therefore\quad B=90°$

73. (a)



Now as $a>b$

$\therefore\quad\angle A>\angle B$

Now from Sine Rule, $\frac{\sin A}{a}=\frac{\sin B}{b}$

$\frac{\sin A}{1+\sqrt{3}}=\frac{\sin B}{2}$

From option (a), $\frac{\sin 75}{1+\sqrt{3}} = \frac{\sin 45}{2}$

$$\frac{\sqrt{6}+\sqrt{2}}{4\left(1+\sqrt{3}\right)} = \frac{1}{2\sqrt{2}}$$

$2\sqrt{12} + 4 = 4 + 4\sqrt{3}$

$4 + 4\sqrt{3} = 4 + 4\sqrt{3}$

$\therefore$ Option (a) is correct.

74. (c) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$$\tan^{-1}\left[\frac{(1+x)+(1-x)}{1-(1+x)(1-x)}\right] = \frac{\pi}{2}$$

$$\Rightarrow \frac{1+x+1-x}{1-(1+x)(1-x)} = \tan\frac{\pi}{2}$$

$$\Rightarrow \frac{2}{1-(1+x)(1-x)} = \frac{1}{0}$$

$\Rightarrow 1-(1+x)(1-x) = 0$

$\Rightarrow (1+x)(1-x) = 1$

$1 - x^2 = 1$

$x^2 = 0$

$x = 0$

$\therefore$ Option (c) is correct.

75. (b) $x = 4\tan^{-1}\left(\frac{1}{5}\right)$

$$= 2\left[2\tan^{-1}\frac{1}{5}\right] = 2\left[\tan^{-1}\frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2}\right]$$

$$= 2\tan^{-1}\left(\frac{\frac{2}{5}\times 25}{24}\right) = 2\tan^{-1}\frac{10}{24} = 2\tan^{-1}\frac{5}{12}$$

$$= \tan^{-1}\left(\frac{2\times\frac{5}{12}}{1-\frac{25}{144}}\right) = \tan^{-1}\frac{120}{119}.$$

76. (c) $x - y = \tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\frac{1}{70}$

$$= \tan^{-1}\left|\frac{\frac{120}{119}-\frac{1}{70}}{1+\left(\frac{120}{119}\times\frac{1}{70}\right)}\right|$$

$$= \tan^{-1}\left|\frac{\frac{8400-119}{8330}}{1+\frac{120}{8330}}\right|$$

$$= \tan^{-1}\left|\frac{\frac{8281}{\cancel{8330}}}{\frac{8450}{\cancel{8330}}}\right| = \tan^{-1}\frac{8281}{8450}$$

$\therefore$ Option (c) is correct.

77. (d) $x - y = \tan^{-1}\frac{8281}{8450} \Rightarrow \tan^{-1}\left|\frac{8281}{8450}\right| + \tan^{-1}\left(\frac{1}{99}\right)$

$$= \tan^{-1}\left|\frac{\frac{8281}{8450}+\frac{1}{99}}{1-\frac{8281}{8450}\times\frac{1}{99}}\right| = \tan^{-1}\left|\frac{\frac{828269}{836550}}{\frac{828269}{836550}}\right| = 1$$

$\tan^{-1} = (1) = \frac{\pi}{4}$

$\therefore$ Option (d) is correct.

78. (a) $2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left[\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right]$

$$= \tan^{-1}\left[\frac{10}{24}\right]$$

$$= \tan^{-1}\left(\frac{5}{12}\right)$$

Let $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right) = x$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{5}{12}\right)-\frac{\pi}{4}\right] = x$$

$$\Rightarrow \tan^{-1}\left(\frac{5}{12}\right)-\frac{\pi}{4} = \tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{5}{12}\right)-\tan^{-1}(1) = \tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left[\frac{\left(\frac{5}{12}-1\right)}{1+\left(\frac{5}{12}\right)(1)}\right] = \tan^{-1}x$$

$$\Rightarrow x = \frac{-7/12}{17/12} = -7/17$$

79. (a) $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$

If $(-1\le x, y\le 1)$ & $(x^2+y^2\le 1)$

$\Rightarrow \quad \sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\frac{3}{5}$

$=\sin^{-1}\left[\frac{4}{5}\sqrt{1-\left(\frac{3}{5}\right)^2}+\frac{3}{5}\sqrt{1-\left(\frac{4}{5}\right)^2}\right]$

$=\sin^{-1}\left[\frac{16}{25}+\frac{9}{25}\right]=\sin^{-1}(1)=\frac{\pi}{2}$

$\therefore$ Statement (1) is correct

Again, $\tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left[\frac{x+y}{1-xy}\right]$

If; $(x>0)$, $(y>0)$ and $(xy>1)$

$\tan^{-1}\left(\sqrt{3}\right)+\tan^{-1}(1)=\pi+\tan^{-1}\left[\frac{\sqrt{3}+1}{1-\sqrt{3}}\right]$

$=\pi+\tan^{-1}\left[\frac{\left(\sqrt{3}+1\right)\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)}\right]$

$=\pi+\tan^{-1}\left(\frac{4+2\sqrt{3}}{-2}\right)=\pi+\tan^{-1}\left[-\left(2+\sqrt{3}\right)\right]$

$=\pi-\tan^{-1}\left(2+\sqrt{3}\right) \qquad \because \tan^{-1}(-x)=-\tan^{-1}x$

$\therefore$ Statement (2) is incorrect

80. (b) Consider any equilateral triangle:
$c=b=a=1$ unit

Take value of p between 1 & 2 i.e., $\frac{3}{2}$

$\therefore \; a^{1/p}+b^{1/p}-c^{1/p}=(1)^{2/3}+(1)^{2/3}-(1)^{2/3}$
$=1+1-1=1>0$

Take value of p greater than 2 i.e; 3.

$\therefore \; a^{1/p}+b^{1/p}-c^{1/p}=(1)^{1/3}+(1)^{1/3}-(1)^{1/3}=1>0.$

$\therefore$ By considering all the options carefully; we came to a conclusion that opton (b) is correct.

81. (c) Given $\cos A+\cos B+\cos C=\sqrt{3}\sin\frac{\pi}{3}$

$\cos A+\cos B+\cos C$

$=2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}+\left(1-2\sin^2\frac{C}{2}\right)$

$=2\sin\frac{C}{2}\cos\frac{A-B}{2}+1-2\sin^2\frac{C}{2}$

$\left(\because \frac{A+B}{2}=90°-\frac{C}{2}\right)$

$=1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\sin\frac{C}{2}\right]$

$=1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right]$

$=1+4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$

$\cos A+\cos B+\cos C=\sqrt{3}\sin\frac{\pi}{3}$

$1+4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}=\sqrt{3}\sin\frac{\pi}{3}$

$\Rightarrow 1+4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}=\sqrt{3}\times\frac{\sqrt{3}}{2}$

$\Rightarrow 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{3}{2}-1$

$\boxed{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{8}}$

82. (d) As we know that

$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{8}$

$\because \left(\frac{A+B}{2}=\frac{\pi}{2}-\frac{C}{2}\right)$

$\therefore \boxed{\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)=\frac{1}{8}}$

83. (b) $\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ for which $\tan^{-1}(\tan\theta)=\theta$.

Hence, statement (1) is incorrect.

if $x\le 1; y\le 1 \,\&\, x^2+y^2\le 1$

$\therefore \sin^{-1}(x)-\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]$

$\Rightarrow \sin^{-1}\frac{1}{3}-\sin^{-1}\frac{1}{5}$

$=\sin^{-1}\left[\frac{1}{3}\sqrt{1-\frac{1}{25}}-\frac{1}{5}\sqrt{1-\frac{1}{9}}\right]$

$=\sin^{-1}\left[\frac{1}{3\times 5}\sqrt{24}-\frac{1}{5\times 3}\sqrt{8}\right]$

$=\sin^{-1}\left[\frac{\sqrt{8\times 3}-\sqrt{8}}{15}\right]=\sin^{-1}\left[\frac{2\sqrt{2}(\sqrt{3}-1)}{15}\right]$

Hence, statement (2) is correct.

84. (d) Statement-1

$\because \left(\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right)$

$\tan^{-1}x+\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{x+\frac{1}{x}}{1-x.\frac{1}{x}}$

$= \tan^{-1}\frac{x^2+1}{0}$

$= \tan^{-1}\infty = \tan^{-1}\tan\frac{\pi}{2}$

$\boxed{\tan^{-1}x \quad \tan^{-1}\frac{1}{x} \quad \frac{\pi}{2}}$

Statement (1) is wrong.
Statement 2,

$\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ $\qquad (x, y) \in (-1, 1)$

Only when $x = y$
Here $x \neq y$.
Statement (2) is also wrong.

85. (b) $\because$ ABC is an equilateral triangle.
$\therefore A = B = C = 60°$
L.H.S. = 3 tan $(A + B)$ tan C
= 3 tan 120° tan 60°
$= 3(-\sqrt{3})(\sqrt{3})$
$= -9 \neq 1$
Hence statement (1) is incorrect.
Statement-2
ABC is a triangle such that $A = 78°$ and $B = 66°$
$C = 180 - (78 + 66) = 180 - 144 = 36°$

$\frac{A}{2} \quad C \quad \frac{78}{2} \quad 36$

$= 39 + 36 = 75°$

$\tan\left(\frac{A}{2}+C\right) < \tan A$

$\Rightarrow \tan 75° < \tan 78°$
Hence statement (2) is correct.
Statement (3)
In a triangle ABC
$A + B + C = 180°$
$A + B = 180° - C$

$\frac{A+B}{2} = \frac{180°-C}{2}$

$\Rightarrow \frac{A+B}{2} = 90 - \frac{C}{2}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(90 - \frac{C}{2}\right)$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$ $\qquad$ ...(1)

$\therefore \tan\left(\frac{A+B}{2}\right).\sin\frac{C}{2} = \cot\frac{C}{2}.\sin\frac{C}{2} = \cos\frac{C}{2}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right).\sin\frac{C}{2} = \cos\frac{C}{2}$

We can see that statement (3) is not correct.
Hence only 2nd statement is correct.

86. (d) We know that

$2\cos^{-1}x \quad \cos^{-1}(2x^2 - 1)$

here, $x = 0.8$

$\therefore \quad 2\cos^{-1}(0.8) \quad \cos^{-1}(2(0.8)^2 - 1)$

$\cos^{-1}(0.28)$

Now, cos $(\cos^{-1}(x)) \quad x.$

$\therefore \cos(\cos^{-1}(0.28)) \quad 0.28$

87. (b) In triangle ABC, $A + B + C = \pi$

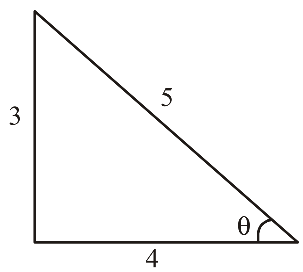
$\Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

$\Rightarrow \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2} - \frac{A}{2}$ $\qquad$ ...(i)

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \cos\frac{A}{2}$

Also, from (i), $\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{A}{2}\right) = \cot\frac{A}{2}$

88. (b) $\sin^{-1}\left(\frac{3}{5}\right) \quad \tan^{-1}\left(\frac{1}{7}\right)$



$\sin\theta \quad \frac{3}{5}$

$\tan\theta \quad \frac{3}{4}$

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right)$

$= \tan^{-1}\left(\frac{\frac{21+4}{28}}{\frac{28-3}{28}}\right)$

$= \tan^{-1}\left(\frac{25}{25}\right) = \tan^{-1}(1) = \frac{\pi}{4}.$

89. (b) Given, $a-2b+c=0 \Rightarrow a+c=2b$ $\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$

$$=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}\times\frac{s(s-c)}{(s-a)(s-b)}}=\sqrt{\frac{s^2}{(s-b)^2}}=\frac{s}{s-b}$$

$$=\frac{2s}{2s-2b}=\frac{a+b+c}{a+b+c-2b}=\frac{2b+b}{2b-b}=\frac{3b}{b}=3.$$

90. (a) Given, $\frac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C}=2.$

Let us take $A=30°, B=60°, C=90°$

$$\frac{\sin^2 30°+\sin^2 60°+\sin^2 90°}{\cos^2 30°+\cos^2 60°+\cos^2 90°}$$

$$=\frac{\frac{1}{4}+\frac{3}{4}+1}{\frac{3}{4}+\frac{1}{4}+0}=\frac{1+1}{1}=\frac{2}{1}=2.$$

So, the given triangle is right-angled triangle.

91. (b) The principal value of $\sin^{-1}x$ lies in it's range.

The range of $\sin^{-1} x$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.

92. (b) In ΔABC, $a=2, b=3$ and $\sin A=\frac{2}{3}$.

We know, $\frac{\sin A}{a}=\frac{\sin B}{b}$

$$\Rightarrow \frac{\frac{2}{3}}{2}=\frac{\sin B}{3}$$

$$\Rightarrow \frac{2}{6}=\frac{\sin B}{3}\Rightarrow \sin B=\frac{6}{6}=1$$

$\Rightarrow B=\sin^{-1}(1)$ ....(1)

$$=\frac{\pi}{2}.$$

93. (c) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$

$$=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}.$$

94. (b) $x, x-y, x+y$ are angles of a triangle.

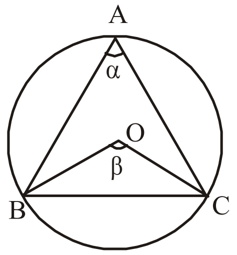
$\tan(x-y), \tan x, \tan(x+y)$ are in G.P.

Now, $x+x-y+x+y=\pi$ (Sum of angles in triangle $=180°=\pi$)

$\Rightarrow 3x=\pi$

$$\Rightarrow x=\frac{\pi}{3}.$$

95. (a) $\angle BAC=\alpha, \angle BOC=\beta$



We know, from figure,

$\beta=2\alpha$

$\therefore \cos\beta=\cos 2\alpha$

$$=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}.$$

96. (b) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)$

We know, $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

So, $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{3}{5}}{1-\left(\frac{1}{4}\right)\left(\frac{3}{5}\right)}\right)$

$$=\tan^{-1}\left(\frac{\frac{5+12}{20}}{1-\frac{3}{20}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{17}{20}}{\frac{17}{20}}\right)=\tan^{-1}(1)=\frac{\pi}{4}.$$

97. (d) $\sin 2A-\sin 2B-\sin 2C$

$=2\cos(A+B)\cdot\sin(A-B)+\sin(2A+2B)$

$=2\cos(A+B)\sin(A-B)+2\sin(A+B)\cos(A+B)$

$=2\cos(A+B)[\sin(A-B)+\sin(A+B)]$

$=-2\cos C[2\sin A.\cos B]$

$=-4\sin A\cos B\cos C$

98. (a) $\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right)=\frac{\pi}{4}$

$$\tan^{-1}\frac{5x}{1-6x^2}=\frac{\pi}{4}$$

$$\frac{5x}{1-6x^2}=1$$

$$1-6x^2=5x$$

$$6x^2+5x-1=0$$

$$(6x-1)(x+1)=0$$

$$\Rightarrow x=\frac{1}{6},-1$$

$$\Rightarrow x=\frac{1}{6}$$

Here $x=-1$ rejected.

99. (b) $y = \cos^{-1}(\sin x)$

$= \cos^{-1}\cos\left(\frac{\pi}{2} - x\right)$

$= \frac{\pi}{2} - x$

From question, slope of the curve $m = \tan\theta$

$\therefore \tan\theta = -1 \Rightarrow \theta = \frac{3\pi}{4}$

100. (d) $\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{4} - \frac{\pi}{2} = \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} - \frac{\pi}{2}$

$= \frac{\pi}{2} - \frac{\pi}{2} = 0$

101. (b) $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-q^2}{1+q^2} = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}p - 2\tan^{-1}q = 2\tan^{-1}x$

$\Rightarrow \tan^{-1}p - \tan^{-1}q = \tan^{-1}x$

$\Rightarrow \tan^{-1}\frac{p-q}{1+pq} = \tan^{-1}x$

$\Rightarrow x = \frac{p-q}{1+pq}$

102. (c) Given, angles of triangle are in ratio 1 : 2 : 3

Consider, A = 30°, B = 60° and C = 90°

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 30^\circ} = \frac{b}{\sin 60^\circ} = \frac{c}{\sin 90^\circ}$

$\Rightarrow \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1}$

$\Rightarrow a : b : c = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$

103. (a) Let $y = \sec^2(\tan^{-1}x)$

Let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta$

$y = \sec^2\theta = 1 + \tan^2\theta = 1 + x^2$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(1 + x^2) = 2x$

Height & Distance 13

1. A vertical pole with height more than 100 m consists of two parts, the lower being one-third of the whole. At a point on a horizontal plane through the foot and 40 m from it, the upper part subtends an angle whose tangent is $\frac{1}{2}$. What is the height of the pole ?
(a) 110m (b) 200m
(c) 120m (d) 150m *[2006-I]*
2. The angle of elevation of the top of a pillar of height h at a point on the ground at a distance x from the pillar is 30°. On walking a distance 'd' towards the pillar the angle of elevation becomes 60°. Then, which one of the following is correct ?
(a) $x = d + h$ (b) $x = \frac{3d}{2}$
(c) $x = \frac{5d}{4}$ (d) $x = 2d$ *[2006-II]*
3. The angle of elevation of the top of a tower EF (F being the foot of the tower) as seen from a point A which is on the same level as F, is α. On advancing towards the foot of the tower the angle of elevation of the top of the tower as seen from a point B such that AB = x, is β. If BF = y, h is the height of the tower and $\alpha + \beta = \frac{\pi}{2}$, then which one of the following is correct ?
(a) $h^2 = x^2 + xy$ (b) $h = y^2 + xy^2$
(c) $h^2 = y^2 + xy$ (d) $h = y + x^2y$ *[2006-II]*
4. The lower 24 m portion of a 50 m tall tower is painted green and the remaining portion red. What is the distance of a point on the ground from the base of the tower where the two different portions of the tower subtend equal angles?
(a) 60m (b) 72m
(c) 90m (d) 120m *[2007-I]*
5. What should be the height of a flag where a 20 feet long ladder reaches 20 feet below the flag (The angle of elevation of the top of the flag at the foot of the ladder is 60°)?
(a) 20 feet (b) 30 feet
(c) 40 feet (d) $20\sqrt{2}$ feet *[2007-II]*
6. PT, a tower of height 2^x metre, P being the foot, T being the top of the tower. A, B are points on the same line with P. If AP = 2^{x+1} m, BP = 192 m and if the angle of elevation of the tower as seen from b is double the angle of the elevation of the tower as seen from A, then what is the value of x?
(a) 6 (b) 7
(c) 8 (d) 9 *[2008-I]*
7. The foot of a tower of height *h* m is in a direct line between two observers *A* and *B*. If the angles of elevation of the top of the tower as seen from *A* and *B* are α and β respectively and if *AB* = *d* m, then what is *h*/*d* equal to? *[2008-II]*
(a) $\frac{\tan(\alpha+\beta)}{(\cot\alpha\cot\beta-1)}$ (b) $\frac{\cot(\alpha+\beta)}{(\cot\alpha\cot\beta-1)}$
(c) $\frac{\tan(\alpha+\beta)}{(\cot\alpha\cot\beta+1)}$ (d) $\frac{\cot(\alpha+\beta)}{(\cot\alpha\cot\beta+1)}$
8. A man observes the elevation of a balloon to be 30°. He, then walks 1 km towards the balloon and finds that the elevation is 60°. What is the height of the balloon? *[2009-I]*
(a) 1 / 2 km (b) $\sqrt{3}/2$ km
(c) 1 / 3 km (d) 1 km
9. The angle of elevation from a point on the bank of a river of the top of a temple on the other bank is 45°. Retreating 50m, the observer finds the new angle of elevation as 30°. What is the width of the river? *[2009-I]*
(a) 50m (b) $50\sqrt{3}$ m
(c) $50/(\sqrt{3}-1)$ m (d) 100m
10. Looking from the top of a 20 *m* high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its bottom is 30°. What is the height of the tower? *[2009-II]*
(a) 50 *m* (b) 60 *m*
(c) 70 *m* (d) 80 *m*
11. The angle of elevation of the top of a flag post from a point 5 m away from its base is 75º. What is the approximate height of the flag post? *[2010-I]*
(a) 15m (b) 17m
(c) 19m (d) 21m

12. Two poles are 10 m and 20 m high. The line joining their tops makes an angle of 15° with the horizontal. What is the approximate distance between the poles? *[2010-II]*
(a) 35.3 m (b) 37.3 m
(c) 41 m (d) 44 m

13. From the top of a lighthouse 120 m above the sea, the angle of depression of a boat is 15°. What is the distance of the boat from the lighthouse? *[2010-II]*
(a) 400 m (b) 421 m
(c) 444 m (d) 460 m

14. A man standing on the bank of a river observes that the angle of elevation of the top of a tree just on the opposite bank is 60°. The angle of elevation is 30° from a point at a distance *y m* from the bank of the river. What is the height of the tree? *[2011-I]*
(a) y m (b) 2y m
(c) $\frac{\sqrt{3}y}{2}m$ (d) $\frac{y}{2}m$

15. At a point 15 m away from the base of a 15 m high house, the angle of elevation of the top is *[2011-II]*
(a) 90° (b) 60°
(c) 45° (d) 30°

16. A tower of height 15 m stands vertically on the ground. From a point on the ground the angle of elevation of the top of the tower is found to be 30°. What is the distance of the point from the foot of the tower? *[2011-II]*
(a) $15\sqrt{3}\ m$ (b) $10\sqrt{3}\ m$
(c) $5\sqrt{3}\ m$ (d) $30\ m$

17. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height *h*. At a point *P* on the plane, the angle of elevation of the bottom of the flag staff is β and that of the top is α. What is the height of the tower? *[2011-II]*
(a) $\frac{h\tan\beta}{\tan\alpha-\tan\beta}$ (b) $\frac{h\tan\beta}{\tan\alpha+\tan\beta}$
(c) $\frac{h\cos\beta}{\cos\alpha-\cos\beta}$ (d) $\frac{h}{\cos(\alpha-\beta)}$

18. An aeroplane flying at a height of 300 m above the ground passes vertically above another plane at an instant when the angles of elevation of two planes from the same point on the ground are 60° and 45° respectively. What is the height of the lower plane from the ground? *[2011-II]*
(a) 50 m (b) $\frac{100}{\sqrt{3}}$ m
(c) $100\sqrt{3}$ m (d) $150(\sqrt{3}+1)$ m

19. The angle of elevation of the tip of a flag staff from a point 10 m due South of its base is 60°. What is the height of the flag staff correct to the nearest meter? *[2012-I]*
(a) 15 m (b) 16 m
(c) 17 m (d) 18 m

20. Two poles are 10 m and 20 m high. The line joining their tips makes an angle of 15° with the horizontal. What is the distance between the poles? *[2012-I]*
(a) $10(\sqrt{3}-1)$m (b) $5(4+2\sqrt{3})$m
(c) $20(\sqrt{3}+1)$m (d) $10(\sqrt{3}+1)$m

21. The angle of elevation of a tower at a level ground is 30°. The angle of elevation becomes θ when 10 m moved towards the tower. If the height of tower is $5\sqrt{3}$ m, then what is θ equal to? *[2012-I]*
(a) 45° (b) 60°
(c) 75° (d) None of the above

22. From the top of a building of height h meter, the angle of depression of an object on the ground is θ. What is the distance (in meter) of the object from the foot of the building? *[2012-I]*
(a) h cotθ (b) h tanθ
(c) h cosθ (d) h sinθ

23. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation α and β respectively. The height of the hill is : *[2012-II]*
(a) $\frac{h\cot\beta}{\cot\beta-\cot\alpha}$ (b) $\frac{h\cot\alpha}{\cot\alpha-\cot\beta}$
(c) $\frac{h\tan\alpha}{\tan\alpha-\tan\beta}$ (d) None of the above

24. From the top of a lighthouse 70 m high with its base at sea level, the angle of depression of a boat is 15°. The distance of the boat from the foot of the lighthouse is: *[2012-II]*
(a) $70(2-\sqrt{3})$ m (b) $70(2+\sqrt{3})$ m
(c) $70(3-\sqrt{3})$ m (d) $70(3+\sqrt{3})$ m

25. The angle of elevation of the top of a tower of height H from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. If h is the height of the other tower, then which one of the following is correct? *[2013-I]*
(a) H = 2h (b) $H=\sqrt{3}h$
(c) H = 3h (d) None of the above

26. A man walks 10 m towards a lamp post and notices that the angle of elevation of the top of the post increases from 30° to 45°. The height of the lamp posts is : *[2013-I]*
(a) 10 m (b) $(5\sqrt{3}+5)$m
(c) $(5\sqrt{3}-5)$m (d) $(10\sqrt{3}+10)$m

27. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30° than when it is 60°. The height of the tower is: *[2013-I]*
(a) 25 m (b) $25\sqrt{3}$ m
(c) 50 m (d) None of these

28. The angle of elevation of the top of a tower from two places situated at distances 21m. and *x* m. from the base of the tower are 45° and 60° respectively. What is the value of *x* ? *[2013-II]*
(a) $7\sqrt{3}$ (b) $7-\sqrt{3}$
(c) $7+\sqrt{3}$ (d) 14

29. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite of bank is 60°. When he retires 40 m. from the bank, he finds the angle to be 30°. What is the breadth of the river ? *[2013-II]*
(a) 60m (b) 40m
(c) 30m (d) 20m

30. From an aeroplane above a straight road the angle of depression of two positions at a distance 20 m apart on the road are observed to be 30° and 45°. The height of the aeroplane above the ground is : *[2014-I]*
(a) $10\sqrt{3}$m (b) $10(\sqrt{3}-1)$m
(c) $10(\sqrt{3}+1)$m (d) 20m

31. A lamp post stands on a horizontal plane. From a point situated at a distance 150 m from its foot, the angle of elevation of the top is 30°. What is the height of the lamp post ? *[2014-II]*
(a) 50m (b) $50\sqrt{3}$ m
(c) $\frac{50}{\sqrt{3}}$ m (d) 100m

32. The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. What is the height of the tower? *[2015-I]*
(a) 10m (b) 20m
(c) 30m (d) 40m

33. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at distances 49 m and 36 m are 43° and 47° respectively. What is the height of the tower? *[2015-I]*
(a) 40m (b) 42m
(c) 45m (d) 47m

34. Two poles are 10 m and 20 m high. The line joining their tops makes an angle of 15° with the horizontal. The distance between the poles is approximately equal to *[2015-II]*
(a) 36.3m (b) 37.3 in
(c) 38.3m (d) 39.3 in

35. A vertical tower standing on a levelled field is mounted with a vertical flag staff of length 3 m. From a point on the field, the angles of elevation of the bottom and tip of the flag staff are 30° and 45° respectively. Which one of the following gives the best approximation to the height of the tower ? *[2015-II]*
(a) 3.90m (b) 4.00m
(c) 4.10m (d) 4.25m

36. The top of a hill when observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. What is the height of the hill? *[2016-II]*
(a) $\frac{h\cot q}{\cot q-\cot p}$ (b) $\frac{h\cot p}{\cot p-\cot q}$
(c) $\frac{2h\tan p}{\tan p-\tan q}$ (d) $\frac{2h\tan q}{\tan q-\tan p}$

37. A moving boat is observed from the top of a cliff of 150 m height. The angle of depression of the boat changes from 60° to 45° in 2 minutes. What is the speed of the boat in metres per hour? *[2016-II]*
(a) $\frac{4500}{\sqrt{3}}$ (b) $\frac{4500(\sqrt{3}-1)}{\sqrt{3}}$
(c) $4500\sqrt{3}$ (d) $\frac{4500(\sqrt{3}+1)}{\sqrt{3}}$

38. From the top of a lighthouse, 100 m high, the angle of depression of a boat is $\tan^{-1}\left(\frac{5}{12}\right)$. What is the distance between the boat and the lighthouse? *[2017-I]*
(a) 120 m (b) 180 m
(c) 240 m (d) 360 m

39. The angle of elevation of a stationary cloud from a point 25 m above a lake is 15° and the angle of depression of its image in the lake is 45°. The height of the cloud above the lake level is *[2017-II]*
(a) 25m (b) $25\sqrt{3}$ m
(c) 50m (d) $50\sqrt{3}$ m

40. The angles of elevation of the top of a tower from the top and foot of a pole are respectively 30° and 45°. If h_T is the height of the tower and h_P is the height of the pole, then which of the following are correct? *[2017-II]*
1. $\frac{2h_P h_T}{3+\sqrt{3}}=h_P^2$ 2. $\frac{h_T-h_P}{\sqrt{3}+1}=\frac{h_P}{2}$
3. $\frac{2(h_P+h_T)}{h_P}=4+\sqrt{3}$

Select the correct answer using the code given below.
(a) 1 and 3 only (b) 2 and 3 only
(c) 1 and 2 only (d) 1, 2 and 3

41. If a flag-staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then what is the angle that the sun makes with the ground? *[2018-I]*
(a) 60° (b) 45° (c) 30° (d) 15°

42. A spherical balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its centre is β. What is the height of the centre of the balloon (neglecting the height of the observer)? *[2018-I]*
(a) $\frac{r\sin\beta}{\sin\left(\frac{\alpha}{2}\right)}$ (b) $\frac{r\sin\beta}{\sin\left(\frac{\alpha}{4}\right)}$
(c) $\frac{r\sin\left(\frac{\beta}{2}\right)}{\sin\alpha}$ (d) $\frac{r\sin\alpha}{\sin\left(\frac{\beta}{2}\right)}$

43. A balloon is directly above one end of a bridge. The angle of depression of the other end of the bridge from the balloon is 48°. If the height of the balloon above the bridge is 122 m, then what is the length of the bridge? *[2018-II]*
(a) 122 sin 48° m (b) 122 tan 42° m
(c) 122 cos 48° m (d) 122 tan 48° m

44. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\frac{\pi}{6}$ and $\frac{\pi}{3}$ respectively. What is the height of the hill? *[2018-II]*
(a) 2h (b) $\frac{3h}{2}$ (c) h (d) $\frac{h}{2}$

45. The angle of elevation of a tower of height h from a point A due South of it is x and from a point B due East of A is y. If AB = z, then which one of the following is correct ? *[2019-I]*
(a) $h^2(\cot^2 y - \cot^2 x) = z^2$
(b) $z^2(\cot^2 y - \cot^2 x) = h^2$
(c) $h^2(\tan^2 y - \tan^2 x) = z^2$
(d) $z^2(\tan^2 y - \tan^2 x) = h^2$

ANSWER KEY

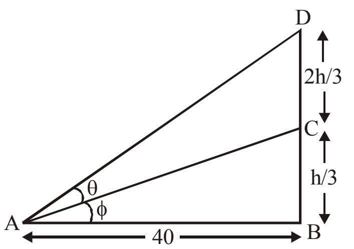
1	(c)	6	(c)	11	(c)	16	(a)	21	(b)	26	(b)	31	(b)	36	(b)	41	(a)
2	(b)	7	(b)	12	(b)	17	(a)	22	(a)	27	(b)	32	(b)	37	(b)	42	(a)
3	(c)	8	(b)	13	(c)	18	(c)	23	(b)	28	(a)	33	(b)	38	(c)	43	(a)
4	(d)	9	(c)	14	(c)	19	(c)	24	(b)	29	(d)	34	(b)	39	(b)	44	(b)
5	(b)	10	(d)	15	(c)	20	(b)	25	(c)	30	(c)	35	(c)	40	(c)	45	(a)

HINTS & SOLUTIONS

1. (c) Let h be the height of pole, upper portion CD subtend angle θ at A.

Then, $\tan\theta = \frac{1}{2}$

Let lower part BC subtend angle ϕ at A then
In Δ ABC,



$$\tan\phi = \frac{BC}{AB} = \frac{h/3}{40} = \frac{h}{120}$$

In Δ ABD,

$$\tan(\theta + \phi) = \frac{BD}{AB}$$

$$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{h}{40}$$

$$\Rightarrow \frac{\frac{1}{2} + \frac{h}{120}}{1 - \frac{h}{240}} = \frac{h}{40}$$

$$\Rightarrow \frac{2(60+h)}{(240-h)} = \frac{h}{40}$$

$\Rightarrow 80(60+h) = 240h - h^2 \Rightarrow 4800 + 80h = 240h - h^2$

$\Rightarrow h^2 - 160h + 4800 = 0 \Rightarrow (h-120)(h-40) = 0$

$\Rightarrow h = 120$

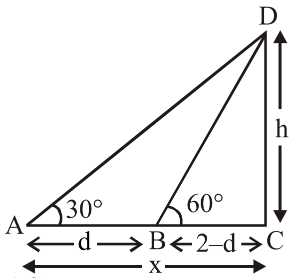
[h = 40 is discarded, since h > 100 is given]

2. (b) Let DC be the pillar of height h and A be the point at a distance x from pillar such that ∠CAD = 30°. On walking a distance d towards pillar (point B) ∠CBD = 60°. So, in ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x-d}$$

$$\Rightarrow h = \sqrt{3}(x-d) \quad ...(i)$$



and in ΔACD,

$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = h\sqrt{3}$$

$\Rightarrow x = 3(x - d)$ (using Eq. (i))

$\Rightarrow x = 3x - 3d$

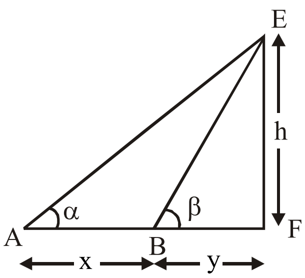
$\Rightarrow 2x = 3d$

$$\Rightarrow x = \frac{3d}{2}$$

3. (c) Let EF be the height of the tower, and FB = Y, AB = x and EF = h.
In ΔBEF,

$$\tan\beta = \frac{EF}{BF}$$

$$\tan\beta = \frac{h}{y} \quad \text{...(i)}$$



and in ΔAFE,

$$\tan\alpha = \frac{EF}{AF}$$

$$\Rightarrow \tan\alpha = \frac{h}{x+y},$$

$$\Rightarrow \tan\left(\frac{\pi}{2} - \beta\right) = \frac{h}{x+y} \quad \left(\text{Given that } \alpha + \beta = \frac{\pi}{2}\right)$$

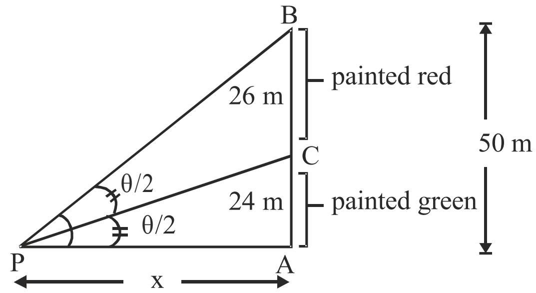
$$\Rightarrow \cot\beta = \frac{h}{x+y} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\tan\beta . \cot\beta = \frac{h}{y} . \frac{h}{(x+y)} \quad (\because \tan\beta \cdot \cot\beta = 1)$$

$$\Rightarrow xy + y^2 = h^2$$

4. (d)



Let the distance, be x, and angle APB = θ, then ∠BPC = ∠APC = θ/2
In triangle Δ APB,

$$\tan\theta = \frac{AB}{x} = \frac{50}{x} \quad \text{...(1)}$$

and in triangle APC

$$\tan\frac{\theta}{2} = \frac{AC}{x} = \frac{24}{x} \quad \text{...(2)}$$

$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}} \quad \text{...(3)}$$

Putting the value of tan θ and tan $\frac{\theta}{2}$
From equation (1) and (2) in equation (3),

$$\frac{50}{x} = \frac{2 \times \frac{24}{x}}{1 - \left(\frac{24}{x}\right)^2}$$

$$\text{or, } \frac{50}{x} = \frac{48}{x} \times \frac{x^2}{x^2 - (24)^2}$$

or, $50\{x^2 - (24)^2\} = 48x^2$

or, $50x^2 - 50 \times (24)^2 = 48x^2$

or, $2x^2 = (24)^2 \times 50$

$x = 25 \times (24)^2$

$x = |5 \times 24| = 120$ m

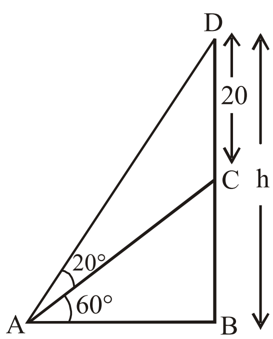
Such a point can exist on the either side of the tower.

5. (b) In ΔABD,

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AB}$$

$$\Rightarrow AB = \frac{h}{\sqrt{3}}$$



$\Rightarrow \quad AB = \frac{h}{3}\sqrt{3}$

Now, in ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 20^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (h-20)^2$

$\Rightarrow \quad h^2 + 3h^2 - 120h = 0$

$\Rightarrow \quad 4h^2 - 120h = 0$

$\Rightarrow \quad h(h-30) = 0$

$h = 0$ or 30

$h = 0$ not possible

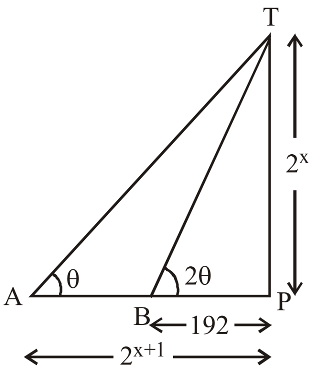
$\Rightarrow \quad h = 30$ ft

6. (c) Let PT be the tower such that

$PT = 2^x$ m, $AP = 2^{x+1}$ m

A and B are two points joining foot of the tower.

T is top of the tower.



Also, given BP = 192 m, $\angle TAP = \theta$ and $\angle TBP = 2\theta$

In ΔPTA

$\tan\theta = \frac{PT}{AP} \Rightarrow \tan\theta = \frac{2^x}{2^{x+1}} = \frac{1}{2}$

Now, in ΔPTB

$\tan 2\theta = \frac{PT}{PB} = \frac{2^x}{192}$

$\Rightarrow \frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{2^x}{192} \Rightarrow \frac{1}{3/4} = \frac{2^x}{192} \Rightarrow \frac{4}{3} \times 192 = 2^x$

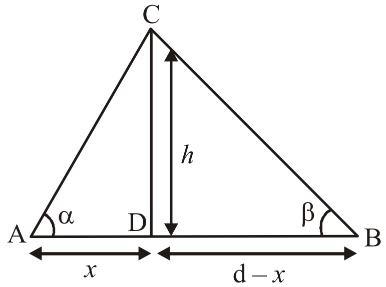
$2^x = 256$

$\Rightarrow 2^x = 2^8 \Rightarrow x = 8$

7. (b) Let $AD = x$

$\therefore DB = d - x$

In ΔADC,



$\tan\alpha = \frac{h}{x}$

$\Rightarrow x = h\cot\alpha$

In ΔCDB,

$\tan\beta = \frac{h}{d-x}$

$\Rightarrow d - x = h\cot\beta$

From equations (i) and (ii), we get

$d = h(\cot\alpha + \cot\beta)$

We know,

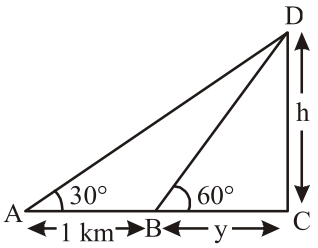
$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha}$

$\Rightarrow \cot\beta + \cot\alpha = \frac{\cot\alpha\cot\beta - 1}{\cot(\alpha+\beta)}$

$\therefore \quad d = h\left[\frac{\cot\alpha\cot\beta - 1}{\cot(\alpha+\beta)}\right]$

$\Rightarrow \frac{h}{d} = \frac{\cot(\alpha+\beta)}{\cot\alpha\cot\beta - 1}$

8. (b) Let the height of the balloon be h and new distance between BC be y as shown in figure given below,



$\tan 60° = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{y} \Rightarrow h = \sqrt{3}y$

and now in ΔADC, $\tan 30° = \frac{CD}{AC}$

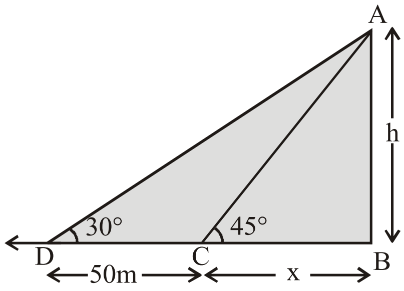
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{1+y}$

$\Rightarrow 1 + y = h\sqrt{3} \Rightarrow 1 + y = 3y$

$\Rightarrow y = \frac{1}{2}$

$\therefore \quad h = \frac{\sqrt{3}}{2}$

9. (c) In ΔABC, $\tan 45° = \dfrac{AB}{BC}$



$\Rightarrow 1 = \dfrac{h}{x}$

$\Rightarrow h = x$ ...(i)

Now, in ΔABD,

$\tan 30° = \dfrac{AB}{BD}$

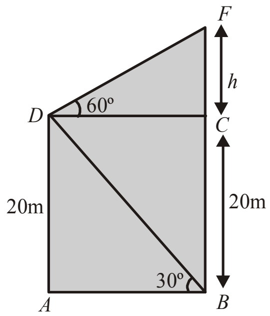
$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+50}$

$\Rightarrow x + 50 = h\sqrt{3}$

$\Rightarrow h + 50 = h\sqrt{3}$ [from Eq. (i)]

$\Rightarrow h = \dfrac{50}{\sqrt{3}-1} m$

10. (d) In ΔABD, $\tan 30° = \dfrac{AD}{AB}$



$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{AD}{AB}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{20}{AB}$

$\Rightarrow AB = 20\sqrt{3}$ m.

In ΔDCF, $\tan 60° = \dfrac{h}{DC} \Rightarrow \sqrt{3} = \dfrac{b}{DC}$

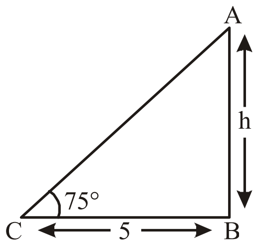
$\Rightarrow h = (\sqrt{3})(20\sqrt{3})$ $(\because AB = DC = 20\sqrt{3})$

$\Rightarrow h = 60$ m

$\therefore$ Height of tower, $BF = 60 + 20 = 80$ m

11. (c) Let h be the height of the flag post where BC is the base.

In ΔABC,



$\tan 75° = \dfrac{AB}{BC} = \dfrac{h}{5}$

$\Rightarrow \tan(45° + 30°) = \dfrac{h}{5}$

$\Rightarrow \dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} = \dfrac{h}{5}$

$\left(\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right)$

$\Rightarrow \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \dfrac{h}{5} \Rightarrow h = \dfrac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2} \times 5$

$\Rightarrow h = 5\left(\dfrac{3+1+2\sqrt{3}}{3-1}\right) = 5(2+\sqrt{3})$

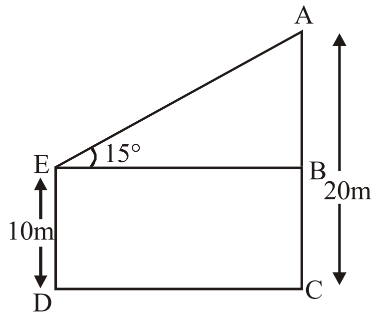
$= 5 \times 3.732 = 18.660$

$= 19$ m (approx.)

12. (b) Let AC and ED be two poles of height 20m and 10m respectively.

Let $\angle AEB = 15°$

To find : DC.



Now, $AB = AC - BC = AC - ED = 20 - 10 = 10$ m.

Now in ΔABE, $(\because BC = ED)$

$\tan 15° = \dfrac{AB}{BE}$

$\Rightarrow \tan(45° - 30°) = \dfrac{10}{BE}$

$\Rightarrow \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \dfrac{10}{BE}$

$\left(\because \tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}\right)$

$\Rightarrow \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{10}{BE}$

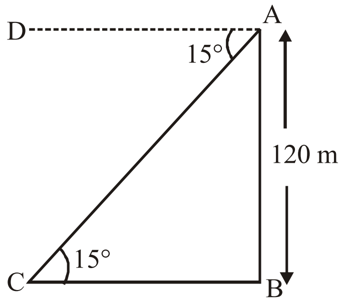
$\Rightarrow BE = 10\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right) = \dfrac{10 \times (\sqrt{3}+1)^2}{2}$

(By rationalizing)

$5(3+1+2\sqrt{3}) = 5(4+2\times1.73) = 5(4+3.46)$

$\Rightarrow CD = BE = 5 \times 7.46 = 37.3$ m

13. (c) Let AB be the light house of 120 m. and c be the boat.
Let $\angle$ DAC = 15° (Angle of depression)
In ΔABC



$$\tan 15° = \frac{AB}{BC}$$

$$\tan(45° - 30°) = \frac{AB}{BC}$$

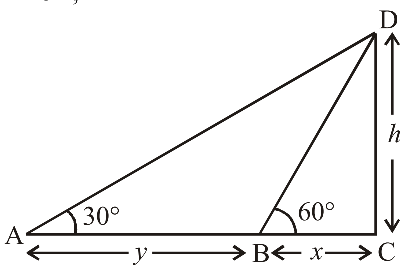
$$\Rightarrow \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \frac{120}{BC}$$

$$\Rightarrow \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{120}{BC}$$

$$\Rightarrow BC = 120\left(\frac{(\sqrt{3}+1)^2}{3-1}\right) = 60(3 + 1 + 2\sqrt{3})$$

$$= 60\,(4 + 2 \times 1.73)$$
$$= 60 \times 7.46 = 447.6 \text{ m} \simeq 444 \text{ m}$$

14. (c) Let DC be the tree of height h metre. Let a man is standing on the point B (bank of a river).
Let BC = x and angle of elevation i.e. $\angle$DBC = 60°.
Also, let AB = y and $\angle$DAC = 30°.
In ΔACD,



$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow x + y = h\sqrt{3} \qquad \text{.........(i)}$$

and in ΔBCD,

$$\tan 60° = \frac{CD}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$$

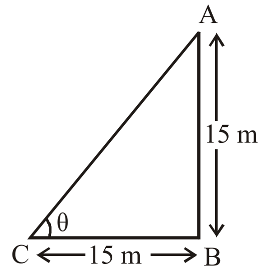
$$\Rightarrow x = \frac{h}{\sqrt{3}} \qquad \text{.........(ii)}$$

From eqns. (i) and (ii),

$$\frac{h}{\sqrt{3}} + y = h\sqrt{3}$$

$$\Rightarrow y = h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}} \quad \Rightarrow h = \frac{\sqrt{3}y}{2} \text{ m}$$

15. (c)



Let AB be the house of height 15 m. Let B be the base of house and C be the point 15 m away from the base of a house. Let 'θ' be the angle of elevation. So, in ΔABC,

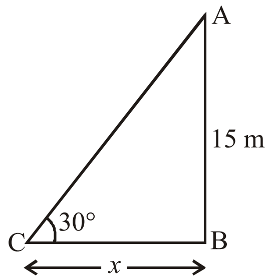
we have $\tan\theta = \frac{AB}{BC} = \frac{15}{15} = 1$

$\Rightarrow \tan\theta = \tan 45°$ $\quad (\because \tan 45° = 1)$

$\Rightarrow \theta = 45°$

Hence, the angle of elevation of the top is 45°

16. (a) Let AB be a tower of height 15 m. Let B be the point on the ground. Let $\angle$ACB = 30° be the angle of elevation. To find BC.

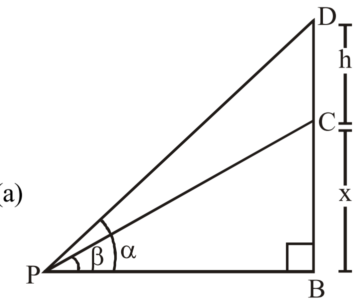


Let BC = xm.

In ΔABC, $\tan 30° = \frac{AB}{BC} = \frac{15}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{15}{x} \Rightarrow x = 15\sqrt{3} \text{ m}$$

17. (a)



Let BC be the vertical tower and CD be the flagstaff so that CD = h

Let P be the point of observation on the plane.

Then, $\angle$BPC = β and $\angle$BPD = α

Let BC = x

Now, $\frac{PB}{x} = \cot\beta \Rightarrow PB = x\cot\beta \qquad \text{...(1)}$

and $\frac{PB}{x+h} = \cot\alpha \Rightarrow PB = (x+h)\cot\alpha$ ...(2)

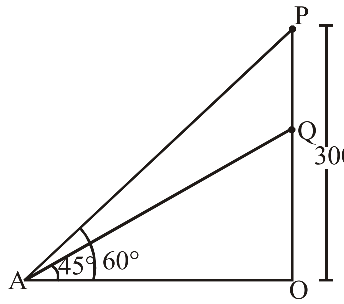
From (1) and (2), we get

$x\cot\beta = (x+h)\cot\alpha \Rightarrow x(\cot\beta - \cot\alpha) = h\cot\alpha$

∴ Height of the tower,

$$x = \frac{h\cot\alpha}{\cot\beta - \cot\alpha} = \frac{h\tan\beta}{\tan\alpha - \tan\beta}$$

18. (c)



Let P and Q be the positions of two aeroplanes when Q is vertically below P and OP = 300 *m*

Let the angles of elevation of P and Q at a point A on the ground be 60° and 45° respectively.

∴ In Δ AOQ,

$$\tan 45° = \frac{OQ}{OA} \Rightarrow OA = OQ$$

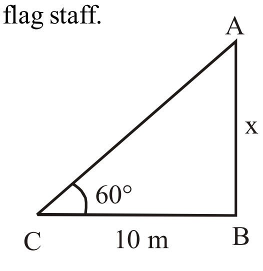
In Δ AOP,

$$\tan 60° = \frac{OP}{OA} = \frac{300}{OA} = \sqrt{3}$$

$$\Rightarrow OA = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

Hence, $OQ = 100\sqrt{3}$ m

19. (c) Let 'A' be the top of the flag staff.
Let 'x' be the height of the flag staff.



In ΔABC, $\tan 60° = \frac{x}{10}$

$$\sqrt{3} = \frac{x}{10}$$

$x = 10\sqrt{3} = 10 \times 1.732$

$= 17.32 \cong 17$

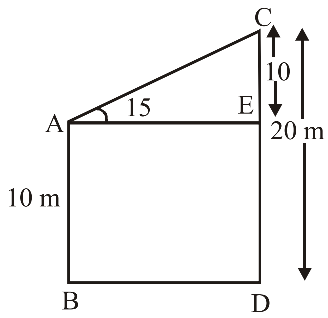
20. (b) Let AB and CD be two poles of height 10 m and 20 m respectively.

In ΔAEC,

$$\frac{CE}{AE} = \tan 15°$$

$$\frac{10}{AE} = \tan(45° - 30°)$$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

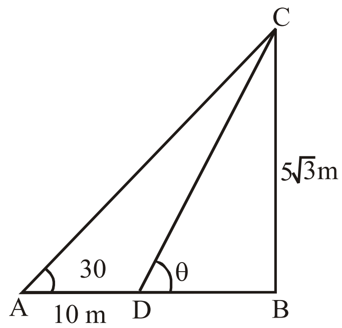


$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\frac{10}{AE} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$AE = \frac{10(\sqrt{3}+1)}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 5(4 + 2\sqrt{3})$$

21. (b) Let BC be the tower of height $5\sqrt{3}$ m
Let AD = 10 m



In Δ ABC,

$$\frac{BC}{AB} = \tan 30°$$

$$\Rightarrow AB = \frac{BC}{\tan 30°} = \frac{5\sqrt{3}}{1/\sqrt{3}} = 15$$

Given

AD = 10 m

∴ DB = (15 – 10)m = 5 m

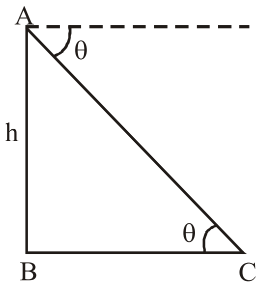
In Δ BCD,

$$\frac{BC}{BD} = \tan\theta$$

$$\frac{5\sqrt{3}}{5} = \tan\theta \Rightarrow \tan\theta = \sqrt{3}$$

$\Rightarrow \theta = 60°$

22. (a)

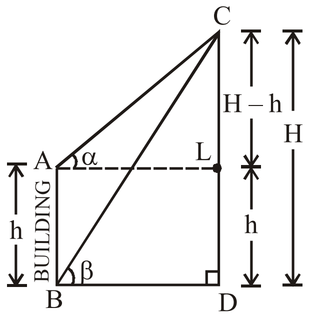


Let AB be the building of height h meter. Let 'θ' be the angle of depression.
To find: BC

In $\Delta ABC,\ \frac{AB}{BC} = \tan\theta$

$\Rightarrow \frac{h}{BC} = \tan\theta \Rightarrow BC = h\cot\theta$

23. (b)



In ΔBDC,

$\tan\beta = \frac{CD}{BD} = \frac{H}{BD}$

$BD = \frac{H}{\tan\beta} = H\cot\beta$ ...(i)

In ΔALC,

$\tan\alpha = \frac{CL}{AL} = \frac{H-h}{BD}$

$BD = (H-h)\cot\alpha$ ...(ii)

from (i) & (ii),

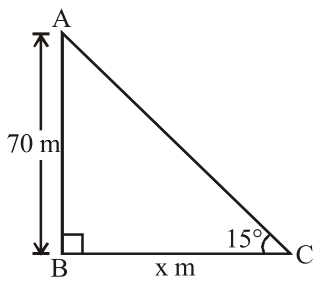
$(H-h)\cot\alpha = H\cot\beta$.

$H\cot\alpha - h\cot\alpha = H\cot\beta$.

$H[\cot\alpha - \cot\beta] = h\cot\alpha$.

$H = \frac{h\cot\alpha}{\cot\alpha - \cot\beta}$

24. (b)



Let AB = lighthouse = 70 m
Let BC = distance of boat from lighthouse
i.e. BC = x m

$\tan 15° = \frac{70}{x}$ ...(i)

$\tan 15° = \tan[45° - 30°]$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$

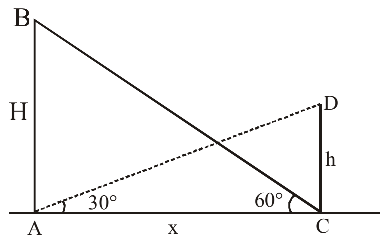
$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{2} = \frac{4-2\sqrt{3}}{2}$

$\tan 15° = 2-\sqrt{3}$

From (i)

$x = \frac{70}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 70\left(2+\sqrt{3}\right)$ m

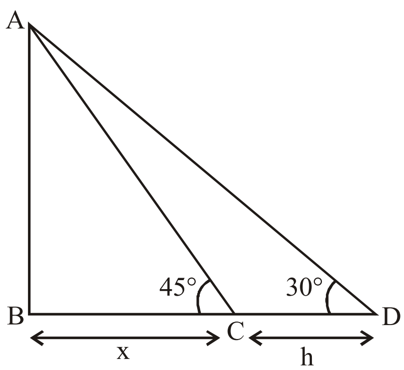
25. (c) Let AB be the tower of height H.
Let CD be the tower of height h.
Let x be the distance between the AB and CD.



from ΔABC, $\frac{H}{x} = \sqrt{3}$ and from ΔACD, $\frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{H}{h} = 3 \Rightarrow H = 3h$.

26. (b)



Let AB be the lamp post of height 'h'. Let BC = x meter
In ΔABC,

$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h$

and in ΔABD,

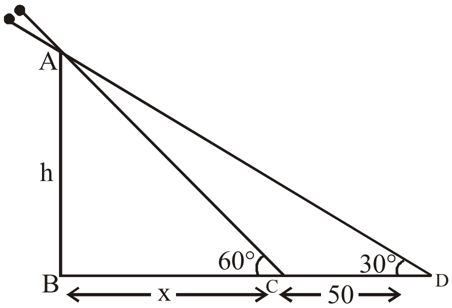
$\tan 30° = \frac{h}{x+10}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+10}$ $(\because x = h)$

$\Rightarrow h + 10 = \sqrt{3}h$

$\Rightarrow h = \left(\frac{10}{\sqrt{3}-1}\right)m = \frac{10\sqrt{3}+10}{2}$

27. (b)



Let AB be the tower and AC & AD be the shadows of the tower. Let h be the height of the tower.

In Δ ABC, $\tan 60° = \frac{h}{x}$ ...(i)

In ΔABD, $\tan 30° = \frac{h}{x+50}$ ..(ii)

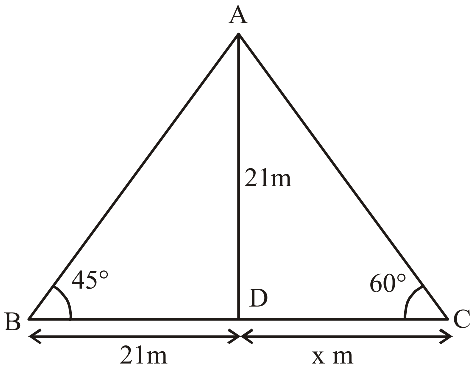
(i) ÷ (ii) $\Rightarrow \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{x+50}{x}$

$\Rightarrow$ $3x = x + 50$

$\Rightarrow$ $x = 25°$

$\Rightarrow$ $h = x\sqrt{3} = 25\sqrt{3}$

28. (a)



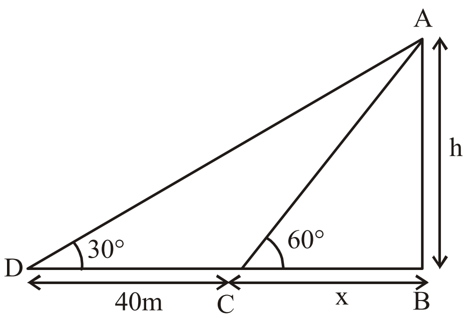
In ΔADB, $\tan 45° = \frac{AD}{BD} = \frac{AD}{21}$

AD = 21 m

In ΔADC, $\tan 60° = \frac{AD}{x}$

$x = \frac{21}{\sqrt{3}} = 7\sqrt{3}$ m

29. (d)



In ΔABC, $\tan 60° = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ ...(1)

In ΔABD, $\tan 30° = \frac{h}{x+40}$

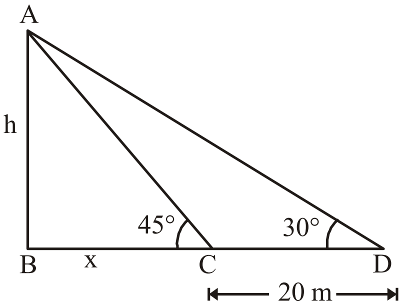
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+40} \Rightarrow x + 40 = \sqrt{3}h$ ...(2)

Putting value of h from equation (1), we get

$x + 40 = 3x$

$x = 20$ m

30. (c)



In Δ ABC, $\tan 45° = \frac{AB}{BC} = \frac{h}{x}$

$1 = \frac{h}{x}$

$h = x$ ...(i)

In ΔABD,

$\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+20}$

$x + 20 = \sqrt{3}h$

$h + 20 = \sqrt{3}h$

$20 = (\sqrt{3} - 1)\,h$

$h = \frac{20}{\sqrt{3}-1}$

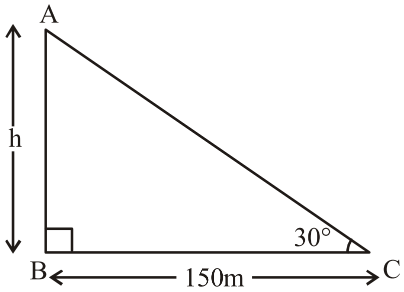
$= \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{20(\sqrt{3}+1)}{2} = 10(\sqrt{3}+1)$ m

Hence the height is $10\,(\sqrt{3}+1)$ m

31. (b) In ΔABC, we have BC = 150 m, AB = h

$\angle C = 30°$



$$\tan 30° = \frac{h}{150}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{150}$$

$$\therefore h = \frac{150}{\sqrt{3}} \text{ m}$$

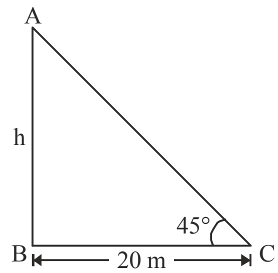
$$= \frac{150 \times \sqrt{3}}{3} = 50\sqrt{3} \text{ m}$$

32. (b) In ΔABC,

AB = h

BC = 20 m

$\angle C = 45°$



$$\therefore \quad \tan 45° = \frac{AB}{BC}$$

$$1 = \frac{h}{20} \qquad [\because \tan 45° = 1]$$

h = 20 m

$\therefore$ Height of the tower = 20 m

$\therefore$ Option (b) is correct.

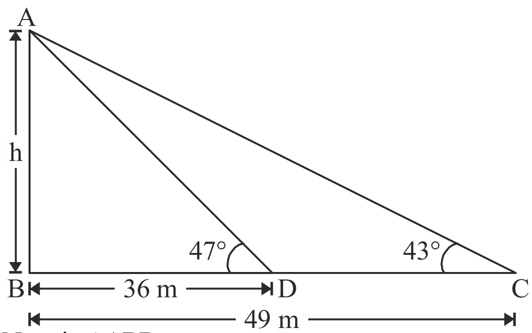
33. (b) AB = h (height of the tower)

BD = 36 m

BC = 49 m

$\angle D = 47°$

$\angle C = 43°$



Now, in ΔABD,

$$\tan 47° = \frac{h}{36 \text{ m}} \qquad \text{...(i)}$$

and in ΔABC,

$$\tan 43° = \frac{h}{49 \text{ m}}$$

$$\tan(90° - 47°) = \frac{h}{49}$$

$$\therefore \quad \cot 47° = \frac{h}{49} \qquad \text{...(ii)}$$

Multiplying equations (i) and (ii)

$$\tan 47°.\cot 47° = \frac{h}{36} \times \frac{h}{49} = 1 = \frac{h^2}{36 \times 49}$$

$h = 6 \times 7 = 42$ m

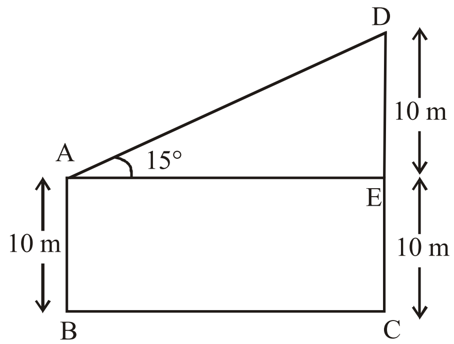
$\therefore$ Option (b) is correct

34. (b) $\tan 15° = DE/AE$

$AE = 10\cot 15°$ ...(1)

$\cot(15°) = \cot(45 - 30°)$

$$= \frac{\cot 45° \cot 30° + 1}{\cot 30° - \cot 45°}$$



$$\cot 15° = \frac{1.\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$$

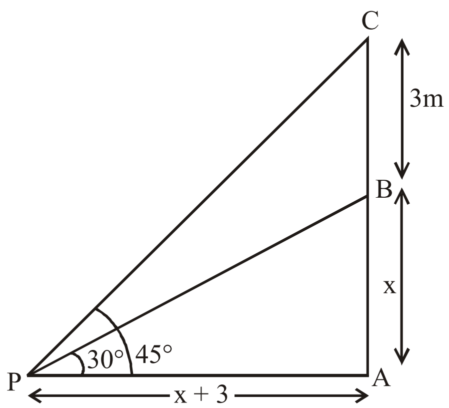
Putting cot 15° in eq (1)

$AE = 10\cot 15°$

$= 10\left(2 + \sqrt{3}\right)$

$= 10(3.73) = 37.3$ m

35. (c) as $\angle CPA = 45°$



so AC = AP = x + 3

$$\tan 30° = \frac{AB}{AP} = \frac{x}{x+3}$$

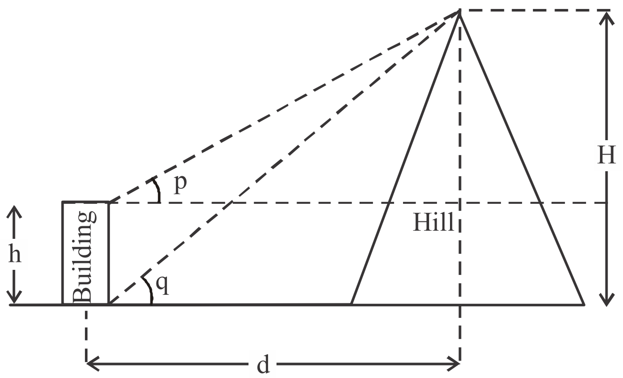
$$\frac{1}{\sqrt{3}} = \frac{x}{x+3}$$

$$x + 3 = \sqrt{3}x$$

$$x = \frac{3}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$x = \frac{3 \times 2.73}{2} = \frac{8.19}{2} = 4.095\text{m} \approx 4.1\text{m}$$

36. (b)



Let height of hill = H
& horizontal distance between building & hill = d

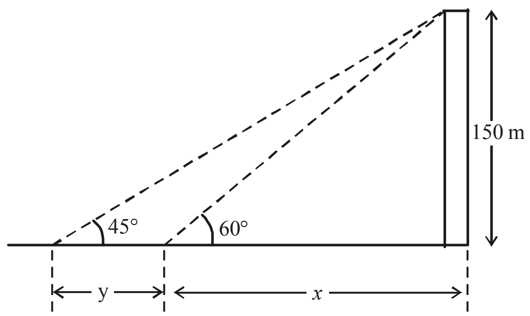
$$\tan q = \frac{H}{d} \Rightarrow d = \frac{H}{\tan q} = H \cot q$$

$$\tan p = \frac{(H-h)}{d} \Rightarrow d = (H-h) \cot p$$

$$\Rightarrow \quad H \cot q = (H-h) \cot p$$

$$H = \frac{h \cot p}{\cot p - \cot q}$$

37. (b)



$$\tan 60° = \frac{150}{x} \Rightarrow x = \frac{150}{\sqrt{3}}$$

$$\text{Also, } \tan 45° = \frac{150}{x+y}$$

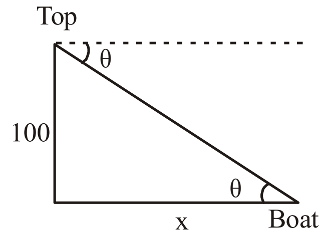
$$\Rightarrow \quad x + y = 150$$

$$\Rightarrow \quad y = 150 - x = 150 - \frac{150}{\sqrt{3}}$$

$$\Rightarrow \quad y = 150\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = \text{distance travelled}$$

$$\text{Speed (in m/hr)} = \frac{150(\sqrt{3}-1)}{\sqrt{3}} \times \frac{60}{2} = 4500\frac{(\sqrt{3}-1)}{\sqrt{3}}$$

38. (c) Given, angle of depression, $\theta = \tan^{-1}\left(\frac{5}{12}\right)$

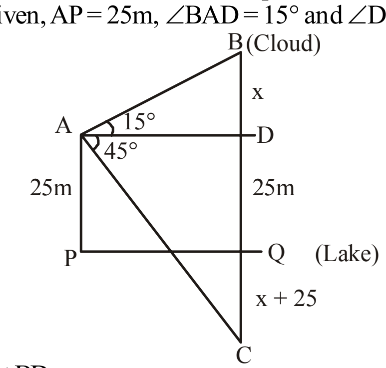


$$\Rightarrow \tan\theta = \frac{5}{12}$$

$$\Rightarrow \frac{100}{x} = \frac{5}{12}$$

$$\Rightarrow x = \frac{100 \times 12}{5} = 240 \text{ m}$$

39. (b) PQ is the lake and A is the point of observation.
Given, AP = 25m, ∠BAD = 15° and ∠DAC = 45°



Let BD = x
Then, DQ = AP = 25m and QC = BD + DQ = x + 25
Also, DC = DQ + QC = 25 + x + 25 = x + 50

$$\text{In } \Delta ADC, \tan 45° = \frac{DC}{AD} = \frac{x+50}{AD}$$

$$\Rightarrow 1 = \frac{x+50}{AD} \Rightarrow AD = x + 50 \qquad \text{....(1)}$$

$$\text{Also, } \tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}.$$

$$\text{In } \Delta ABD, \tan 15° = \frac{BD}{AD}$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{x}{x+50} \qquad \text{(from (1))}$$

$$\Rightarrow (x+50)(\sqrt{3}-1) = x(\sqrt{3}+1)$$

$$\Rightarrow \sqrt{3}x - x + 50\sqrt{3} - 50 = \sqrt{3}x + x$$

$$\Rightarrow 2x = 50\sqrt{3} - 50 = 50(\sqrt{3}-1)$$

$$\Rightarrow x = 25(\sqrt{3}-1)$$

Now, BQ = BD + DQ = x + 25

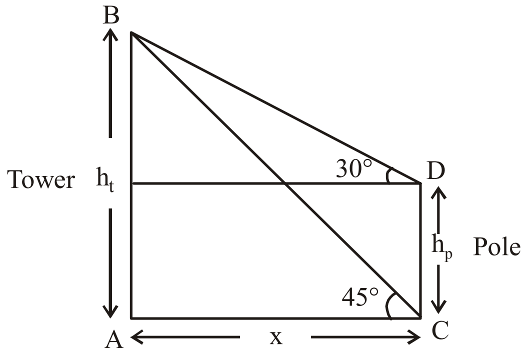
$$= 25(\sqrt{3}-1) + 25$$

$$= 25\sqrt{3} - 25 + 25$$

$$= 25\sqrt{3}.$$

∴ Height of the cloud above the lake $= 25\sqrt{3}$ m.

40. (c) Let the distance between tower and pole be 'x'.



$\tan 45° = \frac{h_t}{x} \Rightarrow \frac{h_t}{x} = 1$

$\Rightarrow h_t = x.$

Now, $\tan 30° = \frac{h_t - h_p}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h_t - h_p}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h_t - h_p}{h_t}$

$\Rightarrow \frac{1}{\sqrt{3}-1} = \frac{h_t - h_p}{h_t - (h_t - h_p)}$

$\Rightarrow \frac{\sqrt{3}+1}{2} = \frac{h_t - h_p}{h_p} \Rightarrow \frac{h_t - h_p}{\sqrt{3}+1} = \frac{h_p}{2}$ ....(1)

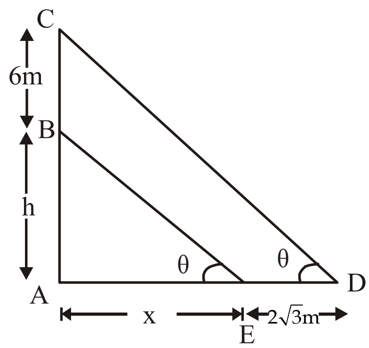
So, statement '2' is correct.

Now, $\frac{h_t - h_p}{h_p} = \frac{\sqrt{3}+1}{2} \Rightarrow \frac{h_t - h_p + 2h_p}{h_p} = \frac{\sqrt{3}+1+4}{2}$

$\Rightarrow \frac{h_t + h_p}{h_p} = \frac{5+\sqrt{3}}{2}$ ....(2)

So, statement '3' is incorrect.

41. (a)



AB is the tower. BC is flag staff.
The angles made by the shadows of tower and flag staff are same.

In $\Delta ABE, \tan\theta = \frac{h}{x}$ ...(1)

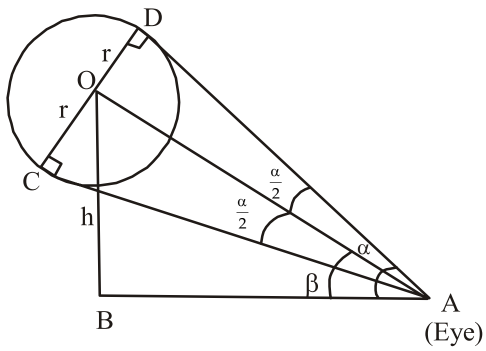
In $\Delta ACD, \tan\theta = \frac{h+6}{x+2\sqrt{3}}$ ...(2)

from (1), (2), $\frac{h}{x} = \frac{h+6}{x+2\sqrt{3}} \Rightarrow hx + 2\sqrt{3}h = hx + 6x$

$\Rightarrow \frac{h}{x} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}} = \sqrt{3}.$

from (1), $\tan\theta = \frac{h}{x} = \sqrt{3} \Rightarrow \theta = 60°.$

42. (a) Let 'A' be the position of eye.
Let 'O' be the centre of spherical balloon.
Let 'h' be the height of centre of balloon.



From figure, in $\Delta OAD, \sin\frac{\alpha}{2} = \frac{OD}{OA} = \frac{r}{OA}$

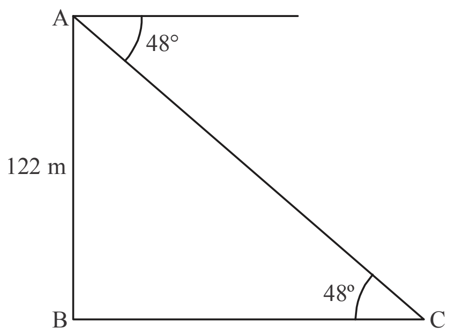
$\Rightarrow OA = \frac{r}{\sin\frac{\alpha}{2}}$ ....(1)

In $\Delta OAB, \sin\beta = \frac{OB}{OA} = \frac{h}{OA}$

$\Rightarrow h = OA \,.\, \sin\beta$

$= \frac{r.\sin\beta}{\sin\frac{\alpha}{2}}$ (from (1))

43. (a) Let the bridge is BC and height above the bridge AB = 122 m.

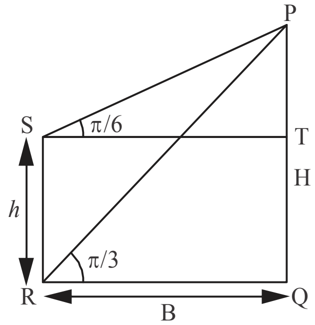


From DABC,

$$\cot 48° = \frac{BC}{AB} = \frac{BC}{122}$$

$$BC = 122 \,.\, \cot(48°)$$
$$= 122 \,.\, \cot(90° - 42°)$$
$$= \boxed{122 \,.\, \tan 42°}$$

44. (b)



From question, In ΔPQR and ΔPST.

$$B = \frac{H}{\tan\frac{\pi}{3}} = \frac{H - h}{\tan\frac{\pi}{6}}$$

$$\Rightarrow \frac{H}{3} = H - h$$

$$\Rightarrow \frac{2}{3}H = h$$

$$\Rightarrow H = \frac{3}{2}h$$

45. (a)

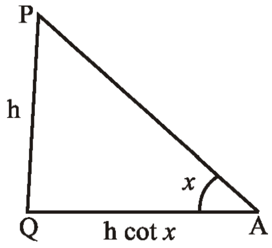


fig. (i)

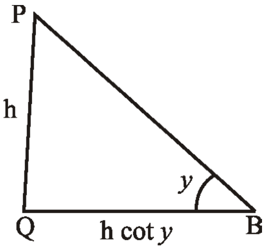


fig. (ii)

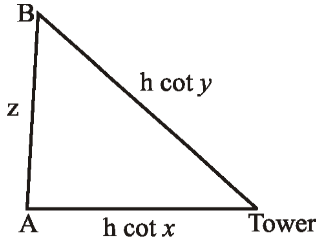


fig. (iii)

from figure (iii), $h^2\cot^2 y = z^2 + h^2\cot^2 x$

$\Rightarrow z^2 = h^2(\cot^2 y - \cot^2 x)$

Functions, Limit, Continuity and Differentiability

14

1. Let R be the set of real numbers and let $f : R \to R$ be a function such that $f(x) = \frac{x^2}{1+x^2}$. What is the range of f?
(a) R (b) $R - \{1\}$
(c) [0, 1] (d) [0, 1) *[2006-I]*

2. Let $f(x) = \frac{1}{\sqrt{18-x^2}}$.
What is the value of $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$?
(a) 0 (b) $-\frac{1}{9}$
(c) $\frac{1}{3}$ (d) $\frac{1}{9}$ *[2006-I]*

3. Let $f(x+y) = f(x).\, f(y)$ and $f(1) = 2$ for all $x, y \in R$, where f(x) is continuous function. What is f ' (1) equal to?
(a) 2 ln 2 (b) ln 2
(c) 1 (d) 0 *[2006-I]*

4. Given $f(x) = \log\left[\frac{(1+x)}{(1-x)}\right]$ and $g(x) = \frac{(3x+x^3)}{(1+3x^2)}$, then what is f [g (x)] equal to?
(a) $-f(x)$ (b) $3[f(x)]$
(c) $[f(x)]^3$ (d) $-3[f(x)]$ *[2006-I]*

5. What is the value of $\lim_{x\to 0}\frac{\sin|x|}{x}$?
(a) 1 (b) – 1
(c) ∞ (d) Limit does not exist *[2006-I]*

6. What is the equivalent definition of the function given by $f(x) = \begin{cases} 2x, x \ge 0 \\ 0, x < 0 \end{cases}$?
(a) $f(x) = |x|$ (b) $f(x) = 2x$
(c) $f(x) = |x| + x$ (d) $f(x) = 2|x|$ *[2006-II]*

7. If $f : R \to R^+$ such that $f(x) = (1/3)^x$, then what is the value of $f^{-1}(x)$?
(a) $(1/3)^x$ (b) 3^x
(c) $\log_{1/3} x$ (d) $\log_x(1/3)$ *[2006-II]*

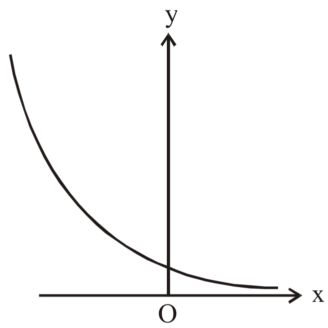
8. What is the value of $\lim_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$ *[2006-II]*
(a) 0 (b) $\frac{5}{4}$
(c) $\frac{5}{16}$ (d) $\frac{25}{4}$

9. If $f(x) = (1 + x)^{5/x}$ is continuous at x = 0, then what is the value of f (0)?
(a) 0 (b) 1
(c) ∞ (d) e^5 *[2006-II]*

10. Consider the following statements:
1. The function f (x) = greatest integer $\le x, x \in R$ is a continuous function.
2. All trigonometric functions are continuous on R.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2 *[2006-II]*

11. If $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]$ exists, then which one of the following correct?
(a) Both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist
(b) $\lim_{x\to a} f(x)$ need not exist but $\lim_{x\to a} g(x)$ must exist
(c) Both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ need not exist
(d) $\lim_{x\to a} f(x)$ must exist but $\lim_{x\to a} g(x)$ need not exist *[2006-II]*

12. If $f(x) = \begin{cases} mx+1 & x \le \frac{\pi}{2} \\ \sin x + n & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then which one of the following is correct?
(a) $m = 1, n = 0$ (b) $m = \frac{n\pi}{2} + 1$
(c) $n = m\left(\frac{\pi}{2}\right)$ (d) $m = n = \frac{\pi}{2}$ *[2006-II]*

13. 

The above curve shows the graph of a^x under which one of the following conditions ?

(a) $a \geq 1$ (b) $a > 1$

(c) $0 < a \leq 1$ (d) $0 < a < 1$ *[2006-II]*

14. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then what is $f\left(\frac{2x}{1-x^2}\right)$ equal to ?

(a) $(f(x))^2$ (b) 1

(c) $2f(x)$ (d) $f\left(\frac{1-x}{1+x}\right)$ *[2006-II]*

15. If $f(x) = (x+1)^{\cot x}$ is continuous at $x = 0$, then what is $f(0)$ equal to?

(a) 1 (b) e

(c) $\frac{1}{e}$ (d) e^2 *[2007-I]*

16. What is the value of $\lim_{x \to \infty}\left(\frac{x-2}{x \quad 2}\right)^{x \quad 2}$?

(a) 0 (b) e^4

(c) e^{-2} (d) e^{-4} *[2007-I]*

17. If the derivative of the function

$$f(x) \quad \begin{cases} ax^2 \quad b & x \quad -1 \\ bx^2 \quad ax \quad 4 & x \geq -1 \end{cases}$$

is every where continuous, then what are the values of a and b?

(a) a =2, b = 3 (b) a =3, b = 2

(c) a =–2, b = –3 (d) a =–3, b = –2 *[2007-I]*

18. If f(x) is differentiable everywhere,then which one of the following is correct?

(a) $|f|$ is differentiable everywhere

(b) $|f|^2$ is differentiable everywhere

(c) $f|f|$ is not differentiable at some points

(d) None of the above *[2007-I]*

19. Let $f: R \to R$ be defined as $f(x) = ax^2 + bx + c$, a ,b, c being fixed non-zero real numbers. Which one of the following statements is correct, in general?

(a) If $b^2 - 4ac > 0$, then $f^{-1}(0)$ does not contain 0

(b) If $b^2 - 4ac < 0$, then $f^{-1}(0)$ must contain 0

(c) If $b^2 - 4ac > 0$, then $f^{-1}(0)$ may contain 0

(d) If $b^2 - 4ac < 0$, then $f^{-1}(0)$ may contain 0 *[2007-I]*

20. If $\frac{x-a}{b \quad c} \quad \frac{x-b}{c \quad a} \quad \frac{x-c}{c \quad b} \quad 3$, then what is the value of x?

(a) 0 (b) 1

(c) a + b + c (d) abc *[2007-I]*

21. If $-x^2 + 3x + 4 > 0$, then which one of the following is correct?

(a) $x \in (-1, 4)$

(b) $x \in [-1, 4]$

(c) $x \in (\infty, -1) \cup (4, \infty)$

(d) $x \in (-\infty, -1] \cup [4, \infty)$ *[2007-I]*

22. Given, $f(x) = x + \frac{1}{x}$, then what is $f^2(x)$ equal to ?

(a) $\frac{x^2+1}{x} + \frac{x}{x^2+1}$ (b) $(x + 1/x)^2$

(c) $x^4 + (1/x^4)$ (d) $x^2 + (1/x^2)$ *[2007-II]*

23. If $f(x) = \begin{cases} 1 & x \text{ is a rational number} \\ 0, & x \text{ is an irrational number,} \end{cases}$ what is/are the value(s) of (f o f) $(\sqrt{3})$?

(a) 0 (b) 1

(c) Both 0 and 1 (d) None of these *[2007-II]*

24. A function f is defined as follows

$f(x) = x^p \cos\left(\frac{1}{x}\right), x \neq 0$

$f(0) = 0$

What conditions should be imposed on p so that f may be continuous at x = 0 ?

(a) p = 0 (b) p > 0

(c) p < 0 (d) No value of p *[2007-II]*

25. What is the value of $\lim_{x \to \infty} \frac{\sin x}{x}$?

(a) 1 (b) 0

(c) ∞ (d) – 1 *[2008-I]*

26. What is $\lim_{x \to 0} \frac{a^x - b^x}{x}$?

(a) $\log\left(\frac{a}{b}\right)$ (b) $\log\left(\frac{b}{a}\right)$

(c) ab (d) log (ab) *[2008-I]*

27. Let $f(x) = \begin{cases} 3x - 4, & 0 \leq x \leq 2 \\ 2x \quad \ell, & 2 \quad x \leq 9 \end{cases}$

If f is continuous at x = 2, then what is the value of ℓ ?

(a) 0 (b) 2

(c) – 2 (d) – 1 *[2008-I]*

28. If f (x) = x and g (x) = | x |, then what is (f + g) (x) equal to ? *[2008-I]*

(a) 0 for all $x \in R$ (b) 2x for all $x \in R$

(c) $\begin{cases} 2x, \text{for } x \geq 0 \\ 0, \ \text{for } x < 0 \end{cases}$ (d) $\begin{cases} 0, \ \text{for } x \geq 0 \\ 2x, \text{for } x < 0 \end{cases}$

29. If $g(x) = \sin x, x \in R$ and $f(x) = \frac{1}{\sin x}, x \in \left(0, \frac{\pi}{2}\right)$ what is (gof) (x) equal to ?
(a) 1 (b) $\frac{1}{\sin(\sin x)}$
(c) $\frac{1}{\sin^2(x)}$ (d) $\sin\left(\frac{1}{\sin x}\right)$ [2008-I]

30. Let $f : R \to R$ be defined as $f(x) = \sin(|x|)$
Which one of the following is correct?
(a) f is not differentiable only at 0
(b) f is differentiable at 0 only
(c) f is differentiable everywhere
(d) f is non-differentiable at many points [2008-I]

31. What is the inverse of the function $y = 5^{\log x}$?
(a) $x = 5^{1/\log y}$ (b) $x = y^{1/\log 5}$
(c) $x = 5^{\log y}$ (d) $x = y^{\log 5}$ [2008-I]

DIRECTIONS (Qs. 32-33) : *The following questions consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers.*

(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. [2008-I]

32. **Assertion (A) :** If $f(x) = \log x$, then $f(x) > 0$ for all $x > 0$.
Reason (R) : $f(x) = \log x$, is defined for all $x > 0$.

33. **Assertion (A) :** $f(x) = x \sin\left(\frac{1}{x}\right)$ is differentiable at $x = 0$.
Reason (R) : f(x) is continuous at $x = 0$.

34. If $f(x) = \log|x|, x \neq 0$, then what is $f'(x)$ equal to? [2008-II]
(a) $\frac{1}{|x|}$ (b) $\frac{1}{x}$
(c) $\frac{-1}{x}$ (d) None of these

35. $\lim_{x \to 0} e^{-1/x}$ is equal to [2008-II]
(a) 0 (b) ∞
(c) e (d) does not exist

36. Let $g : R \to R$ be a function such that, $g(x) = 2x + 5$. Then, what is $g^{-1}(x)$ equal to? [2008-II]
(a) $\frac{x-5}{2}$ (b) $2x - 5$
(c) $x - \frac{5}{2}$ (d) $\frac{x}{2} + \frac{5}{2}$

37. Consider the following statements: [2008-II]
1. $\lim_{x \to 0} \frac{x^2}{x}$ exists.
2. $\left(\frac{x^2}{x}\right)$ is not continuous at $x = 0$
3. $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

38. Let $f(x) = \frac{1}{1 - |1 - x|}$. Then, what is $\lim_{x \to 0} f(x)$ equal to [2008-II]
(a) 0 (b) ∞
(c) 1 (d) -1

39. What is the value of $\lim_{x \to a} \frac{\sqrt{\alpha + 2x} - \sqrt{3x}}{\sqrt{3\alpha + x} - 2\sqrt{x}}$? [2008-II]
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{(3\sqrt{3})}$
(c) $\frac{2}{(3\sqrt{3})}$ (d) $\frac{1}{\sqrt{3}}$

DIRECTIONS (Qs. 40-41) : *The following questions consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers.*

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

40. **Assertion (A) :** The function [2008-II]
$f : (1, 2, 3) \to (a, b, c, d)$ defined by
f= {(1, a), (2, b), (3, c)} has no inverse.
Reason (R) : *f* is not one-one.

41. **Assertion (A) :** $y = 2x + 3$ is a one to one real valued function.
Reason (R) : $x_1 \neq x_2$ [2008-II]
$\Rightarrow y_1 \neq y_2, y_1 = 2x_1 + 3, y_2 = 2x_2 + 3$, for any two real x_1 and x_2

42. The function $f : R \to R$ defined by $f(x) = (x^2 + 1)^{35}$ for all $x \in R$ is [2008-II]
(a) one-one but not onto
(b) onto but not one-one
(c) neither one-one nor onto
(d) both one-one and onto

43. Let $f : R \to R$ be a function defined as $f(x) = x\,|x|$; for each $x \in R$, R being the set of real numbers. Which one of the following is correct? [2009-I]
(a) f is one-one but not onto
(b) f is onto but not one-one
(c) f is both one-one and onto
(d) f is neither one-one nor onto

44. What is the set of all points, where the function $f(x) = \dfrac{x}{1+|x|}$ is differentiable? [2009-I]
(a) $(-\infty, \infty)$ only
(b) $(0, \infty)$ only
(c) $(-\infty, 0) \cup (0, \infty)$ only
(d) $(-\infty, 0)$ only

45. Let $y(x) = ax^n$ and δy denote small change in y. What is limit of $\dfrac{\delta y}{\delta x}$ as $\delta x \to 0$? [2009-I]
(a) 0 (b) 1
(c) anx^{n-1} (d) $ax^n \log(ax)$

46. What is $\lim\limits_{x \to o} \dfrac{\sin^2 ax}{bx}$ (a, b are constants) equal to? [2009-I]
(a) 0 (b) a
(c) a/b (d) Does not exist

47. If $f(x) = \begin{cases} 3x-4, 0 \le x \le 2 \\ 2x+\lambda, 2 < x \le 3 \end{cases}$ [2009-I]
is continuous at $x = 2$, then what is the value of λ?
(a) 1 (b) –1
(c) 2 (d) –2

48. A mapping $f : R \to R$ which is defined as $f(x) = \cos x;\ x \in R$ is [2009-II]
(a) One-one only (b) Onto only
(c) One-one onto (d) Neither one-one nor onto

49. What is $\lim\limits_{x \to \infty} \left(\dfrac{x}{3+x}\right)^{3x}$ equal to? [2009-II]
(a) e (b) e^3
(c) e^{-9} (d) e^9

50. Consider the following function $f : R \to R$ such that $f(x) = x$ if $x \ge 0$ and $f(x) = -x^2$ if $x < 0$. Then, which one of the following is correct? [2009-II]
(a) $f(x)$ is continuous at every $x \in R$
(b) $f(x)$ is continuous at $x = 0$ only
(c) $f(x)$ is discontinous at $x = 0$ only
(d) $f(x)$ is discontinuous at every $x \in R$

51. Which one of the following functions $f : R \to R$ is injective? [2009-II]
(a) $f(x) = |x|$ for all $x \in R$
(b) $f(x) = x^2$ for all $x \in R$
(c) $f(x) = 11$ for all $x \in R$
(d) $f(x) = -x$ for all $x \in R$

52. The function $f(x) = e^x$, $x \in R$ is [2010-I]
(a) onto but not one-one (b) one-one onto
(c) one-one but not onto (d) neither one-one nor onto

53. What is the value of $\lim\limits_{x \to \infty} \left(\dfrac{x+6}{x+1}\right)^{x+4}$ [2010-I]
(a) e (b) e^2
(c) e^4 (d) e^5

54. If $f : R \to R, g : R \to R$ and $g(x) = x+3$ and $(fog)(x) = (x+3)^2$, then what is the value of $f(-3)$? [2010-I]
(a) –9 (b) 0
(c) 9 (d) 3

55. What is the value of $\lim\limits_{x \to 1} \dfrac{(x-1)^2}{|x-1|}$? [2010-I]
(a) 0 (b) 1
(c) –1 (d) The limit does not exist

DIRECTIONS (Qs. 56-58) : *Each item under List I is associated with one or more items under List II.*

	List I (Function)		List II (Property)
A.	$\sin x$	1.	Periodic function
B.	$\cos x$	2.	Non-periodic function
C.	$\tan x$	3.	Continuous at every point on $(-\infty, \infty)$
		4.	Discontinuous function
		5.	Differentiable at every point on $(-\infty, \infty)$
		6.	Not differentiable at every point on $(-\infty, \infty)$
		7.	has period π
		8.	has period 2π
		9.	increases on $\left(0, \frac{\pi}{2}\right)$
		10.	decreases on $\left(0, \frac{\pi}{2}\right)$
		11.	increases on $\left(\frac{\pi}{2}, \pi\right)$
		12.	decreases on $\left(\frac{\pi}{2}, \pi\right)$

56. A is associated with [2010-I]
(a) 1, 3, 5, 8, 9, 12 (b) 2, 4, 6, 8, 10, 11
(c) 1, 3, 5, 7, 10, 11 (d) None of these

57. B is associated with [2010-I]
(a) 2, 3, 5, 8, 9, 12 (b) 1, 3, 5, 8, 10, 12
(c) 1, 3, 5, 8, 9, 12 (d) None of the above

58. C is associated with [2010-I]
(a) 1, 4, 6, 7, 9, 11 (b) 2, 4, 8, 9
(c) 1, 4, 6, 7, 9 (d) None of these

59. Consider the following statements [2010-I]
1. Every function has a primitive.
2. A primitive of a function is unique.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

60. The function $f(x)=\frac{x}{x^2+1}$ from R to R is [2010-I]
(a) one – one as well as onto
(b) onto but not one–one
(c) neither one–one nor onto
(d) one–one but not onto

61. The function $f(x) = x \operatorname{cosec} x$ is [2010-I]
(a) continuous for all values of x
(b) discontinuous everywhere
(c) continuous for all x except at $x = n\pi$, where n is an integer
(d) continuous for all x except at $x = n\pi / 2$, where n is an integer

62. Consider the following statements : [2010-II]
I. $f(x) = |x-3|$ is continuous at $x = 0$.
II. $f(x) = |x-3|$ is differentiable at $x = 0$.
Which of the statements given above is/are correct ?
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

63. Consider the function $f : R \to \{0, 1\}$ such that [2010-II]
$$f(x)=\begin{cases}1, \text{ if } x \text{ is rational}\\ 0, \text{ if } x \text{ is irrational}\end{cases}$$
Which one of the following is correct?
(a) The function is one-one into
(b) The function is many-one into
(c) The function is one-one onto
(d) The function is many-one onto

64. What is the value of $\lim\limits_{x\to 0}\frac{\cos(ax)-\cos(bx)}{x^2}$?
(a) a – b (b) a + b
(c) $\frac{b^2-a^2}{2}$ (d) $\frac{b^2+a^2}{2}$

65. If $f(x) = 2x + 7$ and $g(x) = x^2 + 7$, $x \in R$, then what are values of x for which $fog(x) = 25$? [2010-II]
(a) $-1, 1$ (b) $-2, 2$
(c) $-\sqrt{2}, \sqrt{2}$ (d) None of these

66. What is $\lim\limits_{x\to 0}\frac{a^x-b^x}{x}$ equal to? [2010-II]
(a) $ln\,(ab)$ (b) $\frac{\ln a}{\ln b}$
(c) $ln\left(\frac{a}{b}\right)$ (d) $ln\left(\frac{b}{a}\right)$

67. If the function [2010-II]
$$f(x)=\frac{x(x-2)}{x^2-4},\ x\neq \pm 2$$
is continuous at $x = 2$, then what is $f(2)$ equal to?
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

68. At how many points is the function $f\ x\quad x$ discontinuous? [2010-II]
(a) 1 (b) 2
(c) 3 (d) Infinite

69. If $f(x)=\frac{2}{3}x+\frac{3}{2},\ x\in R,$ [2010-II]
then what is $f^{-1}(x)$ equal to?
(a) $\frac{3}{2}x+\frac{2}{3}$ (b) $\frac{3}{2}x-\frac{9}{4}$
(c) $\frac{2}{3}x-\frac{4}{9}$ (d) $\frac{2}{3}x-\frac{2}{3}$

70. What is $\lim\limits_{x\to\infty}\left(\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}\right)$ equal to? [2011-I]
(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) 0

71. What is the value of k for which the following function $f(x)$ is continuous for all x? [2011-I]
$$f\ x\quad \begin{cases}\frac{x^3-3x\quad 2}{x-1\ ^2}, & \text{for } x\neq 1\\ k & \text{, for } x\quad 1\end{cases}$$
(a) 3 (b) 2
(c) 1 (d) – 1

72. Which one of the following is correct in respect of the function $f(x)=|x|+x^2$ [2011-I]
(a) $f(x)$ is not continuous at $x = 0$
(b) $f(x)$ is differentiable at $x = 0$
(c) $f(x)$ is continuous but not differentiable at $x = 0$
(d) None of the above

73. Consider the following in respect of the function $f(x) = |x - 3|$:
1. $f(x)$ is continuous at $x = 3$
2. $f(x)$ is differentiable at $x = 0$.

Which of the above statements is/are correct? *[2012-I]*
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

74. What is $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$ equal to? *[2012-I]*
(a) 0 (b) 1
(c) 1/2 (d) Limit does not exist

75. What is $\lim_{x\to -2}\left(\frac{x+2}{x^3+8}\right)$ equal to? *[2012-I]*
(a) 1/4 (b) –1/4
(c) 1/12 (d) –1/12

76. If $f[xy] = f[x]\, f[y]$, then $f[t]$ may be of the form:
(a) $t + k$ (b) $ct + k$ *[2012-I]*
(c) $t^k + c$ (d) t^k
where k, c are constants

77. Which one of the following functions is differentiable for all real values of x? *[2012-I]*
(a) $\frac{x}{|x|}$ (b) $x|x|$
(c) $\frac{1}{|x|}$ (d) $\frac{1}{x}$

78. What is $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ equal to ? *[2012-II]*
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) $-\frac{1}{2}$

79. What is $\lim_{x\to 0}\frac{2\ 1-\cos x}{x^2}$ equal to ? *[2012-II]*
(a) 0 (b) 1/2
(c) 1/4 (d) 1

80. Consider the following :
1. $\lim_{x\to 0}\frac{1}{x}$ exists.
2. $\lim_{x\to 0} e^{\frac{1}{x}}$ does not exist.

Which of the above is/are correct? *[2012-II]*
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

81. Which one of the following is correct in respect of the function $f(x) = \frac{x^2}{|x|}$ for $x \neq 0$ and $f(0) = 0$? *[2012-II]*
(a) $f(x)$ is discontinuous every where
(b) $f(x)$ is continuous every where
(c) $f(x)$ is continuous at $x = 0$ only
(d) $f(x)$ is discontinuous at $x = 0$ only

82. What is $\lim_{x\to 2}\frac{x-2}{x^2-4}$ equal to ? *[2012-II]*
(a) 0 (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 1

83. Let $f : R \to R$ be a function whose inverse is $\frac{x+5}{3}$. What is $f(x)$ equal to ? *[2012-II]*
(a) $f(x) = 3x + 5$ (b) $f(x) = 3x - 5$
(c) $f(x) = 5x - 3$ (d) $f(x)$ does not exist

84. Consider the following statements : *[2012-II]*
1. If $f(x) = x^3$ and $g(y) = y^3$ then $f = g$.
2. Identity function is not always a bijection.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

85. Let $A = \{x \in R \mid x \geq 0\}$. A function $f : A \to A$ is defined by $f(x) = x^2$. Which one of the following is correct ?
(a) The function does not have inverse *[2012-II]*
(b) f is its own inverse
(c) The function has an inverse but f is not its own inverse
(d) None of the above

86. Consider the following statements in respect of a function $f(x)$: *[2013-I]*
1. $f(x)$ is continuous at $x = a$ iff $\lim_{x\to a} f(x)$ exists.
2. If $f(x)$ is continuous at a point, then $\frac{1}{f(x)}$ is also continuous at that point.

Which of the above, statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

87. Consider the function $f(x) = \begin{cases} x^2, & x > 2 \\ 3x-2, & x \leq 2 \end{cases}$. Which one of the following statements is correct in respect of the above function? *[2013-I]*
(a) $f(x)$ is derivable but not continuous at $x = 2$.
(b) $f(x)$ is continuous but not derivable at $x = 2$.
(c) $f(x)$ is neither continuous nor derivable at $x = 2$.
(d) $f(x)$ is continuous as well as derivable at $x = 2$.

88. Consider the following statements:

1. $\lim_{x \to 0} \sin \frac{1}{x}$ does not exist.

2. $\lim_{x \to 0} \sin \frac{1}{x}$ exists.

Which of the above statements correct? [2013-I]

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

89. $\lim_{x \to 0} \frac{\sin x - \tan x}{x}$ equal to? [2013-I]

(a) 0 (b) 1
(c) – 1 (d) 1/2

90. What is $\lim_{x \to 0} \frac{1-\sqrt{1+x}}{x}$ equal to? [2013-I]

(a) 1/2 (b) –1/2
(c) 1 (d) –1

91. Consider the following statements:

1. The derivative where the function attains maxima or minima be zero.
2. If a function is differentiable at a point, then it must be continuous at that point.

Which of the above statements is/are correct? [2013-I]

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

92. Let N be the set of natural numbers and f : N → N, be a function given by f (x) = x + 1, x ∈ N. Which one of the following is correct ? [2013-I]

(a) f is one-one and onto
(b) f is one-one but not onto
(c) f is only onto
(d) f is neither one-one nor onto

93. Let f be a function from the set of natural numbers to the set of even natural numbers given by $f(x) = 2x$. Then f is [2013-II]

(a) one to one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

94. Consider the following functions : [2013-II]

1. $f(x) = e^x$, where $x > 0$
2. $g(x) = |x-3|$

Which of the above functions is/are continuous ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

95. What is $\lim_{x \to 2} \frac{2-x}{x^3-8}$ equal to ? [2013-II]

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$

(c) $\frac{1}{12}$ (d) $-\frac{1}{12}$

96. A function $f : R \to R$ is defined as $f(x) = x^2$ for $x \geq 0$ and $f(x) = -x$ for $x < 0$. [2013-II]

Consider the following statements in respect of the above function :

1. The function is continuous at $x = 0$.
2. The function is differentiable at $x = 0$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

97. What is $\lim_{x \to 0} \frac{1-\cos x}{x}$ equal to ? [2013-II]

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

98. What is $\lim_{x \to 0} \frac{\cos x}{\pi - x}$ equal to ? [2013-II]

(a) 0 (b) π

(c) $\frac{1}{\pi}$ (d) 1

99. What is $\lim_{x \to 0} \frac{\sin 2x + 4x}{2x + \sin 4x}$ equal to ? [2013-II]

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

100. Let N denote the set of all non-negative integers and Z denote the set of all integers. The function $f : Z \to N$ given by $f(x) = |x|$ is: [2014-I]

(a) One-one but not onto
(b) Onto but not one-one
(c) Both one-one and onto
(d) Neither one-one nor onto

101. What is $\lim_{x \to 0} \frac{(1+x)^n - 1}{x}$ equal to ? [2014-I]

(a) 0 (b) 1
(c) n (d) $n - 1$

102. What is $\lim_{x \to 0} \frac{x}{\sqrt{1-\cos x}}$ equal to ? [2014-I]

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) Limit does not exist

DIRECTION (Qs. 103-104): *For the next two (02) items that follow :*

Consider the function $f(x) = \frac{1-\sin x}{(\pi - 2x)^2}$

Where $x \neq \frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = \lambda$ [2014-I]

103. What is $\lim_{x \to \frac{\pi}{2}} f(x)$ equal to ?

(a) 1 (b) 1/2
(c) 1/4 (d) 1/8

104. What is the value of λ if the function is continuous at $x \quad \frac{\pi}{2}$?

(a) 1/8 (b) 1/4
(c) 1/2 (d) 1

105. If $f(9) = 9$ and $f'(9) = 4$ then what is $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ equal to ? *[2014-I]*

(a) 36 (b) 9
(c) 4 (d) None of these

106. Consider the following statements : *[2014-I]*

1. The function $f(x) = \sqrt[3]{x}$ is continuous at all x except at $x = 0$.
2. The function $f(x) = [x]$ is continuous at $x = 2.99$ where [.]is the bracket function.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

107. Consider the following statements : *[2014-I]*

1. The function $f(x) = |x|$ is not differentiable at $x = 1$.
2. The function $f(x) = e^x$ is not differentiable at $x = 0$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 108-110): *For the next three (03) items that follow.*

Let $f(x)$ be a function defined in $1 \le x < \infty$ by *[2014-I]*

$$f(x) = \begin{cases} 2 - x & \text{for } 1 \le x \le 2 \\ 3x - x^2 & \text{for } x > 2. \end{cases}$$

108. Consider the following statements :

1. The function is continuous at every point in the interval $(1, \infty)$.
2. The function is differentiable at $x = 1.5$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

109. What is the differentiable coefficient of $f(x)$ at $x = 3$?

(a) 1 (b) 2
(c) –1 (d) –3

110. Consider the following statements :

1. $f'(2 + 0)$ does not exist.
2. $f'(2 - 0)$ does not exist.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

111. The function $f: N \to N$, N being the set of of natural numbers, defined by $f(x) = 2x + 3$ is *[2014-II]*

(a) injective and surjective
(b) injective but not surjective
(c) not injective but surjective
(d) neither injective nor surjective

112. If f(x) = ax + b and g (x) = cx + d such that f[g(x)] = g [f(x)] then which one of the following is correct? *[2014-II]*

(a) f(c) = g (a) (b) f(a) = g (c)
(c) f(c) = g (d) (d) f(d) = g (b)

DIRECTIONS (Qs. 113-115): *For the next three (03) items that follow.*

Consider the function $f(x) \quad \frac{x-1}{x \quad 1}$. *[2014-II]*

113. What $\frac{f(x) \quad 1}{f(x) - 1} \quad x$ is equal to ?

(a) 0 (b) 1
(c) $2x$ (d) $4x$

114. What is $f(2x)$ equal to ?

(a) $\frac{f(x) \quad 1}{f(x) \quad 3}$ (b) $\frac{f(x) \quad 1}{3f(x) \quad 1}$
(c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x) \quad 3}{3f(x) \quad 1}$

115. What is $f(f(x))$ equal to ?

(a) x (b) $-x$
(c) $-\frac{1}{x}$ (d) None of these

DIRECTIONS (Qs. 116-118) : *For the next three (03) items that follow.*

Consider the function $f(x) = \begin{cases} x^2 - 5 & x \le 3 \\ \sqrt{x + 13} & x > 3 \end{cases}$ *[2014-II]*

116. What is $\lim_{x \to 3} f(x)$ equal to ?

(a) 2 (b) 4
(c) 5 (d) 13

117. Consider the following statements :

1. The function is discontinuous at $x = 3$.
2. The function is not differentiable at $x = 0$.

What of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

118. What is the differential coefficient of $f(x)$ at $x = 12$?

(a) 5/2 (b) 5
(c) 1/5 (d) 1/10

119. Consider the function

$$f(x) \begin{cases} \dfrac{\tan kx}{x}, & x \quad 0 \\ 3x+2k^2, & x \geq 0 \end{cases}$$

What is the non-zero value of k for which the function is continuous at $x = 0$?

(a) 1/4 (b) 1/2
(c) 1 (d) 2

120. Consider the following statements :

1.The function $f(x) = [x]$, where [.] is the greatest integer function defined on R, is continuous at all points except at $x=0$. *[2014-II]*

2.The function $f(x) = \sin|x|$ is continuous for all $x \in R$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

121. What is $\lim\limits_{x \to 0} \dfrac{\log_5(1 \quad x)}{x}$ equal to ? *[2014-II]*

(a) 1 (b) $\log_5 e$
(c) $\log_e 5$ (d) 5

122. What is $\lim\limits_{x \to 0} \dfrac{5^x - 1}{x}$ equal to ? *[2014-II]*

(a) $\log_e 5$ (b) $\log_5 e$
(c) 5 (d) 1

123. What is $\lim\limits_{n \to \infty} \dfrac{1 \quad 2 \quad 3 \quad ... \quad n}{1^2 \quad 2^2 \quad 3^2 \quad ...n^2}$ equal to ? *[2014-II]*

(a) 5 (b) 2
(c) 1 (d) 0

DIRECTIONS (Qs. 124-125): *For the next two (2) items that follow.*

Given that $\lim\limits_{x \to \infty}\left(\dfrac{2+x^2}{1+x} - Ax - B\right) = 3.$

124. What is the value of A? *[2015-I]*

(a) –1 (b) 1
(c) 2 (d) 3

125. What is the value of B? *[2015-I]*

(a) –1 (b) 3
(c) –4 (d) –3

126. If $G(x) = \sqrt{25 - x^2}$, then what is $\lim\limits_{x \to 1} \dfrac{G(x) - G(1)}{x - 1}$ equal to? *[2015-I]*

(a) $-\dfrac{1}{2\sqrt{6}}$ (b) $\dfrac{1}{5}$

(c) $-\dfrac{1}{\sqrt{6}}$ (d) $\dfrac{1}{\sqrt{6}}$

127. Consider the following statements: *[2015-I]*

1. $f(x) = [x]$, where [.] is the greatest integer function, is discontinuous at $x = n$, where $n \in Z$.
2. $f(x) = \cot x$ is discontinuous at $x = n\pi$, where $n \in Z$.

which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 or 2

128. If $f(x) = \log_e\left(\dfrac{1+x}{1-x}\right), g(x) = \dfrac{3x+x^3}{1+3x^2}$ and g o f (t) =g(f(t)), then what is $g \circ f\left(\dfrac{e-1}{e+1}\right)$ equal to? *[2015-I]*

(a) 2 (b) 1

(c) 0 (d) $\dfrac{1}{2}$

DIRECTIONS (Qs. 129-130): *For the next two (2) items that follow.*

Given a function

$$f(x) = \begin{cases} -1 & \text{If} \quad x \leq 0 \\ ax+b & \text{If} \quad 0 < x < 1 \\ 1 & \text{If} \quad x \geq 1 \end{cases}$$

where a, b are constants. The function is continuous everywhere.

129. What is the value of a? *[2015-I]*

(a) –1 (b) 0
(c) 1 (d) 2

130. What is the value of b? *[2015-I]*

(a) –1 (b) 1
(c) 0 (d) 2

131. Consider the following functions: *[2015-I]*

1. $f(x) = x^3, x \in \mathbb{R}$
2. $f(x) = \sin x, 0 < x < 2\pi$
3. $f(x) = e^x, x \in \mathbb{R}$

Which of the above functions have inverse defined on their ranges? *[2015-I]*

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

DIRECTIONS (Qs. 132-133): *For the next two (2) items that follow.*

Consider the function

$$f(x) = \begin{cases} \dfrac{\alpha \cos x}{\pi - 2x} & \text{If} \quad x \neq \dfrac{\pi}{2} \\ 3 & \text{If} \quad x = \dfrac{\pi}{2} \end{cases}$$

Which is continuous at $x = \dfrac{\pi}{2}$, where α is a constant.

132. What is the value of α ? *[2015-I]*

(a) 6 (b) 3
(c) 2 (d) 1

133. What is $\lim\limits_{x \to 0} f(x)$ equal to? *[2015-I]*

(a) 0 (b) 3

(c) $\dfrac{3}{\pi}$ (d) $\dfrac{6}{\pi}$

134. If $g(x) = \dfrac{1}{f(x)}$ and f(x) = x, x ≠ 0, then which one of the following is correct [2015-II]
(a) f(f(f(g(g(f(x)))))) = g(g (f (g(f(x)))))
(b) f(f(g(3(g(f(x)))))) = g (g(f(g(f(x))))))
(c) f(g(f(g(g(f(g(x)))))) = g (g(f(g(f(x))))))
(d) f(f(f(f(f(f(x)))))) = f(f(f(g(f(x)))))

135. If $f(x) = \sqrt{25 - x^2}$, then what is $\lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1}$ equal to ? [2015-II]
(a) $\dfrac{1}{5}$ (b) $\dfrac{1}{24}$
(c) $\sqrt{24}$ (d) $-\dfrac{1}{\sqrt{24}}$

136. Consider the function [2015-II]

$$f(x) = \begin{cases} ax - 2 & \text{for } -2 \quad x \quad -1 \\ -1 & \text{for } -1 \le x \le 1 \\ a + 2(x-1)^2 & \text{for } 1 \quad x \quad 2 \end{cases}$$

What is the value of a for which f(x) is continuous at x = –1 and x = 1 ?
(a) –1 (b) 1
(c) 0 (d) 2

137. The function $f(x) = \dfrac{1 - \sin x \quad \cos x}{1 \quad \sin x \quad \cos x}$ is not defined at x = π. The value of f(π) so that f(x) is continuous at x = π is [2015-II]
(a) $-\dfrac{1}{2}$ (b) $\dfrac{1}{2}$
(c) –1 (d) 1

138. Consider the following functions : [2015-II]

1. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } \quad x \neq 0 \\ 0 & \text{if } \quad x \quad 0 \end{cases}$

2. $f(x) = \begin{cases} 2x \quad 5 & \text{if } \quad x \quad 0 \\ x^2 \quad 2x \quad 5 & \text{if } \quad x \le 0 \end{cases}$

Which of the above functions is/are derivable at x = 0 ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

139. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is [2015-II]
(a) [0, ∞) (b) (–∞, 0)
(c) [1, ∞) (d) (–∞, 0]

140. Consider the following statements : [2015-II]
1. The function f (x) = x^2 + 2cos x is increasing in the interval (0, π)
2. The function $f(x) = \ln(\sqrt{1 + x^2} - x)$ is decreasing in the interval (–∞, ∞)

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

141. If f : IR → IR, g : IR → IR be two functions given by f(x) = 2x – 3 and g(x) = x^3 + 5, then $(fog)^{-1}(x)$ is equal to [2015-II]
(a) $\left(\dfrac{x \quad 7}{2}\right)^{\frac{1}{3}}$ (b) $\left(\dfrac{x - 7}{2}\right)^{\frac{1}{3}}$
(c) $\left(x - \dfrac{7}{2}\right)^{\frac{1}{3}}$ (d) $\left(x \quad \dfrac{7}{2}\right)^{\frac{1}{3}}$

142. If $f(x) = \dfrac{\sin(e^{x-2} - 1)}{\ln(x-1)}$, then $\lim\limits_{x \to 2} f(x)$ is equal to [2015-II]
(a) –2 (b) –1
(c) 0 (d) 1

143. Consider the following statements : [2015-II]
Statement 1 : The function f: IR → IR such that f (x) = x^3 for all x ∈ IR is one-one.
Statement 2 : f (a) ⇒ f (b) for all a, b ∈ IR if the function f is one-one.
Which one of the following is correct in respect of the above statements ?
(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1.
(b) Both the statements are true and Statement 2 is *not* the correct explanation of Statement 1.
(c) Statement 1 is true but Statement 2 is false.
(d) Statement 1 is false but Statement 2 is true.

DIRECTIONS (Qs. 144-145) : *For the next two (02) items that follow.*

Consider the function

$$f(x) = \begin{cases} -2\sin x & \text{if } x \le -\dfrac{\pi}{2} \\ A\sin x + B & \text{if } -\dfrac{\pi}{2} \quad x \quad \dfrac{\pi}{2} \\ \cos x & \text{if } x \ge \dfrac{\pi}{2} \end{cases}$$

which is continuous everywhere.

144. The value of A is [2015-II]
(a) 1 (b) 0
(c) –1 (d) –2

145. The value of B is [2015-II]
(a) 1 (b) 0
(c) –1 (d) –2

DIRECTIONS (Qs. 146-147) : *For the next two(2) items that follow:*

Consider the curves

$$f(x) = x|x| - 1 \text{ and } g(x) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \le 0 \end{cases}$$ *[2016-I]*

146. Where do the curves intersect?
(a) At (2, 3) only
(b) At (–1, –2) only
(c) At (2, 3) and (–1, –2)
(d) Neither at (2, 3) nor at (–1, –2)

147. What is the area bounded by the curves?
(a) $\frac{17}{6}$ square units (b) $\frac{8}{3}$ square units
(c) 2 square units (d) $\frac{1}{3}$ square unit

DIRECTIONS (Qs. 148-152) : *For the next five (5) items that follow.*

Consider the function $f(x) = |x-1| + x^2$, where $x \in \mathbf{R}$. *[2016-I]*

148. Which one of the following statements is correct?
(a) f(x) is continuous but not differentiable at x = 0
(b) f(x) is continuous but not differentiable at x = 1
(c) f(x) is differentiable at x = 1
(d) f(x) is not differentiable at x = 0 and x = 1

149. Which one of the following statements is correct?
(a) f(x) is increasing in $\left(-\infty, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, \infty\right)$
(b) f(x) is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right)$
(c) f(x) is increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$
(d) f(x) is decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

150. Which one of the following statements is correct?
(a) f(x) has local minima at more than one point in $(-\infty, \infty)$
(b) f(x) has local maxima at more than one point in $(-\infty, \infty)$
(c) f(x) has local minimum at one point only in $(-\infty, \infty)$
(d) f(x) has neither maxima nor minima in $(-\infty, \infty)$

151. What is the area of the region bounded by x-axis, the curve y = f(x) and the two ordinates $x = \frac{1}{2}$ and x = 1?
(a) $\frac{5}{12}$ square unit (b) $\frac{5}{6}$ square unit
(c) $\frac{7}{6}$ square units (d) 2 square units

152. What is the area of the region bounded by x-axis, the curve y = f(x) and the two ordinates x = 1 and $x = \frac{3}{2}$?
(a) $\frac{5}{12}$ square unit (b) $\frac{7}{12}$ square unit
(c) $\frac{2}{3}$ square unit (d) $\frac{11}{12}$ square unit

DIRECTIONS (Qs. 153-154) : *For the next two (2) items that follow.*

Consider the equation $x + |y| = 2y$. *[2016-I]*

153. Which of the following statements are *not* correct?
1. y as a function of x is not defined for all real x.
2. y as a function of x is not continuous at x = 0.
3. y as a function of x is differentiable for all x.

Select the correct answer using the code given below.
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

154. What is the derivative of y as a function of x with respect to x for x < 0?
(a) 2 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

DIRECTIONS (Qs. 155-156) : *For the next two (2) items that follow*

Consider the function $f(x) = (x-1)^2 (x+1)(x-2)^3$ *[2016-I]*

155. What is the number of points of local minima of the function f(x)?
(a) None (b) One
(c) Two (d) Three

156. What is the number of points of local maxima of the function f(x)?
(a) None (b) One
(c) Two (d) Three

DIRECTIONS (Qs. 157-158) : *For the next two (2) items that follow.*

Consider the function $f(x) = \frac{a^{[x]+x} - 1}{[x] + x}$ where [·] denotes the greatest integer function. *[2016-I]*

157. What is $\lim_{x \to 0^+} f(x)$ equal to?
(a) 1 (b) ln a
(c) $1 - a^{-1}$ (d) Limit does not exist

158. What is $\lim_{x \to 0^-} f(x)$ equal to?
(a) 0 (b) ln a
(c) $1 - a^{-1}$ (d) Limit does not exist

DIRECTIONS (Qs. 159-160) : *For the next two (2) items that follow.*

A function f(x) is defined as follows: *[2016-I]*

$$f(x)=\begin{cases} x+\pi & \text{for } x\in[-\pi,0) \\ \pi\cos x & \text{for } x\in\left[0,\frac{\pi}{2}\right] \\ \left(x-\frac{\pi}{2}\right)^2 & \text{for } x\in\left(\frac{\pi}{2},\pi\right] \end{cases}$$

159. Consider the following statements:
1. The function f(x) is continuous at x = 0.
2. The function f(x) is continuous at $x=\frac{\pi}{2}$.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

160. Consider the following statements:
1. The function f(x) is differentiable at x = 0.
2. The function f(x) is differentiable at $x=\frac{\pi}{2}$.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 161-162) : *For the next two (2) items that follow.*

Let f(x) be the greatest integer function and g(x) be the modulus function. *[2016-I]*

161. What is $(g\circ f)\left(-\frac{5}{3}\right)-(f\circ g)\left(-\frac{5}{3}\right)$ equal to?
(a) –1 (b) 0
(c) 1 (d) 2

162. What is $(f\circ f)\left(-\frac{9}{5}\right)+(g\circ g)(-2)$ equal to?
(a) –1 (b) 0
(c) 1 (d) 2

163. If $\lim_{x\to 0}\phi(x)=a^2$, where $a\neq 0$, then what is $\lim_{x\to 0}\phi\left(\frac{x}{a}\right)$ equal to? *[2016-I]*
(a) a^2 (b) a^{-2}
(c) $-a^2$ (d) –a

164. What is $\lim_{x\to 0} e^{-\frac{1}{x^2}}$ equal to? *[2016-I]*
(a) 0
(b) 1
(c) –1
(d) Limit does not exist

165. What is the domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$? *[2016-II]*
(a) $-\infty, 0$ (b) $0, \infty$
(c) $0<x<1$ (d) $x>1$

166. Consider the following in respect of the function *[2016-II]*

$$f(x)=\begin{cases} 2+x, & x\ge 0 \\ 2-x, & x<0 \end{cases}$$

1. $\lim_{x\to 1} f(x)$ does not exist.
2. f(x) is differentiable at x = 0
3. f(x) is continuous at x = 0

Which of the above statements is/are correct?
(a) 1 only (b) 3 only
(c) 2 and 3 only (d) 1 and 3 only

167. Let $f: A\to R$, where $A = R\setminus\{0\}$ is such that $f(x)=\frac{x+|x|}{x}$.
On which one of the following sets is f(x) continuous? *[2016-II]*
(a) A (b) B = $x\in R : x\ge 0$
(c) C = $x\in R : x\le 0$ (d) D = R

DIRECTIONS (Qs. 168-169) : *Consider the following function for the next two (02) items that follow.*

$$f(x)\begin{cases} 3x^2+12x-1 & -1\le x\le 2 \\ 37-x, & 2<x\le 3 \end{cases}$$ *[2016-II]*

168. Which of the following statements is/are correct?
1. f(x) is increasing in the interval [–1, 2].
2. f(x) is decreasing in the interval (2, 3].

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

169. Which of the following statements are correct?
1. f(x) is continuous at x = 2.
2. f(x) attains greatest value at x = 2.
3. f(x) is differentiable at x = 2.

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

DIRECTIONS (Qs. 170-172) : *Consider the following for the next three (03) items that follow.*

Let f (x) = [x], where [.] is the greatest integer function and g(x) = sin x be two real valued functions over R. *[2016-II]*

170. Which of the following statements is correct?
(a) Both f(x) and g(x) are continuous at x = 0.
(b) f(x) is continuous at x = 0, but g(x) is not continuous at x = 0.
(c) g(x) is continuous at x = 0, but f(x) is not continuous at x = 0.
(d) Both f(x) and g(x) are discontinuous at x = 0.

171. Which one of the following statements is correct?
(a) $\lim_{x\to 0}(fog)(x)$ exists
(b) $\lim_{x\to 0}(gof)(x)$ exists
(c) $\lim_{x\to 0+}(fog)(x)=\lim_{x\to 0-}(gof)(x)$
(d) $\lim_{x\to 0+}(fog)(x)=\lim_{x\to 0+}(gof)(x)$

172. Which of the following statements are correct?
1. (fof) (x) = f(x).
2. (gog) (x) = g(x) only when x = 0.
3. (go (fog)) (x) can take only three values.

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

DIRECTIONS (Qs. 173-174) : *Consider the following for the next two (02) items that follow.*

[2016-II]

Let $f(x) = \begin{cases} \dfrac{e^x - 1}{x}, & x > 0 \\ 0, & x = 0 \end{cases}$ be a real valued function.

173. Which one of the following statements is correct?
(a) f(x) is a strictly decreasing function in (0, x),
(b) f(x) is a strictly increasing function in (0, x),
(c) f(x) is neither increasing nor decreasing in (0, x)
(d) f(x) is not decreasing in (0, x).

174. Which of the following statements is/are correct?
1. f(x) is right continuous at x = 0.
2. f(x) is discontinuous at x = 1.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 175-177) : *Consider the following for the next three (03) items that follow.*

Let $f(x) = \begin{cases} -2, & -3 \le x \le 0 \\ x - 2, & 0 < x \le 3 \end{cases}$ and $g(x) = f\left(|x|\right) + |f(x)|$

[2016-II]

175. Which of the following statements is/are correct?
1. g(x) is differentiable at x = 0.
2. g(x) is differentiable at x = 2.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

176. What is the value of the differential coefficient of g(x) at x = –2?
(a) –1 (b) 0
(c) 1 (d) 2

177. Which of the following statements are correct?
1. g(x) is continuous at x = 0.
2. g(x) is continuous at x = 2.
3. g(x) is continuous at x = –1.

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

178. What is $\lim_{x \to 0} \dfrac{e^x - (1 + x)}{x^2}$ equal to? *[2017-I]*
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

179. The function $f : X \to Y$ defined by $f(x) = \cos x$, where $x \in X$, is one-one and onto if X and Y are respectively equal to *[2017-I]*
(a) $[0, \pi]$ and $[-1, 1]$
(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$
(c) $[0, \pi]$ and $(-1, 1)$
(d) $[0, \pi]$ and $[0, 1]$

180. If $f(x) = \dfrac{x}{x-1}$, then what is $\dfrac{f(a)}{f(a+1)}$ equal to? *[2017-I]*
(a) $f\left(-\dfrac{a}{a+1}\right)$ (b) $f\left(a^2\right)$
(c) $f\left(\dfrac{1}{a}\right)$ (d) $f(-a)$

181. Let $f : [-6, 6] \to R$ be defined by $f(x) = x^2 - 3$. Consider the following : *[2017-I]*
1. $(f \circ f \circ f)(-1) = (f \circ f \circ f)(1)$
2. $(f \circ f \circ f)(-1) - 4(f \circ f \circ f)(1) = (f \circ f)(0)$

Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

182. Let $f(x) = px + q$ and $g(x) = mx + n$. Then $f(g(x)) = g(f(x))$ is equivalent to *[2017-I]*
(a) $f(p) = g(m)$ (b) $f(q) = g(n)$
(c) $f(n) = g(q)$ (d) $f(m) = g(p)$

183. If $F(x) = \sqrt{9 - x^2}$, then what is $\lim_{x \to 1} \dfrac{F(x) - F(1)}{x - 1}$ equal to? *[2017-I]*
(a) $-\dfrac{1}{4\sqrt{2}}$ (b) $\dfrac{1}{8}$
(c) $-\dfrac{1}{2\sqrt{2}}$ (d) $\dfrac{1}{2\sqrt{2}}$

184. Let $f(x) : \begin{cases} x, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$ *[2017-I]*

and

$g(x) : \begin{cases} 0, & x \text{ is rational} \\ x, & x \text{ is irrational} \end{cases}$

If $f : R \to R$ and $g : R \to R$, then $(f - g)$ is
(a) one-one and into
(b) neither one-one nor onto
(c) many-one and onto
(d) one-one and onto

185. Let $f(x)$ be defined as follows : *[2017-I]*

$$f(x) = \begin{cases} 2x + 1, & -3 < x < -2 \\ x - 1, & -2 \le x < 0 \\ x + 2, & 0 \le x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?
(a) It is discontinuous at x = –2 but continuous at every other point.
(b) It is continuous only in the interval (–3, –2).
(c) It is discontinuous at x = 0 but continuous at every other point.
(d) It is discontinuous at every point.

186. Consider the following statements : [2017-I]

1. If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then $\lim_{x\to a} \{f(x)\, g(x)\}$ exists.
2. If $\lim_{x\to a} \{f(x)\, g(x)\}$ exists, then both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

187. Let $f(a) = \frac{a-1}{a+1}$. [2017-I]

Consider the following :

1. $f(2a) = f(a) + 1$ 2. $f\left(\frac{1}{a}\right) = -f(a)$

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

188. Suppose the function $f(x) = x^n, n \neq 0$ is differentiable for all x. Then n can be any element of the interval [2017-I]

(a) $(1, \infty)$ (b) $(0, \infty)$
(c) $\left(\frac{1}{2}, \infty\right)$ (d) None of the above

189. The inverse of the function $y = 5^{\ln x}$ is [2017-II]

(a) $x = y^{\frac{1}{\ln 5}}, y > 0$ (b) $x = y^{\ln 5}, y > 0$
(c) $x = y^{\frac{1}{\ln 5}}, y < 0$ (d) $x = 5 \ln y, y > 0$

190. A function is defined as follows : [2017-II]

$$f(x) = \begin{cases} -\dfrac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Which one of the following is correct in respect of the above function?

(a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$
(b) $f(x)$ is continuous as well as differentiable at $x = 0$
(c) $f(x)$ is discontinuous at $x = 0$
(d) None of the above

191. Consider the following : [2017-II]

1. $x + x^2$ is continuous at $x = 0$
2. $x + \cos\frac{1}{x}$ is discontinuous at $x = 0$
3. $x^2 + \cos\frac{1}{x}$ is continuous at $x = 0$

Which of the above are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

192. A function is defined in $(0, \infty)$ by [2017-II]

$$f(x) = \begin{cases} 1 - x^2 & \text{for } 0 < x \leq 1 \\ \ln x & \text{for } 1 < x \leq 2 \\ \ln 2 - 1 + 0.5x & \text{for } 2 < x < \infty \end{cases}$$

Which one of the following is correct in respect of the derivative of the function, i.e., $f'(x)$?

(a) $f'(x) = 2x$ for $0 < x \leq 1$
(b) $f'(x) = -2x$ for $0 < x \leq 1$
(c) $f'(x) = -2x$ for $0 < x < 1$
(d) $f'(x) = 0$ for $0 < x < \infty$

193. Consider the following statements : [2017-II]

1. Derivative of $f(x)$ may not exist at some point.
2. Derivative of $f(x)$ may exist finitely at some point.
3. Derivative of $f(x)$ may be infinite (geometrically) at some point.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

194. The function $f(x) = |x| - x^3$ is [2017-II]

(a) odd (b) even
(c) both even and odd (d) neither even nor odd

195. If $l_1 = \frac{d}{dx}\left(e^{\sin x}\right)$ [2017-II]

$$l_2 = \lim_{h\to 0} \frac{e^{\sin(x+h)} - e^{\sin x}}{h}$$

$$l_3 = \int e^{\sin x} \cos x\, dx$$

then which one of the following is correct?

(a) $l_1 \neq l_2$ (b) $\frac{d}{dx}(l_3) = l_2$
(c) $\int l_3 dx = l_2$ (d) $l_2 = l_3$

196. If $\lim_{x\to \frac{\pi}{2}} \frac{\sin x}{x} = l$ and $\lim_{x\to\infty} \frac{\cos x}{x} = m$, then which one of the following is correct? [2017-II]

(a) $l = 1, m = 1$ (b) $l = \frac{2}{\pi}, m = \infty$
(c) $l = \frac{2}{\pi}, m = 0$ (d) $l = 1, m = \infty$

197. If x is any real number, then $\frac{x^2}{1+x^4}$ belongs to which one of the following intervals? [2017-II]

(a) $(0, 1)$ (b) $\left(0, \frac{1}{2}\right]$
(c) $\left(0, \frac{1}{2}\right)$ (d) $[0, 1]$

198. The left-hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$ where k is an integer and $[x]$ is the greatest integer function, is [2017-II]

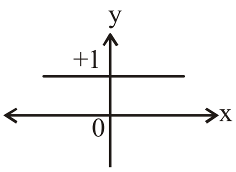
(a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1} (k-1)\pi$
(c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

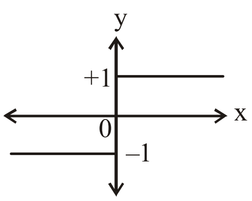
199. If $f(x) = \frac{x}{2} - 1$, then on the interval $[0, \pi]$ which one of the following is correct? *[2017-II]*
   (a) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\frac{1}{f(x)}$ are both continuous.
   (b) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer functin, and $f^{-1}(x)$ are both continuous.
   (c) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\frac{1}{f(x)}$ are both discontinuous.
   (d) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, is discontinuous but $\frac{1}{f(x)}$ is continuous.

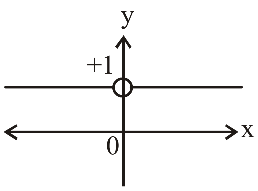
200. The set of all points, where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable, is *[2017-II]*
   (a) $(0, \infty)$ (b) $(-\infty, \infty)$
   (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-1, \infty)$

201. If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then $f(x)$ is *[2017-II]*
   (a) continuous but not differentiable at $x = 0$
   (b) differentiable at $x = 0$
   (c) not continuous at $x = 0$
   (d) None of the above

202. Which one of the following graph represents the function $f(x) = \frac{x}{x}, x \neq 0$? *[2017-II]*

(a) 

(b) 

(c) 

(d) None of the above

203. Let g be the greatest integer function. Then the function $f(x) = (g(x))^2 - g(x)$ is discontinuous at *[2017-II]*
   (a) all integers
   (b) all integers except 0 and 1
   (c) all integers except 0
   (d) all integers except 1

204. Consider the following statements : *[2017-II]*
   Statement I :
   $x > \sin x$ for all $x > 0$
   Statement II :
   $f(x) = x - \sin x$ is an increasing function for all $x > 0$
   Which one of the following is correct in respect of the above statements?
   (a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement II.
   (b) Both Statement I and Statement II are true and Statement II is not the correct explanation of Statement I.
   (c) Statement I is true but Statement II is false
   (d) Statement I is false but Statement II is true

205. If $f(x) = \frac{4x + x^4}{1 + 4x^3}$ and $g(x) = \ln\left(\frac{1+x}{1-x}\right)$, then what is the value of $f \circ g\left(\frac{e-1}{e+1}\right)$ equal to? *[2017-II]*
   (a) 2 (b) 1
   (c) 0 (d) $\frac{1}{2}$

206. Which one of the following is correct in respect of the function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = |x + 1|$? *[2018-I]*
   (a) $f(x^2) = [f(x)]^2$ (b) $f(|x|) = |f(x)|$
   (c) $f(x + y) = f(x) + f(y)$ (d) None of the above

207. Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by $f\ x \quad \frac{x^2}{1 \quad x^2}$. What is the range of the function? *[2018-I]*
   (a) $[0, 1)$ (b) $[0, 1]$
   (c) $(0, 1]$ (d) $(0, 1)$

208. If $f(x) = |x| + |x - 1|$, then which one of the following is correct? *[2018-I]*
   (a) $f(x)$ is continuous at $x = 0$ and $x = 1$
   (b) $f(x)$ is continuous at $x = 0$ but not at $x = 1$
   (c) $f(x)$ is continuous at $x = 1$ but not at $x = 0$
   (d) $f(x)$ is neither continuous at $x = 0$ nor at $x = 1$

209. Consider the function $f\ x \quad \begin{cases} x^2 \ln|x| & x \neq 0 \\ 0 & x \quad 0 \end{cases}$. What is $f'(0)$ equal to? *[2018-I]*
   (a) 0 (b) 1
   (c) -1 (d) It does not exist

210. If $f\ x\ = \frac{x^2 - 9}{x^2 - 2x - 3}, x \neq 3$ is continuous at $x = 3$, then which one of the following is correct? *[2018-I]*
   (a) $f(3) = 0$ (b) $f(3) = 1.5$
   (c) $f(3) = 3$ (d) $f(3) = -1.5$

211. If $f : \mathbb{R} \to S$ defined by $f(x) = 4\sin x - 3\cos x + 1$ is onto, then what is S equal to? *[2018-I]*
   (a) $[-5, 5]$ (b) $(-5, 5)$
   (c) $(-4, 6)$ (d) $[-4, 6]$

212. For f to be a function, what is the domain of f, if $f\ x \quad \frac{1}{\sqrt{|x| - x}}$? *[2018-I]*
   (a) $(-\infty, 0)$ (b) $(0, \infty)$
   (c) $(-\infty, \infty)$ (d) $(-\infty, 0]$

213. What is $\lim_{x\to 0}\frac{\tan x}{\sin 2x}$ equal to? *[2018-I]*

(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) Limit does not exist

214. What is $\lim_{h\to 0}\frac{\sqrt{2x+3h}-\sqrt{2x}}{2h}$ equal to? *[2018-I]*

(a) $\frac{1}{2\sqrt{2x}}$ (b) $\frac{3}{\sqrt{2x}}$
(c) $\frac{3}{2\sqrt{2x}}$ (d) $\frac{3}{4\sqrt{2x}}$

215. If f(x) is an even function, where $f(x)\neq 0$, then which one of the following is correct? *[2018-I]*
(a) $f'(x)$ is an even function
(b) $f'(x)$ is an odd function
(c) $f'(x)$ may be an even or odd function depending on the type of function
(d) $f'(x)$ is a constant function

216. Let $A = x\in R : -1\le x\le 1$ and S be the subset of A × B, defined by $S=\left[\; x, y \in A\times B : x^2 \quad y^2 \quad 1\right]$ *[2018-II]*

Which one of the following is correct?
(a) S is a one-one function from A into B
(b) S is a many-one function from A into B
(c) S is a bijective mapping from A into B
(d) S is not a function

217. If $f(x) = \frac{\sqrt{x-1}}{x-4}$ defines a function of R, then what is its domain? *[2018-II]*

(a) $-\infty, 4 \cup 4, \infty$ (b) $4, \infty$
(c) $1, 4 \cup 4, \infty$ (d) $1, 4 \cup 4, \infty$

218. Consider the function

$$f(x) \begin{cases} \frac{\sin 2x}{5x} \text{ if } x\neq 0 \\ \frac{2}{15} \text{ if } x \quad 0 \end{cases} :$$

Which one of the following is correct in respect of the function? *[2018-II]*
(a) It is not continuous at x = 0
(b) It is continuous at every x
(c) It is not continuous at x = π
(d) It is continuous at x = 0

219. For the function $f(x)=|x-3|$, which of the following is not correct? *[2018-II]*
(a) The function is not continuous at x = – 3
(b) The function is continuous at x = 3
(c) The function is differentiable at x = 0
(d) The function is differentiable at x = – 3

220. If the function $f(x) \quad \frac{2x-\sin^{-1}x}{2x \quad \tan^{-1}x}$ is continuous at each point in its domain, then what is the value of f(0)? *[2018-II]*

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) 2

221. If $f(x)=\sqrt{25-x^2}$, then what is $\lim_{x\to 1}\frac{f \;\; x \; - f \;\; 1}{x-1}$ equal to *[2018-II]*

(a) $-\frac{1}{\sqrt{24}}$ (b) $\frac{1}{\sqrt{24}}$
(c) $-\frac{1}{4\sqrt{3}}$ (d) $\frac{1}{4\sqrt{3}}$

222. What is $\lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta}$ equal to? *[2018-II]*

(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

223. A function $f : A\to R$ is defined by the equation $f(x)=x^2-4x+5$ where A = (1, 4). What is the range of the function? *[2018-II]*

(a) (2, 5) (b) (1, 5)
(c) [1, 5) (d) [1, 5]

224. In which one of the following intervals is the function $f(x)=x^2-5x+6$ decreasing? *[2018-II]*

(a) $-\infty, 2$ (b) $3, \infty$
(c) $-\infty, \infty$ (d) $2, 3$

225. Let f(x + y) = f (x) f(y) and f(x) = 1 + xg (x) ϕ (x), where $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to 0}\phi(x)=b$. Then what is f′ (x) equal to? *[2018-II]*
(a) 1 + ab f(x) (b) 1 + ab
(c) ab (d) abf(x)

226. What is $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x \quad 1}$ to? *[2018-II]*

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$
(c) – 2 (d) – 3

227. A function f defined by f(x) = $\ln(\sqrt{x^2+1}-x)$ is [2019-I]
(a) an even function
(b) an odd function
(c) Both even and odd function
(d) Neither even nor odd function

228. The domain of the function f defined by f(x) = $\log_x 10$ is [2019-I]
(a) x > 10
(b) x > 0 excluding x = 10
(c) x ≥ 10
(d) x > 0 excluding x = 1

229. $\lim_{x\to\infty} \frac{1-\cos^3 4x}{x^2}$ is equal to [2019-I]
(a) 0 (b) 12
(c) 24 (d) 36

230. If f(x) = 3^{1+x}, then f(x) f(y) f(z) is equal to [2019-I]
(a) f(x + y + z) (b) f(x + y + z + 1)
(c) f(x + y + z + 2) (d) f(x + y + z + 3)

231. The domain of the function f(x) = $\sqrt{(2-x)(x-3)}$ is [2019-I]
(a) $(0, \infty)$ (b) $[0, \infty]$
(c) [2, 3] (d) (2, 3)

232. The value of k which makes [2019-I]

$$f(x) = \begin{cases} \sin x & x \neq 0 \\ k & x = 0 \end{cases}$$ continuous at x = 0, is

(a) 2 (b) 1
(c) –1 (d) 0

ANSWER KEY

1	(d)	22	(b)	43	(c)	64	(c)	85	(c)	106	(b)	127	(c)	148	(b)	170	(c)	191	(a)	212	(a)
2	(d)	23	(b)	44	(a)	65	(c)	86	(d)	107	(b)	128	(b)	149	(b)	171	(d)	192	(c)	213	(a)
3	(a)	24	(b)	45	(a)	66	(c)	87	(b)	108	(b)	129	(d)	150	(c)	172	(c)	193	(d)	214	(d)
4	(b)	25	(b)	46	(a)	67	(b)	88	(c)	109	(d)	130	(a)	151	(a)	173	(b)	194	(d)	215	(b)
5	(d)	26	(a)	47	(d)	68	(d)	89	(a)	110	(a)	131	(c)	152	(d)	174	(b)	195	(b)	216	(d)
6	(c)	27	(c)	48	(d)	69	(b)	90	(b)	111	(b)	132	(a)	153	(d)	175	(d)	196	(c)	217	(d)
7	(c)	28	(c)	49	(c)	70	(a)	91	(b)	112	(d)	133	(d)	154	(d)	176	(b)	197	(b)	218	(a)
8	(c)	29	(d)	50	(a)	71	(a)	92	(b)	113	(a)	134	(b)	155	(c)	177	(d)	198	(a)	219	(a)
9	(d)	30	(a)	51	(d)	72	(c)	93	(c)	114	(c)	135	(d)	157	(b)	178	(b)	199	(c)	220	(b)
10	(d)	31	(b)	52	(c)	73	(b)	94	(c)	115	(c)	136	(a)	158	(c)	179	(a)	200	(c)	221	(a)
11	(a)	32	(d)	53	(d)	74	(a)	95	(d)	116	(b)	137	(c)	159	(c)	180	(b)	201	(b)	222	(c)
12	(c)	33	(d)	54	(c)	75	(c)	96	(a)	117	(d)	138	(b)	160	(d)	181	(c)	202	(c)	223	(c)
13	(d)	34	(a)	55	(a)	76	(d)	97	(a)	118	(d)	139	(b)	161	(c)	182	(c)	203	(d)	224	(a)
14	(c)	35	(d)	56	(a)	77	(b)	98	(c)	119	(b)	140	(c)	162	(b)	183	(c)	204	(a)	225	(d)
15	(b)	36	(a)	57	(d)	78	(b)	99	(c)	120	(b)	141	(b)	163	(a)	184	(d)	205	(b)	226	(d)
16	(d)	37	(d)	58	(c)	79	(d)	100	(b)	121	(b)	142	(d)	164	(a)	185	(c)	206	(d)	227	(b)
17	(a)	38	(c)	59	(b)	80	(b)	101	(c)	122	(a)	143	(a)	165	(a)	186	(a)	207	(a)	228	(d)
18	(c)	39	(d)	60	(d)	81	(b)	102	(d)	123	(d)	144	(c)	166	(b)	187	(b)	208	(a)	229	(c)
19	(a)	40	(c)	61	(b)	82	(b)	103	(d)	124	(b)	145	(a)	167	(a)	188	(a)	209	(a)	230	(c)
20	(c)	41	(a)	62	(c)	83	(b)	104	(a)	125	(c)	146	(c)	168	(c)	189	(a)	210	(b)	231	(c)
21	(a)	42	(c)	63	(c)	84	(a)	105	(c)	126	(a)	147	(b)	169	(a)	190	(c)	211	(d)	232	(d)

HINTS & SOLUTIONS

1. (d) $\because \quad f(x)=\frac{x^2}{1+x^2}$

Since, numerator < denominator
$f(x)<1$ for all values of x (negative or positive) and
$f(x)=0$ for $x=0$
So, range of f is $[0,1)$.

2. (d) The given functions is $f(x)=\frac{1}{\sqrt{18-x^2}}$

So, $f(3)=\frac{1}{\sqrt{18-9}}=\frac{1}{3}$

$\Rightarrow \quad \lim_{x\to 3}\frac{f(x)-f(3)}{x-3} \quad \lim_{x\to 3}\frac{\frac{1}{\sqrt{18-x^2}}-\frac{1}{3}}{x-3}$

Putting $x=3$ makes the that of $\frac{0}{0}$ form

$\lim_{x\to 3}-\frac{1}{2}(18-x^2)^{-3/2}(-2x)$

(Applying L' Hospital's Rule)

$=-\frac{1}{2}(9)^{-3/2}(-2\times 3)=\frac{1}{27}\times 3=\frac{1}{9}$

3. (a) Given that $f(1)=2$ and $f(x+y)=f(x)\,f(y)$
These are value for all values of x and y
Puting $x=1$ and $y=1$, we get
$f(2)=f(1)\,.\,f(1)=2.2=2^2$
Similarly,
$f(3)=f(1)\,.\,f(2)=2\,.\,2^2=2^3$
$\Rightarrow \quad f(x)=2^x$
$\Rightarrow \quad f'(x)=2^x\log_e 2$
Hence, $f'(1)=2\log_e 2$

4. (b) Given that, $f(x)=\log\left(\frac{1+x}{1-x}\right)$

and $g(x)=\left(\frac{3x+x^3}{1+3x^2}\right)$

$f[g(x)] \quad \log\left(\frac{1\ \ g(x)}{1-g(x)}\right)$

$=\log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$

$=\log\left(\frac{1+x}{1-x}\right)^3=3\log\left(\frac{1+x}{1-x}\right)=3\,[f(x)]$

5. (d) $\lim_{x\to 0}\frac{\sin|x|}{x}$, LHL is limit when $x<0$

$\text{LHL}=\lim_{x\to 0}\frac{\sin(-x)}{x}=-\lim_{x\to 0}\frac{\sin x}{x}=-1$

RHL is limit when $x>0$

$\text{RHL}=\lim_{x\to 0}\frac{\sin(x)}{x}=1$

So, LHL $\neq$ RHL
Hence, Limit does not exist.

6. (c) The given function is $f(x)=\begin{cases}2x, x\geq 0\\ 0, x<0\end{cases}$

The equation can be re-written as

$f(x)=\begin{cases}x+x, & x\geq 0\\ -x+x & x<0\end{cases}$

Hence, equivalent definition of given function is
$f(x)=|x|+x$

7. (c) Given function is $f(x)=\left(\frac{1}{3}\right)^x$

Let $f(x)=y$, so, $y=\left(\frac{1}{3}\right)^x$

Taking $\log_{1/3}$ on both sides

$\Rightarrow \quad x\times\log_{1/3}\left(\frac{1}{3}\right)=\log_{(1/3)}y$

$x=\log_{(1/3)}y$

$\Rightarrow \quad f^{-1}(x)=\log_{(1/3)}x$

8. (c) $\lim_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$

[multiply denominator and number with x]

We get,

$\lim_{x\to 0}\frac{x^2\sin 5x}{x\sin^2 4x}=\lim_{x\to 0}\frac{\sin 5x}{x}.\frac{x^2}{\sin^2 4x}$

Rearranging to bring a standard form, we get,

$\lim_{x\to 0}\frac{5\sin 5x}{5x}.\frac{(4x)^2}{16\sin^2 4x}$

$=\frac{5}{16}\left(\lim_{x\to 0}\frac{\sin 5x}{5x}\right).\frac{1}{\lim_{x\to 0}\left(\frac{\sin 4x}{4x}\right)^2}=\frac{5}{16}$

9. (d) Given that $f(x) = (1+x)^{5/x}$ and $f(x)$ is continuous at $x = 0$. Value of function at $x = 0$ is same as limit of the function at $x = 0$.

$$f(0) = \lim_{x\to 0}(1+x)^{5/x} = \left\{\lim_{x\to 0}(1+x)^{1/x}\right\}^5 = e^5$$

10. (d) Here, greatest integer function $[x]$ is discontinuous at its integral value of x, cot x and cosec x are discontinuous at $0, \pi, 2\pi$ etc. and tan x and sec x are discontinuous at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ etc. Therefore the greatest integer function and all trigonometric functions are not continuous for $x \in R$
Therefore, neither (1) nor (2) are true.

11. (a) For $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]$ to exist, then both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist.

12. (c) Given function is

$$f(x) = \begin{cases} mx+1, & x \le \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$$

As given this function is continuous at $x = \frac{\pi}{2}$.

So, limit of function when $x \to \frac{\pi}{2} = f\left(\frac{\pi}{2}\right)$

$$\Rightarrow \lim_{x\to \frac{\pi}{2}^+}(\sin x + n) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{h\to 0}\left(\sin\left(\frac{\pi}{2}+h\right)+n\right) = \frac{m\pi}{2}+1$$

$$\Rightarrow \sin\frac{\pi}{2}+n = \frac{m\pi}{2}+1$$

$$\Rightarrow 1+n = \frac{m\pi}{2}+1$$

$$\Rightarrow n = \frac{m\pi}{2}$$

13. (d) The given curve shows the graph of a^x which is decreasing when x is increasing. This happens when $0 < a < 1$.

14. (c) Given that $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$$\text{So, } f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right) = \log\left(\frac{(1 \quad x)^2}{(1-x)^2}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2 = 2\log\left(\frac{1+x}{1-x}\right)$$

$$= 2f(x) \qquad \left[\text{since } f(x) = \log\left(\frac{1+x}{1-x}\right)\right]$$

15. (b) For a function to be continuous at a point the limit should exist and should be equal to the value of the function at that point.
Here point is $x = 0$

and $\lim_{x\to 0} f(x) \quad \lim_{x\to 0}(x \quad 1)^{\cot x}$

$$= \lim_{x\to 0}(1+x)^{\cot x} = \lim_{x\to 0}(1+x)^{\frac{1}{x}.x\cot x}$$

$$= \lim_{x\to 0}(1+x)^{\frac{1}{x}\lim_{x\to 0}\frac{x}{\tan x}} = e^1 = e$$

Since limiting value of $f(x) = e$, when $x \to 0$, $f(0)$ should also be equal to e.

16. (d) $\lim_{x\to 0}\left(\frac{x-2}{x+2}\right)^{x+2}$

can be written as

$$\lim_{x\to 0}\left(\frac{x+2-4}{x+2}\right)^{x+2}$$

$$\text{or, } \lim_{x\to\infty}\left\{1-\frac{4}{(x+2)}\right\}^{x+2}$$

Putting $x+2 = t$
when $x \to \infty, t \to \infty$

$$\text{So, } \lim_{t\to\infty}\left\{1-\frac{4}{t}\right\}^t$$

$$\text{or, } \lim_{t\to\infty}\left\{1-\frac{4}{t}\right\}^{\frac{t}{4}\times 4}$$

$$\text{or, } \lim_{x\to\infty}\left\{\left(1-\frac{4}{t}\right)^{\frac{t}{4}}\right\}^4 = (e^{-1})^4 = e^{-4}$$

17. (a) Derivative of $f(x) = \begin{cases} ax^2 + b & x < -1 \\ bx^2 + ax + a & x \geq -1 \end{cases}$ is

$$f'(x) = \begin{cases} 2ax & x < -1 \\ 2bx + a, & x \geq -1 \end{cases}$$

If $f'(x)$ is continuous everywhere then it is also continuous at $x = -1$

$f'(x)\big|_{x=-1} = -2a = -2b + a$

or, $3a = 2b$ ...(i)

From the given choice

$a = 2, b = 3$ satisfied this equation.

18. (c) If f(x) is differential everywhere then | f | is not differentiable at some point, so, f | f | is not differentiable at some point.

[Example: $f(x) = x$ is differentiable everywhere but $|f(x)| = |x|$ is not differentiable at $x = 0$]

19. (a) As given $f(x) = ax^2 + bx + c$, Let $y = f(x)$. To get $f^{-1}(x)$ we express x is terms of y in equation $y = ax^2 + bx + c$.

$$\frac{y}{a} = x^2 + \frac{b}{a}x + \frac{c}{a} = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$$

$$\Rightarrow \quad x + \frac{b}{2a} = \sqrt{\frac{y}{a} + \frac{b^2}{4a^2} - \frac{c}{a}} = \frac{\pm\sqrt{4ay + b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{4ay + b^2 - 4ac}}{2a}$$

$$\text{or, } f^{-1}(x) = \frac{-b \quad \sqrt{4ax \quad b^2 - 4ac}}{2a}$$

Putting $x = 0$

$$f^{-1}(0) = \frac{-b \pm \sqrt{0 + b^2 - 4ac}}{2a} \neq 0 \quad [\text{if } b^2 - 4ac > 0]$$

so, $f^{-1}(0) \neq 0$. i.e. if $b^2 - 4ac > 0$

$f^{-1}(0)$ does not contain 0, if $b^2 - 4ac > 0$.

20. (c) $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$

Such an equation is possible only, if

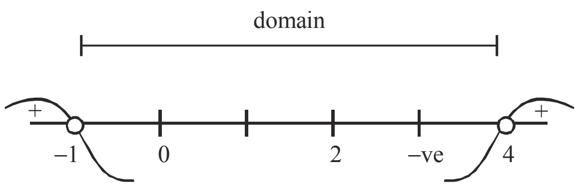
$$\frac{x-a}{b+c} = \frac{x-b}{c+a} = \frac{x-c}{a+b} = 1$$

$\Rightarrow \; x = a + b + c$

21. (a) $-x^2 + 3x + 4 > 0$

$\Rightarrow \; x^2 - 3x - 4 < 0 \Rightarrow \; (x-4)(x+1) < 0$

Noting the sign of expression around – 1 and 4, we use way curve method.



$\Rightarrow \; x \in (-1, 4)$

22. (b) Given function is.

$$f(x) \quad x \quad \frac{1}{x}$$

$f^2(x) = \{f(x)\}^2$

$$\Rightarrow \; f^2(x) \quad \left(x \quad \frac{1}{x}\right)^2$$

23. (b) From the given direction of function

$$f(x) \quad \begin{cases} 1, & x \text{ is a rational number} \\ 0, & x \text{ is an irrational number} \end{cases}$$

$(f\,0\,f)\sqrt{3} \quad f\{f(\sqrt{3})\}$

$= f(0)$ $\qquad (\because \sqrt{3}$ is an irrational number)

$= 1$ $\qquad (\because 0$ is a rational number)

24. (b) Given function is defined as :

$$f(x) \quad \begin{cases} x^p \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0, & x \quad 0 \end{cases}$$

For continuity :

LHS : $\lim\limits_{x\to 0} f(x)$ = RHS $\lim\limits_{x\to 0} f(x) \quad f(0)$

$$\Rightarrow \lim_{x\to 0} f(x) \quad \lim_{x\to 0} x^p \cos\left(\frac{1}{x}\right) \quad 0$$

$$\Rightarrow \lim_{x\to 0} x^p \cos\left(\frac{1}{x}\right) \quad 0$$

$\cos\left(\frac{1}{x}\right)$ is always a finite quantity if $x \to 0$

$\Rightarrow \; x^p = 0$

which is possible only if $p > 0$.

25. (b) Given limit is : $\lim\limits_{x\to\infty} \frac{\sin x}{x}$

Let, $x = \frac{1}{h}$ and as $x \to \infty, h \to 0$

$\therefore \lim\limits_{x\to\infty} \frac{\sin x}{x}$ change to : $\lim\limits_{h\to 0} h \sin\frac{1}{h}$

$= 0$ (value of $\sin\frac{1}{h}$ is finite as it lies between – 1 and 1)

26. (a) Let $L = \lim\limits_{x\to 0} \frac{a^x - b^x}{x}$

This is of $\frac{0}{0}$ form so, L' hospital's rule is applicable.

$$\Rightarrow L = \lim_{x\to 0} \frac{a^x \log a - b^x \log b}{1} \quad \text{(by L' Hospital's rule)}$$

$$= \log a - \log b = \log\frac{a}{b}$$

27. (c) Given function is :

$$f(x) \quad \begin{cases} 3x-4, & 0 \le x \le 2 \\ 2x \quad \ell, & 2 \quad x \le 9 \end{cases}$$

and also given that $f(x)$ is continuous at $x = 2$

For a function to be continuous at a point LHL = RHL = V.F. at that point. $f(2) = 2 =$ V.F.

$\Rightarrow$ RHL : $\lim_{x\to 2}(2x \quad \ell) \quad 3(2)-4$

$\Rightarrow \lim_{h\to 0} \; 2(2 \quad h) \quad \ell \qquad 6-4$

$\Rightarrow 4 \quad \ell \quad 2$

$\Rightarrow \ell \quad -2$

28. (c) Given functions are : $f(x) = x$ and $g(x) = |x|$

$\therefore$ $(f+g)(x) = f(x) + g(x) = x + |x|$

According to defintion of modulus function,

$$(f+g)(x) = \begin{cases} x+x, & x \ge 0 \\ x-x, & x < 0 \end{cases}$$

$$= \begin{cases} 2x, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

29. (d) Given function are :

$g(x) = \sin x$ and $f(x) = \dfrac{1}{\sin x}$

$(gof)(x) = g[f(x)]$

$= \sin f(x)$

$= \sin\left(\dfrac{1}{\sin x}\right)$

30. (a) Given function is : $f(x) = \sin|x|$

$$= \begin{cases} \sin(x), & x \ge 0 \\ \sin(-x), & x < 0 \end{cases}$$

$$= \begin{cases} \sin x, & x \ge 0 \\ -\sin x, & x < 0 \end{cases}$$

LHD at $x = 0 = \lim_{h\to 0}\dfrac{f(0-h)-f(0)}{0-h-0}$

$= \lim_{h\to 0}\dfrac{f(0-h)-f(0)}{-h} \quad \lim_{h\to 0}\dfrac{-\sin(-h)-0}{-h} \quad -1$

RHD at $x = 0$ $= \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{0+h-0}$

$= \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h} \quad \lim_{h\to 0}\dfrac{\sin(h-0)}{h} \quad 1$

LHD $\ne$ RHD

$f(x)$ is not differentiable at $x = 0$.

31. (b) Given equation is $y = 5^{\log x}$

To get the inverse, we express x in terms of y.

Taking log on both the sides,

$\log y = \log x . \log 5$

$\Rightarrow \log x \quad \dfrac{\log y}{\log 5} \quad \log y^{\frac{1}{\log 5}}$

$\Rightarrow x \quad y^{\frac{1}{\log 5}}$

32. (d) (A) : $f(x) = \log x$ for $x = 1$, $f(x) = \log 1 = 0$

(R) : $f(x) = \log x$

and $f(x) \ge 0 \;\forall\; x > 0$

Thus, (A) is false but (R) is true.

33. (d) Given function : $f(x) = x\sin\left(\dfrac{1}{x}\right)$

For differentiability at $x = 0$; LHD = RHD at $x = 0$

LHD $\quad \lim_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0}\dfrac{(-h)\sin\left(-\frac{1}{h}\right)}{-h} \quad \lim_{h\to 0}\dfrac{h\sin\left(\frac{1}{h}\right)}{-h}$

$= \lim_{h\to 0}\sin\left(\dfrac{1}{h}\right) =$ a finite value lies between – 1 and 1 which cannot be qualified exactly.

RHD $= \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\dfrac{h\sin\left(\frac{1}{h}\right)}{h} = \lim_{h\to 0}\sin\left(\dfrac{1}{h}\right)$

= a finite value lies between – 1 and 1 which cannot be qualified exactly.

LHD $\ne$ RHD $\ne$ a definite value.

Hence, $f(x)$ is not differentiable at $x = 0$.

For continuity at $x = 0$:

$\lim_{x\to 0}$ LHL $\quad \lim_{x\to 0}$ RHL $\quad$ V.F. at $x = 0$

LHL $= \lim_{x\to 0} f(0-h) = \lim_{h\to 0} -h\sin\left(-\dfrac{1}{h}\right)$

$= \lim_{h\to 0} h\sin\dfrac{1}{h} = 0$

RHL $= \lim_{x\to 0'} f(0+h) = \lim_{h\to 0} h\sin\dfrac{1}{h} = 0$

$f(0) = 0$

$\Rightarrow$ LHL = RHL = $f(0)$

Hence, $f(x)$ is continuous at $x = 0$

Thus, (A) is false but (R) is true.

34. (a) $f(x) = \log|x|$

$$f'(x) = \frac{1}{|x|}$$

35. (d) LHL $= \lim\limits_{h\to 0^-} e^{\frac{-1}{(0-h)}} \quad \lim\limits_{h\to 0} e^{\frac{1}{h}}$

$= e^{\infty} = \infty$

RHL $= \lim\limits_{h\to 0^+} e^{\frac{-1}{(0+h)}} \quad \lim\limits_{h\to 0} e^{\frac{-1}{h}} = e^{-\infty} = 0$

$\because$ LHL $\neq$ RHL

So, $\lim\limits_{x\to 0} e^{\frac{-1}{x}}$ does not exist.

36. (a) Let $y = 2x + 5$

$\Rightarrow y - 5 = 2x$

$\Rightarrow x = \frac{y-5}{2}$

$\therefore g^{-1}(x) = \frac{x-5}{2}$

37. (d) 1. $\lim\limits_{x\to 0} \frac{x^2}{x} = \lim\limits_{x\to 0}(x) = 0$

2. $\frac{x^2}{x} = x,$

since a polynomial is continuous everywhere, so it is continuous at $x = 0$

3. LHL $= \lim\limits_{h\to 0} \frac{|0-h|}{(0-h)} = \lim\limits_{h\to 0} \frac{h}{-h} = -1$

RHL $= \lim\limits_{h\to 0} \frac{|0+h|}{(0+h)} = \lim\limits_{h\to 0} \frac{h}{h} = 1$

$\because$ LHL $\neq$ RHL

So, it does not exist.

Thus, the statements 1 and 3 are correct.

38. (c) LHL $= \lim\limits_{h\to 0} \frac{1}{1-|1-(1-h)|}$

$= \lim\limits_{h\to 0} \frac{1}{1-|h|} = \lim\limits_{h\to 0} \frac{1}{1-h} = 1$

RHL $= \lim\limits_{h\to 0} \frac{1}{1-|1-(1+h)|}$

$= \lim\limits_{h\to 0} \frac{1}{1-|-h|} \quad \lim\limits_{h\to 0} \frac{1}{1-h} \quad 1$

$\therefore \lim\limits_{x\to 0} f(x) \quad 1$

39. (d) $\lim\limits_{x\to\alpha} \frac{\sqrt{\alpha+2x}-\sqrt{3x}}{\sqrt{3\alpha+x}-2\sqrt{x}}$

$= \lim\limits_{x\to\alpha} \frac{(\sqrt{\alpha+2x})^2-(\sqrt{3x})^2}{\sqrt{\alpha \quad 2x} \quad \sqrt{3x}} \times \frac{\sqrt{3\alpha \quad x} \quad 2\sqrt{x}}{(\sqrt{3\alpha+x})^2-(2\sqrt{x})^2}$

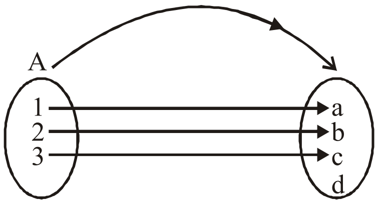
$= \lim\limits_{x\to\alpha} \frac{\alpha+2x-3x}{\sqrt{\alpha+2x}+\sqrt{3x}} \times \frac{\sqrt{3\alpha+x}+2\sqrt{x}}{3\alpha+x-4x}$

$= \lim\limits_{x\to\alpha} \frac{\sqrt{3\alpha+x}+2\sqrt{x}}{\sqrt{\alpha+2x}+\sqrt{3x}} \times \frac{(\alpha-x)}{3(\alpha-x)}$

$= \frac{1}{3} \lim\limits_{x\to\alpha} \frac{\sqrt{3\alpha \quad x} \quad 2\sqrt{x}}{\sqrt{\alpha \quad 2x} \quad \sqrt{3x}}$

$= \frac{1}{3} \frac{\sqrt{4\alpha}+2\sqrt{\alpha}}{\sqrt{3\alpha}+\sqrt{3\alpha}} = \frac{1}{2}\left(\frac{4\sqrt{\alpha}}{2\sqrt{3}\sqrt{\alpha}}\right) = \frac{1}{\sqrt{3}}$

40. (c)



Since, every element of A has only one image but d has no pre-image in A. So, it is one-one function.

Hence, it has no inverse.

41. (a) (A) Given function is

$y = 2x + 3$

Let $y_1 = y_2$ (To show $x_1 = x_2$)

$\Rightarrow 2x_1 + 3 = 2x_2 + 3$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

Hence, $y = 2x + 3$ is one-one real valued function.

(R) Since $y_1 = y_2 \Rightarrow x_1 = x_2$

$\therefore x_1 \neq x_2 \Rightarrow y_1 \neq y_2$

Thus, Both (A) and (R) are true and R is the correct explanation of A.

42. (c) $f(-1) = f(1) = 2^{35}$

Here, two real numbers 1 and -1 have the same image.

So, the function is not one-one and let

$y = (x^2 + 1)^{35}$

$\Rightarrow x = \sqrt{(y)^{1/35} - 1}$

Thus, every real number has no pre image. So, the function is not onto.

Hence, the function is neither one-one nor onto.

43. (c) Given, $f(x) \quad x|x|$

If $f(x_1) \quad f(x_2)$

$\Rightarrow x_1|x_1| \quad x_2|x_2|$

$\Rightarrow x_1 \quad x_2$

$\therefore f(x)$ is one-one.

Also, range of $f(x)$ = co-domain of $f(x)$

$\therefore f(x)$ is onto.

Hence, $f(x)$ is both one-one and onto.

44. (a) Given $f(x) \quad \dfrac{x}{1 \quad |x|}$

$$\begin{cases} \dfrac{x}{1-x}, & x \quad 0 \\ \dfrac{x}{1 \quad x}, & x \geq 0 \end{cases} \qquad \left(\because |x| \quad \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x \quad 0 \end{cases} \right)$$

$\therefore$ LHD $= f'(0^-) \quad \lim_{h\to 0} \dfrac{f(0-h)-f(0)}{-h}$

$$\lim_{h\to 0} \frac{f(-h)-f(0)}{-h}$$

$$\lim_{h\to 0} \frac{\frac{-h}{1+|-h|}-0}{-h} \qquad \lim_{h\to 0} \frac{\frac{-h}{1 \quad h}-0}{-h} \qquad \lim_{h\to 0} \frac{1}{1 \quad h} \quad 1$$

and RHD $= f'(0 \quad)$

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$\lim_{h\to 0} \frac{\frac{h}{1 \quad h}-0}{h} \qquad \lim_{h\to 0} \frac{1}{1 \quad h} \quad 1$$

Since, LHD = RHD

$\therefore \quad f(x)$ is differentiable at $x = 0$

Hence, $f(x)$ is differentiable in $(-\infty, \infty)$.

45. (a) We know $\lim_{\delta x\to 0} \dfrac{\delta y}{\delta x} \quad \left(\dfrac{dy}{dx}\right)_{\text{at } x \quad 0}$

$$= \frac{d}{dx}(ax^n)_{\text{at } x=0} = (an\, x^{n-1})_{\text{at } x=0} = 0$$

46. (a) Consider $\lim_{x\to 0} \dfrac{\sin^2 ax}{bx}$

Multiply and divide by a^2x^2,

$$\Rightarrow \lim_{x\to 0} \frac{\sin^2 ax}{bx} = \lim_{x\to 0}\left[\frac{\sin^2 ax}{bx} \times \frac{a^2x^2}{a^2x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{\sin^2 ax}{a \quad x^2} \times \frac{a^2x^2}{bx}\right]$$

$$= \lim_{x\to 0}\left[\left(\frac{\sin ax}{ax}\right)^2 \times \frac{a^2x}{b}\right]$$

$$= \lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)^2 . \lim_{x\to 0} \frac{a^2x}{b}$$

$$= 1 \times 0 = 0 \qquad \left(\because \lim_{x\to 0} \frac{\sin x}{x} \quad 1\right)$$

47. (d) Given $f(x) = \begin{cases} 3x-4, & 0 \leq x \leq 2 \\ 2x+\lambda, & 2 < x \leq 3 \end{cases}$

Also $f(x)$ is continuous at $x = 2$

$\therefore \quad \lim_{x\to 2} f(x) \quad f(2)$

Now, $f(2) = 3 \times 2 - 4 = 6 - 4 = 2$

$\Rightarrow \quad \lim_{x\to 2}(2x+\lambda) \quad 2$

$\Rightarrow \quad 4 + \lambda = 2$

$\Rightarrow \quad \lambda = -2$

48. (d) Let $x_1, x_2 \in R$

Then, $f(x_1) = f(x_2)$

$\Rightarrow \quad \cos x_1 = \cos x_2$

$\Rightarrow \quad x_1 = 2n\pi \pm x_2$

So, $x_1 \neq x_2$

Hence, cos x is not one-one function.

Now, let y = cos x

We know, $-1 \leq \cos x \leq 1$

$\therefore \quad y \in [-1, 1]$

$[-1, 1] \subset R$. So, cos x is into function, not onto.

Hence, f(x) = cos x is neither one-one nor onto.

49. (c) Consider $\lim_{x\to\infty}\left(\dfrac{x}{3 \quad x}\right)^{3x} \quad \lim_{x\to\infty}\left(\dfrac{1}{1 \quad \frac{3}{x}}\right)^{3x}$

$$= \lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{-3x} = \lim_{x\to\infty}\left[\left(1+\frac{3}{x}\right)^x\right]^{-3}$$

$$= [e^3]^{-3} = e^{-9} \qquad \left(\because \lim_{n\to\infty}\left(1 \quad \frac{\lambda}{n}\right)^n \quad e^{\lambda}\right)$$

50. (a) Given, $f(x) = \begin{cases} x, & x \geq 0 \\ -x^2, & x \quad 0 \end{cases}$

LHL $= \lim_{x\to 0^-} f(x) \quad -\lim_{x\to 0} x^2 \quad 0$

RHL $= \lim_{x\to 0} f(x) \quad \lim_{x\to 0} x \quad 0$

and $f(0) = 0$

Since, LHL = RHL = $f(0)$

$\therefore f(x)$ is continuous at $x = 0$.

Also, $f(x)$ is continuous in the given interval ie. R

Hence, $f(x)$ is continuous in every $x \in R$.

51. (d) A injective function means one-one.

Consider $f(x) = -x$

Let $f(x) = f(y) \ \forall\, x, y \in R$

$\Rightarrow -x = -y \Rightarrow x = y$

For every values of x, we get a different values of f.

Hence, it is injective.

52. (c) Let $f(x) = f(y)$
To show that $f(x)$ is one-one
We have to show that $x = y$
Now, $f(x) = f(y)$

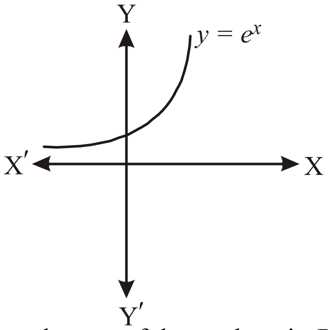
$$\Rightarrow e^x = e^y \Rightarrow \frac{e^x}{e^y} = 1 \Rightarrow e^{x-y} = 1$$

Take log on both side

$$\log e^{x-y} = \log 1$$

$$\Rightarrow x - y = 0 \Rightarrow x = y$$

Hence $f(x)$ one-one $\forall$ x $\in$ R



–2 is an element of the co-domain R. There doesnot exist any element X in the domain R such that $-2 = e^x = f(x)$.
Hence, by definition, f is not a onto function.

53. (d)

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left(\frac{x+5+1}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left(\frac{x+1}{x+1}+\frac{5}{x+1}\right)^{x+4}$$

$$= \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+4}{5}\times\frac{5(x+1)}{x+1}}$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{5\left(\frac{x+4}{x+1}\right)}$$

$$= e^{5\lim_{x\to\infty}\frac{1+\frac{4}{x}}{1+\frac{1}{x}}} = e^5 \qquad \left(\because \lim_{x\to\infty}\frac{1+4/x}{1+1/x} = 1\right)$$

54. (c) Given $fog(x) = (x+3)^2$ and $g(x) = x + 3$

$$fog(x) = (x+3)^2 \Rightarrow f[g(x)] = (x+3)^2$$

$$\Rightarrow f[x+3] = (x+3)^2 \ (\because g(x) = x+3)$$

$$\Rightarrow f(x) = x^2$$

Hence, $f(-3) = (-3)^2 = 9$

55. (a) Let $f(x) = \frac{(x-1)^2}{|x-1|} = \begin{cases}(x-1), & x \ge 1\\ -(x-1), & x < 1\end{cases}$

Now, $\text{LHL} = \lim_{h\to 0} f(1-h)$

$$= \lim_{h\to 0}\left[-(1-h-1)\right] = \lim_{h\to 0} h = 0$$

and $\text{RHL} = \lim_{h\to 0} f(1+h)$

$$= \lim_{h\to 0}(1+h-1) = \lim_{h\to 0} h = 0$$

$\therefore$ LHL = RHL

$\therefore \lim_{x\to 0} f(x) = \text{LHL} = \text{RHL}$

$$\Rightarrow \lim_{x\to 0}\frac{(x-1)^2}{|x-1|} = 0$$

56. (a) sin x is periodic, continuous at every point on $(-\infty, \infty)$, differentiable at every point on $(-\infty, \infty)$, has a period 2π, sin x increases on $\left(0, \frac{\pi}{2}\right)$ and decreases on $\left(\frac{\pi}{2}, \pi\right)$.

57. (d) cos x is periodic, continuous and differentiable at every point on $(-\infty, \infty)$ and has a period 2π, cos x decreases on $\left(0, \frac{\pi}{2}\right)$ and increases on $\left(\frac{\pi}{2}, \pi\right)$.

58. (c) tan x is a periodic function with period π and is discontinuous at $x = \frac{m\pi}{2}$. Also, tan x is not differentiable at every point on $(-\infty, \infty)$ and increases on $\left(0, \frac{\pi}{2}\right)$ and increases on $\left(\frac{\pi}{2}, \pi\right)$.
Hence, option (c) is correct.

59. (b) We know every function does not has a primitive but a primitive of a function is unique.

60. (d) Given $f : R \to R$ defined as $f(x) = \frac{x}{x^2+1}$
Which is a injective function
-ie- $f(x) = \frac{x}{x^2+1}$ is one-one but not onto.

61. (b) The function $f(x) = x$ cosec x is discontinuous everywhere.

62. (c) Let $f(x) = |x-3|$

$$\text{L.H.L} = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}|0-h-3|$$

$$= \lim_{h\to 0}(h+3) = 3$$

$$\text{R.H.L} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}|0+h-3|$$

$$= \lim_{h\to 0}|h-3| = 3$$

Since, L.H.L = R.H.L at $x = 0$
$\therefore f(x) = |x-3|$ is continuous at $x = 0$.

$$\text{Now, LHD} = f'(0^-) = \lim_{h\to 0}\frac{f(0)-f(0-h)}{h}$$

$$= \lim_{h\to 0}\frac{3-(3-h)}{h} = 1$$

$$\text{and RHD} = f'(0^+) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{3+h-3}{h} = 1$$

$\Rightarrow$ LHD = RHD
$\therefore f(x)$ is differentiable
Hence, both statements (I) and (II) are correct.

63. (c) Since, on taking a straight line parallel to x-axis, the group of given function intersect it at one point.

$\therefore$ f(x) is one-one.

and as range of f(x) = Co-domain

$\therefore$ f(x) is onto.

Hence, f(x) is one–one onto.

64. (c) Consider $\lim_{x\to 0}\frac{\cos ax-\cos bx}{x^2}$

$=\lim_{x\to 0}\frac{-a\sin ax+b\sin bx}{2x}$

(using L' Hospital's rule)

$=\lim_{x\to 0}\frac{-a^2\cos ax+b^2\cos bx}{2}$

(using L' Hospital's rule)

$=\frac{b^2-a^2}{2}(\because \cos 0=1)$

65. (c) Let, f(x) = 2x + 7 and g(x) = x^2+7, x∈R

Now fog(x) = f[g(x)] = f(x^2+7)

= 2(x^2+7) + 7 = $2x^2$ + 14 + 7

But fog(x) = 25

$\Rightarrow$ $2x^2+21=25$

$\Rightarrow$ $x^2=2$

$\Rightarrow$ $x=\pm\sqrt{2}$

66. (c) Consider

$\lim_{x\to 0}\frac{a^x-b^x}{x}=\lim_{x\to 0}\frac{a^x\log a-b^x\log b}{1}$

(Using L' Hospital's rule)

$=\log a-\log b=\log\frac{a}{b}$

67. (b) Let $f(x)=\frac{x(x-2)}{x^2-4}=\frac{x(x-2)}{(x-2)(x+2)}=\frac{x}{x+2}$

Since f(x) is continuous at x = 2

$\therefore$ $\lim_{x\to 2}f(x)=f(2)$

$\Rightarrow$ $\lim_{x\to 2}\frac{x}{x+2}=f(2)$

$\Rightarrow$ $f(2)=\frac{2}{4}=\frac{1}{2}$

68. (d) Given, $f(x)=[x]$

Let 'c' be any real number.

f is continuous at $x=c$ if L.H.L. = R.H.L. = $f(c)$

i.e., $\lim_{x\to c^-}f(x)=\lim_{x\to c^+}f(x)=f(c)$

L.H.L. = $\lim_{x\to c^-}f(x)=\lim_{x\to c^-}[x]=c^{-1}$

R.H.L. = $\lim_{x\to c^+}f(x)=\lim_{x\to c^+}[x]=c$

Since, L.H.L. ≠ R.H.L.

f is discontinous for all x ∈ R.

So, $[x]$ is discontinuous at infinite points.

69. (b) Let $f(x)=\frac{2}{3}x+\frac{3}{2}=y(\text{say})=\frac{4x+9}{6}=y$

$\Rightarrow$ $4x+9=6y$

$\Rightarrow$ $x=\frac{6y-9}{4}$

$x=f^{-1}(y)$

$\Rightarrow$ $f^{-1}(x)=\frac{6x-9}{4}=\frac{3x}{2}-\frac{9}{4}$

70. (a) Consider, $\lim_{x\to\infty}\left[\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}\right]$

$=\lim_{x\to\infty}\frac{(\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1})(\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1})}{(\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1})}$

$=\lim_{x\to\infty}\left[\frac{a^2x^2+ax+1-a^2x^2-1}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}\right]$

$=\lim_{x\to\infty}\frac{a}{\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}}$

$=\frac{a}{\sqrt{a^2}+\sqrt{a^2}}=\frac{a}{2a}=\frac{1}{2}$

71. (a) Let $f(x)=\begin{cases}\frac{x^3-3x+2}{(x-1)^2}, & \forall x\neq 1\\ k, & \forall x=1\end{cases}$

and $f(x)$ is continuous.

$\therefore$ $\lim_{x\to 1}f(x)=k$

$\Rightarrow$ $\lim_{x\to 1}\frac{x^3-3x+2}{(x-1)^2}=k$

$\Rightarrow$ $k=\lim_{x\to 1}\frac{3x^2-3}{2(x-1)}$ [By L'Hospitals rule]

$\Rightarrow$ $k=\lim_{x\to 1}\frac{6x}{2}$ [By L'Hospitals rule]

$\Rightarrow$ $k=3$

72. (c) $\because \quad f(x) = |x| + x^2$

$$\Rightarrow f(x) = \begin{cases} x^2 + x, & x \geq 0 \\ x^2 - x, & x \quad 0 \end{cases}$$

$$\text{LHL} = \lim_{x \to 0^-} f(x)$$

$$= \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \; 0-h \;^2 - \; 0-h$$

$$= \lim_{h \to 0} h^2 \quad h \quad 0$$

$$\text{and RHL} = \lim_{x \to 0} f(x) = \lim_{h \to 0} f(0 \quad h)$$

$$= \lim_{h \to 0} (0 \quad h)^2 \quad (0 \quad h) = \lim_{h \to 0} h^2 \quad h \quad 0$$

$\Rightarrow$ LHL = RHL = f(0)

$\Rightarrow f(x)$ is continuous at $x = 0$

$$\text{Now, LHD} = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{h^2 \quad h}{-h} = -\lim_{h \to 0} h \quad 1 = -1$$

$$\text{and, RHD} = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 \quad h}{h} = \lim_{h \to 0} h \quad 1 = 1$$

Thus, LHD $\neq$ RHD

$\Rightarrow \quad f(x)$ is not differentiable at $x = 0$

73. (b) Given f(x) = |x – 3| is not continuous at x = 3 but it is differentiable at x = 0.

74. (a) $\lim_{x \to 0} \left(x^2\right)\left[\sin\left(\frac{1}{x}\right)\right] = 0 \times \lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

= 0 × finite quantity = 0

75. (c) $\lim_{x \to -2}\left(\frac{x+2}{x^3+8}\right) = \lim_{x \to -2} \frac{x+2}{(x)^3 + (2)^3}$

$$= \lim_{x \to -2} \frac{x+2}{(x+2)\left(x^2 - 2x + 4\right)}$$

$$= \frac{1}{(-2)^2 - 2(-2) + 4} = \frac{1}{12}$$

76. (d) By observing the options

Let $f(t) = t^k$

Suppose t = xy

$\therefore$ $f(xy) = (xy)^k = x^k . y^k = f(x).f(y)$

Hence, $f(t) = t^k$ where 'k' is a constant.

77. (b) Since $\frac{x}{|x|}$ is not continuous function $\therefore$ it is not differentiable also.

Also, L.H.D. and R.H.D. at x = 0 not equal.

Thus, only function given in option 'b' gives differentiability for all real values of x.

78. (b) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

$$= \lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1} = \lim_{x \to 0} \frac{1+x-1}{x\left[\sqrt{1+x} + 1\right]}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{1 \quad x} \quad 1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

79. (d) $\lim_{x \to 0} \frac{2(1 - \cos x)}{x^2} = \lim_{x \to 0} \frac{2.2 \sin^2 \frac{x}{2}}{x^2}$

$$= 4 \lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2} \times 2} . \lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2} \times 2}$$

$$= \lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} . \lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} = 1 \times 1 = 1$$

80. (b) $\lim_{x \to 0} \frac{1}{x} \quad \frac{1}{0} \quad \infty$ which does not exist.

Hence, statement-1 is incorrect.

Now, $\lim_{x \to 0} e^{1/x} \quad e^{\infty}$ which also does not exist.

Hence, statement-2 is correct.

81. (b) $f \; x \quad \begin{cases} \frac{x^2}{x}, & x \neq 0 \\ 0 & x \quad 0 \end{cases}$

$$= \begin{cases} \frac{x^2}{x} \quad x, & x \quad 0 \\ 0, & x \quad 0 \\ \frac{x^2}{-x} = -x, & x \quad 0 \end{cases}$$

Now, $\lim_{x \to 0^-} f \; x = \lim_{x \to 0} \; -x \quad 0$

$\lim_{x \to 0} f \; x \quad \lim_{x \to 0} x \quad 0$

and f(0) = 0

So, f(x) is continuous at x = 0

Also, f(x) is continuous for all other values of x.

Hence, f(x) is continuous everywhere.

82. (b) Let $\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{x-2}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{1}{x \quad 2} \quad \frac{1}{2 \quad 2} \quad \frac{1}{4}$

83. (b) Let $y = f^{-1}(x) = \frac{x+5}{3}$

$\Rightarrow 3y - 5 = x$
Now, $y = f^{-1}(x)$
$\Rightarrow x = f(y)$
$\Rightarrow x = 3y - 5$
Hence, $f(x) = 3x - 5$.

84. (a) (1) $f(x) = x^3$ and $g(y) = y^3$.
$\Rightarrow$ f = g is a correct statement.
(2) Identify function is always a bijection.

85. (c) Let $f : A \to A$ defined as $f(x) = x^2$.
Let $f(x) = y \therefore x = f^{-1}(y)$
Now $\Rightarrow x^2 = y$

$x = \sqrt{y}$

Thus, $f^{-1}(y) = \sqrt{y}$

Hence, the function has an inverse but f is not its own inverse.

86. (d) f(x) is continuous if $\lim_{x\to a} f(x)$ exists and equals f(a).
So, statement – 1 is not correct. Also, statement – 2 is incorrect.

87. (b) First we check continuity at x = 2

L. H. L = $\lim_{h\to 0} f(2-h) = \lim_{h\to 0} 3(2-h) - 2$

$= \lim_{h\to 0} 4 - 3h = 4$

R. H. L. = $\lim_{h\to 0} f(2+h) = \lim_{h\to 0} (2+h)^2 = 4$

Also, $f(2) = (2)^2 = 4$
Since, L. H. L = R. H. L = f(2)
$\therefore$ f(x) is continuous at 2.
Now, we check for differentiability

L. H. D at x = 2	R. H. D at x = 2
$f(x) = 3x - 2$	$f(x) = x^2$.
$f'(x) = 3$	$f'(x) = 2x$
$f'(x)\big\|_{x=2} = 3$	$f'(x)\big\|_{x=2} = 4$

Since L. H. D $\neq$ R. H. D
$\therefore$ f(x) is not derivable at x = 2

88. (c) Since $\sin\frac{1}{x}$ is an oscillatory function

$\therefore \lim_{x\to 0} \sin\frac{1}{x}$ has a finite value between – 1 and 1.

Now, At x = 0

L. H. L = $\lim_{h\to 0} f(0-h) = \lim_{h\to 0} (-h)\sin\left(-\frac{1}{h}\right)$

= 0 × a finite value between – 1 and 1
= 0

Similarly R. H. L = $\lim_{h\to 0} f(0+h) = 0$

Also, f(0) = 0

$\therefore \lim_{x\to 0} x\sin\frac{1}{x}$ exists.

Hence Both statements are correct.

89. (a) Consider $\lim_{x\to 0} \frac{\sin x - \tan x}{x}$

$= \lim_{x\to 0} \frac{\cos x - \sec^2 x}{1}$ (By L′ Hospital Rule)

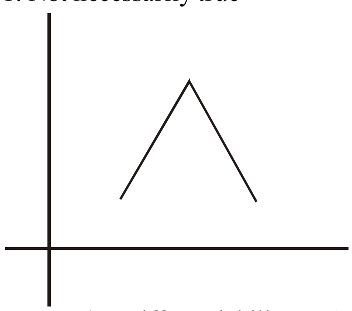
$= \frac{\cos 0 - \sec^2 0}{1} = \frac{1-1}{1} = 0.$

90. (b) Consider $\lim_{x\to 0} \frac{1-\sqrt{1+x}}{x}$

$= \lim_{x\to 0} \frac{1-(1+x)}{x(1+\sqrt{1+x})}$ (By Rationalizing)

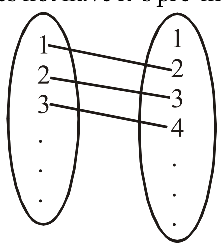
$= \lim_{x\to 0} \frac{-x}{x(1+\sqrt{1+x})} = \frac{-1}{2}$

91. (b) 1. Not necessarily true



2. True ($\because$ Differentiability $\Rightarrow$ Continuity)

92. (b) To show f is one-one.
Let f(x) = f(y) (To show: x = y)
$\Rightarrow x + 1 = y + 1$
$\Rightarrow x = y$
Hence, f is one-one,
Now, 'f ' is not onto because every element of co-domain does not have it's pre-image in domain.



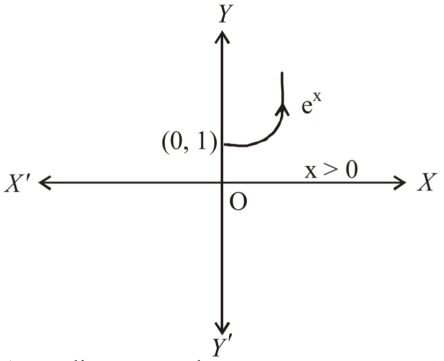
93. (c) 1 2
2 4
3 6
4 8
. .
. .
. .
. .
. .
Domain (x) Co-domain (2x)

For each element in the domain there is only one element in the therefore f(x) is one–one.

No image in the co-domain which has no pre-image in the domain, therefore f(x) is onto

Hence f(x) both is one-one and onto.

94. (c) $f(x) = e^x$, where $x > 0$



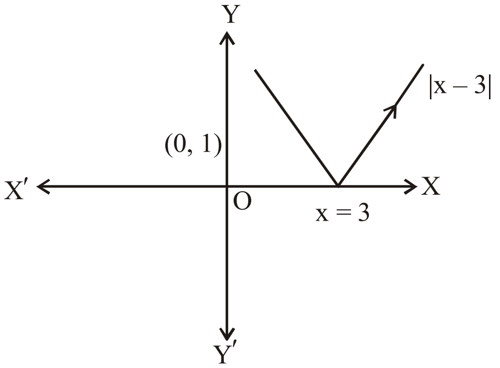
According to graph,

Graph of e^x is not breaking when $x > 0$

Therefore, graph is continuous at $x > 0$

Statement II

$g(x) = |x-3|$



Graph of $|x-3|$ is not breaking but have sharp turn at $x = 3$.

So, it is continuous

95. (d) $\lim\limits_{x\to 2}\frac{2-x}{x^3-8} = \lim\limits_{x\to 2}\frac{2-x}{(x-2)(x^2 \quad 2x \quad 4)}$

$= \lim\limits_{x\to 2}\frac{-1}{x^2 \quad 2x \quad 4}$

Putting $x = 2$, we get

$\frac{-1}{2^2 \quad 4 \quad 4} = -\frac{1}{12}$

96. (a) $f: R \to R, f(x) = \begin{cases} x^2 & , \; x \ge 0 \\ -x & , \; x \quad 0 \end{cases}$

For continuity at $x = 0$

$f(0-0) = \lim\limits_{h\to 0} f(0-h)$

$= \lim\limits_{h\to 0}[(0-h)] \quad \lim\limits_{h\to 0} h \quad 0$

$f(0+0) = \lim\limits_{h\to 0} f(0 \quad h) \quad \lim\limits_{h\to 0}(0 \quad h)^2 \quad 0$ and $f(0) = 0$

f(x) is continuous at $x = 0$

For differentiability at $x = 0$

$\lim\limits_{h\to 0}\frac{-(-h)-0}{-h} = \lim\limits_{h\to 0} = \frac{h}{-h} = -1$

and $\lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{h} \quad \lim\limits_{h\to 0} h \quad 0$

f(x) is not differentiable at $x - 0$

97. (a) $\lim\limits_{x\to 0}\frac{1-\cos x}{x} \quad \lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x}$

$= 2\left[\lim\limits_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}\times 2}.\lim\limits_{x\to 0}\sin\frac{x}{2}\right]$

$= 2\left[\frac{1}{2}\times 0\right] = 0$

98. (c) $\lim\limits_{x\to 0}\frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

99. (c) $\lim\limits_{x\to 0}\frac{\sin 2x \quad 4x}{2x \quad \sin 4x} = \lim\limits_{x\to 0}\frac{\frac{\sin 2x}{x} \quad 4}{2 \quad \frac{\sin 4x}{x}}$

$= \lim\limits_{x\to 0}\frac{2\left(\frac{\sin 2x}{2x}\right) \quad 4}{2 \quad 4\left(\frac{\sin 4x}{4x}\right)}$

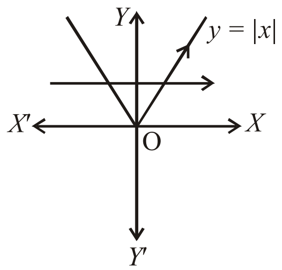
Applying limit, we get $\frac{2 \quad 4}{2 \quad 4} = 1$

100. (b) $f: Z \to N$ and $f(x) = |x|$

When we draw a parallel line to x-axis.

It cuts the curve into more than one point.

Therefore, $f(x) = |x|$ is not one-one. but onto



101. (c) $\lim_{x\to 0}\dfrac{(1+x)^n-1}{x}$

$= \lim_{x\to 0}\dfrac{{}^nC_0+{}^nC_1x+{}^nC_2x^2+....+{}^nC_nx^n-1}{x}$

$= \lim_{x\to 0}\dfrac{x({}^nC_1+{}^nC_2x+....+{}^nC_nx^{n-1})}{x}$

$= \lim_{x\to 0}{}^nC_1+{}^nC_2x+....{}^nC_nx^{n-1}$

Put $x=0 \Rightarrow {}^nC_1=n$

102. (d) $\lim_{x\to 0}\dfrac{x}{\sqrt{1-\cos x}} = \lim_{x\to 0}\dfrac{x}{\sqrt{1-\left(1-2\sin^2\frac{x}{2}\right)}}$

$= \lim_{x\to 0}\dfrac{x}{\sqrt{2\sin^2\frac{x}{2}}} = \dfrac{1}{\sqrt{2}}\lim_{x\to 0}\dfrac{x}{\left|\sin\frac{x}{2}\right|}$

$\text{L.H.L} = f(0-0) = \lim_{h\to 0}\dfrac{x}{\left|\sin\frac{x}{2}\right|}$

$= -\dfrac{1}{\sqrt{2}}\lim_{x\to 0}\dfrac{2\left(\frac{h}{2}\right)}{\sin\frac{h}{2}}$

$= -\dfrac{1}{\sqrt{2}}\times 2\times 1 \qquad \left(\because \lim_{\theta\to 0}\dfrac{\theta}{\sin\theta}=1\right)$

$= -\sqrt{2}$

$\text{RHL} = f(0+0) = \lim_{h\to 0}f(0\quad h)$

$= \dfrac{1}{\sqrt{2}}\lim_{h\to 0}\dfrac{2\left(\frac{h}{2}\right)}{\sin\frac{h}{2}} = \dfrac{1}{\sqrt{2}}\times 2\times 1$

$= \text{LHL} \neq \text{RHL} = \sqrt{2}$

Therefore limit does not exist.

103. (d) $\lim_{x\to\frac{\pi}{2}}f(x) = \lim_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{(\pi-2x)^2}$

$= \lim_{x\to\frac{\pi}{2}}\dfrac{-\cos x}{(\pi-2x)(-2)}$

$= \lim_{x\to\frac{\pi}{2}}\dfrac{\cos x}{4(\pi-2x)}$

and $= \lim_{x\to\frac{\pi}{2}}\dfrac{-\sin x}{4(-2)} = \lim_{x\to\frac{\pi}{2}}\dfrac{\sin x}{8}$

$= \dfrac{1}{8}.\sin\dfrac{\pi}{2} = \dfrac{1}{8}\times 1 \quad \dfrac{1}{8}$

104. (a) Function is continuous at $x = \dfrac{\pi}{2}$

$f\left(\dfrac{\pi}{2}\right) = \lim_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{(\pi-2x)^2} = \dfrac{1}{8}$

105. (c) $\lim_{x\to 0}\dfrac{\sqrt{f(x)}-3}{\sqrt{x}-3} = \lim_{x\to 0}\dfrac{\frac{1}{2\sqrt{f(x)}}f'(x)}{\frac{1}{2\sqrt{x}}.1}$

(By L' Hospital rule)

$= \lim_{x\to 0}\dfrac{f'(x)\times\sqrt{x}}{\sqrt{f(x)}} = \dfrac{f'(9)\times\sqrt{9}}{\sqrt{f(9)}}$

$= \dfrac{4\times 3}{\sqrt{9}} = \dfrac{4\times 3}{3} = 4$

106. (b) LHL $f(2.99-0) = \lim_{h\to 0}(2.99-h)$

$\lim_{h\to 0}(2.99-h) = \lim_{h\to 0}2 = 2$

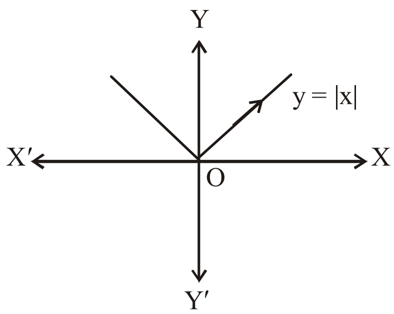
RHL $f(2.99-0) = \lim_{h\to 0}f(2.99+h)$

$= \lim_{h\to 0}(2.99+h) = \lim_{h\to 0}2 = 2$

LHL = RHL

$\therefore$ f(x) is continous at x = 2.99

107. (b) **Statement 1 :** $f(x) = |x|$



From the graph, the curve has sharp turn at x = 0. Therefore, the function $f(x) = |x|$ is not differentiable only x = 0, it is differentiable at x = 1

Statement 2 : $f(x) = e^x$

$Rf'(0) = \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\dfrac{e^{(0+h)}-e^{\circ}}{h} = \lim_{h\to 0}\dfrac{e^h-1}{h}$

Use L'Hospital rule

$= \lim_{h\to 0}\dfrac{e^h-0}{1} = e^{o} = 1$

$f'(0) = \lim_{h\to 0} \frac{1-e^{-h}}{h}$

Use L' Hospital rule

$= \lim_{h\to 0} \frac{e^{-h}}{1} = e^{-0} = 1$

Therefore $f(x) = e^x$ is differentiable at x = 1.

108. (b) **Statement 1 :** Given $f(x) = \begin{cases} 2-x & \text{for } 1 \le x \le 2 \\ 3x - x^2 & \text{for } x > 2 \end{cases}$

function defined in $1 \le x < \infty$

the function is polynomial, so it is continous and differentiable in its domain $[1, \infty] - \{2\}$

LHL $f(2-0) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0} h = 0$

RHL $f(2+0) = \lim_{h\to 0} (2+h)$

$= 6 - 4 = 2$

$f(2) = 2 - 2 = 0 \therefore$ LHL $\ne$ RHL

Statement 2 :

$Rf''(1.5) = \lim_{h\to 0} \frac{f(1.5+h) - f(1.5)}{h}$

$= \lim_{h\to 0} \frac{(2-1.5+h)-(2-1.5)}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$

$Lf'(1.5) = \lim_{h\to 0} \frac{f(1.5-h)-f(1.5)}{-h}$

$= \lim_{h\to 0} \frac{(2-1.5-h)-(2-1.5)}{-h}$

$= \lim_{h\to 0} \frac{h}{-h} = -1$

Therefore, the function is differentiable at x = 1.5

109. (d) $f'(x) = \begin{cases} -1 & \text{for } 1 \le x \le 2 \\ 3-2x & \text{for } x > 2 \end{cases}$

f (x) at x = 3

$f'(3) = 3 - 2(3) = 3 - 6 = -3$

110. (a) $f'(2+0) = \lim_{h\to 0} f'(2+h)$

$= \lim_{h\to 0} 3 - 4 - 2h = -1$

$f'(2-0) = \lim_{h\to 0} f'(2-h) = (3 - 4 - 2h) = -1$

So, $f'(x)$ exist at x = 2

111. (b) Given $f : N \to N$

$\therefore f(x) = 2x + 3 \Rightarrow f'(x) = 2 > 0$

So, f (x) is increasing, $\forall x \in N$.

Hence, f (x) is injective.

Let f(x) = y

$\Rightarrow y = 2x + 3 \Rightarrow x = \frac{y-3}{2}$

This is injective

$\therefore x = \frac{1}{2}$

and $y \in N$ but $x \notin N$

Hence, f(x) is not surjective.

112. (d) f(x) = ax + b and g(x) = cx + d

$f[(g(x))] = a(cx+d) + b$

$= acx + ad + b$

and $g[f(x)] = c(ax+b) + d$

$= acx + dc + d$

Now from $f[(gx)] = g[f(d)]$

$\Rightarrow ad + b = bc + d$

$\Rightarrow f(d) = g(b)$

113. (a) Given $f(x) = \frac{x-1}{x+1}$

Applying componendo and dividendo, we get

$\frac{f(x)+1}{f(x)-1} = \frac{x-1+x+1}{x-1-x-1}$

$\Rightarrow \frac{f(x)+1}{f(x)-1} = -x$

Now, $\frac{f(x)+1}{f(x)-1} + x = -x + x = 0$

114. (c) $f(x) = \frac{x-1}{x+1} \Rightarrow x = \frac{1+f(x)}{1-f(x)}$

$\Rightarrow f(2x) = \frac{2x-1}{2x+1} \Rightarrow f(2x) = \frac{2\left[\frac{[1+f(x)]}{1-f(x)}\right]-1}{2\left[\frac{[1+f(x)]}{1-f(x)}\right]+1}$

$\Rightarrow f(2x) = \frac{3f(x)+1}{f(x)+3}$

115. (c) Given $f(x) = \frac{x-1}{x+1}$

$\Rightarrow f[f(x)] = \frac{f(x)-1}{f(x)+1}$

$\Rightarrow f[f(x)] = -\frac{1}{x}$ $\left[\because x = -\left\{\frac{f(x)+1}{-f(x)-1}\right\}\right]$

116. (b) $f(x)=\begin{cases} x^2-5 & , \quad x \le 3 \\ \sqrt{x \quad 13} & , \quad x \quad 3 \end{cases}$

To find $\lim\limits_{x\to 3} f(x)$

$\text{LHL} = \lim\limits_{x\to 3^-} f(x)$

$= \lim\limits_{x\to 3^-} (x^2-5) = \lim\limits_{x\to(3-h)} [(3-h)^2-5]$

$= \lim\limits_{h\to 0} (9-6h+h^2-5) = 4$

$\text{RHL} = \lim\limits_{x\to 3^+} (\sqrt{x+13})$

$= \lim\limits_{x\to(3 \quad h)} (\sqrt{3 \quad h \quad 13}) = \lim\limits_{h\to 0} (\sqrt{16 \quad h}) = 4$

$\because \lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3} f(x) = 4$

$\therefore \lim\limits_{x\to 3} f(x) = 4$

117. (d) 1. $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3} f(x) = f(3)$

$\therefore \lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3} f(x) = 4$

Therefore f(x) is continuous at x = 4

2. Given $f(x) = x^2 - 5 \; \forall \; x \le 3$

$f'(x) = 2x$

$f'(0) = 0$

So, f (x) is differentiable at x = 0

Therefore, neither statement 1 nor 2 is correct.

118. (d) Given, $f(x) = \sqrt{x \quad 13}, x > 3$

$f'(x) = \dfrac{1}{2\sqrt{x \quad 13}}$

$\Rightarrow f'(12) = \dfrac{1}{2\sqrt{12 \quad 13}} = \dfrac{1}{2\times 5} \quad \dfrac{1}{10}$

119. (b) Given, $f(x)=\begin{cases} \dfrac{\tan kx}{x}, & x \quad 0 \\ 3x+2k^2, & x \ge 0 \end{cases}$

When function is continuous at x = 0, then

$\lim\limits_{x-0^-} f(x) = \lim\limits_{x\to 0} f(x) = f(0)$

$\therefore \lim\limits_{x\to 0-h} \left(\dfrac{\tan kx}{x}\right) = \lim\limits_{x\to 0 \quad h} (3x \quad 2k^2) = 3(0) + 2k^2$

$\Rightarrow \lim\limits_{x\to 0-h} \left[\dfrac{\tan k(0-h)}{(0-h)}\right] = \lim\limits_{x\to 0+h} [3(0+h)+2k^2] = 2k^2$

Therefore, $\lim\limits_{h\to 0} \left(\dfrac{\tan kh}{h}\right) = 2k^2$

$k = 2k^2 \qquad \left[\because \lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1\right]$

Hence, $k = \dfrac{1}{2}$

120. (b) The greatest integer function is continuous at all statement points except integer. Hence, statement 1is incorrect.

Statement 2 : Let h(a) = sin x and g(x) = |x|

hog (x) = sin |x|

$\Rightarrow$ f(x) = hog (x) = sin |x|

Therefore, g(x) is continuous, $\forall$ x $\in$ R and

h (x) is continous $\forall$ x $\in$ R

When both are continuous then hog (x) is also continuous.

Thus, statement 2 is correct.

121. (b) Given equation $\lim\limits_{x\to 0} \dfrac{\log_5(1 \quad x)}{x}$

$= \lim\limits_{x\to 0} \dfrac{\log_e(1+x)}{x \log_e 5} \qquad \left[\because \log_x y \quad \dfrac{\log_e y}{\log_e x}\right]$

$= \dfrac{1}{\log_e 5} \lim\limits_{x\to 0} \dfrac{\log_e(1 \quad x)}{x} = \log_5 e$

$\left[\because \lim\limits_{x\to 0} \dfrac{\log_e(1+x)}{x} = 1, \log_x y = \dfrac{1}{\log_y x}\right]$

122. (a) $\because \lim\limits_{x\to 0} \dfrac{a^x-1}{x} = \text{loe}_e a$

$\therefore \lim\limits_{x\to 0} \dfrac{5^x-1}{x} = \log_e 5$

Hence, option (a) is correct

123. (d) $\lim\limits_{n\to\infty} \dfrac{1+2+3+...+n}{1^2+2^2+3^2+....+n^2} = \lim\limits_{n\to\infty} \dfrac{\dfrac{n(n \quad 1)}{2}}{\dfrac{n(n \quad 1)(2n \quad 1)}{6}}$

$\therefore \lim\limits_{n\to\infty} \dfrac{3}{2n+1} = 0$

Note : $1+2+3+....+n = \dfrac{n(n+1)}{2}$

$1^2+2^2+3^2+....+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Sol. (124-125):

$$\lim_{x\to\infty}\left(\frac{2+x^2}{1+x}-Ax-B\right)=3$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{2+x^2-Ax-B-Ax^2-Bx}{1+x}\right)=3$$

$$\Rightarrow \lim_{x\to\infty}\left[\frac{(1-A)x^2-(A+B)x+2-B}{1+x}\right]=3$$

Applying L'Hospital rule,

$2x(1-A)-(A+B)=3$

Comparing coefficients

$2(1-A)=0$

$\therefore$ $A=1$ and

$-(A+B)=3$

$\therefore$ $B=-3-1=-4$

$A+B=-3$

$A=1, B=-4$

124. (b)

125. (c)

126. (a) $G(x)=\sqrt{25-x^2}$

Now,

$$\lim_{x\to1}\frac{G(x)-G(1)}{x-1}$$

$$\lim_{x\to1}\frac{\sqrt{25-x^2}-\sqrt{24}}{x-1} \qquad (\div \text{ form})$$

Applying L'Hospital rule,

$$\lim_{x\to1}\frac{(-2x)}{2\sqrt{25-x^2}}=\frac{-2}{2\sqrt{25-x^2}}$$

$$=\frac{-2}{2\sqrt{24}}=\frac{-1}{\sqrt{24}}=\frac{-1}{2\sqrt{6}}$$

$\therefore$ Option (a) is correct.

127. (c) From statement-1

From the definition of greatest integer function

$f(x)=[x]$ is discontinuous at $x=n$ for any value of $n\in Z$

$\therefore$ Statement1 is correct

From statement-2

$f(x)=\cot x$

for $x=\pi=\cot\pi=-\infty$

for $x=2\pi$

$f(2\pi)=\cot 2\pi=-\infty$

Hence the function $f(x)=\cot x$ is discontinuous at $x=n\pi$ where $n\in Z$.

$\therefore$ Option (c) is correct.

Sol. (129-130):

$$F(x)=\begin{cases}-1, & x\le 0\\ ax+b, & 0<x<1\\ 1, & x\ge 1\end{cases}$$

At $x=0$,

L.H.L. = R.H.L.

$$\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)$$

$$-1=\lim_{x\to0}(ax+b)$$

$-1=b$

Since the given function is also continuous at $x=1$

L.H.L. = R.H.L.

$$\lim_{x\to1^-}f(x)=\lim_{x\to1^+}f(x)$$

$a+b=1$

$a-1=1$

$a=1+1=2.$

129. (d) Function is continuous at $x=0$

$$\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)$$

$-1=a(0)+b\Rightarrow b=-1$

Now, f(x) is also continuous at $x=1$

$$\lim_{x\to1^-}f(x)=\lim_{x\to1^+}f(x)$$

$a(1)+b=1$

$a=1-b=1+1$

$a=2.$

130. (a) From solution 82, $b=-1$.

132. (a) $f(x)=\begin{cases}\dfrac{\alpha\cos x}{\pi-2x} & \text{if } x\ne\dfrac{\pi}{2}\\ 3 & \text{if } x=\dfrac{\pi}{2}\end{cases}$

For continuity at $x=\frac{\pi}{2}$

$$\text{L.H.L.}=\lim_{x\to\frac{\pi^-}{2}}f(x)=\lim_{x\to\frac{\pi^-}{2}}\frac{\alpha\cos x}{\pi-2x}$$

Put $x=\frac{\pi}{2}-h$ where $x\to\frac{\pi}{2}$, then $h\to0$

$$\therefore \text{L.H.L.}=\lim_{h\to0}\frac{\alpha\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}=\lim_{h\to0}\frac{\alpha\sin h}{\pi-\pi+2h}$$

$\left[\because \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta\right]$

$= \lim_{h\to 0}\frac{2\alpha\sin h}{2h} = \frac{\alpha}{2}\lim_{h\to 0}\frac{\sin h}{h}$

$= \frac{\alpha}{2}\cdot 1 = \frac{\alpha}{2}$ $\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$

and R.H.L. $= \lim_{x\to\frac{\pi^+}{2}} f(x)$ $\lim_{x\to\frac{\pi^+}{2}}\frac{\alpha\cos x}{\pi-2\pi}$

Put $x = \frac{\pi}{2}+h$ when $x\to\frac{\pi}{2}$, then $h\to 0$

$\therefore$ R.H.L. $= \lim_{h\to 0}\frac{\alpha\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)} = \lim_{h\to 0}\frac{\alpha(-\sin h)}{\pi-\pi-2h}$

$= \lim_{h\to 0} -\frac{\alpha\sin h}{-2h}$ $\left[\because \cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta\right]$

$= \frac{\alpha}{2}\lim_{h\to 0}\frac{\sin h}{h} = \frac{\alpha}{2}$ $\left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right)$

Also, $f\left(\frac{\pi}{2}\right)=3$

Since, f(x) is continuous at $x = \frac{\pi}{2}$

$\therefore$ L.H.L. = R.H.L. $= f\left(\frac{\pi}{2}\right)$

$\frac{\alpha}{2}=\frac{\alpha}{2}=3$

$\alpha = 6$

Hence, for $\alpha = 6$, the given function (f) is continuous at $x = \frac{\pi}{2}$.

133. (d) $\lim_{x\to 0} f(x) = \frac{\alpha\cos x}{\pi-2x}$

$= \frac{\alpha(\cos 0)}{\pi-2(0)} = \frac{6}{\pi}$

$\therefore$ Option (d) is correct.

134. (b) f(x) = x

g(x) = 1/x

Putting these values in the options, only (b) is correct.

135. (d) $f(x)=\sqrt{25-x^2}$

$f(1)=\sqrt{24}$

$\Rightarrow \lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$

$\therefore$ It is $\frac{0}{0}$ (undefined condition) so using L'hospital's rule

$\Rightarrow \lim_{x\to 1}\frac{f'(x)-0}{1} = \lim_{x\to 1}\left(\sqrt{25-x^2}\right)'$

$\Rightarrow \lim_{x\to 1}\frac{1}{2}\times\frac{1}{\sqrt{25-x^2}}(-2x)$

$= \frac{1}{2}\times\frac{1}{\sqrt{25-(1)^2}}\times(-2)$

$= -\frac{1}{\sqrt{24}}$

136. (a) $f(x)=\begin{cases} ax-2 & -2<x<-1 \\ -1 & -1\le x\le 1 \\ a+2(x-1)^2 & 1<x<2 \end{cases}$

if f(x) is continuous at x = – 1

then, $\lim_{x\to -1}(ax-2) = \lim_{x\to -1}(-1)$

$\Rightarrow a(-1)-2=-1$

$\Rightarrow \boxed{a=-1}$

if f(x) is continuous at x = 1

then, $\lim_{x\to 1} a+2(x-1)^2 = \lim_{x\to 1} -1$

$\Rightarrow a+2(1-1)^2=-1$

$\Rightarrow \boxed{a=-1}$

137. (c) $\lim_{x\to\pi}\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$

Using L'hospital's rule

$\Rightarrow \lim_{x\to\pi}\frac{-\cos x-\sin x}{\cos x-\sin x}$

$\Rightarrow \frac{-\cos\pi-\sin\pi}{\cos\pi-\sin\pi}$

$\Rightarrow \frac{-(-1)-0}{-1-0}$

$\Rightarrow -1$

138. (b) $f(x)=\begin{cases}\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$

as there is a discontinuity at x = 0, so function is not differentiable at x = 0

$f(x)=\begin{cases}2x+5 & x>0\\ x^2+2x+5 & x\leq 0\end{cases}$

f(0) = 5

$$\text{LHD}=\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{x\to 0}\frac{x^2+2x+5-5}{x}=\lim_{x\to 0}x+2=2$$

$$\text{RHD}=\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{x\to 0}\frac{2x+5-5}{x}=2$$

∴ It is differentiable at x = 0
∴ Only (2) is differentiable at x = 0

139. (b) $f(x)=\frac{1}{\sqrt{|x|-x}}$

$|x|-x\neq 0$

So $\boxed{x<0}$ ...(1)

$x=(-\infty,0)$

again $|x|-x>0$

$|x|>x$
it is possible only when x is negative.

$\boxed{x=(-\infty,0)}$ ...(2)

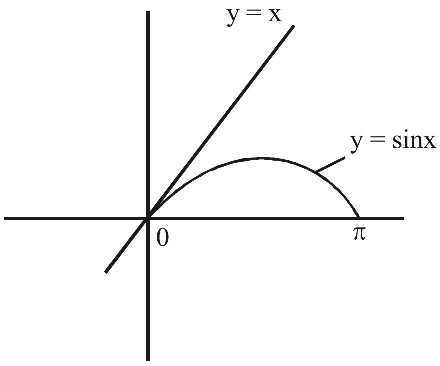
So from eq. (1) & eq. (2)

domain $x=(-\infty,0)$

140. (c) $f(x)=x^2+2\cos x$

$f'(x)=2x-2\sin x$

$=2[x-\sin x]$



Between (0, π); (x – sinx) is always +ve, so f′(x) is always +ve. Hence it is increasing.

$f(x)=\ln\left(\sqrt{1+x^2}-x\right)$

$$f'(x)=\frac{1}{\sqrt{1+x^2}-x}\times\left[\frac{2x}{2\sqrt{(1+x^2)}}-1\right]$$

$$f'(x)=\frac{1}{\sqrt{1+x^2}-x}\times\left(\frac{x}{\sqrt{1+x^2}}-1\right)$$

As f′(x) is –ve always, so this function is decreasing always.

141. (b) $f(x)=2x-3$
$g(x)=x^3+5$
$fog(x)=f[g(x)]=f(x^3+5)$
$=2(x^3+5)-3$
$fog(x)=2x^3+7=y(\text{say})$
$\Rightarrow\ 2x^3=y-7$

$$\Rightarrow\ x=\left[\frac{y-7}{2}\right]^{\frac{1}{3}}$$

$$\Rightarrow\ fog^{-1}(y)=\left[\frac{y-7}{2}\right]^{\frac{1}{3}}$$

$$\Rightarrow\ fog^{-1}(x)=\left[\frac{x-7}{2}\right]^{\frac{1}{3}}$$

142. (d) $f(x)=\frac{\sin\left(e^{x-2}-1\right)}{\ln(x-1)}$

$$\lim_{x\to 2}\frac{\sin\left(e^{x-2}-1\right)}{\ln(x-1)}=L$$

It is $\frac{0}{0}$ (undefined) condition so using L'hospital's rule

$$\Rightarrow L=\lim_{x\to 2}\left[\frac{\left\{\sin\left(e^{x-2}-1\right)\right\}^-}{\left\{\ln(x-1)\right\}^-}\right]$$

$$\Rightarrow L=\lim_{x\to 2}\frac{\cos\left(e^{x-2}-1\right).e^{(x-2)}}{1/(x-1)}$$

$$\Rightarrow L=\lim_{x\to 2}\cos\left(e^{2-2}-1\right)e^{2-2}.(2-1)$$

$\Rightarrow L=\cos(0)\,e^0.1$
$\Rightarrow L=1$

143. (a) $f(x) = x^3$

$f(x_1) = x_1^3$

$f(x_2) = x_2^3$

if $x_1 = x_2$
then $f(x_1) = f(x_2)$
So it is one-one function
Hence option (a) is correct.

144. (c) $\because f(x) \begin{cases} -2\sin x & \text{if} \quad x \le -\pi/2 \\ A\sin x \quad B & \text{if} \quad -\pi/2 \quad x \quad \pi/2 \\ \cos x & \text{if} \quad x \ge \pi/2 \end{cases}$

$\because$ f(x) is continuous every where :

$\therefore \lim\limits_{x \to \pi/2} f(x) \quad \lim\limits_{x \to \pi/2^-} f(x)$

$\Rightarrow \lim\limits_{x \to \pi/2} \cos x \quad \lim\limits_{x \to \pi/2} A\sin x \quad B$

$\Rightarrow A + B = 0$ ... (1)

Also; $\lim\limits_{x \to -\pi/2} f(x) \quad \lim\limits_{x \to -\pi/2^-} f(x)$

$\Rightarrow \lim\limits_{x \to -\pi/2} A\sin x \quad B \quad \lim\limits_{x \to -\pi/2} -2\sin x$

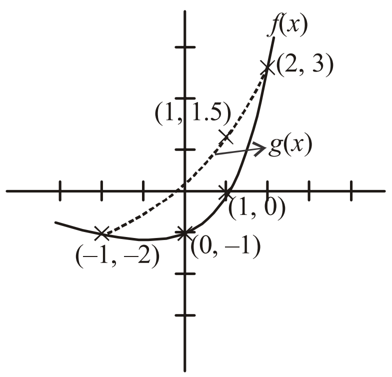
$\Rightarrow -A + B = 2$ ... (2)

is from eq (1) & (2) we get,
$A = -1$

145. (a) Put the value of A in eq. (2) we get :
$\Rightarrow B = 1$

146. (c) $f(x) = x|x| - 1$

$$f(x) = \begin{cases} x^2 - 1 & x > 0 \\ -x^2 - 1 & x \le 0 \end{cases} \qquad g(x) = \begin{cases} \dfrac{3x}{2} & x > 0 \\ 2x & x \le 0 \end{cases}$$



Hence $f(x)$ and $g(x)$ intersects at (–1, –2) and (2, 3).

147. (b) Area $= \int_{-1}^{2} [g(x) - f(x)]\,dx$

$$= \int_{-1}^{0} [g(x) - f(x)]\,dx + \int_{0}^{2} [g(x) - f(x)]\,dx$$

$$= \int_{-1}^{0} (2x + x^2 + 1)\,dx + \int_{0}^{2} \left[\frac{3x}{2} - x^2 + 1\right] dx$$

$$= I_1 + I_2$$

$$I_1 = \left(\frac{2x^2}{2} + \frac{1}{3}x^3 + x\right)_{-1}^{0}$$

$$= (0 + 0 + 0) - \left((-1)^2 + \frac{1}{3}(-1)^3 + (-1)\right)$$

$$= -\left(1 - \frac{1}{3} - 1\right) = \frac{1}{3}$$

$$I_2 = \int_{0}^{2} \left[\frac{3x}{2} - x^2 + 1\right] dx$$

$$= \left[\frac{3}{2}\frac{x^2}{2} - \frac{1}{3}x^3 + x\right]_0^2$$

$$= \left(\frac{3}{4} \times 4 - \frac{1}{3} \times 8 + 2\right) - (0)$$

$$= 5 - \frac{8}{3} = \frac{7}{3}$$

Area $= I_1 + I_2 = \frac{1}{3} + \frac{7}{3}$

$\boxed{\text{Area} = \frac{8}{3}}$ square units.

148. (b) $f(x) = |x - 1| + x^2 \quad \forall \quad x \in R$

$f_1(x) = |x - 1|, \; f_2(x) = x^2$

$f_1(x)$ and $f_2(x)$ both are continuous. Hence $f(x)$ is continuous.
$f(x)$ in differentiable at $x = 0$
$f_1(x)$ is not differentiable at $x = 1$.
Hence $f(x)$ is continuous but not differentiable at $x = 1$.

149. (b) As we know,

$f(x) = |x - 1| + x^2 \quad \forall \; x \in R$

$$f(x) = \begin{cases} x - 1 + x^2 & x \ge 1 \\ 1 - x + x^2 & x < 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2x + 1 \; ; & x \ge 1 \\ 2x - 1 \; ; & x \quad 1 \end{cases}$$

$f(x)$ is in quadratic form (parabola). Hence $f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing $\left(\frac{1}{2}, \infty\right)$.

150. (c) $f(x)$ has local minimum at one point only in $(-\infty, \infty)$.

$$f'(x) = \begin{cases} 2x-1 & ; \quad x<1 \\ 2x+1 & ; \quad x \ge 1 \end{cases}$$

Clearly; for $(x > 1)$; $f'(x) > 0 \ge$ & for $(x < 1)$

$x = \frac{1}{2}$ is the point of local minima

151. (a) $$f(x) = \begin{cases} x^2+x-1 & x \ge 1 \quad \forall \quad x \in R \\ x^2-x+1 & x<1 \end{cases}$$

Hence area required for given region is

$$A_1 = \int_{1/2}^{1} f(x)\, dx$$

$$= \int_{1/2}^{1} (x^2-x+1)\, dx$$

$$= \left[\frac{1}{3}x^3 - \frac{x^2}{2} + x\right]_{1/2}^{1}$$

$$= \left(\frac{1}{3}\times 1 - \frac{1}{2} + 1\right) - \left(\frac{1}{3}\times\frac{1}{8} - \frac{1}{2}\times\frac{1}{4} + \frac{1}{2}\right)$$

$$\boxed{A_1 = \frac{5}{12}}$$ square unit.

152. (d) Area required for given region is

$$A_2 = \int_{1}^{3/2} f(x)\, dx$$

$$= \int_{1}^{3/2} [x^2+x-1]\, dx$$

$$= \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 - x\right]_{1}^{3/2}$$

$$A_2 = \left(\frac{1}{3}\left(\frac{27}{8}\right) + \frac{1}{2}\left(\frac{9}{4}\right) - \frac{3}{2}\right) - \left(\frac{1}{3}(1) + \frac{1}{2}(1) - 1\right)$$

$$\boxed{A_2 = \frac{11}{12}}$$ square unit

153. (d) $x + |y| = 2y$

$x = 2y - |y|$

$2y - |y| = x$

$2y - y = x$ [for $y \ge 0$]

$y = x$

$2y + y = x$ [for $y < 0$]

$3y = x$

$y = \frac{1}{3}x$

$$y = \begin{cases} x & y \ge 0 \\ \frac{1}{3}x & y < 0 \end{cases}$$ function is defined for all value of x.

$$\text{or } y = \begin{cases} x & ; \quad x \ge 0 \\ \frac{1}{3}x & ; \quad x < 0 \end{cases}$$

$\therefore$ by checking

y as a function of x is continuous at $x = 0$, but not differentiable at $x = 0$.

So all of the statements are not correct.

154. (d) $y = \frac{1}{3}x$ for $x < 0$.

Hence $\frac{dy}{dx} = \frac{1}{3}$

Option (d) is correct.

155. (c) $f(x) = (x-1)^2(x+1)(x-2)^3$

$$f'(x) = 2(x-1)(x+1)(x-2)^3 + (x-1)^2(x-2)^3 + (x-1)^2(x+1)3(x-2)^2$$

$$= (x-1)(x-2)^2[2(x+1)(x-2) + (x-1)(x-2) + 3(x-1)(x+1)]$$

$$f'(x) = (x-1)(x-2)^2[2x^2-2x-4+x^2-3x+2+3x^2-3]$$

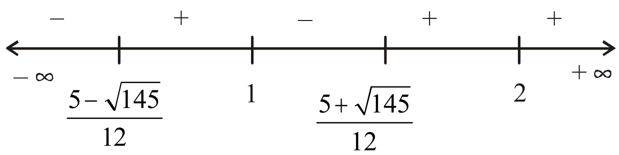
$$= (x-1)(x-2)^2[6x^2-5x-5]$$

For maxima and minima

$f'(x) = 0$

$(x-1)(x-2)^2[6x^2-5x-5] = 0$

$x = 1, 2, 2, \frac{5 \pm \sqrt{145}}{12}$ The change in signs of $f'(x)$ for diffrent values of x is shown:



$\therefore$ Local Minima are

$x = \frac{5-\sqrt{145}}{12}$ & $x = \frac{5+\sqrt{145}}{12}$

156. (b) Local maxima is $[x = 1]$

157. (b) Given $f(x) = \frac{a^{[x]+x} - 1}{[x]+x}$

$$\therefore \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{a^{[0+h]+(0+h)} - 1}{[0+h]+(0+h)}$$

$$\lim_{h\to 0}\frac{a^{[h]\ \ (h)}-1}{[h]\ \ h}$$

$$\lim_{h\to 0}\frac{(a^h-1)}{h}$$

$$=\log_e a$$

158. (c) $\lim_{x\to 0^-} f(x)\quad \lim_{h\to 0}\left[\frac{a^{[0-h]+(0-h)}-1}{[0-h]+(0-h)}\right]$

$$\lim_{h\to 0}\frac{a^{[-h]-h}-1}{[-h]+(-h)}$$

$$\lim_{h\to 0}\frac{a^{-1-h}-1}{-1-h}$$

$$\frac{a^{-1-0}-1}{-1-0}\quad \frac{a^{-1}-1}{-1}$$

$$\lim_{h\to 0^-} f(x)\quad (1-a^{-1})$$

159. (c) Given

$$f(x)\quad \begin{cases}(x+\pi) & \text{for } x\in[-\pi,0)\\ \pi\cos x & \text{for } x\in\left[0,\frac{\pi}{2}\right]\\ \left(x-\frac{\pi}{2}\right)^2 & \text{for } x\in\left(\frac{\pi}{2},\pi\right]\end{cases}$$

For continuity,
$f(a)$ = L.H.L. = R.H.L.
At $x = 0$
$f(0) = \pi\cos 0 = \pi$

L.H.L. = $\lim_{x\to 0^-} f(x-h)$

$$=\lim_{h\to 0} f(-h)$$

$$=\lim_{h\to 0}(-h+\pi)=\pi$$

R.H.L. $\lim_{x\to 0} f(x\quad h)$

$$=\lim_{h\to 0} f(0+h)$$

$$=\lim_{h\to 0}\pi\cos h$$

$$=\pi\cos 0=\pi$$

$f(0)$ = L.H.L. = R.H.L.
Hence function is continuous at $x = 0$.
Statement (1) is correct.

At $x=\frac{\pi}{2}$

L.H.L. = $\lim_{x\to\frac{\pi}{2}^-} f(x-h)$

$$=\lim_{h\to 0} f\left(\frac{\pi}{2}-h\right)$$

$$=\lim_{h\to 0}\pi\cos\left(\frac{\pi}{2}-h\right)$$

$$=\pi\cos\frac{\pi}{2}=0$$

RH.L. = $\lim_{x\to\frac{\pi}{2}^+} f(x+h)$

$$=\lim_{h\to 0} f\left(\frac{\pi}{2}+h\right)$$

$$=\lim_{h\to 0}\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)^2=0$$

$$f\left(\frac{\pi}{2}\right)=\pi\cos\frac{\pi}{2}=0$$

L.H.L. = R.H.L. = $f\left(\frac{\pi}{2}\right)$

Hence function is continuous at $x=\frac{\pi}{2}$

Statement (2) is correct.

160. (d) For differentiability,
L.H.D. = R.H.D.
Thus at $x = 0$

L.H.D. $=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$$=\lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$$

$$=\lim_{h\to 0}\frac{(-h+\pi)-(\pi\cos 0)}{-h}=1$$

L.H.D. = 1

R.H.D. $=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$$=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{\pi\cos h-\pi\cos 0}{h}$$

$$=\lim_{h\to 0}\frac{\pi[\cos h-1]}{h}$$

$$\lim_{h\to 0}\frac{\pi\left[1-\frac{h^2}{2!}+\frac{h^4}{4!}......-1\right]}{h}$$

$$=\lim_{h\to 0}\frac{\pi\left[1-\frac{1}{2}h^2+\frac{1}{24}h^4......-1\right]}{h}$$

$$= \lim_{h\to 0}\frac{\pi\left[-\frac{1}{2}h^2+\frac{1}{24}h^4......\right]}{h} = 0$$

L.H.D. $\neq$ R.H.D.
So at $x = 0$ function is not differentiable.
Statement (1) is not correct.

At $x = \frac{\pi}{2}$

$$f\left(\frac{\pi}{2}\right) = \pi\cos\frac{\pi}{2} = 0$$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{-h}$$

$$= \lim_{h\to 0}\frac{\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)^2-0}{h}$$

$$= \lim_{h\to 0}\frac{h^2}{h} = 0$$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}-h\right)-f\left(\frac{\pi}{2}\right)}{-h}$$

$$= \lim_{h\to 0}\frac{\pi\cos\left(\frac{\pi}{2}-h\right)-0}{-h}$$

$$= \lim_{h\to 0}\pi\left(\frac{\sin h}{-h}\right)$$

$$= -\pi\lim_{h\to 0}\left(\frac{\sin h}{h}\right) = -\pi(1) = -\pi$$

L.H.D. $\neq$ R.H.D.

Hence function is not differentiable at $x = \frac{\pi}{2}$.

Statement (2) is not correct.

Sol. (161-162) :

$f(x) \to$ greatest integer function
$f(x) = [x]$
$g(x) \to$ modulus function
$g(x) = |x|$

161. (c) $gof(x) = g(f(x)) = g([x]) = |[x]|$; $fog(x) = f(g(x)) = f(|x|) = [|x|]$

$$gof\left(-\frac{5}{3}\right) = \left|\left[-\frac{5}{3}\right]\right| ; \; fog\left(-\frac{5}{3}\right) = \left[\left|-\frac{5}{3}\right|\right]$$

$$= |-2| ; \; = \left[\frac{5}{3}\right]$$

$$= 2 ; \; = 1$$

$$gof\left(-\frac{5}{3}\right) - fog\left(-\frac{5}{3}\right) = 2-1 = 1$$

162. (b) $fof(x) = f(f(x)) = f([x])$

$$fof\left(-\frac{9}{5}\right) = f(-2) \qquad \because \left[-\frac{9}{5}\right] = -2$$

$$= [-2] = -2$$

$gog(x) = g(|x|) = ||x|| = |x|$

$gog(-2) = |-2| = 2$

$$(fof)\left(-\frac{9}{5}\right) + gog(-2) = -2+2 = 0$$

163. (a) $\lim_{x\to 0}\phi(x) = a^2 \qquad a \neq 0$

$$\Rightarrow \lim_{x\to 0}\phi\left(\frac{x}{a}\right) = a^2$$

[because function value is constant]

164. (a) $\lim_{x\to 0} e^{-\frac{1}{x^2}} = e^{\frac{-1}{0}} = e^{-\infty} = 0$

165. (a) We know that

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

For domain, $|x| - x > 0$
Case 2 : $x < 0$ Case 1 : $x > 0 \Rightarrow x - x = 0$ (not possible)
$-x - x > 0$
$\Rightarrow -2x > 0$
$\Rightarrow x < 0$
So, $x \in (-\infty, 0)$

166. (b) For $x \geq 1$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} 2 + x = 2 + 1 = 3$

For $x < 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2 + x = 2 + 1 = 3$

So, $\lim_{x\to 1} f(x)$ exist.

At $x = 0$

RHL : $\lim_{h\to 0^+} f(0+h) = \lim_{h\to 0} 2 + h = 2$

LHL $\lim_{h\to 0^-} f(0-h) = \lim_{h\to 0} 2 - h = 2$

$f(0) = 2 + 0 = 2$.
So, RHL = LHL = $f(0)$
$\Rightarrow f(x)$ is continuous at $x = 0$
Differentiability at $x = 0$

$$\text{LHD}: \lim_{h\to 0^-}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0^-}\frac{2+h-2}{-h}$$

$$= \frac{-h}{h} = -1$$

$$\text{RHD}: \lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}\frac{2+h-2}{h} = 1$$

Since LHD $\neq$ RHD
So, $f(x)$ is not differentiable at $x = 0$.

167. (a) For $x \geq 0$

$$f(x) = \frac{x+x}{x} = 2$$

For $x < 0$

$$f(x) = \frac{x-x}{x} = 0$$

$$\lim_{x \to 0^+} f(x) = 2$$

$$\lim_{x \to 0^-} f(x) = 0$$

$f(0) = 2$

$\Rightarrow$ It is discontinuous at $x = 0$.

Option (a) is correct.

168. (c) For $-1 \leq x \leq 2$

$$f(x) = 3x^2 + 12x - 1$$

$$f'(x) = 6x + 12$$

If we take any point in the interval [–1, 2] then

$$f'(1) = 6 \times 1 + 12 = 18 > 0$$

$\Rightarrow f(x)$ is increasing in the interval [–1, 2].

For $2 < x \leq 3$

$$f(x) = 37 - x$$

$$f'(x) = -1 < 0$$

$\Rightarrow f(x)$ is decreasing in the interval (2, 3]

169. (a) For continuity at $x = 2$.

RHL

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 37 - x = 37 - 2 = 3\sigma$$

LHL

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 3x^2 + 12x - 1$$

$= 3(2)^2 + 12 \times 2 - 1 = 12 + 24 - 1 = 35.$

$f(2) = 3 \times 4 + 12 \times 2 - 1 = 12 + 24 - 1 = 35$

So, RHL = LHL

$\Rightarrow f(x)$ is continuous at $x = 2$.

For differentiability at $x = 2$.

$$\text{LHD} = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{h \to 0} \frac{f(2-h) - f(2)}{2 - h - 2}$$

$$= \lim_{h \to 0} \frac{3(2-h)^2 + 12(2-h) - (12 + 24 - 1)}{-h}$$

$$= \lim_{h \to 0} \frac{3h^2 - 24h}{-h} = \lim_{h \to 0} 24 - 3h = 24$$

$$\text{RHD} = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{2 + h - 2}$$

$$= \lim_{h \to 0} \frac{37 - 2 - h - 35}{h} = -1$$

LHD $\neq$ RHD

$\Rightarrow f(x)$ is not differentiable at $x = 2$.

To check 2.

For $x = 2$.

$f(x) = 3x^2 + 12x - 1$

$$= 3\left(x^2 + 4x - \frac{1}{3}\right) = 3\left((x+2)^2 - \frac{13}{3}\right)$$

On putting $x = 2$

$$f(x) = 3\left(16 - \frac{13}{3}\right) = 35$$

On putting $x = 1$

$$f(1) = 3\left((1+2)^2 - \frac{13}{3}\right) = 3\left(9 - \frac{13}{3}\right) = 14$$

So $f(x)$ attains greatest value at $x = 2$.

170. (c) $f(x) = [x]$ and $g(x) = \sin x$

$$\lim_{x \to 0^+} f(x) = [0 + h] = 0$$

$$\lim_{x \to 0^-} f(x) = [0 - h] = -1$$

$f(0) = 0$

$\Rightarrow f(x)$ is not continuous at $x = 0$ and also $g(x)$ is continuous at $x = 0$. (every trignometric function is continuous).

171. (d) $(fog)(x) = [\sin x]$

$$\lim_{x \to 0^+} (fog)(x) = \lim_{x \to 0^+} [\sin x] = [h] \text{ where } h > 0$$

$$\Rightarrow \lim_{x \to 0^+} (fog)(x) = 0$$

$$\lim_{x \to 0^-} (fog)(x) = \lim_{x \to 0^-} [\sin x] = [h] \text{ where } h < 0$$

$$\Rightarrow \lim_{x \to 0^-} (fog)(x) = -1$$

$\Rightarrow \lim_{x \to 0} (fog)(x)$ does not exist.

Now, $(gof)(x) = \sin[x]$

$$\lim_{x \to 0^+} (gof)(x) = \lim_{x \to 0^+} \sin[x] = \sin 0 = 0$$

$$\lim_{x \to 0^-} (gof)(x) = \lim_{x \to 0^-} \sin[x] = \sin(-1) = -0.01745$$

$\Rightarrow \lim_{x \to 0} (gof)(x)$ does not exist.

$$\Rightarrow \lim_{x \to 0^+} (fog)(x) = \lim_{x \to 0^+} (gof)(x)$$

172. (c) $(fof)(x) = [[x]]$ and $f(x) = [x]$

Suppose $x = 0.2$

$\Rightarrow (fof)(x) = [[0.2]] = [0] = 0$

$f(x) = [0.2] = 0$

$\Rightarrow fof(x) = f(x)$

Now, $(gog)(x) = \sin \sin x$ and $g(x) = \sin x$

At $x = 0$

$(gog)(x) = \sin \sin 0 = \sin 0 = 0$

$g(x) = \sin 0 = 0$

$\Rightarrow (gog)(x) = g(x)$ at $x = 0$

and this is true for

$x = n\pi$, where n= 0, 1, 2, 3, 4 ....

$\because (fog)(x) = [\sin x]$

$(go(fog))(x) = \sin[\sin x]$

$\because$ $\sin x$ has value from –1 to 1

If $-1 \le \sin x < 0$.

$(go(fog))(x) = \sin(-1) - \sin(-1)$

If $0 \le \sin x < 1$

$\therefore (go(fog))(x) = \sin(0) = \sin 0$ and

If $\sin x = 1$

$\therefore (go(fog))(x) = \sin(1) = \sin 1$

173. (b) $f(x) = \dfrac{e^x - 1}{x} > 0$

$\Rightarrow \quad f'(x) = \dfrac{x e^x - (e^x - 1)}{x^2} = \dfrac{e^x(x-1)+1}{x^2}$

$= \left(\dfrac{e^x(x-1)}{x^2} + \dfrac{1}{x^2}\right)$, which is a strictly increasing function.

174. (b) For right hand continuity at $x = 0$

RHL $= \lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} f(0+h)$

$$= \lim_{h\to 0} \frac{e^h - 1}{h} = \lim_{h\to 0} \frac{\left(1 + h + \frac{h^2}{2!} + ...\right) - 1}{h}$$

$$= \lim_{h\to 0} \frac{h + \frac{h^2}{2!} + \frac{h^3}{3!} + ...}{h}$$

$$= \lim_{h\to 0} 1 + \frac{h}{2!} + \frac{h^2}{3!} + ... = 1$$

$f(0) = 0$

$\Rightarrow f(x)$ is not right continuous at $x = 0$.

For discontinuity at $x = 1$

RHL $= \lim\limits_{x\to 1^+} f(x) = \lim\limits_{h\to 0} f(1+h)$

$$= \lim_{h\to 0} \frac{e^{1+h} - 1}{1+h}$$

$$= \lim_{h\to 0} \frac{\left(1 + (1+h) + \frac{(1+h)^2}{2} + ...\right) - 1}{1+h}$$

$$= \lim_{h\to 0} \frac{(1+h) + \frac{(1+h)^2}{2!} + \frac{(1+h)^3}{3!} + ...}{(1+h)}$$

$$= \lim_{h\to 0} 1 + \frac{(1+h)}{2!} + \frac{(1+h)^2}{3!} + ...$$

$$= 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...$$

LHL $= \lim\limits_{x\to 1^-} f(x) = \lim\limits_{h\to 0} f(1-h)$

$$= \lim_{h\to 0} \frac{e^{(1-h)} - 1}{1-h}$$

$$= \lim_{h\to 0} \frac{\left(1 + (1-h) + \frac{(1-h)^2}{2!} + ...\right) - 1}{(1-h)}$$

$$= \lim_{h\to 0} 1 + \frac{(1-h)}{2!} + \frac{(1-h)^2}{3!} + ...$$

$$= 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...$$

$$f(1) = \frac{e^1 - 1}{1} = \left(1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\right) - 1$$

$$= \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...$$

$\Rightarrow$ RHL $\ne f(1)$, LHL $\ne f(1)$

So f is discontinuous.

175. (d) $f(x) = \begin{cases} -2, & -3 \le x \le 0 \\ x-2, & 0 < x \le 3 \end{cases}$ and

$g(x) = f(|x|) + |f(x)|$

At $x = 0$

For LHD : $g(x) = -2 + |-2| = -2 + 2 = 0 \Rightarrow g(x) = 0$

LHD $= \lim\limits_{x\to 0^-} \dfrac{g(x) - g(0)}{x - 0} = \lim\limits_{h\to 0} \dfrac{g(-h) - g(0)}{-h}$

$$= \lim_{h\to 0} \frac{0-0}{-h} = \lim_{h\to 0} 0$$

LHD = 0

For RHD : $g(x) = |x| - 2 + |x - 2|$

$g(x) = x - 2 - (x - 2)$ $\quad x > 0$ (and just greater than zero)

$g(x) = x - 2 - x + 2 = 0$

Now g(x) is not continuous at x = 0, hence g(x) is not differentiable at x = 0

At $x = 2$

For LHD :

$g(x) = |x| - 2 + |x - 2| = x - 2 - (x - 2)$

$= x - 2 - x + 2 = 0$

$\therefore$ LHD $= \lim\limits_{x\to 2^-} \dfrac{g(x) - g(2)}{x - 2} = \lim\limits_{x\to 2} \dfrac{0}{x-2} = 0$

For RHD :

$g(x) = |x| - 2 + |x - 2| = x - 2 + x - 2 = 2x - 4$

$\Rightarrow \text{RHD} = \lim\limits_{x \to 2^+} \dfrac{g(x) - g(2)}{x - 2} = \lim\limits_{x \to 2} \dfrac{2x - 4 - 2(2) + 4}{x - 2}$

$= \lim\limits_{x \to 2} \dfrac{2(x-2)}{x-2} = 2$

$\Rightarrow$ LHD $\neq$ RHD

Thus $g(x)$ is not differentiable at $x = 2$.

176. (b) For $x = -2$

$g(x) = -2 + |-2| = -2 + 2$

$\Rightarrow g(x) = 0$

$\Rightarrow$ differential coefficient at $x = -2$ is given as :

$g'(x) = \lim\limits_{h \to 0} \dfrac{g(x+h) - g(x)}{h} = \lim\limits_{h \to 0} \dfrac{0-0}{4} = 0.$

177. (d) At $x = 0$

For LHL : $g(x) = -2 + |-2| = 0$

For RHL : $g(x) = |x| - 2 + |x - 2|$

$g(x) = x - 2 - (x - 2) = 0$

$g(x) = 0$

For $(x = 0)$: $g(x) = -2 + |-2| = 0$

LHL $= \lim\limits_{x \to 0^-} g(x) = 0$

RHL $= \lim\limits_{x \to 0^+} g(x) = 0$

$g(0) = 0$

$\Rightarrow g(x)$ is continuous at $x = 0$

At $x = 2$

For LHL : $g(x) = |x| - 2 + |x - 2|$

$g(x) = x - 2 - (x - 2)$

$g(x) \quad 0$

For RHL : $g(x) = |x| - 2 + |x - 2|$

$g(x) = x - 2 + x - 2$

$2(x) = 2x - 4$

For $(x = 2)$: $g(x) = |x| - 2 + |x - 2|$

LHL $= \lim\limits_{x \to 2^-} g(x) = 0$

RHL $= \lim\limits_{x \to 2^+} g(x) = \lim\limits_{x \to 2} 2x - 4 = 2(2) - 4 = 0$

$g(2) = |2| - 2 + |2 - 2|$

$g(2) = 2 - 2 + 2 - 2 = 0$

$\Rightarrow g(x)$ is continuous at $x = 2$.

At $x = -1$

For LHL : $g(x) = -2 + |-2| = 0$

For RHL : $g(x) = -2 + |-2| = 0$

For $(x = -1)$: $g(x) = -2 + |-2| = 0$

$\therefore$ LHL $= \lim\limits_{x \to -1^-} g(x) = 0$

RHL $= \lim\limits_{x \to -1^+} g(x) = 0$

$g(-1) = 0$

$\Rightarrow g(x)$ is differentiable at $x = -1$.

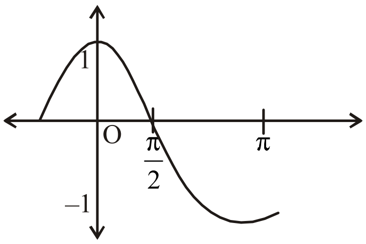
178. (b) $\lim\limits_{x \to 0} \dfrac{e^x - (1+x)}{x^2}$

If we keep $x = 0$, it is $\dfrac{0}{0}$.

So, applying L'Hospital role.

$\lim\limits_{x \to 0} \dfrac{e^x - (1+x)}{x^2} = \dfrac{1}{2} \lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = \dfrac{1}{2} \times 1 = \dfrac{1}{2}.$

179. (a) Observe the cosx graph in the figure.



It is clear that, function is one – one and onto when x and y are [o, π] and [–1, 1]

180. (b) $f(x) = \dfrac{x}{x-1}$

$f(a) = \dfrac{a}{a-1}$

$f(a+1) = \dfrac{a+1}{a+1-1} = \dfrac{a+1}{a}$

$\therefore \quad \dfrac{f(a)}{f(a+1)} = \dfrac{\frac{a}{a-1}}{\frac{a+1}{a}} = \dfrac{a^2}{a^2-1}$

$f(a^2) = \dfrac{a^2}{a^2-1}$

181. (c) $f(x) = x^2 - 3$

$fof(x) = f(f(x)) = f(x^2 - 3) = (2^2 - 3)^2 - 3$

$= 2^4 - 6x^2 + 9 - 3 = x^4 - 6x^2 + 6$

$fof(0) = 0 - 0 + 6 = 6$

$f(f(f(x))) = f(x^4 - 6x^2 + 6) = (x^4 - 6x^2 + 6)^2 - 3$

$fofof(1) = ((-1)^4 - 6(-1)^2 + 6)^2 - 3$

$= (1 - 6 + 6) - 3 = -2$

$fofof(1) = (1^4 - 6(1)^2 + 6)^2 - 3$

$= (1 - 6 + 6) - 3 = -2$

182. (c) $f(x) = px + q$, $g(x) = mx + n$

$f(g(x)) = g(f(x))$

$\Rightarrow f(mx + n) = g(px + q)$

$\Rightarrow p(mx + n) + q = m(px + q) + n$

$\Rightarrow pmx + pn + q = pmx + mq + n$

$\Rightarrow pn + q = mq + n$

$\Rightarrow f(n) = g(q)$

183. (c) $\lim\limits_{x\to1}\dfrac{F(x)-F(1)}{x-1}$

We know, $\lim\limits_{x\to a}\dfrac{F(n)-F(a)}{x-a}=F'(a)$

$\therefore\ F'(x)=\dfrac{d}{dx}\left(\sqrt{9-x^2}\right)$

$=\dfrac{1(0-2x)}{2\sqrt{9-x^2}}=\dfrac{-x}{\sqrt{9-x^2}}$

$F'\ 1\quad \dfrac{-1}{\sqrt{9-1}}\quad \dfrac{-1}{\sqrt{8}}\quad \dfrac{-1}{2\sqrt{2}}$

184. (d) $(f-g)(x)=\begin{cases} x-0=x\,,x \text{ is rational}\\ 0-x=-x\,,x \text{ is irrational}\end{cases}$

Clearly, f – g is one – one and onto.

185. (c) $f(x)=\begin{cases}2x+1, & -3<x<-2\\ x-1, & -2\le x<0\\ x+2, & 0\le x<1\end{cases}$

Here, $f(0^-)=-1$ and $f(0^+)=2$

So, f (x) is discontinuous at x = 0 and continuous at other points.

186. (a) If $\lim\limits_{x\to a} f\ x$ and $\lim\limits_{x\to a} g(x)$ both exists, then

$\lim\limits_{x\to a} f\ x\ .g\ x$ exists. But if $\lim\limits_{x\to a} f(x).g(x)$ exists, then it is not necesary that both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exists.

187. (b) $f(a)=\dfrac{a-1}{a+1}$

$f(2a)=\dfrac{2a-1}{2a+1}$

$f(a)+1=\dfrac{a-1}{a+1}+1=\dfrac{2a}{a+1}$

So, $f(2a)\ne f(a)+1$

Now, $f\left(\dfrac{1}{a}\right)\quad \dfrac{\frac{1}{a}-1}{\frac{1}{a}\quad 1}\quad \dfrac{1-a}{1\quad a}\quad -\left(\dfrac{a-1}{a\quad 1}\right)=-f\ \ a$

188. (a) $f(x)=x^n$, $n\ne0$ This function is differentiable for all values of n, except 0. Hence n $\in$ (1, ∞)

189. (a) $y=h^{\ell n\,x}=e^{\ell n\,5\times\ell n\,x}$

$\Rightarrow \ell n\,y=\ell n\,5\times\ell n\,x$

$\Rightarrow \ell n\,x=\dfrac{1}{\ell n\,5}\cdot\ell n\,y=\ell n y^{\frac{1}{\ell n\,5}}$

$\Rightarrow x=y^{\frac{1}{\ell n\,5}},\ y>0.$

190. (c) $f\ \ x\quad \begin{cases}\dfrac{-x}{\sqrt{x^2}},x\ne0\\ 0,\ x\quad 0\end{cases}$

$=\begin{cases}\dfrac{-x}{|x|},x\ne0\\ 0,\ x=0.\end{cases}$

$f\ \ 0\ \ h\quad \dfrac{-h}{h}\quad -1;\ f\ \ 0-h\quad \dfrac{h}{h}\quad 1$

So, f(x) is discontinuous at x = 0.

191. (a) $x+x^2$ is continuous at x = 0

$x\quad \cos\dfrac{1}{x}$ is discontinuous at x = 0

$x^2\quad \cos\dfrac{1}{x}$ is not continuous at x = 0.

192. (c) $f(x)=\begin{cases}1-x^2 \text{ for } 0<x\le1\\ \ell n\ x \text{ for } 1<x\le2\\ \ell n2-1+0.5x \text{ for } 2<x<\infty\end{cases}$

$f'(x)=-2x$ for $0<x<1$

193. (d) All statements are correct.

194. (d) $f(x)=|x|-x^3$

$f\ \ x\quad \begin{cases}x-x^3,\ x\ge0\\ -x-x^3,x\quad 0\end{cases}\qquad f\ \ -x\quad \begin{cases}x+x^3,\ x\ge0\\ x\quad x^3,x\quad 0\end{cases}$

Neither even nor odd.

195. (b) $l_1=\dfrac{d}{dx}\left(e^{\sin x}\right)$

$l_2=\lim\limits_{h\to0}\dfrac{e^{\sin(x+h)}-e^{\sin x}}{h}$

$l_3=\int e^{\sin x}\cdot\cos x\ dx$

$l_2=\lim\limits_{h\to0}\dfrac{e^{\sin(x+h)}-e^{\sin x}}{h}=\dfrac{d}{dx}\left(e^{\sin x}\right)=l_1$

$l_3=\int e^{\sin x}\cdot\cos x\ dx$

Let $\sin x=t\Rightarrow\cos x\,dx=dt$

$l_3=\int e^t\cdot dt=e^t+c=e^{\sin x}+c.$

$\dfrac{d}{dx}(l_3)=\dfrac{d}{dx}\left(e^{\sin x}+c\right)=\dfrac{d}{dx}\left(e^{\sin x}\right)=l_1=l_2$

196. (c) $\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{x}=l$ and $\lim_{x\to\infty}\frac{\cos x}{x}=m$

$$\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{x}=\frac{1}{\frac{\pi}{2}}=\frac{2}{\pi};\ \lim_{x\to\infty}\frac{\cos x}{x}=0$$

$$\therefore l=\frac{2}{\pi}, m=0$$

197. (b) $y=\frac{x^2}{1+x^4}\Rightarrow y\ge 0.$

Also, $y=\frac{x^2}{1+x^4}=\frac{1}{x^2+\frac{1}{x^2}}$

$$\Rightarrow y\le\frac{1}{2}.$$

$$\therefore \frac{x^2}{1+x^4}\text{ belongs to }\left(0,\frac{1}{2}\right].$$

198. (a) f(x) = [x] sin (πx) at x = k.

Left hand derivative, $\lim_{h\to 0}\frac{f(k)-f(k-h)}{h}$ (k-integer)

$$=\lim_{h\to 0}\frac{[k]\sin k\pi-[k-h]\sin (k-h)\pi}{h}$$

$$=\lim_{h\to 0}\frac{-(k-1)\sin(k-h)\pi}{h}$$

$\sin k\pi=0$ and $\sin(k\pi-\theta)=(-1)^{k-1}\sin\theta$.

$$=\lim_{h\to 0}\frac{-(k-1)-(1)^{k-1}\sin\pi}{h\pi}\times\pi$$

$$=\lim_{h\to 0}\frac{-(k-1)(-1)^{k-1}\sin\pi}{h\pi}\times\pi$$

$$=\pi(k-1)(-1)^k$$

199. (c) $f(x)=\frac{x}{2}-1,\quad [0,\pi]$

$$\tan\cdot f(x)=\tan\left(\frac{x}{2}-1\right)$$

$\frac{1}{f(x)}=\frac{1}{\frac{x}{2}-1}$ is discontinuous at x = 2

tan.f(x) is discontinuous for x = 2 in [0, π]

200. (c) $f(x)=\sqrt{1-e^{-x^2}}$

$$f'(x)=\frac{1}{2\sqrt{1-e^{-x^2}}}.\left(-e^{-x^2}\right)(-2x)$$

$$f'(x)=\frac{xe^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

f' (x) is defined for all values of x, except 0.

∴ f(x) is differentiable on $(-\infty,0)\cup(0,\infty)$

201. (b) $f(x)=x\left(\sqrt{x}-\sqrt{x+1}\right)$

$$f(x)=\frac{x\left(\sqrt{x}-\sqrt{x+1}\right)\left(\sqrt{x}+\sqrt{x+1}\right)}{\left(\sqrt{x}+\sqrt{x+1}\right)}$$

$$=\frac{x(x-x-1)}{\sqrt{x}+\sqrt{x+1}}=\frac{-x}{\sqrt{x}+\sqrt{x+1}}$$

Hence, f(x) is continuous as well as differentiable at x = 0.

202. (c) $f(x)=\frac{x}{x}, x\ne 0$.

∴ The graph is discontinious at x = 0, and correctly shown in option (e).

203. (d) $f(x)=(g(x))^2-g(x)$

Given, g(x) is greatest integer function. So, g(x) = [x].

∴ $f(x)=[x]^2-[x]$

f(x) is discontinuous at every integer except x = 1

205. (b) $f(x)=\frac{4x+x^4}{1+4x^3}, g(x)=\ell n\left(\frac{1+x}{1-x}\right)$

$$g\left(\frac{e-1}{e+1}\right)=\ell n\left(\frac{1+\left(\frac{e-1}{e+1}\right)}{1-\left(\frac{e-1}{e+1}\right)}\right)=\ell n\left(\frac{e+1+e-1}{e+1-e+1}\right)$$

$$=\ell n\left(\frac{2\ell}{2}\right)=\ell n\ e=1$$

$$fo\ g\left(\frac{e-1}{e+1}\right)=f(1)=\frac{4(1)+(1)^4}{1+4(1)^3}=\frac{4+1}{1+4}=\frac{5}{5}=1$$

206. (d) Given, f(x) = |x + 1|.

Let us check all options.

(a) $f(x)^2=|x^2+1|$

$(f(x))^2=(x+1)^2$.

So, $f(x)^2\ne(f(x))^2$

(b) $f(|x|)=||x|+1|$

$|f(x)|=||x+1||=|x+1|$

$f(x)\ne|f(x)|$

(c) $f(x+y)=|x+y+1|$

$f(x)+f(y)=|x+1|+|y+1|$

$f(x+y)\ne f(x+f(y))$

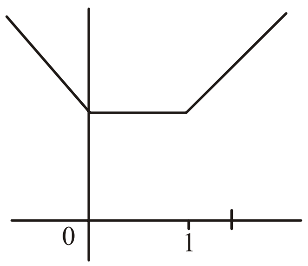
207. (a) $f(x)=\dfrac{x^2}{1+x^2}$

for $x=0, f(0)=0$

for $x=1,\ f(1)=\dfrac{1}{2}=0.5$

So, range is [0, 1)

208. (a) Given function is continuous at x = 0 and 1.



209. (a) $f(x)=\begin{cases} x^2\ell n|x| & x\neq 0 \\ 0 & x=0. \end{cases}$

$$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{h^2.\ell n|h|-0}{h}$$

$$=\lim_{h\to 0} h.\ell n|h|$$

$=0$

210. (b) $f(x)=\dfrac{x^2-9}{x^2-2x-3}$

Given, the function is continuous at x = 3.

$$f(3)=\lim_{x\to 3}\frac{x^2-9}{x^2-2x-3}$$

$$=\lim_{x\to 3}\frac{(x+3)(x-3)}{(x-3)(x+1)}$$

$$=\frac{3+3}{3+1}=\frac{6}{4}=\frac{3}{2}=1.5.$$

211. (d) f(x) = 4 sin x – 3 cos x + 1

To find range of this function, we have to find maximum and minimum values.

We know, for a sin θ + b cos θ + c, max value is $\sqrt{a^2+b^2}+c$ and minimum value is $-\sqrt{a^2+b^2}+c$.

∴ Maximum value

$$=\sqrt{(4)^2+(-3)^2}+1=\sqrt{16+9}+1=5+1=6$$

Minimum value $=-\sqrt{16+9}+1=-5+1=-4$

∴ Range, S ∈ [–4, 6]

212. (a) $f(x)=\dfrac{1}{\sqrt{|x|-x}}$

$|x|-x>0$ since the denominator cannot be zero.

∴ $|x|>x$

for $x>0$, $x>x$ is not possible.

for $x<0$, $|x|>x$

$\Rightarrow 2x<0$

$\Rightarrow x<0.$

∴ Domain is $(-\infty, 0)$

213. (a) $\displaystyle\lim_{x\to 0}\frac{\tan x}{\sin 2x}$

$$=\lim_{x\to 0}\frac{\dfrac{\tan x}{x}}{2\left(\dfrac{\sin 2x}{2x}\right)}=\frac{1}{2}.$$

214. (d) $\displaystyle\lim_{h\to 0}\frac{\sqrt{2x+3h}-\sqrt{2x}}{2h}$

Rationalise the numerator.

$$\lim_{h\to 0}\left(\frac{\sqrt{2x+3h}-\sqrt{2x}}{2h}\times\frac{\sqrt{2x+3h}+\sqrt{2x}}{\sqrt{2x+3h}+\sqrt{2x}}\right)$$

$$=\lim_{h\to 0}\frac{2x+3h-2x}{2h\left(\sqrt{2x+3h}+\sqrt{2x}\right)}$$

$$=\frac{3}{2\left(\sqrt{2x+0}+\sqrt{2x}\right)}=\frac{3}{4\sqrt{2x}}$$

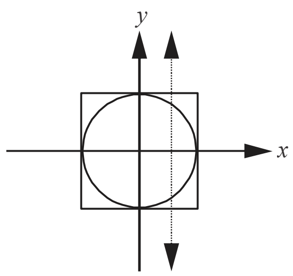
215. (b) f(x) is an even function.

Let's see some examples

(1) If f(x) = cos x, which is even function,
$f'(x)=-\sin x$, which is odd function.

(2) If $f(x)=x^2$, which is even function,
$f'(x)=2x$, which is odd function.

216. (d) S is not a function (By vertical line test)



217. (d) $f(x)=\dfrac{\sqrt{x-1}}{x-4}$

$f(x)$ is defined for $(x-1)\geq 0 \Rightarrow x\geq 1$ and $x-4\neq 0$

$\Rightarrow$ $x\neq 4$

∴ Domain of $f(x)=1\leq x<\infty-\{4\}$

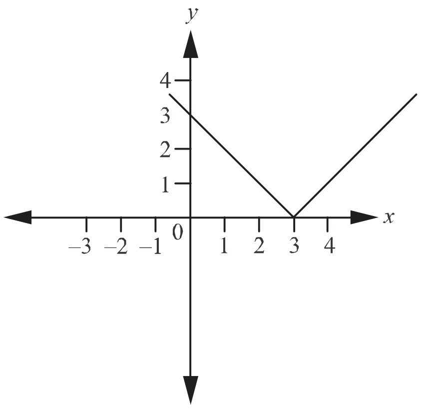
or, $x=[1,4)\cup(4,\infty)$.

218. (a) $\lim_{x\to 0}\frac{\sin(2x)}{5x} = \lim_{x\to 0}\frac{\sin(2x)(2)}{5.(2x)} = \frac{2}{5}$

But at $x = 0, f(x) = \frac{2}{15}$

Hence, $f(x)$ is not continuous at $x = 0$.

219. (a)



$f(x) = |x-3|$

$\Rightarrow f(x) = \begin{cases}(x-3), & x \geq 3\\ (3-x), & x \quad 3\end{cases}$

$f'(x)$ at $x = 3^+ = 1$

$f'(x)$ at $x = 3^- = -1$

Thus, $f(x)$ is not differentiable at $x = 3$ but $f(x)$ is continuous at $x = 3$.

220. (b) $f(0) = \lim_{x\to 0}\frac{2x - \sin^{-1}x}{2x \quad \tan^{-1}x}$

$= \lim_{x\to 0}\frac{2 - \frac{\sin^{-1}x}{x}}{2 \quad \frac{\tan^{-1}x}{x}} \quad \frac{2-1}{2 \quad 1} \quad \frac{1}{3}$

221. (a) $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1} = f'(x)$

Now, $f'(x) \quad \frac{-2x}{2\sqrt{25-x^2}} \quad \frac{-x}{\sqrt{25-x^2}}$

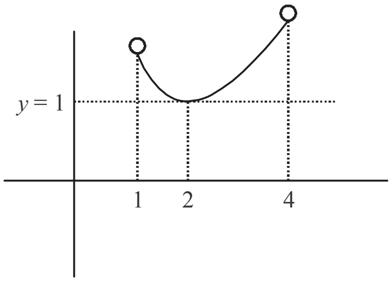
$f'(1) \quad \frac{-1}{\sqrt{25-1}} \quad \frac{-1}{\sqrt{24}}$

222. (c) $\lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta} = \lim_{\theta\to 0}\frac{\sqrt{2}\sin(\theta/2)}{\theta}$

$= \frac{\sqrt{2}}{2} \quad \frac{1}{\sqrt{2}}$

223. (c) $f(x) = x^2 - 4x + 5 = (x-2)^2 + 1$

$f(1) = 2, f(4) = 5$



$\therefore$ Range $f(x) = [1, 5)$

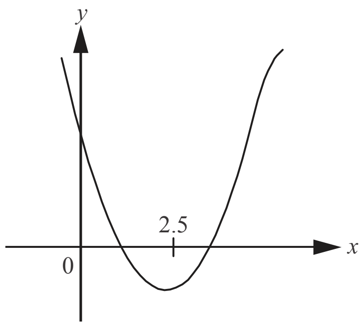
224. (a) $f(x) = x^2 - 5x + 6$

$\Rightarrow f'(x) = 2x - 5$

$\Rightarrow f'(x) < 0$

$\Rightarrow 2x - 5 < 0$

$\Rightarrow x < \frac{5}{2}$



Hence, $f(x)$ is decreasing in $(-\infty, 2.5]$

225. (d) $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{f(x)[f(h)-1]}{h} \quad \lim_{h\to 0}\frac{f(x)[h\,g(h)\phi(h)]}{h}$

$= ab\,f(x)$

226. (d) $\lim_{x\to\frac{\pi}{6}}\frac{(\sin x+1)(2\sin x-1)}{(\sin x-1)(2\sin x-1)} \quad \frac{\frac{1}{2} \quad 1}{\frac{1}{2}-1} \quad -3$

227. (b) $f(x) = \ln(\sqrt{x^2+1}-x)$

$f(-x) = \ln(\sqrt{(-x)^2+1}-(-x)) = \ln(\sqrt{x^2+1}+x)$

$\ln\left(\frac{(\sqrt{x^2 \quad 1} \quad x)(\sqrt{x^2 \quad 1}-x)}{\sqrt{x^2+1}-x}\right)$

$= \ln\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}-x}\right) = \ln\left(\frac{1}{\sqrt{x^2+1}-x}\right)$

$= -\ln(\sqrt{x^2+1}-x) = -f(x)$

So, f(x) is odd function.

228. (d) $f(x) = \log_x 10$
Domain of logarithmic function is $x > 0$ excluding $x = 1$.

229. (c) $\lim_{x\to 0} \frac{1-\cos^3 4x}{x^2}$

The given limit is in $\frac{0}{0}$ form. So, apply L'hospital rule.

$\lim_{x\to 0} \frac{-3\cos^2 4x(-\sin 4x)4}{2x}$

$= \lim_{x\to 0} \frac{12\cos^2 4x \sin 4x}{(2x)2} \times 2$

$= 24. \lim_{x\to 0} \cos^2 4x. \frac{\sin 4x}{4x}$

$= 24(1) = 24$

230. (c) $f(x) = 3^{1+x}$
$f(x) \cdot f(y) \cdot f(z) = 3^{1+x} \cdot 3^{1+y} \cdot 3^{1+z}$
$= 3^{1+x+1+y+1+z} = 3^{3+x+y+z} = 3^{1+x+y+z+2}$
$= f(x+y+z+2)$

231. (c) $f(x) = \sqrt{(2-x)(x-3)}$
Here, $(2-x)(x-3) \geq 0$
$\Rightarrow -(x-2)(x-3) \geq 0$
$\Rightarrow (x-2)(x-3) \leq 0$
$\Rightarrow x \in [2, 3]$

232. (d) $f(x) = \begin{cases} \sin x, & x \neq 0 \\ k, & x = 0 \end{cases}$

Given, f(x) is continuous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \sin x = k$

$\Rightarrow k = 0$

Derivatives

15

1. If $u = \sin^{-1}(x - y)$, $x = 3t$, $y = 4t^3$, then what is the derivative of u with respect to t ?

(a) $3(1 - t^2)$ (b) $3(1-t^2)^{-\frac{1}{2}}$

(c) $5(1-t^2)^{\frac{1}{2}}$ (d) $5(1 - t^2)$ *[2006-I]*

2. If $x = \cos t$, $y = \sin t$, then what is $\frac{d^2y}{dx^2}$ equal to ?

(a) y^{-3} (b) y^3

(c) $-y^{-3}$ (d) $-y^3$ *[2006-I]*

3. If $y = x + e^x$, then, what is $\frac{d^2x}{dy^2}$ equal to ?

(a) e^x (b) $-\frac{e^x}{(1+e^x)^3}$

(c) $-\frac{e^x}{(1+e^x)}$ (d) $-\frac{e^x}{(1+e^x)^2}$ *[2006-I]*

4. What is the derivative of $f(x) = x|x|$?

(a) $|x|+x$ (b) $2x$

(c) $2|x|$ (d) $-2|x|$ *[2006-I]*

5. If $x + y = t - \frac{1}{t}$, $x^2 + y^2 = t^2 + \frac{1}{t^2}$, what is $\frac{dy}{dx}$ equal to ?

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$

(c) $\frac{1}{x^2}$ (d) $-\frac{1}{x^2}$ *[2006-I]*

6. What is the derivative of $f(x) = \sqrt{1-x^2}$ with respect to $g(x) = \sin^{-1}x$, where $|x| \neq 1$?

(a) x (b) $-x$

(c) $\frac{x}{1-x^2}$ (d) $-\frac{x}{1-x^2}$ *[2006-II]*

7. What is the derivative of

$(\log_{\tan x} \cot x)(\log_{\cot x} \tan x)^{-1}$ at $x = \frac{\pi}{4}$?

(a) -1 (b) 0

(c) 1 (d) $\frac{1}{2}$ *[2006-II]*

8. What is the derivative of $\cos^{-1}\left(\frac{2\cos x + 3\sin x}{\sqrt{13}}\right)$?

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{1}{\sqrt{1-x^2}}$

(c) 0 (d) 1 *[2006-II]*

9. What is the derivative of $f(x) = \frac{7x}{(2x-1)(x+3)}$?

(a) $-\frac{3}{(x+3)^2} - \frac{2}{(2x-1)^2}$ (b) $-\frac{3}{(x+3)^2} - \frac{1}{(2x-1)^2}$

(c) $\frac{3}{(x+3)^2} + \frac{1}{(2x-1)^2}$ (d) $\frac{3}{(x+3)^2} + \frac{2}{(2x-1)^2}$

[2006-II]

10. What is the solution of $y' = 1 + x + y^2 + xy^2$, $y(0) = 0$?

(a) $y = \tan^2\left(\frac{x^2}{2} + x\right)$ (b) $y = \tan^2(x^2 + x)$

(c) $y = \tan(x^2 + x)$ (d) $y = \tan\left(\frac{x^2}{2} + x\right)$

[2006-II]

11. If a differentiable function f defined for $x > 0$ satisfies the relation $f(x^2) = x^3$, $x > 0$, then what is the value of $f'(4)$?

(a) 1 (b) 2

(c) 3 (d) 4

[2007-II]

12. What is the derivative of $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$ at $x=1$?

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 1 *[2007-II]*

13. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, what is $\frac{dy}{dx}$ equal to ?

(a) $-\frac{1}{1+x}$ (b) $-\frac{1}{(1+x)^2}$

(c) $\frac{1}{(1+x)^2}$ (d) $\frac{\sqrt{x}}{\sqrt{1+x}}$ *[2007-II]*

14. If $y=f(x)$, $p=\frac{dy}{dx}$ and $q=\frac{d^2y}{dx^2}$, then what is $\frac{d^2x}{dy^2}$ equal to ?

(a) $-\frac{q}{p^2}$ (b) $-\frac{q}{p^3}$

(c) $\frac{1}{q}$ (d) $\frac{q}{p^2}$ *[2008-I]*

15. If $x=\sin t-t\cos t$ and $y=t\sin t+\cos t$, then what is $\frac{dy}{dx}$ at point $t=\frac{\pi}{2}$?

(a) 0 (b) $\frac{\pi}{2}$

(c) $-\frac{\pi}{2}$ (d) 1 *[2008-I]*

16. If $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$, what is $\frac{dy}{dx}$ equal to ?

(a) $\cos^{-1}x+\cos^{-1}\sqrt{1-x^2}$ (b) $\frac{1}{\cos x}+\frac{1}{\cos\sqrt{1-x^2}}$

(c) $\frac{\pi}{2}$ (d) 0 *[2008-I]*

17. If $f(x)=\log_e[\log_e x]$, then what is $f'(e)$ equal to ?

(a) e^{-1} (b) e

(c) 1 (d) 0 *[2008-I]*

18. If $f(x)=e^{\sin(\log\cos x)}$ and $g(x)=\log\cos x$, then what is the derivative of $f(x)$ with respect to $g(x)$?

(a) $f(x)\cos[g(x)]$ (b) $f(x)\sin[g(x)]$

(c) $g(x)\cos[f(x)]$ (d) $g(x)\sin[f(x)]$ *[2008-I]*

19. For the curve $\sqrt{x}+\sqrt{y}=1$, what is the value of $\frac{dy}{dx}$ at $\left(\frac{1}{4},\frac{1}{4}\right)$?

(a) $\frac{1}{2}$ (b) 1

(c) -1 (d) 2 *[2008-I]*

20. If $y=\frac{1}{\log_{10}x}$, then what is $\frac{dy}{dx}$ equal to ?

(a) x (b) $x\log_e 10$

(c) $-\frac{(\log_x 10)^2(\log_{10}e)}{x}$ (d) $x\log_{10}e$ *[2008-I]*

21. If $y=\sin(m\sin^{-1}x)$, what is the value of d^2y/dx^2 at $x=0$?

(a) m (b) m^2 *[2008-II]*

(c) m^2+2 (d) None of these

22. If $x^y=e^{x-y}$, then dy/dx is equal to which one of the following? *[2008-II]*

(a) $\frac{(x-y)}{(1+\log x)^2}$ (b) $\frac{y}{(1+\log x)}$

(c) $\frac{(x+y)}{(1+\log x)}$ (d) $\frac{(\log x)}{(1+\log x)^2}$

23. If $\frac{dy}{dx}=1+x+y+xy$ and $y(-1)=0$, then what is $y(x)$ equal to? *[2009-I]*

(a) $e^{\frac{(1+x)^2}{2}}-1$ (b) $e^{\frac{(1-x)^2}{2}}$

(c) $\log(1+x)-1$ (d) $\log(1-x)$

24. If $f(x)=\tan x+e^{-2x}-7x^3$, then what is the value of $f'(0)$?

(a) -2 (b) -1

(c) 0 (d) 3 *[2009-I]*

25. If $3^x+3^y=3^{x+y}$ then what is $\frac{dy}{dx}$ equal to?

(a) $\frac{3^{x+y}-3^x}{3^y}$ (b) $\frac{3^{x-y}(3^y-1)}{1-3^x}$

(c) $\frac{3^x+3^y}{3^x-3^y}$ (d) $\frac{3^x+3^y}{1+3^{x+y}}$ *[2009-I]*

26. If $f(x)=\sin^2x^2$, then what is $f'(x)$ equal to?

(a) $4x\sin(x^2)\cos(x^2)$ (b) $2\sin(x^2)\cos(x^2)$

(c) $4\sin(x^2)\sin^2x$ (d) $2x\cos^2(x^2)$ *[2009-I]*

27. If f(x) = cos x, g(x) = log x and y = (gof) (x), then what is the value of $\frac{dy}{dx}$ at x = 0? *[2009-I]*
(a) 0 (b) 1
(c) –1 (d) 2

28. If $e^y + xy = e$, then what is the value of $\frac{d^2y}{dx^2}$ at $x = 0$? *[2009-II]*
(a) e^{-1} (b) e^{-2}
(c) e (d) 1

29. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a$, then what is $\frac{dy}{dx}$ equal to? *[2009-II]*
(a) $\sqrt{(1-x^2)(1-y^2)}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$
(c) $\sqrt{\frac{1-x^2}{1-y^2}}$ (d) None of these

30. If $x = \log t$ and $y = t^2 - 1$, then what is $\frac{d^2y}{dx^2}$ at $t = 1$ equal to? *[2009-II]*
(a) 2 (b) 3
(c) –4 (d) 4

31. What is the derivative of $\log_x 5$ with respect to $\log_5 x$? *[2009-II]*
(a) $-(\log_5 x)^{-2}$ (b) $(\log_5 x)^{-2}$
(c) $-(\log_x 5)^{-2}$ (d) $(\log_x 5)^{-2}$

32. A function f is such that $f'(x) = 6 - 4 \sin 2x$ and $f(0) = 3$. What is $f(x)$ equal to? *[2009-II]*
(a) $6x + 2\cos 2x$ (b) $6x - 2\cos 2x$
(c) $6x - 2\cos 2x + 1$ (d) $6x + 2\cos 2x + 1$

33. If $f(x) = e^x$ and $g(x) = \log x$, then what is the value of $(gof)'(x)$? *[2009-II]*
(a) 0 (b) 1
(c) e (d) None of these

34. Let $g(x) = x^3 - 4x + 6$. If $f'(x) = g'(x)$ and $f(1) = 2$, then what is $f(x)$ equal to? *[2009-II]*
(a) $x^3 - 4x + 3$ (b) $x^3 - 4x + 6$
(c) $x^3 - 4x + 1$ (d) $x^3 - 4x + 5$

35. If $y = \sin^{-1}\left(\frac{4x}{1+4x^2}\right)$, then what is $\frac{dy}{dx}$ equal to? *[2010-I]*
(a) $\frac{1}{1+4x^2}$ (b) $-\frac{1}{1+4x^2}$
(c) $\frac{4}{1+4x^2}$ (d) $\frac{4x}{1+4x^2}$

36. What is the differentiation of $\log_x x$ with respect to $\ln x$? *[2010-I]*
(a) 0 (b) 1
(c) $1/x$ (d) x

37. What is the derivative of $x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$? *[2010-I]*
(a) $\sqrt{a^2 - x^2}$ (b) $2\sqrt{a^2 - x^2}$
(c) $\sqrt{x^2 - a^2}$ (d) $2\sqrt{x^2 - a^2}$

38. If $x = t^2$, $y = t^3$, then what is $\frac{d^2y}{dx^2}$ equal to? *[2010-I]*
(a) 1 (b) $\frac{3}{2t}$
(c) $\frac{3}{4t}$ (d) $\frac{3}{2}$

39. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$? *[2010-II]*
(a) $\tan^2 x$ (b) $\cot^2 x$
(c) – 1 (d) 1

40. If $x = k(\theta + \sin\theta)$ and $y = k(1 + \cos\theta)$, then what is the derivative of y with respect to x at $\theta = \pi/2$? *[2010-II]*
(a) – 1 (b) 0
(c) 1 (d) 2

41. If $\sqrt{x} + \sqrt{y} = 2$, then what is $\frac{dy}{dx}$ at y = 1 and x = 1 equal to? *[2010-II]*
(a) 5 (b) 2
(c) 4 (d) – 1

42. If $x = \cos(2t)$ and $y = \sin^2 t$, then what is $\frac{d^2y}{dx^2}$ equal to? *[2010-II]*
(a) 0 (b) sin (2t)
(c) – cos (2t) (d) $-\frac{1}{2}$

43. If $f(x) = 2^x$, then what is $f''(x)$ equal to? *[2011-I]*
(a) $2^x (\ln 2)^2$ (b) $x(x-1)2^{x-2}$
(c) $2^{x+1} (\ln 2)$ (d) $2^x (\log_{10} 2)^2$

44. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$, then what is $\frac{dy}{dx}$ equal to? *[2011-II]*
(a) 1 (b) – 1
(c) x (d) $x^{\frac{1}{2}}$

45. If $y = \ln\sqrt{\tan x}$, then what is the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$? *[2011-II]*
(a) 0 (b) – 1
(c) 1 / 2 (d) 1

46. If $f(x) = x^2 - 6x + 8$ and there exists a point c in the interval [2, 4] such that $f'(c) = 0$, then what is the value of c? *[2012-I]*
(a) 2.5 (b) 2.8
(c) 3 (d) 3.5

47. If $y = \frac{x+1}{x-1}$, then what is $\frac{dy}{dx}$ equal to? *[2012-I]*
(a) $\frac{-2}{x-1}$ (b) $\frac{-2}{(x-1)^2}$
(c) $\frac{2}{(x-1)^2}$ (d) $\frac{2}{(x-1)}$

48. If $y = \cos t$ and $x = \sin t$, then what is $\frac{dy}{dx}$ equal to? *[2012-I]*
(a) xy (b) x/y
(c) – y/x (d) – x/y

49. If $x^m + y^m = 1$ such that $\frac{dy}{dx} = -\frac{x}{y}$, then what should be the value of m ? *[2012-II]*
(a) 0 (b) 1
(c) 2 (d) None of the above

50. Consider the following statements : *[2012-II]*
1. If $y = \ell n(\sec x + \tan x)$, then $\frac{dy}{dx} = \sec x$.
2. If $y = \ell n(\text{cosec } x - \cot x)$, then $\frac{dy}{dx} = \text{cosec } x$.
Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

51. If $f(x) = 2^{\sin x}$, then what is the derivative of f (x) ? *[2012-II]*
(a) $2^{\sin x} \ell n\, 2$ (b) $(\sin x)2^{\sin x-1}$
(c) $(\cos x)2^{\sin x-1}$ (d) None of the above

52. If $y = \ell n(e^{mx} + e^{-mx})$, then what is $\frac{dy}{dx}$ at x = 0 equal to ? *[2012-II]*
(a) – 1 (b) 0
(c) 1 (d) 2

53. If $2x^3 - 3y^2 = 7$, what is $\frac{dy}{dx}$ equal to $(y \neq 0)$? *[2013-I]*
(a) $\frac{x^2}{2y}$ (b) $\frac{x}{2y}$
(c) $\frac{x^2}{y}$ (d) None of the above

54. The derivative of $|x|$ at x = 0 *[2013-I]*
(a) is 1 (b) is – 1
(c) is 0 (d) does not exist

55. If $y = \sin(ax + b)$, then what is $\frac{d^2y}{dx^2}$ at $x = -\frac{b}{a}$, where a, b are constants and $a \neq 0$?
(a) 0 (b) –1
(c) sin (a – b) (d) sin (a + b)

56. If $y = x^x$, what is $\frac{dy}{dx}$ at x = 1 equal to ? *[2013-I]*
(a) 0 (b) 1
(c) – 1 (d) 2

57. What is the differential coefficient of $\log_x x$? *[2013-I]*
(a) 0 (b) 1
(c) $\frac{1}{x}$ (d) x

58. The derivative of $\sec^2 x$ with respect to $\tan^2 x$ is *[2013-I]*
(a) 1 (b) 2
(c) 2 sec x tan x (d) $2\sec^2 x \tan x$

59. What is the derivative of x^3 with respect to x^2 ? *[2013-II]*
(a) $3x^2$ (b) $\frac{3x}{2}$
(c) x (d) $\frac{3}{2}$

60. If $f(x) = 2x^2 + 3x - 5$, then what is $f'(0) + 3f'(-1)$ equal to ? *[2013-II]*
(a) –1 (b) 0
(c) 1 (d) 2

61. What is the derivative of $\sin(\sin x)$? *[2013-II]*
(a) $\cos(\cos x)$ (b) $\cos(\sin x)$
(c) $\cos(\sin x)\cos x$ (d) $\cos(\cos x)\cos x$

62. What is the derivative of $|x-1|$ at $x = 2$? *[2013-II]*
(a) –1 (b) 0
(c) 1 (d) Derivative does not exist

63. What is the derivative of $\sqrt{\frac{1+\cos x}{1-\cos x}}$? *[2014-I]*
(a) $\frac{1}{2}\sec^2\frac{x}{2}$ (b) $-\frac{1}{2}\text{cosec}^2\frac{x}{2}$
(c) $-\text{cosec}^2\frac{x}{2}$ (d) None of these

64. If $z = f\,o\,f(x) = x^2$ where $f(x) = x^2$, then what is $\frac{dz}{dx}$ equal to ? *[2014-I]*
(a) x^3 (b) $2x^3$
(c) $4x^3$ (d) $4x^2$

DIRECTIONS (Qs. 65-67) : *For the next two (02) items that follow* **:**

Consider the curve $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

65. What is $\frac{dy}{dx}$ equal to ? [2014-II]
    (a) $\tan\theta$ (b) $\cot\theta$
    (c) $\sin 2\theta$ (d) $\cos 2\theta$

66. What is $\frac{d^2y}{dx^2}$ equal to ? [2014-II]
    (a) $\sec^2\theta$ (b) $-\text{cosec}^2\theta$
    (c) $\frac{\sec^3\theta}{a\theta}$ (d) None of these

67. If $y = x\ln x + xe^x$, then what is the value of $\frac{dy}{dx}$ at $x = 1$? [2014-II]
    (a) $1 + e$ (b) $1 - e$
    (c) $1 + 2e$ (d) None of these

DIRECTIONS (Qs. 68-69): *For the next two (2) items that follow.*

Given that $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = Ax + B.$ [2015-I]

68. What is the value of A?
    (a) -1 (b) 1
    (c) 2 (d) 4

69. What is the value of B?
    (a) -1 (b) 1
    (c) 2 (d) 4

70. What is the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x$? [2015-I]
    (a) 0 (b) $\frac{1}{2}$
    (c) 1 (d) x

71. The derivative of ln(x+ sin x) with respect to (x+ cos x) is [2015-II]
    (a) $\frac{1+\cos x}{(x+\sin x)(1-\sin x)}$ (b) $\frac{1-\cos x}{(x+\sin x)(1+\sin x)}$
    (c) $\frac{1-\cos x}{(x-\sin x)(1+\cos x)}$ (d) $\frac{1+\cos x}{(x-\sin x)(1-\cos x)}$

72. If $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, where $0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to [2015-II]
    (a) $\frac{1}{2}$ (b) 2
    (c) $\sin x + \cos x$ (d) $\sin x - \cos x$

73. If $x^a y^b = (x-y)^{a+b}$, then the value of $\frac{dy}{dx} - \frac{y}{x}$ is equal to [2015-II]
    (a) $\frac{a}{b}$ (b) $\frac{b}{a}$
    (c) 1 (d) 0

74. If $s = \sqrt{t^2+1}$, then $\frac{d^2s}{dt^2}$ is equal to [2015-II]
    (a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$
    (c) $\frac{1}{s^3}$ (d) $\frac{1}{s^4}$

75. $\int\frac{dx}{1+e^{-x}}$ is equal to [2015-II]
    (a) $1 + e^x + c$ (b) $\ln(1+e^{-x}) + c$
    (c) $\ln(1+e^x) + c$ (d) $2\ln(1+e^{-x}) + c$
    where c is the constant of integration

DIRECTIONS (Qs. 76-79): *For the next four (4) items that follow.*

Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that

$f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$

for $x \in \mathbf{R}$ [2016-I]

76. What is f(1) equal to?
    (a) -2 (b) -1
    (c) 0 (d) 4

77. What is $f'(1)$ equal to?
    (a) -6 (b) -5
    (c) 1 (d) 0

78. What is $f'''(10)$ equal to?
    (a) 1 (b) 5
    (c) 6 (d) 8

79. Consider the following:
    1. $f(2) = f(1) - f(0)$
    2. $f''(2) - 2f'(1) = 12$

    Which of the above is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

80. If $y = \log_{10}x + \log_x 10 + \log_x x + \log_{10}10$ then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to? [2016-I]
    (a) 10 (b) 2
    (c) 1 (d) 0

DIRECTIONS (Qs. 81-83): *For the next three (3) items that follow.*

Consider the following for the next three (03) items that follow:

Let $f(x) = \left[|x| - |x-1|\right]^2$ [2016-II]

81. What is f'(x) equal to when x>1?
    (a) 0 (b) $2x - 1$
    (c) $4x - 2$ (d) $8x - 4$

82. What is f'(x) equal to when 0< x <1?
(a) 0 (b) $2x-1$
(c) $4x-2$ (d) $8x-4$

83. Which of the following equations is/are correct?
1. $f(-2)=f(5)$
2. $f''(-2)+f''(0.5)+f''(3)=4$
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

84. Let $f(x+y)=f(x)f(y)$ for all x and y. Then what is $f'(5)$ equal to [where $f'(x)$ is the derivative of $f(x)$]? *[2017-I]*
(a) $f(5)f'0$ (b) $f(5)-f'(0)$
(c) $f(5)f(0)$ (d) $f(5)+f'(0)$

85. What is the derivative of $\log_{10}\left(5x^2+3\right)$ with respect to x? *[2017-I]*
(a) $\frac{x\log_{10}e}{5x^2+3}$ (b) $\frac{2x\log_{10}e}{5x^2+3}$
(c) $\frac{10x\log_{10}e}{5x^2+3}$ (d) $\frac{10x\log_e 10}{5x^2+3}$

86. If $y=(\cos x)^{(\cos x)^{(\cos x)^{\cdot^{\cdot^{\infty}}}}}$, then $\frac{dy}{dx}$ is equal to *[2017-II]*
(a) $-\frac{y^2\tan x}{1-y\ln(\cos x)}$ (b) $\frac{y^2\tan x}{1+y\ln(\cos x)}$
(c) $\frac{y^2\tan x}{1-y\ln(\sin x)}$ (d) $\frac{y^2\sin x}{1+y\ln(\sin x)}$

87. If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is equal to *[2017-II]*
(a) 0 (b) 1
(c) $\frac{x-1}{x+1}$ (d) $\frac{x+1}{x-1}$

88. If $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to *[2017-II]*
(a) $-\frac{2}{1+x^2}$ for all $|x|<1$
(b) $-\frac{2}{1+x^2}$ for all $|x|>1$
(c) $\frac{2}{1+x^2}$ for all $|x|<1$
(d) None of the above

89. What is the derivative of the function *[2018-I]*
$f(x)=e^{\tan x}+ln(\sec x)-e^{\ell n x}$ at $x=\frac{\pi}{4}$?
(a) $\frac{e}{2}$ (b) e
(c) 2e (d) 4e

90. If $y=e^{x^2}\sin 2x$, then what is $\frac{dy}{dx}$ at $x=\pi$ equal to? *[2018-I]*
(a) $(1+\pi)e^{\pi^2}$ (b) $2\pi e^{\pi^2}$
(c) $2e^{\pi^2}$ (d) e^{π^2}

91. If $y=\tan^{-1}\left(\frac{5-2\tan\sqrt{x}}{2+5\tan\sqrt{x}}\right)$, then what is $\frac{dy}{dx}$ equal to? *[2018-II]*
(a) $-\frac{1}{2\sqrt{x}}$ (b) 1
(c) -1 (d) $\frac{1}{2\sqrt{x}}$

92. What is $\frac{d\sqrt{1-\sin 2x}}{dx}$ equal to, where $\frac{\pi}{4}<x<\frac{\pi}{2}$? *[2018-II]*
(a) $\cos x+\sin x$ (b) $-(\cos x+\sin x)$
(c) $\pm(\cos x+\sin x)$ (d) None of the above

93. If $f(x)=\sin(\cos x)$, then $f'(x)$ is equal to *[2019-I]*
(a) $\cos(\cos x)$ (b) $\sin(-\sin x)$
(c) $(\sin x)\cos(\cos x)$ (d) $(-\sin x)\cos(\cos x)$

94. If $f(x)=\frac{x-2}{x+2}, x\neq -2$, then what is $f^{-1}(x)$ equal to? *[2019-I]*
(a) $\frac{4(x+2)}{x-2}$ (b) $\frac{x+2}{4(x-2)}$
(c) $\frac{x+2}{x-2}$ (d) $\frac{2(1+x)}{1-x}$

ANSWER KEY																			
1	**(b)**	11	**(c)**	21	**(d)**	31	**(a)**	41	**(d)**	51	**(d)**	61	(c)	71	(a)	81	(a)	91	(a)
2	**(c)**	12	**(a)**	22	**(d)**	32	**(d)**	42	**(a)**	52	**(b)**	62	(c)	72	(a)	82	(d)	92	(a)
3	**(b)**	13	**(b)**	23	**(a)**	33	**(b)**	43	**(a)**	53	**(c)**	63	(b)	73	(d)	83	(a)	93	(d)
4	**(c)**	14	**(b)**	24	**(b)**	34	**(d)**	44	**(b)**	54	**(d)**	64	(c)	74	(c)	84	(a)	94	(d)
5	**(c)**	15	**(a)**	25	**(b)**	35	**(c)**	45	**(d)**	55	**(a)**	65	(a)	75	(c)	85	(c)		
6	**(b)**	16	**(d)**	26	**(a)**	36	**(a)**	46	**(c)**	56	**(b)**	66	(c)	76	(d)	86	(a)		
7	**(b)**	17	**(a)**	27	**(a)**	37	**(a)**	47	**(b)**	57	**(a)**	67	(c)	77	(b)	87	(a)		
8	**(d)**	18	**(a)**	28	**(b)**	38	**(c)**	48	**(d)**	58	**(a)**	68	(c)	78	(c)	88	(a)		
9	**(a)**	19	**(c)**	29	**(d)**	39	**(c)**	49	**(c)**	59	(b)	69	(a)	79	(c)	89	(c)		
10	**(d)**	20	**(c)**	30	**(d)**	40	**(a)**	50	**(c)**	60	(b)	70	(b)	80	(d)	90	(c)		

HINTS & SOLUTIONS

1. (b) $u = \sin^{-1}(x - y)$ and $x = 3t,\ y = 4t^3$

So, $u = \sin^{-1}(3t - 4t^3)$

Let $t = \sin\theta \Rightarrow \theta = \sin^{-1} t$,

So, $u = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta) = 3\theta$

Hence, $u = 3\sin^{-1} t$

$$\frac{du}{dt} = 3\frac{1}{\sqrt{1-t^2}} = 3(1-t^2)^{-1/2}$$

2. (c) Given that $x = \cos t,\ y = \sin t$

$$\Rightarrow \frac{dx}{dt} = -\sin t \text{ and } \frac{dy}{dt} = \cot t$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{\cos t}{\sin t} \Rightarrow \frac{dy}{dx} = -\cot t$$

$$\Rightarrow \frac{d^2y}{dx^2} = \operatorname{cosec}^2 t.\frac{dt}{dx} = \operatorname{cosec}^2 t.\frac{1}{-\sin t} = -\frac{1}{\sin^3 t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y^{-3}$$

3. (b) Given that $y = x + e^x$

Differentiating w. r. t. x

$$\frac{dy}{dx} = 1 + e^x$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x}$$

Differentiating w. r. t. y

$$\frac{d^2x}{dy^2} = -\frac{(1)(e^x)}{(1+e^x)^2}.\frac{dx}{dy}$$

$$= -\frac{e^x}{(1+e^x)^2}.\frac{1}{(1+e^x)} = -\frac{e^x}{(1+e^x)^3}$$

4. (c) Given that $f(x) = x|x|$

$f'(x) = x.\frac{|x|}{x} + |x|$ [Since $f'(uv) = u.f'(v) + v.f'(u)$]

$= |x| + |x| = 2|x|$ Here, $u = x,\ v = |x|$

5. (c) Given that $x + y = t - \frac{1}{t}$ and $x^2 + y^2 = t^2 + \frac{1}{t^2}$

$\therefore\ (x + y)^2 = x^2 + y^2 + 2xy$

$$\Rightarrow \left(t - \frac{1}{t}\right)^2 = \left(t^2 + \frac{1}{t^2}\right) + 2xy$$

$-2 = 2xy \Rightarrow xy = -1$

$(x - y)^2 = (x + y)^2 - 4xy$

$$= \left(t - \frac{1}{t}\right)^2 - 4 \times -1 = t^2 + \frac{1}{t^2} - 2 + 4 = \left(t + \frac{1}{t}\right)^2$$

$$x - y = t + \frac{1}{t}$$

$$\Rightarrow x = t,\ y = -\frac{1}{t}$$

$xy = -1$

$\Rightarrow \quad x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x} = \frac{1}{t^2} = \frac{1}{x^2}$

6. (b) Here, $\frac{d(\sqrt{1-x^2})}{d(\sin^{-1}x)} = \frac{\frac{d}{dx}(\sqrt{1-x^2})}{\frac{d}{dx}(\sin^{-1}x)}$ is to be found.

$f(x) = \sqrt{1-x^2}$

So, $f'(x) = \frac{d}{dx}(\sqrt{1-x^2}) = f'(x)\frac{1}{2\sqrt{1-x^2}}(-2x)$

$\Rightarrow \quad f'(x) = \frac{-x}{\sqrt{1-x^2}}$ ... (i)

Also, $g(x) = \sin^{-1}x$

$g'(x) = \frac{1}{\sqrt{1-x^2}}$ ... (ii)

$\therefore \quad \frac{f'(x)}{g'(x)} = \frac{-x/\sqrt{1-x^2}}{1/\sqrt{1-x^2}}$ (using Eqs. (i) and (ii))

$= -x$

7. (b) The given function,

$f(x) = (\log_{\tan x} \cot x)(\log_{\cot x} \tan x)^{-1}$

$= \left(\frac{\log \cot x}{\log \tan x}\right)\left(\frac{\log \tan x}{\log \cot x}\right)^{-1}$

$= \left(\frac{\log \cot x}{\log \tan x}\right)\left(\frac{\log \cot x}{\log \tan x}\right)$

$= \left(\frac{\log \cot x}{\log \tan x}\right)^2 = \left(\frac{\log \frac{1}{\tan x}}{\log \tan x}\right)^2 = \left(-\frac{\log \tan x}{\log \tan x}\right)^2 = 1$

$\Rightarrow \quad f(x) = 1$ (constant function).

$\Rightarrow \quad f'(x) = 0$

and this is true for $0 < x < \frac{\pi}{2}$

$\Rightarrow \quad f'\left(\frac{\pi}{4}\right) = 0$

8. (d) The given function $f(x) = \cos^{-1}\left\{\frac{2\cos x + 3\sin x}{\sqrt{13}}\right\}$

can be written as

$\cos^{-1}\left\{\cos x.\frac{2}{\sqrt{13}} + \sin x.\frac{3}{\sqrt{13}}\right\}$ ...(i)

Let $\frac{2}{\sqrt{13}} = \cos\theta$ and $\frac{3}{\sqrt{13}} = \sin\theta$

$\Rightarrow \tan\theta = \frac{\frac{3}{\sqrt{13}}}{\frac{2}{\sqrt{13}}} = \frac{3}{2}$

So, (i), $\cos^{-1}(\cos x \cos\theta + \sin x \sin\theta)$

where $\theta = \tan^{-1}\left(\frac{3}{2}\right)$

$= \cos^{-1}(\cos(x-\theta)) = x - \theta$

hence, $f'(x) = 1$ ($\because$ θ is a constant)

9. (a) Given function is $f(x) = \frac{7x}{(2x-1)(x+3)}$

Breaking into partial fraction

We get, $f(x) = \frac{1}{2x-1} + \frac{3}{x+3}$

Differentiating w.r.t. x, we get

$f'(x) = -\frac{2}{(2x-1)^2} - \frac{3}{(x+3)^2}$

10. (d) Given differential equation is

$\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \quad \frac{dy}{dx} = (1+x)(1+y^2)$

$\Rightarrow \quad \frac{dy}{1+y^2} = (1+x)\,dx$

$\Rightarrow \quad \tan^{-1} y = \frac{x^2}{2} + x + c$

Given that when $x = 0$, $y(0) = 0$. Hence, $c = 0$

$\Rightarrow \quad y = \tan\left(\frac{x^2}{2} + x\right)$

11. (c) According to given relation.

$\because \quad f(x^2) = x^3$

Putting $x = \sqrt{x}$

$\Rightarrow \quad f(x) = x^{3/2}$

Differentiating both the sides,

$\Rightarrow \quad f'(x) = \frac{3}{2}x^{1/2}$

$\Rightarrow \quad f'(4) = \frac{3}{2}.4^{1/2} = \frac{3}{2}(2) = 3$

12. (a) Let $y=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)=\tan^{-1}\frac{\sqrt{x}-x}{1+\sqrt{x}.x}$

$=\tan^{-1}\sqrt{x}-\tan^{-1}x$

On differentiating w.r.t.x, we get

$$\frac{dy}{dx}=\frac{1}{1+x}.\frac{1}{2\sqrt{x}}-\frac{1}{1+x^2}$$

Now, $\left(\frac{dy}{dx}\right)_{x=1}=\frac{1}{1+1}.\frac{1}{2}-\frac{1}{1+1}=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}.$

13. (b) Given equation

$x\sqrt{1+y}+y\sqrt{1+x}=0$

Can be written as :

$x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y)=y^2(1+x)$

$\Rightarrow x^2+x^2y=y^2+y^2x \quad \Rightarrow x^2-y^2=y^2x-x^2y$

$\Rightarrow (x-y)(x+y)=-xy(x-y)$

$\Rightarrow x+y=-xy \quad \Rightarrow y(1+x)=-x$

$y=\frac{-x}{1+x}$ which is in explicit form.

Differentiating w.r.t.x, we get

$$\frac{dy}{dx}=\frac{(1+x)(-1)+x(1)}{(1+x)^2}=\frac{-1}{(1+x)^2}$$

14. (b) As given, $y=f(x)$, $p=\frac{dy}{dx}$ and $q=\frac{d^2y}{dx^2}$

$$\frac{dx}{dy}=\frac{1}{p}\Rightarrow\frac{d^2x}{dy^2}=\frac{-1}{p^2}\cdot\frac{dp}{dy}$$

$$\frac{dp}{dx}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}=q$$

$$\frac{dp}{dy}=\frac{dp}{dx}.\frac{dx}{dy}=q.\frac{1}{p}=\frac{q}{p}$$

$$\frac{d^2x}{dy^2}=\Rightarrow-\frac{1}{p^2}\times\frac{q}{p}=\frac{-q}{p^3}$$

15. (a) As given :

$x=\sin t-t\cos t$ and $y=t\sin t+\cos t$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt}=\cos t-\{\cos t+t(-\sin t)\}$$

$$\Rightarrow\frac{dx}{dt}=\cos t-\cos t+t\sin t=t\sin t$$

and, $\frac{dy}{dt}=t\cos t+\sin t-\sin t=t\cos t$

Hence, $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{t\cos t}{t\sin t}=\cot t$

$$\Rightarrow\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{2}}=\cot\frac{\pi}{2}=0$$

16. (d) Given function is :

$y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$

On differentiating, w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-1+x^2}}.\frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}=0$$

17. (a) The given funcion is : $f(x)=\log_e[\log_e x]$

Differentiating w.r.t. x, we get

$$f'(x)=\frac{1}{\log_e x}.\frac{1}{x}\Rightarrow f'(e)=\frac{1}{\log_e e}.\frac{1}{e}=\frac{1}{e}=e^{-1}$$

18. (a) Given funcion is : $f(x)=e^{\sin(\log\cos x)}$

Differentiating w.r.t. x

$$f'(x)=e^{\sin(\log\cos x)}.\cos(\log\cos x).\frac{1}{\cos x}(-\sin x)$$

$$=-e^{\sin(\log\cos x)}.\cos(\log\cos x).\tan x$$

and $g(x)=\log\cos x$

$$\therefore g'(x)=\frac{1}{\cos x}(-\sin x)=-\tan x$$

Hence,

$$\frac{f'(x)}{g'(x)}=\frac{-e^{\sin(\log\cos x)}.\cos(\log\cos x).\tan x}{-\tan x}$$

$$=e^{\sin(\log\cos x)}.\cos(\log\cos x)$$

$=f(x).\cos[g(x)]$

19. (c) Given function : $\sqrt{x}+\sqrt{y}=1$

is an implicit function

Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

$$\Rightarrow\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$$

Value of $\frac{dy}{dx}$ at $x=\frac{1}{4},y=\frac{1}{4}$

$$\left(\frac{dy}{dx}\right)_{\left(\frac{1}{4},\frac{1}{4}\right)} = -\sqrt{\frac{1/4}{1/4}} = -1$$

20. (c) Differentiating the given function, $y = \frac{1}{\log_{10} x}$

We get, $\frac{dy}{dx} = -\frac{1}{(\log_{10} x)^2}.\frac{1}{x}\log_{10} e$

$\Rightarrow \frac{dy}{dx} = -\frac{(\log_x 10)^2.\log_{10} e}{x}$

21. (d) $y = \sin(m\sin^{-1} x)$

Then, $\frac{dy}{dx} = \cos(m\sin^{-1} x)\frac{m}{\sqrt{1-x^2}}$

$\therefore \frac{d^2y}{dx^2} = \cos(m\sin^{-1} x).m\left\{\frac{-1}{2}.\frac{(-2x)}{(1-x^2)^{3/2}}\right\}$

$+\frac{m}{\sqrt{1-x^2}}.\{-\sin(m\sin^{-1} x)\}.\frac{m}{\sqrt{1-x^2}}$

$= \frac{m}{\sqrt{1-x^2}}\left[\frac{x}{(1-x^2)}\cos(m\sin^{-1} x)\right.$

$\left.-\frac{m}{\sqrt{1-x^2}}\sin(m\sin^{-1} x)\right]$

Now, $\frac{d^2y}{dx^2}$ at $x = 0$ is $m[0-0] = 0$ $\quad (\because \sin^{-1} 0 = 0)$

22. (d) $x^y = e^{x-y}$

Taking log both sides, we get

$\Rightarrow y.\log x = x - y$

$\Rightarrow y = \frac{x}{1+\log x}$

$\Rightarrow \frac{dy}{dx} = \frac{(1+\log x) - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$

$= \frac{(1+\log x)-1}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$

23. (a) Let $\frac{dy}{dx} = 1 + x + y + xy$

$\Rightarrow \frac{dy}{dx} = (1+x)(1+y)$

$\Rightarrow \frac{dy}{1+y} = dx(1+x)$

$\Rightarrow \int\frac{1}{1+y}dy = \int(1+x)dx$

$\Rightarrow \log(1+y) = x + \frac{x^2}{2} + c$

Given

At $x = -1, y = 0$

$\Rightarrow \log 1 = -1 + \frac{1}{2} + c$

$\Rightarrow c = \frac{1}{2}$

$\therefore \log(1+y) = x + \frac{x^2}{2} + \frac{1}{2} = \frac{(1+x)^2}{2}$

$\Rightarrow 1 + y = e^{\frac{(1+x)^2}{2}}$

$\Rightarrow y = e^{\frac{(1+x)^2}{2}} - 1$

24. (b) Given $f(x) = \tan x + e^{-2x} - 7x^3$

On differentiating w.r.t. x, we get

$f'(x) = \sec^2 x - 2e^{-2x} - 21x^2$

Put $x = 0$

$\Rightarrow f'(0) = \sec^2 0 - 2e^0 - 21\times 0 = 1 - 2 = -1$

25. (b) $3^x + 3^y = 3^{x+y}$

On differentiating w.r.t. x, we get

$3^x \log 3 + 3^y \log 3\frac{dy}{dx} = 3^{x+y}\log 3\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \log 3\left[3^x + 3^y\frac{dy}{dx}\right] = \log 3\cdot 3^{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}(-3^{x+y} + 3^y) = 3^{x+y} - 3^x$

$= 3^x.3^y - 3^x = 3^x(3^y - 1)$

$\Rightarrow \frac{dy}{dx} = \frac{3^x(3^y-1)}{3^y(1-3^x)} = \frac{3^{x-y}(3^y-1)}{(1-3^x)}$

26. (a) Given $f(x) = \sin^2 x^2$

$\therefore f'(x) = 2\sin x^2 \cos x^2.2x$

$= 4x \sin x^2 \cos x^2$

27. (a) Given $f(x) = \cos x$ and $g(x) = \log x$

Consider $y = gof(x)$

$= g\{f(x)\}$

$= \log(f(x))$

$= \log(\cos x)$

$\therefore \quad \frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) = -\tan x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = -\tan 0 = 0$

28. (b) Given $e^y + xy = e$

On differentiating w.r.t. x, we get

$e^y \frac{dy}{dx} + y + x\frac{dy}{dx} = 0$ ...(i)

At $x = 0$ we get $e^y + 0 . y = e \Rightarrow e^y = e \Rightarrow y = 1$

$\therefore$ By putting $y = 1$ in equation (i)

we get

$e\frac{dy}{dx} + 1 + 0 = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{e}$

Again differentiating Eq. (i), we get

$e^y \frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

$\Rightarrow \frac{d^2y}{dx^2}(e^y + x) + e^y\left(\frac{dy}{dx}\right)^2 + \frac{2dy}{dx} = 0$

Now, At $x = 0, y = 1$

$\frac{d^2y}{dx^2}(e+0) + e\left(-\frac{1}{e}\right)^2 + 2\left(-\frac{1}{e}\right) = 0$

$\Rightarrow e\frac{d^2y}{dx^2} - \frac{1}{e} = 0$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{e^2} = e^{-2}$

29. (d) Let $\sqrt{1-x^2} + \sqrt{1-y^2} = a$

On differentiating w.r.t. x, we get

$\frac{1}{2\sqrt{1-x^2}}(-2x) + \frac{1(-2y)}{2\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{-x}{\sqrt{1-x^2}} - \frac{y}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{-x}{\sqrt{1-x^2}} = \frac{y}{\sqrt{1-y^2}}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}\sqrt{\frac{1-y^2}{1-x^2}}$

30. (d) Let $x = \log t$ and $y = t^2 - 1$

$x = \log t$

$\Rightarrow 2x = 2\log t$

$\Rightarrow 2x = \log t^2$

$\Rightarrow 2x = \log(y+1) \Rightarrow e^{2x} = y + 1$

On differentiating w.r.t. x, twice, we get

$e^{2x}2 = \frac{dy}{dx} \Rightarrow 4e^{2x} = \frac{d^2y}{dx^2}$

At $t = 1, x = 0$

$\frac{d^2y}{dx^2} = 4e^{2(0)} = 4 \qquad (\because e^0 = 1)$

31. (a) Let $u_1 = \log_x 5$ and $u_2 = \log_5 x$

$\Rightarrow u_1 = \frac{\log_e 5}{\log_e x}$ and $u_2 = \frac{\log_e x}{\log_e 5}$

On differentiating w.r.t. x, we get

$\frac{du_1}{dx} = \left[\frac{\log_e x(0) - \left(\frac{1}{x}\right)}{(\log_e x)^2}\right]\log_e 5 = -\frac{\log_e 5}{x(\log_e x)^2}$

and $\frac{du_2}{dx} = \frac{1}{x\log_e 5}$

$\therefore \frac{du_1}{du_2} = \frac{du_1/dx}{du_2/dx} = -\frac{\log_e 5}{x(\log_e x)^2} \times x\log_e 5$

$= -\left(\frac{\log_e 5}{\log_e x}\right)^2 = -(\log_x 5)^2 = -(\log_5 x)^{-2}$

32. (d) Given,

$f'(x) = 6 - 4\sin 2x$ and $f(0) = 3$

Consider $f'(x) = 6 - 4\sin 2x$

Integrate both sides w.r.t x

$\int f'(x)dx = \int(6 - 4\sin 2x)dx$

$f(x) = 6x - \frac{4(-\cos 2x)}{2} + c$

Where 'c' is constant of integration

$\Rightarrow f(x) = 6x + 2\cos 2x + c$

By using $f(0) = 3$, we have

$3 = f(0) = 6.0 + 2\cos 0 + c$

$\Rightarrow 3 = 2 + c \Rightarrow c = 1$

Hence, $f(x) = 6x + 2\cos 2x + 1$

33. (b) Let $f(x) = e^x$, $g(x) = \log x$

Consider $(g \circ f)(x) = g[f(x)]$

$= \log f(x)$ (By defn of $g(x)$)

$= \log(e^x)$ $(\because f(x) = e^x)$

$= x$ $(\because \log e = 1)$

Now, $(gof)'(x) = 1$

34. (d) Given, $g(x) = x^3 - 4x + 6$

But $f'(x) = g'(x)$

$\Rightarrow \int f'(x)dx = \int g'(x)dx$

$\Rightarrow f(x) = g(x) + c$

$\therefore f(x) = x^3 - 4x + 6 + c$ where 'c' is a constant.

Now, $f(1) = 2$

$\Rightarrow 2 = f(1) = (1)^3 - 4(1) + 6 + c$

$\Rightarrow 2 = 1 - 4 + 6 + c$

$\Rightarrow c = -1$

$\therefore f(x) = x^3 - 4x + 6 - 1 = x^3 - 4x + 5$

35. (c) Let $y = \sin^{-1}\left(\frac{4x}{1+4x^2}\right) = \sin^{-1}\left(\frac{2.2x}{1+(2x)^2}\right)$

Put $2x = \tan\theta \Rightarrow \theta = \tan^{-1}2x$

$$\therefore\ y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta) = 2\theta \left(\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= 2\tan^{-1}2x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2}{1+(2x)^2}.2 = \frac{4}{1+4x^2}$$

ALTERNATE SOLUTION

$$y = \sin^{-1}\left(\frac{4x}{1+4x^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{4x}{1+4x^2}\right)^2}} \times \frac{(1+4x^2)4 - 4x(8x)}{(1+4x^2)^2}$$

$$= \frac{1+4x^2}{\sqrt{(1+4x^2)^2 - 16x^2}} \times \frac{4+16x^2-32x^2}{(1+4x^2)^2}$$

$$= \frac{4-16x^2}{(1+4x^2)\sqrt{1-8x^2+16x^4}}$$

$$= \frac{4-16x^2}{(1+4x^2)(1-4x^2)}$$

$$= \frac{(2)^2-(4x)^2}{(1+4x^2)(1-2x)(1+2x)} = \frac{(2+4x)2}{(1+4x^2)(1+2x)} = \frac{4}{1+4x^2}$$

36. (a) Let $u = \log_x x = 1$ $(\because \log_a a = 1)$

Differentiate w.r.t 'x'

$$\frac{du}{dx} = 0$$

and Let $v = \ln x \Rightarrow \frac{dv}{dx} = \frac{1}{x}$

$$\therefore\ \frac{du}{dv} = \frac{du/dx}{dv/dx} = 0$$

37. (a) Let $y = x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)$

Differentiate both side w.r.t 'x' we get

$$\frac{dy}{dx} = x\frac{1}{2\sqrt{a^2-x^2}}.(-2x) + \frac{a^2}{\sqrt{1-\frac{x^2}{a^2}}}.\frac{1}{a}$$

$$= \frac{-x^2}{\sqrt{a^2-x^2}} + \frac{a^2.a}{\sqrt{a^2-x^2}}.\frac{1}{a}$$

$$= \frac{a^2-x^2}{\sqrt{a^2-x^2}} = \sqrt{a^2-x^2}$$

38. (c) Let $x = t^2$ and $y = t^3$

$$\Rightarrow\ \frac{dx}{dt} = 2t \text{ and } \frac{dy}{dt} = 3t^2$$

$$\therefore\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t$$

$$\Rightarrow\ \frac{d^2y}{dx^2} = \frac{3}{2}.\frac{dt}{dx} = \frac{3}{2}.\frac{1}{2t} \left(\because \frac{dx}{dt} = 2t\right)$$

$$= \frac{3}{4t}$$

39. (c) Let $u = \sin^2 x$ and $v = \cos^2 x$

$$\Rightarrow\ \frac{du}{dx} = 2\sin x\cos x = \sin 2x$$

and $\frac{dv}{dx} = -2\sin x\cos x = -\sin 2x$

$$\therefore\ \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-\sin 2x}{\sin 2x} = -1$$

40. (a) Let $x = k(\theta + \sin\theta)$ and $y = k(1 + \cos\theta)$

Differentiate both the functions w.r.t. 'θ'

$$\Rightarrow\ \frac{dx}{d\theta} = k(1+\cos\theta)$$

and $\frac{dy}{d\theta} = -k\sin\theta$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$

$= \dfrac{-k\sin\theta}{k(1+\cos\theta)} = \dfrac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = -\tan\dfrac{\theta}{2}$

($\because \sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$)

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = -\tan\dfrac{\pi}{4} = -1$

41. (d) Let $\sqrt{x} + \sqrt{y} = 2$

Differentiate w.r.t.x, we get

$\dfrac{1}{2\sqrt{x}} + \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx} = 0$ .....(1)

Put y = 1, x = 1 in equation (1)

$\dfrac{1}{2} + \dfrac{1}{2}\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = -1$

42. (a) Let $x = \cos 2t$ and $y = \sin^2 t$

Differentiate both the functions w.r.t. 't'

$\dfrac{dx}{dt} = -2\sin 2t$ and $\dfrac{dy}{dt} = 2\sin t\cos t = \sin 2t$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\sin 2t}{-2\sin 2t} = -\dfrac{1}{2}$

$\Rightarrow \dfrac{d^2y}{dx^2} = 0$

43. (a) Let $f(x) = 2^x$

On differentiating w.r.t. x, we get

$f'(x) = 2^x(\ln 2)$

On again differentiating w.r.t. x, we get

$f''(x) = 2^x(\ln 2)^2$

44. (b) Let $y = \left(1 + x^{1/4}\right)\left(1 + x^{1/2}\right)\left(1 - x^{1/4}\right)$

$= \left(1 + x^{1/4}\right)\left(1 - x^{1/4}\right)\left(1 + x^{1/2}\right)$

$= \left(1 - x^{1/2}\right)\left(1 + x^{1/2}\right)$

$\left(\because (a+b)(a-b) = a^2 - b^2\right)$

$= (1-x)\ \left(\because (a+b)(a-b) = a^2 - b^2\right)$

$\Rightarrow y = 1 - x$

Differentiate both side w.r.t 'x' $\dfrac{dy}{dx} = -1$

45. (d) Let $y = \ln\sqrt{\tan x}$

Differentiate both side w.r.t 'x'

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\tan x}} \cdot \dfrac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$

Now, $\dfrac{dy}{dx}$ at $x = \pi/4$

$= \dfrac{1}{\sqrt{\tan\frac{\pi}{4}}} \times \dfrac{1}{2\sqrt{\tan\frac{\pi}{4}}} \times \dfrac{1}{\cos^2(\pi/4)}$

$= \dfrac{1}{2} \times 1 \times \dfrac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = \dfrac{1}{2} \times 1 \times 2 = 1$

46. (c) Let $f(x) = x^2 - 6x + 8$

$f'(x) = 2x - 6$

$f'(c) = 2c - 6$

$f'(c) = 0$

$\Rightarrow 2c - 6 = 0 \Rightarrow c = 3$

47. (b) Let $y = \dfrac{x+1}{x-1}$

Differentiate both the side w.r.t 'x'

$\dfrac{dy}{dx} = \dfrac{(x-1)(1) - (x+1)1}{(x-1)^2}$

$= \dfrac{x - 1 - x - 1}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

48. (d) Let $y = \cos t$, $x = \sin t$

$\dfrac{dy}{dt} = -\sin t$, $\dfrac{dx}{dt} = \cos t$

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = -\dfrac{\sin t}{\cos t} = -\dfrac{x}{y}$

49. (c) Let $x^m + y^m = 1$

Differentiate both the sides w.r.t 'x'

$m.\, x^{m-1} + m.\, y^{m-1}.\dfrac{dy}{dx} = 0$

$\Rightarrow mx^{m-1} = -my^{m-1}.\dfrac{dy}{dx}$

$\Rightarrow \dfrac{x^{m-1}}{y^{m-1}} = -\dfrac{dy}{dx}$

$\Rightarrow \quad \left(\frac{x}{y}\right)^{m-1} = \frac{x}{y} \qquad \left(\because \frac{dy}{dx} = -\frac{x}{y}\right)$

$\Rightarrow \quad m - 1 = 1 \Rightarrow m = 2$

50. (c) (1) $y = \ell n\,(\sec x + \tan x)$

$$\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\sec x\,(\sec x + \tan x)}{\sec x + \tan x} = \sec x$$

$y = \ell n(\mathrm{cosec}\, x - \cot x)$

$$\frac{dy}{dx} = \frac{\mathrm{cosec}\, x(\mathrm{cosec}\, x - \cot x)}{\mathrm{cosec}\, x - \cot x} = \mathrm{cosec}\, x$$

Hence, both statements are correct.

51. (d) Let $f(x) = 2^{\sin x}$.

$f'(x) = 2^{\sin x}.\ \ell n\, 2 \cos x.$

52. (b) y = ln(emx + e–mx)

Þ $\frac{dy}{dx} = \frac{1}{e^{mx} + e^{-mx}} \cdot \frac{d}{dx}$(emx + e–mx)

$$= \frac{me^{mx} - me^{-mx}}{e^{mn} + e^{-mx}} = \frac{m(e^{mx} - e^{-mx})}{e^{mx} + e^{-mx}}$$

$$= \frac{m\left[e^{mn} - \frac{1}{e^{mx}}\right]}{e^{mx} + \frac{1}{x^{nx}}} = \frac{m[e^{2mx} - 1]}{e^{2mx} + 1}$$

so, $\left.\frac{dy}{dx}\right|_{x=0} = \frac{m(e^0 - 1)}{e^0 + 1} = m(0) = 0$

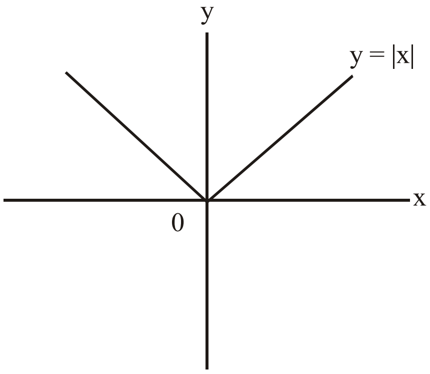
53. (c) Let $2x^3 - 3y^2 = 7$

Differentiate both side, w.r.t. 'x'

$$6x^2 - 6y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{y}$$

54. (d)



Since $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

$\therefore$ R. H. D of |x| = 1 at x = 0

and L. H. D of |x| = –1 at x = 0

Now, R. H. D $\neq$ L. H. D at x = 0

Hence, the derivative of |x| at x = 0 does not exist.

55. (a) Let $y = \sin(ax + b)$

$$\Rightarrow \frac{dy}{dx} = a \cos(ax + b)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -a^2 \sin(ax + b)$$

Now, $\frac{d^2y}{dx^2}$ at $x = -\frac{b}{a}$ is

$$-a^2 \sin\left(a\left(-\frac{b}{a}\right) + b\right) = -a^2 \sin 0 = 0$$

56. (b) Let $y = x^x$

Take log on both the sides

$\Rightarrow \ell n y = x \cdot \ell n x.$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = 1 + \ell n\, x \qquad \Rightarrow \frac{1}{y}.\frac{dy}{dx} = \ell n\, e + \ell n\, x$

$\Rightarrow \frac{dy}{dx} = y\ \ell n\, ex = = (x^x)\ell n\, ex$

$\left.\frac{dy}{dx}\right|_{x=1} = 1.\ \ \ell n\, e = 1$

57. (a) Let $y = \log_x x$

$\Rightarrow \quad y = 1$ (for $x > 0$ and $x \neq 1$)

On differentiating both the side w.r.t 'x', we get $\frac{dy}{dx} = 0$

58. (a) Let $u = \sec^2 x,\ v = \tan^2 x$

To find : $\frac{du}{dv}$.

Now, $\frac{du}{dx} = 2\ \sec x.\ \sec x.\ \tan x$

and $\frac{dv}{dx} = 2 \tan x.\ \sec^2 x$

Thus, $\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = \frac{2 \sec x.\ \sec x.\ \tan x}{2 \tan x \sec^2 x} = 1$

59. (b) $U = x^3$

$\frac{dU}{dx} = 3x^2$ ...(1)

$V = x^2$

$\frac{dV}{dx} = 2x$ ...(2)

From (1) and (2)

$\frac{dU}{dV} = \frac{3x^2}{2x} = \frac{3}{2}x$

60. (b) $F(x) = 2x^2 + 3x - 5$

$F'(x) = 4x + 3$

$F'(0) + 3F'(-1) = 3 + 3(-4 + 3) = 0$

61. (c) $\frac{d}{dx}\sin(\sin x) = \cos(\sin x).\cos x$

62. (c) $f(x) = |x - 1|$

Redefined the function f(x)

$f(x) = \begin{cases} 1-x, & x<1 \\ x-1, & x>1 \end{cases}$

$f'(x) = \begin{cases} -1; & x<1 \\ 1; & x>1 \end{cases}$

$\therefore f'(2) = 1$

63. (b) Let $y = \sqrt{\frac{1+\cos x}{1-\cos x}} = \frac{\sqrt{2}\cos\frac{x}{2}}{\sqrt{2}\sin\frac{x}{2}} = \cot\frac{x}{2}$

$\frac{dy}{dx} = -\text{cosec}^2\frac{x}{2}.\frac{1}{2} = -\frac{1}{2}\text{cosec}^2\frac{x}{2}$

64. (c) $z = fof(x) = f(x^2) = x^4$

$\frac{dz}{dx} = 4x^3$

65. (a) Given, $x = a(\cos\theta + \theta\sin\theta)$ and

$y = a(\sin\theta - \theta\cos\theta)$

$\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta) = a\theta\cos\theta$

$\frac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta) = a\theta\sin\theta$

Now $\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$

66. (c) Given $x = a(\cos\theta + \theta\sin\theta)$

$y = a(\sin\theta - \theta\cos\theta)$ and we have $\frac{dy}{dx} = \tan\theta$

According to question $\frac{d^2y}{dx^2} = \sec^2\theta\,\frac{d\theta}{dx}$

$= \sec^2\theta\left(\frac{1}{a\theta\cos\theta}\right)$

$\left[\because \frac{dx}{d\theta} = a\theta\cos\theta\right]$

Hence, $\frac{d^2y}{dx^2} = \frac{\sec^3\theta}{a\theta}$

67. (c) $y = x\ln x + xe^x$

After differentiating both sides with respect to x,

we get $\frac{dy}{dx} = x.\frac{1}{x} + \log x + xe^x + e^x$

or $= 1 + \log x + xe^x + e^x$

Therefore $\left(\frac{dy}{dx}\right)_{x=1} = 1 + \log 1 + 1.e^1 + e^1 = 1 + 2e$

$[\because \log 1 = 0]$

Sol. (68-69)

Given $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right)$

$= \frac{d}{dx}\left[\frac{1+x+x^2+x^4-x}{1+x+x^2}\right]$

$= \frac{d}{dx}\left[1+\frac{x^4-x}{x^2+x+1}\right]$

$= \frac{d}{dx}\left[1+\frac{x(x^3-1)}{x^2+x+1}\right]$

$= \frac{d}{dx}\left[1+x(x-1)\right]$

$= \frac{d}{dx}\left[1+x^2-x\right] = 2x - 1$ ... (i)

Now comparing equation (i) with AX + B, we get A = 2 and B = –1.

68. (c)

69. (a)

70. (b) Let $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ and $u = \tan^{-1}x$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

Then, $y = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right] = \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2},\ \cos\frac{\theta}{2}}\right]$$

$$\left(\because 1-\cos\theta = 2\sin^2\frac{\theta}{2} \text{ and } \sin x = 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right)$$

$$= \tan^{-1}\left[\tan\frac{\theta}{2}\right]$$

$$\Rightarrow\ y = \frac{\theta}{2} \Rightarrow y = \frac{\tan^{-1}x}{2} \qquad [\because \theta = \tan^{-1}x]$$

$$\Rightarrow\ y = \frac{u}{2}$$

$$\frac{dy}{du} = \frac{1}{2}$$

$\therefore$ Option (b) is correct.

71. (a) $\ln(x+\sin x) = y$ (say)

$$\frac{dy}{dx} = \frac{1}{(x+\sin x)}(1+\cos x)$$

$$= \frac{(1+\cos x)}{(x+\sin x)}$$

$x + \cos x = z$ (say)

$$\frac{dz}{dx} = (1-\sin x)$$

derivative of $\ln(x+\sin x)$ w.r.t $(x+\cos x)$ is

$$\frac{dy}{dz} = \frac{(1+\cos x)}{(x+\sin x)(1-\sin x)}$$

72. (a) $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$$y = \cot^{-1}\left[\frac{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}} + \sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}} - \sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}\right]$$

$$y = \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$$

$$y = \cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$$

$$y = \cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right] = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2}}$$

73. (d) $x^a y^b = (x-y)^{a+b}$

taking log both the sides.

$$\log\left(x^a y^b\right) = \log(x-y)^{(a+b)}$$

$a\log x + b\log y = (a+b)\log(x-y)$

differentiating both sides w.r.t 'x'.

$$\frac{a}{x}+\frac{b}{y}\frac{dy}{dx} = \frac{(a+b)}{(x-y)}\left[1-\frac{dy}{dx}\right]$$

$$\frac{dy}{dx}\left[\frac{b}{y}+\frac{a+b}{x-y}\right] = \frac{a+b}{x-y}-\frac{a}{x}$$

$$\frac{dy}{dx}\left[\frac{bx-by+ay+by}{y(x-y)}\right] = \frac{ax+bx-ax+ay}{x(x-y)}$$

$$\frac{dy}{dx}\left[\frac{bx+ay}{y}\right] = \frac{bx+ay}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\boxed{\frac{dy}{dx}-\frac{y}{x} = 0}$$

74. (c) $s = \sqrt{t^2+1}$

$$\Rightarrow \frac{ds}{dt} = \frac{t}{\sqrt{t^2+1}}$$

$$\Rightarrow \frac{d^2s}{dt^2} = \frac{1}{\sqrt{\left(t^2+1\right)^3}}$$

$$\Rightarrow \boxed{\frac{d^2s}{dt^2} = \frac{1}{s^3}}$$

75. (c) $\int \frac{dx}{1+e^{-x}}$

$\Rightarrow \int \frac{e^x}{e^x+1} dx$

Let $e^x + 1 = t$

$e^x dx = dt$

$= \int \frac{dt}{t}$

$\Rightarrow \log t + c \Rightarrow \log(e^x + 1) + c$

76. (d) $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ ...(1)

$f'(x) = 3x^2 + 2x f'(1) + f''(2)$ ...(2)

$f''(x) = 6x + 2f'(1)$ ...(3)

$f'''(x) = 6$ ...(4)

$f'(1) = 3 + 2f'(1) + f''(2)$ ...(5)

$f'(2) = 12 + 2f'(1)$ ...(6)

Using (6) in (5), we get

$f'(1) = 3 + 2f'(1) + 12 + 2f''(1)$

$-3f'(1) = 15$

$f'(1) = -5$

Using this value in eqn (6) we get

$f'(2) = 12 + 2 \times (-5)$

$f''(2) = 2$

Using $x = 3$ in eqn (4),

$f'''(3) = 6$

Putting value of $f'(1) + f''(2)$ and $f'''(3)$ in eqn (1)

We get

$f(x) = x^3 + x^2(-5) + x(2) + 6$

$= x^3 - 5x^2 + 2x + 6$

Putting $x = 1$

$f(1) = (1)^3 - 5(1)^2 + 2(1) + 6$

$f(1) = 4$

77. (b) $f'(1) = -5$

78. (c) $f'''(10) = 6$

79. (c) 1. $f(1) - f(0) = 4 - 6$

$= -2$

$f(2) = 8 - 20 + 4 + 6 = -2$

Hence $f(2) = f(1) - f(0)$.

$\therefore$ Statement (1) is correct.

2. $f''(2) - 2f'(1) = 2 - 2(-5)$

$f''(2) - 2f'(1) = 12$

$\therefore$ Statement (2) is correct.

80. (d) $y = \log_{10}x + \log_x 10 + \log_x x + \log_{10} 10$

$y = \log_{10}x + \log_x 10 + 1 + 1$

Differentiating equation w.r.t. x

$$\frac{dy}{dx} = \frac{1}{x\log_e 10} - \frac{1}{(\log_{10} x)^2} \cdot \frac{1}{(x\log 10)}$$

$$= \frac{1}{x\log_e 10}\left[1 - \frac{1}{(\log_{10} x)^2}\right]$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10\log_e 10}[1-1] = 0$$

$$\left[\begin{array}{l} \text{Note: } \log_x 10 = \frac{\log_{10} 10}{\log_{10} x} = \frac{1}{\log_{10} x} \\ \frac{d}{dx}\left[\frac{1}{\log_{10} x}\right] = -(\log_{10} x)^{-2} \times \frac{1}{x\log_e 10} \\ \qquad = -\frac{1}{(\log_{10} x)^2 x \log_e 10} \end{array}\right]$$

Sol. (Qs. 81 to 83)

Given $f(x) = (|x| - |x+1|)^2$

$$f(x) = \begin{cases} 1 & x \le 0 \\ (2x-1)^2 & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}$$

81. (a) When $x > 1$

$f(x) = 1$

$f'(x) = 0$

82. (d) When $0 < x < 1$

$f(x) = (2x-1)^2$

$f'(x) = 2(2x-1)\cdot 2 = 4(2x-1)$

$f'(x) = 8x - 4$

83. (a) For $x = -2$

$f(x) = 1$ so $f(-2) = 1$

For $x = 5$

$f(x) = 1 \Rightarrow f(5) = 1$

Hence $f(-2) = f(5)$

Now, for $x = -1$

$f''(x) = 0$

$f''(-2) = 0$

For $x = 0.5$

$f''(x) = 8 \Rightarrow f''(0.5) = 8$

For $x = 3$

$f''(x) = 0 \Rightarrow f''(3) = 0$

$\Rightarrow f''(-2) + f''(0.5) + f''(3) = 8 \ne 4$

Only statement 1 is correct.

84. (a) $f(x+y) = f(x)\cdot f(y)$

Let $f(x) = a^x$

$f(x+y) = a^{x+y} = a^x \cdot a^y = f(x) \cdot f(y)$.

$f(5) = a^5$

$f'(5) = a^5 \cdot \log a$

$= f(5) \cdot f'(0)$

$f'(0) = a^0 \cdot \log a$

$= \log a$

85\. (c) $y=\log_{10}(5x^2+3)$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\log_{10}\left(5x^2+3\right)\right)=\frac{1}{5x^2+3}\times\log_{10}e\times 10x$$

$$=\frac{10x\log_{10}e}{5x^2+3}$$

86\. (a) $y=(\cos x)^{(\cos x)^{\cos x\cdots}}$

$\Rightarrow y=(\cos x)^y$

$\Rightarrow \log y = y\,.\,\log\cos x$

Differentiating on both sides, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=y(-\tan x)+\log\cos x\cdot\frac{dy}{dx}$$

$$\Rightarrow\frac{dy}{dx}\left(\frac{1}{y}-\log\cos x\right)=-y\tan x$$

$$\Rightarrow\frac{dy}{dx}=\frac{-y\tan x}{\frac{1}{y}-\log\cos x}=\frac{-y^2\tan x}{1-y\log\cos x}$$

87\. (a) $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$=\cos^{-1}\left(\frac{x-1}{x+1}\right)+\sin^{-1}\left(\frac{x+1}{x-1}\right)$$

$$=\frac{\pi}{2}\quad\left(\because\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right)\quad\therefore\frac{dy}{dx}=0$$

88\. (a) $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x=\tan\theta.\Rightarrow\theta=\tan^{-1}x$

$$y=\cos^{-1}\left(\frac{2\tan\theta}{\left(1+\tan^2\theta\right)}\right)=\cos^{-1}(\sin 2\theta)$$

$$=\cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right)\quad=\frac{\pi}{2}-2\theta.$$

$$\therefore\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\pi}{2}-2\theta\right)$$

$$=\frac{d}{dx}\left(\frac{\pi}{2}-2\tan^{-1}x\right)$$

$$\frac{dy}{dx}=\frac{-2}{1+x^2},\ \text{when } |x|<1$$

89\. (c) $f(x)=e^{\tan x}+\ell n(\sec x)-e^{\ell n\, x}$

$$f'(x)=e^{\tan x}.\sec^2 x+\frac{1}{\sec x}.\sec x\tan x-1$$

$$\left(\because e^{\ln x}=x\text{ and }\frac{d}{dx}(x)=1\right)$$

$$f'\left(\frac{\pi}{4}\right)=e^{\tan\frac{\pi}{4}}.\sec^2\frac{\pi}{4}+\tan\frac{\pi}{4}-1$$

$=2e+1-1$

$=2e.$

90\. (c) $y=e^{x^2}.\sin 2x$

$$\frac{dy}{dx}=2.e^{x^2}.\cos 2x+2xe^{x^2}.\sin 2x$$

$$=2e^{x^2}(\cos 2x+x\sin 2x)$$

$$\left.\frac{dy}{dx}\right|_{x=\pi}=2e^{\pi^2}(\cos 2\pi+\pi.\sin 2\pi)$$

$$=2e^{\pi^2}(1+0)$$

$$=2.e^{\pi^2}$$

91\. (a) $y=\tan^{-1}\left(\frac{5-2\tan\sqrt{x}}{2+5\tan\sqrt{x}}\right)$

$$=\tan^{-1}\left(\frac{\frac{5}{2}-\tan\sqrt{x}}{1+\left(\frac{5}{2}\right)\tan\sqrt{x}}\right)$$

$$=\tan^{-1}\frac{5}{2}-\tan^{-1}\tan\sqrt{x}$$

$$=\tan^{-1}\frac{5}{2}-\sqrt{x}$$

$$\therefore\quad\frac{dy}{dx}=-\frac{1}{2\sqrt{x}}$$

92\. (a) For $\frac{\pi}{4}<x<\frac{\pi}{2}$,

$\sqrt{1-\sin 2x}=(\sin^2 x+\cos^2 x+2\sin x.\cos x)^{1/2}$

$=(\sin x-\cos x)$

$\therefore$ Differentiation $=\cos x+\sin x$

93\. (d) $f(x)=\sin(\cos x)$

$f'(x)=\cos(\cos x).(-\sin x)$

$=-\sin x.\cos(\cos x)$

94\. (d) $f(x)\,y=4\frac{x-2}{x+2}, x\neq-2$

$$\frac{y}{1}=\frac{x-2}{x+2}\Rightarrow\frac{y+1}{y-1}=\frac{x-2+x+2}{x-2-x-2}$$

$$\Rightarrow\frac{y+1}{y-1}=\frac{2x}{-4}$$

$$\Rightarrow\frac{y+1}{y-1}=\frac{-x}{2}\Rightarrow x=-2\left(\frac{y+1}{y-1}\right)$$

Now, $y=\frac{-2(x+1)}{x-1}=\frac{2(x+1)}{1-x}$

Application of Derivatives 16

1. Under what conditions is the tangent to a given curve at a point perpendicular to x-axis ?
(a) $\frac{dy}{dx}=0$ (b) $\frac{dy}{dx}=1$
(c) $\frac{dx}{dy}=0$ (d) $\frac{d^2y}{dx^2}=1$ *[2006-I]*

2. If $f(x)=(x-x_0)\,\phi(x)$ and $\phi(x)$ is continuous at $x=x_0$, then what is $f'(x_0)$ equal to ?
(a) $\phi'(x_0)$ (b) $\phi(x_0)$
(c) $x_0\phi(x_0)$ (d) $2\phi(x_0)$ *[2006-I]*

3. The sum of two numbers is 20. What are the numbers if the product of the square of one and the cube of the other is maximum?
(a) 6, 14 (b) 15, 5
(c) 12, 8 (d) 10, 10 *[2006-I]*

4. What is the slope of the normal at the point $(at^2, 2at)$ of the parabola $y^2=4ax$?
(a) $\frac{1}{t}$ (b) t
(c) $-t$ (d) $-\frac{1}{t}$ *[2006-I]*

5. Which one of the following statements is not correct ?
(a) The derivative of f (x) at x = a is the slope of the graph of f (x) at the point [a, f (a)]
(b) f (x) has a positive derivative at x = a means f (x) increases as x increases from 'a'
(c) The sum of two differentiable functions is differentiable
(d) If a function is continuous at a point, it is also differentiable at the same point. *[2006-II]*

6. Which one of the following statements is correct in respect of the curve $4y-x^2-8=0$?
(a) The curve is increasing in (– 4, 4)
(b) The curve is increasing in (– 4, 0)
(c) The curve is increasing in (0, 4)
(d) The curve is decreasing in (– 4, 4) *[2006-II]*

7. What is the minimum value of $px+qy$ $(p>0, q>0)$ when $xy=r^2$?
(a) $2r\sqrt{pq}$ (b) $2pq\sqrt{r}$
(c) $-2r\sqrt{pq}$ (d) $2rpq$ *[2006-II]*

8. What is /are the critical points(s) of the function $f(x)=x^{2/3}(5-2x)$ on the interval [–1, 2]?
(a) 1 only (b) 0, 1
(c) $\frac{3}{2}$ only (d) $0, \frac{3}{2}$ *[2007-I]*

9. Match List I with List II and select the correct answer using the code given below the lists:

	List I		List II
(a)	$f(x)=\cos x$	1.	The graph cuts y-axis in infinite number of points
(b)	$f(x)=\ln x$	2.	The graph cuts x -axis in two point
(c)	$f(x)=x^2-5x+4$	3.	The graph cuts y-axis in only one point
(d)	$f(x)=e^x$	4.	The graph cuts x-axis in only one point
		5.	The graph cuts x-axis in infinite number of points

Codes:

	(A)	(B)	(C)	(D)
(a)	1	4	5	3
(b)	1	3	5	4
(c)	5	4	2	3
(d)	5	3	2	4

[2007-I]

10. If x + y = 12, what is the maximum value of xy?
(a) 25 (b) 36
(c) 49 (d) 64 *[2007-I]*

11. What is the x-coordinate of the point on the curve $f(x)=\sqrt{x}\,(7x-6)$, where the tangent is parallel to x-axis?
(a) $-\frac{1}{3}$ (b) $\frac{2}{7}$
(c) $\frac{6}{7}$ (d) $\frac{1}{2}$ *[2007-I]*

12. If $\sin x\cos y=\frac{1}{2}$, then what is the value of $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$?
(a) –4 (b) –2
(c) –6 (d) 0 *[2007-I]*

13. What is the interval in which the function $f(x)\ \sqrt{9-x^2}$ is increasing? $(f(x)>0)$
(a) $0<x<3$ (b) $-3<x<0$
(c) $0<x<9$ (d) $-3<x<3$ *[2007-I]*

14. A wire 34 cm long is to be bent in the form of a quadrilateral of which each angle is 90°. What is the maximum area which can be enclosed inside the quadrilateral?
(a) 68 cm^2 (b) 70 cm^2
(c) 71.25 cm^2 (d) 72. 25 cm^2 *[2007-I]*

15. Which one of the following is correct ? The function $f(x)=(x-1)e^x+1$ is *[2007-II]*
(a) negative for all $x>0$ (b) positive for all $x>0$
(c) increasing for all x (d) decreasing for all x

16. The motion of a particle is described as $s=2-3t+4t^3$. What is the acceleration of the particle at the point where its velocity is zero?
(a) 0 (b) 4 unit
(c) 8 unit (d) 12 unit *[2007-II]*

17. What is the product of two parts of 20, such that the product of one part and the cube of the other is maximum?
(a) 75 (b) 91
(c) 84 (d) 96 *[2007-II]*

18. What is the maximum slope of the curve $y=-x^3+3x^2+2x-27$?
(a) 1 (b) 2
(c) 5 (d) -23 *[2007-II]*

19. What is the area of the largest rectangular field which can be enclosed with 200 m of fencing ?
(a) 1600 m^2 (b) 2100 m^2
(c) 2400 m^2 (d) 2500 m^2 *[2008-I]*

20. What is the smallest value of m for which $f(x)=x^2+mx+5$ is increasing in the interval $1\le x\le 2$?
(a) $m=0$ (b) $m=-1$
(c) $m=-2$ (d) $m=-3$ *[2008-I]*

21. What is the maximum value of x . y subject to the condition $x+y=8$?
(a) 8 (b) 16
(c) 24 (d) 32 *[2008-I]*

22. What is the equation of the curve whose slope at any point is equal to $2x$ and which passes through the origin? *[2008-II]*
(a) $y(1-x)=x^2$ (b) $y^2(1+x^2)=x^4$
(c) $y^2=(x+1)^2$ (d) $y=x^2$

23. What is the maximum value of the function $\log x-x$? *[2008-II]*
(a) -1 (b) 0
(c) 1 (d) ∞

24. A rectangular box with a cover is to have a square base. The volume is to be 10 cubic cm. The surface area of the box in terms of the side x is given by which one of the following functions? *[2008-II]*
(a) $f(x)=(40/x)+2x^2$ (b) $f(x)=(40/x)+x^2$
(c) $f(x)=(40/x)+x$ (d) $f(x)=(60/x)+2x$

25. $f(x)=\cos x$ is monotonic decreasing under which one of the following conditions? *[2008-II]*
(a) $0<x<\frac{\pi}{2}$ only (b) $\frac{\pi}{2}<x<\pi$ only
(c) $0<x<\pi$ (d) $0<x<2\pi$

26. What is the minimum value of $2x^2-3x+5$? *[2008-II]*
(a) 0 (b) 3/4
(c) 31/4 (d) 31/8

27. **Assertion (A) :** The tangent to the curve $y=x^3-x^2-x+2$ at (1, 1) is parallel to the x-axis.
Reason (R) : The slope of the tangent to the curve at (1, 1) is zero. *[2009-I]*
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

28. The function $f(x)=x^2-2x$ increases for all *[2009-I]*
(a) $x>-1$ only (b) $x<-1$ only
(c) $x>1$ only (d) $x<1$ only

29. Let a and b be two distinct roots of a polynomial equation $f(x)=0$. Then there exists at least one root lying between a and b of the polynomial equation. *[2009-I]*
(a) $f(x)=0$ (b) $f'(x)=0$
(c) $f''(x)=0$ (d) None of these

30. The profit function, in rupees, of a firm selling x items $x\ge 0$ per week is given by $P(x)=-3500+(400-x)x$. How many items should the firm sell so that the firm has maximum profit? *[2009-I]*
(a) 400 (b) 300
(c) 200 (d) 100

31. A stone thrown vertically upward satisfies the equation $s=64t-16t^2$, where s is in meter and t is in second. What is the time required to reach the maximum height? *[2009-I]*
(a) 1s (b) 2s
(c) 3s (d) 4s

32. If $f(x)=3x^2+6x-9$, then *[2009-I]*
(a) $f(x)$ is increasing in $(-1, 3)$
(b) $f(x)$ is decreasing in $(3, \infty)$
(c) $f(x)$ is increasing in $(-\infty, -1)$
(d) $f(x)$ is decreasing in $(-\infty, -1)$

33. If $x\cos\theta+y\sin\theta=2$ is perpendicular to the line $x-y=3$, then what is one of the value of θ? *[2009-I]*
(a) $\pi/6$ (b) $\pi/4$
(c) $\pi/2$ (d) $\pi/3$

34. The function $y=\tan^{-1}x-x$ *[2009-II]*
(a) is always decreasing
(b) is always increasing
(c) first increases and then decreases
(d) first decreases and then increases

35. The velocity v of a particle at any instant t moving in a straight line is given by $v = s + 1$ where s metre is the distance travelled in t second. What is the time taken by the particle to cover a distance of 9m? *[2009-II]*
(a) 1 s (b) (log 10) s
(c) 2 (log 10) s (d) 10 s

36. The velocity of telegraphic communication is given by $v = x^2 \log(1/x)$, where x is the displacement. For maximum velocity, x equals to? *[2009-II]*
(a) $e^{1/2}$ (b) $e^{-1/2}$
(c) $(2e)^{-1}$ (d) $2e^{-1/2}$

37. What is the maximum point on the curve $x = e^x y$?
(a) $(1, e)$ (b) $(1, e^{-1})$ *[2010-I]*
(c) $(e, 1)$ (d) $(e^{-1}, 1)$

38. A balloon is pumped at the rate of 4cm^3 per second. What is the rate at which its surface area increases and radius is 4 cm? *[2010-I]*
(a) 1 cm^2/s? (b) 2 cm^2/s
(c) 3 cm^2/s (d) 4 cm^2/s

39. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then which one of the following is correct?
(a) $k < 3$ (b) $k \le 3$ *[2010-I]*
(c) $k > 3$ (d) $k \ge 3$

40. Given two squares of sides x and y such that $y = x + x^2$. What is the rate of change of area of the second square with respect to the area of the first square? *[2010-I]*
(a) $1 + 3x + 2x^2$ (b) $1 + 2x + 3x^2$
(c) $1 - 2x + 3x^2$ (d) $1 - 2x - 3x^2$

41. **Statement I :** $y = -\tan^{-1}(x^{-1}) + 1$ is an increasing function of x.
Statement II : $\frac{dy}{dx}$ is positive for all values of x.
Which one of the following is correct in respect of the above statements? *[2010-I]*
(a) Both statements I and II are independently correct and statement II is the correct explanation of statement I
(b) Both statements I and II are independently correct but statement II is not the correct explanation of statement I
(c) Statement I is correct but statement II is false.
(d) Statement I is false but statement II is correct.

42. What is the least vlaue of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 2.5]$? *[2010-I]*
(a) –3 (b) 8
(c) –19 (d) –16.5

43. What is the interval over which the function $f(x) = 6x - x^2, x > 0$ is increasing ? *[2010-II]*
(a) $(0, 3)$ (b) $(3, 6)$
(c) $(6, 9)$ (d) None of these

44. If f and g are two increasing functions such that fog is defined, then which one of the following is correct? *[2010-II]*
(a) fog is always an increasing function
(b) fog is always a decreasing function
(c) fog is neither an increasing nor a decreasing function
(d) None of the above

45. For a point of inflection of $y = f(x)$, which one of the following is correct? *[2010-II]*
(a) $\frac{dy}{dx}$ must be zero
(b) $\frac{d^2y}{dx^2}$ must be zero
(c) $\frac{dy}{dx}$ must be non-zero
(d) $\frac{d^2y}{dx^2}$ must be non- zero

46. What is the value of p for which the function
$$f(x) = p \sin x + \frac{\sin 3x}{3}$$
has an extremum at $x = \frac{\pi}{3}$? *[2010-II]*
(a) 0 (b) 1
(c) –1 (d) 2

47. If at any instant t, for a sphere, r denotes the radius, S denotes the surface area and V denotes the volume, then what is $\frac{dV}{dt}$ equal to? *[2010-II]*
(a) $\frac{1}{2} S \frac{dr}{dt}$ (b) $\frac{1}{2} r \frac{dS}{dt}$
(c) $r \frac{dS}{dt}$ (d) $\frac{1}{2} r^2 \frac{dS}{dt}$

48. The function $f(x) = k \sin x + \frac{1}{3} \sin 3x$ has maximum value at $x = \frac{\pi}{3}$, what is the value of k? *[2011-I]*
(a) 3 (b) $\frac{1}{3}$
(c) 2 (d) $\frac{1}{2}$

49. Consider the following statements in respect of the function
$f(x) = x^3 - 1,\ x \in [-1, 1]$ *[2011-I]*
I. $f(x)$ is increasing in $[-1, 1]$
II. $f(x)$ has no root in $(-1, 1)$.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

50. The largest value of $2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 2$ occurs when *[2011-I]*
(a) $x = -2$ (b) $x = -1$
(c) $x = 2$ (d) $x = 0$

51. The function $y = f(x) = mx + c$ has *[2011-II]*
(a) maximum point but no minimum point
(b) minimum point but no maximum point
(c) both maximum and minimum points
(d) neither maximum point nor minimum point

52. At an extreme point of a function $f(x)$, the tangent to the curve is *[2011-II]*
 (a) parallel to the x-axis
 (b) perpendicular to the x-axis
 (c) inclined at an angle 45° to the x-axis
 (d) inclined at an angle 60° to the x-axis
53. The point in the interval $(0, 2\pi)$ where $f(x) = e^x \sin x$ has maximum slope is *[2011-II]*
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$
54. If the rate of change in volume of spherical soap bubble is uniform, then the rate of change of surface area varies as *[2011-II]*
 (a) square of radius
 (b) square root of radius
 (c) inversely proportional to radius
 (d) cube of the radius
55. If $f(x) = x \ell n x$, then $f(x)$ attains minimum value at which one of the following points? *[2011-II]*
 (a) $x = e^{-2}$ (b) $x = e$
 (c) $x = e^{-1}$ (d) $x = 2e^{-1}$
56. What are the points on the curve $x^2 + y^2 - 2x - 3 = 0$ where the tangents are parallel to x-axis? *[2011-II]*
 (a) $(1, 2)$ and $(1, -2)$ (b) $(0, \sqrt{3})$ and $(0, -\sqrt{3})$
 (c) $(3, 0)$ and $(-3, 0)$ (d) $(2, 1)$ and $(2, -1)$
57. Which one of the following statement is correct? *[2012-I]*
 (a) The derivative of a function f(x) at a point will exist if there is one tangent to the curve y = f(x) at that point and the tangent is parallel to y-axis
 (b) The derivative of a function f(x) at a point will exist if there is one tangent to the curve y = f(x) at that point and the tangent must be parallel to x-axis
 (c) The derivative of a function f(x) at a point will exist if there is one and only one tangent to the curve y= f[x] at that point and the tangent is not parallel to y-axis
 (d) None of the above
58. How many tangents are parallel to x-axis for the curve $y = x^2 - 4x + 3$? *[2012-I]*
 (a) 1
 (b) 2
 (c) 3
 (d) No tangent is parallel to x-axis
59. What is the rate of change of $\sqrt{x^2 + 16}$ with respect to x^2 at $x = 3$? *[2012-I]*
 (a) 1/5 (b) 1/10
 (c) 1/20 (d) None of the above
60. What is the slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at $t = 2$? *[2012-I]*
 (a) 7/6 (b) 6/7
 (c) 1 (d) 5/6
61. Which one of the following statement is correct? *[2012-I]*
 (a) e^x is an increasing function
 (b) e^x is a decreasing function
 (c) e^x is neither increasing nor decreasing function
 (d) e^x is a constant function
62. The radius of a circle is uniformly increasing at the rate of 3 cm/s. What is the rate of increase in area, when the radius is 10 cm? *[2012-II]*
 (a) $6\pi\ cm^2/s$ (b) $10\pi\ cm^2/s$
 (c) $30\pi\ cm^2/s$ (d) $60\pi\ cm^2/s$
63. The function $f(x) = x^3 - 3x^2 + 6$ is an increasing function for: *[2012-II]*
 (a) $0 < x < 2$ (b) $x < 2$
 (c) $x > 2$ or $x < 0$ (d) all x
64. What is the minimum value of $|x|$? *[2012-II]*
 (a) -1 (b) 0
 (c) 1 (d) 2
65. The function $f(x) = x^2 - 4x$, $x \in [0, 4]$ attains minimum value at *[2013-I]*
 (a) $x = 0$ (b) $x = 1$
 (c) $x = 2$ (d) $x = 4$
66. The curve $y = xe^x$ has minimum value equal to *[2013-I]*
 (a) $-\frac{1}{e}$ (b) $\frac{1}{e}$
 (c) $-e$ (d) e
67. The maximum value of the function $f(x) = x^3 + 2x^2 - 4x + 6$ exists at *[2013-II]*
 (a) $x = -2$ (b) $x = 1$
 (c) $x = 2$ (d) $x = -1$
68. The minimum value of the function $f(x) = |x - 4|$ exists at *[2013-II]*
 (a) $x = 0$ (b) $x = 2$
 (c) $x = 4$ (d) $x = -4$
69. What is the slope of the tangent to the curve $y = \sin^{1}(\sin^2 x)$ at $x = 0$? *[2014-I]*
 (a) 0 (b) 1
 (c) 2 (d) None of these

DIRECTIONS (Qs. 70-71): *For the next two (02) items that follow*

Consider the curve $y = e^{2x}$. *[2014-I]*

70. What is the slope of the tangent to the curve at (0, 1)?
 (a) 0 (b) 1
 (c) 2 (d) 4
71. Where does the tangent to the curve at (0, 1) meet the *x*-axis?
 (a) (1, 0) (b) (2, 0)
 (c) (–1/2, 0) (d) (1/2, 0)

DIRECTIONS (Qs. 72-73): *For the next two (02) items that follow*

Consider the function $f(x)\ \ \frac{x^2 - x\ \ 1}{x^2\ \ x\ \ 1}$ *[2014-I]*

72. What is the maximum value of the function?
 (a) 1/2 (b) 1/3
 (c) 2 (d) 3
73. What is the minimum value of the function?
 (a) 1/2 (b) 1/3
 (c) 2 (d) 3

DIRECTIONS (Qs. 74-75): *For the next two (02) items that follow*

A rectangular box is to be made from a sheet of 24 inch length and 9 inch width cutting out identical squares of side length x from the four corners and turning up the sides. *[2014-II]*

74. What is the value of x for width the vulume is maximum ?
(a) 1 inch (b) 1.5 inch
(c) 2 inch (d) 2.5 inch

75. What is the maximum volume of the box ?
(a) 200 cubic inch (b) 400 cubic inch
(c) 100 cubic inch (d) None of these

DIRECTIONS (Qs. 76-78): *For the next two (02) items that follow*

A cylinder is inscribed in a sphere of radius *r*. *[2014-II]*

76. What is the height of the cylinder of maximum volume ?
(a) $\frac{2r}{\sqrt{3}}$ (b) $\frac{r}{\sqrt{3}}$
(c) 2r (d) $\sqrt{3}r$

77. What is the radius of the cylinder of maximum volume ?
(a) $\frac{2r}{\sqrt{3}}$ (b) $\frac{\sqrt{2}r}{\sqrt{3}}$
(c) r (d) $\sqrt{3}r$

78. Consider the following statements: *[2015-I]*
1. $y = \frac{e^x + e^{-x}}{2}$ is an increasing function on $[0, \infty)$.
2. $y = \frac{e^x - e^{-x}}{2}$ is an increasing function on $(-\infty, \infty)$.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 79-80): *For the next two (02) items that follow*

Consider the function $f(x) = \frac{x^2 - 1}{x^2 + 1}$, where $x \in \mathbb{R}$ *[2015-I]*

79. At what value of x does f(x) attain minimum value?
(a) –1 (b) 0 (c) 1 (d) 2

80. What is the minimum value of f(x)?
(a) 0 (b) $\frac{1}{2}$ (c) –1 (d) 2

DIRECTION (Q. 81): *For the next one (01) item that follow*

Consider the function *[2015-I]*

$$f(x) = 0 \cdot 75x^4 - x^3 - 9x^2 + 7$$

81. What is the maximum value of the function?
(a) 1 (b) 3 (c) 7 (d) 9

82. Consider the following statements: *[2015-I]*
1. The function attains local minima at x = – 2 and x = 3.
2. The function increases in the interval (–2, 0).

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 83-85): *For the next three (03) items that follow*

Consider the parametric equation *[2015-I]*

$$x = \frac{a(1-t^2)}{1+t^2}, y = \frac{2at}{1+t^2}$$

83. What does the equation represent?
(a) It represents a circle of diameter a
(b) It represents a circle of radius a
(c) It represents a parabola
(d) None of the above

84. What is $\frac{dy}{dx}$ equal to?
(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $\frac{x}{y}$ (d) $-\frac{x}{y}$

85. What is $\frac{d^2y}{dx^2}$ equal to?
(a) $\frac{a^2}{y^2}$ (b) $\frac{a^2}{x^2}$ (c) $-\frac{a^2}{x^2}$ (d) $-\frac{a^2}{y^3}$

86. The function $f(x) = \frac{x^2}{e^x}$ monotonically increasing if *[2015-II]*
(a) x < 0 only (b) x > 2 only
(c) 0 < x < 2 (d) $x \in (-\infty, 0) \cup (2, \infty)$

87. Consider the following statements : *[2015-II]*
1. f(x) = ln x is an increasing function on $(0, \infty)$.
2. $f(x) = e^x - x(\ln x)$ is an increasing function on $(1, \infty)$.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 88-89): *For the next two (02) items that follow*

Consider the function *[2015-II]*

$$f(x) = \left(\frac{1}{x}\right)^{2x^2}, \text{ where } x > 0$$

88. At what value of x does the function attain maximum value? *[2015-II]*
(a) e (b) $\sqrt{e}$ (c) $\frac{1}{\sqrt{e}}$ (d) $\frac{1}{e}$

89. The maximum value of the function is *[2015-II]*
(a) e (b) $e^{\frac{2}{e}}$ (c) $e^{\frac{1}{e}}$ (d) $\frac{1}{e}$

DIRECTIONS (Qs. 90-91): *For the next two (02) items that follow*

Consider $f'(x) = \frac{x^2}{2} - kx \quad 1$ such that f(0) = 0 and f(3) = 15 *[2015-II]*

90. The value of k is
(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $-\frac{5}{3}$ (d) $-\frac{3}{5}$

91. $f''\left(-\frac{2}{3}\right)$ is equal to
(a) –1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

DIRECTIONS (Qs. 92-93): *For the next two (02) items that follow*

Consider the function *[2015-II]*

$f(x) = -2x^3 - 9x^2 - 12x + 1$

92. The function f(x) is an increasing function in the interval
(a) $(-2, -1)$ (b) $(-\infty, -2)$
(c) $(-1, 2)$ (d) $(-1, \infty)$

93. The function f(x) is a decreasing function in the interval
(a) $(-2, -1)$ (b) $(-\infty, -2)$ only
(c) $(-1, \infty)$ only (d) $(-\infty, -2) \cup (-1, \infty)$

DIRECTIONS (Qs. 94-96): *For the next three (03) items that follow*

Consider the function $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$ *[2016-I]*

94. What is the maximum value of the function $f(\theta)$?
(a) 1 (b) 2 (c) 2 (d) 4

95. What is the minimum value of the function $f(\theta)$?
(a) 0 (b) 1 (c) 2 (d) 3

96. Consider the following statements:
1. $f(\theta) = 2$ has no solution.
2. $f(\theta) = \frac{7}{2}$ has a solution.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 97-98): *For the next two (02) items that follow*

Consider the equation

$k \sin x + \cos 2x = 2k - 7$ *[2016-I]*

97. If the equation possesses solution, then what is the minimum value of k?
(a) 1 (b) 2 (c) 4 (d) 6

98. If the equation possesses solution, then what is the maximum value of k?
(a) 1 (b) 2 (c) 4 (d) 6

99. Which one of the following statements is correct in respect of the function $f(x) = x^3 \sin x$? *[2016-II]*
(a) It has local maximum at x = 0.
(b) It has local minimum at x = 0.
(c) It has neither maximum nor minimum at x = 0.
(d) It has maximum value 1.

100. The maximum value of $\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$ in the interval $\left(0, \frac{\pi}{2}\right)$ is attained at *[2017-I]*
(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

101. What is the length of the longest interval in which the function $f(x) = 3\sin x - 4\sin^3 x$ is increasing? *[2017-I]*
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

102. What is the maximum value of the function $f(x) = 4\sin^2 x + 1$? *[2017-I]*
(a) 5 (b) 3 (c) 2 (d) 1

103. Let $f(x) = x + \frac{1}{x}$, when $x \in (0, 1)$. Then which one of the following is correct? *[2017-I]*
(a) $f(x)$ fluctuates in the interval
(b) $f(x)$ increases in the interval
(c) $f(x)$ decreases in the interval
(d) None of the above

104. Consider the following statements : *[2017-II]*
1. $\frac{dy}{dx}$ at a point on the curve given slope of the tangent at that point.
2. If a(t) denotes acceleration of a particle, then $\int a(t)\,dt + c$ gives velocity of the particle.
3. If s(t) gives displacement of a particle at time t, then $\frac{ds}{dt}$ gives its acceleration at that instant.

Which of the above statements is/are correct?
(a) 1 and 2 only (b) 2 only
(c) 1 only (d) 1, 2 and 3

105. Which one of the following is correct in respect of the function $f(x) = x(x-1)(x+1)$? *[2017-II]*
(a) The local maximum value is larger than local minimum value
(b) The local maximum value is smaller than local minimum value
(c) The function has no local maximum
(d) The function has no local minimum

106. The maximum value of $\frac{\ln x}{x}$ is *[2017-II]*
(a) e (b) $\frac{1}{e}$
(c) $\frac{2}{e}$ (d) 1

107. Match List-I with List-II and select the correct answer using the code given below the lists : *[2017-II]*

List-I (Function)		**List-II (Maximum value)**	
A.	$\sin x + \cos x$	1.	$\sqrt{10}$
B.	$3\sin x + 4\cos x$	2.	$\sqrt{2}$
C.	$2\sin x + \cos x$	3.	5
D.	$\sin x + 3\cos x$	4.	$\sqrt{5}$

Code :

	A	B	C	D
(a)	2	3	1	4
(b)	2	3	4	1
(c)	3	2	1	4
(d)	3	2	4	1

108. A cylindrical jar without a lid has to be constructed using a given surface area of a metal sheet. If the capacity of the jar is to be maximum, then the diameter of the jar must be k times the height of the jar. The value of k is *[2017-II]*
(a) 1 (b) 2
(c) 3 (d) 4

109. The maximum value of *[2018-I]*
$\sin\left(x+\frac{\pi}{5}\right)+\cos\left(x+\frac{\pi}{5}\right)$, where $x \in \left(0, \frac{\pi}{2}\right)$, is attained at
(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{15}$
(c) $\frac{\pi}{10}$ (d) $\frac{\pi}{2}$

110. What is the maximum value of $16 \sin\theta - 12 \sin^2\theta$? *[2018-I]*
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{16}{3}$ (d) 4

111. Which one of the following is correct in respect of the function *[2018-II]*
$f(x) = x\sin x + \cos x + \frac{1}{2}\cos^2 x$?
(a) It is increasing in the interval $\left(0, \frac{\pi}{2}\right)$
(b) It remain constant in the interval $\left(0, \frac{\pi}{2}\right)$
(c) It is decreasing in the interval $\left(0, \frac{\pi}{2}\right)$
(d) It is decreasing in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

112. A flower in the form of a sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible area, then what is the radius of the sector? *[2018-II]*
(a) 25 m (b) 20 m (c) 10 m (d) 5 m

113. What is the minimum value of $[x(x-1)+1]^{\frac{1}{3}}$, where $0<x<1$? *[2018-II]*
(a) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$ (b) 1 (c) $\frac{1}{3}$ (d) $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

114. If $y = |\sin x|^{|x|}$, then what is the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$? *[2018-II]*
(a) $\frac{2^{-\frac{\pi}{6}}\left(6\,\ell n2-\sqrt{3}\pi\right)}{6}$
(b) $\frac{2^{\frac{\pi}{6}}\left(6\,\ell n2+\sqrt{3}\pi\right)}{6}$
(c) $\frac{2^{-\frac{\pi}{6}}\left(6\,\ell n2+\sqrt{3}\pi\right)}{6}$
(d) $\frac{2^{\frac{\pi}{6}}\left(6\,\ell n2-\sqrt{3}\pi\right)}{6}$

115. A given quantity of metal is to be cast into a half cylinder (i,e, with a rectangular base and semicircular ends). If the total surface area is to be minimum, then the ratio of the height of the half cylinder to the diameter of the semicircular ends is *[2019-I]*
(a) $\pi : (\pi+2)$ (b) $(\pi+2) : \pi$
(c) 1 : 1 (d) None of the above

ANSWER KEY

1	**(c)**	13	**(b)**	25	**(c)**	37	**(b)**	49	**(a)**	61	**(a)**	73	(b)	85	(d)	97	(b)	109	(a)
2	**(b)**	14	**(d)**	26	**(d)**	38	**(b)**	50	**(b)**	62	**(d)**	74	(c)	86	(c)	98	(d)	110	(c)
3	**(c)**	15	**(c)**	27	**(a)**	39	**(c)**	51	**(d)**	63	**(c)**	75	(a)	87	(c)	99	(b)	111	(a)
4	**(c)**	16	**(d)**	28	**(c)**	40	**(a)**	52	**(a)**	64	**(b)**	76	(a)	88	(c)	100	(a)	112	(c)
5	**(d)**	17	**(a)**	29	**(b)**	41	**(a)**	53	**(a)**	65	**(c)**	77	(b)	89	(c)	101	(a)	113	(a)
6	**(c)**	18	**(c)**	30	**(c)**	42	**(c)**	54	**(c)**	66	**(a)**	78	(c)	90	(c)	102	(a)	114	(a)
7	**(a)**	19	**(d)**	31	**(b)**	43	**(a)**	55	**(c)**	67	(a)	79	(b)	91	(d)	103	(c)	115	(a)
8	**(a)**	20	**(c)**	32	**(d)**	44	**(a)**	56	**(a)**	68	(c)	80	(c)	92	(a)	104	(a)		
9	**(c)**	21	**(b)**	33	**(b)**	45	**(b)**	57	**(c)**	69	(a)	81	(c)	93	(d)	105	(a)		
10	**(b)**	22	**(d)**	34	**(a)**	46	**(d)**	58	**(a)**	70	(c)	82	(c)	94	(d)	106	(b)		
11	**(b)**	23	**(a)**	35	**(b)**	47	**(b)**	59	**(b)**	71	(c)	83	(b)	95	(d)	107	(b)		
12	**(a)**	24	**(a)**	36	**(b)**	48	**(c)**	60	**(b)**	72	(d)	84	(d)	96	(c)	108	(b)		

HINTS & SOLUTIONS

1. (c) Tangent to a given curve at a point is perpendicular to x-axis gives $\frac{dy}{dx} = \tan\frac{\pi}{2}$

$\Rightarrow \frac{dx}{dy} = \cot\frac{\pi}{2} = 0$

2. (b) Given that $f(x) = (x - x_0)\,\phi(x)$
Differentiating w. r. t. x
$f'(x) = (x - x_0)\,\phi'(x) + \phi(x)(1)$
Putting $x = x_0$
$f'(x_0) = \phi(x_0)$

3. (c) Let the numbers are x and y.
So, $x + y = 20$; Let $P = x^2 y^3$ (As given)
$= x^2 (20 - x)^3$
Differentiating w. r. t. x

$\frac{dP}{dx} = x^2.3(20-x)^2(-1) + (20-x)^3.2x$
$= (20-x)^2[-3x^2 + 40x - 2x^2]$
$= (20-x)^2[40x - 5x^2]$

$\frac{d^2P}{dx^2} = (20-x)^2[40 - 10x] + (40x - 5x^2)\,2(20-x)(-1)$

$\frac{dp}{dx} = 0$ for maxima or minima.
So, $(20-x)^2[40x - 5x^2] = 0$
$\Rightarrow (20-x)^2 \times (x)(40 - 5x) = 0 \Rightarrow x = 20, 0, 8$

We get, $\left(\frac{d^2P}{dx^2}\right)_{x=8} < 0$; $\left(\frac{d^2p}{dx^2}\right)_{x=0} > 0$ and

$\left(\frac{d^2p}{dx^2}\right)_{x=20} = 0$

Hence, P is maximum at $x = 8$
and, Numbers are 12 and 8.

4. (c) Equation of parabola is
$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\therefore \frac{dy}{dx} = \frac{2a}{y}$, [slope of tangent]

So, slope of normal $= -\left(\frac{dx}{dy}\right)_{(at^2, 2at)}$

$= -\left(\frac{y}{2a}\right) = -\frac{2at}{2a} = -t$

5. (d) If a function is continuous at a point, it need not be differentiable at the same point. Example, $f(x) = |x|$ is continuous at $x = 0$ but $f(x)$ is not differentiable at $x = 0$

6. (c) Given that $4y - x^2 - 8 = 0$

$\Rightarrow y = \frac{x^2 + 8}{4}$

Differentiating w.r.t. x

$\frac{dy}{dx} = \frac{2x}{4}$

For increasing function

$\frac{dy}{dx} > 0$

So, $\frac{2x}{4} > 0 \Rightarrow x > 0$

Thus, the curve is increasing in (0, 4).

7. (a) Given that $xy = r^2$

$\Rightarrow y = \frac{r^2}{x}$

Let $S = px + qy = px + \frac{qr^2}{x}$

$\Rightarrow \frac{dS}{dx} = p - \frac{qr^2}{x^2}$

$\frac{dS}{dx} = 0$ for maximum or minimum.

So, $0 = p - \frac{qr^2}{x^2}$

$\Rightarrow x^2 = \frac{qr^2}{p} \Rightarrow x = \pm\sqrt{\frac{q}{p}}.r$

Now, $\frac{d^2S}{dx^2} = \frac{2qr^2}{x^3}$

At $x = +\sqrt{\frac{q}{p}}.r$ $\quad \frac{d^2S}{dx^2} > 0$

Hence, S is minimum at $x = \sqrt{\frac{q}{p}}.r$

$\Rightarrow y = \frac{r^2}{\sqrt{\frac{q}{p}}.r} = \sqrt{\frac{p}{q}}.r$

Minimum value of $px + qy = p.\sqrt{\frac{q}{p}}.r + q\sqrt{\frac{p}{q}}.r$

$= \sqrt{pq}\,r + \sqrt{pq}\,r = 2r\sqrt{pq}$

8. (a) $f(x) = x^{2/3}(5-2x)$

or, $f(x) = 5x^{2/3} - 2x^{5/3}$

differentiating both the sides.

$f'(x) = 5 \times \frac{2}{3}x^{-1/3} - 2 \times \frac{5}{3}x^{2/3}$

or, $f'(x) = \frac{10}{3}(x^{-1/3} - x^{2/3})$

To get the critical value.

$f'(x) = 0$

so, $x^{-1/3} - x^{2/3} = 0 \Rightarrow x^{-1/3}(1-x) = 0$

$\Rightarrow \quad 1 - x = 0$ as $x^{-1/3} \neq 0$

or, x = 1 is the only value in the interval $[-1, 2]$

9. (c) (A) Graph of $f(x) = \cos x$ cuts x-axis at infinite number of points. (5 of list II)

(B) Graph of $f(x) = \ln x$ cuts x-axis in only one point. (4 of list II)

(C) Graph of $f(x) = x^2 - 5x + 4$ cuts x axis in two points (2 of list II)

(D) Graph of $f(x) = e^x$ cuts y-axis in only one point. (3 of list II)

10. (b) Given $x + y = 12$

$y = 12 - x$

so, $xy = x(12 - x) = 12x - x^2$

Let $f(x) = 12x - x^2$

$f'(x) = 12 - 2x$

To get maximum or minimum value

$f'(x) = 0$ and $f''(x) < 0$ it is maximum

$f''(x) = -2 < 0$,

so, $f'(x) = 0$ will give maximum value.

so, $12 - 2x = 0 \Rightarrow x = 6$ and $x + y = 12 \Rightarrow y = 6$

Hence, $y = 6$

and $f(x) = 12x - x^2 = 12 \times 6 - 36 = 36$

11. (b) $f(x) = \sqrt{x}\,(7x - 6) = 7x^{3/2} - 6x^{1/2}$

$f'(x) = 7 \times \frac{3}{2}x^{1/2} - 6 \times \frac{1}{2}x^{-1/2}$

When tangent is parallel to x axis $f'(x) = 0$

or, $\frac{21}{2}x^{1/2} - 3x^{-1/2} \quad 0$

$\frac{21}{2}\sqrt{x} = \frac{3}{\sqrt{x}}$

or, $7x = 2 \Rightarrow x = \frac{2}{7}$

12. (a) $\sin x \cos y = \frac{1}{2}$

Differentiating both the sides

$\sin x(-\sin y)\frac{dy}{dx} + \cos y \cos x = 0$

$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y} = \cot x \cot y$

$\frac{d^2y}{dx^2} = \cot x(-\text{cosec}^2 y)\frac{dy}{dx} + \cot y(-\text{cosec}^2 x)$

when the point is $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

then $x = \frac{\pi}{4}$ and $y = \frac{\pi}{4}$

$\frac{dy}{dx} = \cot\frac{\pi}{4}.\cot\frac{\pi}{4} = 1$

So, $\frac{d^2y}{dx^2} = \cot\frac{\pi}{4}\left(-\text{cosec}^2\frac{\pi}{4}\right).1 + \cot\frac{\pi}{4}\left(-\text{cosec}^2\frac{\pi}{4}\right)$

$= 1(-2) + 1.(-2) = -4$

13. (b) $f(x) = \sqrt{9 - x^2}$

$f'(x) = \frac{1}{2\sqrt{9-x^2}} \times (-2x) = -\frac{x}{\sqrt{9-x^2}}$

For function to be increasing

$-\frac{x}{\sqrt{9-x^2}} > 0$

or $-x > 0$ or $x < 0$

but $\sqrt{9-x^2}$ is defined only when

$9 - x^2 > 0$ or $x^2 - 9 < 0$

$(x+3)(x-3) < 0$

i.e. $-3 < x < 3$

$-3 < x < 3 \cap x < 0$

$\Rightarrow -3 < x < 0$

14. (d) Let one side of quadrilateral be x and another side be y

so, $2(x + y) = 34$

or, $(x + y) = 17$ ...(i)

We know from the basic principle that for a given perimeter square has the maximum area, so, x = y and putting this value in equation (i)

$x = y = \frac{17}{2}$

Area $= x\,.\,y = \frac{17}{2} \times \frac{17}{2} = \frac{289}{4} = 72.25$

15. (c) The given function is

$f(x) = (x-1)e^x + 1$

$\Rightarrow \quad f'(x) = (x-1)e^x + e^x$

$= (x - 1 + 1)e^x = xe^x$

Thus, it is clear that f (x) is increasing for all x.

16. (d) Given rule is :

Distance, $s = 2 - 3t + 4t^3$

$\Rightarrow$ Velocity $\frac{ds}{dt} \quad -3 \quad 12t^2$

$\Rightarrow$ Acceleration $= \frac{d^2s}{dt^2} = 24t$

Since, velocity is zero

$\therefore \quad \frac{ds}{dt} \quad 0$

$\Rightarrow \quad 0 = -3 + 12t^2 \quad \Rightarrow \quad t = \sqrt{\frac{3}{12}} = \frac{1}{2}$

Acceleration (when velocity is zero)

$\Rightarrow \quad \frac{d^2s}{dt^2} = 24t = 24 \times \frac{1}{2} = 12$ unit

17. (a) Let 20 be divided in two parts such that first part = x

$\therefore$ Second part = 20 – x

Now, assume that

$P = x^3(20 - x)$

$= 20x^3 - x^4$

Now, $\frac{dP}{dx} = 60x^2 - 4x^3$

and $\frac{d^2P}{dx^2} = 120x - 12x^2$

Put $\frac{dP}{dx}$ 0 for maxima or minima

$\Rightarrow \frac{dP}{dx}$ 0

$\Rightarrow \quad 4x^2(15 - x) = 0 \quad \Rightarrow \quad x = 0, x = 15$

$\therefore \quad \left(\frac{d^2P}{dx^2}\right)_{x=15} = 120 \times 15 - 12 \times (225)$

$= 1800 - 2700 = -900 < 0$

$\therefore$ P is a maximum at x = 15.

$\therefore$ First part = 15

and second part = 20 – 15 = 5

Required product = 15 × 5 = 75

18. (c) The equation of curve is given as :

$y = -x^3 + 3x^2 + 2x - 27$

On differentiating w.r.t.x, we get

$\frac{dy}{dx} = -3x^2 \quad 6x \quad 2$

This represents slope of the curve at any point.

Let $A \quad \frac{dy}{dx} \quad -3x^2 \quad 6x \quad 2$

$\Rightarrow \frac{dA}{dx} = -6x \quad 6$

and $\frac{d^2A}{dx^2} = -6$

Put $\frac{dA}{dx}$ 0 for maxima or minima.

$-6x + 6 = 0$

$\Rightarrow$ x = 1

Now, $\left(\frac{d^2A}{dx^2}\right)_{x=1} \quad 1 = -6 < 0$

$\therefore$ A is maximum at x = 1

$\therefore$ Maximum slope of curve = – 3 + 6 + 2 = 5

19. (d) Area of largest rectangular field for a given perimeter for is possible if length and breadth of rectangular field are equal i.e. it is a square

$\Rightarrow 4x = 200 \quad \Rightarrow x = \frac{200}{4} = 50\text{m}$

$\therefore$ Area of largest rectangular field = 50 × 50 = 2500m^2

Aliter:

Let length and breadth of rectangular field be x and y respectively

$\therefore \quad 2(x + y) = 200$

$\Rightarrow \quad y = 100 - x$

and area, A = xy

$= x(100 - x) = 100x - x^2$

$\because \frac{dA}{dx} = 100 - 2x$

Put $\frac{dA}{dx} = 0$ for maxima or minima

$100 - 2x = 0$

$\Rightarrow \quad x = 50 \Rightarrow y = 50$

Now, $\frac{d^2A}{dx^2} = -2 < 0$, which shows maximum, indepenent of values of x and y, but only when they are equal.

$\therefore$ A is maximum at x = 50.

Hence, required area = 50 (100 – 50) = 50 × 50 = 2500m^2

20. (c) $f(x) = x^2 + mx + 5$

Consider, option (c) m = –2.

$f(x) = x^2 - 2x + 5 \quad \Rightarrow \quad f'(x) = 2x - 2$

It is clear that $f'(x) > 0$ in interval $1 \le x \le 2$.

$\therefore \quad m = -2$

21. (b) The given condition is :

$x + y = 8 \quad \Rightarrow y = 8 - x$

and let, the product of x and y be,

P = xy

$\Rightarrow P = x(8 - x) = 8x - x^2$

Differentiating w.r.t. x

$\Rightarrow \frac{dP}{dx} = 8 - 2x$

and $\frac{d^2P}{dx^2} = -2 < 0$

Put $\frac{dP}{dx} = 0$, for maxima or minima

$8 - 2x = 0$

$\Rightarrow x = \frac{8}{2} = 4$ and y = 4

and $\left(\frac{d^2P}{dx^2}\right) < 0$ when x = y

$\therefore$ P is maximum at x = 4

Maximum value of P = 4. (8 – 4) = 4.4 = 16

22. (d) Slope at the point $(x, y) = 2x$

$\therefore \frac{dy}{dx} = 2x$

$\Rightarrow \int dy = \int 2x\,dx$

$\Rightarrow y = x^2 + c$

Given that, it passes through the origin.

So, $0 = (0)^2 + c$

$\Rightarrow c = 0$

Then, $y = x^2$

23. (a) Let $y = \log x - x$

$\therefore \frac{dy}{dx} = \frac{1}{x} - 1$ and $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$

For maximum and minimum value of y

$\frac{dy}{dx} = \frac{1}{x} - 1 = 0 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

For $x = 1$, $\frac{d^2y}{dx^2} = -ve$

Thus, the value of given function is maximum for $x = 1$

So, the maximum value of the function $= \log(1) - 1 = -1$

24. (a) Let the height of rectangular box be y cm.

$\therefore$ Volume $= x \times x \times y$

$\Rightarrow y = \frac{10}{x^2}$ ...(i)

Now, surface area of box

$= 2(x^2 + xy + yx) = 2(x^2 + 2xy)$

$= 2\left(x^2 + \frac{20}{x}\right)$ From equation (i)

$= 2x^2 + \frac{40}{x}$

25. (c) Given, $f(x) = \cos x$,

$\Rightarrow f'(x) = -\sin x$

Now $f'(x) = 0$

$\Rightarrow -\sin x = 0$

$\Rightarrow \sin x = \sin(0)$

$\Rightarrow x = n\pi$

Clearly, $f'(x) < 0$, when $0 < x < \pi$

Hence $f(x)$ is decreasing when $0 < x < \pi$

26. (d) Let $y = 2x^2 - 3x + 5$

$\Rightarrow \frac{dy}{dx} = 4x - 3$

and $\frac{d^2y}{dx^2} = 4$

For maximum and minimum value of y

$\frac{dy}{dx} = 0$

$\Rightarrow 4x - 3 = 0$

$\Rightarrow x = \frac{3}{4}$

Since, for every value of $x, \frac{d^2y}{dx^2} = +ve$

So, minimum value of $2x^2 - 3x + 5$ is minimum

$x = \frac{3}{4}$.

$\therefore$ The minimum value $= 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) \quad 5$

$= \frac{9}{8} - \frac{9}{4} \quad 5 = \frac{9 - 18 \quad 40}{8} = \frac{31}{8}$

27. (a) Given equation of the curve is $y = x^3 - x^2 - x + 2$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 3 - 2 - 1 \quad 0$

Since, $\frac{dy}{dx}$ at (1, 1) is 0

$\therefore$ slope of the tangent = 0.

Hence, The equation of tangent at (1, 1) is

$y - 1 = 0(x - 1) \Rightarrow y = 1$

ie, parallel to x-axis.

Both A and R true and R is the correct explanation of A.

28. (c) Given $f(x) = x^2 - 2x$

On differentiating w.r.t 'x', we get $f'(x) = 2x - 2$

$f(x)$ is increasing, if $f'(x) > 0$

$\Rightarrow 2x - 2 > 0$

$\Rightarrow x > 1$

29. (b) Rolle's theorem says between any two roots of a polynomial $f(x)$, there is always a root of its derivative $f'(x)$

Therefore between a and b. There exist at least one root of the polynomial equation $f'(x) = 0$

30. (c) $P(x) = -3500 + (400 - x)x = -3500 + 400x - x^2$

On differentiating w.r.t. x, we get

$P'(x) = 400 - 2x$

Put $P'(x) = 0$ for maxima or minima

$\Rightarrow 400 - 2x = 0$

$\Rightarrow x = 200$

Now $P''(x) = -2x$

$\Rightarrow P''(200) = -400 < 0$

$\therefore P(x)$ is maximum at $x = 200$

Hence 200 items should the firm sell so that the firm has maximum profit.

31. (b) Given equation is $s = 64t - 16t^2$

$\therefore$ On differentiating w.r.t. t, we get

$\frac{ds}{dt} = 64 - 32t$

Put $\frac{ds}{dt} \quad 0$ for maximum height

$\Rightarrow 64 - 32t = 0 \Rightarrow t = 2$

Now, $\frac{d^2s}{dt^2} = -32$

At $t = 2$, $\frac{d^2s}{dt^2} = -32$

Since, $\left(\frac{d^2s}{dt^2}\right)_{t=2} < 0$

$\therefore$ Required time = 2 second

32. (d) Given $f(x) = 3x^2 + 6x - 9$
On differentiating w.r.t. x, we get
$f'(x) = 6x + 6$
$f'(x) < 0$
$\Rightarrow 6x + 6 < 0$
$\Rightarrow 6x < -6$
$\Rightarrow x < -1$
Hence $f(x)$ is decreasing in $(-\infty, -1)$

33. (b) Consider a line
$x\cos\theta + y\sin\theta \quad 2$
$\Rightarrow y\sin\theta = -x\cos\theta + 2$
$\Rightarrow y = -x\frac{\cos\theta}{\sin\theta} \quad \frac{2}{\sin\theta}$
$\Rightarrow y = -x\cot\theta + 2\operatorname{cosec}\theta$
On comparing this equation with
$y = mx + c$ we get
slope of line $x\cos\theta + y\sin\theta = 2$ is $-\cot\theta$
Also, we have a line $x - y = 3$
$\Rightarrow y = x - 3$
slope of line $x - y = 3$ is 1.
Since, both the lines are perpendicular to each other
$\therefore$ Product of their slopes $= -1$
$\Rightarrow (-\cot\theta)(1) = -1$
$\Rightarrow \cot\theta = 1 = \cot\frac{\pi}{4}$
$\Rightarrow \theta \quad \frac{\pi}{4}$

34. (a) Let $y = \tan^{-1}x - x$
On differentiating w.r.t.x, we get
$$\frac{dy}{dx} = \frac{1}{1+x^2} - 1 = \frac{1-1-x^2}{1+x^2} = \frac{-x^2}{1+x^2}$$
$\Rightarrow \frac{dy}{dx} < 0, \forall x \in R$
Hence, function is always decreasing.

35. (b) Given velocity is $v = s + 1$
Since, velocity $= \frac{ds}{dt}$
$\therefore \frac{ds}{dt} = s + 1 \Rightarrow \frac{ds}{s+1} = dt$
Integrate both side we get
$\log(s+1) = t$
At $s = 9$ m,
$t = \log(10)$ second

36. (b) Given, velocity is
$v = x^2\log\frac{1}{x} \quad = -x^2\log x$ where x is displacement.

For maximum velocity, $\frac{dv}{dx} = 0$

Now, $\frac{dv}{dx} = -x^2\frac{1}{x} + \log x(-2x)$
$= -x - 2x\log x$

$\frac{dv}{dx} = 0 \Rightarrow -x - 2x\log x = 0 \Rightarrow x = -2x\log x$

$\Rightarrow \frac{-1}{2} = \log x$

$\Rightarrow x = e^{-\frac{1}{2}}$

Hence, for maximum velocity $x = e^{-1/2}$

37. (b) Given curve is $x = e^x y$
Which can be rewritten as $y = xe^{-x}$
On differentiating w.r.t. x, we get
$\frac{dy}{dx} = -xe^{-x} + e^{-x}$
Put $\frac{dy}{dx} = 0$ for maxima or minima
$\Rightarrow -xe^{-x} + e^{-x} = 0$
$\Rightarrow e^{-x}(1-x) = 0$
Since e^{-x} can not be zero
$\therefore 1 - x = 0 \Rightarrow x = 1$
Now, $\frac{d^2y}{dx^2} = -e^{-x} + xe^{-x} - e^{-x} = xe^{-x} - 2e^{-x}$
$= e^{-x}(x-2)$
$\Rightarrow \left(\frac{d^2y}{dx^2}\right) < 0$
$\therefore y$ is maximum at $x = 1$.
Thus, when $x = 1$
then $y = e^{-1}$
Hence, maximum point on the curve $x = e^x y$ is $(1, e^{-1})$.

38. (b) Let r be the radius of balloon.
Balloon is like a sphere and volume of sphere $= \frac{4}{3}\pi r^3$
$\therefore V = \frac{4}{3}\pi r^3$
Differentiate both side w.r.t 't'
$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt}$
$\Rightarrow 4 = \frac{4}{3}\pi.3(4)^2\frac{dr}{dt} \qquad \left(\because \frac{dV}{dt} = 4cm^3/s\right)$
$\Rightarrow \frac{dr}{dt} = \frac{1}{16\pi} \qquad (1)$

Now, surface area of balloon $= S = 4\ \pi r^2$

$$\frac{dS}{dt} = 4\pi.2r\frac{dr}{dt}$$

$$= 4\pi.2\times 4\frac{1}{16\pi}\left(\text{from }(1)\right) = 2cm^2/s$$

39. (c) Given $f(x) = kx^3 - 9x^2 + 9x + 3$

On differentiating w.r.t. x, we get

$f'(x) = 3kx^2 - 18x + 9$

For a function to be monotonically increasing.

$b^2 - 4ac < 0$

Here, $a = 3k$, $b = -18$, $c = 9$

$\therefore\ b^2 - 4ac = (-18)^2 - 4(3k)(9) = (-18)(-18) - (3k)18\times 2$

$\Rightarrow\ 36 - 12k < 0$

$\Rightarrow\ k > 3$

40. (a) Let x be the side of first square and y be the side of second square.

$\therefore$ Area of first square, $A_1 = x^2$

and area of second square, $A_2 = y^2$

$= \left(x + x^2\right)^2 \left(\because y = x + x^2\right)$

$= x^2 + x^4 + 2x^3$

Now, $\frac{dA_1}{dx} = 2x$ and, $\frac{dA_2}{dx} = 2x + 4x^3 + 6x^2$

Hence, the Rate of change of area of the second square with respect to the area of the first square

$$= \frac{dA_2}{dA_1} = \frac{2x + 4x^3 + 6x^2}{2x} = 1 + 2x^2 + 3x$$

41. (a) Let $y = -\tan^{-1}\left(x^{-1}\right) + 1$

$$\therefore\ \frac{dy}{dx} = -\frac{1}{1 + x^{-2}}(-1)x^{-2} = \frac{1}{1 + x^2}$$

Since, $\frac{dy}{dx}$ is positive for all values of x. Therefore, y is an increasing function of x.

Hence, option (a) is correct.

42. (c) Let $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\Rightarrow\ f'(x) = 6x^2 - 6x - 12$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow\ 6x^2 - 6x - 12 = 0\ \Rightarrow\ x^2 - x - 2 = 0$

$\Rightarrow\ (x-2)(x+1) = 0$

$\Rightarrow\ x = 2,\ x = -1$

Now, $f''(x) = 12x - 6$

$\Rightarrow\ f''(2) = 24 - 6 = 18 > 0$

Since, $f''(x)$ is $+ve\ at\ x = 2$

$\therefore f(x)$ is minimum at $x = 2$.

Hence, minimum value is

$f(2) = 2(2)^3 - 3(2)^2 - 12\times 2 + 1$

$= 16 - 12 - 24 + 1 = -19$

43. (a) Given $f(x) = 6x - x^2, x > 0$

Differentiate w.r.t. x both side we get

$f'(x) = 6 - 2x$

As we know $f(x)$ will be increasing function, if

$f'(x) > 0\ \ \therefore 6 - 2x > 0 \Rightarrow x < 3$

Thus, required interval is (0,3).

44. (a) Product of two increasing function is always an increasing function.

$\therefore$ *fog* is always an increasing function

45. (b) Let y = f(x)

Now, for a point of inflection of y = f(x),

$\frac{d^2y}{dx^2}$ must be zero.

46. (d) Let f x p sin x $\frac{\sin 3x}{3}$

Differentiate both side w.r.t (x).

$\Rightarrow$ f′ x p cos x $\frac{3\cos 3x}{3}$ p cos x cos 3x

It is given that f(x) has extremum value at x = π/3

$\therefore\ f'\left(\frac{\pi}{3}\right) = 0$

$\Rightarrow\ p\cos\frac{\pi}{3} + \cos\pi\ \ 0$

$\Rightarrow\ \frac{p}{2} - 1 = 0\ \Rightarrow p\ \ 2$

47. (b) Surface area of sphere $S = 4\pi r^2$

Differentiate both sides w.r.t. t'

$$\Rightarrow\ \frac{dS}{dt}\ \ \frac{8\pi r dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{8\pi r}\cdot\frac{ds}{dt} \quad ...(i)$$

and Volume = $V = \frac{4}{3}\pi r^3$

$$\Rightarrow\ \frac{dV}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\frac{4\pi r^2}{8\pi r}.\frac{dS}{dt}\ \ \frac{1}{2}r\frac{dS}{dt} \quad \text{(from (i)}$$

48. (c) Let $f(x) = k\sin x + \frac{1}{3}\sin 3x$

Differentiate both side w.r.t. 'x'

$\Rightarrow\ f'(x) = k\cos x + \frac{3}{3}\cos 3x$

Put $f'(x) = 0$

$\Rightarrow\ k\cos x + \cos 3x = 0$

Since the $f(x)$ has maximum value at $x = \frac{\pi}{3}$

$\therefore \quad k\cos\frac{\pi}{3} + \cos 3\left(\frac{\pi}{3}\right) = 0$

$\Rightarrow \quad \frac{k}{2} + (-1) \quad 0 \Rightarrow \quad k = 2$

49. (a) Since $f(x)$ is an increasing function in [– 1, 1] and it has a root in (– 1, 1).

$\therefore$ Only statement I is correct.

50. (b) Let $f(x) = 2x^3 - 3x^2 - 12x + 5$

We find, $f(-2), f(-1), f(0), f(1), f(2)$.

Now, $f(-2) = -16 - 12 + 24 + 5 = 1$

$f(-1) = -2 - 3 + 12 + 5 = 12$

$f(0) = 5$

$f(1) = 2 - 3 - 12 + 5 = -8$

and $f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 5$

$= 16 - 12 - 24 + 5 = 21 - 36 = -15$

$\therefore$ Largest value of $2x^3 - 3x^2 - 12x + 5$ is at $x = -1$

51. (d) Let $f(x) = mx + c$

then, $f'(x) = m$

So, $f'(x) \neq 0$ for any real value of x.

Hence, $f(x)$ has neither maximum point nor minimum point.

52. (a) At an extreme point of a function $f(x)$, slope is always zero.

Thus, At an extreme point of a function $f(x)$, the tangent to the curve is parallel to the x-axis.

53. (a) Given $f(x) = e^x \sin x$

$\Rightarrow f'(x) = e^x \cos x + e^x \sin x$

$\Rightarrow$ slope $= e^x(\cos x + \sin x)$

Now, $\frac{d}{dx} \quad \cos x \quad \sin x \quad 0$

$\Rightarrow -\sin x + \cos x = 0$

$\Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

54. (c) Let volume $= V = \frac{4}{3}\pi r^3$ ...(1)

and surface area $= S = 4\pi r^2$ ...(2)

Now, (1) $\Rightarrow \frac{dv}{dt} = \frac{4}{3} \times 3\pi r^2 \times \frac{dr}{dt}$

$= 4\pi r^2 \frac{dr}{dt}$ ...(3)

(2) $\Rightarrow \frac{ds}{dt} = 4\pi \times 2 \times r\frac{dr}{dt} = \frac{8\pi r^2}{r}\frac{dr}{dt}$

$= \frac{2}{r}\left[4\pi r^2 \frac{dr}{dt}\right] = \frac{2}{r}\frac{dv}{dt}$ (from 3)

55. (c) Let $f(x) = x \ln x$

$f'(x) = \frac{x}{x} + \ln x = 1 + \ln x$

Put $f'(x) = 0 \Rightarrow 1 + \ln x = 0$

$\Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$

Now, $f''(x) = \frac{1}{x}$

$f''(x)\big|_{x=e^{-1}} = \frac{1}{e^{-1}} = e > 0$

Hence, $f(x)$ attains minimum value at $x = e^{-1}$.

56. (a) Given equation curve is

$x^2 + y^2 - 2x - 3 = 0$ ... (1)

On differentiating we get

$2x + 2y\frac{dy}{dx} - 2 = 0$

$\Rightarrow y\frac{dy}{dx} = -x + 1$

$\Rightarrow \frac{dy}{dx} = -\frac{x+1}{y}$

Since, tangent is parallel to x-axis

$\therefore \frac{dy}{dx} = 0 \Rightarrow -\frac{x+1}{y} = 0 \Rightarrow x = 1$

$\therefore$ From equation (1), we have

$1 + y^2 - 2 - 3 = 0 \Rightarrow y = \pm 2$

Hence, required points are (1, 2) and (1, –2).

57. (c) It is obvious

58. (a) Let $y = x^2 - 4x + 3$

Differentiate both sides w.r.t. 'x'

$\frac{dy}{dx} = 2x - 4$

So, slope $= 2x - 4$

Since, tangent is || to x-axis

$\therefore$ slope $= 0$

$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 2x - 4 = 0 \Rightarrow x = \frac{4}{2} = 2$

$\Rightarrow$ one tangent

59. (b) Let $u = \sqrt{x^2 + 16}$, $v = x^2$

$\Rightarrow u^2 = x^2 + 16, \quad \frac{dv}{dx} = 2x$

$\Rightarrow 2u\frac{du}{dx} = 2x$

$\Rightarrow \frac{du}{dx} = \frac{x}{\sqrt{x^2+16}}$

Now, required rate of change $= \frac{du}{dv}$

$= \frac{du}{dx} \times \frac{dx}{dv} = \frac{x}{\sqrt{x^2+16}} \times \frac{1}{2x} = \frac{1}{2\sqrt{x^2+16}}$

Now, $\frac{du}{dv}\bigg|_{x=3} = \frac{1}{2 \times 5} = \frac{1}{10}$

60. (b) $\frac{dx}{dt} = 2t + 3,\ \frac{dy}{dt} = 4t - 2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$$

$$\text{slope at } t = 2 = \frac{dy}{dx} / t = 2 = \frac{4(2)-2}{2(2)+3} = \frac{6}{7}$$

61. (a) From the graph of e^x it is clear that e^x is an increasing function.
Also Domain = R
and Range = R^+

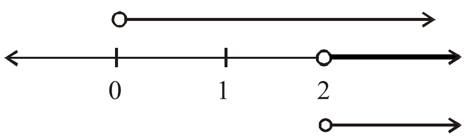
62. (d) Given $\frac{dr}{dt} = 3$

Let A = Area of circle = πr^2.

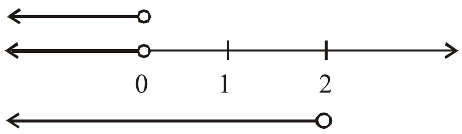
$$\therefore \frac{dA}{dt} = 2\pi r. \frac{dr}{dt} = 6\pi r$$

$$\text{Now, } \left.\frac{dA}{dt}\right|_{r=10} = 6 \times 10 \times \pi = 60\pi \text{ cm}^2/\text{s}$$

63. (c) For increasing function,
$f'(x) = 3x^2 - 6x > 0$
$\Rightarrow x(x-2) > 0$
Either both factors x and $(x - 2)$ are simultaneously +ve simultaneously –ve.
Case - I: $x > 0,\ x - 2 > 0$
$\Rightarrow x > 0, \ , x > 2$



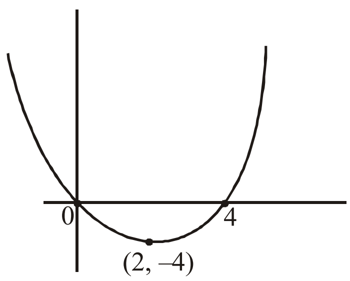
i.e. $x > 2$
Case - II: $x < 0, \quad x - 2 < 0$
$\Rightarrow x < 0, \ x < 2$



i.e. $x < 0$
From both cases $x > 2$ or $x < 0$.

64. (b) Minimum value of $|x|$ is 0.

65. (c) Given function attains minimum at $x = 2$



$\because f(2) = 4 - 8 = -4$

66. (a) Let $y = xe^x$.
Differentiate both side w.r.t. 'x'.

$$\Rightarrow \frac{dy}{dx} = e^x + xe^x = e^x(1+x)$$

Put $\frac{dy}{dx} = 0$

$\Rightarrow e^x(1+x) = 0 \quad \Rightarrow x = -1$

$$\text{Now, } \frac{d^2y}{dx^2} = e^x + e^x(1+x) = e^x(x+2)$$

$$\left(\frac{d^2y}{dx^2}\right)_{(x=-1)} = \frac{1}{e} + 0 > 0$$

Hence, $y = xe^x$ is minimum function and

$$y_{min} = -\frac{1}{e}.$$

67. (a) $f(x) = x^3 + 2x^2 - 4x + 6$
$f'(x) = 3x^2 + 4x - 4$
$f''(x) = 6x + 4$
Put $f'(x) = 0$
$3x^2 + 4x - 4 = 0$

$x = -2, \frac{2}{3}$

$f''(-2) = 6x - 2 + 4 = -8 < 0$

$f''\left(\frac{2}{3}\right) = 6 \times \frac{2}{3} + 4 = 8 > 0$

Value is maximum at $x = -2$

68. (c) At $x = 4$, $f(x) = 0$

69. (a) $y = \sin^{-1}(\sin^2 x)$

$$\frac{dy}{dx} = \frac{2\sin x \cos x}{\sqrt{1-\sin^4 x}} \Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sqrt{1-\sin^4 x}}$$

at $x = 0$, $\frac{dy}{dx} = 0$

70. (c) $y = e^{2x}$

$$\frac{dy}{dx} = 2e^{2x}$$

$$\left.\frac{dy}{dx}\right|_{(x,\,y)=(0,\,1)} = 2e^0 = 2$$

71. (c) Equation of line passing through (0, 1) and slope = 2
$y - 1 = 2(x - 0)$
$y = 2x + 1$
let line meets at $(x_1, 0)$

$$0 = 2x_1 + 1 \Rightarrow x_1 = -\frac{1}{2}$$

Tangent to the curve at (0, 1) meets the x-axis at $\left(-\frac{1}{2}, 0\right)$

72. (d) $f(x) = \dfrac{x^2 - x \quad 1}{x^2 \quad x \quad 1}$

$f'(x) = \dfrac{(x^2 + x + 1)(2x - 1) - (x^2 - x + 1)(2x + 1)}{(x^2 + x + 1)^2}$

$= \dfrac{2x^2 - 2}{(x^2 \quad x \quad 1)^2}$

Put $f'(x) = 0$

$2x^2 - 2 = 0$

$x = \pm 1$

$f''(x) = \dfrac{(x^2 \quad x \quad 1)^2(4x) - 2(2x^2 - 2)(x^2 \quad x \quad 1)(2x \quad 1)}{(x^2 \quad x \quad 1)^4}$

$f''(-1) = \dfrac{-36}{81} < 0$

$f(x)$ is maximum at $x = -1$

$f(-1) = 3$

73. (b) $f''(1) = \dfrac{36}{81} \quad 0$

$f(x)$ is minimum at $x = 1$

$f(1) = \dfrac{1}{3}$

74. (c) Volume of the box = V

$V = (24 - 2x)(9 - 2x).x$ ($\because$ height of box = x inch)

$= (216 - 48x - 18x + 4x^2).x$

$V(x) = 4x^3 - 66x^2 + 216x$

$\Rightarrow V'(x) = 12x^2 - 132x + 216$

For maximum, put $V'(x) = 0$

$\Rightarrow 12x^2 - 132x + 216 = 0$

$\Rightarrow x^2 - 11x + 18 = 0$

$\Rightarrow x^2 - 9x - 2x + 18 = 0$

$\Rightarrow x(x - 9) - 2(x - 9) = 0$

$\Rightarrow (x - 9)(x - 2) = 0$

$\Rightarrow x = 9$ or $x = 2$

Now $V''(x) = 24x - 132$

$\therefore V''(9) = 216 - 132 = 84 > 0$

and $V''(2) = 48 - 132 = -84 < 0$

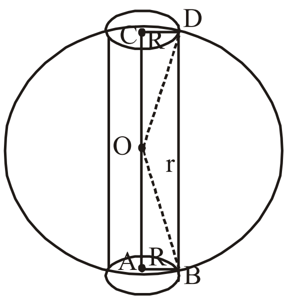
Thus, volume is maximum when x = 2 inch.

75. (a) Volume of box = $(24 - 4)(9 - 4).2$

$= 20 \times 5 \times 2 = 200$ cu inch

76. (a) Let h be the height, R be the radius and V be the volume of cylinder.

In ΔOAB, we have



$r^2 = R^2 \quad \left(\dfrac{h}{2}\right)^2$ ...(i)

$\left(\because OA = \dfrac{h}{2} \text{ as } \Delta OAB \cong \Delta OCD\right)$

Clearly, $V = \pi R^2 h$

$\Rightarrow V(h) = \pi\left(r^2 - \dfrac{h^2}{4}\right)h$ [using eq. (i)]

$\Rightarrow V(h) = \pi\left(r^2 h - \dfrac{h^3}{4}\right)$

$\Rightarrow V'(h) = \pi\left(r^2 - \dfrac{3h^2}{4}\right)$ ...(ii)

For maximum put $V'(h) = 0$

$\Rightarrow r^2 = \dfrac{3h^2}{4} \quad \Rightarrow h^2 = \dfrac{4r^2}{3}$

$\Rightarrow h = \dfrac{2r}{\sqrt{3}}$ ($\because h > 0$)

Differentiating eq. (ii) w.r.t. h, we get

$V''(h) = \pi\left(\dfrac{-6h}{4}\right)$

$\Rightarrow V''\left(\dfrac{2r}{\sqrt{3}}\right) = \pi\left(\dfrac{-6}{4} \times \dfrac{2r}{\sqrt{3}}\right) \quad 0$

Thus, the volume is maximum when $h = \dfrac{2r}{\sqrt{3}}$.

77. (b) Volume of cylinder is maximum when $h = \dfrac{2r}{\sqrt{3}}$

By using the relation $r^2 = R^2 \quad \left(\dfrac{h}{2}\right)^2$, we get

$R^2 = r^2 - \dfrac{h^2}{4} = r^2 - \dfrac{4r^2}{12}$

$R^2 = \dfrac{12r^2 - 4r^2}{12} = \dfrac{8r^2}{12} \quad \dfrac{2r^2}{3}$

$\Rightarrow R = \sqrt{\dfrac{2r^2}{3}} \quad \dfrac{\sqrt{2}r}{\sqrt{3}}$ ($\because R > 0$)

78. (c) From statement -1

$$y = \frac{e^x + e^{-x}}{2}$$

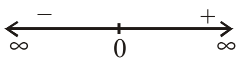
$\therefore \quad \frac{dy}{dx} = \frac{1}{2}[e^x - e^{-x}]$

Now equating $\frac{dy}{dx}$ to zero we get,

$e^x - e^{-x} = 0$

$\therefore \quad e^x = e^{-x}$

$\therefore \quad \frac{e^x}{e^{-x}} = 1$



$\therefore \quad e^{2x} = 1$

$\therefore \quad 2x = 0$

$\therefore \quad x = 0$

The given function is increasing on interval $[0, \infty]$

From Statement -2

$$y = \frac{e^x - e^{-x}}{2}$$

$\frac{dy}{dx} = \frac{1}{2}[e^x - e^x]$

Now equating $\frac{dy}{dx}$ to zero, we get.

$e^x + e^{-x} = 0$

$\therefore \quad e^x + \frac{1}{e^x} = 0$

$\therefore \quad e^{2x} + 1 = 0$

$\therefore \quad e^{2x} = -1$

Hence, the given function is increacing from $[-\infty, \infty]$

$\therefore$ Both statement are correct

$\therefore$ Option (c) is correct.

79. (b) Given $f(x) = \frac{x^2 - 1}{x^2 + 1}$

In order to find the value of 'x', where f(x) is maximum or minimum; equation $f'(x)$ equal to zero.

$$\therefore \quad f'(x) = \frac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$\therefore \quad f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x[x^2+1-x^2+1]}{(x^2+1)^2}$$

$$\therefore \quad f'(x) = \frac{2x[2]}{(x^2+1)^2}$$

Now equating $f'(x)$ to zero, we get

$4x = 0$

$\therefore \quad x = 0$

Hence, f(x) attain minimum value at $x = 0$

$\therefore$ Option (b) is correct.

80. (c) f(x) is minimum at $x = 0$

$\therefore \quad f(0) = \frac{-1}{1} = -1$

$\therefore$ minimum value of f(x) is –1

$\therefore$ Option (c) is correct.

81. (c) $f(x) = 0.75x^4 - x^3 - 9x^2 + 7$

$f'(x) = 4 \times 0.75\,x^3 - 3x^2 - 18x$

$= 3x(x^2 - x - 6)$

$= 3x(x+2)(x-3)$

for maximum value,

$f'(x) = 0$

$3x(x+2)(x-3) = 0$

$x = 0, -2, 3.$

$f(0) = 7$

$f(-2) = 0.75(-2)^4 - (-2)^3 - 9(-2)^2 + 7$

$= 2 + 8 - 36 + 7 = -9$

$f(3) = 0.75(3)^4 - (3)^3 - 9(3)^2 + 7$

$= 60.75 - 27 - 81 + 7 = -40.25$

$\therefore$ maximum value of f(x) is 7

$\therefore$ Option (c) is correct.

82. (c) **From Statement 1:**

Function attain local minima at $x = -2$ and $x = 3$

As we have,

$f'(x) = 3x^3 - 3x^2 - 18x$

Now, $f''(x) = 9x^2 - 6x - 18$

For $x = -2$,

$f''(-2) = 9(-2)^2 - 6(-2) - 18$

$= 36 + 12 - 18 = 48 - 18 = 30 > 0$

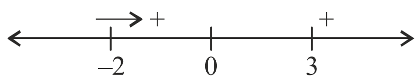
For $x = 3$,

$f''(3) = 9(3)^2 - 6(3) - 18$

$= 81 - 36 = 45 > 0$

$\therefore$ Statement 1 is correct

From Statement 2:



The function increases in the interval (–2, 0)

$\therefore$ Option (c) is correct.

83. (b) $x^2+y^2=\left[\frac{a(1-t^2)}{1+t^2}\right]^2+\left[\frac{2at}{1+t^2}\right]^2$

$$=\frac{a^2(1-t^2)^2}{(1+t^2)^2}+\frac{4a^2t^2}{(1+t^2)^2}$$

$$=\frac{a^2(1+t^4-2t^2)+4a^2t^2}{(1+t^2)^2}$$

$$=\frac{a^2(1+t^4+2t^2)}{(1+t^2)^2}=\frac{a^2(1+t^2)^2}{(1+t^2)^2}=a^2$$

Hence, the given equation represent a circle of radius (a)

$\therefore$ Option (b) is correct.

84. (d) Here, $x^2+y^2=a^2$

$\therefore$ $2x+2y\frac{dy}{dx}=0$

$\therefore$ $\frac{dy}{dx}=\frac{2x}{-2y}=-\frac{x}{y}$

$\therefore$ Option (d) is correct.

85. (d) $\frac{d^2y}{dx^2}=\frac{y\frac{d}{dx}(-x)-(-x)\frac{dy}{dx}}{y^2}$

$$=\frac{-y+x\frac{dy}{dx}}{y^2}=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}$$

$$=\frac{-y^2-x^2}{y^3}=\frac{-(x^2+y^2)}{y^3}=-\frac{a^2}{y^3}$$

$\therefore$ Option (d) is correct.

86. (c) $f(x)=\frac{x^2}{e^x}$

$$f'(x)=\frac{2x.e^x-e^x.x^2}{(e^x)^2}$$

$$f'(x)=\frac{2x-x^2}{e^x}$$

as e^x is always positive and for monotonically increasing; $2x-x^2>0$

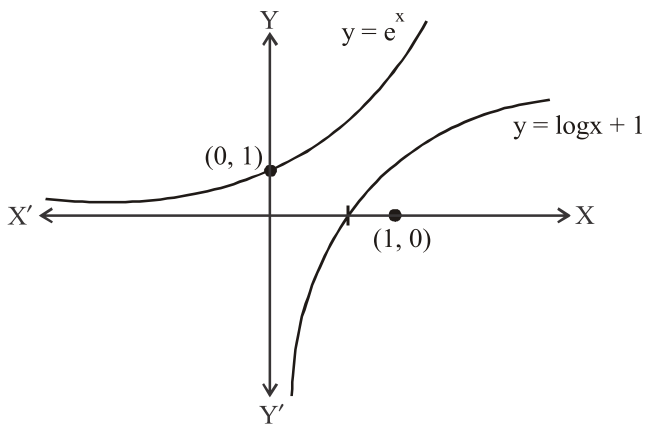
$\Rightarrow$ $x^2-2x<0 \Rightarrow x(x-2)<0 \Rightarrow x=(0,2)$

87. (c) $f(x)=\log x$

Clearly f(x) is increasing on $(0,\infty)$

$f(x)=e^x-x\log x$

$$f'(x)=e^x-(\log x+1)$$



From the figure it is clear that $f'(x)>0$ on $(1,\infty)$.
So both statements (1) & (2) are correct.

88. (c) $f(x)=\left(\frac{1}{x}\right)^{2x^2}=y$ (say)

$$\log y=2x^2\log\left(\frac{1}{x}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=2\left[2x.\log\left(\frac{1}{x}\right)+x^2.\frac{1}{1/x}.\left(\frac{-1}{x^2}\right)\right]$$

$$\frac{dy}{dx}=2y\left[2x\log\left(\frac{1}{x}\right)-x\right]$$

For max. or min. value $\frac{dy}{dx}=0$

$$2x\log\left(\frac{1}{x}\right)-x=0$$

$$x\left[2\log\left(\frac{1}{x}\right)-1\right]=0$$

$\because x\neq 0$

$\Rightarrow$ $2\log\left(\frac{1}{x}\right)=1$ $\Rightarrow$ $\frac{1}{x}=e^{\frac{1}{2}}$

$\Rightarrow$ $\boxed{x=e^{-\frac{1}{2}}}$

Again $\frac{d^2y}{dx^2}=\left\{2xy\left[\log\left(\frac{1}{x}\right)-1\right]\right\}$

$$=(2xy)'\left(\log\frac{1}{x}-1\right)+2xy\left[x.\left(-\frac{1}{x^2}\right)\right]$$

$$=(2xy)'\left(\log\frac{1}{x}-1\right)-2y$$

$$=2\left[(y+xy^1)\left(\log\frac{1}{x}-1\right)-y\right]<0$$

So at $x=e^{-1/2}$ function is maximum.

89. (c) $f(x)=\left(\frac{1}{x}\right)^{2x^2}$

$f\left(e^{-\frac{1}{2}}\right)=\left(e^{\frac{1}{2}}\right)^{2\times e^{-1}}$

$=\left(e^{\frac{1}{2}}\right)^{\frac{2}{e}} = e^{\frac{1}{2}\times\frac{2}{e}} = e^{1/e}$

90. (c) $f'(x)=\frac{x^2}{2}-kx+1$

$f(0)=0;\ f(3)=15$

$f(x)=\frac{1}{6}x^3-\frac{k}{2}x^2+x+c$

Putting $x=0$

$\boxed{f(0)=c=0}$

$f(x)=\frac{x^3}{6}-\frac{k}{2}x^2+x$

Putting $x=3$

$f(3)=\frac{(3)^3}{6}-\frac{k}{2}(3)^2+3$

$15=\frac{9}{2}-\frac{9}{2}k+3 \quad \Rightarrow k=-\frac{5}{3}$

91. (d) $\because f'(x)=\frac{x^2}{2}+\frac{5}{3}x+1$

$\Rightarrow f''(x)=x+\frac{5}{3}$

$f''\left(-\frac{2}{3}\right)=\frac{-2}{3}+\frac{5}{3}=1$

So option (d) is correct.

92. (a) $f(x)=-2x^3-9x^2-12x+1$

$f'(x)=-6x^2-18x-12$

for increasing function $f'(x)>0$

$\Rightarrow -(6x^2+18x+12)>0$

$\Rightarrow (x^2+3x+2)<0$

$\Rightarrow (x+2)(x+1)<0$

$x=(-2,-1)$

93. (d) For decreasing function $f'(x)<0$

$\Rightarrow -(6x^2+18x+12)<0$

$\Rightarrow (x^2+3x+2)>0$

$\Rightarrow (x+2)(x+1)>0$

$x=(-\infty,-2)\cup(-1,\infty)$

Sol. (94-96)

$f(\theta)=4(\sin^2\theta+\cos^4\theta)$

$=4(\sin^2\theta+\cos^2\theta(1-\sin^2\theta))$

$=4(\sin^2\theta+\cos^2\theta-\sin^2\theta\cos^2\theta)$

$=4\left(1-\frac{1}{4}\sin^2 2\theta\right) \qquad [\because \sin 2\theta=2\sin\theta\cos\theta]$

94. (d) For maximum value of $f(\theta)$, $\sin^2 2\theta$ should be minimum. i.e. $\sin^2 2\theta=0$

$f(\theta)|_{max}=4(1-0)=4$

95. (d) For minimum value of $f(\theta)$, $\sin^2 2\theta$ should be maximum i.e. $\sin^2 2\theta=1$.

$f(\theta)|_{min}=4\left(1-\frac{1}{4}(1)\right)=4\times\frac{3}{4}=3$

96. (c) $f(\theta)=2$

$4\left(1-\frac{1}{4}\sin^2 2\theta\right)=2$

$\Rightarrow 1-\frac{1}{4}\sin^2 2\theta=\frac{2}{4} \qquad \Rightarrow -\frac{1}{4}\sin^2 2\theta=+\frac{1}{2}-1$

$\Rightarrow -\frac{1}{4}\sin^2 2\theta=-\frac{1}{2} \qquad \Rightarrow \sin^2 2\theta=2$

$\Rightarrow \sin 2\theta=\pm\sqrt{2}$

Since sin θ cannot have vlaue greater than 1 & less than –1.

Hence $f(\theta)=2$ has no solution.

$f(\theta)=\frac{7}{2}$

$4\left(1-\frac{1}{4}\sin^2 2\theta\right)=\frac{7}{2}$

$\Rightarrow\left(1-\frac{1}{4}\sin^2 2\theta\right)=\frac{7}{8} \qquad \Rightarrow -\frac{1}{4}\sin^2 2\theta=\frac{7}{8}-1$

$\Rightarrow -\frac{1}{4}\sin^2 2\theta=-\frac{1}{8} \qquad \Rightarrow \sin^2 2\theta=\frac{1}{2}$

$\Rightarrow \sin 2\theta=\pm\frac{1}{\sqrt{2}} \qquad \Rightarrow \sin 2\theta=\pm\sin\frac{\pi}{4}$

$\Rightarrow \sin 2\theta=\sin(\pm\pi/4) \qquad \Rightarrow 2\theta=\pm\pi/4$

$\boxed{\theta=(\pm)\frac{\pi}{8}}$

Hence $f(\theta)=\frac{7}{2}$ has a solution.

97. (b) $K\sin x+\cos 2x=2K-7$

$K\sin x+(1-2\sin^2 x)=(2K-7)$

$2\sin^2 x-K\sin x+(2K-8)=0$

This is a quadratic equation in $\sin x$.

$\sin x = \dfrac{-(-K) \pm \sqrt{K^2 - 4(2)(2K-8)}}{2 \times 2}$

For minimum value of K

$\sin x = -1$

$\Rightarrow \dfrac{K \pm \sqrt{K^2 - 16K + 64}}{4} = -1$

$\Rightarrow (\pm)\sqrt{K^2 - 16K \quad 64} \quad -K - 4$

Squaring both sides, we get

$K^2 - 16K + 64 = K^2 + 16 + 8K$

$\Rightarrow 24K = 48 \quad \Rightarrow K = 2$

98. (d) For maximum value of K

$\sin x = 1$

$\dfrac{K \pm \sqrt{K^2 - 16K + 64}}{4} = 1$

$\Rightarrow (\pm)\sqrt{K^2 - 16K \quad 64} \quad (-K \quad 4)$

Squaring both sides, we get

$K^2 - 16K + 64 = K^2 + 16 - 8K$

$\Rightarrow 8K = 48 \quad \Rightarrow K = 6$

99. (b) $f(x) = x^3 \sin x$

$f'(x) = 3x^2 \sin x + x^3 \cos x$

$f'(x) = 0$

$\Rightarrow 3x^2 \sin x + x^3 \cos x = 0$

$\Rightarrow x^2(3\sin x + x\cos x) = 0$

$\Rightarrow x = 0,\ 3\sin x + x\cos x = 0$ ...(1)

Put $x = 0$ in (1)

$3 \sin x = 0 \Rightarrow \sin x = 0$

$f''(x) = 6x\sin x + 3x^2\cos x + 3x^2\cos x + x^3(-\sin x)$

$f''(0) = 0$

So, f(x) has min. at $x = 0$.

100. (a) $\sin\left(x + \dfrac{\pi}{6}\right) + \cos\left(x + \dfrac{\pi}{6}\right)$

$= \sqrt{2}\left[\dfrac{1}{\sqrt{2}}\sin\left(x + \dfrac{\pi}{6}\right) + \dfrac{1}{\sqrt{2}}\cos\left(x + \dfrac{\pi}{6}\right)\right]$

$= \sqrt{2}\left[\sin\left(x + \dfrac{\pi}{6}\right)\cos\dfrac{\pi}{4} + \cos\left(x + \dfrac{\pi}{6}\right)\sin\dfrac{\pi}{4}\right]$

$= \sqrt{2}\left[\sin\left(x + \dfrac{\pi}{6} + \dfrac{\pi}{4}\right)\right]$

[$\because$ sin (A + B) = sin A cos B + cosASinB]

$= \sqrt{2}\left[\sin\left(x + \dfrac{5\pi}{12}\right)\right]$

Given interval is $\left(0, \dfrac{\pi}{2}\right)$

For, maximum value $x + \dfrac{5\pi}{12} = \dfrac{\pi}{2}$

$\Rightarrow x = \dfrac{\pi}{2} - \dfrac{5\pi}{12} = \dfrac{6\pi - 5\pi}{12} = \dfrac{\pi}{12}$

101. (a) $f(x) = 3\sin x - 4\sin^3 x = \sin 3x$

sin 3x increases in $\left[\dfrac{-\pi}{6}, \dfrac{\pi}{6}\right]$

So, interval length $= \dfrac{\pi}{3}$

102. (a) We know, $-1 \le \sin x \le 1$

$\Rightarrow 0 \le \sin^2 x \le 1$

$\Rightarrow 0 \le 4\sin^2 x \le 4$

$\Rightarrow 1 \le 4\sin^2 x + 1 \le 5$

103. (c) $f(x) = x + \dfrac{1}{x}$

$f'\ x\ = 1 - \dfrac{1}{x^2} \quad \dfrac{x^2 - 1}{x^2} \quad \dfrac{x-1\ \ x\ \ 1}{x^2}$

For $x \in (0,1)$, $f'(x) < 0$

$\Rightarrow$ f(x) decreases

104. (a) Statements (1), (2) are correct.

105. (a) $f(x) = x(x-1)(x+1) = x(x^2 - 1)$

Differentiating, we get

$f'(x) = x(2x) + (x^2 - 1) = 2x^2 + x^2 - 1 = 3x^2 - 1$

Again differentiating

$f''(x) = 6x$

At $f'(x) = 0 \Rightarrow 3x^2 - 1 = 0 \Rightarrow x^2 = \dfrac{1}{3} \Rightarrow x = \pm\dfrac{1}{\sqrt{3}}$.

Maximum value of $f(x) = f\left(\dfrac{-1}{\sqrt{3}}\right) = \dfrac{-1}{\sqrt{3}}\left(\left(\dfrac{-1}{\sqrt{3}}\right)^2 - 1\right)$

$= \dfrac{-1}{\sqrt{3}}\left(\dfrac{1}{3} - 1\right) \quad \dfrac{-1}{\sqrt{3}}\left(\dfrac{-2}{3}\right) \quad \dfrac{2}{3\sqrt{3}}$

Minimum value of $f\ \ x \quad f\left(\dfrac{1}{\sqrt{3}}\right) \quad \dfrac{1}{\sqrt{3}}\left(\dfrac{1}{3} - 1\right) \quad \dfrac{-2}{3\sqrt{3}}$

106. (b) $f\ \ x \quad \dfrac{\ell nx}{x}$

$f'(x) = \dfrac{\left(\dfrac{1}{x}\right)x - \ell nx}{x^2} = \dfrac{1 - \ell nx}{x^2}$

$f'(x) = 0 \Rightarrow \dfrac{1 - \ell nx}{x^2} = 0 \Rightarrow 1 - \ell nx = 0$

$\Rightarrow \ell nx \quad 1 \quad \Rightarrow x = e.$

107. (b) $f(x) = \sin x + \cos x$.

We know, the maximum value of a sin x + b cos x is $\sqrt{a^2 \quad b^2}$.

$\therefore$ Maximum value of sin x + cos x is $\sqrt{1 \quad 1} \quad \sqrt{2}$.

$A \to (2)$

Maximum value of 3 sin x + 4 cos x is $\sqrt{9 \quad 16} \quad 5$

$B \to (3)$

Maximum value of 2 sin x + cos x is $\sqrt{4+1} = \sqrt{5}$

$C \to (4)$

Maximum value of sin x + 3 cos x is $\sqrt{1+9} = \sqrt{10}$

$D \to (1)$

108. (b) The capacity of the jar will be maximum if height and radius of the cylinder are equal.

$\therefore$ height = radius ....(1)

Given diameter is k times the height of the jar.

Diameter = 2 × radius

= 2 × height ....(from (1))

$\therefore$ k = 2.

109. (a) Let $y = \sin\left(x + \frac{\pi}{5}\right) + \cos\left(x + \frac{\pi}{5}\right)$

$\Rightarrow \frac{dy}{dx} = \cos\left(x + \frac{\pi}{5}\right) - \sin\left(x + \frac{\pi}{5}\right)$

for maximum value, $\frac{dy}{dx} = 0$.

$\therefore \cos\left(x + \frac{\pi}{5}\right) - \sin\left(x + \frac{\pi}{5}\right) = 0$

$\Rightarrow \frac{\cos\left(x + \frac{\pi}{5}\right)}{\sin\left(x + \frac{\pi}{5}\right)} = 1$

$\Rightarrow \cot\left(x + \frac{\pi}{5}\right) = \cot\frac{\pi}{4}$

$\Rightarrow x + \frac{\pi}{5} = \frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4} - \frac{\pi}{5} = \frac{5\pi - 4\pi}{20} = \frac{\pi}{20}$.

110. (c) $f(\theta) = 16 \sin\theta - 12 \sin^2\theta$

$f'(\theta) = 16 \cos\theta - 24 \sin\theta \cos\theta$

$f'(\theta) = 0 \Rightarrow \cos\theta\,(16 - 24 \sin\theta) = 0$

$\Rightarrow \cos\theta = 0$ (or) $16 - 24 \sin\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$ (or) $\sin\theta = \frac{2}{3}$

$f\left(\frac{\pi}{2}\right) = 16 \sin\frac{\pi}{2} - 12 \sin^2\frac{\pi}{2} = 16 - 12 = 4$

$f\left(\sin\theta = \frac{2}{3}\right) = 16\left(\frac{2}{3}\right) - 12\left(\frac{2}{3}\right)^2$

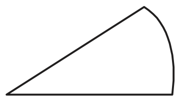
$= \frac{32}{3} - \frac{48}{9} = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$.

111. (a) $f(x) = x \sin x + \cos x + \frac{1}{2}\cos^2 x$

$\Rightarrow f'(x) = x \cos x + \sin x - \sin x - \sin x \cos x$

$= \cos x (x - \sin x) > 0$ in $\left(0, \frac{\pi}{2}\right)$

112. (c) $\ell + 2r = 40$



Area is maximum, when $\ell + 2r = 20 \Rightarrow r = 10$

113. (a) $[x(x-1)+1]^{1/3}$ is minimum when $x^2 - x + 1$ is minimum, $0 \le x \le 1$

$x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \left(1 - \frac{1}{4}\right)$

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\therefore$ minimum value of $x^2 - x + 1 = \frac{3}{4}$

$\Rightarrow$ Required value $= \left(\frac{3}{4}\right)^{1/3}$

114. (a) $y = (-\sin x)^{-x}$

$\frac{dy}{dx} = (-\sin)^{-x}\left[\frac{x}{\sin x}(-\cos x) + \log(-\sin x)\cdot(-1)\right]$

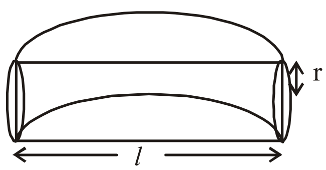
$\left[\frac{dy}{dx}\right]_{x = -\frac{\pi}{6}} = \left(\frac{1}{2}\right)^{\pi/6}\left[-\frac{\pi}{6}\sqrt{3} - \log\frac{1}{2}\right]$

$= 2^{-\pi/6}\left(\frac{6\log 2 - \sqrt{3}\pi}{6}\right)$

115. (a) Volume of half cylinder with height 'l' and radius 'r'

$= \frac{1}{2}\pi r^2 l$ ...(1)

Surface area of half cylinder.



$S = l \times 2r + 2\left(\frac{1}{2}\pi r^2\right) + 2\left(\frac{1}{2}\pi r l\right)$

$S = 2rl + \pi r^2 + \pi r l$

$S = \frac{4V}{\pi r} + \frac{\pi . 2V}{\pi r} + \pi r^2$ (from (1))

$\frac{ds}{dr} \Rightarrow \frac{-4V}{\pi r^2} - \frac{2V}{r^2} + 2\pi r = 0$

$\Rightarrow 2\pi r = \frac{4V}{\pi r^2} + \frac{2V}{r^2}$

$\Rightarrow 2\pi r = \frac{4.\pi r^2 l}{2\pi r^2} + \frac{2}{r^2}.\frac{1}{2}\pi r^2 l$

$\Rightarrow \frac{l}{2r} = \frac{\pi}{\pi + 2}$

Indefinite Integration

17

1. If $f(x) = \ln(x - \sqrt{1+x^2})$, then what is $\int f''(x)\,dx$ equal to ? *[2006-I]*

(a) $\frac{1}{(x - \sqrt{1+x^2})} + c$ (b) $-\frac{1}{\sqrt{1+x^2}} + c$

(c) $-\sqrt{1+x^2} + c$ (d) $\ln(x - \sqrt{1+x^2}) + c$

2. If $\int \sec x \operatorname{cosec} x\,dx = \log|g(x)| + c$, then what is g (x) equal to ? *[2006-II]*

(a) sin x cos x (b) $\sec^2 x$

(c) tan x (d) log | tan x |

3. What is the value of $\int \frac{dx}{(x^2+a^2)(x^2+b^2)}$? *[2007-I]*

(a) $\int \frac{[\{\tan^{-1}(x/a)\}/a - \{\tan^{-1}(x/b)\}/b]}{(a^2+b^2)} + c$

(b) $\int \frac{[\{\tan^{-1}(x/a)\}/a + \{\tan^{-1}(x/b)\}/b]}{(a^2+b^2)} + c$

(c) $\int \frac{[\{\tan^{-1}(x/a)\}/a + \{\tan^{-1}(x/b)\}/b]}{(b^2-a^2)} + c$

(d) $\int \frac{[\{\tan^{-1}(x/a)\}/a + \{\tan^{-1}(x/b)\}/b]}{(b^2-a^2)} + c$

4. What is the value of $\int (\sqrt{x} + x)^{-1} dx$? *[2007-I]*

(a) In $(x + \sqrt{x}) + c$ (b) $2\ln(1+\sqrt{x}) + c$

(c) $2\ln(x+\sqrt{x}) + c$ (d) $2\ln(x-\sqrt{x}) + c$

5. What is the value of $\int \frac{e^x(1+x)}{\sin^2(xe^x)}\,dx$? *[2007-II]*

(a) $-e^x \cot x + c$

(b) $\cos^2(x e^x) + c$

(c) $\log \sin(x e^x) + c$

(d) $-\cot(x e^x) + c$

6. What is $\int \log(x+1)\,dx$ is equal to ?

(a) $x \log(x+1) - x + c$ (b) $(x+1)\log(x+1) - x + c$

(c) $\frac{1}{x+1} + c$ (d) $\frac{\log(x+1)}{x+1} + c$ *[2008-I]*

7. If $\int \frac{dx}{f(x)} = \log\{f(x)\}^2 + c$, then what is f (x) equal to ?

(a) $2x + \alpha$ (b) $x + \alpha$

(c) $\frac{x}{2} + \alpha$ (d) $x^2 + \alpha$ *[2008-I]*

8. What is $\int (e^x + 1)^{-1} dx$ equal to? *[2008-II]*

(a) $\ln(e^x+1) + c$ (b) $\ln(e^{-x}+1) + c$

(c) $-\ln(e^{-x}+1) + c$ (d) $-(e^x+1) + c$

9. What is $\int \frac{d\theta}{\sin^2\theta + 2\cos^2\theta - 1}$ equal to? *[2008-II]*

(a) $\tan\theta + c$ (b) $\cot\theta + c$

(c) $\frac{1}{2}\tan\theta + c$ (d) $\frac{1}{2}\cot\theta + c$

10. What is $\int \sin x \log(\tan x)\,dx$ equal to? *[2008-II]*

(a) $\cos x \log \tan x + \log \tan(x/2) + c$

(b) $-\cos x \log \tan x + \log \tan(x/2) + c$

(c) $\cos x \log \tan x + \log \cot(x/2) + c$

(d) $-\cos x \log \tan x + \log \cot(x/2) + c$

DIRECTION (Q. 11) : *The following item consists of two statements, one labelled the Assertion (A) and the other labelled the Reason (R). You are to examine these two statements carefully and decide if the Assertion (A) and Reason (R) are individually true and if so, whether the reason is a correct explanation of the Assertion. Select your answer using the codes given below.*

11. **Assertion (A) :** $\int \frac{e^x}{x}(1 + x\log x)dx + c = e^x \log x$

Reason (R) : $\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$

[2009-I]

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

12. What is $\int \tan^2 x \sec^4 x\, dx$ equal to? [2009-I]

(a) $\frac{\sec^5 x}{5}+\frac{\sec^3 x}{3}+c$ (b) $\frac{\tan^5 x}{5}+\frac{\tan^3 x}{3}+c$

(c) $\frac{\tan^5 x}{5}+\frac{\sec^3 x}{3}+c$ (d) $\frac{\sec^5 x}{5}+\frac{\tan^3 x}{3}+c$

13. What $\int \sec x^{\circ}\, dx$ is equal to? [2009-I]

(a) $\log(\sec x^{\circ}+\tan x^{\circ})+c$

(b) $\frac{\pi \log \tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)}{180^{\circ}}+c$

(c) $\frac{180^{\circ} \log \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\pi}+c$

(d) $\frac{180^{\circ} \log \tan\left(\frac{\pi}{4}+\frac{x}{360^{\circ}}\right)}{\pi}+c$

14. What is $\int \frac{a \quad b \sin x}{\cos^2 x} dx$ equal to? [2009-II]

(a) $a\sec x + b\tan x + c$ (b) $a\tan x + b\sec x + c$
(c) $a\cot x + b\operatorname{cosec} x + c$ (d) $a\operatorname{cosec} x + b\cot x + c$

15. What is $\int \frac{\log x}{1 \quad \log x \quad ^2} dx$ equal to? [2009-II]

(a) $\frac{1}{1 \quad \log x \quad ^3} \quad c$ (b) $\frac{1}{1 \quad \log x \quad ^2} \quad c$

(c) $\frac{x}{1 \quad \log x} \quad c$ (d) $\frac{x}{1 \quad \log x \quad ^2} \quad c$

Where c is a constant.

16. What is $\int e^{\ln x} \sin x\, dx$ equal to? [2010-I]

(a) $e^{\ln x}(\sin x - \cos x) + c$ (b) $(\sin x - x\cos x) + c$
(c) $(x\sin x + \cos x) + c$ (d) $(\sin x + x\cos x) - c$
Where 'c' is a constant of integration.

17. What is $\int \frac{x^4+1}{x^2+1} dx$ equal to? [2010-I]

(a) $\frac{x^3}{3} - x + 4\tan^{-1} x + c$ (b) $\frac{x^3}{3} + x + 4\tan^{-1} x + c$

(c) $\frac{x^3}{3} - x + 2\tan^{-1} x + c$ (d) $\frac{x^3}{3} - x - 4\tan^{-1} x + c$

Where 'c' is a constant of integration.

18. If $\int x^2 \ln x\, dx = \frac{x^3}{m}\ln x + \frac{x^3}{n} + c,$ then what are the values of m and n respectively? [2010-I]

(a) $\frac{1}{3}, -\frac{1}{9}$ (b) $3, -9$
(c) $3, 9$ (d) $3, 3$
where c is a constant of integration.

19. What is $\int \frac{1}{1+e^x} dx$ equal to? [2010-I]

(a) $x - \log x + c$ (b) $x - \log(\tan x) + c$
(c) $x - \log(1+e^x) + c$ (d) $\log(1+e^x) + c$
where c is a constant of integration.

20. What is $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$ equal to? [2010-II]

(a) $2e^{\sqrt{x}}\left(x - 2\sqrt{x} + 2\right) + c$

(b) $2e^{\sqrt{x}}\left(x + 2\sqrt{x} + 2\right) + c$

(c) $2e^{\sqrt{x}} \quad x + 2\sqrt{x} - 2 \quad c$

(d) $2e^{\sqrt{x}}\left(x - 2\sqrt{x} - 2\right) + c$

Where 'c' is a constant of integration.

21. What is $\int \sec^n x \tan x\, dx$ equal to ? [2010-II]

(a) $\frac{\sec^n x}{n} + c$ (b) $\frac{\sec^{n-1} x}{n-1} + c$

(c) $\frac{\tan^n x}{n} + c$ (d) $\frac{\tan^{n-1} x}{n-1} + c$

Where 'c' is a constant of integration.

22. What is $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ equal to? [2010-II]

(a) $xe^x + c$ (b) $\cos(xe^x) + c$
(c) $\tan(xe^x) + c$ (d) $x\operatorname{cosec}(xe^x) + c$
Where c is a constant of integration.

23. What is $\int e^x\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$ equal to [2011-I]

(a) $xe^x + C$ (b) $e^x\left(\sqrt{x}\right) + C$

(c) $2e^x\left(\sqrt{x}\right) + C$ (d) $2xe^x + C$

(where C is a constant of integration.)

24. What is $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ equal to? [2011-I]

(a) $\frac{\cos\sqrt{x}}{2} + C$ (b) $2\cos\sqrt{x} + C$

(c) $\frac{-\cos\sqrt{x}}{2} + C$ (d) $-2\cos\sqrt{x} + C$

25. What is $\int \sin^{-1}(\cos x)\, dx$ equal to ? *[2011-I]*
(a) $\frac{x\pi}{2}-\frac{x^2}{2}+K$ (b) $\frac{\pi}{2}+\frac{x^2}{2}+K$
(c) $-\frac{x\pi}{2}-\frac{x^2}{2}+K$ (d) $\frac{\pi}{2}-\frac{x^2}{2}+K$
Where K is a constant of integration.

26. What is $\int \frac{dx}{\sin^2 x \cos^2 x}$ equal to? *[2011-II]*
(a) $\tan x + \cot x + c$ (b) $\tan x - \cot x + c$
(c) $(\tan x + \cot x)^2 + c$ (d) $(\tan x - \cot x)^2 + c$
where c is the constant of integration.

27. Consider the following: *[2012-I]*
1. $\int \ell n\, 10\, dx = x + c$ 2. $\int 10^x\, dx = 10^x + c$
where c is the constant of integration. Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

28. What is $\int (x^2+1)^{\frac{5}{2}} x\, dx$ equal to? *[2012-I]*
(a) $(x^2+1)^{\frac{7}{2}}+c$ (b) $\frac{2}{7}(x^2+1)^{\frac{7}{2}}+c$
(c) $\frac{1}{7}(x^2+1)^{\frac{7}{2}}+c$ (d) None of the above
where c is a constant of integration.

29. What is $\int a^x e^x\, dx$ equal to ? *[2012-II]*
(a) $\frac{a^x e^x}{\ell n a}\ \ c$ (b) $a^x e^x + c$
(c) $\frac{a^x e^x}{\ell n\ \ ae}\ \ c$ (d) None of the above
where c is the constant of integration.

30. What is $\int \frac{\ell n x}{x} dx$ equal to ? *[2012-II]*
(a) $\frac{\ell n x^{\ 2}}{2}\ \ c$ (b) $\frac{\ell n x}{2}\ \ c$
(c) $(\ell n x)^2 + c$ (d) None of the above

31. What is $\int \left(\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right)dx$ equal to ? *[2012-II]*
(a) 2 cosec 2x + c (b) – 2 cot 2x + c
(c) 2 sec 2x + c (d) – 2 tan 2x + c
where c is the constant of integration.

32. What is $\int e^{\ell nx}\, dx$ equal to ? *[2013-I]*
(a) $xe^{\ell nx} + c$ (b) $-xe^{-\ell nx} + c$
(c) $x + c$ (d) $\frac{x^2}{2}+c$
where c is constant of integration.

33. What is $\int \frac{dx}{x\, \ell n\, x}$ equal to ? *[2013-I]*
(a) $\ell n(\ell n x) + c$ (b) $\ell n\ x + c$
(c) $(\ell n x)^2 + c$ (d) None of the above

34. What is $\int \frac{dx}{\sqrt{4\ \ x^2}}$ equal to ? *[2013-II]*
(a) $\ell n\left|\sqrt{4\ \ x^2}\ \ x\right|\ \ c$ (b) $\ell n\left|\sqrt{4+x^2}-x\right|\ \ c$
(c) $\sin^{-1}\left(\frac{x}{2}\right)\ \ c$ (d) None of these

35. What is $\int \sin^2 x\, dx + \int \cos^2 x\, dx$ equal to ? *[2013-II]*
(a) $x + c$ (b) $\frac{x^2}{2}\ \ c$
(c) $x^2 + c$ (d) None of these
Where *c* is an arbitary constant.

36. What is $\int e^{e^x} e^x dx$ equal to ? *[2013-II]*
(a) $e^{e^x}\ \ c$ (b) $2e^{e^x}\ \ c$
(c) $e^{e^x} e^x\ \ c$ (d) $2e^{e^x} e^x\ \ c$

37. What is $\int (x\cos x\ \ \ \sin x)dx$ equal to ? *[2013-II]*
(a) $x \sin x + c$ (b) $x \cos x + c$
(c) $-x \sin x + c$ (d) $-x \cos x + c$
Where *c* is an arbitrary constant.

38. What is the equation of a curve passing through (0, 1) and whose differential equation is given by $dy = y \tan x\, dx$? *[2014-I]*
(a) $y = \cos x$ (b) $y = \sin x$
(c) $y = \sec x$ (d) $y = \text{cosec}\, x$

DIRECTIONS (Qs. 39-40) : *For the next two (02) items that follow :*

Consider $\int x \tan^{-1} x\, dx\ \ \ A(x^2\ \ \ 1)\tan^{-1} x\ \ \ Bx\ \ \ C,$
where C is the constant of integration. *[2014-II]*

39. What is the value of *A* ?
(a) 1 (b) 1/2
(c) –1/2 (d) 1/4

40. What is the value of *B* ?
(a) 1 (b) 1/2
(c) –1/2 (d) 1/4

DIRECTIONS (Qs. 41-42) : *For the next two (02) items that follow:*

Consider the function $f''(x) = \sec^4 x + 4$ with $f(0) = 0$ and $f'(0) = 0$. *[2014-II]*

41. What is $f'(x)$ equal to ?
(a) $\tan x - \frac{\tan^3 x}{3}\ \ 4x$ (b) $\tan x\ \ \frac{\tan^3 x}{3}\ \ 4x$
(c) $\tan x - \frac{\sec^3 x}{3}\ \ 4x$ (d) $-\tan x - \frac{\tan^3 x}{3}\ \ 4x$

42. What is $f(x)$ equal to ?

(a) $\frac{2\ln\sec x}{3}+\frac{\tan^2 x}{6}+2x^2$

(b) $\frac{3\ln\sec x}{2}+\frac{\cot^2 x}{6}+2x^2$

(c) $\frac{4\ln\sec x}{3}+\frac{\sec^2 x}{6}+2x^2$

(d) $\ln\sec x+\frac{\tan^4 x}{12}+2x^2$

43. What is $\int\frac{xe^x dx}{(x+1)^2}$ equal to? [2015-I]

(a) $(x+1)^2 e^x+c$ (b) $(x+1)e^x+c$

(c) $\frac{e^x}{x+1}+c$ (d) $\frac{e^x}{(x+1)^2}+c$

where c is the constant integration.

DIRECTIONS (Qs. 44-45): *For the next two (2) items that follow.*

The integral $\int\frac{dx}{a\cos x+b\sin x}$ is of the form $\frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$.

44. What is r equal to? [2015-I]

(a) a^2+b^2 (b) $\sqrt{a^2+b^2}$

(c) $a+b$ (d) $\sqrt{a^2-b^2}$

45. What is α equal to?

(a) $\tan^{-1}\left(\frac{a}{b}\right)$ (b) $\tan^{-1}\left(\frac{b}{a}\right)$

(c) $\tan^{-1}\left(\frac{a+b}{a-b}\right)$ (d) $\tan^{-1}\left(\frac{a-b}{a+b}\right)$

46. What is $\int\frac{dx}{\sqrt{x^2+a^2}}$ equal to? [2015-I]

(a) $\ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right|+c$ (b) $\ln\left|\frac{x-\sqrt{x^2+a^2}}{a}\right|+c$

(c) $\ln\left|\frac{x^2+\sqrt{x^2+a^2}}{a}\right|+c$ (d) None of these

where c is the constant of integration.

47. What is $\int\frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx$ equal to? [2016-II]

(a) $\sqrt{\frac{x^4+x^2+1}{x}}+c$ (b) $\sqrt{x^4+2-\frac{1}{x^2}}+c$

(c) $\sqrt{x^2+\frac{1}{x^2}+1}+c$ (d) $\sqrt{\frac{x^4-x^2+1}{x}}+c$

48. What is $\int e^{\sin x}\frac{x\cos^3 x-\sin x}{\cos^2 x}dx$ equal to? [2016-II]

(a) $x \quad \sec x \quad e^{\sin x} \quad c$ (b) $x-\sec x \quad e^{\sin x} \quad c$

(c) $x \quad \tan x \quad e^{\sin x} \quad c$ (d) $x-\tan x \quad e^{\sin x} \quad c$

49. What is $\int\frac{dx}{x\left(x^7+1\right)}$ equal to? [2017-I]

(a) $\frac{1}{2}\ln\left|\frac{x^7-1}{x^7+1}\right|+c$ (b) $\frac{1}{7}\ln\left|\frac{x^7 \quad 1}{x^7}\right| \quad c$

(c) $\ln\left|\frac{x^7-1}{7x}\right|+c$ (d) $\frac{1}{7}\ln\left|\frac{x^7}{x^7+1}\right|+c$

50. What is $\int\frac{\left(x^{e-1}+e^{x-1}\right)dx}{x^e+e^x}$ equal to? [2017-I]

(a) $\frac{x^2}{2}+c$ (b) $\text{In}(x+e)+c$

(c) $\text{In}(x^e+e^x)+c$ (d) $\frac{1}{e}\text{In}\left(x^e+e^x\right)+c$

51. Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements : [2017-I]

Statement 1 : The function $f(x)$ satisfies $f(x+\pi)=f(x)$ for all real x.

Statement 2 : $\sin^2(x+\pi)=\sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

52. What is $\int\tan^{-1}(\sec x+\tan x)dx$ equal to? [2017-II]

(a) $\frac{\pi x}{4}+\frac{x^2}{4}+c$ (b) $\frac{\pi x}{2}+\frac{x^2}{4}+c$

(c) $\frac{\pi x}{4} \quad \frac{\pi x^2}{4} \quad c$ (d) $\frac{\pi x}{4}-\frac{x^2}{4}+c$

53. $\int(\text{In } x)^{-1}dx-\int(\text{In } x)^{-2}dx$ is equal to [2017-II]

(a) $x(\text{In } x)^{-1}+c$ (b) $x(\text{In } x)^{-2}+c$

(c) $x(\text{In } x)+c$ (d) $x(\text{In } x)^2+c$

54. What is $\int\frac{dx}{2^x-1}$ equal to? [2018-I]

(a) $\ln(2^x-1)+c$ (b) $\frac{\ln \; 1-2^{-x}}{\ln 2} \quad c$

(c) $\frac{\ln \; 2^{-x}-1}{2\ln 2} \quad c$ (d) $\frac{\ln \; 1 \quad 2^{-x}}{\ln 2} \quad c$

55. What is $\int \sin^3 x \cos x \, dx$ equal to? *[2018-II]*
 (a) $\cos^4 x + c$ (b) $\sin^4 x + c$
 (c) $\frac{1-\sin^2 x \quad ^2}{4} \quad c$ (d) $\frac{1-\cos^2 x \quad ^2}{4} \quad c$
 Where c is the constant of integration.

56. What is $\int e^{\text{In}(\tan x)} dx$ equal to? *[2018-II]*
 (a) In $|\tan x| \quad c$ (b) In $|\sec x| \quad c$
 (c) $\tan x \quad c$ (d) $e^{\tan x} \quad c$
 Where c is the constant of integration.

57. What is $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ equal to? *[2018-II]*
 (a) $c + \frac{1}{ab} \tan^{-1}\left(\frac{a \tan x}{b}\right)$
 (b) $c - \frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{a}\right)$
 (c) $c + \frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{a}\right)$
 (d) None of these

58. What is $\int \ln(x^2) dx$ equal to ? *[2019-I]*
 (a) $2x \ln(x) - 2x + c$ (b) $\frac{2}{x} + c$
 (c) $2x \ln(x) + c$ (d) $\frac{2 \ln(x)}{x} - 2x + c$

59. What is $\int e^{x \ln(a)} dx$ equal to ? *[2019-I]*
 (a) $\frac{a^x}{\ln(a)} + c$ (b) $\frac{e^x}{\ln(a)} + c$
 (c) $\frac{e^x}{\ln(ae)} + c$ (d) $\frac{ae^x}{\ln(a)} + c$

ANSWER KEY

1	**(b)**	7	**(c)**	13	**(a)**	19	**(c)**	25	**(a)**	31	**(a)**	37	(a)	43	(c)	49	(d)	55	(d)
2	**(c)**	8	**(c)**	14	**(b)**	20	**(a)**	26	**(b)**	32	**(d)**	38	(c)	44	(b)	50	(d)	56	(b)
3	**(d)**	9	**(a)**	15	**(c)**	21	**(a)**	27	**(d)**	33	**(a)**	39	(b)	45	(a)	51	(b)	57	(a)
4	**(b)**	10	**(b)**	16	**(b)**	22	**(c)**	28	**(c)**	34	(a)	40	(c)	46	(a)	52	(a)	58	(a)
5	**(d)**	11	**(b)**	17	**(c)**	23	**(b)**	29	**(c)**	35	(a)	41	(b)	47	(c)	53	(a)	59	(a)
6	**(b)**	12	**(b)**	18	**(b)**	24	**(d)**	30	**(a)**	36	(a)	42	(a)	48	(b)	54	(b)		

HINTS & SOLUTIONS

1. (b) Given that $f(x) = \ln(x - \sqrt{1+x^2})$

$\int f''(x) dx \quad f'(x) \quad c$ where c is a constant

$$= \frac{1}{(x-\sqrt{1+x^2})} \cdot \left(1 - \frac{2x}{2\sqrt{1+x^2}}\right) + c$$

$$= \frac{-(x-\sqrt{1+x^2})}{(\sqrt{1+x^2})(x-\sqrt{1+x^2})} + c = -\frac{1}{\sqrt{1+x^2}} + c$$

2. (c) Let $I = \int \sec x . \text{cosec}\, x \, dx = \int \frac{2}{2 \sin x \cos x} dx$

$$= 2\int \frac{1}{\sin 2x} dx = 2\int \frac{1}{\frac{2 \tan x}{1+\tan^2 x}} \quad [\because \sin 2x = \frac{2 \tan x}{1+\tan^2 x}]$$

$$= \int \frac{\sec^2 x}{\tan x} dx$$

Let $\tan x = t \Rightarrow \sec^2 dx = dt$

So, $I = \int \frac{dt}{t} = \log|t| + c = \log|\tan x| + c$

$g(x) = \tan x$

But $\int \sec x \,\text{cosec}\, x \, dx = \log|g(x)| + c$

$\therefore \quad g(x) = \tan x$

3. (d) The given integral is $\int \frac{dx}{(x^2+a^2)(x^2+b^2)}$

Breaking the expression under integral into partial fraction

$$\frac{1}{(x^2+a^2)(x^2+b^2)}$$

$$= \left(\frac{1}{(x^2+a^2)} - \frac{1}{(x^2+b^2)}\right) \times \frac{1}{b^2-a^2}$$

The given integral is

$$\frac{1}{(b^2-a^2)} \int \left(\frac{1}{(x^2+a^2)} - \frac{1}{(x^2+b^2)}\right) dx$$

$$= \frac{1}{(b^2-a^2)} \left[\int \frac{1}{x^2+a^2} dx - \int \frac{1}{x^2+b^2} dx\right]$$

$$= \frac{1}{(b^2-a^2)} \left\{ \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a} - \frac{\tan^{-1}\left(\frac{x}{b}\right)}{b} \right\} + c$$

4. (b) $\int (x+\sqrt{x})^{-1} dx = \int \frac{1}{(x+\sqrt{x})} dx$

$= \int \frac{1}{(\sqrt{x}\cdot\sqrt{x}+\sqrt{x})}\cdot dx$

Let $\sqrt{x}+1 = t$

then, $\frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

$\therefore \int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx = \int \frac{2dt}{t} = 2 \log t + c$

$= 2 \log(1+\sqrt{x}) + c$

5. (d) Let the given integral be

$I = \int \frac{e^x(1+x)}{\sin^2(xe^x)} dx$

Put $x e^x = t$ and $e^x(1+x)\,dx = dt$

$\Rightarrow \quad I = \int \frac{dt}{\sin^2 t} = \int \operatorname{cosec}^2 t\, dt$

$= -\cot t + c = -\cot(x e^x) + c$

6. (b) Let $I = \int \log(x+1)dx$

Let $x + 1 = t$

$\Rightarrow \quad dx = dt$

$\Rightarrow \quad I = \int 1.\log t dt$

Integrating by parts, taking log t as first function

$\Rightarrow I = t\log t - \int \frac{1}{t}.t dt + c_1$

$= t\log t - \int 1 dt + c_1 = t\log t - t + c1$

$= (x+1)\log(x+1) - x - 1 + c_1$

$= (x+1)\log(x+1) - x + c \qquad [\because c = c_1 - 1]$

7. (c) We check from the given options one by one. Options (a) and (b) do not satisfy. We check option (c).

Let $f(x) = \frac{x}{2} + \alpha$

$\therefore \int \frac{dx}{\frac{x}{2}+\alpha} = \int \frac{2dx}{(x+2\alpha)}$

$= 2\log(x+2\alpha) + c_1 = \log(x+2\alpha)^2 + c_1$

$= \log\left(\frac{x}{2}+\alpha\right)^2 + \log 2^2 + c_1 = \log\left(\frac{x}{2}+\alpha\right)^2 + c$

8. (c) Let $I = \int (e^x+1)^{-1} dx = \int \frac{1}{e^x+1} dx = \int \frac{e^{-x}}{1+e^{-x}} dx$

Let $1+e^{-x} = t \Rightarrow -e^{-x}dx = dt$

$\therefore \quad I = -\int \frac{1}{t} dt = -\log t + c = -\log(1+e^{-x}) + c$

9. (a) Let $I = \int \frac{d\theta}{\sin^2\theta + 2\cos^2\theta - 1}$

$= \int \frac{d\theta}{1-\cos^2\theta + 2\cos^2\theta - 1} = \int \frac{d\theta}{\cos^2\theta}$

$= \int \sec^2\theta d\theta = \tan\theta + c$

10. (b) $\int \sin x \log(\tan x) dx$

$= -\cos x \log\tan x - \int (-\cos x)\frac{1}{\tan x}.\sec^2 x\, dx$

$= -\cos x \log\tan x + \int \frac{1}{\sin x} dx$

$= -\cos x \log(\tan x) + \int \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}} dx$

Let $t = \tan\frac{x}{2}$

$\Rightarrow \frac{dx}{dt} = \frac{2}{1+t^2} \Rightarrow dx = \frac{2}{1+t^2}\cdot dt$

So, $-\cos x\cdot\log(\tan x) + \int \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}}\cdot dx$

$= -\cos x\cdot\log(\tan x) + \int \frac{1+t^2}{2t}\cdot\frac{2}{1+t^2}\cdot dt$

$= -\cos x\log\tan x + \int \frac{1}{t}.dt$

$= -\cos x\log\tan x + \log(t) + c$

$= -\cos x\log\tan x + \log\tan\left(\frac{x}{2}\right) + c$

11. (b) (A) Consider $\int \frac{e^x}{x}(1+x\log x)dx$

$= \int \frac{e^x}{x} dx + \int e^x \log x\, dx$

$= e^x\log x - \int e^x\log x\, dx + \int e^x \log x\, dx = e^x \log x$

(R) $\int e^x[f(x)+f'(x)]dx$

$= \int e^x f(x)\,dx + \int e^x f'(x)\,dx$

$= e^x f(x) - \int e^x f'(x)\,dx + \int e^x f'(x)\,dx = e^x f(x) + c$

Both A and R are true but R is not the correct explanation of A.

12. (b) Let $I = \int \tan^2 x\sec^4 x\, dx$

Let $\tan x = t$

$\Rightarrow \quad \sec^2 x\, dx = dt$

$\therefore \quad I = \int \tan^2 x.\sec^2 x.\sec^2 x.dx$

$= \int \tan^2 x(1+\tan^2 x)\sec^2 x.dx$

$\therefore \quad I = \int t^2(1+t^2)dt = \int (t^2+t^4)dt$

$= \frac{t^5}{5} + \frac{t^3}{3} + c = \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + c$

13. (a) $\int \sec x^\circ \cdot dx = \int \frac{\sec x^\circ \cdot (\sec x^\circ + \tan x^\circ)}{\sec x^\circ + \tan x^\circ} \cdot dx$

Let $u = \sec x^\circ + \tan x^\circ$

$\Rightarrow \frac{du}{dx} = \sec x^\circ + \tan x^\circ + \sec^2 x^\circ$

$\Rightarrow du = (\sec x^\circ \times \tan x^\circ + \sec^2 x^\circ) \cdot dx$

$\therefore \int \frac{\sec x^\circ \cdot (\sec x^\circ + \tan x^\circ)}{\sec x^\circ + \tan x^\circ} dx$

$= \int \frac{\sec^2 x^\circ + \sec x^\circ \cdot \tan x^\circ}{\sec x^\circ + \tan x^\circ} \cdot dx$

$= \int \frac{du}{u} = \log (u) + C = \log(\sec x^\circ + \tan x^\circ) + C$

14. (b) Consider $\int \frac{a \quad b \sin x}{\cos^2 x} dx \quad \int \left(\frac{a}{\cos^2 x} \quad \frac{b \sin x}{\cos^2 x} \right) dx$

$= \int (a \sec^2 x + b \tan x \sec x) dx = a \tan x + b \sec x + c$

15. (c) Let $I = \int \frac{\log x}{(1 + \log x)^2} dx$

Put $\log x = t \Rightarrow \frac{1}{x} dx \quad dt$

$\Rightarrow dx = x \cdot dt \Rightarrow dx = e^t \cdot dt \qquad (\because x = e^t)$

$I = \int \frac{e^t t}{(1+t)^2} dt \quad = \int \frac{e^t . (t+1-1)}{(1+t)^2} dt$

$\int \frac{e^t (1 \quad t)}{(1 \quad t)^2} dt - \int \frac{e^t}{(1 \quad t)^2} dt \quad = \int \frac{e^t}{1+t} dt - \int \frac{e^t}{(1+t)^2} dt$

$= \frac{e^t}{1+t} - \int -e^t \frac{1}{(1+t)^2} dt - \int \frac{e^t}{(1+t)^2} dt$

$= \frac{e^t}{1 \quad t} \quad \int e^t \frac{1}{(1 \quad t)^2} dt - \int \frac{e^t}{(1 \quad t)^2} dt$

$= \frac{e^t}{1+t} + c \quad = \frac{x}{1 + \log x} + c$

16. (b) Let I $= \int e^{\ln x} \sin x \, dx = \int x \sin x \, dx \; \left(\because e^{\log a} = a \right)$

$= -x \cos x + \int 1 . \cos x \, dx = (\sin x - x \cos x) + c$

17. (c) $\int \frac{x^4 + 1}{x^2 + 1} dx = \int \frac{x^4 + 2 - 1}{x^2 + 1} dx = \int \left(\frac{x^4 - 1}{x^2 + 1} + \frac{2}{x^2 + 1} \right) dx$

$= \int \left[\frac{(x^2 - 1)(x^2 + 1)}{x^2 + 1} + \frac{2}{x^2 + 1} \right] dx$

$= \int \left(x^2 - 1 + \frac{2}{x^2 + 1} \right) dx \quad = \frac{x^3}{3} - x + 2 \tan^{-1} x + c$

18. (b) $\int x^2 \ln x \, dx = \ln x \frac{x^3}{3} - \int \frac{1}{x} . \frac{x^3}{3} dx$

$= \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx \quad = \frac{x^3}{3} \ln x - \frac{1}{3} . \frac{x^3}{3} + c$

$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + c \qquad \text{....(1)}$

But $\int x^2 \ln x \, dx = \frac{x^3}{m} \ln x + \frac{x^3}{n} + c$

On comparing with (1), we get
$m = 3$ and $n = -9$

19. (c) $\int \frac{1}{1 + e^x} dx = \int \frac{1}{e^x \left(1 + \frac{1}{e^x} \right)} dx = \int \frac{e^{-x}}{e^{-x} + 1} dx$

Put $1 + e^{-x} = t$

$-e^{-x} \; dx = dt$

$\therefore \int \frac{e^{-x}}{1 + e^{-x}} dx = -\int \frac{dt}{t} = -\log t + c$

$= -\log \left(1 + e^{-x} \right) + c$

$= -\log \left(\frac{1 \quad e^x}{e^x} \right) \quad c \quad = x - \log \left(1 + e^x \right) + c$

$(\because \log e^x = x)$

20. (a) Let $I = \int \sqrt{x} e^{\sqrt{x}} dx$

Put, $\sqrt{x} = t$
Differentiate both side w.r.t (t)

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt \quad \Rightarrow dx = 2t \, dt$

$\therefore \quad I = \int t e^t 2t \; dt = 2 \int t^2 e^t dt$

By parts, let first function = t^2 and second function = e^t.

$= 2 \left[t^2 e^t - \int 2t \; e^t dt \right] = 2 \left[t^2 e^t - 2\{ t e^t - \int e^t dt \} \right]$

$= 2 \; [t^2 e^t - 2t \; e^t + 2e^t] + c$

where c is a constant of integration.

$= 2[x e^{\sqrt{x}} - 2\sqrt{x} \, e^{\sqrt{x}} + 2e^{\sqrt{x}}] + c$

$= 2e^{\sqrt{x}} (x - 2\sqrt{x} + 2) + c$

21. (a) Let I $= \int \sec^n x \tan x dx$.

Put, $\sec x = t \Rightarrow \sec x \tan x dx = dt \Rightarrow dx = \frac{dt}{t}$

$\therefore \quad I = \int t^n . \frac{dt}{t} = \int t^{n-1} dt = \frac{t^n}{n} + c = \frac{\sec^n x}{n} + c$

where ‘c’ is a constant of integration.

22. (c) Let $\mathrm{I} \quad \int \frac{\mathrm{e^x \quad 1 \quad x}}{\mathrm{\cos^2 \quad xe^x}} \mathrm{dx}$

Put, $\mathrm{xe^x = t \Rightarrow e^x (1 + x) \, dx = dt}$

$\therefore \quad \mathrm{I} \quad \int \frac{\mathrm{dt}}{\mathrm{\cos^2 t}} \quad \int \mathrm{\sec^2 tdt} \quad \mathrm{\tan t} \quad \mathrm{c}$

where ‘c’ is a constant of integration.
$= \tan (\mathrm{xe^x}) + \mathrm{c}$

23. (b) Let $I = \int e^x\left(\sqrt{x} \quad \frac{1}{2\sqrt{x}}\right)dx$

$= \int e^x\sqrt{x}\,dx \quad \int e^x.\frac{1}{2\sqrt{x}}dx$

$= e^x.\sqrt{x} - \int e^x.\frac{1}{2\sqrt{x}}dx \quad \int e^x.\frac{1}{2\sqrt{x}}dx$

$= e^x.\sqrt{x} + C$

where C is constant of integration.

24. (d) Let $I = \int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}\,dx \quad dt \Rightarrow \frac{1}{\sqrt{x}}\,dx \quad 2dt$

$\therefore \quad I = 2\int \sin t\,dt = -2\cos t \quad C = -2\cos\sqrt{x} \quad C$

where 'C' is a constant of integration.

25. (a) Let $I = \int \sin^{-1}(\cos x)\,dx$

$= \int \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right]dx = \int\left(\frac{\pi}{2} - x\right)dx$

$= \frac{x\pi}{2} - \frac{x^2}{2} \quad K$

where K is a constant of integration.

26. (b) Let $I = \int \frac{dx}{\sin^2 x\cos^2 x} = \int \frac{(\sin^2 x + \cos^2 x)\,dx}{\sin^2 x\cos^2 x}$

$= \int\left[\frac{\sin^2 x}{\sin^2 x\cos^2 x} + \frac{\cos^2 x}{\sin^2 x\cos^2 x}\right]dx$

$= \int\left[\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right]dx$

$= \int\left(\sec^2 x + \operatorname{cosec}^2 x\right)dx$

$= \tan x - \cot x + c$ where c is constant of Integration.

27. (d) (1) Let $I = \int \ell n 10\,dx = \ell n 10 \int dx = [\ell n 10]x + c$

(2) Let $I = \int 10^x\,dx = \frac{10^x}{\log_e 10} + c = \frac{10^x}{\ell n\,10} + c$

28. (c) Let $I = \int x(x^2+1)^{5/2}dx$

Put $x^2 + 1 = t$

$2xdx = dt$

$xdx = \frac{dt}{2}$

$\therefore \quad I = \int (t)^{5/2}\frac{dt}{2} = \frac{1}{2}\left(\frac{t^{7/2}}{7/2}\right) + c = \frac{1}{7}\left(x^2+1\right)^{7/2} + c$

29. (c) Let $I \quad \int \underset{I}{a^x}\,\underset{II}{e^x}\,dx$

$I = a^x\int e^x\,dx - \int a^x\,\ell n\,a\,e^x\,dx$

$I = a^x\,e^x - \ell n\,a\int a^x\,e^x\,dx$

$I = a^x e^x - \ell n\,a.\,(I)$

$\Rightarrow (1 + \ell n\,a)\,I = a^x e^x.$

$\Rightarrow I \quad \frac{a^x\,e^x}{\ell n\;ae} \quad c \quad \because \ell n\,e \quad 1$

30. (a) Let $I = \int \frac{\ell nx}{x}\,dx$

Put $\ell n\,x = t$

$\frac{1}{x}dx = dt$

$\therefore \; I = \int t\,dt = \frac{t^2}{2} + c$

where c is the constant of integration.

$= \frac{\ell n\,x\;\;^2}{2} \quad c$

31. (a) Let $I = \int\left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}\right)dx$

$= \int\left(\sec^2 x - \operatorname{cosec}^2 x\right)dx$

$= \int \sec^2 x\;dx - \int \operatorname{cosec}^2 x\;dx$

$= \tan x + \cot x + c$

$= \tan x + \frac{1}{\tan x} + c = \frac{\tan^2 x + 1}{\tan x} + c = \frac{\sec^2 x}{\tan x} + c$

$= \frac{2}{2\sin x\cos x} + c = \frac{2}{\sin 2x} + c = 2\operatorname{cosec} 2x + c$

32. (d) Let $I = \int e^{\ell nx}\,dx = \int x\,dx = \frac{x^2}{2} + c$

33. (a) Let $I = \int \frac{dx}{x\,\ell n\,x}$

Put $\ell n\,x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore \quad I = \int \frac{1}{t}dt = \ell nt + c = \ell n(\ell nx) + c$

where c is the constant of integration.

34. (a) $\int \frac{dx}{\sqrt{4 \quad x^2}} = \int \frac{dx}{\sqrt{2^2 \quad x^2}} = \ln|\sqrt{4+x^2} + x| + C$

35. (a) $\int \sin^2 x\,dx + \int \cos^2 x\,dx = \int (\sin^2 x + (\cos^2 x)\,dx$

$\int dx = x + c$

36. (a) $I = \int \; e^{e^x}e^x dx$

Let $e^x = y \Rightarrow e^x\,dx = dy$

$dx = \frac{dy}{e^x}$

$I = \int \; e^y e^x\frac{dy}{e^x} = \int \; e^y dy = e^y + c$

$I = e^{e^x} \quad c$

37. (a) $\int (x\cos x + \sin x)\,dx = \int x\cos x\,dx + \int \sin x\,dx$

$= x\sin x - \int \sin x\,dx + \int \sin x\,dx = x\sin x + c$

38. (c) $dy = y\tan x\,dx$

$\int \frac{dy}{y} = \int \tan x\,dx$

$\log|y| = \log|\sec x| + \log|c|$

$\log|y| = \log|c\sec x|$

$y = c\sec x$

at $x = 0, y = 1$

$y = c$

Solution is given by

$y = \sec x$

Sol. (39–40)

Given, $\int x\tan^{-1} x\,dx \quad A(x^2 \quad 1)\tan^{-1} x \quad Bx \quad C$

where, C is the constant of integration

Consider, $\int \underset{\text{II}}{x}\,\underset{\text{I}}{\tan^{-1}} x\,dx$

$= \tan^{-1} x.\frac{x^2}{2} - \int \frac{d}{dx}(\tan^{-1} x).\frac{x^2}{2}dx$

(using integration by parts)

$= \frac{x^2.\tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1 \quad x^2}dx$

$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\left(\int \left(\frac{1+x^2-1}{1 \quad x^2}\right)dx\right)$

$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\left(\int dx - \int \frac{dx}{1+x^2}\right)$

$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}(x - \tan^{-1} x) \quad C$

$= \frac{x^2\tan^{-1} x}{2} - \frac{x}{2} \quad \frac{\tan^{-1} x}{2} \quad C$

$= \frac{1}{2}(x^2+1)\tan^{-1} x - \frac{x}{2} \quad C$

39. (b) $A = \frac{1}{2}$, hence option (b) is correct.

40. (c) $B = -\frac{1}{2}$, hence option (c) is correct.

41. (b) $f'(x) = \int f''(x)dx \quad C_1$

$= \int (\sec^4 x \quad 4)dx \quad C_1$

$= \int \sec^2 x\sec^2 x\,dx + \int 4dx + C_1$

$= \int (1+\tan^2 x)\sec^2 x\,dx + 4x + C_1 = I_1 + 4x + C_1$

Put $\tan x = t$ in the integral I_1, then

$\sec^2 x\,dx = dt$

$\therefore I_1 = \int (1 \quad t^2)dt \quad t \quad \frac{t^3}{3} \quad C'$

$= \tan x \quad \frac{\tan^3 x}{3} \quad C'$

$\therefore f'(x) = \tan x \quad \frac{\tan^3 x}{3} \quad 4x \quad C'$

where, $C = C_1 + C'$

$\because f'(0) = 0 \Rightarrow C \; 0$

Thus, $f'(x) = \tan x \quad \frac{\tan^3 x}{3} \quad 4x$

42. (a) $f(x) = \int f'(x)dx \quad C_2$

$= \int \left(\tan x \quad \frac{\tan^3 x}{3} \quad 4x\right)dx \quad C_2$

$= \int \tan x\,dx + \frac{1}{3}\int \tan^3 x\,dx + 4\int x\,dx + C_2$

$= \int \tan x\,dx + \frac{1}{3}\int \tan x(\sec^2 x - 1)dx + 4.\frac{x^2}{2} + C_2$

$= \frac{2}{3}\int \tan x\,dx + \frac{1}{3}\int \tan x.\sec^2 x\,dx + 2x^2 + C_2$

$= \frac{2}{3}\ln(\sec x) + \frac{1}{3}I_2 + 2x^2 + C_2$

Consider $I_2 = \int \tan x\sec^2 x\,dx$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow I_2 = \int t\,dt = \frac{t^2}{2} + C_3 = \frac{\tan^2 x}{2} + C_3$

$\therefore f(x) = \frac{2}{3}\ln(\sec x) \quad \frac{1}{6}\tan^2 x \quad 2x^2 \quad C_4$

Here, $C_4 = C_2 \quad \left(\frac{C_3}{3}\right)$

$\because f(0) = 0$

$\Rightarrow 0 = \frac{2}{3}\ln(1) \quad 0 \quad 0 \quad C_4$

$\Rightarrow C_4 = 0$

$\therefore f(x) = \frac{2}{3}\ln(\sec x) \quad \frac{1}{6}\tan^2 x \quad 2x^2$

43. (c) $\int \frac{xe^x}{(1+x)^2}dx$

$= \int e^x\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right]dx$

$= \frac{e^x}{1+x} + C \qquad \left\{\int e^x\left(f(x) + f'(x)\right)dx = e^x f(x)\right\}$

44. (b) Given that, $\int \frac{dx}{a\cos x + b\sin x} = \frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$

Let a = r sin α, b = r cos α

$$\int \frac{dx}{r\sin\alpha\cos x + r\cos\alpha\sin x} = \frac{1}{r}\int\frac{1}{\sin(x+\alpha)}$$

$$= \frac{1}{r}\int \text{cosec}(x+\alpha)dx = \frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$$

$a = r\sin\alpha \Rightarrow a^2 = r^2\sin^2\alpha$ ... (i)

$b = r\cos\alpha \Rightarrow b^2 = r^2\cos^2\alpha$ ... (ii)

Adding (i) and (ii), we get

$r^2 = a^2 + b^2$

$\Rightarrow$ $r = \sqrt{a^2+b^2}$.

45. (a) $a = r\sin\alpha$ ... (i)

$b = r\cos\alpha$ ... (ii)

Dividing (i) from (ii),

$$\frac{a}{b} = \tan\alpha$$

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right).$$

46. (a) $\int \frac{dx}{\sqrt{x^2+a^2}}$

Let x = a tan u

dx = a $\sec^2$ u du

$$= \int \frac{a\sec^2 u\,du}{\sqrt{a^2\tan^2 u + a^2}}$$

$$= a\int\frac{\sec^2 u\,du}{\sqrt{a^2(1+\tan^2 u)}} \Rightarrow \frac{a}{a}\int\frac{\sec^2 u\,du}{\sec u}$$

$$= \int\frac{\sec^2 u\,du}{\sec u} = \int \sec u\,du$$

$$= \ln[\tan(u) + \sec(u)] + c \quad \left[\because \int \sec x\,dx = \tan x\right]$$

$$= \ln\left[\frac{x}{\sqrt{a^2}} + \sqrt{1+\frac{x^2}{a^2}}\right] + c$$

$$= \ln\left[\frac{x}{a} + \frac{\sqrt{a^2+x^2}}{a}\right] + c$$

$$= \ln\left[\frac{x+\sqrt{x^2+a^2}}{a}\right] + c$$

∴ Option (a) is correct.

47. (c) Take option (a)

$$I_1 = \sqrt{\frac{x^4+x^3+1}{x}} + C$$

$$\frac{dI_1}{dx} = \frac{d}{dx}\left[(x^3+x^2+x^{-1})^{1/2} + C\right]$$

$$\frac{dI_2}{dx} \quad \frac{1}{2}(x^3 \quad x^2 \quad x^{-1})^{-1/2}(3x^2 \quad 2x - x^{-2})$$

$$\frac{1}{2}\left[\frac{3x^2+2x-\frac{1}{x^2}}{\sqrt{x^3 \quad x^2 \quad \frac{1}{x}}}\right]$$

$$\frac{dI_2}{dx} \quad \frac{1}{2}\left[\frac{3x^4+2x^3-1}{\frac{x^2}{\sqrt{x}}\sqrt{x^4 \quad x^3 \quad 1}}\right]$$

Take option (b) :

$$I_2 = \sqrt{x^4+2-\frac{1}{x^2}} + C$$

$$\frac{dI_2}{dx} = \frac{1}{2}[x^4+2-x^{-2}]^{-1/2}[4x^3+0+2x^{-3}]$$

$$\frac{1}{2}\left[\frac{4x^3 \quad \frac{2}{x^3}}{\sqrt{x^4+2-\frac{1}{x^2}}}\right] \quad \frac{2x^6 \quad 1}{\frac{x^3}{x}\sqrt{x^6+2x^2-1}}$$

$$\frac{dI_2}{dx} \quad \frac{2x^6 \quad 1}{x^2\sqrt{x^6+2x-1}}$$

Take option (c) :

$$I_3 = \sqrt{x^2+x^{-2}+1} + C$$

$$\frac{dI_3}{dx} = \frac{1}{2}[x^2+x^{-2}+1]^{-1/2}[2x-2x^{-3}+0]$$

$$\frac{1}{2}\left[\frac{2x-\frac{2}{x^3}}{\sqrt{x^2 \quad \frac{1}{x^2} \quad 1}}\right] \quad \frac{1}{2}\left[\frac{2(x^4-1)}{x^3\sqrt{\frac{x^4 \quad 1 \quad x^2}{x^2}}}\right]$$

$$\frac{dI_3}{dx} = \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}$$

48. (b) Let us differentiate all the options one by one to get the expression in the question whose integral is to be found.

Here $xe^{\sin x}$ is the common term in all the options. So, let us differentiate it first.

Let $l = xe^{\sin x}$

$$\Rightarrow \frac{dl}{dx} = e^{\sin x}[x\cos x + 1]$$

$$\Rightarrow \frac{dl}{dx} = \frac{e^{\sin x}}{\cos^2 x}[x\cos^3 x + \cos^2 x]$$

Let $m = \sec x e^{\sin x}$

$$\Rightarrow \frac{dm}{dx} = \sec x e^{\sin x} \cdot \cos x + e^{\sin x} \sec x \tan x$$

$$\Rightarrow \frac{dm}{dx} = e^{\sin x}\left[1 + \frac{\sin x}{\cos^2 x}\right]$$

$$\Rightarrow \frac{dm}{dx} = \frac{e^{\sin x}}{\cos^2 x}[\cos^2 x + \sin x]$$

Differentiation of option (a) is

$$= \frac{e^{\sin x}}{\cos^2 x}[x\cos^3 x + \cos^2 x + \cos^2 x + \sin x]$$

$$= \frac{e^{\sin x}}{\cos^2 x}[x\cos^3 x + 2\cos^2 x + \sin x]$$

Differentiation of option (b) is

$$= \frac{e^{\sin x}}{\cos^2 x}[x\cos^3 x + \cos^2 x - \cos^2 x - \sin x]$$

$$= \frac{e^{\sin x}}{\cos^2 x}[x\cos^3 x - \sin x]$$

$\therefore$ Option (b) is correct.

49. (d) $\int \frac{dx}{x\left(x^7+1\right)} = \int \frac{x^6}{x^7\left(x^7+1\right)}.dx$

Let $x^7 = t$

$$\Rightarrow 7x^6.dx = dt \Rightarrow x^6 dx = \frac{dt}{7}$$

Then $\int \frac{x^6 dx}{x^7(x^7+1)} \quad = \frac{1}{7}\int \frac{dt}{t(t+1)}$

$$= \frac{1}{7}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$$

$$= \frac{1}{7}\left[\ell n\,|t| - \ell n\,|t+1|\right] \quad c$$

$$\frac{1}{7}\ell n\left|\frac{t}{t \quad 1}\right| \quad c$$

$$\frac{1}{7}\ell n\left|\frac{x^7}{x^7 \quad 1}\right| \quad c$$

50. (d) $\int \frac{\left(x^{e-1} + e^{x-1}\right)dx}{x^e + e^x}$

Put $x^e \quad e^x \quad t \Rightarrow ex^{e-1} \quad e^x \quad \frac{dt}{dx}$

$$\Rightarrow \quad ex^{e-1} \quad e^x \;\; dx \quad dt$$

$$\therefore \int \frac{\left(x^{e-1} + e^{x-1}\right)dx}{x^e + e^x} = \frac{1}{e}\int \frac{e.x^{e-1} + e^x}{x^e + e^x}.dx$$

$$\frac{1}{e}\int \frac{dt}{t} \quad \frac{1}{e}.\ln t \quad c$$

$$= \frac{1}{e}.\ln\left(x^e + e^x\right) + c$$

51. (b) $f \;\; x \quad \int \sin^2 x.dx \quad \int \frac{1 - \cos 2x}{2}.dx$

$$= \frac{1}{2}x - \frac{\sin 2x}{2}\cdot\frac{1}{2} + c = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

(i) $f(\pi + x) = \frac{1}{2}(\pi + x) - \frac{1}{4}\sin 2(\pi + x)$

$$= \frac{1}{2}\pi + \frac{1}{2}x - \frac{1}{2}\sin(2\pi + 2x) + c$$

$$= \frac{x}{2} - \frac{1}{2}\sin 2x + c = f(x)$$

So, statement 1 is true.

(ii) $\sin^2(\pi + x) = \sin^2 x$

$(-\sin x)^2 = \sin^2 x$

$\Rightarrow \sin^2 x = \sin^2 x$

So, statement 2 is true

52. (a) $\int \tan^{-1} \;\; \sec x \quad \tan x \;\; dx$

$$= \int \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)dx \quad \because \sec x \quad \tan x \quad \frac{1 \quad \sin x}{\cos x}$$

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \quad = \int\left(\frac{\pi}{4} \quad \frac{x}{2}\right)dx$$

$$= \frac{\pi x}{4} + \frac{x^2}{4} + c.$$

53. (a) $\int(\ell n\, x)^{-1}dx - \int(\ell n\, x)^{-2}dx$

$$= \int\left[\frac{1}{\ell n\, x} - \frac{1}{(\ell n\, x)^2}\right].dx.$$

Put $\ell n\, x = t \Rightarrow x = e^t$

$dx = e^t \,.\, dt.$

$$\therefore \int\left[\frac{1}{\ell n\, x} - \frac{1}{(\ell n\, x)^2}\right]dx = \int\left(\frac{1}{t} - \frac{1}{t^2}\right).e^t.dt$$

$$= \int e^t.\left(\frac{1}{t} - \frac{1}{t^2}\right).dt$$

$$= \frac{e^t}{t} + c = \frac{x}{\ell n\, x} + c$$

$$= x(\ell n\, x)^{-1} + c.$$

54. (b) $\int \frac{dx}{2^x - 1} = \int \frac{dx}{\frac{1}{2^{-x}}}$

$= \int \frac{2^{-x}}{1-2^{-x}}.dx$

Let $1 - 2^{-x} = t$

$\Rightarrow 2^{-x}.\log 2 = \frac{dt}{dx} \Rightarrow 2^{-x} = \frac{1}{\log 2}.\frac{dt}{dx} \Rightarrow 2^{-x}.dx \quad \frac{dt}{\log 2}$

$\therefore \int \frac{2^{-x}}{1-2^{-x}} dx \quad \frac{1}{\log 2}\int \frac{dt}{t} \quad \frac{1}{\log 2} \log t \quad c$

$= \frac{1}{\log 2} \log 1 - 2^{-x} \quad c.$

55. (d) Let $t = \sin x \Rightarrow dt = \cos x\, dx$

$\int \sin^3 x \cos x\, dx = \int t^3 dt \quad \frac{t^4}{4} \quad c \quad \frac{\sin^4 x}{4} \quad c$

$= \frac{(1-\cos^2 x)^2}{4} \quad c$

56. (b) $\int e^{\ell n(\tan x)} dx = \int \tan x\, dx$

$= \ell n |\sec x| \quad c$

57. (a) $I = \frac{1}{a^2}\int \frac{\sec^2 x}{\tan^2 x \quad \left(\frac{b}{a}\right)^2}$

$= \frac{1}{a^2} \times \frac{a}{b} \tan^{-1}\left(\frac{a \tan x}{b}\right) \quad c$

$= c \quad \frac{1}{ab} \tan^{-1}\left(\frac{a \tan x}{b}\right)$

58. (a) $\int \ln(x^2) \cdot dx = 2 \cdot \int \ln x \cdot dx$

$= 2\int 1. \ln x.dx = 2 \,[\ln x.x - \int \frac{1}{x}.x.dx$

$= 2\,(x. \ln x - x) + c = 2x \ln x - 2x + c$

59. (a) $\int e^{x \ln(a)} \cdot dx = \int e^x.dx = \frac{a^x}{\ln(a)} + c$

Definite Integration & Its Application

18

1. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$ and $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then what is the value of B ?

(a) $\frac{2}{\pi}$ (b) $\frac{4}{\pi}$

(c) 0 (d) 1 *[2006-I]*

2. If m and n are integers, then what is the value of $\int_0^{\pi} \sin mx \sin nx\,dx$, if $m \neq n$?

(a) 0 (b) $\frac{1}{m+n}$

(c) $\frac{1}{m-n}$ (d) mn *[2006-I]*

3. What is the area under the curve $y = |x| + |x-1|$ between $x = 0$ and $x = 1$?

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2 *[2006-I]*

4. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.

Assertion (A) : $\int_1^e \ln^2 x\,dx = e - 2$

Reason (R) : $I_n = \int_1^e \ln^n x\,dx = e - nI_{n-1}$

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation A
(c) A is true but R is false
(d) A is false but R is true *[2006-II]*

5. What is the value of $\int_0^1 (x-1)e^{-x}dx$?

(a) 0 (b) e

(c) $\frac{1}{e}$ (d) $\frac{-1}{e}$ *[2006-II]*

6. If $\int_{\ln 2}^{x} (e^x - 1)^{-1}\,dx = \ln\frac{3}{2}$, then what is the value of x ?

(a) e^2 (b) $\frac{1}{e}$

(c) ln 4 (d) 1 *[2007-I]*

7. If $\int_{-3}^{2} f(x)dx = \frac{7}{3}$ and $\int_{-3}^{9} f(x)dx = -\frac{5}{6}$, then what is the value of $\int_2^9 f(x)dx$?

(a) $-\frac{19}{6}$ (b) $\frac{19}{6}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$ *[2007-I]*

8. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.

Assertion(A) : $\int_0^{\pi} \sin^7 x\,dx = 2\int_0^{\pi/2} \sin^7 x\,dx$

Reason(R) : $\sin^7 x$ is an odd function

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation A
(c) A is true but R is false
(d) A is false but R is true *[2007-I]*

9. What is the area enclosed by the curve $2x^2 + y^2 = 1$?
(a) 2π (b) π
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{\sqrt{2}}$ *[2007-II]*

10. What is the value of integral $I = \int_0^1 x(1-x)^9 dx$?
(a) $\frac{1}{110}$ (b) $\frac{1}{111}$
(c) $\frac{1}{112}$ (d) $\frac{1}{119}$ *[2007-II]*

11. What is the value of $\int_{-1}^{1} x|x| dx$?
(a) 2 (b) 1
(c) $\frac{1}{4}$ (d) 0 *[2007-II]*

12. What is the value of $\int_0^{\pi/2} \cos^8 x\, dx$?
(a) $\frac{35\pi}{256}$ (b) $\frac{70}{256}$
(c) $\frac{16}{35}$ (d) $\frac{8\pi}{35}$ *[2007-II]*

13. What is $\int_a^b \frac{\log x}{x} dx$ equal to?
(a) $(1/2) \log(ab) . \log(b/a)$
(b) $\log b / \log a$
(c) $\log(b/a)$
(d) $(1/2) \log[(a+b)/ab]$ *[2007-II]*

14. What is the area of the region bounded by the line $3x - 5y = 15$, $x = 1$, $x = 3$ and x-axis in sq unit?
(a) $\frac{36}{5}$ (b) $\frac{18}{5}$
(c) $\frac{9}{5}$ (d) $\frac{3}{5}$ *[2008-I]*

15. What is the value of $\int_0^1 xe^{x^2} dx$?
(a) $\frac{(e-1)}{2}$ (b) $e^2 - 1$
(c) $2(e-1)$ (d) $e - 1$ *[2008-I]*

16. What is the area of the ellipse $4x^2 + 9y^2 = 1$ *[2008-II]*
(a) 6π (b) $\frac{\pi}{36}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{\sqrt{6}}$

17. The value of $\int_{-2}^{2} (ax^3 + bx + c)\, dx$ depends on which of the following? *[2008-II]*
(a) Values of x only
(b) Values of each of a, b and c
(c) Value of c only
(d) Value of b only

18. What are the values of p which satisfy the equation $\int_0^p (3x^2 + 4x - 5)\, dx = p^3 - 2$? *[2008-II]*
(a) 1/2 and 2 (b) –1/2 and 2
(c) 1/2 and –2 (d) –1/2 and –2

19. What is the value of $\int_0^{\pi/2} \log(\tan x)\, dx$? *[2009-I]*
(a) 0 (b) 1
(c) –1 (d) $\pi/4$

20. What is $\int_0^1 x(1-x)^n dx$ equal to? *[2009-I]*
(a) $\frac{1}{n(n+1)}$ (b) $\frac{1}{(n+1)(n+2)}$
(c) 1 (d) 0

21. What is the value of k if the area bounded by the curve $y = \sin kx$, $y = 0$, $x = \pi/k$, $x = \pi/(3k)$ is 3 sq unit? *[2009-II]*
(a) 1/2 (b) 1
(c) 3/2 (d) 2

22. If $f(x) = a + bx + cx^2$, then what is $\int_0^1 f(x)\, dx$ equal to? *[2009-II]*
(a) $[f(0) + 4f(1/2) + f(1)]/6$
(b) $[f(0) + 4f(1/2) + f(1)]/3$
(c) $[f(0) + 4f(1/2) + f(1)]$
(d) $[f(0) + 2f(1/2) + f(1)]/6$

23. What is the area bounded by the curve $y = 4x - x^2 - 3$ and the x-axis? *[2009-II]*
(a) 2/3 sq unit (b) 4/3 sq unit
(c) 5/3 sq unit (d) 4/5 sq unit

24. What is $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$? *[2010-I]*
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) 0

25. What is the area enclosed between the curves $y^2 = 12x$ and the lines $x = 0$ and $y = 6$? *[2010-I]*
(a) 2 sq unit (b) 4 sq unit
(c) 6 sq unit (d) 8 sq unit

26. What is $\int_{-\pi/4}^{\pi/4} \tan^3 x\, dx$ equal to? *[2010-I]*
(a) $\sqrt{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) 0

27. What is the value of $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin x \cos x}$? *[2010-II]*

(a) $2\,ln\,\sqrt{3}$ (b) $ln\,\sqrt{3}$
(c) $2\,ln\,3$ (d) $4\,ln\,3$

28. What is the value of $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$? *[2010-II]*

(a) $e\left(\frac{e}{2}-1\right)$ (b) $e(e-1)$
(c) $e-\frac{1}{e}$ (d) 0

29. What is the area under the curve $f(x) = xe^x$ above the x-axis and between the lines $x = 0$ and $x = 1$? *[2010-II]*

(a) $\frac{1}{2}$ sq unit (b) 1 sq unit
(c) $\frac{3}{2}$ sq unit (d) 2 sq unit

30. What is the area bounded by the curve $y = x^2$ and the line $y = 16$? *[2010-II]*
(a) 32/3 (b) 64/3
(c) 256/3 (d) 128/3

31. What is the area of the region bounded by the curve $f(x)=1-\frac{x^2}{4}$, $x \in [-2, 2]$, and the x-axis? *[2010-II]*

(a) $\frac{8}{3}$ sq unit (b) $\frac{4}{3}$ sq unit
(c) $\frac{2}{3}$ sq unit (d) $\frac{1}{3}$ sq unit

32. What is the value of the integral $\int_{-1}^{1} |x|\,dx$? *[2010-II]*
(a) 1 (b) 0
(c) 2 (d) -1

33. What is the area bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$? *[2011-I]*

(a) $\left(e+\frac{1}{e}\right)$ sq unit (b) $\left(e-\frac{1}{e}\right)$ sq unit
(c) $\left(e+\frac{1}{e}-2\right)$ sq unit (d) $\left(e-\frac{1}{e}-2\right)$ sq unit

34. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$ then what is $I_n + I_{n-2}$ equal to? *[2011-I]*

(a) $\frac{1}{n}$ (b) $\frac{1}{(n-1)}$
(c) $\frac{n}{(n-1)}$ (d) $\frac{1}{(n-2)}$

35. What is $\int_0^{\pi} \frac{dx}{1+2\sin^2 x}$ equal to? *[2011-I]*

(a) π (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{\sqrt{3}}$ (d) $\frac{2\pi}{\sqrt{3}}$

36. If $f(x)$ is an even function, then what is $\int_0^{\pi} f(\cos x)\,dx$ equal to? *[2011-I]*

(a) 0 (b) $\int_0^{\frac{\pi}{2}} f(\cos x)\,dx$
(c) $2\int_0^{\frac{\pi}{2}} f(\cos x)\,dx$ (d) 1

37. What is the area between the curve $y = \cos 3x,\ 0 \le x \le \frac{\pi}{6}$ and the co-ordinate axes? *[2011-II]*

(a) 1 square unit (b) $\frac{1}{2}$ square unit
(c) $\frac{1}{3}$ square unit (d) $\frac{1}{4}$ square unit

38. What is the area enclosed by the equation $x^2 + y^2 = 2$? *[2011-II]*
(a) 4π square units (b) 2π square units
(c) $4\pi^2$ square units (d) 4 square units

39. If $\int_1^2 \left\{K^2 + (4-4K)x + 4x^3\right\}dx \le 12$, then which one of the following is correct? *[2011-II]*
(a) $K = 3$ (b) $0 \le K < 3$
(c) $K \le 4$ (d) $K = 0$

40. What is the area bounded by the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ $(x, y \ge 0)$ and the coordinate axes? *[2011-II]*

(a) $\frac{5a^2}{6}$ (b) $\frac{a^2}{3}$
(c) $\frac{a^2}{2}$ (d) $\frac{a^2}{6}$

41. What is $\int_{-\pi/2}^{\pi/2} |\sin x|\,dx$ equal to? *[2012-I]*

(a) 2 (b) 1
(c) π (d) 0

42. The area bounded by the curve x = f[y], the y-axis and the two lines y = a and y = b is equal to: *[2012-I]*
(a) $\int_a^b y\,dx$ (b) $\int_a^b y^2\,dx$
(c) $\int_a^b x\,dy$ (d) None of the above

43. What is $\int_0^1 \frac{\tan^{-1}}{1+x^2}dx$ equal to? *[2012-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$
(c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{32}$

44. What is $\int_{-1}^{1} x|x|\,dx$ equal to ? *[2012-II]*
(a) 2 (b) 1
(c) 0 (d) – 1

45. What is $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$ equal to ? *[2012-II]*
(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{32}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

46. What is $\int_0^{\pi/2} \sin 2x\, \ell n(\cot x)\,dx$ equal to ? *[2012-II]*
(a) 0 (b) $\pi \ell n 2$
(c) $-\pi \ell n 2$ (d) $\frac{\pi \ell n 2}{2}$

47. What is the area of the portion of the curve y = sin x, lying between x = 0, y = 0 and x = 2π ? *[2012-II]*
(a) 1 square unit (b) 2 square units
(c) 4 square units (d) 8 square units

48. What is the area of the region bounded by the lines y = x, y = 0 and x = 4 ? *[2012-II]*
(a) 4 square units (b) 8 square units
(c) 12 square units (d) 16 square units

49. What $\int_0^2 \frac{dx}{x^2+4}$ equal to ? *[2013-I]*
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{8}$ (d) None of the above

50. What is $\int_{-a}^{a}\left(x^3+\sin x\right)dx$ equal to *[2013-I]*
(a) a (b) 2a
(c) 0 (d) 1

51. What is $\int_0^1 xe^x\,dx$ equal to *[2013-I]*
(a) 1 (b) – 1
(c) 0 (d) e

52. What is $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^5 x\cos^3 x}{x^4}dx$ is equal to ? *[2013-I]*
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{8}$ (d) 0

53. What is the area of the region enclosed by y = 2 |x| and y = 4? *[2013-I]*
(a) 2 square unit (b) 4 square unit
(c) 8 square unit (d) 16 square unit

54. What is the area of the parabola $y^2 = x$ bounded by its latus rectum? *[2013-I]*
(a) $\frac{1}{12}$ square unit (b) $\frac{1}{6}$ square unit
(c) $\frac{1}{3}$ square unit (d) None of the above

55. What is $\int_1^2 \ell n\, x\, dx$ equal to ? *[2013-II]*
(a) $\ell n 2$ (b) 1
(c) $\ell n\left(\frac{4}{e}\right)$ (d) $\ell n\left(\frac{e}{4}\right)$

56. What is the area bounded by the lines $x = 0$, $y = 0$ and $x + y + 2 = 0$? *[2013-II]*
(a) $\frac{1}{2}$ square unit (b) 1 square unit
(c) 2 square units (d) 4 square units

57. What is the area of the parabola $x^2 = y$ bounded by the line $y = 1$? *[2013-II]*
(a) $\frac{1}{3}$ square unit (b) $\frac{2}{3}$ square unit
(c) $\frac{4}{3}$ square units (d) 2 square units

58. What is the area bounded by $y = \tan x$, $y = 0$ and $x = \frac{\pi}{4}$? *[2013-II]*
(a) ℓn 2 square units (b) $\frac{\ell n 2}{2}$ square units
(c) 2 (ℓn 2)square units (d) None of these

59. What is $\int_0^2 e^{\ln x} dx$ equal to ? [2013-II]
(a) 1 (b) 2
(c) 4 (d) None of these

60. What is the derivative of $\sqrt{\frac{1+\cos x}{1-\cos x}}$? [2014-I]
(a) $\frac{1}{2}\sec^2\frac{x}{2}$ (b) $-\frac{1}{2}\text{cosec}^2\frac{x}{2}$
(c) $-\text{cosec}^2\frac{x}{2}$ (d) None of these

61. What is $\int_0^1 \frac{e^{\tan^{-1}x}dx}{1+x^2}$ equal to ? [2014-I]
(a) $e^{\frac{\pi}{4}}-1$ (b) $e^{\frac{\pi}{4}}+1$
(c) $e-1$ (d) e

DIRECTIONS (Qs. 62-63): *For the next two (02) items that follow.*

Consider the integrals

$$I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} \text{ and } I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}dx}{\sqrt{\sin x}+\sqrt{\cos x}}$$ [2014-I]

62. What is $I_1 - I_2$ equal to ?
(a) 0 (b) $2I_1$
(c) π (d) None of the above

63. What is I_1 equal to ?
(a) π/24 (b) π/18
(c) π/12 (d) π/6

64. What is $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\sin x\, dx$ equal to ? [2014-I]
(a) 0 (b) 2
(c) –2 (d) π

65. What is $\int_0^{\frac{\pi}{2}} \ln(\tan x)dx$ equal to ? [2014-I]
(a) ln 2 (b) –ln 2
(c) 0 (d) None of these

66. What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum ? [2014-II]
(a) $2b^2/3$ square unit (b) $4b^2/3$ square unit
(c) b^2 square unit (d) $8b^2/3$ square unit

DIRECTIONS (Qs. 67-69): *For the next three (03) items that follow.*

Consider $I = \int_0^{\pi} \frac{xdx}{1+\sin x}$ [2014-II]

67. What is I equal to ?
(a) –π (b) 0
(c) π (d) 2π

68. What is $\int_0^{\pi} \frac{(\pi - x)dx}{1+\sin x}$ equal to ?
(a) π (b) π/2
(c) 0 (d) 2π

69. What is $\int_0^{\pi} \frac{dx}{1+\sin x}$ equal to ?
(a) 1 (b) 2
(c) 4 (d) –2

DIRECTIONS (Qs. 70-71): *For the next two (02) items that follow.*

Consider the integral $I = \int_0^{\pi} \ln(\sin x)dx$ [2014-II]

70. What is $\int_0^{\pi/2} \ln(\sin x)dx$ equal to ?
(a) $4I$ (b) $2I$
(c) I (d) $I/2$

71. What is $\int_0^{\pi/2} \ln(\cos x)dx$ equal to ?
(a) $I/2$ (b) I
(c) $2I$ (d) $4I$

72. What is $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ equal to? [2014-II]
(a) $2ab$ (b) $2\pi ab$
(c) $\frac{\pi}{2ab}$ (d) $\frac{\pi}{ab}$

73. The area of a triangle, whose vertices are (3, 4), (5, 2) and the point of intersection of the lines x = a and y = 5, is 3 square units. What is the value of a? [2015-I]
(a) 2 (b) 3
(c) 4 (d) 5

DIRECTIONS (Qs. 74-75): *For the next two (2) items that follows.*

Consider the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

74. What is the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}$ and the circle? [2015-I]
(a) $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$ (b) $\frac{\pi}{2}-\frac{\sqrt{3}}{2}$
(c) $\frac{\pi}{3}-\frac{1}{2}$ (d) None of these

75. What is the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}y$ and the circle? [2015-I]

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (d) None of these

DIRECTIONS (Qs. 76-77): *For the next two (2) items that follow.*

Consider the curves $y = \sin x$ and $y = \cos x$.

76. What is the area of the region bounded by the above two curves and the lines x = 0 and $x = \frac{\pi}{4}$? [2015-I]

(a) $\sqrt{2} - 1$` (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) 2

77. What is the area of the region bounded by the above two curves and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$? [2015-I]

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $2\sqrt{2}$ (d) 2

DIRECTIONS (Qs. 78-81): *For the next four (4) items that follow.*

Consider the integral $I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x} dx$, where m is a positive integer.

78. What is I_1 equal to? [2015-I]

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

79. What is $I_2 + I_3$ equal to? [2015-I]

(a) 4 (b) 2

(c) 1 (d) 0

80. What is I_m equal to? [2015-I]

(a) 0 (b) 1

(c) m (d) 2m

81. Consider the following: [2015-I]

1. $I_m - I_{m-1}$ is equal to 0.
2. $I_{2m} > I_m$

Which of the above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

82. The area of the figure formed by the lines ax + by + c = 0, ax–by + c = 0, ax + by – c = 0 and ax – by – c = 0 is [2015-II]

(a) $\frac{c^2}{ab}$ (b) $\frac{2c^2}{ab}$

(c) $\frac{c^2}{2ab}$ (d) $\frac{c^2}{4ab}$

83. The value of [2015-II]

$\int_a^b \frac{x^7 + \sin x}{\cos x} dx$ where a + b = 0 is

(a) 2b – a sin (b – a) (b) a + 3bcos (b – a)

(c) sin a – (b – a) cos b (d) 0

84. If 0 < a < b, then $\int_a^b \frac{|x|}{x} dx$ is equal to [2015-II]

(a) |b| – |a| (b) |a| – |b|

(c) $\frac{|b|}{|a|}$ (d) 0

85. $\int_0^{2\pi} \sin^5\left(\frac{x}{4}\right) dx$ is equal to [2015-II]

(a) $\frac{8}{15}$ (b) $\frac{16}{15}$

(c) $\frac{32}{15}$ (d) 0

86. $\int_{-1}^{1} x|x| dx$ is equal to [2015-II]

(a) 0 (b) $\frac{2}{3}$

(c) 2 (d) –2

87. The area bounded by the coordinate axes and the curve $\sqrt{x} + \sqrt{y} = 1$, is [2015-II]

(a) 1 square unit (b) $\frac{1}{2}$ square unit

(c) $\frac{1}{3}$ square unit (d) $\frac{1}{6}$ square unit

DIRECTIONS (Qs. 88-89) : *For the next two (02) items that follow :*

Consider the integrals

$$A = \int_0^{\pi} \frac{\sin x\, dx}{\sin x + \cos x} \text{ and } B = \int_0^{\pi} \frac{\sin x\, dx}{\sin x - \cos x}$$

88. Which one of the follwing is correct ? [2015-II]

(a) A = 2B (b) B = 2A

(c) A = B (d) A = 3B

89. What is the value of B? [2015-II]

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) π

DIRECTIONS (Qs. 90-91) : *For the next two (2) items that follow:*

Consider the functions

$f(x) = xg(x)$ and $g(x) = \left[\frac{1}{x}\right]$

Where $[\cdot]$ is the greatest integer function.

90. What is $\int_{\frac{1}{3}}^{\frac{1}{2}} g(x)\,dx$ equal to? *[2016-I]*

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{5}{18}$ (d) $\frac{5}{36}$

91. What is $\int_{\frac{1}{3}}^{1} f(x)\,dx$ equal to ? *[2016-I]*

(a) $\frac{37}{72}$ (b) $\frac{2}{3}$

(c) $\frac{17}{72}$ (d) $\frac{37}{144}$

92. What is *[2016-I]*

$$\int_{-2}^{2} x\,dx - \int_{-2}^{2} [x]\,dx$$

equal to, where $[\cdot]$ is the greatest integer function?

(a) 0 (b) 1

(c) 2 (d) 4

93. If $\int_{-2}^{5} f(x)dx = 4$ and $\int_{0}^{5} \{1 + f(x)\}dx = 7$, then what is $\int_{-2}^{0} f(x)dx$ equal to? *[2016-I]*

(a) –3 (b) 2

(c) 3 (d) 5

94. What is $\int_{0}^{4\pi} |\cos x|\,dx$ equal to? *[2016-I]*

(a) 0 (b) 2

(c) 4 (d) 8

95. What is the area bounded by the curves $|y| = 1 - x^2$? *[2016-II]*

(a) $\frac{4}{3}$ square units (b) $\frac{8}{3}$ square units

(c) 4 square units (d) $\frac{16}{3}$ square units

96. If $\int_{0}^{\pi/2} \frac{dx}{3\cos x + 5} = k\cot^{-1} 2$, then what is the value of K ? *[2016-II]*

(a) 1/4 (b) 1/2

(c) 1 (d) 2

97. What is $\int_{1}^{3} \left|1 - x^4\right| dx$ equal to? *[2016-II]*

(a) $-232/5$ (b) $-116/5$

(c) $116/5$ (d) $232/5$

98. What is $\int_{0}^{\frac{\pi}{2}} \frac{d\theta}{1 + \cos\theta}$ equal to? *[2017-I]*

(a) $\frac{1}{2}$ (b) 1

(c) $\sqrt{3}$ (d) None of the above

99. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = 2$, then what is $\int_{0}^{a} f(x)g(x)dx$ equal to? *[2017-I]*

(a) $\int_{0}^{a} g(x)dx$ (b) $\int_{0}^{a} f(x)dx$

(c) $2\int_{0}^{a} f(x)dx$ (d) 0

100. What is the maximum area of a triangle that can be inscribed in a circle of radius a? *[2017-I]*

(a) $\frac{3a^2}{4}$ (b) $\frac{a^2}{2}$

(c) $\frac{3\sqrt{3}a^2}{4}$ (d) $\frac{\sqrt{3}a^2}{4}$

101. What is $\int_{e^{-1}}^{e^2} \left|\frac{\ln x}{x}\right| dx$ equal to? *[2017-I]*

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) 3 (d) 4

102. What is $\int_{0}^{2\pi} \sqrt{1 + \sin\frac{x}{2}}\,dx$ equal to? *[2017-II]*

(a) 8 (b) 4

(c) 2 (d) 0

103. The area bounded by the curve $|x| + |y| = 1$ is *[2017-II]*

(a) 1 square unit (b) $2\sqrt{2}$ square units

(c) 2 square units (d) $2\sqrt{3}$ square units

104. Let $f(n) = \left[\frac{1}{4} + \frac{n}{1000}\right]$, where [x] denote the integral part of x. Then the value of $\sum_{n=1}^{1000} f(n)$ is *[2017-II]*

(a) 251 (b) 250

(c) 1 (d) 0

105. The value of $\int_{0}^{\frac{\pi}{4}} \sqrt{\tan x}\,dx + \int_{0}^{\frac{\pi}{4}} \sqrt{\cot x}\,dx$ is equal to *[2017-II]*

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{\sqrt{2}}$

106. What is the area of the region bounded by the parabolas $y^2 = 6(x-1)$ and $y^2 = 3x$? [2018-I]

(a) $\frac{\sqrt{6}}{3}$ (b) $\frac{2\sqrt{6}}{3}$

(c) $\frac{4\sqrt{6}}{3}$ (d) $\frac{5\sqrt{6}}{3}$

DIRECTIONS (Qs. 107-109) : *Consider the following information for the next three (03) items that follow.*

Three sides of a trapezium are each equal to 6 cm. Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be the angle between a pair of adjacent sides.

107. If the area of the trapezium is the maximum possible, then what is α equal to? [2018-I]

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{5}$

108. If the area of the trapezium is maximum, what is the length of the fourth side? [2018-I]

(a) 8 cm (b) 9 cm

(c) 10 cm (d) 12 cm

109. What is the maximum area of the trapezium? [2018-I]

(a) $36\sqrt{3}\ cm^2$ (b) $30\sqrt{3}\ cm^2$

(c) $27\sqrt{3}\ cm^2$ (d) $24\sqrt{3}\ cm^2$

110. What is $\int_0^{\pi} e^x \sin x\, dx$ equal to? [2018-I]

(a) $\frac{e^{\pi}+1}{2}$ (b) $\frac{e^{\pi}-1}{2}$

(c) $e^{\pi}+1$ (d) $\frac{e^{\pi}+1}{4}$

111. What is $\int_1^e x\, ln\, x\, dx$ equal to? [2018-I]

(a) $\frac{e+1}{4}$ (b) $\frac{e^2+1}{4}$

(c) $\frac{e-1}{4}$ (d) $\frac{e^2-1}{4}$

112. What is $\int_0^{\sqrt{2}} [x^2]dx$ equal to (where [.] is the greatest integer function)? [2018-I]

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$

(c) $2\left(\sqrt{2}-1\right)$ (d) $\sqrt{3}-1$

113. What is the value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin \times \tan x)dx$? [2018-I]

(a) $-\frac{1}{\sqrt{2}} + \ell n\left(\frac{1}{\sqrt{2}}\right)$ (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) $\sqrt{2}$

114. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then what are the values of a and b respectively? [2018-I]

(a) –1, 1 (b) 1, 1

(c) 0, 0 (d) 2, –2

115. What is $\int_0^1 x(1-x)^9 dx$ equal to? [2018-I]

(a) $\frac{1}{110}$ (b) $\frac{1}{132}$

(c) $\frac{1}{148}$ (d) $\frac{1}{240}$

116. What is $\int_a^b [x]\, dx + \int_a^b [-x]\, dx$ equal to, where [.] is the greatest integer function? [2018-II]

(a) b – a (b) a – b

(c) 0 (d) 2(b – a)

117. What is $\int_2^8 |x-5| dx$ equal to? [2018-II]

(a) 2 (b) 3

(c) 4 (d) 9

118. What is $\int_{-1}^{1} \left\{\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right\} dx$ equal to? [2018-II]

(a) 0 (b) $-\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

119. $\int_0^{\pi/2} |\sin x - \cos x|\, dx$ is equal to [2019-I]

(a) 0 (b) $2(\sqrt{2}-1)$

(c) $2\sqrt{2}$ (d) $2(\sqrt{2}+1)$

120. $\int_0^{\pi/2} e^{\sin x} \cos x\, dx$ is equal to [2019-I]

(a) $e+1$ (b) $e-1$

(c) $e+2$ (d) e

121. What is the area of one of the loops between the curve y = c sin x and x -axis ? [2019-I]

(a) c (b) 2c

(c) 3c (d) 4c

ANSWER KEY

1	(c)	13	(a)	25	(c)	37	(c)	49	(c)	61	(a)	73	(d)	85	(c)	97	(d)	109	(c)	121	(b)
2	(a)	14	(b)	26	(d)	38	(b)	50	(c)	62	(a)	74	(a)	86	(a)	98	(b)	110	(a)		
3	(b)	15	(a)	27	(b)	39	(a)	51	(a)	63	(c)	75	(a)	87	(d)	99	(b)	111	(b)		
4	(a)	16	(c)	28	(a)	40	(d)	52	(d)	64	(b)	76	(a)	88	(c)	100	(c)	112	(a)		
5	(d)	17	(c)	29	(b)	41	(a)	53	(c)	65	(c)	77	(a)	89	(b)	101	(b)	113	(c)		
6	(c)	18	(a)	30	(c)	42	(c)	54	(b)	66	(d)	78	(a)	90	(b)	102	(a)	114	(a)		
7	(a)	19	(a)	31	(a)	43	(d)	55	(c)	67	(c)	79	(d)	91	(a)	103	(c)	115	(a)		
8	(b)	20	(b)	32	(a)	44	(c)	56	(c)	68	(a)	80	(a)	92	(c)	104	(a)	116	(b)		
9	(d)	21	(a)	33	(c)	45	(b)	57	(c)	69	(b)	81	(a)	93	(b)	105	(d)	117	(d)		
10	(a)	22	(a)	34	(b)	46	(a)	58	(b)	70	(d)	82	(b)	94	(d)	106	(c)	118	(c)		
11	(d)	23	(b)	35	(c)	47	(b)	59	(b)	71	(a)	83	(d)	95	(b)	107	(c)	119	(b)		
12	(a)	24	(c)	36	(c)	48	(b)	60	(b)	72	(c)	84	(a)	96	(b)	108	(d)	120	(b)		

HINTS & SOLUTIONS

1. (c) Given function $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$

Differentiating w. r. t. x

$$f'(x) = A\cos\left(\frac{\pi x}{2}\right).\frac{\pi}{2}$$

$$f'\left(\frac{1}{2}\right) = \sqrt{2} = A\left(\cos\frac{\pi}{4}\right)\frac{\pi}{2} = A.\frac{1}{\sqrt{2}}.\frac{\pi}{2}$$

$$\Rightarrow \quad A = \frac{(\sqrt{2}\times\sqrt{2})\times 2}{\pi} = \frac{4}{\pi}$$

Now, $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$

$$\Rightarrow \quad \int_0^1 \left\{A\sin\left(\frac{\pi x}{2}\right) + B\right\}dx = \frac{2\times 4}{\pi^2}$$

$$\Rightarrow \quad \left[-A\cos\frac{\pi x}{2}.\frac{2}{\pi} + Bx\right]_0^1 = \frac{8}{\pi^2}$$

$$\Rightarrow \quad -\frac{4}{\pi}.\frac{2}{\pi}\cos\frac{\pi}{2} + B + \frac{4}{\pi}.\frac{2}{\pi}\cos 0 = \frac{8}{\pi^2}$$

$$\Rightarrow \quad B + \frac{8}{\pi^2} = \frac{8}{\pi^2} \quad \Rightarrow B = 0$$

2. (a) The given integral

$$\int_0^{\pi} \sin mx.\sin nx\,dx$$

$$= \int_0^{\pi} \sin m(\pi - x).\sin n(\pi - x)\,dx$$

$$= \int_0^{\pi} \sin mx.\sin nx\,dx$$

So, $\int_0^{\pi} \sin mx.\sin nx\,dx$

$$= 2\int_0^{\pi/2} \sin mx.\sin nx\,dx$$

$$= 2\int_0^{\pi/2} \frac{1}{2}[\cos(mx - nx) - \cos(mx + nx)]dx$$

$$= \int_0^{\pi/2} [\cos(m-n)x - \cos(m+n)x]dx$$

$$= \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n}\right]_0^{\pi/2} = 0$$

3. (b) $|x|$ for $x \geq 0$
$= x$ and $|x - 1|$ for $x \leq 1$
$= -(x - 1)$,

so, $\int_0^1 (|x| + |x - |1||) =$ required area

$$a = \int_0^1 x\,dx - \int_0^1 (x - 1)\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^2}{2} - x\right]_0^1 = \frac{1}{2} - \left(\frac{1}{2} - 1\right) = 1 \text{ sq units}$$

4. (a) Assertion (A) : $\int_1^e \ln^2 x \cdot dx = \int_1^e \ln^2 x \cdot 1\,dx$

$$\int \ln^2 x \cdot 1dx = x \cdot \ln^2(x) - \int 2 \cdot \ln x \cdot dx$$
$$= x \cdot \ln^2 x - 2x \cdot \ln x + 2x$$

$$\therefore \int_1^e \ln^2 x \cdot dx = [x \cdot \ln^2 x - 2x\ln x + 2x]_1^e$$

$= (e \cdot \ln^2 e - 2e \cdot \ln e + 2e) - (\ln^2 1 - 2\ln 1 + 2)$
$= e - 2e + 2e - 0 + 0 - 2 \quad (\because \ln e = 1 \text{ and } \ln 1 = 0)$
$= e - 2$
So, A is true.

Reason (R) : $I_n = \int_1^e \ln^n x \cdot dx$

$\therefore \int \ln^n x \cdot dx = x \times \ln^n x - n \int \ln^{n-1} x \cdot dx$
$= x \cdot \ln^n x - n \times I_{n-1}$

So, $\int_1^e \ln^n x \cdot dx = [x \cdot \ln^n x - n \times I_{n-1}]_1^e$
$= e \cdot \ln^n e - \ln^n 1 - n \times I_{n-1}$
$= e - n \times I_{n-1} \quad (\because \ln e = 1 \text{ and } \ln 1 = 0)$
So, R is true and R is correct explanation of A.

5. (d) Given integral is $I = \int_0^1 (x-1) e^{-x} dx$

Integrating by parts taking (x –1) as first function

We get, $I = [(x-1)\{-e^{-x}\}]_0^1 - \int_0^1 1.(-e^{-x}) dx$

$= -(1-1)\frac{1}{e} + (-1)e^0 + [-e^{-x}]_0^1 = -1 - \frac{1}{e} + 1 = -\frac{1}{e}$

6. (c) Let $I = \int_{\ell n 2}^{x} (e^x - 1)^{-1} dx$

$= \int_{\ell n 2}^{x} \frac{1}{e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow e^x = t + 1$

$e^x dx = dt \Rightarrow dx = \frac{dt}{e^x}$

or, $dx = \frac{dt}{t+1}$

when $x = \ell n 2$, $t = e^{\ln 2} - 1 = 2 - 1 = 1$

and $I = \int_1^t \frac{1}{t(t+1)} dt$

breaking into partial fractions.

$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$

and $I = \int_1^t \left(\frac{1}{t} - \frac{1}{t+1}\right) dt = \left[\log_e^t - \log_e(t+1)\right]_1^t$

or $I = \left[\log_e \frac{t}{t+1}\right]_1^t = \log_e \frac{t}{t+1} - \log_e \frac{1}{2}$

$= \log_e \frac{2t}{t+1} = \log_e \frac{3}{2}$

$\left[\text{Since}, \int_{\ell n 2}^{x} (e^x - 1)^{-1} dx = \log_e \frac{3}{2}\right]$

So, $\frac{2t}{t+1} = \frac{3}{2}$

or, $4t = 3t + 3 \Rightarrow t = 3$

So, $e^x - 1 = 3, e^x = 4 \Rightarrow x = \ln 4$

7. (a) Value of the integral $\int_2^9 f(x)dx$

$= \int_{-3}^{9} f(x)dx - \int_{-3}^{2} f(x)\,dx \quad$ ...(i)

Given, $\int_{-3}^{9} f(x)\,dx = \frac{-5}{6}$ and $\int_{-3}^{2} f(x)dx = \frac{7}{3}$

Putting these values in equation (i)

$\int_2^9 f(x)dx = \frac{-5}{6} - \frac{7}{3} = -\frac{19}{6}$

8. (b) $\int_0^{\pi} \sin^7 x\,dx = 2\int_0^{\pi/2} \sin^7 x\,dx$

sin x is an odd function and for an odd function

$\int_0^a f(x)\,dx = 2\int_0^{a/2} f(x)\,dx$

Hence, $\int_0^{\pi} \sin^7 x\,dx = 2\int_0^{\pi/2} \sin^7 x\,dx$ is true.

So, A and R both are individually true but R is not the correct explanation of A.

9. (d) Given equation of curve $2x^2 + y^2 = 1$ is an ellipse which can be written as $\frac{x^2}{y^2} + \frac{y^2}{1} = 1$

Area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $A = \pi ab$ sq unit

Here, $a = \frac{1}{\sqrt{2}}, b = 1$.

$\therefore$ Required area $= \pi.\frac{1}{\sqrt{2}}.1 = \frac{\pi}{\sqrt{2}}$ sq unit

10. (a) Let the given integral be,

$I = \int_0^1 x(1-x)^9 dx$

Put $1 - x = t \Rightarrow dx = -dt$ and $x = 1 - t$
when $x = 0, t = 1$ and when $x = 1, t = 0$

$\Rightarrow I = \int_1^0 (1-t)\, t^9 (-dt)$

$= -\int_1^0 (-t^{10} + t^9)dt = -\int_1^0 (-t^{10} + t^9)dt$

$\left[\frac{-t^{11}}{11} + \frac{t^{10}}{10}\right]_0^1 = \frac{-1}{11} + \frac{1}{10} = \frac{-10+11}{110} = \frac{1}{110}$

11. (d) Let $f(x)=x|x|$

$f(-x)=-x|-x|=-x|x|=-f(x)$

$\Rightarrow$ $x|x|$ is an odd function

Hence, $\int_{-1}^{1} x|x|\,dx = 0$

12. (a) Let, $I = \int_0^{\pi/2} \cos^8 x\,dx$

Given integral can be also be written as :

$I = \int_0^{\pi/2} \sin^{\circ} x \cos^8 x\,dx,$ which is known as Gamma function.

Solution is : $\int_0^{\pi/2} \sin^m x \cos^n x\,dx$

$= \dfrac{[(m-1)(m-3)....2 \text{ or } 1][(n-1)(n-3)....2 \text{ or } 1]}{[(m+n)(m+n-2)....2 \text{ or } 1]} \cdot \theta$

If m and n both are even then RHS should be multiplied by $\frac{\pi}{2}$ here, $m = 0$, $n = 8$

$\Rightarrow \quad I = \dfrac{(8-1)(8-3)(8-5)...(8-7)}{8.(8-2)(8-4)(8-6)} \dfrac{\pi}{2}$

$= \dfrac{7.5.3.1}{8.6.4.2} \dfrac{\pi}{2} = \dfrac{35\pi}{256}$

13. (a) Let the given integer be

$I = \int_a^b \dfrac{\log x}{x}\,dx$

Put $\log x = t$ and $\frac{dx}{x} = dt$ when $x = a$, $t = \log a$ and if $x = b$, $t = \log b$.

$\therefore \quad I = \int_{\log a}^{\log b} t\,dt = \left[\dfrac{t^2}{2}\right]_a^b$

$= \dfrac{1}{2}[(\log b)^2 - (\log a)^2]$

$= \dfrac{1}{2}[(\log b + \log a)(\log b - \log a)]$

$= \dfrac{1}{2}\log(ab)\log\left(\dfrac{b}{a}\right)$

14. (b) The given equation of line can be rewritten as

$\dfrac{x}{5} - \dfrac{y}{3} = 1$ and $y = \dfrac{3x-15}{5}$

$\therefore$ Required area $= \int_1^3 y\,dx$

$= \int_1^3 \left(\dfrac{3x-15}{5}\right) dx = \dfrac{1}{5}\int_1^3 (3x-15)\,dx$

$= \dfrac{1}{5}\left[\dfrac{3x^2}{2} - 15x\right]_1^3 = \dfrac{1}{5}\left[\dfrac{27}{2} - 45 - \dfrac{3}{2} + 15\right]$

$= \dfrac{1}{5}\left[\dfrac{24}{3} - 30\right] = \dfrac{1}{5}[12-30]$

$= \dfrac{-18}{5} = \dfrac{18}{5}$ sq. unit (neglecting -ve sign)

15. (a) Let $I = \int_0^1 x e^{x^2} \cdot dx$

Let $x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \dfrac{dt}{2}$

when $x = 0, t = 0$ then $x = 1, t = 1$

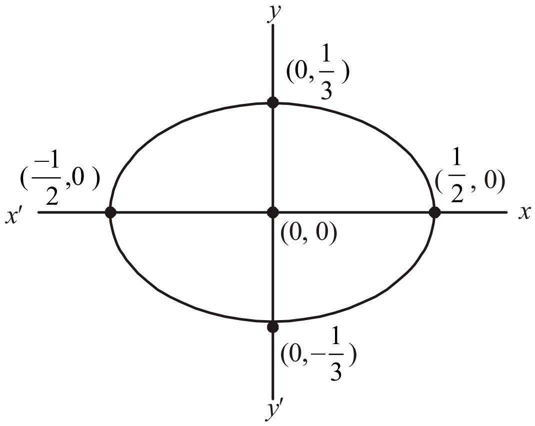
$\Rightarrow I = \dfrac{1}{2}\int_0^1 e^t dt = \dfrac{1}{2}\left[e^t\right]_0^1$

$= \dfrac{1}{2}\left[e^{x^2}\right]_0^1 = \dfrac{1}{2}[e - e^0] = \dfrac{e-1}{2}$

16. (c) Given $4x^2 + 9y^2 = 1$

$\Rightarrow \dfrac{x^2}{\left(\frac{1}{2}\right)^2} + \dfrac{y^2}{\left(\frac{1}{3}\right)^2} = 1$

$\therefore \quad a = \dfrac{1}{2}$ and $b = \dfrac{1}{3}$



Now, area of ellipse $= 4\int_0^{1/2} y\,dx$

$= 4\int_0^{1/2} \sqrt{\dfrac{1-4x^2}{9}}\,dx = \dfrac{4}{3}\int_0^{1/2} \sqrt{1-(2x)^2}\,dx$

$= \dfrac{2}{3}\int_0^1 \sqrt{1-t^2}\,dt = \dfrac{2}{3}\left[\dfrac{1}{2}t\sqrt{1-t^2} + \dfrac{1}{2}\sin^{-1}(t)\right]_0^1$

$= \dfrac{2}{3}\left[0 + \dfrac{1}{2}\sin^{-1}(1) - \dfrac{1}{2}\sin^{-1}(0)\right]$

$= \dfrac{2}{3}\left[\dfrac{1}{2} \times \dfrac{\pi}{2} - \dfrac{1}{2} \times 0\right] = \dfrac{\pi}{6}$

17. (c) $\int_{-2}^{2}(ax^3+bx+c)dx$

$=\left[\frac{ax^4}{4}+\frac{bx^2}{2}+\frac{cx}{1}\right]_{-2}^{2}$

$=\left[\frac{a(16)}{4}+\frac{b(4)}{2}+2c\right]-\left[\frac{a(16)}{4}+\frac{b(4)}{2}-2c\right]=4c$

So, the value of given integral depends on the value of c only

18. (a) Given equation,

$\int_0^p(3x^2+4x-5)dx=p^3-2$

$\Rightarrow\left[\frac{3x^3}{3}+\frac{4x^2}{2}-5x\right]_0^p=p^3-2$

$\Rightarrow p^3+2p^2-5p=p^3-2$

$\Rightarrow 2p^2-5p+2=0$

$\Rightarrow 2p^2-4p-p+2=0$

$\Rightarrow 2p(p-2)-1(p-2)=0$

$\Rightarrow (p-2)(2p-1)=0$

$\Rightarrow p-2=0, 2p-1=0$

Hence the values of p are $\frac{1}{2}$ and 2.

19. (a) Let $I=\int_0^{\pi/2}\log(\tan x)dx$ ...(i)

and $I=\int_0^{\pi/2}\log\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}dx$

[By the property of definite integral which says $\int_0^a f(x)dx=\int_0^a f(a-x)dx$]

$\Rightarrow I=\int_0^{\pi/2}\log(\cot x)dx$ ...(ii)

By adding equation (i) and (ii), we get

$2I=\int_0^{\pi/2}\log(\tan x)dx+\int_0^{\pi/2}\log(\cot x)dx$

$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log(\tan x\cot x)dx$

$[\because \log m+\log n=\log(mn)]$

$=\int_0^{\frac{\pi}{2}}\log(\tan x.\frac{1}{\tan x})\,dx=\int_0^{\frac{\pi}{2}}\log 1\,dx=0$

$\Rightarrow I=0$

20. (b) Let $I=\int_0^1 x(1-x)^n dx$

Put $1-x=t\Rightarrow dx=-dt$

when $x=0$ then $t=1$

when $x=1$ then $t=0$

$\therefore\ I=-\int_1^0(1-t)t^n dt=\int_0^1(t^n-t^{n+1})\,dt$

$=\left[\frac{t^{n+1}}{n+1}-\frac{t^{n+2}}{n+2}\right]_0^1$

$=\frac{1}{n+1}-\frac{1}{n+2}=\frac{1}{(n+1)(n+2)}$

21. (a) Given $x=\pi/k, x=\frac{\pi}{3k}$

and $y=\sin kx$

Let the required area be A

So, A = 3 (Given)

Therefore Area, $A=\int_{\pi/3k}^{\pi/k}\sin kx\,dx$

$\Rightarrow 3=-\left[\frac{\cos kx}{k}\right]_{\pi/3k}^{\pi/k}$

$\Rightarrow 3=-\frac{1}{k}\left[\cos\pi-\cos\frac{\pi}{3}\right]$

$\Rightarrow 3=-\frac{1}{k}\left[-1-\frac{1}{2}\right]$

$(\because \cos\pi=\cos(\pi/2+\pi/2)$

$=-\sin\pi/2$ and $\cos 60^\circ=\frac{1}{2})$

$\Rightarrow 3=\frac{3}{2k}\Rightarrow k=\frac{1}{2}$

22. (a) Given, $f(x)=a+bx+cx^2$

$\therefore\ \int_0^1 f(x)dx=\int_0^1(a+bx+cx^2)dx$

$=\left[ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right]_0^1$

$=a+\frac{b}{2}+\frac{c}{3}$ ..(i)

Here, $f(0)=a$, $f\left(\frac{1}{2}\right)=a+\frac{b}{2}+\frac{c}{4}$

and $f(1)=a+b+c$

Now, $\frac{f(0)+4f\left(\frac{1}{2}\right)+f(1)}{6}$

$=\frac{a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)+a+b+c}{6}$

$=\frac{a+4\left(\frac{4a+2b+c}{4}\right)+a+b+c}{6}$

$= \dfrac{a+4a+2b+c+a+b+c}{6} = \dfrac{6a+3b+2c}{6}$

$= a + \dfrac{b}{2} + \dfrac{c}{3}$

$\therefore$ From Eqs. (i) and (ii), we get

$\int_0^1 f(x)dx = \dfrac{f(0)+4f\left(\frac{1}{2}\right)+f(1)}{6}$

23. (b) Given curve is $y = 4x - x^2 - 3$
Since, area bounded by x-axis
$\therefore\ y = 0$
$\Rightarrow 4x - x^2 - 3 = 0 \Rightarrow x^2 - 4x + 3 = 0$
$\Rightarrow x^2 - 3x - x + 3 = 0 \Rightarrow (x-3)(x-1) = 0$
$\Rightarrow x = 1, 3$

$\therefore$ Required area $= \int_1^3 (4x - x^2 - 3)dx$

$= \left.\dfrac{4x^2}{2} - \dfrac{x^3}{3} - 3x\right|_1^3 = \left(\dfrac{36}{2} - \dfrac{27}{3} - 9\right) - \left(\dfrac{4}{2} - \dfrac{1}{3} - 3\right)$

$= (18-9-9) - \left(2 - \dfrac{10}{3}\right) = 0 - \left(\dfrac{-4}{3}\right) = \dfrac{4}{3}$ sq. unit.

24. (c) Let $I = \int_0^{\pi/2} \dfrac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$ ...(i)

$= \int_0^{\pi/2} \dfrac{\sin^3(\pi/2 - x)}{\sin^3(\pi/2 - x) + \cos^3(\pi/2 - x)} dx$

By using the property $\int_0^a f(x)dx = \int_0^a f(a-x)\, dx$

$\Rightarrow\ I \int_0^{\pi/2} \dfrac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \left(\begin{array}{l} \because \sin(\pi/2 - \theta) = \cos\theta \text{ and} \\ \cos(\pi/2 - \theta) = \sin\theta \end{array}\right)$

On adding Eqs. (*i*) and (*ii*), we get

$2I = \int_0^{\pi/2} \dfrac{\sin^3 x\, dx}{\sin^3 x + \cos^3 x} + \int_0^{\pi/2} \dfrac{\cos^3 x\, dx}{\sin^3 x + \cos^3 x}$

$= \int_0^{\pi/2} \dfrac{\sin^3 x + \cos^3 x\, dx}{\sin^3 x + \cos^3 x}$

$2I = \int_0^{\pi/2} 1\, dx \Rightarrow 2I = [x]_0^{\pi/2} = \dfrac{\pi}{2} \Rightarrow I = \dfrac{\pi}{4}$

25. (c) Equation of given curve is $y^2 = 12x$
At $y = 6$, $36 = 12x \Rightarrow x = 3$

$\therefore$ Required area $= \int_0^3 (y_1 - y_2)\, dx$ where y_1 represents line and y_2 represents the curve.

$= \int_0^3 \left(6 - \sqrt{12x}\right) dx = [6x]_0^3 - \sqrt{12}\left[\dfrac{2x^{3/2}}{3}\right]_0^3$

$= [6 \times 3] - \dfrac{\sqrt{12} \times 2 \times \sqrt{27}}{3} = 18 - 12 = 6$ sq unit

26. (d) We know

$\int_{-a}^{a} f(x)dx = \begin{cases} 0 \text{ if } f(x) \text{ is odd.} \\ 2\int_0^a f(x)dx \text{ if } f(x) \text{ is even} \end{cases}$

Since, $\tan^3 x$ is an odd function

$\therefore\ \int_{-\pi/4}^{\pi/4} \tan^3 x\ dx = 0$

27. (b) Let $I = \int_{\pi/6}^{\pi/4} \dfrac{dx}{\sin x \cos x}$

multiply and divide by 2

$= 2\int_{\pi/6}^{\pi/4} \dfrac{dx}{2\sin x \cos x} = 2\int_{\pi/6}^{\pi/4} \dfrac{dx}{\sin 2x}$

$= 2\int_{\pi/6}^{\pi/4} \operatorname{cosec} 2x\, dx = 2[\log \tan x]_{\pi/6}^{\pi/4} \cdot \dfrac{1}{2}$

$= [\log \tan \pi/4 - \log \tan \pi/6]$

$= \log 1 - \log \dfrac{1}{\sqrt{3}} = 0 - \log \dfrac{1}{\sqrt{3}}$

$= \log\sqrt{3}\ (\because \log 1 = 0)$

28. (a) Let $I = \int_1^2 e^x \left(\dfrac{1}{x} - \dfrac{1}{x^2}\right) dx$

$= \int_1^2 e^x (f(x) + f'(x))dx$ where $f(x) = \dfrac{1}{x}$

$= e^x f(x)\Big|_1^2$

$\therefore\ I = \left.\dfrac{e^x}{x}\right|_1^2 = \dfrac{e^2}{2} - e = e\left(\dfrac{e}{2} - 1\right)$

29. (b) Given curve is $f(x) = xe^x$, $x = 0$ and $x = 1$.

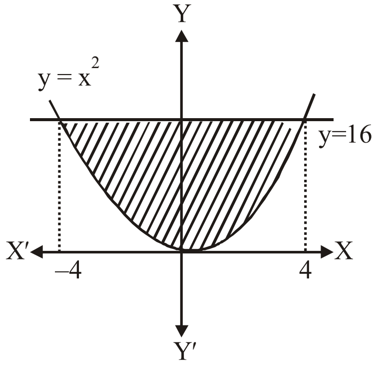
So, Required Area $= \int_0^1 f(x)\, dx = \int_0^1 xe^x dx$

Let x be the first function and e^x be the second function then by parts

$= \left[xe^x - \int e^x dx\right]_0^1 = \left[xe^x - e^x\right]_0^1$

$= (e - e) - (0 - 1) = 1$ sq unit

30. (c) Given, equations of curves are
$y = x^2$ ...(i)
and $y = 16$ ...(ii)
On solving Eqs. (i) and (ii), we get
$x^2 = 16 \Rightarrow x = 4, -4$
$\therefore$ Points of intersection are (4, 16), and (– 4, 16).



Required area = $\int_{-4}^{4}(16-x^2)dx = 2\int_0^4(16-x^2)dx$

$= 2\left[16x-\frac{x^3}{3}\right]_0^4 = 2\left[64-\frac{64}{3}\right] = 2\times 64\times\frac{2}{3}$

$= \frac{256}{3}$ sq unit

31. (a) Required area = $\int_{-2}^{2}\left(1-\frac{x^2}{4}\right)dx$

Since, $\left(1-\frac{x^2}{4}\right)$ is an even function therefore

$\int_{-2}^{2}\left(1-\frac{x^2}{4}\right)dx = 2\int_0^2\left(1-\frac{x^2}{4}\right)dx$

$= 2\left[x-\frac{x^3}{12}\right]_0^2 = 2\left[2-\frac{2^3}{12}\right]$

$= 2\left(2-\frac{2}{3}\right) = \frac{8}{3}$ sq unit

32. (a) Let I = $\int_{-1}^{1}|x|dx = -\int_{-1}^{0}x dx + \int_0^1 x dx$

Since, $|x| = \begin{cases} x & \text{if} \quad x \ge 0 \\ -x & \text{if} \quad x < 0 \end{cases}$

therefore $|x| = -x$ when x lies, between –1 and 0.
and $|x| = x$ when x lies between 0 and 1.

$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = -\left[-\frac{1}{2}\right]+\left[\frac{1}{2}\right] = 1$

33. (c) Given equations of curves are $y = e^x$ and $y = e^{-x}$.

$\Rightarrow e^x = \frac{1}{e^x} \quad \Rightarrow e^{2x} = e^0 \quad \Rightarrow x = 0$

Also, equation of straight line gives $x = 1$

$\therefore$ Required area = $\int_0^1 (e^x - e^{-x})dx$

$= \left[e^x + e^{-x}\right]_0^1 = e + e^{-1} - (e^0 + e^{-0})$

$= \left(e + \frac{1}{e} - 2\right)$ sq unit

34. (b) Let $I_n = \int_0^{\pi/4} \tan^n x\, dx$

Consider,

$I_n + I_{n-2} = \int_0^{\pi/4}\tan^n x\,dx + \int_0^{\pi/4}\tan^{n-2}x\,dx$

$= \int_0^{\pi/4}\tan^{n-2}x(\tan^2 x+1)\,dx$

$= \int_0^{\pi/4}\sec^2 x\tan^{n-2}x\,dx$

Put $\tan x = t$
$\sec^2 x\, dx = dt$

when $x = 0$ then $t = 0$ and when $x = \frac{\pi}{4}$, $t = 1$

$\therefore I_n + I_{n-2} = \int_0^1 t^{n-2}\,dt$

$= \left.\frac{t^{n-2+1}}{n-2+1}\right|_0^1 = \left.\frac{t^{n-1}}{n-1}\right|_0^1 = \frac{1}{n-1}[1-0] = \frac{1}{n-1}$

35. (c) $I = \int_0^{\pi}\frac{dx}{1+2\sin^2 x} = 2\int_0^{\pi/2}\frac{dx}{1+2\sin^2 x}$

$= 2\int_0^{\pi/2}\frac{\sec^2 x\,dx}{\sec^2 x + 2\tan^2 x} = 2\int_0^{\pi/2}\frac{\sec^2 x\,dx}{1+3\tan^2 x}$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$
When x = 0, t = 0

When $x = \frac{\pi}{2}$, $t = \infty$

$\therefore \quad I = 2\int_0^{\infty}\frac{dt}{1+3t^2} = \frac{2}{3}\int_0^{\infty}\frac{dt}{t^2+\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{2}{3}\times\sqrt{3}\left[\tan^{-1}\frac{t}{1/\sqrt{3}}\right]_0^{\infty}$

$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\sqrt{3}\,t\right]_0^{\infty}$

$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\infty - \tan^{-1}0\right]$

$= \frac{2}{\sqrt{3}}\times\frac{\pi}{2} = \frac{\pi}{\sqrt{3}}$

36. (c) Since $f(x)$ is an even function therefore

$$\int_0^{\pi} f(x)\,dx = 2\int_0^{\pi/2} f(x)\,dx$$

Hence, $\int_0^{\pi} f(\cos x)\,dx = 2\int_0^{\pi/2} f(\cos x)\,dx$

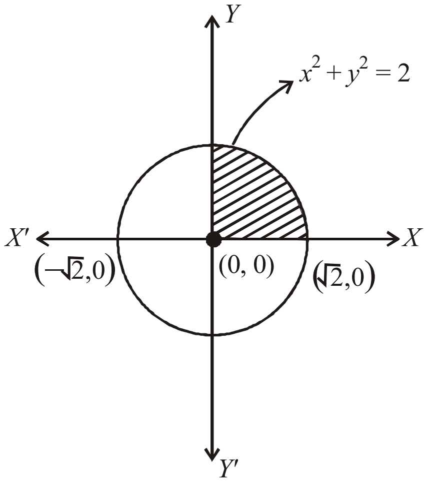
37. (c) Required Area $= \int_0^{\pi/6} \cos 3x\, dx$

$$= \left.\frac{\sin 3x}{3}\right|_0^{\pi/6} = \frac{\sin 3\left(\pi/6\right)}{3} - \sin 0$$

$$= \frac{1}{3}\sin \pi/2 - 0 \;= \frac{1}{3}(1) = \frac{1}{3} \text{ sq. unit.}$$

38. (b) Given equation of circle is $x^2 + y^2 = 2$

$\Rightarrow y = \sqrt{2 - x^2}$



Required area = 4 × Area of shaded portion

$$= 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\; dx \qquad \text{...(i)}$$

$\sqrt{2 - x^2}\,dx$

Let $x = \sqrt{2}\sin t \;\Rightarrow\; t = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$dx = \sqrt{2}\cos t \cdot dt$

$$\therefore \int \sqrt{2 - x^2}\cdot dx = \int \sqrt{2 - 2\sin^2 t}\cdot \sqrt{2}\cos t \cdot dt$$

$$= \int \sqrt{2\cos^2 t}\cdot \sqrt{2}\cos t \cdot dt$$

$$= 2\int \cos^2 t \cdot dt$$

We know,

$$\int \cos^n x \cdot dx = \frac{n-1}{n}\int \cos^{n-2}(x)dx + \frac{\cos^{n-1} x \cdot \sin x}{n}$$

$$= 2\left[\frac{\cos t \cdot \sin t}{2} + \frac{1}{2}\int 1 \cdot dt\right] = 2\left[\frac{\cos t \cdot \sin t}{2} + \frac{t}{2}\right]$$

$= \cos t \sin t + t$

$$= \cos\left(\sin^{-1}\frac{x}{\sqrt{2}}\right) \times \sin\left(\sin^{-1}\frac{x}{\sqrt{2}}\right) + \sin^{-1}\frac{x}{\sqrt{2}}$$

$$= \sqrt{1 - \frac{x^2}{2}}\cdot \frac{x}{\sqrt{2}} + \sin^{-1}\frac{x}{\sqrt{2}}$$

$$= \frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}$$

$$\therefore\; 4\cdot\int_0^{\sqrt{2}} \sqrt{2 - x^2}\cdot dx = 4\left[\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right]_0^{\sqrt{2}}$$

$$= 4\left[0 + \sin^{-1}\frac{\sqrt{2}}{\sqrt{2}} - 0 - \sin^{-1}\frac{0}{\sqrt{2}}\right]$$

$$= 4[\sin^{-1} 1 - \sin^{-1} 0] = 4\left(\frac{\pi}{2} - 0\right) = 2\pi \text{ sq. units.}$$

39. (a) Let $\int_1^2 \left\{K^2 + (4 - 4K)x + 4x^3\right\} dx \le 12$

$$\Rightarrow \left. K^2 x + \frac{(4 - 4K)x^2}{2} + \frac{4x^4}{4}\right|_1^2 \le 12$$

$\Rightarrow [2K^2 + (2 - 2K)(4) + 16] - [K^2 + (2 - 2K) + 1] \le 12$

$\Rightarrow (2K^2 + 8 - 8K + 16) - (K^2 - 2K + 3) \le 12$

$\Rightarrow K^2 - 6K + 21 \le 12$

$\Rightarrow K^2 - 6K + 9 \le 0 \Rightarrow (K - 3)^2 \le 0$

$\Rightarrow K = 3$

40. (d) Area bounded by curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ (x, y ≥ 0) and coordinate axes is

$$= \int_0^a y\, dx = \int_0^a a + x - 2\sqrt{a}\sqrt{x}\; dx$$

$$\left(\because \sqrt{y} = \sqrt{a} - \sqrt{x} \Rightarrow y = a + x - 2\sqrt{a}\sqrt{x}\right)$$

$$= \left. ax + \frac{x^2}{2} - \frac{2\sqrt{a}\, x^{3/2}}{3/2}\right|_0^a$$

$$= a^2 + \frac{a^2}{2} - \frac{4}{3}a^2 = \frac{3a^2}{2} - \frac{4}{3}a^2$$

$$= \frac{9a^2 - 8a^2}{6} = \frac{a^2}{6} \text{ sq. unit}$$

41. (a) Let $I = \int_{-\pi/2}^{\pi/2} |\sin x| \, dx$

Consider, $|\sin x| = \begin{cases} \sin x, 0 < x < \dfrac{\pi}{2} \\ -\sin x, -\dfrac{\pi}{2} < x < 0 \end{cases}$

$I = \int_{-\pi/2}^{0} -\sin x \, dx + \int_{0}^{\pi/2} \sin x \, dx$

$= -[-\cos x]_{-\pi/2}^{0} + (-\cos x)\Big|_{0}^{\pi/2}$

$= (\cos 0 - \cos(-\pi/2)) + (-\cos \pi/2 - (-\cos 0))$

$= (1 - 0) + (-0 + 1) = 2.$

42. (c) Required Area $= \int_{y=a}^{b} f(y) \, dy = \int_{a}^{b} x \, dy$

43. (d) Let $I = \int_{0}^{1} \dfrac{\tan^{-1}}{1+x^2} dx$

Put $\tan^{-1} x = t$

$\dfrac{1}{1+x^2} dx = dt$

$x = 0, \Rightarrow t = 0$

$x = 1, \Rightarrow t = \pi/4$

$\therefore \; I = \int_{0}^{\pi/4} t dt = \dfrac{t^2}{2}\Big|_{0}^{\pi/4} = \dfrac{\pi^2}{32}$

44. (c) Let $I = \int_{-1}^{1} x |x| \, dx = \int_{-1}^{0} x(-x) \, dx + \int_{0}^{1} x(x) \, dx$

$\left(\because |x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases} \right)$

$= -\int_{-1}^{0} x^2 \, dx + \int_{0}^{1} x^2 \, dx$

$= \dfrac{-x^3}{3}\Big|_{-1}^{0} + \dfrac{x^3}{3}\Big|_{0}^{1} = -\left(0 + \dfrac{1}{3}\right) + \dfrac{1}{3} = 0$

45. (b) Let $I = \int_{0}^{1} \dfrac{\tan^{-1} x}{1+x^2} dx$

Let, $\tan^{-1} x = t$

$\dfrac{1}{1+x^2} dx = dt$

Also, $x = 0 \to t = 0$

$x = 1 \to t = \dfrac{\pi}{4}$

$\therefore \; I = \int_{0}^{\frac{\pi}{4}} t \, dt = \dfrac{t^2}{2}\Big|_{0}^{\frac{\pi}{4}} = \dfrac{\pi^2}{32}$

46. (a) Let $I = \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\cot x) \, dx$

$= \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\cos x) \, dx - \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\sin x) \, dx$

$\left(\because \cot x = \dfrac{\cos x}{\sin x} \right)$

$= \int_{0}^{\frac{\pi}{2}} \sin\left[2\left(\dfrac{\pi}{2} + x\right)\right] \ell n \cos\left(\dfrac{\pi}{2} + x\right) dx$

$- \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\sin x) \, dx$

$= \int_{0}^{\frac{\pi}{2}} \sin(\pi + 2x) \, \ell n(\sin x) \, dx - \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\sin x) \, dx$

$= \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\sin x) \, dx - \int_{0}^{\frac{\pi}{2}} \sin 2x \, \ell n(\sin x) \, dx = 0$

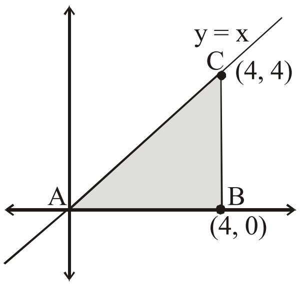
47. (b) Required area $= \int_{0}^{2\pi} \sin x \, dx$

$= -\cos x\Big|_{0}^{2\pi} = -\cos 2\pi - (-\cos 0)$

$= -\cos(\pi + \pi) + 1 = -[-\cos \pi] + 1$

$= +\cos\left(\dfrac{\pi}{2} + \dfrac{\pi}{2}\right) + 1 = \sin \dfrac{\pi}{2} + 1 = 1 + 1 = 2$ sq. units.

48. (b)



Area of shaded region $= \dfrac{1}{2} \times AB \times BC$

$= \dfrac{1}{2} \times 4 \times 4 = 8$ sq. units.

49. (c) Let $I=\int_0^2 \frac{dx}{x^2+4}=\int_0^2 \frac{dx}{x^2+(2)^2}$

$=\frac{1}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^{-2}=\frac{1}{2}\left[\tan^{-1}(1)-\tan^{-1}(0)\right]$

$=\frac{1}{2}\left[\frac{\pi}{4}-0\right]=\frac{\pi}{8}$

50. (c) Let $I=\int_{-a}^{a}\left(x^3+\sin x\right)dx$

Let $f(x)=x^3+\sin x$

$f(-x)=(-x)^3+\sin(-x)=-x^3-\sin x$

$=-(x^3+\sin x)=-f(x)$

Since f (x) is an odd function

$\therefore \quad \int_{-a}^{a} f(x)\,dx=0$

51. (a) Let $I=\int_0^1 \underset{\text{I}}{x}\,\underset{\text{II}}{e^x}\,dx = x\,e^x-\int_0^1 1.\,e^x\,dx=\left[xe^x-e^x\right]_0^1$

$=(e-e)-[0-1]=1$

52. (d) Let $f(x)=\frac{\sin^5 x\cos^3 x}{x^4}$

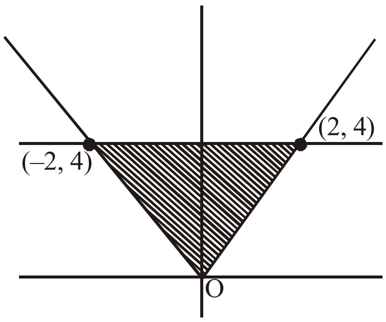
$f(-x)=\frac{\sin^5(-x)\cos^3(-x)}{(-x)^4}$

$=\frac{-\sin^5 x\cos^3 x}{x^4}=-f(x)$

$\Rightarrow$ f (x) is an odd function.

Hence, $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^5 x\cos^3 x}{x^4}dx=0$

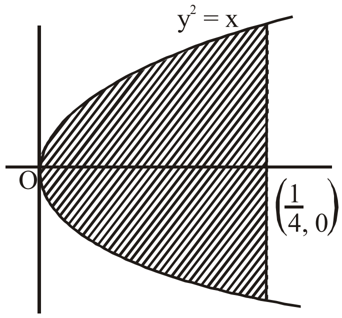
53. (c)



Required Area $=2\int_{y=0}^{4}\frac{y}{2}dy=\left.\frac{y^2}{2}\right|_0^4=\frac{16}{2}=8$ sq. unit.

or Area = 2 × 4 = 8 sq. unit.

54. (b) Required Area $=2\int_0^{\frac{1}{4}}\sqrt{x}\,dx$



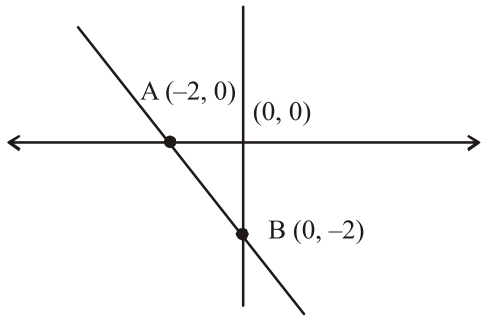
$=2.\frac{2}{3}\left[x^{3/2}\right]_0^{1/4}=\frac{4}{3}\left[\frac{1}{8}-0\right]=\frac{1}{6}$ sq. unit.

55. (c) $\int_1^2 \ln x\,dx=\left[x\ln x-x\right]_1^2$

$=2\ln 2-2-\ln 1+1$

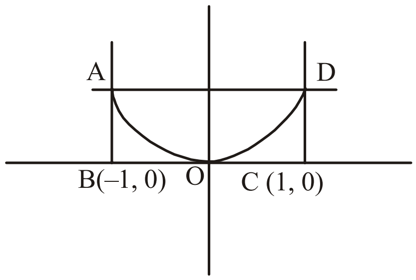
$=\ln 4-\ln e=\ln\left(\frac{4}{e}\right)$

56. (c)



Area of $\Delta OAB=\frac{1}{2}\times 2\times 2=2$ square units

57. (c)



Area of ABCD = 2 × 1 = 2 sq. units

Area of AOD $=\int_{-1}^{1}x^2dx=\left.\frac{x^3}{3}\right|_{-1}^{1}=\frac{2}{3}$ sq. units

Required area $=2-\frac{2}{3}=\frac{4}{3}$ sq. units

58. (b) Required area $= \int_0^{\frac{\pi}{4}} \tan x \, dx$

$= \ln|\sec x|_0^{\frac{\pi}{4}} = \ln\sqrt{2} = \frac{\ln 2}{2}$

59. (b) $\int_0^2 e^{\ln x} dx = \int_0^2 x \, dx = \left|\frac{x^2}{2}\right|_0^2 = 2$

60. (b) Let $y = \sqrt{\frac{1+\cos x}{1-\cos x}}$

$= \frac{\sqrt{2}\cos\frac{x}{2}}{\sqrt{2}\sin\frac{x}{2}} = \cot\frac{x}{2}$

$\frac{dy}{dx} = -\operatorname{cosec}^2\frac{x}{2}.\frac{1}{2} = -\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$

61. (a) $I = \int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2} dx$

Let $\tan^{-1} x = t$

$\frac{1}{1+x^2} dx = dt$

Lower limit $\rightarrow t = \tan^{-1} 0 = 0$

upper limit $\rightarrow t = \tan^{-1}, = \pi/4$

$\therefore \int_0^{\pi/4} e^t - dt = \left[e^t\right]_0^{\pi/4}$

$e^{\pi/4} - e^o \Rightarrow e^{\pi/4} - 1$

$= (-a\cos\theta, -b\sin\theta)$

62. (a) $I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Hence, $I_1 = I_2$

$\therefore I_1 - I_2 = 0$

63. (c) Adding I_1 and I_2

$I_1 + I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$

$= [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{6} = \frac{\pi}{12}$ $\quad (\because I_1 + I_2 = 2I)$

64. (b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\sin x \, dx = 2\int_0^{\frac{\pi}{2}} x\sin x \, dx$

{x sin x is an even function}

$= 2[-x\cos x + \sin x]_0^{\frac{\pi}{2}} = 2$

65. (c) $I = \int_0^{\frac{\pi}{2}} \ln(\tan x) \, dx$ ...(i)

$I = \int_0^{\frac{\pi}{2}} \ln\left(\tan\left(\frac{\pi}{2}-x\right)\right) dx$

$= \int_0^{\frac{\pi}{2}} \ln\cot x \, dx$ ...(ii)

Adding equations (i) and (ii)

$2I = \int_0^{\frac{\pi}{2}} \ln(\tan x.\cot x) dx$

$2I = 0$

$I = 0$

66. (d) Given equation $y^2 = 4bx = 2\int_0^b \sqrt{4bx} \, dx$

$= 4\sqrt{b} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^b = \frac{8\sqrt{b}}{3}\left[b^{\frac{3}{2}} - 0\right]$

$\therefore$ area of parabola bounded by its latus rectum

$= \frac{8b^2}{3}$ sq. units

Sol. (67–69)

Given, $I = \int_0^{\pi} \frac{x \, dx}{1+\sin x}$ ...(i)

$= \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \int_0^{\pi} \frac{(\pi - x)}{1+\sin x} dx$ ...(ii)

$[\because \sin(\pi - x) = \sin x]$

Adding eqs. (i) and (ii), we get

$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin x}$ ...(iii)

$\Rightarrow 2I = 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$

$\left[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{if } f(2a-x) = f(x)\right]$

$$\Rightarrow I = \pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow I = \pi\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2}+1+2\tan\frac{x}{2}}$$

$$\Rightarrow I = \pi\int_0^{\frac{\pi}{2}} \frac{\left(\sec^2\frac{x}{2}\right)dx}{\left(\tan\frac{x}{2}+1\right)^2}$$

Let $\tan\frac{x}{2}+1 = t$

$\Rightarrow \sec^2\frac{x}{2}.\frac{1}{2}dx = dt$

$\Rightarrow \sec^2\frac{x}{2}dx = 2dt$

When $x = 0$, then $t = 1$ and when $x = \frac{\pi}{2}$, then $t = 2$

$$\therefore I = 2\pi\int_1^2 \frac{dt}{t^2} = 2\pi\left[\frac{t^{-2+1}}{-2+1}\right]_1^2 = -2\pi\left[\frac{1}{t}\right]_1^2$$

$$= -2\pi\left[\frac{1}{2}-1\right]$$

$$= -2\pi\left(-\frac{1}{2}\right) = \pi$$

67. (c) According to the explanation, $I = \pi$

68. (a) Let $I_1 = \int_0^{\pi} \frac{(\pi-x)dx}{1+\sin x}$

$$= \int_0^{\pi} \frac{[\pi-(\pi-x)]dx}{1+\sin(\pi-x)}$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x} = \pi \qquad [\because \sin(\pi-x) = \sin x]$$

69. (b) From eq. (iii).

$$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin x}$$

$$\Rightarrow \int_0^{\pi} \frac{dx}{1+\sin x} = \frac{2}{\pi}I$$

$$\Rightarrow \int_0^{\pi} \frac{dx}{1+\sin x} = \frac{2}{\pi}\times\pi = 2 \qquad (\therefore I = \pi)$$

Sol. (70 –71)

Consider $I = \int_0^{\pi} \ln(\sin x)dx$

$$I = \int_0^{\pi} \ln(\sin x)dx$$

$$= 2\int_0^{\frac{\pi}{2}} \ln(\sin x)dx \qquad ...(i)$$

$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{if } f(2a-x) = f(x)\right]$$

$$= 2\int_0^{\frac{\pi}{2}} \ln\left[\sin\left(\frac{\pi}{2}-x\right)\right]dx$$

$$\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$= 2\int_0^{\frac{\pi}{2}} \ln(\cos x)dx \qquad ...(ii)$$

70. (d) From eq. (i),

$$I = 2\int_0^{\frac{\pi}{2}} \ln(\sin x)dx$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \ln(\sin x)dx = \frac{1}{2}I$$

71. (a) From eq. (ii), we have

$$I = 2\int_0^{\frac{\pi}{2}} \ln(\cos x)dx$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \ln(\cos x)dx = \frac{1}{2}I$$

72. (c) Let $I = \int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x\,dx}{a^2+b^2\tan^2 x}$$

[divide numerator and denominator by $\cos^2 x$]

Put $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

When $x = 0$, then $t = 0$ and when $x = \frac{\pi}{2}$, then $t = \infty$

$$\therefore I = \int_0^{\infty} \frac{dt}{a^2+b^2t^2} = \frac{1}{b^2}\int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2+t^2}$$

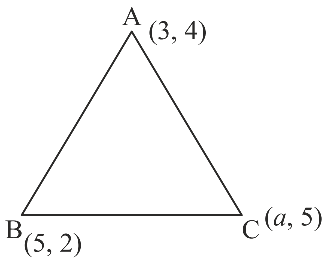
$$= \frac{1}{b^2}\frac{1}{\left(\frac{a}{b}\right)}\left[\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^{\infty}$$

$$\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C\right]$$

$$= \frac{1}{ab}[\tan^{-1}(\infty) - \tan^{-1}(0)]$$

$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2ab}$$

73. (d) Area of $\Delta ABC = 3$ sq. unit



$\therefore \quad \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 5 & 2 & 1 \\ a & 5 & 1 \end{vmatrix} = 3$

$\therefore \quad \begin{vmatrix} 3 & 4 & 1 \\ 5 & 2 & 1 \\ a & 5 & 1 \end{vmatrix} = 6$

$\therefore \quad 3(2-5) - 4(5-a) + 1(25-2a) = 6$

$-9 - 20 + 4a + 25 - 2a = 6$

$2a = 10$

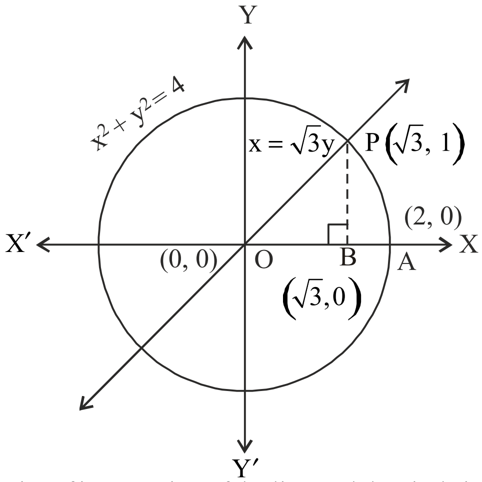
$a = 5$

$\therefore$ Option (d) is correct.

For (74-75)

$x^2 + y^2 = 4$ and $x = \sqrt{3}y$

$P(\sqrt{3}, 1)$ = First quadrant



The point of intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$.

Area of Δ OPA = Area of Δ OPB + Area of PAB

$$= \frac{1}{2} \times OB \times PB = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$$

Area of PAB $\int_{\sqrt{3}}^{2} y\,dx$

$$= \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx = \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$\left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2}[\sqrt{4-3}] - 2\sin^{-1}\frac{\sqrt{3}}{2}\right]$$

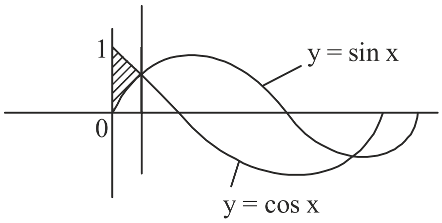
$$= \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

74. (a)

75. (a) Area enclosed by x-axis, the line $x = \sqrt{3}.y$, and the circle $x^2 + y^2 = 4$ in the first quadrant

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

76. (a)



Area of shaded region $= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}}$$

$$= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1)\right]$$

$$= \left(\sqrt{2} - 1\right) \text{ sq. units.}$$

77. (a)

78. (a) $I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x}\,dx$

$\therefore \quad I_1 = \int_0^{\pi} \frac{\sin 2x}{\sin x}\,dx = \int_0^{\pi} \frac{2\sin x \cos x}{\sin x}\,dx$

$$= 2\int_0^{\pi} \cos x\,dx = 2[\sin x]_0^{\pi} = 2[\sin \pi - \sin 0]$$

$$= 2(0) = 0$$

79. (d) $I_2 = \int_0^{\pi} \frac{\sin 4x}{\sin x}\,dx = \int_0^{\pi} \frac{2\sin 2x \cos 2x}{\sin x}\,dx$

$$= 2\int_0^{\pi} \frac{2\sin x \cos x \cos 2x}{\sin x}\,dx$$

$$= 4\int_0^{\pi} \cos x\left(1 - 2\sin^2 x\right)dx$$

$= 4\int_0^{\pi} \cos x \, dx - 8\int_0^{\pi} \cos x \sin^2 x \, dx = 0$

Let $\sin x = t$

$\cos x \, dx = dt$

$= 4(\sin \pi - \sin 0) - 0 = 0$

$$I_3 = \int_0^{\pi} \frac{\sin 6x}{\sin x} dx$$

$$= \int_0^{\pi} \frac{2\left[3 \sin x - 4 \sin^3 x\right] \cos 3x}{\sin x} dx$$

$$= \int_0^{\pi} 2\left(3 - 4 \sin^2 x\right) \cos 3x \, dx$$

$$= \int_0^{\pi} 6 \cos 3x \, dx - \int_0^{\pi} 8 \sin^2 x \cos 3x \, dx$$

$$= \frac{6 \sin 3x}{3}\int_0^{\pi} - 8 \cdot \frac{1}{2}\left[\int_0^{\pi} \sin x \left(\sin 4x - \sin 2x\right) dx\right]$$

$$= 6 - \frac{8}{2}\left[\int_0^{\pi} \left(\sin x \sin 4x - \sin x \cdot \sin 2x\right) dx\right]$$

$$= \frac{-8}{2}\left[\frac{1}{2}\int_0^{\pi} \cos 3x - \cos 5x - \cos x + \cos 3x \, dx\right]$$

$$= -2\left[\frac{2 \sin 3x}{3} - \frac{\sin 5x}{5} - \sin x\right]_0^{\pi} = 0$$

Hence, $I_2 + I_3 = 0 + 0 = 0$

80. (a) $I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x} dx$

$$= \int_0^{\pi} \frac{\sin 2m(\pi - x)}{\sin(\pi - x)} dx$$

$$= \int_0^{\pi} \frac{\sin(2m\pi - 2mx)}{\sin x} dx$$

$$= \int_0^{\pi} \frac{-\sin 2mx}{\sin x} dx$$

$$I_m = -\int_0^{\pi} \frac{\sin 2mx}{\sin x} dx$$

$2I_m = 0 \Rightarrow I_m = 0.$

81. (a) $I_{2m} > I_m$ is wrong statement

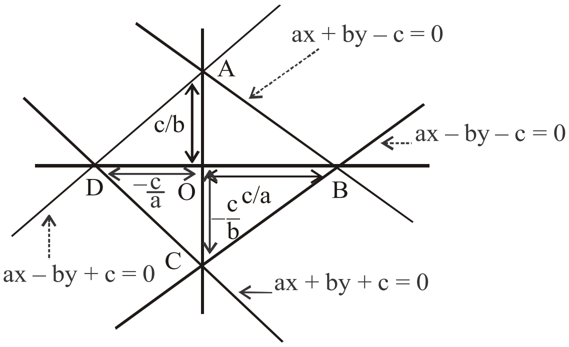
Because, $I_m = I_{m-1} = ... = I_n$

Thus,

$I_m - I_{m-1} = 0$ is the only correct statement.

82. (b) Area of triangle

$$\Delta AOB = \frac{1}{2} \times \frac{c}{b} \times \frac{c}{a} = \frac{c^2}{2ab}$$



Total area

$= 4 \times$ area ΔAOB

$$= 4 \times \frac{c^2}{2ab}$$

$$= \frac{2c^2}{ab}$$

83. (d) $a + b = 0 \Rightarrow a = -b$

$$I = \int_{-b}^{b} \frac{x^7 + \sin x}{\cos x}$$

Using property

$$\int_{-a}^{a} f(x) dx = \begin{cases} \int_0^{2a} f(x) dx & ; \text{if } f(x) \text{ is even} \\ 0 & ; \text{if } f(x) \text{ is odd} \end{cases}$$

$$f(x) = \frac{x^7 + \sin x}{\cos x}$$

$$f(-x) = \frac{(-x)^7 + \sin(-x)}{\cos(-x)} = \frac{-x^7 - \sin x}{\cos x}$$

$$= -\left[\frac{x^7 + \sin x}{\cos x}\right]$$

$= -f(x)$

So f(x) is odd hence

$I = 0$

84. (a) $\int_a^b \frac{|x|}{x} dx$

when $x \geq 0$

$$\Rightarrow \int_a^b \frac{x}{x} dx$$

$$\Rightarrow \int_a^b (1) dx$$

$= [x]_a^b = |b| - |a|$

when $x < 0$; as $0 < a < b$; x will not lie between a and b so

$\int_a^b \frac{|x|}{x} dx = 0$ for $x < 0$

85. (c) $\int_0^{2\pi} \sin^5\left(\frac{x}{4}\right) dx = \int_0^{2\pi}\left(1-\cos^2\frac{x}{4}\right)\left(1-\cos^2\frac{x}{4}\right)\sin\frac{x}{4}.dx$

Put $\cos\left(\frac{x}{4}\right) = t$

$\Rightarrow -\sin\left(\frac{x}{4}\right).\frac{dx}{4} = dt$

$\Rightarrow \sin\left(\frac{x}{4}\right) dx = -4dt$

$\Rightarrow \int_0^{2\pi} \sin^5\left(\frac{x}{4}\right) dx = -4\int(1-t^2)(1-t^2)\,dt$

$= -4\int(1+t^4-2t^2)\,dt$

$= -4\left[t+\frac{t^5}{5}-\frac{2t^3}{3}\right]$

$= -4\left[\cos\left(\frac{x}{4}\right)+\frac{\cos^5\left(\frac{x}{4}\right)}{5}-\frac{2\cos^3\left(\frac{x}{4}\right)}{3}\right]_0^{2\pi}$

$= -4\left[(0+0-0)-\left(1+\frac{1}{5}-\frac{2}{3}\right)\right] = \frac{32}{15}$

86. (a) $\int_{-1}^{1} x|x|dx$

$= \int_{-1}^{0} x|x|dx + \int_0^1 x|x|dx$

$= \int_{-1}^{0} x(-x)dx + \int_0^1 x.xdx$

$= -\int_{-1}^{0} x^2dx + \int_0^1 x^2dx$

$= -\left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_0^1$

$= -\left[0-\frac{(-1)^3}{3}\right]+\frac{1}{3}\left[(1)^3-(0)^3\right]$

$= -\frac{1}{3}+\frac{1}{3} = 0$

87. (d) Area $= \int y\,dx$

$= \int_0^1\left(1-\sqrt{x}\right)^2 dx$

[$\because$ curve makes the intercept of 1 on both axes]

$= \int_0^1\left(1+x-2\sqrt{x}\right)dx$

$= [x]_0^1 + \frac{1}{2}\left[x^2\right]_0^1 - \frac{4}{3}\left[x^{\frac{3}{2}}\right]_0^1$

$= 1+\frac{1}{2}-\frac{4}{3} = \frac{3}{2}-\frac{4}{3} = \frac{1}{6}$ sq unit

88. (c) $A = \int_0^{\pi}\frac{\sin x}{\sin x+\cos x}dx$

Using property

$A = \int_0^{\pi}\frac{\sin(\pi-x)}{\sin(\pi-x)+\cos(\pi-x)}dx$

$A = \int_0^{\pi}\frac{\sin x}{\sin x-\cos x}dx = B$

$A = B$

89. (b) $B = \int_0^{\pi}\frac{\sin x}{\sin x-\cos x}\times\frac{(\sin x+\cos x)}{(\sin x+\cos x)}dx$

$= -\int_0^{\pi}\frac{\sin^2 x+\sin x\cos x}{\cos 2x}dx$

$= \frac{-1}{2}\int_0^{\pi}\frac{2\sin^2 x}{\cos 2x}dx - \frac{1}{2}\int_0^{\pi}\frac{2\sin x\cos x}{\cos 2x}dx$

$= \frac{-1}{2}\int_0^{\pi}\frac{1-\cos 2x}{\cos 2x}\,dx - \frac{1}{2}\int_0^{\pi}\tan 2x\,dx$

$= \frac{-1}{2}\int_0^{\pi}\sec 2x\,dx + \frac{1}{2}\int_0^{\pi}dx - \frac{1}{2}\int_0^{\pi}\tan 2x\,dx$

$= \frac{-1}{2}\left[\frac{\log|\sec 2x+\tan 2x|}{2}\right]_0^{\pi} + \frac{1}{2}[x]_0^{\pi} - \frac{1}{2}\left[\frac{\log|\sec 2x|}{2}\right]_0^{\pi}$

$= \frac{-1}{4}[\log(1+0)-\log(1+0)]+\frac{1}{2}[\pi+0]-\frac{1}{4}[\log(1)-\log(1)]$

$= 0+\pi/2-0 = \pi/2$

90. (b) $\int_{1/3}^{1/2} g(x)\,dx$

$g(x) = \begin{cases} 2, & \text{if } \frac{1}{3} < x \le \frac{1}{2} \\ 3, & \text{if } x = \frac{1}{3} \end{cases}$

As g(x) is a gretest integer function so value of g(x) in integral limit will be

So $\int_{1/3}^{1/2} g(x)\,dx = \int_{1/3}^{1/2} 2\,dx$

$= 2[x]_{1/3}^{1/2} = 2\left[\frac{1}{2}-\frac{1}{3}\right] = \frac{1}{3}$

91. (a) $\int_{1/3}^{1} f(x)\,dx = \int_{1/3}^{1/2} f(x)\,dx + \int_{1/2}^{1} f(x)\,dx$ ...(1)

f(x) = xg(x)

$g\left(\frac{1}{3}\right) = 3 \quad g\left(\frac{1}{2}\right) = 2 \quad g(1) = 1$

The value of g(x) in value $\left(\frac{1}{2}, \frac{1}{3}\right)$ will be 2 and in range $\left(\frac{1}{2}, 1\right)$ it will be 1

form (1)

$\int_{1/3}^{1} f(x)\,dx = \int_{1/3}^{1/2} xg(x)\,dx + \int_{1/2}^{1} xg(x)\,dx.$

$= \int_{1/3}^{1/2} x \cdot 2\,dx + \int_{1/2}^{1} x \times 1\,dx.$

$= \left[x^2\right]_{1/3}^{1/2} + \left[\frac{x^2}{2}\right]_{1/2}^{1}$

$= \left[\frac{1}{4} - \frac{1}{9}\right] + \frac{1}{2}\left[1 - \frac{1}{4}\right]$

$= \frac{5}{36} + \frac{3}{8} = \frac{37}{72}$

92. (c) $\int_{-2}^{2} x\,dx - \int_{-2}^{2} [x]\,dx$

$= \left[\frac{x^2}{2}\right]_{-2}^{2} - \int_{-2}^{-1} [x]\,dx - \int_{-1}^{0} [x]\,dx - \int_{0}^{1} [x]\,dx - \int_{1}^{2} [x]\,dx$

$= \frac{1}{2}[4 - 4] - (-2) - (-1) - 0 - (1)$

$= 2 + 1 - 1 = 2$

93. (b) $\int_{-2}^{5} f(x)\,dx = 4$ and $\int_{0}^{5} \{1 + f(x)\}\,dx = 7$

$\int_{0}^{5} f(x)\,dx = 7 - \int_{0}^{5} 1\,dx = 7 - 5 = 2$ ...(1)

$\int_{-2}^{0} f(x)\,dx = ?$

$\int_{-2}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx = 4$

$\int_{-2}^{0} f(x)\,dx = 4 - \int_{0}^{5} f(x)\,dx$

$= 4 - 2 = 2.$ [Using eqn. (1)]

$\int_{-2}^{0} f(x)\,dx = 2$

94. (d) $\int_{0}^{4\pi} |\cos x|\,dx = 4\int_{0}^{\pi} |\cos x|\,dx$

$= 4\left[\int_{0}^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx\right]$

$= 4\left[(\sin x)_0^{\pi/2} - (\sin x)_{\pi/2}^{\pi}\right]$

$= 4\left[\sin\frac{\pi}{2} - 0 - \sin\pi + \sin\frac{\pi}{2}\right]$

$= 4\left[2\sin\frac{\pi}{2}\right] = 8$

95. (b) Since $|y| = \begin{cases} y & y > 0 \\ -y & y < 0 \\ 0 & y = 0 \end{cases}$

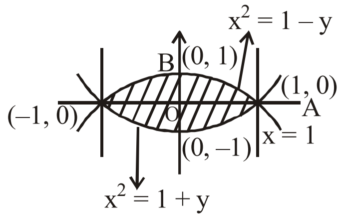
For $y > 0 \Rightarrow y = 1 - x^2$

For $y < 0 \Rightarrow y = x^2 - 1$

For $y = 0 \Rightarrow x = \pm 1$

So area under the curve

= 4 × Area under the region OABO (symmetry)



$= 4 \times \int_{0}^{1} 1 - x^2\,dx$

$= 4 \times \left[x - \frac{x^3}{3}\right]_0^1$

$= 4\left(1 - \frac{1}{3}\right) = 4 \times \frac{2}{3} = \frac{8}{3}$ sq. units

96. (b) $I = \int_{0}^{\pi/2} \frac{dx}{3\cos x + 5}$

$I = \int_{0}^{\pi/2} \frac{dx}{3\left[\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right] + 5}$

$I = \int_{0}^{\pi/2} \frac{\left(1 + \tan^2\frac{x}{2}\right)dx}{3 - 3\tan^2\frac{x}{2} + 5 + 5\tan^2\frac{x}{2}}$

$I = \int_{0}^{\pi/2} \frac{\sec^2\frac{x}{2}\,dx}{2\tan^2\frac{x}{2} + 8}$

$I = \frac{1}{2}\int_{0}^{\pi/2} \frac{\sec^2\frac{x}{2}\,dx}{\tan^2\frac{x}{2} + 2^2}$

Put $\tan\frac{x}{2} = y$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dy$$

$$\Rightarrow I = \int_0^1 \frac{dy}{y^2+2^2}$$

$$\Rightarrow I = \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right)$$

$$\Rightarrow I = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) - 0$$

Also $I = \frac{1}{2}\tan^{-1}\frac{1}{2} = k\cot^{-1}(2)$

$$\left(\because \tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right)\right)$$

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) = k\tan^{-1}\left(\frac{1}{2}\right)$$

$$\therefore k = \frac{1}{2}$$

97. (d) $I = \int_1^3 |1-x^4|\,dx$

$$I = \int_1^3 -(1-x^4)dx \qquad \left(\because |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}\right)$$

$$I = \int_1^3 (x^4-1)dx \Rightarrow I = \left[\frac{x^5}{5} - x\right]_1^3$$

$$I = \left(\frac{3^5}{5} - 3\right) - \left(\frac{1^5}{5} - 1\right) \Rightarrow I = \frac{232}{5}$$

98. (b)

$$\int_0^{\pi/2} \frac{d\theta}{1+\cos\theta} = \int_0^{\pi/2} \frac{d\theta}{2\cos^2\left(\frac{\theta}{2}\right)} = \frac{1}{2}\int_0^{\pi/2} \sec^2\frac{\theta}{2}.d\theta$$

$$= \frac{1}{2}\left[\frac{\tan\theta/2}{1/2}\right]_0^{\pi/2}$$

$$= \tan\frac{\pi}{4} - \tan 0$$

$$= 1 - 0 = 1$$

99. (b) $I = \int_0^a f(x).g(x)\,dx$

$$I = \int_0^a f(a-x).g(a-x).dx$$

$$I = \int_0^a f(x).[2-g(x)].dx$$

$\therefore$ f(a–x) = f(x) and g(a–x) = 2g(x)

$$I = \int_0^a 2.f(x).dx - \int_0^a f(x).g(x).dx$$

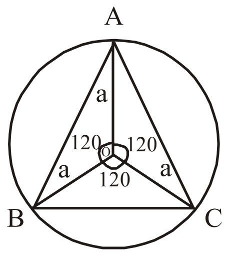
$$I = 2\cdot\int_0^a f(x)\cdot - I$$

$$\Rightarrow 2I = 2\int_0^a f(x)dx$$

$$\therefore I = \int_0^a f(x)\,dx$$

100. (c) For area of triangle to be maximum, it should be equilateral triangle.

Area of Δ OAB $= \frac{1}{2}ab\sin\theta$



$$= \frac{1}{2}.a.a.\sin\theta$$

$$= \frac{1}{2}a^2.\sin 120°$$

$$= \frac{\sqrt{3}}{4}a^2$$

Area of triangle OAB, OBC, OAC
= Area of triangle ABC

$$= 3 \times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{4}a^2$$

101. (b) $\int_{e^{-1}}^{e^2} \left|\frac{\log x}{x}\right| dx = \int_{e^{-1}}^{e^0} \frac{-\log x}{x}dx + \int_{e^0}^{e^2} \frac{\log x}{x}dx$

$$= \frac{-1}{2}\left[(\log x)^2\right]_{e^{-1}}^{e^0} + \frac{1}{2}\left[(\log x)^2\right]_{e^0}^{e^2}$$

$$= \frac{-1}{2}\left[0 - \left(\log e^{-1}\right)^2\right] + \frac{1}{2}\left[\left(\log e^2\right)^2 - 0\right]$$

$$= \frac{-1}{2}(-1) + \frac{1}{2}(2)^2$$

$$= \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$$

102. (a) $\int_0^{2\pi}\sqrt{1+\sin\frac{x}{2}}\cdot dx$

$\sin\frac{x}{2}=\sin 2\left(\frac{x}{4}\right)=2\sin\frac{x}{4}\cos\frac{x}{4}$

$\therefore \int_0^{2\pi}\sqrt{\sin^2\frac{x}{4}+\cos^2\frac{x}{4}+2\sin\frac{x}{4}\cos\frac{x}{4}}$

$=\int_0^{2\pi}\sqrt{\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)^2}\cdot dx$

$=\int_0^{2\pi}\left|\sin\frac{x}{4}+\cos\frac{x}{4}\right|\cdot dx$

$=4\left[-\cos\frac{x}{4}+\sin\frac{x}{4}\right]_0^{2\pi}$

$=4\left[-\left(\cos\frac{2\pi}{4}-\cos 0\right)+\sin\left(\frac{2\pi}{4}-\sin 0\right)\right]$

$=4[-(-1)+(1)]\ =4\times 2=8.$

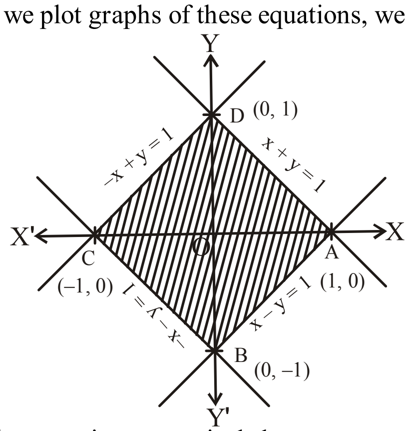
103. (c) $|x|+|y|=1$

We know, $|x|=\begin{cases}x, & x\ge 0\\ -x, & x<0\end{cases}$

$\therefore |x|+|y|=1$ is

$\begin{cases}x+y=1 & \text{for } x>0, y>0\\ -x+y=1 & \text{for } x<0, y>0\\ x-y=1 & \text{for } x>0, y<0\\ -x-y=1 & \text{for } x<0, y<0\end{cases}$

If we plot graphs of these equations, we get



The curve is symmetrical about x any y-axis.

$\therefore$ Area = 4 × Area of AOD

$=4\times\int_0^1 y dx=4\times\int_0^1(1-x)dx=4\left[x-\frac{x^2}{2}\right]_0^1$

$=4\left[1-\frac{1^2}{2}-\left(0-\frac{0^2}{2}\right)\right]$

$=4\left(1-\frac{1}{2}\right)=4\times\frac{1}{2}=2$

104. (a) $f(n)=\left[\frac{1}{4}+\frac{n}{1000}\right]$

$\sum_{n=1}^{1000}f(n)=\left[\frac{1}{4}+\frac{1}{1000}\right]+\left[\frac{1}{4}+\frac{2}{1000}\right]+....+\left[\frac{1}{4}+\frac{1000}{1000}\right]$

$=[0.25+0.001]+[0.25+0.002]+....+[0.25+1]$

We get '0' for all values of n from 1 to 750.

From n = 750, we get all the values as 1.

So,

$\sum_{n=1}^{1000}f(n)=0+0+0+....+\left[\frac{1}{4}+\frac{750}{1000}\right]+\left[\frac{1}{4}+\frac{751}{1000}\right]+....[1.25]$

$=1+1+1+....$ (251 times)

$=251.$

105. (d) $\int_0^{\frac{\pi}{4}}\sqrt{\tan x}\ dx+\int_0^{\frac{\pi}{4}}\sqrt{\cot x}\cdot dx$

$=\int_0^{\frac{\pi}{4}}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

$=\int_0^{\frac{\pi}{4}}\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$

$=\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{\sqrt{\sin x}\cdot\sqrt{\cos x}}\cdot dx$

$=\sqrt{2}\int_0^{\frac{\pi}{4}}\frac{(\sin x+\cos x)}{\left(\sqrt{1-\left(\sin^2 x+\cos^2 x-2\sin x\cos x\right)}\right)}\cdot dx$

$=\sqrt{2}\int_0^{\frac{\pi}{4}}\frac{(\sin x+\cos x)}{\sqrt{1-(\sin x-\cos x)^2}}\cdot dx$

Put $\sin x-\cos x=t\Rightarrow(\cos x+\sin x)\,dx=dt$

When $x=0$, $t=-1$ and $x=\frac{\pi}{4}, t=0.$

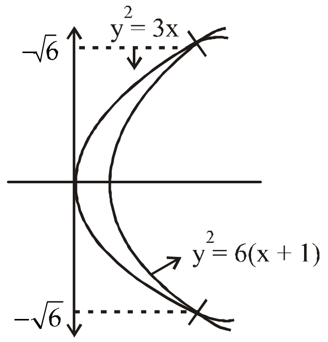
$=\sqrt{2}\int_{-1}^{0}\frac{1}{\sqrt{1-t^2}}\cdot dt=\sqrt{2}\left(\sin^{-1}t\right)_{-1}^{0}$

$=\sqrt{2}\left[\sin^{-1}(0)-\sin^{-1}(-1)\right]$

$=\sqrt{2}\left[0-\left(\frac{-\pi}{2}\right)\right]$

$=\frac{\pi}{\sqrt{2}}.$

106. (c) Given parabolas are $y^2 = 6(x-1)$ ....(1)
and $y^2 = 3x$ ....(2)



$y^2 = 6(x-1)$
$\Rightarrow \quad 6x - 6 = y^2$

$$\Rightarrow x = \frac{y^2+6}{6} = \frac{y^2}{6} + 1$$

Also, $y^2 = 3x \Rightarrow x = \frac{y^2}{3}$.

Solving (1), (2), $3x = 6(x-1)$
$\Rightarrow 3x = 6x - 6$
$\Rightarrow 3x = 6 \Rightarrow x = 2.$

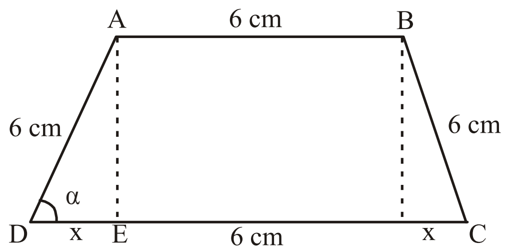
$y^2 = 6 \Rightarrow y = \pm\sqrt{6}$.

$\therefore$ Area =

$$\int_{-\sqrt{6}}^{\sqrt{6}} \left(\frac{y^2}{6} + 1 - \frac{y^2}{3}\right).dy = 2\int_0^{\sqrt{6}} \left(1 - \frac{y^2}{6}\right) dy = 2\left[y - \frac{y^3}{18}\right]_0^{\sqrt{6}}$$

$$= 2\left[\sqrt{6} - \frac{\left(\sqrt{6}\right)^3}{18}\right] = 2 \times \frac{2\sqrt{6}}{3} = \frac{4\sqrt{6}}{3}.$$

107. (c) Length of three sides of trapezium = 6 cm
Let AE be height of trapezium.



In ΔADE, $AD^2 = AE^2 + DE^2$
$\Rightarrow 6^2 = AE^2 + x^2$
$\Rightarrow AE^2 = 36 - x^2$

$$\Rightarrow AE = \sqrt{36 - x^2}$$

Area of trapezium $= \frac{1}{2}h(a+b)$

$$= \frac{1}{2}\sqrt{36-x^2}\,(6+6+2x)$$

$$A = (6+x)\sqrt{36-x^2}.$$

Given, Area of trapezium is maximum.

$$\frac{dA}{dx} = \frac{d}{dx}\left[(6+x).\sqrt{36-x^2}\right]$$

$$= (6+x).\left(\frac{-2x}{2\sqrt{36-x^2}}\right) + \sqrt{36-x^2}$$

$$= \sqrt{36-x^2} - \frac{x(6+x)}{\sqrt{36-x^2}}$$

$$= \frac{36-x^2-6x-x^2}{\sqrt{36-x^2}} = \frac{36-6x-2x^2}{\sqrt{36-2x^2}}$$

$$\frac{dA}{dx} = 0 \Rightarrow 36 - 6x - 2x^2 = 0$$

$\Rightarrow \quad 2x^2 + 6x - 36 = 0$
$\Rightarrow \quad x^2 + 3x - 18 = 0$
$\Rightarrow \quad x^2 + 6x - 3x - 18 = 0$
$\Rightarrow \quad x(x+6) - 3(x+6) = 0$
$\quad (x+6)(x-3) = 0$
$\Rightarrow \quad x = -6$ or 3. Since x cannot be negative. So, $x = 3$.

$\therefore$ In ΔADE, $\cos\alpha = \frac{x}{6} = \frac{3}{6} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$.

108. (d) Fourth side, $DC = x + 6 + x$
$= 3 + 6 + 3$
$= 12.$

109. (c) Area $= (6+x)\sqrt{36-x^2}$

$$= (6+3)\sqrt{36-3^2}$$
$$= 9\sqrt{36-9}$$
$$= 9\sqrt{27}$$
$$= 9\sqrt{9\times 3}$$
$$= 27\sqrt{3}.$$

110. (a) $\int_0^{\pi} e^x \sin x.dx = I$

$$I = \left(\sin x.e^x\right)_0^{\pi} - \int_0^{\pi} \cos x.e^x.dx$$

$$I = \left(\sin\pi.e^{\pi} - \sin 0.e^0\right) - \left\{\left[\cos x.e^x\right]_0^{\pi} - \int_0^{\pi} \sin x.e^x.dx\right\}$$

$$I = 0 - \left\{\left(\cos\pi.e^{\pi} - \cos 0.e^0\right) - I\right\}$$

$I = -[-e^{\pi} - 1] - I$

$$\Rightarrow 2I = e^{\pi} + 1 \Rightarrow I = \frac{e^{\pi}+1}{2}.$$

111. (b) $\int_1^e x.\ell n\, x.dx = \left[\ell nx.\frac{x^2}{2}\right]_1^e - \int_1^e \frac{1}{x}.x^2.dx$

$= \left(\frac{e^2}{2} - 0\right) - \frac{1}{2}\left(\frac{1}{2}\right)\left(x^2\right)_1^e = \frac{e^2}{2} - \frac{1}{4}\left(e^2 - 1\right) = \frac{2e^2 - e^2 + 1}{4}$

$= \frac{e^2 + 1}{4}.$

112. (a) $\int_0^{\sqrt{2}} \left[x^2\right] dx$

$= \int_0^1 \left[x^2\right] dx + \int_1^{\sqrt{2}} \left[x^2\right] dx$

$= 0 + \int_1^{\sqrt{2}} 1.dx$ (∵ [ ] is greatest integer function)

$(x)_1^{\sqrt{2}} = \sqrt{2} - 1.$

113. (c) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} (\sin x - \tan x)\, dx.$

Let f(x) = sin x – tan x
f(–x) = sin (–x) – tan (–x)
= –sin x + tan x = –(sin x – tan x)
= –f(x).
So, f(x) is an odd function.

We know $\int_{-a}^{a} f(x).dx = 0$, if f(x) is odd function.

114. (a)

$\int_a^b x^3.dx = 0$

$\Rightarrow \left(\frac{x^4}{4}\right)_a^b = 0$

$\Rightarrow b^4 - a^4 = 0$

$\Rightarrow \left(b^2 - a^2\right)\left(b^2 + a^2\right) = 0$

$\Rightarrow b^2 - a^2 = 0 (or) b^2 + a^2 = 0$

$\Rightarrow (b + a)(b - a) = 0 (or) b^2 + a^2 = 0$

$\Rightarrow b = -a (or) a = b (or) b^2 + a^2 = 0$

$\int_a^b x^2.dx = \frac{2}{3}$

$\left(\frac{x^3}{3}\right)_a^b = \frac{2}{3}$

$\Rightarrow b^3 - a^3 = 2$

Since, b = –a.

$\Rightarrow b^3 - \left(-b^3\right) = 2$

$\Rightarrow 2b^3 = 2$

$\Rightarrow b^3 = 1$

$\Rightarrow b = 1, a = -1.$

115. (a) $\int_0^1 x(1 - x)^9 .dx$

$= \int_0^1 (1 - x)(1 - (1 - x))^9 .dx \quad \left(\because \int_0^a f(x).dx = \int_0^a f(a - x).dx\right)$

$= \int_0^1 (1 - x).x^9.dx$

$= \int_0^1 \left(x^9 - x^{10}\right).dx$

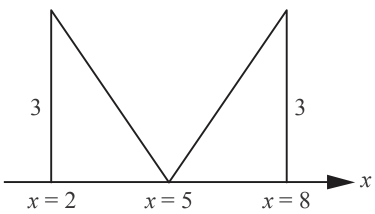
$= \left(\frac{x^{10}}{10} - \frac{x^{11}}{11}\right)_0^1$

$= \frac{1}{10} - \frac{1}{11} = \frac{11 - 10}{110} = \frac{1}{110}.$

116. (b) $\int_a^b [x]dx + \int_a^b [-x]dx$

$= \int_a^b ([x] + [-x])\, dx = \int_a^b (-1)dx = a - b$

117. (d) $\int_2^8 |\, x - 5\,|\, dx = 2 \times \frac{1}{2} \times 3 \times 3 = 9$



118. (c) $\int_{-1}^{1} \left(\frac{d}{dx} \tan^{-1} \frac{1}{x}\right) dx$

$= \int_{-1}^{1} \left(\frac{d}{dx} \cot^{-1} x\right) dx$

$= \int_{-1}^{1} \frac{-1}{(1 + x^2)} dx = -2\int_0^1 \frac{1}{1 + x^2} dx$

$= -2[\tan^{-1} x]_0^1 = -2 \times \frac{\pi}{4} = -\frac{\pi}{2}$

119. (b) $\int_0^{\pi/2} |\sin x - \cos x|\, dx$

$= \int_0^{\pi/4} (\sin x - \cos x).dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x).dx$

$= \int_0^{\pi/4} (\cos x - \sin x)\cdot dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\cdot dx$

$= (\sin x + \cos x)_0^{\pi/4} + (-\cos x - \sin x)_{\pi/4}^{\pi/2}$

$= 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$

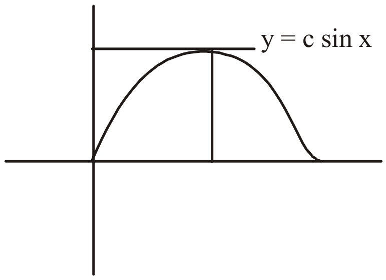
120. (b) $\int_0^{\frac{\pi}{2}} e^{\sin x}.\cos x\, dx$

Let sin x = t $\Rightarrow$ cosx. dx = dt

$\therefore \int_0^{\frac{\pi}{2}} e^{\sin x}.\cos x\, dx = \int_0^1 .e^t .dt$

$= (e^t)_0^1 = e^1 - e^0 = e - 1$

121. (b) Area of one of loop between y = c sin x and x-axis



= c + c = 2c

Differential Equation

19

1. What does the solution of the differential equation $xdy - ydx = 0$ represent ?
   (a) Rectangular hyperbola
   (b) Straight line passing through (0, 0)
   (c) Parabola with vertex at (0, 0)
   (d) Circle with centre at (0, 0) *[2006-I]*
2. Which one of the following differential equations represents the system of circles touching y-axis at the origin ?
   (a) $\frac{dy}{dx} = x^2 - y^2$ (b) $2xy\frac{dy}{dx} = y^2 - x^2$
   (c) $2xy\frac{dy}{dx} = x^2 - y^2$ (d) $\frac{dy}{dx} = y^2 - x^2$ *[2006-I]*
3. What is the solution of the differential equation $\frac{dy}{dx} = \frac{y}{(x + 2y^3)}$?
   (a) $y(1 - xy) = cx$ (b) $y^3 - x = cy$
   (c) $x(1 - xy) = cy$ (d) $x(1 + xy) = cy$ *[2006-I]*
4. If $y^2 = p(x)$ is a polynomial of degree 3, then what is $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right]$ equal to ?
   (a) $p'(x)p'''(x)$ (b) $p''(x)p'''(x)$
   (c) $p(x)p'''(x)$ (d) A constant *[2006-I]*
5. What is the degree of the equation $\left[\frac{d^2y}{dx^2}\right] = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$?
   (a) 1 (b) 2
   (c) 3 (d) 4 *[2006-I]*
6. What are the order and degree respectively of the differential equation $y = x\frac{dy}{dx} + \frac{dx}{dy}$?
   (a) 1, 1 (b) 1, 2
   (c) 2, 1 (d) 2, 2 *[2006-II]*
7. What is the equation of the curve passing through the origin and satisfying the differential equation $dy = (y\tan x + \sec x)dx$?
   (a) $y = x\cos x$ (b) $y\cos x = x$
   (c) $xy = \cos x$ (d) $y\sin x = x$ *[2007-I]*
8. What is the solution of the differential equation $\frac{dy}{dx} = \sec(x + y)$? *[2007-I]*
   (a) $y + \tan(x + y) = c$ (b) $y - \tan\left\{\frac{(x+y)}{2}\right\} = c$
   (c) $y + \tan\left\{\frac{(x+y)}{2}\right\} = c$ (d) $y + \tan\left\{\frac{(x-y)}{2}\right\} = c$
9. For what value of k, does the differential equation $\frac{dy}{dx} = ky$ represent the law of natural decay?
   (a) –5 (b) 0
   (c) 0.01 (d) $(10)^{-1}$ *[2007-I]*
10. What is the solution of the differential equation $(x + y)(dx - dy) = dx + dy$? *[2007-I]*
    (a) $x + y + \ln(x + y) = c$ (b) $x - y + \ln(x + y) = c$
    (c) $y - x + \ln(x + y) = c$ (d) $y - x - \ln(x - y) = c$
11. What is the degree of the differential equation $k\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{3/2}$, where k is a constant?
    (a) 1 (b) 2
    (c) 3 (d) 4 *[2007-I]*
12. Under which one of the following conditions does the solution of $\frac{dy}{dx} = \frac{ax + b}{cy + d}$ represent a parabola?
    (a) $a = 0, c = 0$ (b) $a = 1, b = 2, c \neq 0$
    (c) $a = 0, c \neq 0, b \neq 0$ (d) $a = 1, c = 1$ *[2007-I]*
13. A radioactive element disintegrates at a rate proportional to the quantity of substance Q present at any time t. What is the differential equation of the disintegration ?
    (a) $\frac{dQ}{dt} = -Q$ (b) $\frac{dQ}{dt} = -kQ, k < 0$
    (c) $\frac{dQ}{dt} = -kQ, k > 0$ (d) $\frac{dQ}{dt} = Q$ *[2007-II]*
14. What is the solution of the differential equation $(x + y)(dx - dy) = dx + dy$? *[2007-II]*
    (a) $2\log(x + y) = c(y - x)$ (b) $(y - x) + \log(x + y) = c$
    (c) $\left(\frac{y}{x}\right) + \left[\log\left(\frac{y}{x}\right)\right] = c$ (d) None of these

15. What is the only solution of the initial value problem $y' = t(1+y), y(0) = 0$?

(a) $y = -1 + e^{t^2/2}$ (b) $y = 1 + e^{t^2/2}$
(c) $y = -t$ (d) $y = t$ *[2007-II]*

16. What is the differential equation of the curve $y = ax^2 + bx$?

(a) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

(b) $x^2\frac{d^2y}{dx^2} - y\left(\frac{dy}{dx}\right)^2 + 2 = 0$

(c) $(1-x^2)\frac{d^2y}{dx^2} - \left(y\frac{dy}{dx}\right)^2 = 0$

(d) None of the above *[2007-II]*

17. What is the degree of the differential equation

$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}$? *[2007-II]*

(a) 4 (b) 3
(c) 2 (d) 1

18. If $f(x) = \sqrt{x+\sqrt{x+\sqrt{x+\sqrt{....\infty}}}}$, then what is $f'(x)$ equal to?

(a) $\frac{1}{1-2f(x)}$ (b) $\frac{1}{2f(x)-1}$

(c) $\frac{1}{1+2f(x)}$ (d) $\frac{1}{2+f(x)}$ *[2007-II]*

19. What is the solution of the differential equation

$\frac{dy}{dx} = xy + x + y + 1$? *[2008-I]*

(a) $y = \frac{x^2}{2} + x + c$ (b) $\log(y+1) = \frac{x^2}{2} + x + c$
(c) $y = x^2 + x + c$ (d) $\log(y+1) = x^2 + x + c$

20. What are the order and degree, respectively of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{5/6} = \left(\frac{dy}{dx}\right)^{1/3}$?

(a) 2, 1 (b) 2, 5

(c) $2, \frac{5}{6}$ (d) $1, \frac{1}{3}$ *[2008-I]*

21. What is the solution of the differential equation
$-\text{cosec}^2(x+y)\,dy = dx$? *[2008-II]*

(a) $y - c = \sin(x+y)$ (b) $x - c = \sin(x+y)$
(c) $y - c = \tan(x+y)$ (d) None of the above

22. What are the order and degree respectively of the differential equation

$(d^4y/dx^4)^3\}^{2/3} - 7x(d^3y/dx^3)^2 = 8$? *[2008-II]*

(a) 3, 2 (b) 4, 3
(c) 4, 2 (d) 3, 3

23. What is the solution of the differential equation
$x\,dy - y\,dx = xy^2 dx$? *[2008-II]*

(a) $yx^2 + 2x = 2cy$ (b) $y^2x + 2y = 2cx$
(c) $y^2x^2 + 2x = 2cy$ (d) None of these

24. What does the solution of the differential equation $x\,dy - y\,dx = 0$ represent? *[2008-II]*

(a) Rectangular hyperbola
(b) Straight line passing through the origin
(c) Parabola whose vertex is at origin
(d) Circle whose centre is at origin

25. What is the order of the differential equation ?

$\frac{dy}{dx} + y = \frac{1}{\left(\frac{dy}{dx}\right)}$ *[2008-II]*

(a) –1 (b) 0 (c) 1 (d) 2

26. Rate of growth of bacteria is proportional to the number of bacteria present at that time. If x is the number of bacteria present at any instant t, then which one of the following is correct? (Take proportional constant equal to 1) *[2008-II]*

(a) $x = \log t$ (b) $x = ce^t$
(c) $e^x = t$ (d) $x = \sqrt{t}$

27. What is the solution of the differential equation $\frac{dy}{dx} = e^{x-y}\left(e^{y-x} - e^y\right)$? *[2009-I]*

(a) $y = x - e^x + c$ (b) $y = x + e^x + c$
(c) $y = e^{x-y} - e^y + c$ (d) None of these

28. What are the degree and order respectively of differential equation of the family of rectangular hyperbolas whose axis of symmetry are the coordinate axis? *[2009-I]*

(a) 1, 1 (b) 1, 2
(c) 2, 1 (d) 2, 2

29. What does the equation $x\,dy = y\,dx$ represent? *[2009-II]*

(a) A family of circles (b) A family of parabolas
(c) A family of hyperbolas (d) A family of straight lines

30. What is the solution of the differential equation
$xdy - y\,dx = xy^2dx$? *[2009-II]*

(a) $y + x^{-2} = c$ (b) $y^2 + 2x^{-1} = c$
(c) $y + x^{-1} = c$ (d) $x^2 + 2xy^{-1} = c$

31. When a and b are eliminated from the equation $xy = ae^x + be^{-x}$, the resulting differential equation is of *[2009-II]*

(a) first order and first degree
(b) first order and second degree
(c) second order and first degree
(d) second order and second degree

32. What is the solution of the differential equation
$3e^x \tan y\,dx + (1+e^x)\sec^2 y\,dy = 0$? *[2010-I]*

(a) $(1+e^x)\tan y = c$ (b) $(1+e^x)^3 \tan y = c$
(c) $(1+e^x)^2 \tan y = c$ (d) $(1+e^x)\sec^2 y = c$

where c is a constant of integration.

33. What is the differential equation for $y^2 = 4a(x - a)$?
(a) $yy' - 2xyy' + y^2 = 0$ [2010-I]
(b) $yy'(yy' + 2x) + y^2 = 0$
(c) $yy'(yy' - 2x) + y^2 = 0$
(d) $yy' - 2xyy' + y = 0$

34. What is the degree of the differential equation
$$\frac{d^2y}{dx^2} - \sqrt{1+\left(\frac{dy}{dx}\right)^3} = 0\ ?$$ [2010-I]
(a) 1 (b) 2
(c) 3 (d) 6

35. The growth of a quantity $N(t)$ at any instant t is given by $\frac{dN(t)}{dt} = \alpha N(t)$. Given that $N(t) = ce^{kt}$, c is a constant. What is the value of α? [2010-I]
(a) c (b) k
(c) $c + k$ (d) $c - k$

36. What is the solution of the differential equation
$$a\left(x\frac{dy}{dx} + 2y\right) = xy\frac{dy}{dx}?$$ [2010-I]
(a) $x^2 = kye^{\frac{y}{a}}$ (b) $yx^2 = kye^{\frac{y}{a}}$
(c) $y^2x^2 = kye^{\frac{y^2}{a}}$ (d) None of the above

37. What is the degree of the differential equation [2010-II]
$$\left(1+\frac{dy}{dx}\right)^4 = \left(\frac{d^2y}{dx^2}\right)^2 ?$$
(a) 1 (b) 2
(c) 4 (d) 8

38. What is the general solution of
$(1 + e^x)\, y\, dy = e^x\, dx$? [2010-II]
(a) $y^2 = \ell n\,[c^2\,(e^x + 1)^2]$ (b) $y = \ell n\,[c\,(e^x + 1)]$
(c) $y^2 = \ell n\,[c\,(e^x + 1)]$ (d) None of these
Where 'c' is a constant of integration

39. Which one of the following is the differential equation to family of circles having centre at the origin? [2010-II]
(a) $\left(x^2 - y^2\right)\frac{dy}{dx} = 2xy$ (b) $\left(x^2 + y^2\right)\frac{dy}{dx} = 2xy$
(c) $\frac{dy}{dx} = \left(x^2 + y^2\right)$ (d) $xdx + ydy = 0$

40. What does the solution of the differential equation $x\frac{dy}{dx} = y$ represent? [2010-II]
(a) Family of straight lines through the origin
(b) Family of circles with their centres at the origin
(c) Family of parabolas with their vertices at the origin
(d) Family of straight lines having slope 1 and not passing through the origin

41. What does the differential equation $y\frac{dy}{dx} + x = k$ (where k is a constant) represents? [2010-II]
(a) A family of circles having centre on the y-axis.
(b) A family of circles having centre on the x-axis.
(c) A family of circles touching the x-axis
(d) A family of ellipses.

42. What is the differential equation to family of parabolas having their vertices at the origin and foci on the x-axis?
(a) $y = 2xy'$ (b) $x = 2yy'$ [2010-II]
(c) $xy = y'$ (d) $x = yy'$

43. What is the solution of the differential equation
$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0?$$ [2011-I]
(a) $\sin^{-1} y + \sin^{-1} x = C$ (b) $\sin^{-1} y - \sin^{-1} x = C$
(c) $2\sin^{-1} y + \sin^{-1} x = C$ (d) $2\sin^{-1} y - \sin^{-1} x = C$
Where C is a constant.

44. What is the differential equation of all parabolas whose axes are parallel to Y-axis? [2011-I]
(a) $\frac{d^3y}{dx^3} = 0$ (b) $\frac{d^2x}{dy^2} = C$
(c) $\frac{d^3x}{dy^3} = 1$ (d) $\frac{d^3y}{dx^3} = C$
(where C is a constant).

45. If the solution of the differential equation [2011-I]
$$\frac{dy}{dx} = \frac{ax+3}{2y+f}$$
represents a circle, then what is the value of a?
(a) 2 (b) 1
(c) -2 (d) -1

46. What is the degree of the following differential equation? [2011-I]
$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$$
(a) 1 (b) 2
(c) 3 (d) 4

47. What does the differential equation $y\frac{dy}{dx} + x = a$ (where a is a constant) represent? [2011-I]
(a) A set of circles having centre on the Y-axis
(b) A set of circles having centre on the X-axis
(c) A set of ellipses
(d) A pair of straight lines

48. What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\left(\frac{d^2y}{dx^2}\right) + 5\left(\frac{dy}{dx}\right) = 0 ?$$ [2011-II]

(a) 3 (b) 2
(c) 2 / 3 (d) Not defined

49. What is the equation of the curve passing through the point $\left(0, \frac{\pi}{3}\right)$ satisfying the differential equation $\sin x \cos y\, dx + \cos x \sin y\, dy = 0$? [2011-II]

(a) $\cos x \cos y = \frac{\sqrt{3}}{2}$ (b) $\sin x \sin y = \frac{\sqrt{3}}{2}$
(c) $\sin x \sin y = \frac{1}{2}$ (d) $\cos x \cos y = \frac{1}{2}$

50. What is the solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$? [2012-I]

(a) xy = c (b) x = cy
(c) y = cx (d) None of the above

51. What is the degree of the differential equation $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1}$? [2012-I]

(a) 1 (b) 2
(c) – 1 (d) Degree does not exist.

52. Which one of the following differential equations is not linear? [2012-I]

(a) $\frac{d^2y}{dx^2} + 4y = 0$ (b) $x\frac{dy}{dx} + y = x^3$
(c) $(x - y)^2 \frac{dy}{dx} = 9$ (d) $\cos^2 x \frac{dy}{dx} + y = \tan x$

53. What is the degree of the differential equation $\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$? [2012-II]

(a) 6 (b) 3
(c) 2 (d) 1

54. Consider a differential equation of order m and degree n. Which one of the following pairs is not feasible ? [2012-II]

(a) (3, 2) (b) (2, 3/2)
(c) (2, 4) (d) (2, 2)

55. The differential equation representing the family of curves $y = a \sin(\lambda x + \alpha)$ is : [2012-II]

(a) $\frac{d^2y}{dx^2} + \lambda^2 y = 0$ (b) $\frac{d^2y}{dx^2} - \lambda^2 y = 0$
(c) $\frac{d^2y}{dx^2} + \lambda y = 0$ (d) None of the above

56. The differential equation $y\frac{dy}{dx} + x = a$ where 'a' is any constant represents : [2012-II]

(a) A set of straight lines (b) A set of ellipses
(c) A set of circles (d) None of the above

57. For the differential equation $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$, which one of the following is not its solution ? [2012-II]

(a) $y = x - 1$ (b) $4y = x^2$
(c) $y = x$ (d) $y = -x - 1$

58. What is the general solution of the differential equation $x^2\, dy + y^2\, dx = 0$? [2012-II]

(a) x + y = c (b) xy = c
(c) c(x + y) = xy (d) None of the above

where c is the constant of integration.

59. What is the general solution of the differential equation $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$? [2012-II]

(a) $\sin y = c(1 - e^x)$ (b) $\cos y = c(1 - e^x)$
(c) $\cot y = c(1 - e^x)$ (d) None of the above

where c is the constant of integration

60. What is the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^{3/5} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 5 = 0 ?$$ [2013-I]

(a) 5 (b) 4
(c) 3 (d) 2

61. The general solution of the differential equation $x\frac{dy}{dx} + y = 0$ is? [2013-I]

(a) xy = c (b) x = cy
(c) x + y = c (d) $x^2 + y^2 = c$

62. The general solution of the differential equation $\ell n\left(\frac{dy}{dx}\right) + x = 0$ is? [2013-I]

(a) $y = e^{-x} + c$ (b) $y = -e^{-x} + c$
(c) $y = e^x + c$ (d) $y = -e^x + c$

63. The differential equation of the curve y = sin x is [2013-I]

(a) $\frac{d^2y}{dx^2} + y\frac{dy}{dx} + x = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$
(c) $\frac{d^2y}{dx^2} - y = 0$ (d) $\frac{d^2y}{dx^2} + x = 0$

64. The degree and order respectively of the differential equation $\frac{dy}{dx} = \frac{1}{x + y + 1}$ are [2013-I]

(a) 1, 1 (b) 1, 2
(c) 2, 1 (d) 2, 2

65. What is the order of the differential equation [2013-II]

$$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 y = 0 ?$$

(a) 1 (b) 2
(c) 3 (d) Undefined

66. $y = 2\cos x + 3\sin x$ satisfies which of the following differential equations ? *[2013-II]*

1. $\frac{d^2y}{dx^2} + y = 0$ 2. $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} = 0$

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

67. The differential equation of all circles whose centres are at the origin is *[2013-II]*

(a) $\frac{dy}{dx} = \frac{y}{x}$ (b) $\frac{dy}{dx} = \frac{x}{y}$

(c) $\frac{dy}{dx} = -\frac{x}{y}$ (d) None of the above

68. The solution of $\frac{dy}{dx} = |x|$ is : *[2014-I]*

(a) $y = \frac{x|x|}{2} + c$ (b) $y = \frac{|x|}{2} + c$

(c) $y = \frac{x^2}{2} + c$ (d) $y = \frac{x^3}{2} + c$

Where c is an arbitary constant

69. What is the solution of $\frac{dy}{dx} + 2y = 1$ satisfying $y(0) = 0$? *[2014-I]*

(a) $y = \frac{1 - e^{-2x}}{2}$ (b) $y = \frac{1 + e^{-2x}}{2}$

(c) $y = 1 + e^x$ (d) $y = \frac{1 + e^x}{2}$

70. What is the general solution of the differential equation $x\,dy - y\,dx = y^2$? *[2014-I]*

(a) $x = cy$ (b) $y^2 = cx$
(c) $x + xy - cy = 0$ (d) None of these

DIRECTIONS (Qs. 71 - 73): *(For the next three (03) items that follow) :*

The general solution of the differential equation $(x^2 + x + 1)\,dy + (y^2 + y + 1)\,dx = 0$ is $(x + y + 1) = A(1 + Bx + Cy + Dxy)$ where B, C and D are constants and A is parameter. *[2014-I]*

71. What is B equal to ?

(a) –1 (b) 1
(c) 2 (d) None of these

72. What is C equal to ?

(a) 1 (b) –1
(c) 2 (d) None of these

73. What is D equal to ?

(a) –1 (b) 1
(c) –2 (d) None of these

74. What is the number of arbitrary constants in the particular solution of differential equation of third order ? *[2014-I]*

(a) 0 (b) 1
(c) 2 (d) 3

75. Consider the following statements in respect of the differential equation *[2014-I]*

$$\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$$

1. The degree of the differential equation is not defined.
2. The order of the differential equation is 2.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

76. What is the degree of the differential equation *[2014-II]*

$$\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2 ?$$

(a) 1 (b) 2
(c) 3 (d) 4

77. What is the solution of the equation *[2014-II]*

$$\ln\left(\frac{dy}{dx}\right) + x = 0 ?$$

(a) $y + e^x = c$ (b) $y - e^{-x} = c$

(c) $y + e^{-x} = c$ (d) $y - e^x = c$

78. Eliminating the arbitrary constants B and C in the expresion

$$y = \frac{2}{3C}(Cx - 1)^{3/2} + B, \text{ we get}$$

(a) $x\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$

(b) $2x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^2$

(c) $\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 1$

(d) $\left(\frac{dy}{dx}\right)^2 + 1 = \frac{d^2y}{dx^2}$

79. What is the solution of the differential equation *[2015-I]*

$$\frac{ydx - xdy}{y^2} = 0 ?$$

(a) $xy = c$ (b) $y = cx$
(c) $x + y = c$ (d) $x - y = c$

where c is an arbitrary constant.

80. What is the solution of the differential equation *[2015-I]*

$$\sin\left(\frac{dy}{dx}\right) - a = 0 ?$$

(a) $y = x\sin^{-1} a + c$ (b) $x = y\sin^{-1} a + c$
(c) $y = x + x\sin^{-1} a + c$ (d) $y = \sin^{-1} a + c$

where c is an arbitrary constant.

81. What is the solution of the differential equation

$\frac{dx}{dy}+\frac{x}{y}-y^2=0$? [2015-I]

(a) $xy=x^4+c$ (b) $xy=y^4+c$
(c) $4xy=y^4+c$ (d) $3xy=y^3+c$
where c is an arbitrary constant.

82. Consider the following statements: [2015-I]

1. The general solution of $\frac{dy}{dx}=f(x)+x$ is of the form $y=g(x)+c$, where c is an arbitrary constant.

2. The degree of $\left(\frac{dy}{dx}\right)^2=f(x)$ is 2.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

83. The degree of the differential equation [2015-II]

$\frac{dy}{dx}-x=\left(y-x\frac{dy}{dx}\right)^{-4}$ is

(a) 2 (b) 3
(c) 4 (d) 5

84. The solution of $\frac{dy}{dx}=\sqrt{1-x^2-y^2+x^2y^2}$ is [2015-II]

(a) $\sin^{-1}y=\sin^{-1}x+c$
(b) $2\sin^{-1}y=\sqrt{1-x^2}+\sin^{-1}x+c$
(c) $2\sin^{-1}y=x\sqrt{1-x^2}+\sin^{-1}x+c$
(d) $2\sin^{-1}y=x\sqrt{1-x^2}+\cos^{-1}x+c$
where c is an arbitrary constant.

85. The differential equation of the family of circles passing through the origin and having centres on the x-axis is [2015-II]

(a) $2xy\frac{dy}{dx}=x^2-y^2$ (b) $2xy\frac{dy}{dx}=y^2-x^2$
(c) $2xy\frac{dy}{dx}=x^2+y^2$ (d) $2xy\frac{dy}{dx}+x^2+y^2=0$

86. The order and degree of the differential equation of parabolas having vertex at the origin and focus at (a, 0) where $a>0$, are respectively [2015-II]
(a) 1, 1 (b) 2, 1
(c) 1, 2 (d) 2, 2

87. What are the order and degree respectively of the differential equation whose solution is $y=cx+c^2-3c^{3/2}+2$, where c is a parameter? [2016-I]
(a) 1, 2 (b) 2, 2
(c) 1, 3 (d) 1, 4

88. Let f(x) be a function such that $f'\left(\frac{1}{x}\right)+x^3f'(x)=0$, What is $\int_{-1}^{1}f(x)\,dx$ equal to? [2016-II]

(a) 2 f(1) (b) 0
(c) 2 f(–1) (d) 4 f(1)

89. What are the degree and order respectively of the differential equation satisfying $e^{y\sqrt{1-x^2}+x\sqrt{1-y^2}}=ce^x$, $\left(\text{where } c>0, |x|<1, |y|<1\right)$? [2016-II]

(a) 1, 1 (b) 1, 2
(c) 2, 1 (d) 2, 2

90. If $x\,dy=y\,dx+y^2\,dy, y>0$ and $y(1)=1$, then what is y (–3) equal to? [2016-II]
(a) 3 only (b) –1 only
(c) Both –1 and 3 (d) Neither –1 nor 3

91. What is the order of the differential equation $\frac{dx}{dy}+\int y\,dx=x^3$? [2016-II]

(a) 1 (b) 2
(c) 3 (d) Cannot be determined

92. Which one of the following differential equations represents the family of straight lines which are at unit distance from the origin? [2016-II]

(a) $\left(y-x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$

(b) $\left(y+x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$

(c) $\left(y-x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$

(d) $\left(y+x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$

93. What is $\frac{d^2x}{dy^2}$ equal to? [2017-I]

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (b) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ (d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

94. If $xdy=y(dx+ydy)$; $y(1)=1$ and $y(x)>0$, then what is y(–3) equal to? [2017-I]
(a) 3 (b) 2
(c) 1 (d) 0

95. What are the degree and order respectively of the differential equation $y=x\left(\frac{dy}{dx}\right)^2+\left(\frac{dx}{dy}\right)^2$? [2017-I]

(a) 1, 2 (b) 2, 1
(c) 1, 4 (d) 4, 1

96. What is the differential equation corresponding to $y^2 - 2ay + x^2 = a^2$ by eliminating a? *[2017-I]*
(a) $(x^2 - 2y^2)p^2 - 4\,pxy - x^2 = 0$
(b) $(x^2 - 2y^2)p^2 + 4\,pxy - x^2 = 0$
(c) $(x^2 + 2y^2)p^2 - 4\,pxy - x^2 = 0$
(d) $(x^2 + 2y^2)p^2 - 4\,pxy + x^2 = 0$

where $p = \frac{dy}{dx}$.

97. What is the general solution of the differential equation $ydx - (x + 2y^2)\,dy = 0$? *[2017-I]*
(a) $x = y^2 + cy$ (b) $x = 2cy^2$
(c) $x = 2y^2 + cy$ (d) None of the above

98. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) - a = 0$? *[2017-I]*
(a) $y = xe^a + c$ (b) $x = ye^a + c$
(c) $y = \ln x + c$ (d) $x = \ln y + c$

99. The general solution of $\frac{dy}{dx} = \frac{ax + h}{by + k}$ represents a circle only when *[2017-II]*
(a) $a = b = 0$ (b) $a = -b \neq 0$
(c) $a = b \neq 0,\ h = k$ (d) $a = b \neq 0$

100. The order and degree of the differential equation
$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2\left[\frac{d^2y}{dx^2}\right]^2$ are respectively *[2017-II]*
(a) 3 and 2 (b) 2 and 2
(c) 2 and 3 (d) 1 and 3

101. The differential equation of minimum order by eliminating the arbitrat constants A and C in the equatic $y = A[\sin(x + C) + \cos(x + C)]$ is *[2017-II]*
(a) $y'' + (\sin x + \cos x)y' = 1$
(b) $y'' = (\sin x + \cos x)y'$
(c) $y'' = (y')^2 + \sin x \cos x$
(d) $y'' + y = 0$

102. The solution of the differential equation $\frac{dy}{dx} = \frac{y\phi'(x) - y^2}{\phi(x)}$ is *[2017-II]*
(a) $y = \frac{x}{\phi(x) + c}$ (b) $y = \frac{\phi(x)}{x} + c$
(c) $y = \frac{\phi(x) + c}{x}$ (d) $y = \frac{\phi(x)}{x + c}$

103. What is the solution of the differential equation $x\,dy - y\,dx = 0$? *[2018-I]*
(a) $xy = c$ (b) $y = cx$
(c) $x + y = c$ (d) $x - y = c$

104. Which one of the following differential equations has a periodic solution? *[2018-I]*
(a) $\frac{d^2x}{dt^2} + \mu x = 0$ (b) $\frac{d^2x}{dt^2} - \mu x = 0$
(c) $x\frac{dx}{dt} + \mu t = 0$ (d) $\frac{dx}{dt} + \mu xt = 0$
where $\mu > 0$.

105. The order and degree of the differential equation $y^2 = 4a\,(x - a)$, where 'a' is an arbitrary constant, are respectively *[2018-I]*
(a) 1, 2 (b) 2, 1
(c) 2, 2 (d) 1, 1

106. What is the solution of $(1 + 2x)\,dy - (1 - 2y)\,dx = 0$? *[2018-I]*
(a) $x - y - 2xy = c$ (b) $y - x - 2xy = c$
(c) $y + x - 2xy = c$ (d) $x + y + 2xy = c$

107. What are the order and degree, respectively, of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 = y^4 + \left(\frac{dy}{dx}\right)^5$? *[2018-I]*
(a) 4, 5 (b) 2, 3
(c) 3, 2 (d) 5, 4

108. The differential equation of the family of curves $y = p\cos(ax) + q\sin(ax)$, where p, q are arbitrary constants, is *[2018-II]*
(a) $\frac{d^2y}{dx^2} - a^2y = 0$ (b) $\frac{d^2y}{dx^2} - ay = 0$
(c) $\frac{d^2y}{dx^2} + ay = 0$ (d) $\frac{d^2y}{dx^2} + a^2y = 0$

109. The equation of the curve passing through the point $(-1, -2)$ which satisfies $\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$ is *[2018-II]*
(a) $17x^2y - 6x^2 + 3x^5 - 2 = 0$
(b) $6x^2y + 17x^2 + 2x^5 - 3 = 0$
(c) $6xy - 2x^2 + 17x^5 + 3 = 0$
(d) $17x^2y + 6xy - 3x^5 + 5 = 0$

110. What is the order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x} + d$, where a, b, c and d are arbitrary constants? *[2018-II]*
(a) 1 (b) 2
(c) 3 (d) 4

111. What is the solution of the differential equation $ln\left(\frac{dy}{dx}\right) = ax + by$? *[2018-II]*
(a) $a\,e^{ax} + b\,e^{by} = c$
(b) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{by} = c$
(c) $a\,e^{ax} + b\,e^{-by} = c$
(d) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{-by} = c$

112. If $u = e^{ax}\sin bx$ and $v = e^{ax}\cos bx$, then what is $u\frac{du}{dx} + v\frac{dv}{dx}$ equal to? *[2018-II]*
(a) $a\,e^{2ax}$ (b) $(a^2 + b^2)e^{ax}$
(c) $ab\,e^{2ax}$ (d) $(a + b)e^{ax}$

113. If $y = \sin(\ell nx)$, then which one of the following is correct? [2018-II]

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} = 0$

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(d) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

114. What is the solution of the differential equation $\frac{dx}{dy} = \frac{x+y+1}{x+y-1}$? [2018-II]

(a) $y - x + 4\ln(x+y) = c$
(b) $y + x + c\ln(x+y) = c$
(c) $y - x + \ln(x+y) = c$
(d) $y + x + 2\ln(x+y) = c$

115. The solution of the differential equation [2019-I]

$\frac{dy}{dx} = \cos(y-x) + 1$ is

(a) $e^x[\sec(y-x) - \tan(y-x)] = c$
(b) $e^x[\sec(y-x) + \tan(y-x)] = c$
(c) $e^x\sec(y-x)\tan(y-x) = c$
(d) $e^x = c\sec(y-x)\tan(y-x)$

116. If $y = a\cos 2x + b\sin 2x$, then [2019-I]

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} + 2y = 0$

(c) $\frac{d^2y}{dx^2} - 4y = 0$ (d) $\frac{d^2y}{dx^2} + 4y = 0$

117. The differential equation of the system of circles touching the y-axis at the origin is [2019-I]

(a) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$ (b) $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

(c) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$ (d) $x^2 - y^2 - 2xy\frac{dy}{dx} = 0$

118. Consider the following in respect of the differential equation :

$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 9y = x$ [2019-I]

1. The degree of the differential equation is 1.
2. The order of the differential equation is 2.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

119. What is the general solution of the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$? [2019-I]

(a) $x^2 + y^2 = c$ (b) $x^2 - y^2 = c$
(c) $x^2 + y^2 = cxy$ (d) $x + y = c$

ANSWER KEY

1	(b)	13	(c)	25	(c)	37	(b)	49	(d)	61	(a)	73	(c)	85	(b)	97	(c)	109	(b)
2	(b)	14	(b)	26	(b)	38	(a)	50	(a)	62	(b)	74	(d)	86	(a)	98	(a)	110	(d)
3	(b)	15	(a)	27	(a)	39	(d)	51	(b)	63	(b)	75	(c)	87	(d)	99	(b)	111	(d)
4	(c)	16	(a)	28	(a)	40	(a)	52	(a)	64	(a)	76	(c)	88	(c)	100	(b)	112	(a)
5	(d)	17	(c)	29	(d)	41	(b)	53	(d)	65	(a)	77	(c)	89	(a)	101	(d)	113	(c)
6	(b)	18	(b)	30	(d)	42	(a)	54	(b)	66	(a)	78	(b)	90	(a)	102	(d)	114	(c)
7	(a)	19	(b)	31	(c)	43	(a)	55	(a)	67	(c)	79	(b)	91	(b)	103	(b)	115	(a)
8	(b)	20	(b)	32	(b)	44	(a)	56	(c)	68	(a)	80	(a)	92	(c)	104	(a)	116	(c)
9	(a)	21	(d)	33	(c)	45	(c)	57	(c)	69	(a)	81	(c)	93	(c)	105	(*)	117	(c)
10	(c)	22	(c)	34	(b)	46	(b)	58	(c)	70	(*)	82	(c)	94	(a)	106	(a)	118	(c)
11	(b)	23	(a)	35	(b)	47	(b)	59	(d)	71	(a)	83	(d)	95	(d)	107	(c)	119	(a)
12	(c)	24	(b)	36	(d)	48	(b)	60	(c)	72	(b)	84	(c)	96	(a)	108	(d)		

HINTS & SOLUTIONS

1. (b) Given that x dy – ydx = 0

Dividing both the sides by x^2

$$\Rightarrow \frac{xdy - ydx}{x^2} = 0$$

$$\Rightarrow d\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{y}{x} = c \Rightarrow y = cx, \text{ where c is a constant.}$$

Thus, it a straight line passing through (0, 0).

Aliter :

Given, x dy – y dx = 0

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

Integrating both the sides, $\int \frac{dy}{y} = \int \frac{dx}{x} + \log c$

log y = log x + log c where c is constant

$\Rightarrow$ y = cx

2. (b) Since, circle is touching y-axis at origin its center lies on x-axis. Let the centre be (a, 0). Its radius = a

$$(x - a)^2 + y^2 = a^2$$

$$x^2 + a^2 - 2ax + y^2 = a^2$$

$$\Rightarrow x^2 + y^2 - 2ax = 0 \qquad ...(i)$$

$$a = \frac{x^2 + y^2}{2x}$$

Differentiating both sides

Now, $2x + 2y\frac{dy}{dx} - 2a = 0$

$$\Rightarrow 2x + 2y\frac{dy}{dx} - \frac{x^2 + y^2}{x} = 0$$

$$\Rightarrow 2x^2 + 2xy\frac{dy}{dx} - x^2 - y^2 = 0$$

$$\Rightarrow 2xy\frac{dy}{dx} = y^2 - x^2$$

3. (b) $y^3 - x = cy$

$$\Rightarrow 3y^2\frac{dy}{dx} - 1 = c\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(3y^2 - c) = 1$$

$$\Rightarrow \frac{dy}{dx}\left(3y^2 - \frac{y^3 - x}{y}\right) = 1$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{3y^3 - y^3 + x}{y}\right) = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x + 2y^3}$$

4. (c) Given that $y^2 = p(x)$

Differentiating

$$\Rightarrow 2yy_1 = p'(x) \qquad \left[\text{here } y_1 = \frac{dy}{dx}\right]$$

$$\Rightarrow 2y_1 = \frac{p'(x)}{y}$$

Differentiating again,

$$\Rightarrow 2y_2 = \frac{yp''(x) - p'(x)y_1}{y^2}, \quad \left[y_2 = \frac{d^2y}{dx^2}\right]$$

$$\Rightarrow 2y_2 = \frac{yp''(x) - \frac{p'(x).p'(x)}{2y}}{y^2}$$

$$= \frac{2y^2p''(x) - p'(x))^2}{2y^3}$$

$$\Rightarrow 2y^3y_2 = \frac{1}{2}[2y^2p''(x) - (p'(x))^2]$$

$$\Rightarrow 2y^3y_2 = \frac{1}{2}[2p(x)p''(x) - (p'(x))^2]$$

$$\Rightarrow 2\frac{d}{dx}(y^3y_2)$$

$$= \frac{1}{2}[2p'(x)p''(x) + 2p(x)p'''(x) - 2p'(x)p''(x)]$$

$$= p(x)p'''(x)$$

5. (d) The given differential equation is :

$$\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{1/4}$$

This can be re-written as by squaring both the sides to the power 4 to make it a polynomial of derivative.

$$\left[\frac{d^2y}{dx^2}\right]^4 = \left[y + \left(\frac{dy}{dx}\right)^2\right]$$

Power of highest derivatives 4,

So, degree of the equation is 4.

6. (b) The given differential equation is

$$y = x\frac{dy}{dx} + \frac{dx}{dy}$$

Multiplying both the sides by $\frac{dy}{dx}$

We get $\left(\frac{dy}{dx}\right)y = x\left(\frac{dy}{dx}\right)^2 + 1$

$\Rightarrow \quad x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) + 1 = 0$

Hence, order and degree of differential equation are 1 and 2.

7. (a) The differential equation

$dy = (y \tan x + \sec x)\, dx$ can be written as

$$\frac{dy}{dx} = y \tan x + \sec x$$

or, $\frac{dy}{dx} - y \tan x = \sec x$

which is of the form $\frac{dy}{dx} + P(x).y = Q(x)$

Here $P(x) = -\tan x$ and $Q(x) = \sec x$

Integrating factor IF $= e^{\int P(x)dx}$

IF $= e^{\int -\tan x\, dx} = e^{\int -\frac{\sin x}{\cos x} dx}$

Putting $\cos x = t$

$-\sin x\, dx = dt$

If $= e^{\int \frac{dt}{t}} = e^{\log_e t} = t = \cos x$

The solution is

$y.Q(x) = \int I.F.Q(x)dx + c$

or, $y.\sec x = \int \cos x.\sec x\, dx + c$

or, $y.\sec x = \int dx + c$

or, $y.\sec x = x + c$

Since the curve passes through the origin.

$0 = 0 + c \Rightarrow c = 0$

and $y \sec x = x$

or, $y = x \cos x$

8. (b) In the equation $\frac{dy}{dx} = \sec(x+y)$

Let $x + y = v$

So, $1 + \frac{dy}{dx} = \frac{dv}{dx}$ or $\frac{dy}{dx} = \frac{dv}{dx} - 1$

and $\frac{dv}{dx} - 1 = \sec v$

$\frac{dv}{dx} = 1 + \sec v = \frac{1+\cos v}{\cos v}$

or, $\frac{\cos v}{1+\cos v} dv = dx$

$\cos v = \cos^2\frac{v}{2} - \sin^2\frac{v}{2}$

and $1 + \cos v = 2\cos^2\frac{v}{2}$

so, $\left(\frac{2\cos^2\frac{v}{2} - 2\sin^2\frac{v}{2}}{2\cos^2\frac{v}{2}}\right)dv = dx$

$\left(1 - \tan^2\frac{v}{2}\right)dv = 2dx$

or, $\left\{1 - \left(\sec^2\frac{v}{2} - 1\right)\right\}dv = 2dx$

or, $\left(2 - \sec^2\frac{v}{2}\right)dv = 2dx$

Integrating on both the sides

$2\int dv - \int \sec^2\frac{v}{2} dv = 2\int dx + c_1$

where c_1 is a constant.

$2v - 2\tan\frac{v}{2} = 2x + c_1$

$2(x+y) - 2\tan\frac{x+y}{2} = 2x + c_1$

or, $2x + 2y - 2\tan\frac{x+y}{2} = 2x + c_1$

$y - \tan\left(\frac{x+y}{2}\right) = \frac{c_1}{2} = c,$

[c is a constant]

so, $y - \tan\frac{x+y}{2} = c$

9. (a) $\frac{dy}{dx} = ky$ or $\frac{dy}{y} = kdx$

Integrating both the sides

$\int \frac{dy}{y} = k\int dx + \log c$ (where c is a constant)

$\log y = kx + \log c \Rightarrow \log y - \log c = kx$

or, $\log\left(\frac{y}{c}\right) = kx$

or, $\frac{y}{c} = e^{kx}$

or, $y = c.e^{kx}$

The equation will show a decay. If value of k is negative. Only option (a) shows negative value of k.

10. (c) Differential equation is

$(x + y)(dx - dy) = dx + dy$

dividing by dx on both the sides

$$(x+y)\left(1-\frac{dy}{dx}\right) = 1+\frac{dy}{dx}$$

Putting $x + y = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx} \text{ and } \frac{dy}{dx} = \frac{dv}{dx}-1$$

The equation changes to

$$v\left\{1-\left(\frac{dv}{dx}-1\right)\right\} = \frac{dv}{dx}$$

$$v\left(2-\frac{dv}{dx}\right) = \frac{dv}{dx}$$

$$2v - v\frac{dv}{dx} = \frac{dv}{dx}$$

$$2v = (1+v)\frac{dv}{dx}$$

$$\left(\frac{1+v}{v}\right)dv = 2dx$$

$$\text{or, } \left(\frac{1}{v}+1\right)dv = 2dx$$

Integrating on both the sides,

$$\int\frac{dv}{v}+\int dv = 2\int dx + c$$

$\log v + v = 2x + c$

Putting $v = x + y$

$\log(x + y) + x + y = 2x + c$

or, $\log(x + y) + y - x = c$

or, $y - x + \log(x + y) = c$

11. (b) In the given equation,

$$K.\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^3\right]^{3/2}$$

Squaring both the sides,

$$K^2.\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^3\right]^3$$

Degree of a differential equation is the highest power of the highest derivative in equation when derivatives are expressed as polynomial. Here degree of differential equation is 2.

12. (c) Given: $\frac{dy}{dx} = \frac{ax+b}{cy+d}$

or, $(cy + d)\,dy = (ax + b)dx$

Integrating both the sides.

$$c.\int ydy + d\int dy = a\int x\,dx + b\int dx + K \quad \text{[K is constant integration]}$$

$$\text{or, } c.\frac{y^2}{2}+d.y = a\frac{x^2}{2}+b.x+K$$

or, $cy^2 + 2d.y = ax^2 + 2b.x + 2K$

This equation will represent a parabola when either, the coefficient of x^2 or the coefficient of y^2 is zero, but not both.

Thus either $c = 0$ or $a = 0$ but not both.

From the choice given, $a = 0$, $c \neq 0$ and $b \neq 0$.

13. (c) A radioactive element disintegrates at a rate proportional to the quantity of substance Q present at any time t.

$$\Rightarrow \frac{dQ}{dt} \propto -Q$$

$$\Rightarrow \frac{dQ}{dt} = -kQ, k > 0 \text{ is a constant.}$$

This is required differential equation.

14. (b) Given differential equation is :

$(x + y)(dx - dy) = dx + dy$

$\Rightarrow (x + y)dx - (x + y)dy = dx + dy$

$\Rightarrow (x + y - 1)dx = (x + y + 1)dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

Let $x + y = v$ and $\frac{dy}{dx} = \frac{dv}{dx}-1$

$$\therefore \frac{dv}{dx}-1 = \frac{v-1}{v+1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{v-1}{v+1}+1$$

$$\Rightarrow \frac{dv}{dx} = \frac{v-1+v+1}{v+1}$$

$$\Rightarrow \frac{v+1}{2v}dv = dx$$

$$\Rightarrow \frac{1}{2}\int 1dv + \frac{1}{2}\int\frac{1}{v}dv = \int 1dx$$

$$\Rightarrow \frac{1}{2}v+\frac{1}{2}\log v = x + c_1$$

$\Rightarrow x + y + \log(x + y) = 2x + c \quad (\because 2c_1 = c = \text{costant})$

$\Rightarrow (y - x) + \log(x + y) = c$

15. (a) Given, equation is : $y' = t(1+y)$

i.e., $\frac{dy}{dt} = t(1+y)$

$\Rightarrow \int \frac{1}{1+y} dy = \int t dt \Rightarrow \log(1+y) = \frac{t^2}{2} + c$

As per initial conditions

$y(0) = 0$ when $t = 0, y = 0$

$\Rightarrow \log 1 = c \Rightarrow c = 0$

$\therefore \log(1+y) = \frac{t^2}{2} \Rightarrow 1+y = e^{t^2/2}$

$\Rightarrow y = -1 + e^{t^2/2}$

which is required solution.

16. (a) Given, equation is :

$y = ax^2 + bx$ ....(1)

Differentiating w.r.t. x,

$\Rightarrow \frac{dy}{dx} = 2ax + b$ ....(2)

Differentiation g w.r.t. x,

$\Rightarrow \frac{d^2y}{dx^2} = 2a$

$\Rightarrow a = \frac{1}{2}\frac{d^2y}{dx^2}$ ....(3)

From (2) and (3)

$b = \frac{dy}{dx} - x.\frac{d^2y}{dx^2},$

Putting values of a and b in equation (1):

$y = \frac{1}{2}\frac{d^2y}{dx^2}.x^2 + x\left(\frac{dy}{dx} - x\frac{d^2y}{dx^2}\right)$

$\Rightarrow 2y = x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2x^2\frac{d^2y}{dx^2}$

$\Rightarrow x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

17. (c) The given differential equation is

$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}$

To express it as a polynomial of derivatives we square both side,

$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2$

Highest derivative has power = 2

Degree of differential equation = 2.

18. (b) Given function is :

$f(x) = \sqrt{x+\sqrt{x+\sqrt{x\sqrt{\infty}}}}$

$\Rightarrow f(x) = \sqrt{x+f(x)} \Rightarrow (f(x))^2 = x + f(x)$

On differentiating both sides wrt. x, we get

$2f(x)f'(x) = 1 + f'(x)$

$f'(x)\{2f(x)-1\} = 1$

$f'(x) = \frac{1}{2f(x)-1}$

19. (b) The given differential equation is :

$\frac{dy}{dx} = xy + x + y + 1 \Rightarrow \frac{dy}{dx} = (x+1)(y+1)$

Separating variables,

$\Rightarrow \frac{1}{(1+y)}dy = (x+1)dx \Rightarrow \log(1+y) = \frac{x^2}{2} + x + c$

20. (b) Given differential equation is :

$\left(\frac{d^2y}{dx^2}\right)^{5/6} = \left(\frac{dy}{dx}\right)^{1/3}$

Raising both the side to power of 6, to make it a polynomial of derivatives.

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^5 = \left(\frac{dy}{dx}\right)^{6/3} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^5 = \left(\frac{dy}{dx}\right)^2$

Highest derivative has power of 5. So, the order and degree of given differential equation are 2 and 5 respectively.

21. (d) $-\text{cosec}^2(x+y)dy = dx$

$\Rightarrow \frac{dy}{dx} = -\sin^2(x+y)$

Put $x + y = t$

$\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$ Þ $\frac{dy}{dx} = \frac{dt}{dx} - 1$

$\therefore \frac{dt}{dx} - 1 = -\sin^2(t)$

$\Rightarrow \frac{dt}{dx} = 1 - \sin^2 t \Rightarrow \frac{dt}{dx} = \cos^2 t = \frac{1}{\sec^2 t}$

$\Rightarrow \int \sec^2 t\, dt = \int dx \Rightarrow \tan t = x - c$

22. (c) $\left\{\left(\frac{d^4y}{dx^4}\right)^3\right\}^{\frac{2}{3}} - 7x\left(\frac{d^3y}{dx^3}\right)^2 = 8$

$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 - 7x\left(\frac{d^3y}{dx^3}\right)^2 = 8$

$\therefore$ The order and degree of the given differential equation are 4 and 2 respectively.

23. (a) Given, $x\,dy - y\,dx = xy^2 dx$

$$\Rightarrow \frac{x\,dy - y\,dx}{y^2} = x\,dx \Rightarrow \frac{y\,dx - x\,dy}{y^2} = -x dx$$

$$\Rightarrow \int d\left(\frac{x}{y}\right) = -\int x\,dx \Rightarrow \frac{x}{y} = \frac{-x^2}{2} + c = \frac{-x^2 + 2c}{2}$$

$$\Rightarrow \quad yx^2 + 2x = 2cy$$

24. (b) $x\,dy - y\,dx = 0$

$\Rightarrow \quad x\,dy = y\,dx$

$$\Rightarrow \quad \int \frac{1}{y} dy = \int \frac{1}{x} dx$$

$\Rightarrow \quad \log y = \log x + \log c$

$\Rightarrow \quad \log y = \log cx$

$\Rightarrow \quad y = cx$

Thus, the solution of equation $x\,dy - y\,dx = 0$ represents straight line passing through the origin.

25. (c) $\dfrac{dy}{dx} + y = \dfrac{1}{\left(\dfrac{dy}{dx}\right)} \Rightarrow \left(\dfrac{dy}{dx}\right)^2 + y\left(\dfrac{dy}{dx}\right) = 1$

Hence required order of differential equation = 1

26. (b) Rate of growth of bacteria $\propto$ number of bacteria present at that time

$$\Rightarrow \quad \frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = x \quad (\because \text{Proportional constant} = 1)$$

$$\Rightarrow \quad \int \frac{1}{x} dx = \int dt$$

$\Rightarrow \quad \log x = t + \log c$

$\Rightarrow \quad \log x - \log c = t$

$$\Rightarrow \quad \log\left(\frac{x}{c}\right) = t$$

$$\Rightarrow \quad \frac{x}{c} = e^t$$

$x = ce^t$

27. (a) Given differential equation is

$$\frac{dy}{dx} = e^{x-y}\left(e^{y-x} - e^y\right) = e^{-y}.e^y\left(e^x.e^{-x} - e^x\right)$$

$$\Rightarrow \quad \int 1 dy = \int (1 - e^x) dx$$

$\Rightarrow \quad y = x - e^x + c$

28. (a) The equation of family of rectangular hyperbola is $xy = c^2$.

On differentiating w.r.t. x, we get

$$y + x\frac{dy}{dx} = 0$$

Thus, the order and degree of differential equation are 1 and 1 respectively.

29. (d) Given equation is $x\,dy = y\,dx$

By separating the variable we get

$$\frac{dy}{y} = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$\Rightarrow \log y = \log x + \log c$

$\Rightarrow y = cx$

It represents a family of straight lines.

30. (d) Given differential equation is $x\,dy - y\,dx = xy^2\,dx$

Which can be rewritten as $\dfrac{x\,dy - y\,dx}{y^2} = x\,dx$

$$\Rightarrow -d\left(\frac{x}{y}\right) = x\,dx$$

On integrating both sides, we get

$$\int -d\left(\frac{x}{y}\right) = \int x\,dx$$

$$-\frac{x}{y} = \frac{-c}{2} + \left(\frac{-c}{2}\right) \qquad \left(\because \frac{-c}{2} \text{ is constant}\right)$$

$$\Rightarrow \frac{c}{2} = \frac{x^2}{2} + \frac{x}{y} \Rightarrow \frac{c}{2} = \frac{x^2 y + 2x}{2y}$$

$$\Rightarrow c = \frac{x^2 y}{y} + \frac{2x}{y} \Rightarrow c = x^2 + \frac{2x}{y}$$

$\Rightarrow x^2 + 2xy^{-1} = c$

31. (c) Given equation is

$xy = ae^x + be^{-x}$ .... (1)

Differentiate both side, w.r.t. 'x'

$$x\frac{dy}{dx} + y = ae^x - be^{-x}$$

Again differentiate both side w.r.t 'x'

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = ae^x + be^{-x}$$

$$\Rightarrow x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = xy \text{ (from (1))}$$

Hence, this is the differential equation of second order and first degree.

32. (b) $3e^x \tan y\,dx + \left(1 + e^x\right)\sec^2 y\,dy = 0$

By separating the variables, we get

$$3e^x dx = \frac{-\left(1 + e^x\right)\sec^2 y}{\tan y} dy$$

$$\Rightarrow \quad \frac{3e^x}{1 + e^x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrate on both sides,

$$\Rightarrow \int \frac{3e^x}{1+e^x}\,dx + \int \frac{\sec^2 y}{\tan y}\,dy = 0$$

$$\Rightarrow 3\log\left(1+e^x\right) + \log\tan y = \log c$$

$$\Rightarrow \log\left(1+e^x\right)^3 \tan y = \log c$$

$(\because \log m + \log n = \log mn)$

$$\Rightarrow \left(1+e^x\right)^3 \tan y = c$$

33. (c) Given curve is $y^2 = 4a(x-a)$ ...(i)
On differentiating w.r.t. x, we get
$2yy' = 4a$

$$\Rightarrow a = \frac{yy'}{2}$$

On putting the value of a in Eq. (i), we get

$$y^2 = 4\left(\frac{yy'}{2}\right)\left(x - \frac{yy'}{2}\right) = yy'(2x - yy')$$

$$\Rightarrow yy'(yy' - 2x) + y^2 = 0$$

34. (b) Given differential equation is $\frac{d^2y}{dx^2} - \sqrt{1+\left(\frac{dy}{dx}\right)^3} = 0$

$$\Rightarrow \frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^3}$$

On squaring both the sides,

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^3$$

Since, degree of the differential equation is the power of highest order derivative.
Therefore from above it is clear that degree of equation is 2.

35. (b) Given $N(t) = ce^{kt}$
Diff. both side w.r.t. 't'

$$\therefore \frac{dN(t)}{dt} = \frac{d}{dt}ce^{kt} = k\left(ce^{kt}\right)$$

$$= k\left[N(t)\right] \quad \text{(by Defn. of N(}t\text{))}$$

But $\frac{dN(t)}{dt} = \alpha\, N(t)$ (given)

$$\Rightarrow \alpha = k$$

36. (d) Given differential equation is $a\left(x\frac{dy}{dx} + 2y\right) = xy\frac{dy}{dx}$

$$\Rightarrow ax\frac{dy}{dx} - xy\frac{dy}{dx} = -2ay$$

$$\Rightarrow (xy - ax)\frac{dy}{dx} = 2ay$$

$$\Rightarrow x(y-a)\frac{dy}{dx} = 2ay$$

$$\Rightarrow x(y-a)dy = 2ay\,dx$$

$$\Rightarrow \frac{(y-a)}{y}dy = \frac{2a}{x}dx$$

$$\Rightarrow \left(1 - \frac{a}{y}\right)dy = \frac{2a}{x}dx$$

$$dy - \frac{a}{y}dy = \frac{2a}{x}dx$$

Integrate on both side

$$\int dy - a\int \frac{1}{y}\,dy = 2a\int \frac{1}{x}\,dx$$

$$y - a\log y = 2a\log x + \log c$$

$$\Rightarrow y = a\log x^2 yc$$

$$\Rightarrow x^2 y = ke^{y/a} \quad (\because c = k = \text{constant})$$

37. (b) The given differential equation is

$$\left(1+\frac{dy}{dx}\right)^4 = \left(\frac{d^2y}{dx^2}\right)^2$$

From above it is clear that degree of given differential equation is 2.
Because degree is the power of highest order derivative.

38. (a) The given differential equation is

$$(1+e^x)y\,dy = e^x dx$$

By separating the variable, we get

$$y dy = \frac{e^x}{1+e^x}dx$$

Integrating on both the sides,

$$\Rightarrow \int y dy = \int\left(\frac{e^x}{1+e^x}\right)dx$$

$$\Rightarrow \frac{y^2}{2} = \log(1+e^x) + \log c$$

$$\Rightarrow y^2 = 2\log[c(1+e^x)]$$

$(\because \log m + \log n = \log mn)$

$$\Rightarrow y^2 = \log[c^2(1+e^x)^2]$$

39. (d) The equation of family of circles having centres at the origin is
$x^2 + y^2 = r^2$
where 'r' is the radius.
Differentiate both side w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$2xdx + 2y\,dy = 0$

$\Rightarrow \quad xdx + ydy = 0.$

which is required differential equation.

40. (a) Given differential equation is

$$x\frac{dy}{dx} = y$$

By separating the variables, we get

$$\frac{dy}{y} = \frac{dx}{x}$$

Intergrate both the sides, we get

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$\Rightarrow \quad \log y = \log x + \log c$

$\Rightarrow \quad y = xc$

which is a family of straight lines through the origin.

41. (b) Given differential equation is $y\frac{dy}{dx} + x = k$

$\Rightarrow \quad y\frac{dy}{dx} = k - x$

$\Rightarrow \quad y\,dy = (k - x)\,dx$

Integrate on both side, we get

$$\int ydy = \int (k - x)dx$$

$\Rightarrow \quad \frac{y^2}{2} = kx - \frac{x^2}{2} + c$

$\Rightarrow \quad x^2 + y^2 - 2\,kx - c = 0$

Which represents a family of circles whose centre lies on the x-axis.

42. (a) Let the equation of parabola is

$y^2 = 4ax$ ...(i)

On differentiating w.r.t.x, we get

$2yy' = 4a$

$\Rightarrow \quad \frac{1}{2}yy' = a$

put the value of 'a' in equation (i), we get

$$y^2 = \frac{4}{2}yy'\,x$$

$\Rightarrow \quad y = 2xy'$

43. (a) The differential equation is

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$\Rightarrow \quad \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

$\Rightarrow \quad \frac{dy}{\sqrt{1-y^2}} = \frac{-dx}{\sqrt{1-x^2}}$

$\Rightarrow \quad \int \frac{1}{\sqrt{1-y^2}}dy + \int \frac{1}{\sqrt{1-x^2}}dx = 0$

$\Rightarrow \sin^{-1} y + \sin^{-1} x = C$

44. (a) The general equation of all parabolas where axes are parallel to Y-axis, is

$y = Ax^2 + Bx + C$ ... (i)

where A, B and C are arbitrary constants.

On differentiating eq. (i) w.r.t. x, we get

$$\frac{dy}{dx} = 2Ax + B \quad \text{... (ii)}$$

On differentiating eq. (ii) w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 2A \quad \text{... (iii)}$$

On differentiating eq. (iii) w.r.t. x, we get

$$\frac{d^3y}{dx^3} = 0$$

45. (c) Given differential equation is

$$\frac{dy}{dx} = \frac{ax+3}{2y+f}$$

By separating the variable we get

$(2y + f)\,dy = (ax + 3)\,dx$

Integrate on both side,

$$\int (2y + f)dy = \int (ax + 3)dx$$

$\Rightarrow \quad y^2 + fy = \frac{ax^2}{2} + 3x$

This equation represents a circle, if

$$-1 = \frac{a}{2} \Rightarrow a = -2$$

46. (b) The given differential equation can be rewritten as

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$$

$\therefore$ Degree of differential equation is 2.

($\because$ Degree is the power of the highest order derivative)

47. (b) Given differential equation is

$$\frac{ydy}{dx} + x = a$$

$\Rightarrow \quad ydy + xdx = adx$

Integrate on both sides, we get

$$\int y\,dy + \int x\,dx = \int a\,dx$$

$\Rightarrow \quad \frac{y^2}{2} + \frac{x^2}{2} = ax + c$

where c is a constant of integration.

$\Rightarrow \quad y^2 + x^2 - 2ax = c$

This represents a circle whose centre is on the X-axis.

48. (b) Degree of a differential equation is the power to which the highest derivative is raised when it is expressed as polynomial of derivatives.
Given equation is

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\left(\frac{d^2y}{dx^2}\right) + 5\left(\frac{dy}{dx}\right) + 4 = 0$$

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^{2/3} = 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4$$

Cube on both side,

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left[3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right]^3$$

Hence, degree = 2

49. (d) Given differential equation is
$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$
$\Rightarrow \sin x \cos y\, dx = -\cos x \sin y\, dy$

$$\Rightarrow \frac{\sin x}{\cos x} dx = -\frac{\sin y}{\cos y} dy$$

Integrate on both side

$$\int \frac{\sin x}{\cos x} dx = -\int \frac{\sin y}{\cos y} dy$$

$\Rightarrow -\log(\cos x) = \log(\cos y) + \log c$
where $\log c$ is constant of integration.
$\Rightarrow -\log c = \log(\cos y) + \log(\cos x)$

$$\frac{1}{c} = \cos y \cos x \quad \ldots (1)$$

Since, this curve passing through $\left(0, \frac{\pi}{3}\right)$

$\therefore$ it satisfies equation (1)

So, $\frac{1}{c} = \cos\frac{\pi}{3} . \cos 0$

$\frac{1}{c} = \frac{1}{2} \times 1 \Rightarrow c = 2$

Hence, required equation of curve is $\cos x \cos y = \frac{1}{2}$

50. (a) $\frac{dy}{dx} + \frac{y}{x} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

$$\Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0 \Rightarrow \int \frac{dy}{y} + \int \frac{dx}{x} = 0$$

$\Rightarrow \log y + \log x = \log c$
$\Rightarrow \log xy = \log c$
$\Rightarrow xy = c$

51. (b) Given differential equation is

$$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1}$$

Multiply by $\frac{dy}{dx}$

$$y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 1$$

Since power of highest order derivative is 2.
$\therefore$ degree = 2

52. (a) Differential equation given in option (a) is not linear because differential coefficient $\frac{dy}{dx}$ has exponent 2.

53. (d) Given differential equation

$$\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right) - \frac{dy}{dx} + y = 0$$

Since exponent of highest order derivative is 1 therefore degree = 1

54. (b) Degree of differential equation is always a positive integer.

$\therefore \left(2, \frac{3}{2}\right)$ can not be the feasible.

55. (a) Let $y = a\sin(\lambda x + \alpha)$

$$\Rightarrow \frac{dy}{dx} = \lambda\, a \cos(\lambda\, x + \alpha)$$

Again differentiating on both side we get

$$\frac{d^2y}{dx^2} = -\lambda^2\, a \sin(x + \alpha)$$

$$\Rightarrow \frac{d^2y}{dx^2} + \lambda^2\, y = 0$$ Required equation.

56. (c) Given diff. equation is

$$y\frac{dy}{dx} + x = a$$

$$\Rightarrow y\frac{dy}{dx} = a - x$$

$\Rightarrow y\,dy = (a - x)\,dx$

$$\int y\,dy = \int (a - x)\,dx$$

$$\Rightarrow \frac{y^2}{2} = ax - \frac{x^2}{2} + k$$

$\Rightarrow x^2 + y^2 - 2ax = 2k$
Which represents a set of circles.

57. (c) Given differential equation is

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$$

From the options only option (c) does not satisfy the given diff equation.
Hence, y = x is not a solution of given diff equation.

58. (c) Given differential equation is $x^2\,dy + y^2\,dx = 0$

$\Rightarrow\ x^2\,dy = -y^2\,dx$

$$\Rightarrow\ \frac{dy}{y^2} + \frac{dx}{x^2} = 0$$

$$\Rightarrow\ \int y^{-2}\,dy + \int x^{-2}\,dx = 0$$

$$\Rightarrow\ \frac{y^{-2+1}}{-2+1} + \frac{x^{-2+1}}{-2+1} = a$$ where 'a' is a constant of integration

$$\Rightarrow\ -\frac{1}{y} - \frac{1}{x} = a$$

$-(x + y) = axy\ \Rightarrow\ c(x + y) = xy$

where $c = -\frac{1}{a}$ is a constant of integration.

59. (d) Given diff equation can be written as

$$\frac{e^x}{1-e^x}\,dx = -\frac{\sec^2 y}{\tan y}\,dy$$

On integrating both the sides, we get

$$\int \frac{e^x}{1-e^x}\,dx = -\int \frac{\sec^2 y}{\tan y}\,dy$$

$\Rightarrow\ -\log(1 - e^x) = -\log(\tan y) + \log c$

$\Rightarrow\ \log(\tan y) = \log c + \log(1 - e^x)$

$\Rightarrow\ \log(\tan y) = \log[c(1 - e^x)]$

$\Rightarrow\ \tan y = c(1 - e^x)$

Where 'c' is the constant of integration.

60. (c) Consider differential equation

$$\left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 5 = 0$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}} = 5\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 5$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^3 = \left(5\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 5\right)^5$$

So, highest order derivative = 4, degree = 3

61. (a) Given differential equation is

$$x\frac{dy}{dx} + y = 0$$

$\Rightarrow\ x\,dy + y\,dx = 0$

$\Rightarrow\ x\,dy = -y\,dx$

$$\Rightarrow\ \frac{dy}{y} = -\frac{dx}{x}$$

On integrating both side we get

$\ell n\, y = -\ell n\, x + \ell n c$

$$\Rightarrow \left(y = \frac{c}{x}\right)$$

62. (b) Let $\ell n\left(\frac{dy}{dx}\right) + x = 0 \Rightarrow \ell n\left(\frac{dy}{dx}\right) = -x$

$$\Rightarrow \frac{dy}{dx} = e^{-x}$$

Integrate both the side,

$y = -e^{-x} + c$

63. (b) Given curve is y = sin x

Differentiate both the sides w.r.t 'x'.

$$\Rightarrow \frac{dy}{dx} = \cos x \Rightarrow \frac{d^2y}{dx^2} = -\sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y$$

$$\therefore\ \ y + \frac{d^2y}{dx^2} = 0$$

64. (a) Since order of the highest derivative in the given diff. equation is 1 and exponent of the derivative is also 1 therefore degree and order is (1, 1).

65. (a) Highest order derivative, present in the differential equations is $\left(\frac{dy}{dx}\right)$, therefore its order is one.

66. (a) y = 2 cos x + 3 sin x

$$\frac{dy}{dx} = -2\sin x + 3\cos x$$

$$\frac{d^2y}{dx} = -2\cos x - 3\sin x$$

$= -(2\cos x + 3\sin x)$

$= -y$

$$\frac{d^2y}{dx^2} + y = 0$$

67. (c) $x^2 + y^2 = r^2$ [equation of circle]

Differentiating both sides w.r.t. x.

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

68. (a) $\frac{dy}{dx} = |x|$

$$\frac{dy}{dx} = x \text{ for } x \ge 0\,;\ \frac{dy}{dx} = -x \text{ for } x < 0$$

$\int dy = \int x\,dx$

$y = \frac{x^2}{2} + C_1$ ...(i); $\int dy = -1\,x\,dx$

$y = -\frac{x^2}{2} + C_1$ ...(ii)

From (i) and (ii)

$y = \frac{x|x|}{2} + C$

69. (a) $\frac{dy}{dx} + 2y = 1 \Rightarrow \frac{dy}{dx} = 1 - 2y$

$\int \frac{dy}{1-2y} = \int dx$

$-\frac{1}{2}\log|1-2y| = x + C$

at $x = 0, y = 0$

$-\frac{1}{2}\log 1 = 0 + C \Rightarrow C = 0$

$1 - 2y = e^{-2x}$

$y = \frac{1-e^{-2x}}{2}$

70. (*) Differential equation $x\,dy - y\,dx = y^2$

$= (y\,dx - x\,dy) = y^2$

$\therefore d\left(\frac{x}{y}\right) = 0$

$\frac{x}{y} = C \therefore x = Cy$

For (71 - 73)

$(x^2 + x + 1)\,dy + (y^2 + y + 1)\,dx = 0$

$(x^2 + x + 1)\,dy = -(y^2 + y + 1)\,dx$

$$\frac{dx}{(1+x+x^2)} + \frac{dy}{(1+y+y^2)} = 0$$

$$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \int \frac{dy}{\left(y+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 0$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}(C_1)$$

$$\Rightarrow \tan^{-1}\left\{\frac{\left(\frac{2x+1}{\sqrt{3}}\right) + \left(\frac{2y+1}{\sqrt{3}}\right)}{1 - \left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2y+1}{\sqrt{3}}\right)}\right\} = \tan^{-1} C_1$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \frac{\sqrt{3}[(2x+1)+(2y+1)]}{3-(2x+1)(2y+1)} = C_1$$

$$\Rightarrow \frac{2\sqrt{3}(x+y+1)}{-4xy-2y-2x+2} = C_1$$

$$\Rightarrow 2\sqrt{3}(x+y+1) = C_1(2-2x-2y-4xy)$$

$$\Rightarrow 2\sqrt{3}(x+y+1) = 2C_1(1-x-y-2xy)$$

$$\Rightarrow (x+y+1) = \frac{C_1}{\sqrt{3}}(1-x-y-2xy)$$

$(x+y+1) = A(1+Bx+Cy+Dxy)$

71. (a) $B = -1$

72. (b) $C = -1$

73. (c) $D = -2$

74. (d) Particular solution of D. ε. of third order have three arbitary constant.

75. (c) **Statement 1:** Differential equation is not a polynomial equation in its derivatives. So, its degree is not defined.
Statement 2 : The highest order derivative in the given polynomial is 2.

76. (c) Consider the given differential equation,

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} = \left(\frac{d^2y}{dx^2}\right)^2 \quad ...(i)$$

In order to find degree, differential equation should be free from fractional indices.

Now, squaring eqn (i) both sides, we get

$$\left(\frac{d^3y}{dx^3}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^4$$

Since, the power of highest order derivative is 3, therefore degree = 3.

77. (c) Consider the given differential equation

$$\ln\left(\frac{dy}{dx}\right) + x = 0$$

$$\Rightarrow \ln\left(\frac{dy}{dx}\right) = -x$$

$$\Rightarrow \frac{dy}{dx} = e^{-x}$$

on separating the variables, we get

$dy = e^{-x}\,dx$,

on integrating both sides, we get

$$\int dy = \int e^{-x}dx$$

$$\Rightarrow y = \frac{e^{-x}}{-1} + C = -e^{-x} + C$$

$$\Rightarrow y + e^{-x} = C$$

78. (b) $y = \frac{2}{3C}(Cx-1)^{\frac{3}{2}} + B$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2}{3C}.\frac{3}{2}(Cx-1)^{\frac{1}{2}}.C + 0 = (Cx-1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = (Cx-1)^{\frac{1}{2}}$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^2 = Cx - 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + 1 = Cx \quad \text{...(i)}$$

Now, on differentiating w.r.t. x, we get

$$2\left(\frac{dy}{dx}\right).\frac{d^2y}{dx^2} = C$$

From eq. (i)

$$\left(\frac{dy}{dx}\right)^2 + 1 = 2x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$$

79. (b) $\frac{y\,dx - x\,dy}{y^2} = 0$

$\therefore$ $y\,dx - x\,dy = 0$

$\therefore$ $\frac{dy}{y} = \frac{dx}{x}$

Now integrating both sides,

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$\Rightarrow$ $\log y = \log x + \log c$

$\Rightarrow$ $\log y = \log c\,x$

$\therefore$ $y = cx$

$\therefore$ Option (b) is correct.

80. (a) $\sin\left(\frac{dy}{dx}\right) - a = 0$

$$\sin\left(\frac{dy}{dx}\right) = a$$

$\Rightarrow$ $\frac{dy}{dx} = \sin^{-1} a$

$dy = \sin^{-1} a\,dx$

Now integrating both sides,

$$\int dy = \int \sin^{-1} a\,dx$$

$y = x\sin^{-1} a + c$

$\therefore$ Option (a) is correct.

81. (c) $\frac{dx}{dy} + \frac{x}{y} - y^2 = 0$

$$\frac{dx}{dy} + \frac{x}{y} = y^2$$

This is a linear differential equation of the form

$$\frac{dx}{dy} + P_1 x = Q_1$$

Here, $P = \frac{1}{y}$ and $Q = y^2$

$\therefore$ I.F. $= e^{\int P\,dy} = e^{\int \frac{1}{y}dy} = e^{\log y} = y$

So, required solution is

$x \cdot y = \int y^2 \cdot y\,dy + c$

$xy = \int y^3\,dy + c$

$xy = \frac{y^4}{4} + c$

$4xy = y^4 + c$

$\therefore$ Option (c) is correct.

82. (c) **Statement 1:**

The general solution of $\frac{dy}{dx} = f(x) + x$

Now integrating on both side

$$\int \frac{dy}{dx} = \int [f(x) + x]dx$$

$\therefore$ $y = f(x) + C$

$\therefore$ Statement 1 is correct.

Statement 2:

$$\frac{dy}{dx} = f(x) + c$$

Squaring both sides,

$$\left(\frac{dy}{dx}\right)^2 = (f(x) + c)^2$$

$$\left(\frac{dy}{dx}\right)^2 = \left[f(x)^2 + 2c\,f(x) + c^2\right]$$

Hence, the differential equation is of order –1 and degree –2.

$\therefore$ both Statements 1 and 2 are correct.

$\therefore$ Option (c) is correct.

83. (d) $\because \frac{dy}{dx} - x = \left(y - x\frac{dy}{dx}\right)^{-4}$

$$\Rightarrow \left(\frac{dy}{dx} - x\right)\left(y - x.\frac{dy}{dx}\right)^4 = 1$$

$\therefore$ Order of the above differential equation = 1 & degree = 5

84. (c) $\because \frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$

$\frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}.dx$

$= \int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2}.dx$ [integrating b/s]

$= \sin^{-1}\left(\frac{y}{1}\right) = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right) + c$

$= 2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + c$

85. (b) Eq. of family of circles passing through the origin & having centres on the x-axis is :

$x^2 + y^2 + 2gx = 0$ ... (1)

$2x + 2y.\frac{dy}{dx} + 2g = 0$ [on differentiating]

$\Rightarrow g = -\left[x + y\frac{dy}{dx}\right]$

Putting the value of (g) in eq. (1) we get;

$2xy\frac{dy}{dx} = y^2 - x^2$

86. (a) The eq. of parabolas having vertex at (0, 0) & focus at (a, 0), where (a > 0) is :

$y^2 = 4ax$ ... (1)

$2y.\frac{dy}{dx} = 4a$ [on differentiating]

On putting the value of (4a) in eq. (1) we get,

$2x.\frac{dy}{dx} - y = 0$

in order =1 & degree = 1.

87. (d) Given :

Solution of differential equation is

$y = cx + c^2 - 3c^{\frac{3}{2}} + 2$ ...(1)

To find order and degree of differential equation, we will find differential equation first.

Now differentiating equation (1) w.r.t. x and putting value of c to remove it, we get

$\frac{dy}{dx} = c$

$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 - 3\left(\frac{dy}{dx}\right)^{\frac{3}{2}} + 2$

$\Rightarrow (y-2)^2 + \left(\frac{dy}{dx}\right)^4 + (2x-9)\left(\frac{dy}{dx}\right)^3$

$+(x^2 - 2y + 4)\left(\frac{dy}{dx}\right)^2 + (-2xy + 4x)\frac{dy}{dx} = 0$

Hence order of differential equation is 1 and degree is 4.

88. (c)

89. (a) $e^{y\left(\sqrt{1-x^2} + x\sqrt{1-y^2}\right) - x = c}$

$\Rightarrow y\sqrt{1-x^2} + x\sqrt{1-y^2} - x = \log c$

$\Rightarrow \frac{dy}{dx}\sqrt{1-x^2} + y.\frac{1}{2\sqrt{1-x^2}}(-2x) + \sqrt{1-y^2}$

$+ x.\frac{1}{2\sqrt{1-y^2}}(-2y).\frac{dy}{dx} - 1 = 0$

$\Rightarrow \frac{dy}{dx}\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}.\frac{dy}{dx} = 1$

Degree = 1, order =1

90. (a) Given, $xdy = ydx + y^2dy$

$\Rightarrow 1 = \frac{y}{x}\frac{dx}{dy} + \frac{y^2}{x}$

$\Rightarrow \frac{dx}{dy} + y = \frac{x}{y}$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = -y$ ...(1)

$P = -\frac{1}{y}, Q = -y$

$IF = e^{\int Pdy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$

Now, Solution of d.E.

$x(I.F.) = \int (Q.I.F.)\, dy$

$\frac{x}{y} = \int \frac{1}{y}(-y)dy + C$

$\Rightarrow \frac{x}{y} = \int -1\, dy + C$

$\Rightarrow \frac{x}{y} = -y + C$

$y(1) = 1$

$\frac{1}{1} = -1 + C \Rightarrow C = 2$

$\Rightarrow \frac{x}{y} = -y + 2 \Rightarrow x = -y^2 + 2y$

$\Rightarrow y(-3) \Rightarrow -3 = -y^2 + 2y$

$\Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow y = \dfrac{+2 \pm \sqrt{4+12}}{2} = \dfrac{2 \pm 4}{2}$

$\Rightarrow y = 3, -1$

Since $y > 0$ so $y = 3$.

91. (b) $\dfrac{dx}{dy} + \int y \cdot dx = x^3 \Rightarrow \int y \cdot dx = x^3 - \dfrac{dx}{dy}$

$\Rightarrow 1 + \dfrac{dy}{dx}\left(\int y \cdot dx\right) = x^3 \cdot \dfrac{dy}{dx}$

Differentiate both sides w.e.t. x

$\Rightarrow 0 + \dfrac{dy}{dx}(y) + \left(\int y \cdot dx\right)\left(\dfrac{d^2y}{dx^2}\right) = x^3 \cdot \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}(2x^2)$

$\Rightarrow y \cdot \dfrac{dy}{dx} + \dfrac{d^2y}{dx^2}\left[x^3 - \dfrac{dx}{dy}\right] = x^3 \cdot \dfrac{d^2y}{dx^2} + 2x^2 \dfrac{dy}{dx}$

$\Rightarrow y\dfrac{dy}{dx} + x^3 \dfrac{d^2y}{dx^2} - \left(\dfrac{dx}{dy}\right)\left(\dfrac{d^2y}{dx^2}\right) = x^3 \dfrac{d^2y}{dx^2} + 2x^2 \dfrac{dy}{dx}$

$\Rightarrow y\dfrac{dy}{dx} - \dfrac{dx}{dy} \cdot \dfrac{d^2y}{dx^2} = 2x^2 \bullet \dfrac{dy}{dx}$

Multiplying both side by $\dfrac{dy}{dx}$

$y\left(\dfrac{dy}{dx}\right)^2 - \dfrac{d^2y}{dx^2} = 2x^2\left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow \dfrac{d^2y}{dx^2} + (2x^2 - y)\left(\dfrac{dy}{dx}\right)^2 = 0$

Order = 2, degree = 1.

92. (c) $y = mx + c$ (Equation of straight line)

$\dfrac{dy}{dx} = m$ and $mx - y + c = 0$ is at unit distance from origin.

$\therefore \dfrac{|m(0) - (0) + c|}{\sqrt{m^2 + (-1)^2}} = 1 \Rightarrow c = \sqrt{1+m^2}$

Now :

$\left[y - x\dfrac{dy}{dx}\right]^2 = [mx + c - xm]^2 \Rightarrow c^2 = 1 + m^2$

also,

$\left[y + x\dfrac{dy}{dx}\right]^2 = [mx + c + mx]^2 = [2mx + \sqrt{1+m^2}]^2$

also, $1 - \left(\dfrac{dy}{dx}\right)^2 = 1 - m^2$ and $1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + m^2$

$\Rightarrow \left[y - x\dfrac{dy}{dx}\right]^2 = 1 + \left(\dfrac{dy}{dx}\right)^2$

93. (c) Let $y = f(x) \Rightarrow \dfrac{dy}{dx} = f^1(x) \Rightarrow \dfrac{dx}{dy} = \dfrac{1}{f'(x)}$

$\dfrac{d^2x}{dy^2} = \dfrac{\frac{-d}{dy} f'(x)}{\left(f'(x)\right)^2}$ ...(i)

$\dfrac{\frac{-d}{dy} f'(x)}{\left(f'(x)\right)^2} = \dfrac{\frac{-d}{dy}\left(\frac{dy}{dx}\right) \cdot \frac{dx}{dx}}{\left(f'(x)\right)^2} = \dfrac{\frac{-d}{dx}\left(f'(x)\right) \cdot \frac{dx}{dy}}{\left(\frac{dy}{dx}\right)^2}$

$\Rightarrow \dfrac{-f'(x) \cdot \frac{dx}{dy}}{\left(\frac{dy}{dx}\right)^2} = \dfrac{-\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^3} = \dfrac{-d^2y}{dx^2}\left(\dfrac{dy}{dx}\right)^{-3}$

94. (a) $x\,dy = y\,dx + y^2\,dy$

$\Rightarrow x \cdot \dfrac{dy}{dx} = y + y^2 \cdot \dfrac{dy}{dx} \quad \Rightarrow \dfrac{y - x \cdot \frac{dy}{dx}}{y^2} = \dfrac{-dy}{dx}$

Integrating both sides

$\dfrac{x}{y} = -y + c$

Given, $x = 1, y = 1$

$\Rightarrow \dfrac{1}{1} = -1 + c \quad \Rightarrow c = 2$

$\therefore \dfrac{x}{y} + y = 2$

$\Rightarrow x + y^2 = 2y$

$\Rightarrow -3 + y^2 - 2y = 0$

$\Rightarrow y^2 - 2y - 3 = 0$

$y = 3, -1$

$\therefore y = 3$

95. (d) $y = x\left(\dfrac{dy}{dx}\right)^2 + \left(\dfrac{dx}{dy}\right)^2$

$= x\left(\dfrac{dy}{dx}\right)^2 + \left(\dfrac{dy}{dx}\right)^{-2}$

$\Rightarrow y\left(\dfrac{dy}{dx}\right)^2 = x \cdot \left(\dfrac{dy}{dx}\right)^4 + 1$

So, degree = 4, order = 1

96. (a) $y^2 - 2ay + x^2 = a^2$

$\Rightarrow y^2 - 2(y)(a) + a^2 + x^2 = a^2 + a^2$

$\Rightarrow (y - a)^2 + x^2 = 2a^2$ ...(i)

Dift w.r.t x

$2(y - a) \cdot y^1 + 2x = 0$

$\Rightarrow (y-a).p = -x \qquad \left(\text{Let } \frac{dy}{dx} = y^1 = P\right)$

$\Rightarrow y - a = \frac{-x}{p}$ ...(ii)

$\Rightarrow a = y + \frac{x}{p}$ ...(iii)

$(i) \Rightarrow \left(\frac{-x}{p}\right)^2 + x^2 = 2\left(y + \frac{x}{p}\right)^2$ (from (ii) (iii))

$= 2\left(\frac{(py+x)^2}{p2}\right)$

$\Rightarrow \frac{x^2 + p^2x^2}{\cancel{p^2}} = \frac{2\left(p^2y^2 + x^2 + 2pxy\right)}{\cancel{p^2}}$

$\Rightarrow x^2 + p^2x^2 - 2p^2y^2 - 2x^2 - 4pxy = 0$
$\Rightarrow (x^2 - 2y^2)p^2 - 4pxy - x^2 = 0$

97. (c) $ydx - (x + 2y^2)dy = 0$
$\Rightarrow ydx - xdy - 2y^2dy = 0$

$\Rightarrow y - x\cdot\frac{dy}{dx} = 2y^2.\frac{dy}{dx}$

$\Rightarrow \frac{y - x.\frac{dy}{dx}}{y^2} = 2.\frac{dy}{dx}$

Integrating

$\Rightarrow \frac{x}{y} = 2y + c$

$\Rightarrow x = 2y^2 + cy$

98. (a) $\ln\left(\frac{dy}{dx}\right) - a = 0$

$\Rightarrow \ln\left(\frac{dy}{dx}\right) = a \qquad \Rightarrow \frac{dy}{dx} = e^a$

$\Rightarrow \int dy = \int e^a.dx \qquad \Rightarrow y = e^a . x + c$

99. (b) $\frac{dy}{dx} = \frac{ax+h}{by+k}$
$\Rightarrow (by + k)dy = (ax + h)dx$

Intergrating, we get $\int(by+k)dy = \int(ax+h)dx$

$\Rightarrow \frac{by^2}{2} + ky = \frac{ax^2}{2} + hx$
$\Rightarrow by^2 + 2ky = ax^2 + 2hx$
$\Rightarrow ax^2 - by^2 + 2hx - 2ky = 0$

This represents circle only when $a = -b \neq 0$.

100. (b) Degree of a differential equation is power of the highest order derivative.
In the given differential equation,

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2\left[\frac{d^2y}{dx^2}\right]^2$$

order = 2 and degree = 2.

101. (d) $y = A[\sin(x + c) + \cos(x + c)]$
$y' = A[\cos(x + c) - \sin(x + c)]$
$y'' = -A[\sin(x + c) + \cos(x + c)]$
$= -y$
$\therefore y'' + y = 0$

102. (d) $\frac{dy}{dx} = \frac{y\phi'(x) - y^2}{\phi(x)} = \frac{y\phi'(x)}{\phi(x)} - \frac{y^2}{\phi(x)}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2\cdot\phi'(x)}{y\cdot\phi(x)} - \frac{y^2}{\phi(x)}$

$\Rightarrow \frac{1}{y^2}\left(\frac{dy}{dx}\right) = \frac{\phi'(x)}{y\cdot\phi(x)} - \frac{1}{\phi(x)}$

$\Rightarrow \frac{1}{y^2}\left(\frac{dy}{dx}\right) - \frac{1}{y}\cdot\frac{\phi'(x)}{\phi(x)} = \frac{-1}{\phi(x)}$

Let $t = \frac{-1}{y} \Rightarrow \frac{dt}{dx} = \frac{1}{y^2}\left(\frac{dy}{dx}\right)$

$\Rightarrow \frac{dt}{dx} + t\cdot\frac{\phi'(x)}{\phi(x)} = \frac{-1}{\phi(x)}$

$\text{I.F} = e^{\int\frac{\phi'(x)}{\phi(x)}\cdot dx} = e^{\log\phi(x)} = \phi(x)$

Solution of differential equation is

$t\cdot\phi(x) = \int\frac{-1}{\phi(x)}\times\phi(x)\cdot dx$

$\Rightarrow \frac{-1}{y}\phi(x) = -x - c \Rightarrow \frac{\phi(x)}{y} = x + c \Rightarrow y = \frac{\phi(x)}{x} + c.$

103. (b) $xdy - ydx = 0.$
$\Rightarrow xdy = ydx$

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

$\Rightarrow \log y = \log x + \log c$
$\Rightarrow \log y - \log x = \log c$

$\Rightarrow \log\left(\frac{y}{x}\right) = \log c$

$\Rightarrow \frac{y}{x} = c \Rightarrow y = cx.$

104. (a)

105. (*)

106. (a) $(1+2x)\,dy-(1-2y)\,dx=0 \Rightarrow (1+2x)\,dy=(1-2y)\,dx$

$$\Rightarrow \frac{dy}{1-2y}=\frac{dx}{1+2x}$$

Integrating both sides, $\int\frac{dy}{1-2y}=\int\frac{dx}{1+2x}$

$$\Rightarrow \frac{-1}{2}\log(1-2y)=\frac{1}{2}\log(1+2x)+\frac{1}{2}\log c$$

$\Rightarrow \log(1-2y)+\log(1+2x)=\log c$

$\Rightarrow (1+2x)(1-2y)=c$

$\Rightarrow 1+2x-2y-4xy=c$

$\Rightarrow 2x-2y-4xy=c$

$\Rightarrow x-y-2xy=c.$

107. (c) $\left(\frac{d^3y}{dx^3}\right)^2=y^4+\left(\frac{dy}{dx}\right)^5$

order = 3, degree = 2

108. (d) $y=p\cos ax+q\sin ax$

$\Rightarrow \frac{dy}{dx}=-p\,a\sin ax+qa\cos ax$

$\Rightarrow \frac{d^2y}{dx^2}=-p\,a^2\cos ax-qa^2\sin ax=-a^2y$

$\Rightarrow \frac{d^2y}{dx^2}+a^2y=0$

109. (b) $\frac{dy}{dx}=-x^2-\frac{1}{x^3}$

$\Rightarrow \int dy=\int\left(-x^2-\frac{1}{x^3}\right)dx$

$\Rightarrow y=-\frac{x^3}{3}+\frac{1}{2x^2}+c$

Putting (–1, –2), we get

$-2=\frac{1}{3}+\frac{1}{2}+c \Rightarrow C=-\frac{17}{6}$

$\therefore\ y=-\frac{x^3}{3}+\frac{1}{2x^2}-\frac{17}{6}$

$\Rightarrow 6x^2y=-2x^5+3-17x^2$

$\Rightarrow 6x^2y+17x^2+2x^5-3=0$

110. (d) Order = 4 ($\because$ No. of arbitrary constants = 4)

111. (d) $\frac{dy}{dx}=e^{ax+by}=e^{ax}.e^{by}$

$\Rightarrow e^{ax}\,dx-e^{-by}\,dy=0$

Integrating both sides,

$\frac{1}{a}e^{ax}+\frac{1}{b}e^{-by}=c$

112. (a) $u\frac{du}{dx}+v\frac{dv}{dx}=e^{ax}\sin bx[ae^{ax}\sin bx+be^{ax}\cos bx]$

$+\,e^{ax}\cos bx[ae^{ax}\cos bx-be^{ax}\sin bx]$

$=e^{2ax}[a\sin^2 bx+b\sin bx\cos bx+$

$a\cos^2 bx-b\sin bx\cos bx]$

$=ae^{2ax}$

113. (c) $y=\sin(\log x)$

$\Rightarrow \frac{dy}{dx}=\frac{\cos(\log x)}{x}$

$\Rightarrow x\left(\frac{dy}{dx}\right)=\cos(\log x)$

Again differentiating,

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-\frac{\sin\log x}{x}=\frac{-y}{x}$

$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

114. (c) Let $t=x+y \Rightarrow \frac{dt}{dy}=\frac{dx}{dy}+1$

So, $\frac{dt}{dy}-1=\frac{t+1}{t-1}$

$\Rightarrow \frac{dt}{dy}=\frac{t+1+t-1}{t-1}=\frac{2t}{t-1}$

$\Rightarrow \int\frac{t-1}{t}dt=2\int dy$

$\Rightarrow t-\log t=2y+C_1$

$\Rightarrow x+y-\log(x+y)-2y=C_1$

$\Rightarrow y-x+\log(x+y)=-C_1=C$

115. (a) $\frac{dy}{dx}=\cos(y-x)+1$ ...(1)

Let $y-x=t \Rightarrow \frac{dy}{dx}-1=\frac{dt}{dx}$

$\therefore (1) \Rightarrow \frac{dt}{dx}+1=\cos t+1$

$\Rightarrow \frac{dt}{dx}=\cos t$

$\Rightarrow \sec t\,.\,dt=1.\,dx$

$\Rightarrow \int\sec t\cdot dt=\int 1\cdot dx$

$\Rightarrow \log|\sec t+\tan t|=x+c$

$\Rightarrow -\log|\sec t-\tan t|=x+c$

$\Rightarrow \log|\sec t-\tan t|=-x-c$

$\Rightarrow \log(\sec(y-x)-\tan(y-x)=-x-c$

$\Rightarrow \sec(y-x)-\tan(y-x)=e^{-x}.\,e^{-c}$

$\Rightarrow e^x(\sec(y-x)-\tan(y-x))=c.$

116. (c) The equation of circle touching y-axis at origin is

$(x-\alpha)^2+(y-0)^2=\alpha^2$

$\Rightarrow x^2 + \alpha^2 - 2\alpha x + y^2 = \alpha^2$
$\Rightarrow x^2 + y^2 - 2\alpha x = 0$

$$\Rightarrow x + \frac{y^2}{x} - 2\alpha = 0$$

Differentiating, $1 + \dfrac{x.2y.\frac{dy}{dx} - y^2}{x^2} = 0$

$$\Rightarrow x^2 + 2xy.\frac{dy}{dx} - y^2 = 0$$

117. (c) The equation of circle touching y-axis at origin is
$(x - \alpha)^2 + (y - 0)^2 = \alpha^2$
$\Rightarrow x^2 + \alpha^2 - 2\alpha x + y^2 = \alpha^2$
$\Rightarrow x^2 + y^2 - 2\alpha x = 0$

$$\Rightarrow x + \frac{y^2}{x} - 2\alpha = 0$$

Differentiating, $1 + \dfrac{x \cdot 2y \cdot \frac{dy}{dx} - y^2}{x^2} = 0$

$$\Rightarrow x^2 + 2xy \cdot \frac{dy}{dx} - y^2 = 0$$

118. (c) $\dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 + 9y = x$

Order =2
Degree = 1

119. (a) $\dfrac{dy}{dx} + \dfrac{x}{y} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

$\Rightarrow y\,dy + x\,dx = 0$

$$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} + c = 0$$

$\Rightarrow y^2 + x^2 + 2c = 0$
$\Rightarrow x^2 + y^2 = c.$

Matrices & Determinants

20

1. $A_{(\alpha)} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}, A_{(\beta)} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$.

   Which one of the following is correct ?
   (a) $A_{(-\alpha)} A_{(-\beta)} = A_{(\alpha+\beta)}$
   (b) $A_{(-\alpha)} A_{(\beta)} = A_{(\beta-\alpha)}$
   (c) $A_{(\alpha)} + A_{(-\beta)} = A_{\{-(\beta-\alpha)\}}$
   (d) $A_{(\alpha)} + A_{(\beta)} = A_{(\alpha+\beta)}$ *[2006-I]*

2. If $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

   What is the maximum value of f (x) ?
   (a) 2 (b) 4
   (c) 6 (d) 8 *[2006-I]*

3. If the matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular, then what is one of the values of θ ?
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
   (c) π (d) 0 *[2006-I]*

4. For what values of k, does the system of linear equations
   $x + y + z = 2, 2x + y - z = 3, 3x + 2y + kz = 4$
   have a unique solution ?
   (a) $k = 0$ (b) $-1 < k < 1$
   (c) $-2 < k < 2$ (d) $k \neq 0$ *[2006-I]*

5. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$

   If AB = BA, then what is the value of x ?
   (a) – 1 (b) 0
   (c) 1 (d) Any real number *[2006-I]*

6. If a matrix B is obtained from a square matrix A by interchanging any two of its rows, then what is |A + B| equal to
   (a) 2 |A| (b) 2 |B|
   (c) 0 (d) |A| – |B| *[2006-I]*

7. Let $A = (a_{ij})_{n \times n}$ and adj $A = (\alpha_{ij})$

   If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 2 & 3 & -1 \end{bmatrix}$, what is the value of α_{23} ?
   (a) 1 (b) – 1
   (c) 8 (d) – 8 *[2006-I]*

8. Let A and B be two invertible square matrices each of order n. What is adj (AB) equal to ?
   (a) (adj A) (adj B) (b) (adj A) + (adj B)
   (c) (adj A) – (adj B) (d) (adj B) (adj A) *[2006-I]*

9. M is a matrix with real entries given by

   $$M = \begin{bmatrix} 4 & k & 0 \\ 6 & 3 & 0 \\ 2 & t & k \end{bmatrix}$$

   Which of the following conditions guarantee the invertibility of M ?
   1. $k \neq 2$ 2. $k \neq 0$
   3. $t \neq 0$ 4. $t \neq 1$

   Select the correct answer using the code given below :
   (a) 1 and 2 (b) 2 and 3
   (c) 1 and 4 (d) 3 and 4 *[2006-I]*

10. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ be a square matrix of order 3. Then for any positive integer n, what is A^n equal to ?
    (a) A (b) $3^n A$
    (c) $(3^{n-1}) A$ (d) 3A *[2006-I]*

11. Let A and B be two matrices such that AB is defined. If AB = 0, then which one of the following can be definitely concluded ?
    (a) A = 0 or B = 0
    (b) A = 0 and B = 0
    (c) A and B are non-zero square matrices
    (d) A and B cannot both be non-singular *[2006-I]*

12. If A is a matrix of order p × q and B is a matrix of order s × t, under which one of the following conditions does AB exist ?
    (a) p = t (b) p = s
    (c) q = t (d) q = s *[2006-II]*

13. If A is a square matrix such that $A - A^T = 0$, then which one of the following is correct ?
(a) A must be a null matrix
(b) A must be a unit matrix
(c) A must be a scalar matrix
(d) None of the above [2006-II]

14. What is the largest value of a third order determinant whose elements are 0 or 1 ?
(a) 0 (b) 1
(c) 2 (d) 3 [2006-II]

15. What is the inverse of $A = \begin{bmatrix} 1+i & 1+i \\ -1+i & 1-i \end{bmatrix}$?
(a) $\frac{1}{4}\begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$ (b) $\frac{1}{4}\begin{bmatrix} 1+i & -1+i \\ 1+i & -1-i \end{bmatrix}$
(c) $\frac{1}{4}\begin{bmatrix} 1+i & 1-i \\ -1-i & 1+i \end{bmatrix}$ (d) $\frac{1}{4}\begin{bmatrix} 1+i & 1-i \\ -1-i & -1+i \end{bmatrix}$
[2006-II]

16. In respect of the equation
$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ c-5 \end{bmatrix}$$
correctly match List I with List II and select the correct answer using the code given below the lists:

List I (Value of c)	List II (Nature of the Equation)
A. 5	1. The equation has no solution
B. 10	2. The equation has a unique solution
C. 15	3. The equation has an infinite set of solutions
	4. The equation has two infinite sets of independent solutions

Code:

	A	B	C
(a)	4	2	3
(b)	1	1	3
(c)	2	2	4
(d)	4	1	3

[2006-II]

17. If $A^{-1} = \begin{bmatrix} 1 & -2 \\ -2 & 2 \end{bmatrix}$, what is det (A) ?
(a) 2 (b) -2
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ [2006-II]

18. From the matrix equation AB = AC, which one of the following can be concluded ?
(a) B = C for any matrix A
(b) B = C, if A is singular
(c) B = C, if A is non-singular
(d) A = B = C for any matrix A [2006-II]

19. What is the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ if $a^3 + b^3 + c^3 = 0$?
(a) 0 (b) 1
(c) 3 abc (d) – 3 abc [2006-II]

20. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a 2 × 2 matrix and $f(x) = x^2 - x + 2$ is a polynomial, then what is f (A) ?
(a) $\begin{bmatrix} 1 & 7 \\ 1 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 6 \\ 0 & 8 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 6 \\ 0 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 6 \\ 0 & 7 \end{bmatrix}$ [2006-II]

21. If A is a non-null row matrix with 5 columns and B is a non-null column matrix with 5 rows, how many rows are there in A × B ?
(a) 1 (b) 5
(c) 10 (d) 25 [2006-II]

DIRECTIONS (Qs. 22-23) : *The following questions consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

22. **Assertion(A):** If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $(A + B)^2 = A^2 + B^2 + 2AB$.
Reason(R): In the above AB = BA

23. **Assertion(A):** If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}$ and $B = \begin{pmatrix} \cos\alpha & \cos\alpha \\ \sin\alpha & \sin\alpha \end{pmatrix}$, then $AB \neq 1$.
Reason(R): The product of two matrices can never be equal to an identity matrix.

24. If A is any 2 × 2 matrix such that $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} A = \begin{bmatrix} -1 & 0 \\ 6 & 3 \end{bmatrix}$ then what is A equal to?
(a) $\begin{bmatrix} -5 & 1 \\ -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & -2 \\ 1 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} -5 & -2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 2 \\ -2 & -1 \end{bmatrix}$ [2007-I]

25. If A is a 3 × 3 matrix such that |A|=4, then what is A(adj A) equal to?
(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}$

(d) Cannot be determined, as data is insufficient *[2007-I]*

26. If $A = \begin{bmatrix} x & x^2 & 1+x^2 \\ y & y^2 & 1+y^2 \\ z & z^2 & 1+z^2 \end{bmatrix}$ where x, y, z are distinct what is |A|?

(a) 0
(b) $x^2y - y^2x + xyz$
(c) $(x-y)(y-z)(z-x)$
(d) xyz *[2007-I]*

27. Under which of the following condition(s), will the matrix

$A = \begin{bmatrix} 0 & 0 & q \\ 2 & 5 & 1 \\ 8 & p & p \end{bmatrix}$ be singular?

1. $q = 0$
2. $p = 0$
3. $p = 20$

Select the correct answer using the code given below:
(a) 1 and 2 (b) 3 only
(c) 1 and 3 (d) 1 or 3 *[2007-I]*

28. Consider the following statements:
1. If det A = 0, then det (adj A) = 0
2. If A is non- singular, then $\det(A^{-1}) = (\det A)^{-1}$
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2 *[2007-I]*

29. Let A be an $m \times n$ matrix. Under which one of the following conditions does A^{-1} exist?
(a) $m = n$ only (b) $m = n$ and $\det A \neq 0$
(c) $m = n$ and $\det A = 0$ (d) $m \neq n$ *[2007-I]*

30. Let A and B be two matrices of order $n \times n$. Let A be non-singular and B be singular. Consider the following:
1. AB is singular
2. AB is non-singular
3. $A^{-1}B$ is singular
4. $A^{-1}B$ is non singular

Which of the above is/ are correct?
(a) 1 and 3 (b) 2 and 4 only
(c) 1 only (d) 3 only *[2007-I]*

31. Let A be a square matrix of order $n \times n$ where $n \geq 2$. Let B be a matrix obtained from A with first and second rows interchanged. Then which one of the following is correct?
(a) det A = det B (b) det A = –det B
(c) A = B (d) A = – B *[2007-I]*

32. What should be the value of k so that the system of linear equations $x - y + 2z = 0$, $kx - y + z = 0$, $3x + y - 3z = 0$ does not possess a unique solution?
(a) 0 (b) 3
(c) 4 (d) 5 *[2007-I]*

33. The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ satisfies which one of the following polynomial equations?
(a) $A^2 + 3A + 2I = 0$ (b) $A^2 + 3A - 2I = 0$
(c) $A^2 - 3A - 2I = 0$ (d) $A^2 - 3A + 2I = 0$
[2007-II]

34. For how many values of k, will the system of equations $(k+1)x + 8y = 4k$ and $kx + (k+3)y = 3k - 1$, have an infinite number of solutions ?
(a) 1 (b) 2
(c) 3 (d) None of the above
[2007-II]

35. For what value of p, is the system of equations :
$p^3x + (p+1)^3 y = (p+2)^3$
$px + (p+1)y = p + 2$
$x + y = 1$
consistent ?
(a) $p = 0$ (b) $p = 1$
(c) $p = -1$ (d) For all $p > 1$ *[2007-II]*

36. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then what is the value of x ?
(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) 2 *[2007-II]*

37. Let $A = [a_{ij}]_{m \times m}$ be a matrix and $C = [c_{ij}]_{m \times m}$ be another matrix where c_{ij} is the cofactor of a_{ij}. Then, what is the value of $|AC|$?
(a) $|A|^{m-1}$ (b) $|A|^{m}$
(c) $|A|^{m+1}$ (d) Zero *[2007-II]*

38. If ω is the cube root of unity, then what is one root of the equation

$\begin{vmatrix} x^2 & -2x & -2\omega^2 \\ 2 & \omega & -\omega \\ 0 & \omega & 1 \end{vmatrix} = 0$?

(a) 1 (b) –2
(c) 2 (d) ω *[2007-II]*

39. If $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, then what is A^n equal to ?

(a) $\begin{bmatrix} 2^n & 2^n \\ 2^n & 2^n \end{bmatrix}$ (b) $\begin{bmatrix} 2n & 2n \\ 2n & 2n \end{bmatrix}$

(c) $\begin{bmatrix} 2^{2n-1} & 2^{2n-1} \\ 2^{2n-1} & 2^{2n-1} \end{bmatrix}$ (d) $\begin{bmatrix} 2^{2n+1} & 2^{2n+1} \\ 2^{2n+1} & 2^{2n+1} \end{bmatrix}$

[2007-II]

40. If the least number of zeroes in a lower triangular matrix is 10, then what is the order of the matrix ?
(a) 3×3 (b) 4×4
(c) 5×5 (d) 10×10 *[2007-II]*

41. If the inverse of $\begin{bmatrix} 1 & p & q \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & -p & -q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then what is the value of x ?
(a) 1 (b) Zero
(c) – 1 (d) $\frac{1}{p}+\frac{1}{q}$ *[2007-II]*

42. If $AB = \begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$, then what is the value of the determinant of the matrix B ?
(a) 4 (b) – 6
(c) $-\frac{1}{4}$ (d) – 28 *[2007-II]*

43. The determinant

$$\begin{vmatrix} a+b+c & a+b & a \\ 4a+3b+2c & 3a+2b & 2a \\ 10a+6b+3c & 6a+3b & 3a \end{vmatrix}$$

is independent of which one of the following?
(a) a and b (b) b and c
(c) a and c (d) All of these *[2007-II]*

44. If $X = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$, and I is a 2×2 identity matrix, then $X^2 - 2X + 3I$ equals to which one of the following ?
(a) – I (b) –2X
(c) 2X (d) 4X *[2008-I]*

45. If the matrix B is the adjoint of the square matrix A and α is the value of the determinant of A, then what is AB equal to ?
(a) α (b) $\left(\frac{1}{\alpha}\right)I$
(c) I (d) αI *[2008-I]*
where I is identity matrix

46. What is the determinant

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$$ equal to ?

(a) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$ (d) $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$

[2008-I]

47. If $x^2 + y^2 + z^2 = 1$, then what is the value of

$$\begin{vmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -x & 1 \end{vmatrix} = ?$$

(a) 0 (b) 1
(c) 2 (d) $2 - 2xyz$ *[2008-I]*

48. If $|A_{n \times n}| = 3$ and $|\text{adj } A| = 243$, what is the value of n ?
(a) 4 (b) 5
(c) 6 (d) 7 *[2008-I]*

49. Under what condition does A (BC) = (AB) C hold, where A, B, C are three matrices ?
(a) AB and BC both must exist
(b) Only Ab must exist
(c) Only BC must exist
(d) Always true *[2008-I]*

50. If A is matrix of order 3×2 and B is matrix of order 2×3, then what is $|kAB|$ equal to (where k is any scalar quantity)?
(a) $k|AB|$ (b) $k^2|AB|$
(c) $k^3|AB|$ (d) $|AB|$ *[2008-I]*

51. If $\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}^{-1} \begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, then which one of the following is correct ?
(a) $x = 5, y = 14$ (b) $x = -5, y = 14$
(c) $x = -5, y = -14$ (d) $x = 5, y = -14$ *[2008-I]*

52. Which one of the following statements is correct ? The system of linear equations,
$2x + 3y = 4$ and $4x + 6y = 7$, has
(a) no solution
(b) a unique solution
(c) exactly 3 solutions
(d) an infinite number of solutions *[2008-I]*

53. Suppose the system of equations
$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$
has a unique solution (x_0, y_0, z_0). If $x_0 = 0$, then which one of the following is correct ?

(a) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ (b) $\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = 0$

(c) $\begin{vmatrix} d_1 & a_1 & c_1 \\ d_2 & a_2 & c_2 \\ d_3 & a_3 & c_3 \end{vmatrix} = 0$ (d) $\begin{vmatrix} d_1 & a_1 & b_1 \\ d_2 & a_2 & b_2 \\ d_3 & a_3 & b_3 \end{vmatrix} = 0$

[2008-I]

54. If a, b, c are in GP, then what is the value of

$$\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix}?$$

(a) 0 (b) 1
(c) – 1 (d) None of these [2008-I]

55. If adj $A=\begin{bmatrix} a & 0 \\ -1 & b \end{bmatrix}$ and $ab \neq 0$, then what is the value of $|A^{-1}|$? [2008-II]

(a) 1 (b) ab
(c) $1/\sqrt{ab}$ (d) $1/ab$

56. If $l+m+n=0$, then the system of equations [2008-II]

$-2x+y+z=l$
$x-2y+z=m$
$x+y-2z=n$

has

(a) a trivial solution (b) no solution
(c) a unique solution (d) infinitely many solutions

57. If $(a_1/x)+(b_1/y)=c_1$, $(a_2/x)+(b_2/y)=c_2$ [2008-II]

$\Delta_1=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $\Delta_2=\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$, $\Delta_3=\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$,

then (x, y) is equal to which one of the following?

(a) $(\Delta_2/\Delta_1, \Delta_3/\Delta_1)$ (b) $(\Delta_3/\Delta_1, \Delta_2/\Delta_1)$
(c) $(-\Delta_1/\Delta_2, -\Delta_1/\Delta_3)$ (d) $(-\Delta_1/\Delta_2, -\Delta_1/\Delta_3)$

58. What is the value of $\begin{vmatrix} \sin 10° & -\cos 10° \\ \sin 80° & \cos 80° \end{vmatrix}$? [2008-II]

(a) 0 (b) 1
(c) –1 (d) 1/2

59. If $\begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 16 \\ 0 & 0 & 1+p \end{vmatrix}=20$, then what is the value of p? [2008-II]

(a) 0 (b) 1
(c) 2 (d) 5

60. If A and B are two matrices such that $AB = A$ and $BA = B$, then which one of the following is correct? [2008-II]

(a) $(A^T)^2=A^T$ (b) $(A^T)^2=B^T$
(c) $(A^T)^2=(A^{-1})^{-1}$ (d) None of the above

61. If $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}A=\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, then what is the matrix A? [2008-II]

(a) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

62. Under which one of the following condition does the system of equations [2009-I]

$kx+y+z=k-1$
$x+ky+z=k-1$
$x+y+kz=k-1$

have no solution?

(a) $k=1$ (b) $k \neq -2$
(c) $k=1$ or $k=-2$ (d) $k=-2$

63. Let $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ where a, b are natural numbers, then which one of the following is correct? [2009-I]

(a) There exist more than one but finite number of B's such that $AB=BA$
(b) There exists exactly one B such that $AB=BA$
(c) There exist infinitely many B's such that $AB=BA$
(d) There cannot exist any B such that $AB=BA$

64. Consider a matrix $M=\begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k \end{bmatrix}$ and the following statements

Statement A : Inverse of M exists.
Statement B : $k \neq 0$

Which one of the following in respect of the above matrix and statement is correct? [2009-I]

(a) A implies B, but B does not imply A
(b) B implies A, but A does not imply B
(c) Neither A implies B nor B implies A
(d) A implies B as well as B implies A

65. If $\begin{vmatrix} y & x & y+z \\ z & y & x+y \\ x & z & z+x \end{vmatrix}=0$, then which one of the following is correct? [2009-I]

(a) Either $x+y=z$ or $x=y$
(b) Either $x+y=-z$ or $x=z$
(c) Either $x+z=y$ or $z=y$
(d) Either $z+y=x$ or $x=y$

66. What is the value of k, if [2009-I]

$$\begin{vmatrix} k & b+c & b^2+c^2 \\ k & c+a & c^2+a^2 \\ k & a+b & a^2+b^2 \end{vmatrix}=(a-b)(b-c)(c-a)?$$

(a) 1 (b) –1
(c) 2 (d) 0

67. Which one of the following is correct in respect of the matrix [2009-I]

$$A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}?$$

(a) A^{-1} does not exist (b) $A=(-1)I$
(c) A is a unit matrix (d) $A^2=I$

68. If $A=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$, then what is A (adj A) equal to ? *[2009-I]*

(a) $\begin{bmatrix}0 & 10\\ 10 & 0\end{bmatrix}$ (b) $\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$

(c) $\begin{bmatrix}1 & 10\\ 10 & 1\end{bmatrix}$ (d) $\begin{bmatrix}10 & 1\\ 1 & 10\end{bmatrix}$

69. What is the inverse of $\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$? *[2009-I]*

(a) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}-1 & 0 & 1\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{bmatrix}$

70. Consider the following statements in respect of symmetric matrices A and B *[2009-I]*

1. AB is symmetric.
2. A^2+B^2 is symmetric.

Which of the above statement(s) is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

71. *The following item consists of two statements, one labelled the Assertion (A) and the other labelled the Reason (R). You are to examine these two statements carefully and decide if the Assertion (A) and Reason (R) are individually true and if so, whether the reason is a correct explanation of the Assertion. Select your answer using the codes given below.*

Assertion (A) : $M=\begin{bmatrix}5 & 10\\ 4 & 8\end{bmatrix}$ is invertible. *[2009-I]*

Reason (R) : M is singular.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

72. If X and Y are the matrices of order 2 × 2 each and $2X-3Y=\begin{bmatrix}-7 & 0\\ 7 & -13\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}9 & 13\\ 4 & 13\end{bmatrix}$, then what is Y equal to? *[2009-II]*

(a) $\begin{bmatrix}1 & 3\\ -2 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}$

(c) $\begin{bmatrix}3 & 2\\ -1 & 5\end{bmatrix}$ (d) $\begin{bmatrix}3 & 2\\ 1 & -5\end{bmatrix}$

73. If a, b, c, are non-zero real numbers and *[2009-II]*

$$\begin{bmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{bmatrix}=0,$$

then what is the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$?

(a) 2 (b) 1
(c) –1 (d) 0

74. If a matrix A is symmetric as well as anti-symmetric, then which one of the following is correct? *[2009-II]*

(a) A is a diagonal matrix (b) A is a null matrix
(c) A is a unit matrix (d) A is a triangular matrix

75. If $A=\begin{bmatrix}1 & -2 & -3\\ 2 & 1 & -2\\ 3 & 2 & 1\end{bmatrix}$, then which one of the following is correct? *[2009-II]*

(a) A is symmetric matrix
(b) A is anti- symmetric matrix
(c) A is singular matrix
(d) A is non-singular matrix

76. $A=\begin{vmatrix}2a & 3r & x\\ 4b & 6s & 2y\\ -2c & -3t & -z\end{vmatrix}=\lambda\begin{vmatrix}a & r & x\\ b & s & y\\ c & t & z\end{vmatrix}$, then what is the value of λ ? *[2009-II]*

(a) 12 (b) –12
(c) 7 (d) –7

77. What is the value of $\begin{vmatrix}1-i & \omega^2 & -\omega\\ \omega^2+i & \omega & -i\\ 1-2i-\omega^2 & \omega^2-\omega & i-\omega\end{vmatrix}$, where ω is the cube root of unity? *[2009-II]*

(a) –1 (b) 1
(c) 2 (d) 0

78. If $A=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}$, where ω is cube root of unity, then what is A^{100} equal to? *[2009-II]*

(a) A (b) $-A$
(c) Null matrix (d) Identity matrix

79. A matrix X has $(a+b)$ rows and $(a+2)$ columns; and a matrix Y has $(b+1)$ rows and $(a+3)$ columns. If both XY and YX exist, then what are the values of a, b respectively? *[2009-II]*
(a) 3, 2 (b) 2, 3
(c) 2, 4 (d) 4, 3

80. If $\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = 2$, then what is the value of the determinant $\begin{vmatrix} 6a & 3b & 15c \\ 2l & m & 5n \\ 2p & q & 5r \end{vmatrix}$? *[2010-I]*
(a) 10 (b) 20
(c) 40 (d) 60

81. Let A = $\begin{bmatrix} 5 & 6 & 1 \\ 2 & -1 & 5 \end{bmatrix}$. Let there exist a matrix B such that $AB = \begin{bmatrix} 35 & 49 \\ 29 & 13 \end{bmatrix}$. What is B equal to? *[2010-I]*
(a) $\begin{bmatrix} 5 & 1 & 4 \\ 2 & 6 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 6 & 3 \\ 5 & 1 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} 5 & 2 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 5 \\ 6 & 1 \\ 3 & 4 \end{bmatrix}$

82. Consider the following statements *[2010-I]*
1. If $A' = A$; then A is a singular matrix, where A' is the transpose of A.
2. If A is a square matrix such that $A^3 = I$, then A is non-singular.

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

83. If the system of equations $2x + 3y = 7$ and $2ax + (a+b)y = 28$ has infinitely many solutions, then which one of the following is correct? *[2010-I]*
(a) $a = 2b$ (b) $b = 2a$
(c) $a = -2b$ (d) $b = -2a$

84. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b? *[2010-I]*
(a) 1 (b) 3
(c) 6 (d) 0

85. What is the value of *[2010-I]*
$$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \cos 45^\circ & \sin 45^\circ \end{vmatrix} \times \begin{vmatrix} \cos 45^\circ & \cos 15^\circ \\ \sin 45^\circ & \sin 15^\circ \end{vmatrix}?$$
(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{2}$
(c) $-\frac{1}{4}$ (d) $-\frac{3}{4}$

86. Let A be an $n \times n$ matrix. If $\det(\lambda A) = \lambda^s \det(A)$, what is the value of s? *[2010-I]*
(a) 0 (b) 1
(c) –1 (d) n

87. If A be a real skew- symmetric matrix of order n such that $A^2 + I = 0$, I being the identity matrix of the same order as that of A, then what is the order of A? *[2010-I]*
(a) 3 (b) Odd
(c) Prime number (d) Even

88. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \left[a_{ij}\right]$, where $i, j = 1, 2$, If its inverse matrix is $\left[b_{ij}\right]$, what is b_{22}? *[2010-I]*
(a) –2 (b) 1
(c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

89. If $\begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix}$ is not an invertible matrix, then what is the value of λ? *[2010-II]*
(a) –1 (b) 0
(c) 1 (d) 2

90. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $C = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$, then which one of the following is **not** correct? *[2010-II]*
(a) $A^2 = B^2$ (b) $B^2 = C^2$
(c) $AB = C$ (d) $AB = BA$

91. If $x + iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$, then what is $x - iy$ equal to?
(a) $3 + i$ (b) $1 + 3i$ *[2010-II]*
(c) $3i$ (d) 0

92. If $|A| = 8$, where A is square matrix of order 3, then what is $|\text{adj } A|$ equal to? *[2010-II]*
(a) 16 (b) 24
(c) 64 (d) 512

93. Consider the following statements in respect of a square matrix A and its transpose A^T. *[2010-II]*
1. $A + A^T$ is always symmetric.
2. $A - A^T$ is always anti-symmetric

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

94. If a matrix A is such that [2010-II]

$$3A^3 + 2A^2 + 5A + I = 0,$$

Then what is A^{-1} equal to?

(a) $-(3A^2 + 2A + 5)$ (b) $3A^2 + 2A + 5I$
(c) $3A^2 - 2A - 5I$ (d) $(3A^2 + 2A - 5I)$

95. Let A and B be matrices of order 3×3. If $AB = 0$, then which of the following can be concluded? [2010-II]

(a) $A = 0$ and $B = 0$ (b) $|A| = 0$ and $|B| = 0$
(c) Either $|A| = 0$ or $|B| = 0$ (d) Either $A = 0$ or $B = 0$

96. If A is a square matrix, then what is adj $A^T - (adj\ A)^T$ equal to? [2010-II]

(a) $2|A|$ (b) $2|A|I$
(c) Null Matrix (d) Unit Matrix

97. What is the value of

$$\begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix},$$

where ω is the cube root of unity? [2010-II]

(a) 0 (b) 1
(c) 2 (d) 3

98. If the matrix

$$A = \begin{bmatrix} 2-x & 1 & 1 \\ 1 & 3-x & 0 \\ -1 & -3 & -x \end{bmatrix}$$

is singular, then what is the solution set S? [2011-I]

(a) $S = \{0, 2, 3\}$ (b) $S = \{-1, 2, 3\}$
(c) $S = \{1, 2, 3\}$ (d) $S = \{2, 3\}$

99. Consider the following statements. [2011-I]

I. The inverse of a square matrix, if it exists, is unique.
II. If A and B are singular matrices of order n, then AB is also a singular matrix of order n.

Which of the statements given above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

100. What is the value of the determinant [2011-I]

$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}?$$

(a) $x+2$ (b) x^2+2
(c) 2 (d) -2

101. If 5 and 7 are the roots of the equation

$$\begin{vmatrix} x & 4 & 5 \\ 7 & x & 7 \\ 5 & 8 & x \end{vmatrix} = 0,$$ then what is the third root? [2011-I]

(a) -12 (b) 9
(c) 13 (d) 14

102. Find the value of k in which the system of equations $kx + 2y = 5$ and $3x + y = 1$ has no solution? [2011-I]

(a) 0 (b) 3
(c) 6 (d) 15

103. If the matrix

$$A = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

is such that $A^2 = I$, then which one of the following is correct? [2011-I]

(a) $\alpha = 0, \beta = 1$ or $\alpha = 1, \beta = 0$
(b) $\alpha = 0, \beta \neq 1$ or $\alpha \neq 1, \beta = 1$
(c) $\alpha = 1, \beta \neq 0$ or $\alpha \neq 1, \beta = 1$
(d) $\alpha \neq 0, \beta \neq 0$

104. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

such that $A^2 = B$, then what is the value of α? [2011-I]

(a) -1 (b) 1
(c) 2 (d) 4

105. $A = \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, then which of the following is/are correct? [2011-I]

I. AB is defined II. BA is defined
III. $AB = BA$

Select the correct answer using the codes given below.

(a) Only I (b) Only II
(c) Both I and II (d) I, II and III

106. The simultaneous equations $3x + 5y = 7$ and $6x + 10y = 18$ have [2011-II]

(a) no solution
(b) infinitely many solutions
(c) unique solution
(d) any finite number of solutions

107. The roots of the equation $\begin{vmatrix} x & \alpha & 1 \\ \beta & x & 1 \\ \beta & \gamma & 1 \end{vmatrix} = 0$ are independent of

[2011-II]

(a) α (b) β
(c) γ (d) α, β and γ

108. What is the value of the determinant

$$\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}?$$ [2011-II]

(a) $a^3 + b^3 + c^3$ (b) $3bc$
(c) $a^3 + b^3 + c^3 - 3abc$ (d) 0

109. If $\begin{vmatrix} p & -q & 0 \\ 0 & p & q \\ q & 0 & p \end{vmatrix} = 0$, then which one of the following is correct? [2011-II]
(a) p is one of the cube roots of unity
(b) q is one of the cube roots of unity
(c) $\frac{p}{q}$ is one of the cube roots of unity
(d) None of the above

110. If $a^{-1} + b^{-1} + c^{-1} = 0$ such that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$, then what is λ equal to? [2011-II]
(a) $-abc$ (b) abc
(c) 0 (d) 1

111. Consider the following statements in respect of the square matrices A and B of same order: [2011-II]
1. A and B are non-zero and $AB = 0 \Rightarrow$ either $|A| = 0$ or $|B| = 0$
2. $AB = 0 \Rightarrow A = 0$ or $B = 0$
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

112. For what value of x does
$$\begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix} = (0) \text{ hold?}$$ [2011-II]
(a) -1 (b) 1
(c) 9/8 (d) $-9/8$

113. Consider the following statements: [2012-I]
1. every zero matrix is a square matrix.
2. A matrix has a numerical value.
3. A unit matrix is a diagonal matrix.
Which of the above statements is/are correct?
(a) 2 only (b) 3 only
(c) 2 and 3 (d) 1 and 3

114. If a matrix A has inverses B and C, then which one of the following is correct? [2012-I]
(a) B may not be equal to C
(b) B should be equal to C
(c) B and C should be unit matrices
(d) None of the above

115. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ then what is determinant of AB? [2012-I]
(a) 0 (b) 1
(c) 10 (d) 20

116. What is $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ equal to? [2012-I]
(a) $4abc$ (b) $4a^2bc$
(c) $4a^2b^2c^2$ (d) $-4a^2b^2c^2$

117. A and B are two matrices such that AB = A and BA = B then what is B^2 equal to? [2012-I]
(a) B (b) A
(c) I (d) $-I$
where I is the identity matrix

118. The sum and product of matrices A and B exist. Which of the following implications are necessarily true?
1. A and B are square matrices of same order.
2. A and B are non-singular matrices
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

119. If A is a square matrix such that $A^2 = I$ where I is the identity matrix, then what is A^{-1} equal to? [2012-I]
(a) A + 1 (b) Null matrix
(c) A (d) Transpose of A

120. If two rows of a determinant are identical, then what is the value of the determinant? [2012-I]
(a) 0 (b) 1
(c) -1 (d) can be any real value

121. If $\begin{vmatrix} 8 & -5 & 1 \\ 5 & x & 1 \\ 6 & 3 & 1 \end{vmatrix} = 2$ then what is the value of x? [2012-I]
(a) 4 (b) 5
(c) 6 (d) 8

122. What is the order of the product
$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} ?$$ [2012-I]
(a) 3×1 (b) 1×1
(c) 1×3 (d) 3×3

123. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$, then what is $B^{-1}A^{-1}$ equal to? [2012-I]
(a) $\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -3 \\ 1 & -2 \end{bmatrix}$

124. If each element in a row of a determinant is multiplied by the same factor r, then the value of the determinant : *[2012-II]*
(a) is multiplied by r^3. (b) is increased by 3r
(c) remains unchanged (d) is multiplied by r

125. The inverse of a diagonal matrix is a: *[2012-II]*
(a) symmetric matrix (b) skew-symmetric matrix
(c) diagonal matrix (d) None of the above

126. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \end{bmatrix}$, then which one of the following is correct ? *[2012-II]*
(a) B is the inverse of A (b) B is the adjoint of A
(c) B is the transpose of A (d) None of the above

127. If the sum of the matrices $\begin{bmatrix} x \\ x \\ y \end{bmatrix}, \begin{bmatrix} y \\ y \\ z \end{bmatrix}$ and $\begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix}$ is the matrix $\begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$, then what is the value of y ? *[2012-II]*
(a) – 5 (b) 0
(c) 5 (d) 10

128. If the matrix AB is a zero matrix, then which one of the following is correct ? *[2012-II]*
(a) A must be equal to zero matrix or B must be equal to zero matrix.
(b) A must be equal to zero matrix and B must be equal to zero matrix.
(c) It is not necessary that either A is zero matrix or B is zero matrix.
(d) None of the above

129. If the matrix $\begin{bmatrix} \alpha & 2 & 2 \\ -3 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ is not invertible, then :
(a) $\alpha = -5$ (b) $\alpha = 5$ *[2012-II]*
(c) $\alpha = 0$ (d) $\alpha = 1$

130. The value of the determinant $\begin{vmatrix} x^2 & 1 & y^2+z^2 \\ y^2 & 1 & z^2+x^2 \\ z^2 & 1 & x^2+y^2 \end{vmatrix}$ is : *[2012-II]*
(a) 0 (b) $x^2+y^2+z^2$
(c) $x^2+y^2+z^2+1$ (d) None of the above

131. A square matrix $[a_{ij}]$ such that $a_{ij} = 0$ for $i \neq j$ and $a_{ij} = k$ where k is a constant for $i = j$ is called : *[2012-II]*
(a) diagonal matrix, but not scalar matrix
(b) scalar matrix
(c) unit matrix
(d) None of the above

132. If A and B are two non-singular square matrices such that AB = A, then which one of the following is correct? *[2013-I]*
(a) B is an identity matrix (b) $B = A^{-1}$
(c) $B = A^2$ (d) Determinant of B is zero

133. What is the value of the minor of the element 9 in the determinant
$\begin{vmatrix} 10 & 19 & 2 \\ 0 & 13 & 1 \\ 9 & 24 & 2 \end{vmatrix}$? *[2013-I]*
(a) – 9 (b) – 7
(c) 7 (d) 0

134. The roots of the equation $\begin{vmatrix} 1 & t-1 & 1 \\ t-1 & 1 & 1 \\ 1 & 1 & t-1 \end{vmatrix} = 0$ are
(a) 1, 2 (b) –1, 2 *[2013-I]*
(c) 1, –2 (d) –1, –2

135. The value of the determinant $\begin{vmatrix} m & n & p \\ p & m & n \\ n & p & m \end{vmatrix}$ *[2013-I]*
(a) is a perfect cube (b) is a perfect square
(c) has linear factor (d) is zero

136. The determinant of a orthogonal matrix is: *[2013-I]*
(a) ± 1 (b) 2
(c) 0 (d) ± 2

137. If D is determinant of order 3 and D′ is the determinant obtained by replacing the elements of D by their cofactors, then which one of the following is correct? *[2013-I]*
(a) $D' = D^2$ (b) $D' = D^3$
(c) $D' = 2D^2$ (d) $D' = 3D^3$

138. Consider the following statements:
1. A matrix is not a number
2. Two determinants of different order may have the same value.

Which of the above statements is/are correct? *[2013-I]*
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

139. Consider the following statements : *[2013-II]*
1. The product of two non-zero matrices can never be identity matrix.
2. The product of two non-zero matrices can never be zero matrix.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1nor 2

140. Consider the following statements : [2013-II]

1. The matrix $\begin{pmatrix} 1 & 2 & 1 \\ a & 2a & 1 \\ b & 2b & 1 \end{pmatrix}$ is singular.

2. The matrix $\begin{pmatrix} c & 2c & 1 \\ a & 2a & 1 \\ b & 2b & 1 \end{pmatrix}$ is non-singular.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

141. The cofactor of the element 4 in the determinant

$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 8 & 9 \end{vmatrix}$ is [2013-II]

(a) 2 (b) 4
(c) 6 (d) –6

142. If A is a square matrix of order 3 with $|A| \neq 0$, then which one of the following is correct ? [2013-II]

(a) $|adjA| = |A|$ (b) $|adjA| = |A|^2$
(c) $|adjA| = |A|^3$ (d) $|adjA|^2 = |A|$

143. If $A = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

where $i = \sqrt{-1}$, then which one of the following is correct? [2013-II]

(a) $AB = -C$
(b) $AB = C$
(c) $A^2 = B^2 = C^2 = I$, where I is the identity matrix
(d) $BA \neq C$

144. If $2A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, then what is A^{-1} equal to ? [2013-II]

(a) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ (b) $\frac{1}{2}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

(c) $\frac{1}{4}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ (d) None of these

145. If $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \times \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 17 & \lambda \end{pmatrix}$, then what is λ equal to ? [2013-II]

(a) 7 (b) –7
(c) 9 (d) –9

146. What is the value of the determinant [2013-II]

$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$?

(a) 0 (b) abc
(c) $ab + bc + ca$ (d) $abc\,(a+b+c)$

147. Consider the following statements in respect of the matrix

$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ [2014-I]

1. The matrix A is skew-symmetric.
2. The matrix A is symmetric.
3. The matrix A is invertible.

Which of the above statements is/are correct ?
(a) 1 only (b) 3 only
(c) 1 and 3 (d) 2 and 3

148. Consider two matrices $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{bmatrix}$.

Which one of the following is correct ? [2014-I]
(a) B is the right inverse of A
(b) B is the left inverse of A
(c) B is the both sided inverse of A
(d) None of the above

149. One of the roots of [2014-I]

$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$ is :

(a) abc (b) $a+b+c$
(c) $-(a+b+c)$ (d) $-abc$

150. If A is any matrix, then the product AA is defined only when A is a matrix of order $m \times n$ where : [2014-I]
(a) $m > n$ (b) $m < n$
(c) $m = n$ (d) $m \leq n$

151. The determinant of an odd order skew symmetric matrix is always : [2014-I]
(a) Zero (b) One
(c) Negative (d) Depends on the matrix

152. If any two adjacent rows or columns of a determinant are intercharged in position, the value of the determinant : [2014-I]
(a) Becomes zero (b) Remains the same
(c) Changes its sign (d) Is doubled

153. If $a \neq b \neq c$ are all positive, then the value of the determinant

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is [2014-II]

(a) non-negative (b) non-positive
(c) negative (d) positive

154. Let A and B be two matrices such that $AB = A$ and $BA = B$. Which of the following statements are correct ? [2014-II]
1. $A^2 = A$
2. $B^2 = B$
3. $(AB)^2 = AB$

Select the correct answer using the code given below :
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

155. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, where $i = \sqrt{-1}$, then what is x equal to ? [2014-II]
(a) 3 (b) 2
(c) 1 (d) 0

156. If the matrix A is such that $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$, then what is A equal to ? [2014-II]
(a) $\begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -4 \\ 0 & -1 \end{pmatrix}$

157. Consider the following statements : [2014-II]
1. Determinant is a square matrix.
2. Determinant is a number associated with a square matrix.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

158. If A is an invertible matrix, then what is det (A^{-1}) equal to ? [2014-II]
(a) det A (b) $\frac{1}{\det A}$
(c) 1 (d) None of the above

159. From the matrix equation $AB = AC$, Where A, B, C are the square matrices of same order, we can conclude $B = C$ provided [2014-II]
(a) A is non-singular. (b) A is singular.
(c) A is symmetric. (d) A is skew symmetric.

160. If $A = \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is symmetric, then what is x equal to ? [2014-II]
(a) 2 (b) 3
(c) –1 (d) 5

161. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then which one of the following is correct? [2014-II]
(a) $\frac{a}{b}$ is one of the cube roots of unity.
(b) $\frac{a}{b}$ is one of the cube roots of –1.
(c) a is one of the cube roots of unity.
(d) b is one of the cube roots of unity.

162. If A and B are square matrices of second order such that $|A| = -1$, $|B| = 3$, then what is $|3AB|$ equal to ? [2014-II]
(a) 3 (b) –9
(c) –27 (d) None of these

163. Which one of the following matrices is an elementary matrix? [2015-I]
(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 2 \end{bmatrix}$

164. If $A = \begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}$ then that is $A + 3A^{-1}$ equal to? [2015-I]
(a) $3I$ (b) $5I$
(c) $7I$ (d) None of these

where I is the identity matrix order 2.

165. The matrix $\begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$ is [2015-I]
(a) symmetric (b) skew-symmetric
(c) Hermitian (d) skew-Hermitian

166. Consider the following in respect of two non-singular matrices A and B of same order: [2015-I]
1. det $(A + B) = \det A + \det B$
2. $(A + B)^{-1} = A^{-1} + B^{-1}$

Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

167. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ satisfy the equation $AX = B$, then the matrix A is equal to [2015-I]
(a) $\begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 26 \\ 4 & 17 \end{bmatrix}$
(c) $\begin{bmatrix} -7 & -4 \\ 26 & 13 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & 26 \\ -6 & 23 \end{bmatrix}$

168. Let

$$A=\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}, B=\begin{bmatrix} 2 \\ -1 \end{bmatrix} \text{ and } C=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

If $AB = C$, then what is A^2 equal to? [2015-I]

(a) $\begin{bmatrix} 6 & -10 \\ 4 & 26 \end{bmatrix}$ (b) $\begin{bmatrix} -10 & 5 \\ 4 & 24 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -6 \\ -4 & -20 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -7 \\ -5 & 20 \end{bmatrix}$

169. The value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} \text{ is}$$ [2015-I]

(a) $x+y$ (b) $x-y$
(c) xy (d) $1+x+y$

170. If $E(\theta)=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $E(\alpha)E(\beta)$ is equal to [2015-I]

(a) $E(\alpha\beta)$ (b) $E(\alpha-\beta)$
(c) $E(\alpha+\beta)$ (d) $-E(\alpha+\beta)$

171. The matrix $A=\begin{bmatrix} 1 & 3 & 2 \\ 1 & x-1 & 1 \\ 2 & 7 & x-3 \end{bmatrix}$ will have inverse for every real number x *except* for [2015-II]

(a) $x=\dfrac{11\pm\sqrt{5}}{2}$ (b) $x=\dfrac{9\pm\sqrt{5}}{2}$

(c) $x=\dfrac{11\pm\sqrt{3}}{2}$ (d) $x=\dfrac{9\pm\sqrt{3}}{2}$

172. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, where $a \neq b \neq c$, then the value of abc [2015-II]
(a) cannot be less than 1
(b) is greater than –8
(c) is less than –8
(d) must be greater than 8

173. Consider the following statements in respect of the determinant [2015-II]

$$\begin{vmatrix} \cos^2\dfrac{\alpha}{2} & \sin^2\dfrac{\alpha}{2} \\ \sin^2\dfrac{\beta}{2} & \cos^2\dfrac{\beta}{2} \end{vmatrix}$$

where α, β are complementary angles

1. The value of the determinant is $\dfrac{1}{\sqrt{2}}\cos\left(\dfrac{\alpha-\beta}{2}\right)$.
2. The maximum value of the determinant is $\dfrac{1}{\sqrt{2}}$.

Which of the above statements is/ are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

174. If $A=\begin{bmatrix} 1 & 0 & -2 \\ 2 & -3 & 4 \end{bmatrix}$, then the matrix X for which $2X+3A=0$ holds true is [2015-II]

(a) $\begin{bmatrix} -\frac{3}{2} & 0 & -3 \\ -3 & -\frac{9}{2} & -6 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{2} & 0 & -3 \\ 3 & -\frac{9}{2} & -6 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{2} & 0 & 3 \\ 3 & \frac{9}{2} & 6 \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{3}{2} & 0 & 3 \\ -3 & \frac{9}{2} & -6 \end{bmatrix}$

175. If $A=\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ then which of the following is/are correct ? [2015-II]
1. *A* and *B* commute.
2. *AB* is a null matrix.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

176. If A is an invertible matrix of order n and k is any positive real number, then the value of $[\det(kA)]^{-1}\det A$ is [2015-II]
(a) k^{-n} (b) k^{-1}
(c) k^{n} (d) nk

177. If A is an orthogonal matrix of order 3 and $B=\begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 2 \\ 2 & 5 & 0 \end{bmatrix}$, then which of the following is/are correct ? [2015-II]
1. $|AB| = \pm 47$
2. AB = BA

Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

178. If a, b, c are real numbers, then the value of the determinant

$$\begin{vmatrix} 1-a & a-b-c & b+c \\ 1-b & b-c-a & c+a \\ 1-c & c-a-b & a+b \end{vmatrix} \text{ is}$$ [2015-II]

(a) 0 (b) (a – b) (b – c) (c – a)
(c) $(a+b+c)^2$ (d) $(a+b+c)^3$

DIRECTIONS (Qs. 179 - 180) : *For the next two (2) items that follow.*

Consider the function

$$f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix},$$ where p is a constant.

179. What is the value of f'(0)? *[2016-I]*
(a) p^3 (b) $3p^3$
(c) $6p^3$ (d) $-6p^3$

180. What is the value of p for which f"(0)=0? *[2016-I]*
(a) $-\frac{1}{6}$ or 0 (b) –1 or 0
(c) $-\frac{1}{6}$ or 1 (d) –1 or 1

181. If A is a square matrix, then what is $adj(A^{-1}) - (adj\ A)^{-1}$ equal to? *[2016-I]*
(a) 2|A| (b) Null matrix
(c) Unit matrix (d) None of the above

182. Consider the following in respect of the matrix $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$: *[2016-I]*
1. $A^2 = -A$
2. $A^3 = 4A$

Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

183. Which of the following determinants have value 'zero'? *[2016-I]*

1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

Select the correct answer using the code given below.
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

184. The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions? *[2016-I]*
(a) $k \neq 1$ and $k \neq -2$ (b) $k \neq 1$ and $k \neq 2$
(c) $k \neq -1$ and $k \neq -2$ (d) $k \neq -1$ and $k \neq 2$

185. If A is semy square matrix of order 3 and det A= 5, then what is $\det[(2A)^{-1}]$ equal to? *[2016-II]*
(a) 1/10 (b) 2/5
(c) 8/5 (d) 1/40

186. What is $[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ equal to ? *[2016-II]*

(a) $[ax + hy + gz \quad h + b + f \quad g + f + c]$

(b) $\begin{bmatrix} a & h & g \\ hx & by & fz \\ g & f & c \end{bmatrix}$

(c) $\begin{bmatrix} ax + hy + gz \\ hx + by + fz \\ gx + fy + cz \end{bmatrix}$

(d) $[ax + hy + gz \quad hx + by + fz \quad gx + fy + cz]$

DIRECTIONS (Qs. 187-188) : *Consider the following for the next two (02) items that follow:* *[2016-II]*

Let $ax^3 + bx^2 + cx + d = \begin{vmatrix} x+1 & 2x & 3x \\ 2x+3 & x+1 & x \\ 2-x & 3x+4 & 5x-1 \end{vmatrix}$ then

187. What is the value of c?
(a) – 1 (b) 34
(c) 35 (d) 50

188. What is the value of a + b + c + d ?
(a) 62 (b) 63
(c) 65 (d) 68

189. If $m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $n = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then what is the value of the determinant of $m\cos\theta - n\sin\theta$? *[2016-II]*
(a) – 1 (b) 0
(c) 1 (d) 2

190. If $f\{x\} = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then which of the following are correct ? *[2016-II]*
1. $f(\theta) \times f(\phi) = f(\theta + \phi)$
2. The value of the determinant of the matrix $f(\theta) \times f(\phi)$ is 1.
3. The determinant of f(x) is an even function.

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

191. Which of the following are correct in respect of the system of equations $x + y + z = 8$, $x - y + 2z = 6$ and $3x - y + 5z = k$?
1. They have no solution. if k = 15. *[2016-II]*
2. They have infinitely many solutions, if k = 20.
3. They have unique solution, if k = 25

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

192. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, then which of the following is/are correct? *[2016-II]*

1. $AB(A^{-1}B^{-1})$ is a unit matrix.
2. $(AB)^{-1} = A^{-1}B^{-1}$

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 only 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 193-194) : *Consider the following for the next two (02) items that follow:*

For the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$ and $2x + 3y + \lambda z = \mu$ *[2016-II]*

193. Under what condition does the above system of equations have infinitely many solutions ?
(a) $\lambda = 5$ and $\mu \neq 9$ (b) $\lambda = 5$ and $\mu = 9$
(c) $\lambda = 9$ and $\mu = 5$ (d) $\lambda = 9$ and $\mu \neq 5$

194. Under what condition does the above system of equations have unique solutions? *[2016-II]*
(a) $\lambda = 5$ and $\mu = 9$
(b) $\lambda \neq 9$ and $\mu = 7$ only
(c) $\lambda \neq 5$ and μ has any real value
(d) λ has any real value and $\mu \neq 9$

195. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $\det(A^3) = 125$, then α is equal to
(a) ± 1 (b) ± 2 *[2017-I]*
(c) ± 3 (d) ± 5

196. If B is a non-singular matrix and A is a square matrix, then the value of $\det(B^{-1}AB)$ is equal to *[2017-I]*
(a) det (B) (b) det (A)
(c) $\det(B^{-1})$ (d) $\det(A^{-1})$

197. If $a \neq b \neq c$, then one value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$ is given by *[2017-I]*

(a) a (b) b
(c) c (d) 0

198. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then what is AA^T equal to (where A^T is the transpose of A)? *[2017-I]*
(a) Null matrix (b) Identify matrix
(c) A (d) –A

199. The equations *[2017-I]*
$x + 2y + 3z = 1$
$2x + y + 3z = 2$
$5x + 5y + 9z = 4$
(a) have the unique solution
(b) have infinitely many solutions
(c) are inconsistent
(d) None of the above

200. $A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.
If AB = C, then what is A^2 equal to? *[2017-I]*
(a) $\begin{bmatrix} 4 & 8 \\ -4 & -16 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -4 \\ 8 & -16 \end{bmatrix}$
(c) $\begin{bmatrix} -4 & -8 \\ 4 & 12 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

201. What is the value of the determinant *[2017-I]*

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}?$$

(a) $1 + x + y + z$ (b) 2xyz
(c) $x^2y^2z^2$ (d) $2x^2y^2z^2$

202. If $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = 0$, then which one of the following is correct? *[2017-I]*
(a) $\frac{x}{y}$ is one of the cube roots of unity
(b) x is one of the cube roots of unity
(c) y is one of the cube roots of unity
(d) $\frac{x}{y}$ is one of the cube roots of –1

203. Consider the set A of all matrices of order 3×3 with entries 0 or 1 only. Let B be the subset of A consisting of all matrices whose determinant is 1. Let C be the subset of A consisting of all matrices whose determinant is –1. Then which one of the following is correct? *[2017-I]*
(a) C is empty
(b) B has as many elements as C
(c) $A = B \cup C$
(d) B has thrice as many elements as C

204. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then what is A^3 equal to? *[2017-I]*
(a) $\begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos^3\theta & \sin^3\theta \\ -\sin^3\theta & \cos^3\theta \end{bmatrix}$
(c) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$ (d) $\begin{bmatrix} \cos^3\theta & -\sin^3\theta \\ \sin^3\theta & \cos^3\theta \end{bmatrix}$

205. What is the order of *[2017-I]*

$$(x\ y\ z)\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}?$$

(a) 3×1 (b) 1×1
(c) 1×3 (d) 3×3

206. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the value of A^4 is [2017-I]

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

207. The matrix A has x rows and x + 5 columns. The matrix B has y rows and 11 – y columns. Both AB and BA exist. What are the values of x and y respectively? [2017-II]
(a) 8 and 3 (b) 3 and 4
(c) 3 and 8 (d) 8 and 8

208. If A is a square matrix, then the value of adj $A^T - (\text{adj } A)^T$ is equal to [2017-II]
(a) A
(b) $2|A|I$, where I is the identity matrix
(c) null matrix whose order is same as that of A
(d) unit matrix whose order is same as that of A

209. The value of the determinant $\begin{vmatrix} \cos^2\frac{\theta}{2} & \sin^2\frac{\theta}{2} \\ \sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{vmatrix}$ for all values of θ, is [2017-II]
(a) 1 (b) cos θ
(c) sin θ (d) cos 2θ

210. If a, b, c are non-zero real numbers, then the inverse of the matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ is equal to [2017-II]

(a) $\begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$ (b) $\frac{1}{abc}\begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

(c) $\frac{1}{abc}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\frac{1}{abc}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

211. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + kz = k^2$ has no solution if k equals [2017-II]
(a) 0 (b) 1
(c) –1 (d) –2

212. The value of the determinant $\begin{vmatrix} 1-\alpha & \alpha-\alpha^2 & \alpha^2 \\ 1-\beta & \beta-\beta^2 & \beta^2 \\ 1-\gamma & \gamma-\gamma^2 & \gamma^2 \end{vmatrix}$ is equal to [2017-II]

(a) $(\alpha-\beta)(\beta-\gamma)(\alpha-\gamma)$
(b) $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$
(c) $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$
(d) 0

213. The adjoint of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ is [2017-II]

(a) $\begin{bmatrix} -1 & 6 & 2 \\ -2 & 1 & -4 \\ 6 & 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 1 & 2 \\ 4 & -1 & 2 \\ 6 & 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & 2 & 1 \\ 4 & -2 & 1 \\ 3 & 1 & -6 \end{bmatrix}$

214. If $A = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$, then which one of the following is correct? [2017-II]
(a) $A^2 = -2A$ (b) $A^2 = -4A$
(c) $A^2 = -3A$ (d) $A^2 = 4A$

215. If p + q + r = a + b + c = 0, then the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals [2017-II]
(a) 0 (b) 1
(c) pa + qb + rc (d) pa + qb + rc + a + b + c

216. What is the inverse of the matrix [2018-I]

$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$?

(a) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

217. If A is a 2 × 3 matrix and AB is a 2 × 5 matrix, then B must be a [2018-I]
(a) 3 × 5 matrix (b) 5 × 3 matrix
(c) 3 × 2 matrix (d) 5 × 2 matrix

218. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $A^2 - kA - I_2 = O$, where I_2 is the 2 × 2 identity matrix, then what is the value of k? [2018-I]
(a) 4 (b) –4
(c) 8 (d) –8

219. A square matrix A is called orthogonal if [2018-II]
(a) $A = A^2$ (b) $A' = A^{-1}$
(c) $A = A^{-1}$ (d) $A = A'$
Where A' is the transpose of A.

220. For a square matrix A, which of the following properties hold? [2018-II]
1. $(A^{-1})^{-1} = A$
2. $\det(A^{-1}) = \dfrac{1}{\det A}$
3. $(\lambda A)^{-1} \lambda A^{-1}$ where λ is a scalar

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

221. Which one of the following factors does the expansions of the determinant $\begin{pmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^3 & 27 \end{pmatrix}$ contain? [2018-II]
(a) $x - 3$ (b) $x - y$
(c) $Y - 3$ (d) $x - 3y$

222. What is the adjoint of the matrix
$\begin{pmatrix} \cos(-\theta) - \sin(-\theta) \\ -\sin(-\theta) \cos(-\theta) \end{pmatrix}$? [2018-II]
(a) $\begin{pmatrix} \cos\theta - \sin\theta \\ -\sin\theta \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta \sin\theta \\ \sin\theta \cos\theta \end{pmatrix}$
(c) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

223. If A and B are two invertible square matrices of same order, then what is $(AB)^{-1}$ equal to? [2018-II]
(a) $B^{-1}A^{-1}$ (b) $A^{-1}B^{-1}$
(c) $B^{-1}A$ (d) $A^{-1}B$

224. If a + b + c = 0, then one of the solution of [2018-II]
$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is
(a) $x = a$
(b) $x = \sqrt{\dfrac{3(a^2+b^2+c^2)}{2}}$
(c) $x = \sqrt{\dfrac{2(a^2+b^2+c^2)}{3}}$
(d) $x = 0$

225. What should be the value of x so that the matrix $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$ does not have an inverse? [2018-II]
(a) 16 (b) – 16
(c) 8 (d) – 8

226. The system of equation [2018-II]
$2x + y - 3z = 5$
$3x - 2y + 2z = 5$ and
$5x - 3y - z = 16$
(a) is inconsistent
(b) is consistent, with a unique solution
(c) is consistent, with infinitely many solutions
(d) has its solution lying along x-axis in three - dimensional space

227. If u, v and w (all positive) are the p^{th}, q^{th} and r^{th} terms of a GP, the determinant of the matrix [2018-II]
$\begin{pmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{pmatrix}$ is
(a) 0
(b) 1
(c) $(p-q)(q-r)(r-p)$
(d) $\ln u \times \ln v \times \ln w$

228. Consider the following in respect of matrices A, B and C of same order: [2018-II]
1. $(A + B + C)' = A' + B' + C'$
2. $(AB)' = A'B'$
3. $(ABC)' = C'B'A'$

Where A' is the transpose of the matrix A. Which of the above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

229. Let matrix B be the adjoint of a square matrix A, ℓ be the identity matrix of same order as A. If k ($\neq 0$) is the determinant of the matrix A, then what is AB equal to? [2018-II]
(a) ℓ (b) $k\,\ell$
(c) $k^2\,\ell$ (d) $(1/k)\ell$

230. What is the determinant of the matrix $\begin{pmatrix} x & y & y+z \\ z & z & z+x \\ y & z & x+y \end{pmatrix}$? [2018-II]
(a) $(x-y)(y-z)(z-x)$ (b) $(x-z)(z-x)$
(c) $(y-z)(z-x)$ (d) $(z-x)^2(x+y+z)$

231. If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$$

then which one of the following is correct? *[2018-II]*

(a) The triangle ABC is isosceles
(b) The triangle ABC is equilateral
(c) The triangle ABC is scalene
(d) No conclusion can be drawn with regard to the nature of the triangle

232. Consider the following in respect of matrices A and B of same order: *[2018-II]*

1. $A^2 - B^2 = (A+B)(A-B)$
2. $(A-I)(I+A) = 0 \Leftrightarrow A^2 = I$

Where I is the identity matrix and O is the null matrix.

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

233. What is the area of the triangle with vertices *[2018-II]*

$$\left(x_1, \frac{1}{x_1}\right), \left(x_2, \frac{1}{x_2}\right), \left(x_3, \frac{1}{x_3}\right)?$$

(a) $|(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)|$

(b) 0

(c) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1 x_2 x_3}\right|$

(d) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1 x_2 x_3}\right|$

234. If $B = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, then what is adjoint of B equal to? *[2019-I]*

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (d) It does not exist

235. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the matrix A is/an *[2019-I]*

(a) Singular matrix (b) Involutory matrix
(c) Nilpotent matrix (d) Idempotent matrix

236. If A is an identity matrix of order 3, then its inverse (A^{-1}) *[2019-I]*

(a) is equal to null matrix (b) is equal to A
(c) is equal to 3A (d) does not exist

237. A is a square matrix of order 3 such that its determinant is 4. What is the determinant of its transpose? *[2019-I]*

(a) 64 (b) 36
(c) 32 (d) 4

238. If A is a square matrix of order n > 1, then which one of the following is correct? *[2019-I]*

(a) det (–A) = det A (b) det (–A) = $(-1)^n$ det A
(c) det (–A) = – det A (d) det (–A) = n det A

DIRECTION (Qs. 239 - 240) : *Consider the following for the next 02 (two) items :*

Let A and B be (3 × 3) matrices with det A = 4 and det B = 3.

239. What is det (2AB) equal to? *[2019-I]*

(a) 96 (b) 72 (c) 48 (d) 36

240. What is det $(3AB^{-1})$ equal to? *[2019-I]*

(a) 12 (b) 18 (c) 36 (d) 48

ANSWER KEY

1	(d)	25	(b)	49	(a)	73	(c)	97	(a)	121	(d)	145	(b)	169	(c)	193	(b)	217	(a)
2	(c)	26	(c)	50	(c)	74	(b)	98	(a)	122	(b)	146	(a)	170	(c)	194	(c)	218	(a)
3	(a)	27	(d)	51	(c)	75	(d)	99	(a)	123	(b)	147	(a)	171	(a)	195	(c)	219	(b)
4	(d)	28	(c)	52	(a)	76	(b)	100	(d)	124	(d)	148	(b)	172	(b)	196	(b)	220	(a)
5	(b)	29	(b)	53	(b)	77	(d)	101	(a)	125	(c)	149	(c)	173	(c)	197	(d)	221	(a)
6	(c)	30	(b)	54	(a)	78	(a)	102	(c)	126	(c)	150	(c)	174	(d)	198	(b)	222	(a)
7	(c)	31	(b)	55	(a)	79	(b)	103	(a)	127	(b)	151	(a)	175	(b)	199	(a)	223	(a)
8	(d)	32	(d)	56	(d)	80	(d)	104	(b)	128	(c)	152	(c)	176	(a)	200	(d)	224	(d)
9	(a)	33	(c)	57	(c)	81	(c)	105	(d)	129	(a)	153	(c)	177	(a)	201	(c)	225	(b)
10	(c)	34	(c)	58	(b)	82	(b)	106	(a)	130	(a)	154	(d)	178	(a)	202	(d)	226	(b)
11	(c)	35	(c)	59	(b)	83	(b)	107	(a)	131	(b)	155	(d)	179	(d)	203	(b)	227	(a)
12	(d)	36	(b)	60	(a)	84	(c)	108	(c)	132	(a)	156	(a)	180	(a)	204	(a)	228	(c)
13	(d)	37	(c)	61	(d)	85	(c)	109	(c)	133	(b)	157	(b)	181	(b)	205	(b)	229	(b)
14	(c)	38	(b)	62	(c)	86	(d)	110	(b)	134	(b)	158	(b)	182	(b)	206	(a)	230	(d)
15	(a)	39	(c)	63	(c)	87	(a)	111	(a)	135	(c)	159	(a)	183	(d)	207	(c)	231	(a)
16	(b)	40	(b)	64	(d)	88	(d)	112	(d)	136	(a)	160	(d)	184	(a)	208	(c)	232	(b)
17	(d)	41	(a)	65	(b)	89	(c)	113	(b)	137	(a)	161	(b)	185	(d)	209	(b)	233	(d)
18	(c)	42	(b)	66	(a)	90	(d)	114	(b)	138	(c)	162	(c)	186	(d)	210	(a)	234	(a)
19	(c)	43	(b)	67	(d)	91	(d)	115	(a)	139	(d)	163	(b)	187	(c)	211	(d)	235	(b)
20	(b)	44	(c)	68	(b)	92	(c)	116	(c)	140	(a)	164	(c)	188	(b)	212	(b)	236	(b)
21	(a)	45	(d)	69	(b)	93	(c)	117	(a)	141	(c)	165	(d)	189	(c)	213	(b)	237	(d)
22	(a)	46	(b)	70	(b)	94	(a)	118	(a)	142	(b)	166	(d)	190	(d)	214	(b)	238	(b)
23	(c)	47	(c)	71	(d)	95	(d)	119	(c)	143	(a)	167	(a)	191	(a)	215	(a)	239	(a)
24	(c)	48	(c)	72	(c)	96	(c)	120	(a)	144	(d)	168	(a)	192	(d)	216	(a)	240	(c)

HINTS & SOLUTIONS

1. (d) As given $A(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

and $A(\beta) = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

Hence,

$$A(\alpha).A(\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & \sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix}$$

$$= \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{vmatrix} = A_{(\alpha+\beta)}$$

2. (c) $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

Applying $C_1 \to C_1 + C_2$

$$= \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

(Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

$$= \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$f(x) = 2 + 4\sin 2x$

$\therefore$ $-1 \le \sin 2x \le 1$, maximum value of $\sin 2x = 1$

Thus, maximum value of $f(x) = 2 + 4 = 6$

3. (a) A matrix is singular if value of its determinant is zero.

Given that matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular,

$$\begin{vmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 0 \Rightarrow \cos^2\theta - \sin^2\theta = 0 = \cos\frac{\pi}{2}$

$\Rightarrow \cos 2\theta = \cos\frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

4. (d) The given system of equations is

$x + y + z = 2$ ...(i)

$2x + y - z = 3$ ...(ii)

and $3x + 2y + kz = 4$ ...(iii)

This system has a unique solution if

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

We get $\begin{vmatrix} 1 & 0 & 0 \\ 3 & -1 & -3 \\ 3 & -1 & k-3 \end{vmatrix} \neq 0$

$\Rightarrow -1(k-3) - 3 \neq 0$ or $-k + 3 - 3 \neq 0 \quad \Rightarrow k \neq 0$

5. (b) As given $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{vmatrix} 1 & x \\ 0 & 1 \end{vmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & x \\ 0 & -1 \end{bmatrix}$

and $BA = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -x \\ 0 & -1 \end{bmatrix}$

Given that AB = BA

We have, $\begin{bmatrix} 1 & x \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -x \\ 0 & -1 \end{bmatrix}$

$\Rightarrow x = -x \Rightarrow 2x = 0 \Rightarrow x = 0$

6. (c) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Let rows 1 and 2 be interchanged.

and $B = \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}$

$A + B = \begin{bmatrix} a+d & b+e & c+f \\ a+d & b+e & c+f \\ 2g & 2h & 2i \end{bmatrix}$

$|A + B| = \begin{vmatrix} a+d & b+e & c+f \\ a+d & b+e & c+f \\ 2g & 2h & 2i \end{vmatrix}$

$= 0$ (since two rows are identical)

7. (c) $\because A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 2 & 3 & -1 \end{bmatrix}$

Since, adj A = (α_{ij}) so, $\alpha_{ij} = a_{ji}$ of A.

$\therefore \alpha_{23} = a_{32} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ 4 & 4 \end{vmatrix} = -(4 - 12) = 8$

8. (d) Since, A and B be two invertible square matrices each of order n, then $(AB)^{-1} = (B^{-1})(A^{-1})$

$\Rightarrow \frac{\text{adj}(AB)}{|AB|} = \frac{\text{adj } B}{|B|}.\frac{\text{adj } A}{|A|}$

Since $|AB| = |B||A|$

adj (AB) = (adj B) (adj A)

9. (a) As given $M = \begin{bmatrix} 4 & k & 0 \\ 6 & 3 & 0 \\ 2 & t & k \end{bmatrix}$

M will be invertible, if

$\begin{bmatrix} 4 & k & 0 \\ 6 & 3 & 0 \\ 2 & t & k \end{bmatrix} \neq 0$

$\Rightarrow k \neq 0$ or $k(12 - 6k) \neq 0$

$\Rightarrow k \neq 0, k \neq 2$

Thus, statement (1) and (2) are correct.

10. (c) Given matrix is $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

So, $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

and $A^3 = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= 3\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 9\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= 3^2 A$

Similarly $A^4 = 3^3 A$. Hence, $A^n = 3^{n-1} A$

11. (c) Since, AB is defined, neither A nor B is singular i.e., they are non-zero matrix and if AB = 0 both A and B are square matrix. So, A and B are non-zero square matrices.

12. (d) If A is a matrix of order $p \times q$ and B is a matrix of order $s \times t$, then AB will exist when the number of column in A is equal to the number of rows in B

$\Rightarrow q = s$

13. (d) Since, A is a square matrix and

$A - A^T = 0 \Rightarrow A = A^T$.

A is a symmetric matrix.

Considering following two points.

1. No two rows or two columns should be identical and

2. There should be two 1's and one 0, in every row or column.

Such determinant can be found.

14. (c) In the third order determinant $\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$ which satisfied these conditions.

Its value is 2, which is largest if the elements of a determinant are 0 or 1.

15. (a) We know that

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\text{adj}\, A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

and $|A| = ad - bc$

$$A^{-1} = \frac{\text{adj}A}{|A|} \text{ if } |A| \neq 0$$

$$\text{So, } A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Given that $A = \begin{bmatrix} 1+i & 1+i \\ -1+i & 1-i \end{bmatrix}$

Then adj $A = \begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$

and, $|A| = \begin{vmatrix} 1+i & 1+i \\ -1+i & 1-i \end{vmatrix}$

$= (1 - i^2) - (i^2 - 1) = (1 - i^2 - i^2 + 1) = 1 + 1 + 1 + 1 = 4$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$$

16. (b) $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ c-5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+3y \\ 4x+6y \end{bmatrix} = \begin{bmatrix} 5 \\ c-5 \end{bmatrix}$

$\Rightarrow 2x + 3y = 5$ ...(1)

and $4x + 6y = c - 5$ ...(2)

$\Rightarrow$ Solving equation (1) and (2) c = 15

Now, if c = 15, equations 1 and 2 becomes

$\Rightarrow 2x + 3y = 5$ and $4x + 6y = 10$

This shows that the equation has an infinite set of solutions. For other values of c, equation has no solution.

17. (d) We know that $\det(A) = \dfrac{1}{\det(A^{-1})}$

$$A^{-1} = \begin{bmatrix} 1 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\Rightarrow |A^{-1}| = \begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = 2 - 4 = -2 \;\Rightarrow \det(A) = -\frac{1}{2}$$

18. (c) If AB = AC, then

B = C, if A is non-singular.

19. (c) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= abc - a^3 - b^3 + abc + abc - c^3 = 3abc - (a^3 + b^3 + c^3)$

Given that $a^3 + b^3 + c^3 = 0$

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc.$$

20. (b) Given that, $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2+6 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix}$$

Since, $f(x) = x^2 - x + 2$

Putting A in place of x

$f(A) = A^2 - A + 2I$

$$= \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1-1+2 & 8-2+0 \\ 0-0+0 & 9-3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ 0 & 8 \end{bmatrix}$$

21. (a) If there is matrix of m rows and n columns and another with n rows and k columns, their product will be a matrix of m rows and k column.

A is a non-null matrix with one row and 5 columns and B is a non-null matrix with 5 rows and one column. Therefore number of row in A × B is 1.

22. (a) $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+0 & 0+3 \\ 1+0 & 0+4 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2+0 & 3+0 \\ 0+1 & 0+4 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

Also,

$$A + B = \begin{pmatrix} 3 & 3 \\ 1 & 5 \end{pmatrix}$$

$$(A+B)^2 = \begin{pmatrix} 9+3 & 9+15 \\ 3+5 & 3+25 \end{pmatrix} = \begin{pmatrix} 12 & 24 \\ 8 & 28 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4+3 & 6+12 \\ 2+4 & 3+16 \end{pmatrix} = \begin{pmatrix} 7 & 18 \\ 6 & 19 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$A^2 + B^2 + 2AB$

$= \begin{pmatrix} 7 & 18 \\ 6 & 19 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 2\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 12 & 24 \\ 8 & 28 \end{pmatrix} = (A+B)^2$

So, Assertion A is correct
R is AB = BA
Hence, R is correct.
Since this leads from Assersion A, then both A and R are individually true and R is the correct explanation of A.

23. (c) $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}$

$B = \begin{pmatrix} \cos\alpha & \cos\alpha \\ \sin\alpha & \sin\alpha \end{pmatrix}$

$AB = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \cos\alpha \\ \sin\alpha & \sin\alpha \end{pmatrix}$

$\begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \cos^2\alpha + \sin^2\alpha \\ \cos^2\alpha + \sin^2\alpha & \cos^2\alpha + \sin^2\alpha \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq I,$

So, A is true. Since product of two matrix may be equal to identity matrix.
so, R is false and A is true.

24. (c) Let $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = B$

Then $BA = \begin{bmatrix} -1 & 0 \\ 6 & 3 \end{bmatrix}$

$\Rightarrow \; A = B^{-1}\begin{bmatrix} -1 & 0 \\ -6 & 3 \end{bmatrix}$

$|B| = 3,$

$\text{adj } B = \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$

$B^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \; A = \frac{1}{3}\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 6 & 3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3-12 & -6 \\ 6 & 3 \end{bmatrix}$

$= \begin{bmatrix} -5 & -2 \\ 2 & 1 \end{bmatrix}$

Aliter:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

then $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 6 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+2c & b+2d \\ 0+3c & 0+3d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 6 & 3 \end{bmatrix}$

$\Rightarrow$ 3c = 6 or c = 2
3d = 3 or d = 1,
a + 2 × 2 = – 1 or a = – 5
b + 2 × 1 = 0, b = – 2

So, $A = \begin{bmatrix} -5 & -2 \\ 2 & 1 \end{bmatrix}$

25. (b) We know that $A^{-1} = \frac{(\text{adj}A)}{|A|}$

or, $A A^{-1} = \frac{A.Ad_jA}{|A|}$

or, $I_n = \frac{A.Ad_jA}{|A|}$

$A(\text{adj } A) = |A|\, I_n = |A|\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

26. (c) $A = \begin{bmatrix} x & x^2 & 1+x^2 \\ y & y^2 & 1+y^2 \\ z & z^2 & 1+z^2 \end{bmatrix}$

$|A| = \begin{vmatrix} x & x^2 & 1+x^2 \\ y & y^2 & 1+y^2 \\ z & z^2 & 1+z^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$|A| = \begin{vmatrix} x-y & (x-y)(x+y) & (x-y)(x+y) \\ y-z & (y-z)(y+z) & (y-z)(y+z) \\ z & z^2 & 1+z^2 \end{vmatrix}$

$= (x-y)\,(y-z)\begin{vmatrix} 1 & x+y & x+y \\ 1 & y+z & y+z \\ z & z^2 & 1+z^2 \end{vmatrix}$

Applying $C_3 \to C_3 - C_2$

$|A| = (x-y)\,(y-z)\begin{vmatrix} 1 & x+y & 0 \\ 1 & y+z & 0 \\ z & z^2 & 1 \end{vmatrix}$

$= (x-y)\,(y-z)\,[1\{y+z-(x+y)]$
$= (x-y)\,(y-z)\,(z-x)$

27. (d) $A = \begin{bmatrix} 0 & 0 & q \\ 2 & 5 & 1 \\ 8 & p & p \end{bmatrix}$

Matrix A will be singular, when |A| = 0, |A| will be zero when either one row or one column is zero.

(1) For q = 0

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 5 & 1 \\ 8 & p & p \end{bmatrix}$$

$\Rightarrow |A| = 0$

$\therefore$ A is singular.

(2) For p = 0

$$A = \begin{bmatrix} 0 & 0 & q \\ 2 & 5 & 5 \\ 8 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow |A| = q\begin{vmatrix} 2 & 5 \\ 8 & 0 \end{vmatrix} = -40q$$

$\therefore$ A is not singular.

(3) For p = 20

$$|A| = q\begin{vmatrix} 2 & 5 \\ 8 & 20 \end{vmatrix} = 40 - 40 = 0$$

$\therefore$ A is singular.

Thus codes (1) or (3) are correct.

28. (c) We know that, adj A and A has same value of determinant, if det A = 0, then det (adj A) = 0

so, statement (1) is correct.

Also If A is a matrix the determinant of A^{-1} equals inverse of determinant A, so, $\det(A^{-1}) = (\det A)^{-1}$, if A is non singular; Statement 2 is correct.

Thus both (1) and (2) are correct.

29. (b) Let a be an $m \times n$ matrix, then A^{-1} will exist if m = n since only square matrix has determinant and det A $\neq 0$

[Since $A^{-1} = \frac{\text{adj} A}{|A|}$]

30. (b) If A is non-singular and B is singular, then AB and $A^{-1}B$ are non-singular. Statements (2) and (4) are correct.

31. (b) A be a square matrix of order $n \times n$ where $n \geq 2$. B be a matrix obtained from A with first and second rows interchanged. Then, det A = – det B. Since interchanging any two rows makes the sign change with same value.

32. (d) Equations are

$x - y + 2x = 0$ ...(i)

$kx - y + z = 0$ ...(ii)

$3x + y - 3x = 0$ ...(iii)

System of equations posses a unique solution, if

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ k & -1 & 1 \\ 3 & 1 & -3 \end{vmatrix} \neq 0$$

Applying $C_1 \to C_1 + C_2$ and $C_2 \to C_2 + \frac{1}{2}C_3$

$$|A| = \begin{vmatrix} 0 & 0 & 2 \\ k-1 & -\frac{1}{2} & 1 \\ 4 & -\frac{1}{2} & -3 \end{vmatrix} = -\frac{1}{2}\begin{vmatrix} 0 & 0 & 2 \\ k-1 & -1 & 1 \\ 4 & -1 & -3 \end{vmatrix}$$

$$= \frac{1}{2} \times 2[(k-1)(-1) - (4)(-1)] , 0$$

$\Rightarrow -(-k + 1 + 4) \neq 0 \Rightarrow k - 5 \neq 0 \Rightarrow k \neq 5$

Thus, the system does not posses unique solution, if k = 5

33. (c) Given that, $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1+4 & 2+4 \\ 2+4 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix}$$

Let $A^2 + xA + yI = 0$ where x and y are constant.

$$\Rightarrow \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} x & 2x \\ 2x & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5+x+y & 6+2x \\ 6+2x & 8+2x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So, $6 + 2x = 0 \Rightarrow x = -3$

$5 + x + y = 0 \Rightarrow y = -5 - x = -2$

$\Rightarrow A^2 - 3A - 2I = 0$

34. (c) System of equation is given as :

$(k+1)x + 8y = 4k$ .... (1)

and $kx + (k+3)y = 3k - 1$ ....(2)

Here, $a_1 = k + 1$, $b_1 = 8$, $c_1 = 4k$, $a_2 = k$, $b_2 = k + 3$ and $c_2 = 3k - 1$

Such a system of equations will have infinite number of solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e., $\frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$

Taking last two we get

$8(3k-1) = 4k(k+3)$

$\Rightarrow 24k - 8 = 4k^2 + 12k$

$\Rightarrow 4k^2 - 12k + 8 = 0$

$\Rightarrow k^2 - 3k + 2 = 0$

$\Rightarrow (k-1)(k-2)\backslash = 0$

$\Rightarrow k = 1, 2$

Taking first two $(k+1)(k+3) = 8k$

$\Rightarrow k^2 + 4k + 3 - 8k = 0 \Rightarrow k^2 - 4k + 3 = 0$

$\Rightarrow (k-1)(k-3) = 0$

So, k = 1,3.

Combining both, k = 1, 2, 3.

Thus, this system have 3 values of k.

35. (c) The given system of equations are :

$p^3x + (p+1)^3 y = (p+2)^3$ ....(1)

$px + (p+1)y = (p+2)$ ....(2)

$x + y = 1$ ....(3)

This system is consistent, if values of x and y from first two equation satisfy the third equation.

$$\text{which} \Rightarrow \begin{vmatrix} p^3 & (p+1)^3 & (p+2)^3 \\ p & (p+1) & (p+2) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$

$$\Rightarrow \begin{vmatrix} p^3 & (p+1)^3 - p^3 & (p+2)^3 - p^3 \\ p & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow 2(p+1)^3 - 2p^3 - (p+2)^3 + p^3 = 0$

$\Rightarrow 2(p^3 + 1 + 3p^2 + 3p) - 2p^3 - (p^3 + 8 + 12p + 6p^2) + p^3 = 0$

$\Rightarrow 2p^3 + 2 + 6p^2 + 6p - 2p^3 - p^3 - 8 - 12p - 6p^2 + p^3 = 0$

$\Rightarrow -6 - 6p = 0$

$\Rightarrow p = -1$

36. (b) Given matrices are :

$$A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \text{and } A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$AA^{-1} = I$

$$\Rightarrow \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 2x = 1$$

37. (c) Let $A = [a_{ij}]_{m \times m}$ be a matrix and $C = [c_{ij}]_{m \times m}$ be another matrix where cij is the cofactor of a_{ij}.

$\therefore$ The value of $|AC| = |A|^{m+1}$

38. (b) Given matrix is :

$$\begin{vmatrix} x^2 & -2x & -2\omega^2 \\ 2 & \omega & -\omega \\ 0 & \omega & 1 \end{vmatrix} = 0$$

By $C_2 \to C_2 + C_3$, we get

$$\Rightarrow \begin{vmatrix} x^2 & -2x - 2\omega^2 & -2\omega^2 \\ 2 & 0 & -\omega \\ 0 & 1+\omega & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x^2 & -2x - 2\omega^2 & -2\omega^2 \\ 2 & 0 & -\omega \\ 0 & -\omega^2 & 1 \end{vmatrix} = 0$$

$[\because 1 + \omega = -\omega^2]$

$$\Rightarrow \omega^2 \begin{vmatrix} x^2 & -2\omega^2 \\ 2 & -\omega \end{vmatrix} + 1 \begin{vmatrix} x^2 & -2x - 2\omega^2 \\ 2 & -0 \end{vmatrix} = 0$$

$\Rightarrow \omega^2(-\omega x^2 + 4\omega^2) - (-4x - 4\omega^2) = 0$

$\Rightarrow -x^2 + 4\omega + 4x + 4\omega^2 = 0$

$\Rightarrow -x^2 + 4\omega - 4x - 4 - 4\omega = 0 \Rightarrow -x^2 - 4x - 4 = 0$

$\Rightarrow (x+2)^2 = 0 \Rightarrow x = -2$

39. (c) Given matrix is :

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4+4 & 4+4 \\ 4+4 & 4+4 \end{bmatrix} = \begin{bmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 16+16 & 16+16 \\ 16+16 & 16+16 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 32 \\ 32 & 32 \end{bmatrix} = \begin{bmatrix} 2^5 & 2^5 \\ 2^5 & 2^5 \end{bmatrix}$$

Going this way we get

$$A^4 = \begin{bmatrix} 2^7 & 2^7 \\ 2^7 & 2^7 \end{bmatrix} \Rightarrow A^n = \begin{bmatrix} 2^{2n-1} & 2^{2n-1} \\ 2^{2n-1} & 2^{2n-1} \end{bmatrix}$$

40. (b) Number of zeroes in a lower triangular matrix of order $n \times n$ is

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

Number of zeros = 10

$\Rightarrow \frac{n(n+1)}{2} = 10 \Rightarrow n^2 + n - 20 = 0$

$\Rightarrow (n+5)(n-4) = 0 \Rightarrow n = 4 \text{ or } -5$

(– 5 is meaningless)

$\Rightarrow n = 4. \Rightarrow$ order of the matrix is 4×4

41. (a) Let $A = \begin{bmatrix} 1 & p & q \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -p & -q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AA^{-1} = I$

$$\Rightarrow \begin{bmatrix} 1 & p & q \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -p & -q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow x = 1$

42. (b) Given that, $AB = \begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3a+2c & 3b+2d \\ a+2c & b+2d \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$$

$\Rightarrow 3a + 2c = 4$ and $a + 2c = 4$ ....(1)

and $3b + 2d = 11$ and $b + 2d = 5$ ....(2)

From equation set (1) $a = 0$ and $c = 2$ and from equation set (2), $b = 3$ and $d = 1$

$$\Rightarrow B = \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$$

Hence $|B| = \begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix} = 0 - 6 = -6$

43. (b) Let $D = \begin{vmatrix} a+b+c & a+b & a \\ 4a+3b+2c & 3a+2b & 2a \\ 10a+6b+3c & 6a+3b & 3a \end{vmatrix}$

By $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we get :

$$\Rightarrow \begin{vmatrix} a+b+c & a+b & a \\ 2a+b & a & 0 \\ 7a+3b & 3a & 0 \end{vmatrix}$$

By $C_1 \to C_1 - C_2$ gives :

$$\Rightarrow \begin{vmatrix} c & a+b & a \\ a+b & a & 0 \\ 4a+3b & 3a & 0 \end{vmatrix}$$

Again by $R_3 \to R_3 - 3R_2$, we get :

$$D = \begin{vmatrix} c & a+b & a \\ a+b & a & 0 \\ a & 0 & 0 \end{vmatrix}$$

$= a\{0.(a+b) - a.a\}$

$= -a^3$ which is independent of b and c.

44. (c) Given matrix is :

$$X = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\therefore X^2 = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2-6 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$$

So, the given expression is :

$$X^2 - 2X + 3I = \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix} - 2\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} -2 & +4 \\ 0 & -6 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+3 & -8+4 \\ 0 & 9-6+3 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 6 \end{bmatrix} = 2\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = 2X$$

45. (d) Since, adjoint of the square matrix A is B $\Rightarrow \frac{B}{|A|} = A^{-1}$

$$\Rightarrow \frac{AB}{|A|} = AA^{-1} = I$$

$\Rightarrow AB = |A|\, I \Rightarrow AB = \alpha I$

46. (b) $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$

$R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$ and divide whole determinant by abc

$$= \frac{1}{abc}\begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^2 \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

47. (c) Let $D = \begin{vmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -x & 1 \end{vmatrix}$

Expanding along R_1

$$\Rightarrow D = 1\begin{vmatrix} 1 & x \\ -x & 1 \end{vmatrix} - z\begin{vmatrix} -z & x \\ y & 1 \end{vmatrix} - y\begin{vmatrix} -z & 1 \\ y & -x \end{vmatrix}$$

$= (1+x^2) - z(-z-xy) - y(xz-y)$
$= 1 + x^2 + z^2 + xyz - xyz + y^2$
$= 1 + x^2 + y^2 + z^2 = 1 + 1 = 2$

48. (c) As given : $|A_{n\times n}| = 3$ and $|\text{adj}A| = 243$
Dterminant of adjoint A is given by :
$|\text{adj } A| = |A_{n\times n}|^{n-1}$
$\Rightarrow 243 = 3^{n-1} \Rightarrow 3^5 = 3^{n-1} \Rightarrow n-1 = 5 \Rightarrow n = 6$

49. (a) AB and BC both must exit, to hold the condition A (BC) = (AB) C

50. (c) As given of A and B are 3 × 2 and 2 × 3 respectively.
$\Rightarrow$ order of AB is 3 × 3
$\Rightarrow |kAB| = k^3 |AB|$

51. (c) The given equation is : $\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}^{-1} \begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Multiplying both sides by $\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$, we get

$$\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}^{-1} \begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -x \\ -y \end{bmatrix} = \begin{bmatrix} -5 \\ 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} -5 \\ 14 \end{bmatrix} \qquad \Rightarrow x = -5 \text{ and } y = -14$$

52. (a) Slope of both the lines are same and intercepts are different. So, the given equations represent the two parallel lines. Hence the system of linear equations has no solution.

53. (b) The given system of equations is
$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
and $a_3x + b_3y + c_3z = d_3$

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

This system has a unique solution x_0, y_0, z_0 if $\Delta \neq 0$

and $x_0 = \dfrac{\Delta x}{\Delta} = 0 \Rightarrow \Delta x = 0$

$$\Rightarrow \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = 0$$

54. (a) Since, a, b, c are in GP.
$\Rightarrow b^2 = ac$
Explanding the determinant we get,

$$\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix}$$

$$= a\begin{vmatrix} c & b+c \\ b+c & 0 \end{vmatrix} - b\begin{vmatrix} b & b+c \\ a+b & 0 \end{vmatrix} + (a+b)\begin{vmatrix} b & c \\ a+b & b+c \end{vmatrix}$$

$= -a(b+c)^2 + b(a+b)(b+c) + (a+b)(b^2 + bc - ac - bc)$
$= -a(b^2 + c^2 + 2bc) + b(ab + ac + b^2 + bc)$
$= -ab^2 - ac^2 - 2abc + ab^2 + 2abc + b^2c \quad (\because b^2 = ac)$
$= -ac^2 + b^2c = -ac^2 + ac.c.$
$= -ac^2 + ac^2 = 0$

55. (a) For 2 × 2 matrix,
$|A| = |\text{adj } A|$
$= (ab - 0) = ab$

$$\therefore A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{ab}.\begin{pmatrix} a & 0 \\ -1 & b \end{pmatrix}$$

$$|A^{-1}| = \frac{1}{ab}(ab) = 1$$

56. (d) Here, $A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$,

$$B = \begin{bmatrix} 1 \\ m \\ n \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore |A| = -2\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}$$

$= -2(4-1) - 1(-2-1) + (1+2)$
$= -6 + 3 + 3 = 0$

Now, adj $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

$\therefore (\text{adj } A).B = 0$
So, the given system of equations has an infinitely many solutions.

57. (c) Let $\dfrac{1}{x} = u, \dfrac{1}{y} = v$
$\therefore a_1u + b_1v = c_1$ and $a_2u + b_2v = c_2$
Using the method of cross multiplication

$$\frac{u}{b_1c_2 - b_2c_1} = \frac{v}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{\frac{1}{x}}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{\frac{1}{y}}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\frac{\frac{1}{x}}{\Delta_2}=\frac{\frac{1}{y}}{\Delta_3}=-\frac{1}{\Delta_1}$$

$\therefore\ \frac{1}{x}=-\frac{\Delta_2}{\Delta_1}$

and $\frac{1}{y}=-\frac{\Delta_3}{\Delta_1}$

$\Rightarrow x=-\frac{\Delta_1}{\Delta_2}$ and $y=-\frac{\Delta_1}{\Delta_3}$

58. (b) $\begin{vmatrix}\sin 10° & -\cos 10° \\ \sin 80° & \cos 80°\end{vmatrix}$

$= \sin 10° \cos 80° + \sin 80° \cos 10°$
$= \sin 10° \sin 10° + \cos 10° \cos 10°$
$= \sin^2 10° + \cos^2 10° = 1$

59. (b) $\begin{vmatrix}2 & 4 & 0 \\ 0 & 5 & 16 \\ 0 & 0 & 1+p\end{vmatrix}=20$

On expanding along C_1,
$2\{5(1+p)-0\}=20$
$\Rightarrow\ 1+p=2$
$\Rightarrow\ p=1$

60. (a) Let A and B be two matrices such that $AB = A$ and $BA = B$
Now, consider $AB = A$
Take Transpose on both side
$(AB)^T = A^T$
$\Rightarrow\ A^T = B^T . A^T$ ...(1)
Now, BA = B
Take, Transpose on both side
$(BA)^T = B^T$
$\Rightarrow B^T = A^T . B^T$ ...(2)
Now, from equation (1) and (2). we have
$A^T = (A^T . B^T) A^T$
$A^T = A^T (B^T A^T)$
$= A^T (AB)^T$ $\quad (\because (AB)^T = B^T = B^T A^T)$
$= A^T . A^T$
Thus, $A^T = (A^T)^2$

61. (d) Let $B=\begin{bmatrix}1 & 3 \\ 0 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & -1 \\ 0 & 1\end{bmatrix}$

then $B^{-1}=\begin{bmatrix}1 & -3 \\ 0 & 1\end{bmatrix}$

Now, $BA = C$

$\Rightarrow B^{-1}\ BA = B^{-1} C$
$\Rightarrow\ A = B^{-1}C$

$$=\begin{bmatrix}1 & -3 \\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & -1 \\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -4 \\ 0 & 1\end{bmatrix}$$

62. (c) The given system of equations are
$kx+y+z=k-1$
$x+ky+z=k-1$
$x+y+kz=k-1$

$$A=\begin{bmatrix}k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k\end{bmatrix}, B=\begin{bmatrix}k-1 \\ k-1 \\ k-1\end{bmatrix} \text{ and } x=\begin{bmatrix}x \\ y \\ z\end{bmatrix}$$

Now, $|A|=\begin{vmatrix}k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k\end{vmatrix}$

$= k(k^2-1)-1(k-1)+1(1-k)$
$= k^3-k-k+1+1-k$
$= k^3-3k+2$

The given system of equations has no solution, if

$|A|=0$
$\Rightarrow k^3-3k+2=0$
$\Rightarrow (k-1)^2(k+2)=0$
$\Rightarrow k=1$ or $k=-2$

63. (c) $AB=\begin{bmatrix}1 & 2 \\ 3 & 4\end{bmatrix}\begin{bmatrix}a & 0 \\ 0 & b\end{bmatrix}=\begin{bmatrix}a & 2b \\ 3a & 4b\end{bmatrix}$

and $BA=\begin{bmatrix}a & 0 \\ 0 & b\end{bmatrix}\begin{bmatrix}1 & 2 \\ 3 & 4\end{bmatrix}=\begin{bmatrix}a & 2a \\ 3b & 4b\end{bmatrix}$

If $AB = BA$

$\Rightarrow \begin{bmatrix}a & 2b \\ 3a & 4b\end{bmatrix}=\begin{bmatrix}a & 2a \\ 3b & 4b\end{bmatrix}\Rightarrow a=b$

From the above it is clear that there exist infinitely many B's such that AB = BA.

64. (d) Given $M=\begin{bmatrix}3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k\end{bmatrix}$

Now $|M|=\begin{vmatrix}3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k\end{vmatrix}=k(3-8)=-5k$

From statement II, $k \neq 0$ then inverse of M exist (statement I). Thus, statement A implies B as well as B implies A.

65. (b) Given, $\begin{vmatrix} y & x & y+z \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$$

$$\Rightarrow (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$$

Applying $C_2 \to C_1 - C_2$, $C_3 \to C_3 - 2C_1$

$$\Rightarrow (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ z & z-y & x+y-2z \\ x & z-x & z-x \end{vmatrix} = 0$$

$$\Rightarrow (x+y+z)\begin{vmatrix} z-y & x+y-2z \\ z-x & z-x \end{vmatrix} = 0$$

$\Rightarrow (x+y+z)(z-x)(z-y-x-y+2z) = 0$

$\Rightarrow x+y = -z$ or $z = x$

66. (a) Let $\begin{vmatrix} k & b+c & b^2+c^2 \\ k & c+a & c^2+a^2 \\ k & a+b & a^2+b^2 \end{vmatrix} = \Delta$

$$= k\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= k\begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ b^2+c^2 & a^2-b^2 & a^2-c^2 \end{vmatrix}$$

$$= k\begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ b^2+c^2 & (a-b)(a+b) & (a-c)(a+c) \end{vmatrix}$$

$$= k(a-b)(a-c)\begin{vmatrix} 1 & 1 \\ a+b & a+c \end{vmatrix}$$

$= k(a-b)(a-c)(a+c-a-b)$

$= k(a-b)(b-c)(c-a)$

But given $\Delta = (a-b)(b-c)(c-a)$

Thus, $k = 1$

67. (d) Given, $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = -1(-1) = 1 \neq 0$$

$\therefore$ A^{-1} exists

Now, $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow A^2 = I$

68. (b) Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

We have

If A is a square matrix of order n then

$A(\text{adj } A) = |A|.I_n$

Here, $n = 2$

$\therefore$ $A(\text{adj } A) = I_2|A|$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}(12-2) = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

69. (b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\therefore$ $|A| = [0(0) - 0(0) + 1(-1)] = -1$

and adj $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}\text{adj } A$

$$= -\frac{1}{1}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

70. (b) We know, a matrix A is said to be symmetric matrix if $A' = A$ where '′' represents the transpose.

Consider $(AB)' = B'A'$

Since, $(AB)' \neq AB$

$\therefore$ AB is not symmetric.

and consider $(A^2 + B^2)' = (A')^2 + (B')^2 = A^2 + B^2$

$\therefore$ $A^2 + B^2$ is symmetric.

71. (d) (A) Consider $M = \begin{bmatrix} 5 & 10 \\ 4 & 8 \end{bmatrix}$

Now, $|M| = \begin{vmatrix} 5 & 10 \\ 4 & 8 \end{vmatrix} = 40 - 40 = 0$

Since, $|M| = 0$

$\therefore$ M is not invertible.

(R) Since, determinant of M is zero therefore M is singular matrix.

Therefore, A is false and R is true.

72. (c) Let X and Y be two matrices of order 2 × 2 each.

Given, $2X - 3Y = \begin{bmatrix} -7 & 0 \\ 7 & -13 \end{bmatrix}$ ...(i)

and $3X + 2Y = \begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix}$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 2, we get

$6X - 9Y = \begin{bmatrix} -21 & 0 \\ 21 & -39 \end{bmatrix}$ ...(iii)

$6X + 4Y = \begin{bmatrix} 18 & 26 \\ 8 & 26 \end{bmatrix}$ ...(iv)

On subtracting Eqs. (iii) from (iv), we get

$13Y = \begin{bmatrix} 39 & 26 \\ -13 & 65 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$

73. (c) Given $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = 0$

$\Rightarrow$ Expanding along R_3, $1(ab) + c(b + ab + a) = 0$

$\Rightarrow ab + bc + ca + abc = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$

74. (b) Since, matrix A is symmetric and anti-symmetric therefore

$A' = A$ and $A' = -A$

$\Rightarrow A = -A \Rightarrow 2A = 0$

$\Rightarrow A$ is a null matrix

75. (d) Here we see that its diagonal elements are not zero, so it is not anti-symmetric matrix.

Now, $|A| = 1(1+4) + 2(2+6) - 3(4-3)$
$= 5 + 16 - 3 = 18 \neq 0$

Hence, it is non-singular matrix.

76. (b) Given, $\begin{vmatrix} 2a & 3r & x \\ 4b & 6s & 2y \\ -2c & -3t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$

Taking 2 common from C_1 and 3 from C_2 in LHS

$\therefore 2 \times 3 \begin{vmatrix} a & r & x \\ 2b & 2s & 2y \\ -c & -t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$

Taking 2 common from R_2 and – 1 from R_3 in LHS

$\therefore -12 \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$

$\Rightarrow \lambda = -12$

77. (d) Let $\Delta = \begin{vmatrix} 1-i & \omega^2 & -\omega \\ \omega^2 + i & \omega & -i \\ 1 - 2i - \omega^2 & \omega^2 - \omega & i - \omega \end{vmatrix}$

Applying $R_3 \to R_1 - R_2 - R_3$

$= \begin{vmatrix} 1-i & \omega^2 & -\omega \\ \omega^2 + i & \omega & -i \\ 0 & 0 & 0 \end{vmatrix} = 0$

($\because$ one row of determinant is zero)

78. (a) Given, $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

$A^3 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix}$

Similarly, $A^{100} = \begin{bmatrix} \omega^{100} & 0 \\ 0 & \omega^{100} \end{bmatrix}$

$= \begin{bmatrix} (\omega^3)^{33}.\omega^1 & 0 \\ 0 & (\omega^3)^{33}.\omega^1 \end{bmatrix}$

$= \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} (\because \omega^3 = 1) = A$

79. (b) The order of a given matrices are $[X]_{(a+b)\times(a+2)}$ and $[Y]_{(b+1)\times(a+3)}$
As $[XY]$ and $[YX]$ exist
$\therefore\ a+2=b+1$ and $a+3=a+b$
$\Rightarrow a+3=a+b$
$\Rightarrow b=3$
Hence, $a=3+1-2=2$

80. (d) Let $\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = 2$

Consider $\begin{vmatrix} 6a & 3b & 15c \\ 2l & m & 5n \\ 2p & q & 5r \end{vmatrix} = 2\times5 \begin{vmatrix} 3a & 3b & 3c \\ l & m & n \\ p & q & r \end{vmatrix}$

$= 10\times3 \begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}$

$= 30 \begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = 30\times2 = 60$

81. (c) $A = \begin{bmatrix} 5 & 6 & 1 \\ 2 & -1 & 5 \end{bmatrix}$ and let $B = \begin{bmatrix} 5 & 2 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}$

$\therefore\ AB = \begin{bmatrix} 5 & 6 & 1 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 25+6+4 & 10+36+3 \\ 10-1+20 & 4-6+15 \end{bmatrix}$

$= \begin{bmatrix} 35 & 49 \\ 29 & 13 \end{bmatrix}$

Hence, option (c) is correct.

ALTERNATE SOLUTION

Given $A = \begin{bmatrix} 5 & 6 & 1 \\ 2 & -1 & 5 \end{bmatrix}_{2\times3}$ and $AB = \begin{bmatrix} 35 & 49 \\ 29 & 13 \end{bmatrix}_{2\times2}$

Since, order of A = 2 × 3
and order of AB = 2 × 2
$\therefore$ order of B = 3 × 2

Let $B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$\therefore\ AB = \begin{bmatrix} 5 & 6 & 1 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$\begin{bmatrix} 35 & 49 \\ 29 & 13 \end{bmatrix} = \begin{bmatrix} 5a+6c+e & 5b+6d+f \\ 2a-c+5e & 2b-d+5f \end{bmatrix}$

$\Rightarrow\ 5a+6c+e=35$

$5b+6d+f=49$

$2a-c+5e=29$

$2b-d+5f=13$

On solving above four equations we get
a = 5, b = 2, c = 1, d = 6, e = 4 and f = 3.

Hence $B = \begin{bmatrix} 5 & 2 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}$

82. (b) If $A' = A$ where A' is transpose of matrix then $|A| = |A'|$

But it is not neccessary that $|A| = 0$
i.e. A is singular matrix
Hence, statement 1 is wrong.
Given $A^3 = I$

$|A^3| = |I| = 1$

$\Rightarrow\ |A| = 1$

Thus, A is non-singular matrix.
Hence, only statement 2 is correct.

83. (b) The given system of equations has infinitely many solution, then

$\frac{2}{2a} = \frac{3}{a+b} = \frac{7}{28}$

$\Rightarrow$ a = 4 and $12 = a+b$
$\Rightarrow$ a = 4 and $b = 8 \Rightarrow b = 2a$

ALTERNATE SOLUTION : Given equations are
$2x+3y=7$
$2ax+(a+b)y=28$

Matrix form by these equations is $\begin{bmatrix} 2 & 3 \\ 2a & (a+b) \end{bmatrix}$

As we know if value of determinant is zero then system of equations have infinitely many solutions.

So, $\begin{vmatrix} 2 & 3 \\ 2a & a+b \end{vmatrix} = 0$

$\Rightarrow\ 2a + 2b - 3 \times 2a = 0$

$\Rightarrow\ 2a + 2b - 6a = 0$

$\Rightarrow\ 2b - 4a = 0 \Rightarrow b = 2a$

84. (c) The equation of given lines are

$3y + 4x = 1$ ...(i)

$y = x + 5$ ...(ii)

and $5y + bx = 3$ ...(iii)

On solving Eqs. (i) and (ii), we get

$x = -2$ and $y = 3$

If these lines are concurrent, then these values must satisfy the third equation

$15 - 2b = 3 \Rightarrow\ 2b = 12 \Rightarrow\ b = 6$

ALTERNATE SOLUTION :

Given equation of lines are

$3y + 4x - 1 = 0$

$x + 5 - y = 0$ and

$bx + 5y - 3 = 0$

Since, the given lines are concurrent

$\therefore$ The value of determinant made by coeff of equations is 0.

ie, $\begin{vmatrix} 4 & 3 & -1 \\ 1 & -1 & 5 \\ b & 5 & -3 \end{vmatrix} = 0$

$\Rightarrow\ 4(3-25) - 3(-3-5b) - 1(5+b) = 0$

$\Rightarrow\ -88 + 4 + 14b = 0$

$\Rightarrow\ -84 = -14b$

$\Rightarrow\ b = 6.$

85. (c) Consider $\begin{vmatrix} \cos 15° & \sin 15° \\ \cos 45° & \sin 45° \end{vmatrix} \times \begin{vmatrix} \cos 45° & \cos 15° \\ \sin 45° & \sin 15° \end{vmatrix}$

$= (\sin 45° \cos 15° - \cos 45° \sin 15°)$

$\times(\cos 45° \sin 15° - \sin\ 45° \cos 15°)$

$= -\sin(45° - 15°) \times \sin(45° - 15°)$

(using $\sin(A - B) = -\cos A \sin B + \sin A \cos B$)

$= -\sin 30° \times \sin 30° = -\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$

86. (d) We know, if A is an $n \times n$ matrix, then

$\det(\lambda A) = \lambda^s \det(A)$

But given $\det(\lambda A) = \lambda^s \det A$

$\Rightarrow\ s = n$

87. (a) We know if A is a real skew-symmetric matrix of order n such that $A^2 + I = 0$, then order of A is 3.

88. (d) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$|A| = 4 \times 1 - 2 \times 3 = 4 - 6 = -2$

$\therefore\ A^{-1} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow\ [b_{ij}] = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

$\Rightarrow\ b_{22} = \frac{1}{2}$

89. (c) Since, the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix}$ is not an invertible matrix.

therefore it's determinant is zero.

$\Rightarrow\ \begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix} = 0$

$\Rightarrow\ 1(-8\lambda - 10) + 3(2\lambda - 20) + 2(4 + 32) = 0$

$\Rightarrow\ -8\lambda - 10 + 6\lambda - 60 + 72 = 0$

$\Rightarrow\ -2\lambda + 2 = 0 \Rightarrow \lambda = 1$

90. (d) Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $C = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ..... (1)

$B^2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ...... (2)

From (1) and (2), we have

$A^2 = B^2$

Now, $C^2 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

.... (3)

From (2) and (3), we have $B^2 = C^2$

Now, $AB = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = C$

Now, we find

$BA = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \neq C$

Hence $AB \neq BA$

91. (d) Let $x+iy=\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 20 & 3 & i\end{vmatrix}$

$\Rightarrow$ $x+iy=6i(3i^2+3)+3i(4i+20)+1(12-60i)$

$=-12+60i+12-60i=0$ ($\because i^2=-1$)

$\Rightarrow$ $x+iy=0$

$\Rightarrow$ $x=0$ and $y=0$

Hence, $x-iy=0-i(0)=0$

92. (c) Let $|A|=8$ and A is a square matrix of order 3.

We know that $|\text{adj } A|=|A|^{n-1}$. I where '$n$' is the order of the matrix A.

$\therefore$ $|\text{adj } A|=8^{3-1}=8^2=64$

93. (c) We know, that A matrix 'A' is said to be symmetric $A=A^T$ and anti- symmetric if $A=-A^T$

Now, consider $(A+A^T)^T=A^T+(A^T)^T=A^T+A$

$\Rightarrow$ $A+A^T$ is always symmetric

Now, consider $(A-A^T)^T=A^T-(A^T)^T=A^T-A$ $=-(A-A^T)$

$\Rightarrow$ $A-A^T$ is always anti- symmetric.

94. (a) Let A be a matrix such that $3A^3+2A^2+5A+I=0$

Post multiply by A^{-1} on both the sides, we get

$3A^3A^{-1}+2A^2A^{-1}+5AA^{-1}+IA^{-1}=0$

$\Rightarrow$ $3A^2+2A+5I+A^{-1}=0$

$\Rightarrow$ $A^{-1}=-(3A^2+2A+5I)$

95. (d) If $AB=0$, then it may be concluded that either $A=0$ or $B=0$

But, it should be noticed that it is not necessary that either $A=0$ or $B=0$.

96. (c) We know $(\text{adj } A^T)=(\text{adj } A)^T$

$\Rightarrow$ $(\text{adj } A^T)-(\text{adj } A)^T=$ Null matrix

97. (a) Consider $\begin{vmatrix}1 & \omega & 2\omega^2\\ 2 & 2\omega^2 & 4\omega^3\\ 3 & 3\omega^3 & 6\omega^4\end{vmatrix}=\begin{vmatrix}1 & \omega & 2\omega^2\\ 2 & 2\omega^2 & 4\\ 3 & 3 & 6\omega\end{vmatrix}$

($\because \omega^3=1$ and $\omega^4=\omega$)

$=2\times 3\begin{vmatrix}1 & \omega & 2\omega^2\\ 1 & \omega^2 & 2\\ 1 & 1 & 2\omega\end{vmatrix}$

$=6[1(2\omega^3-2)-\omega(2\omega-2)+2\omega^2(1-\omega^2)]$

$=6[0-2\omega^2-2\omega+2\omega^2-2\omega]=0$

98. (a) Let $A=\begin{bmatrix}2-x & 1 & 1\\ 1 & 3-x & 0\\ -1 & -3 & -x\end{bmatrix}$

Since, this matrix is singular.

$\therefore$ $|A|=0$

$\Rightarrow \begin{vmatrix}2-x & 1 & 1\\ 1 & 3-x & 0\\ -1 & -3 & -x\end{vmatrix}=0$

Applying $R_2\to R_2+R_3$

$\Rightarrow \begin{vmatrix}2-x & 1 & 1\\ 0 & -x & -x\\ -1 & -3 & -x\end{vmatrix}=0$

$\Rightarrow (2-x)(x^2-3x)-1(-x)+1(-x)=0$

$\Rightarrow (2-x)(x)(x-3)=0$

$\Rightarrow x=2,0,3$

Hence, solution set $S=\{0,2,3\}$

99. (a) The inverse of a square matrix if it exists, is unique but if A and B are singular matrices of order n, then AB is not a singular matrices of order n.

Hence, only statement I is correct.

100. (d) Let $\Delta=\begin{vmatrix}x+1 & x+2 & x+4\\ x+3 & x+5 & x+8\\ x+7 & x+10 & x+14\end{vmatrix}$

By applying $C_2\to C_2-C_1$, $C_3\to C_3-C_1$

$=\begin{vmatrix}x+1 & 1 & 3\\ x+3 & 2 & 5\\ x+7 & 3 & 7\end{vmatrix}$

By applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_2$, we get

$=\begin{vmatrix}x+1 & 1 & 3\\ 2 & 1 & 2\\ 4 & 1 & 2\end{vmatrix}$

$=(x+1)(0)-1(4-8)+3(2-4)$

$=4-6=-2$

101. (a) Given, $\begin{vmatrix}x & 4 & 5\\ 7 & x & 7\\ 5 & 8 & x\end{vmatrix}=0$

$\Rightarrow x(x^2-56)-4(7x-35)+5(56-5x)=0$

$\Rightarrow x^3-56x-28x+140+280-25x=0$

$\Rightarrow x^3-109x+420=0$

$\Rightarrow (x-5)(x-7)(x+12)=0$

$\Rightarrow x=-12$

Hence, the third root is -12.

102. (c) We know,

System of a pair of linear equations in two variables are given as

$a_1x+b_1y+c_1=0$ ... (i)

and $a_2x+b_2y+c_2=0$ ... (ii)

This system has no solution if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Now, on comparing given equations with (i) and (ii), we get

$$a_1=k, a_2=3, b_1=2, b_2=1, c_1=-5, c_2=-1$$

For no solution,

$$\frac{k}{3}=\frac{2}{1}\Rightarrow k=6$$

103. (a) Let $A=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$

$\therefore\quad A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$

$=\begin{bmatrix}\alpha^2+\beta^2 & 2\alpha\beta\\ 2\alpha\beta & \alpha^2+\beta^2\end{bmatrix}$

Now, $A^2=I$

$\Rightarrow\quad\begin{bmatrix}\alpha^2+\beta^2 & 2\alpha\beta\\ 2\alpha\beta & \alpha^2+\beta^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow\quad\alpha^2+\beta^2=1,\qquad\alpha\beta=0$

$\Rightarrow\quad\alpha=0,\beta=1$

or $\quad\beta=0,\alpha=1$

104. (b) Let $A=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$

$\Rightarrow\quad A^2=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$

$\Rightarrow\quad A^2=\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$

But it is given that

$A^2=B$

$\Rightarrow\quad\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$\Rightarrow\quad\alpha+1=2$

$\Rightarrow\quad\alpha=1$

105. (d) Let $A=\begin{bmatrix}3 & 1\\ 0 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$

Now, $AB=\begin{bmatrix}3 & 1\\ 0 & 4\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}=\begin{bmatrix}3 & 5\\ 0 & 8\end{bmatrix}$

and $BA=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ 0 & 4\end{bmatrix}=\begin{bmatrix}3 & 5\\ 0 & 8\end{bmatrix}$

$\Rightarrow\quad AB=BA$

Hence, all the three statements are correct.

106. (a) Given system of equations

$3x+5y=7$ and $6x+10y=18$

This system can be written as

$AX=B$ where

$$A=\begin{pmatrix}3 & 5\\ 6 & 10\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}, B=\begin{pmatrix}7\\ 18\end{pmatrix}$$

Now, $|A|=30-30=0$

and $(\text{adj } A)\ B=\begin{pmatrix}10 & -5\\ -6 & 3\end{pmatrix}\begin{pmatrix}7\\ 18\end{pmatrix}=\begin{pmatrix}-20\\ -96\end{pmatrix}\neq 0$

$\therefore$ system of equations is inconsistent.

$\Rightarrow$ system of equations have no solution.

107. (a) Given $\begin{vmatrix}x & \alpha & 1\\ \beta & x & 1\\ \beta & \gamma & 1\end{vmatrix}=0$

$\Rightarrow x(x-\gamma)-\alpha(0)+1(\gamma\beta-x\beta)=0$

$\Rightarrow x^2-x\gamma+\gamma\beta-x\beta=0$

$\Rightarrow x^2-(\gamma+\beta)x+\gamma\beta=0$

$\Rightarrow x=\dfrac{(\gamma+\beta)\pm\sqrt{\gamma^2+\beta^2-2\gamma\beta}}{2}$

($\because$ roots of Quad. eqn. $ax^2+bx+c=0$ are $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$\Rightarrow x=\dfrac{(\gamma+\beta)\pm(\gamma-\beta)}{2}$

$\Rightarrow x=\dfrac{\gamma+\beta+\gamma-\beta}{2},\dfrac{\gamma+\beta-\gamma+\beta}{2}$

$\Rightarrow x=\gamma,\beta.$

Hence, roots of the equation $\begin{vmatrix}x & \alpha & 1\\ \beta & x & 1\\ \beta & \gamma & 1\end{vmatrix}=0$ are independent of α.

108. (c) Given $\begin{vmatrix}a-b & b+c & a\\ b-c & c+a & b\\ c-a & a+b & c\end{vmatrix}$

$C_1\to C_1-C_3.$

$=\begin{vmatrix}-b & b+c & a\\ -c & c+a & b\\ -a & a+b & c\end{vmatrix}$

$C_2 \to C_2 + C_1.$

$$= \begin{vmatrix} -b & c & a \\ -c & a & b \\ -a & b & c \end{vmatrix} = (-1)\begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$$= -1\left[b\left(ac - b^2\right) - c\left(c^2 - ab\right) + a\left(bc - a^2\right)\right]$$

$$= -\left[abc - b^3 - c^3 + abc + abc - a^3\right]$$

$$= a^3 + b^3 + c^3 - 3abc.$$

109. (c) Let $\begin{vmatrix} p & -q & 0 \\ 0 & p & q \\ q & 0 & p \end{vmatrix} = 0$

$\Rightarrow p(p^2) + q(-q^2) + 0 = 0$
$\Rightarrow p^3 - q^3 = 0$
$\Rightarrow p^3 = q^3$

$\Rightarrow \dfrac{p^3}{q^3} = 1 \Rightarrow \left(\dfrac{p}{q}\right)^3 = 1$

$\Rightarrow \dfrac{p}{q}$ is one of the cube roots of unity.

110. (b) Let $a^{-1} + b^{-1} + c^{-1} = 0$

$\Rightarrow \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 0$

$\Rightarrow \dfrac{bc + ac + ab}{abc} = 0$

$\Rightarrow ab + bc + ca = 0$ ...(1)

Consider $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

$\Rightarrow (1+a)[(1+b)(1+c) - 1] - (1+c-1) + (1-1-b) = \lambda$
$\Rightarrow (1+a)(c+b+bc) - c - b = \lambda$
$\Rightarrow bc + ac + ab + abc = \lambda$
$\Rightarrow abc = \lambda$ (using (1))

111. (a) If the matrix AB is zero then it is not necessary that either $A = 0$ or $B = 0$
therefore statement 2 is incorrect.
Let $AB = 0$

$\Rightarrow |AB| = 0$

$\Rightarrow |A||B| = 0 \Rightarrow$ either $|A| = 0$ or $|B| = 0$

112. (d) Given

$$\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1+9+4 & 3 & 6+2 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 14 & 3 & 8 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow 9 + 8x = 0 \Rightarrow x = \dfrac{-9}{8}$

Hence, for $x = \dfrac{-9}{8}$

$\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix} = (0)$ holds.

113. (b) Only statement - 3 is correct

Unit matrix $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

114. (b) Since, Inverse is unique.
$\therefore$ B should be equal to C.

115. (a) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix}$

So, $|AB| = 0$ ($\because$ one column is zero)

116. (c) Consider $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

Take out 'a', 'b' and 'c' common from R_1, R_2 and R_3 respectively.

$$= abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Now, take out 'a', 'b' and 'c' common from C_1, C_2 and C_3 respectively.

$$= a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Applying $R_2 \to R_2 + R_1,\ R_3 \to R_3 + R_1$

$$= a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix} = a^2b^2c^2\,[(-1)(-4)]$$

$= 4a^2b^2c^2$.

117. (a) Given AB = A and BA = B

Consider $B = BA = B(AB)\ (\because AB = A)$

$= (BA).B = B.B = B^2$

Hence, $B^2 = B$

118. (a) (1) Since sum of matrices exist therefore A and B are square matrices of same order.

(2) Non-singularity of A and B does not depend on sum and product of A and B.

119. (c) Let $A^2 = I$

$\Rightarrow A^2A^{-1} = IA^{-1}$

$\Rightarrow A = A^{-1}$

120. (a) It is a property.

121. (d) Since $\begin{vmatrix} 8 & -5 & 1 \\ 5 & x & 1 \\ 6 & 3 & 1 \end{vmatrix} = 2$

$\Rightarrow 8(x-3) + 5(5-6) + 1(15-6x) = 2$

$\Rightarrow 8x - 24 - 5 + 15 - 6x = 2$

$\Rightarrow 2x = 16$

$\Rightarrow x = 8$

122. (b) Given $\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Consider $\begin{bmatrix} x & y & z \end{bmatrix}_{1\times 3}\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3\times 3}$

$\Rightarrow$ order of product = 1 × 3

Now, order of product = 1 × 3 and order of $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3\times 1$

$\therefore$ Required order = 1 × 1

123. (b) $|A| = -1,\ |B| = 1$

$$A^{-1} = \frac{1}{|A|}\text{Adj}\,A = \frac{1}{-1}\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\text{adj}\,B = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

124. (d) As we know that if each element of any row (or column) of a determinant is multiplied by the same number, then the value of determinant is multiplied by that number.

125. (c) The inverse of a diagonal matrix is a diagonal matrix.

126. (c) We have,

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \end{bmatrix} = B$$

$\therefore$ B is the transpose of A

127. (b) We have,

$$\begin{bmatrix} x \\ x \\ y \end{bmatrix} + \begin{bmatrix} y \\ y \\ z \end{bmatrix} + \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$$

$\Rightarrow x + y + z = 10$ .... (i)

$x + y = 5$ .... (ii)

$y + z = 5$ .... (iii)

from (ii), $x = 5 - y$

from (iii), $z = 5 - y$

$\therefore$ from (i),

$5 - y + y + 5 - y = 10$

$\Rightarrow 10 - y = 10 \Rightarrow \boxed{y = 0}$

128. (c) If matrix AB = 0 then it is not necessary that either A is zero matrix or B is zero matrix.

129. (a) Let $A = \begin{bmatrix} \alpha & 2 & 2 \\ -3 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} \alpha & 2 & 2 \\ -3 & 0 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$|A| = \alpha(0 + 4) - 2(-3 - 4) + 2(3 - 0) = 4\alpha + 20$

Since A^{-1} does not exist,

$\therefore\ |A| = 0$

$4\alpha + 20 = 0$

$4\alpha = -20$

$\boxed{\alpha = -5}$

130. (a) Let $\Delta = \begin{vmatrix} x^2 & 1 & y^2+z^2 \\ y^2 & 1 & z^2+x^2 \\ z^2 & 1 & x^2+y^2 \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$.

$$\Delta = \begin{vmatrix} x^2+y^2+z^2 & 1 & y^2+z^2 \\ x^2+y^2+z^2 & 1 & z^2+x^2 \\ x^2+y^2+z^2 & 1 & x^2+y^2 \end{vmatrix}$$

$$\Delta = \left(x^2+y^2+z^2\right)\begin{vmatrix} 1 & 1 & y^2+z^2 \\ 1 & 1 & z^2+x^2 \\ 1 & 1 & x^2+y^2 \end{vmatrix}$$

$\Delta = 0 \left[\because C_1 \ \& \ C_2 \text{ are identical}\right]$

131. (b) Scalar Matrix.
We know that, $A = [a_{ij}]_{nxn}$ is called a scalar matrix if $a_{ij} = 0$ for $i \neq j$ and $a_{ij} = k$ for $i = j$ [where k is constant]

132. (a) Since, A and B are two non-singular matrices therefore their determinant is non-zero.
$\therefore$ A^{-1} and B^{-1} defined.
Consider $AB = A \Rightarrow A^{-1} AB = A^{-1} A \Rightarrow B = I$

133. (b) Minor of element 9 = $\begin{vmatrix} 19 & 2 \\ 13 & 1 \end{vmatrix} = 19 - 26 = -7$

134. (b) $1(t-2) - (t-1)[(t-1)^2 - 1] + 1(t-2) = 0$
$\Rightarrow \ (t-2) - t(t-1)(t-2) + (t-2) = 0$
$\Rightarrow \ (t-2)[1-(t-1)(t)+1] = 0$
$\Rightarrow \ (t-2)(t^2-t-2) = 0$
$\Rightarrow \ (t-2)(t-2)(t+1) = 0$
$\Rightarrow \ t = 2, t = -1$
Hence, required roots are –1, 2.

135. (c) Consider $\begin{vmatrix} m & n & p \\ p & m & n \\ n & p & m \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$.

$$= \begin{vmatrix} m+n+p & n & p \\ p+m+n & m & n \\ n+p+m & p & m \end{vmatrix}$$

Take m + n + p common from C_1.

$$= (m+n+p)\begin{vmatrix} 1 & n & p \\ 1 & m & n \\ 1 & p & m \end{vmatrix}$$

$$= (m+n+p)\left[\left(m^2+n^2+p^2\right) - mn - np - pm\right]$$

Hence, value of the determinant has linear factor.

136. (a) Let A be orthogonal matrix, therefore $AA^T = 1$
$\Rightarrow \ |AA^T| = 1 \Rightarrow |A|.\,|A^T| = 1$
$\Rightarrow \ |A|^2 = 1$
$\Rightarrow \ |A| = \pm 1$

137. (a) $D' =$ cofactor D
$\Rightarrow \ |D'| = |\text{cofactor } D|$
$\Rightarrow \ |D'| = |D|^{3-1}$.
$\Rightarrow \ |D'| = |D|^2$.
So $D' = D^2$.

138. (c) Both the statements are correct.

139. (d) If $B = A^{-1}$, then $AB = I$ (identity matrix)
Therefore, statement 1 is false.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, then $IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = 0$

Therefore, statement 2 is not correct.

140. (a) 1. $\begin{pmatrix} 1 & 2 & 1 \\ a & 2a & 1 \\ b & 2b & 1 \end{pmatrix} = 2\begin{pmatrix} 1 & 1 & 1 \\ a & a & 1 \\ b & b & 1 \end{pmatrix} = 2\begin{vmatrix} 1 & 1 & 1 \\ a & a & 1 \\ b & b & 1 \end{vmatrix} = 0$

Hence matrix is singular.

2. $\begin{vmatrix} c & 2c & 1 \\ a & 2a & 1 \\ b & 2b & 1 \end{vmatrix} = 2\begin{vmatrix} c & c & 1 \\ a & a & 1 \\ b & b & 1 \end{vmatrix} = 2\begin{vmatrix} c & c & 1 \\ a & a & 1 \\ b & b & 1 \end{vmatrix} = 0$

Hence matrix is singular.

141. (c) Co-factor of 4 = $(-1)^3 (2 \times 9 - 3 \times 8) = -(-6) = 6$

142. (b) $|\text{adj } A| = |A|^{n-1}$ {n is order of square matrix}
If A is square matrix of order 3, then $|\text{adj } A| = |A|^2$

143. (a) $AB = \begin{pmatrix} i & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$

$AB = -\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = -C$

144. (d) Given, $2A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$$A = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{pmatrix}$$

$$|A| = \frac{1}{4}$$

$$\text{adj } A = \begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{-3}{2} & 1 \end{pmatrix}$$

$$A^{-1} = 4\begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{-3}{2} & 1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ -6 & 4 \end{pmatrix}$$

145. (b) $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \times \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 17 & \lambda \end{pmatrix}$

$$\begin{pmatrix} 1 & -1 \\ 17 & -7 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 17 & \lambda \end{pmatrix}$$

$\lambda = -7$

146. (a) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ac & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Applying $C_3 \to C_2 + C_3$

$$\begin{vmatrix} 1 & bc & ab+bc+ac \\ 1 & ac & ab+bc+ac \\ 1 & ab & ab+bc+ac \end{vmatrix}$$

$$= (ab+bc+ac)\begin{vmatrix} 1 & bc & 1 \\ 1 & ac & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$= (ab+bc+ac) \times 0 = 0$

147. (a) $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = -A$$

Hence, A is skew symmetric matrix

$$|A| = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} = 1(-6) - 2(-3) = -6 + 6 = 0$$

Therefore A is non-invertible.

148. (b) I. $AB = \begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{vmatrix}_{3\times 2} \begin{vmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{vmatrix}_{2\times 3}$

$$= \begin{vmatrix} 5 & 4 & -12 \\ 4 & 5 & -12 \\ 3 & 3 & -8 \end{vmatrix}_{3\times 3}$$

II. $BA = \begin{vmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{vmatrix}_{2\times 3} \begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{vmatrix}_{3\times 2}$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}_{2\times 2}$$

Here, B is not the right inverse of A but B is the left inverse of A.

149. (c) $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

Applying, $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} (a+b+c+x) & b & c \\ (a+b+c+x) & x+b & c \\ (a+b+c+x) & b & c+x \end{vmatrix} = 0$$

$$(a+b+c+x)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$(a+b+c+x)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$(a+b+c+x)\,1.\,x^2 = 0$

$x = 0, -(a+b+c) \quad (\because x \neq 0)$

150. (c) A A is defined only when A is a matrix of order $m \times n$ where $m = n$.
$A \times A = (m \times n)(m \times n) = (m \times n)(n \times n)$ if $m = n$
$= m \times n = n \times n$ or $m \times m$.
= A is a square matrix.

151. (a) We know that, elements of principal diagonals of a skew-symmetric matrix are all zero.

$$A = \begin{vmatrix} 0 & a & b \\ -a & 0 & -c \\ -b & c & 0 \end{vmatrix}_{3\times 3} \Rightarrow |A| \begin{vmatrix} 0 & a & b \\ -a & 0 & -c \\ -b & c & 0 \end{vmatrix}$$

$= abc - abc = 0$

152. (c) If any two adjacent rows or columns of a determinant are interchanged in position, the value of the determinant changes its sign.

153. (c) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$

(Applying $C_1 \to C_1 + C_2 + C_3$)

$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$

[on taking $(a+b+c)$ common from C_1]

$= (a+b+c)\,[1\,(bc - a^2) - b\,(b-a) + c\,(a-c)]$

$= (a+b+c)\,[bc - a^2 - b^2 + ab + ac - c^2]$

$= (a+b+c)\,[-(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= -\frac{1}{2}(a+b+c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2]$

Hence, the determinant value is Negative

154. (d) We have, $AB = A$

$\therefore \quad A^2 = (AB).(AB) = A.(BA)B$

$= ABB$ $\quad (\because BA = B)$

$= AB = A$ $\quad (\because AB = A)$

Also, $B^2 = (BA).(BA) = B.(AB).A$

$= B.A.A$ $\quad (\because AB = A)$

$= B.A = B$ $\quad (\because BA = B)$

Again, $(AB)^2 = (AB).(AB) = A.(BA)B$

$= A.B.B$ $\quad (\because BA = B)$

$= A.B = A$ $\quad (\because AB = A)$

155. (d) $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$= 6i\,[3i^2 + 3] + 3i\,[4i + 20] + 1\,[12 - 60i]$

$= 6i\,[-3 + 3] + 12i^2 + 60i + 12 - 60i$

$= -12 + 12 = 0 = x + iy$

$\therefore x = 0$

156. (a) $\because \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$

Let $B = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ and $|B| = 1$

$\therefore B^{-1} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$ $\quad \left(\because A^{-1} = \frac{1}{|A|}\text{adj}\,A\right)$

$\therefore A = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$

157. (b) 1. We know that, determinant is not a square matrix, so it is not a true statement.
2. It is true that, determinant is a number associated with a square matrix.
Hence, Statement 2 is correct

158. (b) $\det(A^{-1}) = \frac{1}{\det A}$

159. (a) From the matrix equation, $AB = AC$, where A, B and C are the square matrices of same order.
We can conclude $B = C$ provided and A is non-singular.

160. (d) $\because A = A'$

$\Rightarrow \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix} = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$

$\Rightarrow 2x - 3 = x + 2$

$\therefore x = 5$

161. (b) $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix}$

$= a\,[a^2 - 0] - b\,[-b^2] + 0$

$= a^3 + b^3 = 0$

$\Rightarrow a^3 = -b^3$

$\Rightarrow \left(\frac{a}{b}\right)^3 = -1$

Hence, $\frac{a}{b}$ is one of the cube roots of -1

162. (c) We know that, $|kA| = k^n|A|$, where n is order of matrix A.

$\therefore |3AB| = 3^2|A||B|$ $\quad (\because |AB| = |A||B|)$

$= 9(-1)(3)$

$= -27$ $\quad (\because |A| = -1,\ |B| = 3)$

163. (b) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

An elementary matrix has each diagonal element 1. So, option (b) is correct answer.

164. (c) $A = \begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}\text{adj}(A)$

$= \frac{1}{(10-7)}\begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 3A^{-1} = \begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$

Now, $A + 3A^{-1} = \begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 7I$ where I is Identity Matrix.

165. (d) The given matrix $A = \begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$

Now, from options:

From option (a): For *Symmetric matrix*

$A^T = A$

Now, $A^T = \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix} \neq A$

∴ The given matrix is not symmetric

∴ option (a) is wrong.

From option (b): For *Skew-symmetric matrix*

$A^T = -A$

$= \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix} \neq -A$

∴ Given matrix is not skew-symmetric

∴ option (b) is wrong.

From option (c): For *Hermitian matrix*

$A^T = \overline{A}$, where $\overline{A}$ is conjugate of matrix A

$\overline{A} = \begin{bmatrix} 0 & -4-i \\ 4-i & 0 \end{bmatrix} \neq A^T$

∴ option (c) is wrong.

From option (d): For *Skew-Hermitian matrix*

The diagonal element of a skew-hermitian matrix are pure imaginary or zero.

$A = \begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$

Here, diagonal element indicates that the given matrix is skew-hermitian matrix.

∴ option (d) is correct.

166. (d) Non-singular matrix is a matrix whose determinate Value is non-zero.

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Here, A and B are non-singular matrix

Now from Statement 1:

$A + B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

$\det(A+B) = 12 - 4 = 8$

Now $\det(A) = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$

and $\det(B) = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$

Now, $\det(A+B) = 4 \neq \det(A+B)$

∴ Statement 1 is wrong.

Now from Statement 2:

$(A+B)^{-1} = \dfrac{1}{|A+B|} \text{adj}(A+B)$

$= \dfrac{1}{8}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$

and $A^{-1} = \dfrac{1}{|A|} \text{adj}(A) = \dfrac{1}{1}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

and $B^{-1} = \dfrac{1}{|B|} \text{adj}(B) = \dfrac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Now, $A^{-1} + B^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix}$

$= \begin{bmatrix} \frac{8}{3} & \frac{-4}{3} \\ \frac{-4}{3} & \frac{5}{3} \end{bmatrix} \neq (A+B)^{-1}$

∴ Statement 2 is wrong

∴ Option (d) is correct.

167. (a) $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

Now, AX = B

∴ $\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$

⇒ $\begin{bmatrix} 3p+q & -4p-q \\ 3r+s & -4r-s \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$

$3p + q = 5$ ... (i)

$-4p - q = 2$ ... (ii)

$3r + s = -2$ ... (iii)

$-4r - s = 1$ ... (iv)

From equations (i) and (ii), we get

$-p = 7$

∴ $p = -7$

⇒ $q = 5 - 3(-7)$

$q = 26$

From equations (iii) and (iv),

$-r = -1$

∴ $r = 1$

⇒ $s = -2 - 3 = -5$

∴ $s = -5$

Hence, $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$

∴ Option (a) is correct.

168. (a) $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$

$B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Here AB = C

$\therefore \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2(x+y) & -y \\ 4x & -x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$2x + y = 3$ ... (i)

$3x + y = 2$ ... (ii)

From equations (i) and (ii), we get

$x = -1$ and $y = 5$

$\therefore A = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

$= \begin{bmatrix} 16-10 & 20-30 \\ -8+12 & -10+36 \end{bmatrix} = \begin{bmatrix} 6 & -10 \\ 4 & 26 \end{bmatrix}$

$\therefore$ Option (a) is correct.

169. (c) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$= [(1+x)(1+y) - 1] - 1(1+y-1) + 1(1-1-x)$

$= 1 + x + y + xy - 1 - y - x$

$= xy$

$\therefore$ Option (c) is correct.

170. (c) $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Now $E(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

and $E(\beta) = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

Now $E(\alpha)\,E(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha\cdot\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

$= E(\alpha+\beta)$

$\therefore$ Option (c) is correct.

171. (a) $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & x-1 & 1 \\ 2 & 7 & x-3 \end{bmatrix}$

$|A| = 1[(x-1)(x-3) - 7] - 3[(x-3) - 2] + 2[7 - 2(x-1)]$

$= x^2 - 11x + 29$

If inverse will not exist then $|A| = 0$

$x^2 - 11x + 29 = 0$

$x = \dfrac{11 \pm \sqrt{5}}{2}$

172. (b) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} > 0$

$\Rightarrow a(bc-1) - 1(c-1) + 1(1-b) > 0$

$\Rightarrow abc - a - c + 1 + 1 - b > 0$

$\Rightarrow abc + 2 - (a+b+c) > 0$

$\Rightarrow abc > (a+b+c) - 2$

Let; $a = -1$; $b = 0$ & $c = 1$

Then; $0 > -2$ [which is correct]

Hence, $abc = 0$

$\therefore$ After considering all the option; (b) is correct option.s

173. (c) $\alpha + \beta = 90°$

$\begin{vmatrix} \cos^2\dfrac{\alpha}{2} & \sin^2\dfrac{\alpha}{2} \\ \sin^2\dfrac{\beta}{2} & \cos^2\dfrac{\beta}{2} \end{vmatrix}$

$= \cos^2\dfrac{\alpha}{2}\cos^2\dfrac{\beta}{2} - \sin^2\dfrac{\alpha}{2}\sin^2\dfrac{\beta}{2}$

$= \left(\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2} + \sin\dfrac{\alpha}{2}\sin\dfrac{\beta}{2}\right)\left(\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2} - \sin\dfrac{\alpha}{2}\sin\dfrac{\beta}{2}\right)$

$= \cos\dfrac{(\alpha-\beta)}{2} \times \cos\dfrac{(\alpha+\beta)}{2}$

$= \cos\dfrac{(\alpha-\beta)}{2} \times \cos\dfrac{(90°)}{2}$

$= \cos\dfrac{(\alpha-\beta)}{2} \times \dfrac{1}{\sqrt{2}}$

Maximum value of $\cos\left(\dfrac{\alpha-\beta}{2}\right)$ is 1. So maximum value of determinent is $\left(\dfrac{1}{\sqrt{2}}\right)$

So both 1 and 2 are correct.

174. (d) $\because 2X + 3A = 0$

$\Rightarrow x = \dfrac{-3}{2}A$

$$\Rightarrow x = \frac{-3}{2}\begin{bmatrix} 1 & 0 & -2 \\ 2 & -3 & 4 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -\frac{3}{2} & 0 & 3 \\ -3 & \frac{9}{2} & -6 \end{bmatrix}$$

175. (b) $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$; $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -8 & 7 & -10 \\ 48 & -42 & 60 \\ 40 & -35 & 50 \end{bmatrix}$$

as $AB \neq BA$

So A and B are not commute.

176. (a) If A is a matrix that is invertible then det- (kA) will be k^n. det(A), where n is the order.

$$\therefore \quad [\det(KA)]^{-1}\det(A)$$

$$= \quad [(K)^n \times \det(A)]^{-1}.\det(A)$$

$$= \quad K^{-n} \times \frac{1}{\det(A)} \times \det(A)$$

$$= \quad K^{-n}$$

177. (a) The determinent of a orthogonal matrix is always ± 1

$|A| = \pm 1$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 2 \\ 2 & 5 & 0 \end{bmatrix}$$

$|B| = -10 - 2(-4) + 3(-15)$

$= -47$

$|AB| = |A|\,|B|$

$= (\pm 1)(-47)$

$= \pm 47$

178. (a) $\begin{vmatrix} 1-a & a-b-c & b+c \\ 1-b & b-c-a & c+a \\ 1-c & c-a-b & a+b \end{vmatrix}$

apply $C_2 \to C_2 + C_3$

$$\begin{vmatrix} 1-a & a & b+c \\ 1-b & b & c+a \\ 1-c & c & a+b \end{vmatrix}$$

apply $C_1 \to C_1 + C_2$

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

apply $C_3 \to C_2 + C_3$

$$(a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0$$

179. (d) $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$+\begin{vmatrix} x^3 & \sin x & \cos x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f'(0) = \begin{vmatrix} 0 & \cos 0 & -\sin 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$= -6p^3$$

180. (a) $f''(x) = \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$f''(0) = \begin{vmatrix} 0 & 0 & -\cos 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$= -1\ (6p^2 + p)$

$f''(0) = 0$

$-(6p^2 + p) = 0$

$p(6p+1)=0$

$p=0$ or $p=-\dfrac{1}{6}$

181. (b) Let $[A]=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1}=\frac{1}{|A|}(adj.\ A)$$

$$=\frac{1}{(ad-bc)}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T=\frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$adj(A^{-1})=\frac{1}{(ad-bc)}\begin{bmatrix} a & c \\ b & d \end{bmatrix}^T=\frac{1}{(ad-bc)}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad ...(1)$$

$$adjA=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$|adj\ A|=(ad-bc)$

$$(adj\ A)^{-1}=\frac{1}{|adj\ A|}\begin{bmatrix} a & c \\ b & d \end{bmatrix}^T=\frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad ...(2)$$

Subtracting eqn. (1) and (2),

$$adj(A^{-1})-(adj\ A)^{-1}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= null matrix.

182. (b) $A=\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$$A.A=\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}=-2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$A^2=-2A$

$$A^2.A=-2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$=-2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}=4\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$A^3=4A$

Hence $A^2\neq -A,\ A^3=4A$

183. (d) 1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}=\begin{vmatrix} 40 & 1 & 5 \\ 72 & 7 & 9 \\ 24 & 5 & 3 \end{vmatrix} \quad C_1\to C_1-C_2$

$$=8\begin{vmatrix} 5 & 1 & 5 \\ 9 & 7 & 9 \\ 3 & 5 & 3 \end{vmatrix}=0$$

$\because$ two columns are same so value of determinant is zero.

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}=\begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix} \quad C_2\to C_2+C_3$

$$=(a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}=0$$

$\because$ two columns are same so value of determinant is zero.

3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}=0$

$\because$ diagonal is zero so value of determinant is zero.

184. (a) Linear equations

$Kx+y+z=1$
$x+Ky+z=1$
$x+y+Kz=1$

$$\begin{bmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$A\ X\ =\ B$

$\boxed{X=A^{-1}B}$

Linear equantion will have unique solution when A^{-1} exist :

$|A|\neq 0$

$$\begin{bmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{bmatrix}\neq 0$$

$K(K^2-1)-1(K-1)+1(1-K)\neq 0$

$\Rightarrow K^3-K-K+1+1-K\neq 0$

$\Rightarrow K^3-3K+2\neq 0$

$(K-1)(K^2+K-2)\neq 0$

$(K-1)(K-1)(K+2)\neq 0$

$K\neq 1,\ K\neq 1$ and $K\neq -2$

$\Rightarrow K\neq 1$ and $K\neq -2$

185. (d) $|A|=5\Rightarrow |2A|=2^3\times 5=40$

$\Rightarrow |A^{-1}| = \frac{1}{|A|} \Rightarrow |(2A)^{-1}| = \frac{1}{|2A|} = \frac{1}{40}$

186. (d) $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

$= \begin{bmatrix} ax+hy+gz & hx+by+fz & gx+fy+cz \end{bmatrix}$

187. (c) $ax^3 + bx^2 + cx + d$

$= (x+1)[(x+1)(5x-1) - x(3x+4)] - 2x[(2x+3)(5x-1) - x(2-x)] + 3x[(2x+3)(3x+4) - (2-x)(x+1)]$

$\Rightarrow ax^3 + bx^2 + cx + d = x^3 + 28x^2 + 35x - 1$

$\Rightarrow c = 35$

188. (b) $a + b + c + d = 63$

189. (c) $m\cos\theta - n\sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} - \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

det. $(m\cos\theta - n\sin\theta) = \cos^2\theta - (-\sin^2\theta)$

$= \cos^2\theta + \sin^2\theta = 1.$

190. (d) $f(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ & $f(\phi) \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(\theta) \times f(\phi) = \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - \sin\theta\cos\phi & 0 \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(using Trigonometric Identities)

$\Rightarrow f(\theta) \times f(\phi) = f(\theta + \phi)$.

Also, det, $[f(\theta) \times f(\phi)] = 1[\cos^2(\theta+\phi) - (-\sin^2(\theta+\phi))]$

$= \cos^2(\theta+\phi) + \sin^2(\theta+\phi) = 1.$

& det.$(f(x)) = (\cos^2 x - (-\sin^2 x) = \cos^2 x + \sin^2 x = 1$.

For $x = -x$

det.$(f(-x)) = \cos^2(-x) + \text{Sin}^2(-x) = 1$

Hence, det.$(f(-x))$ = det.$(f(x))$

Hence, det.$(f(x))$ is even function.

191. (a) Here $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ k \end{bmatrix}$ or $AX = B$.

$|A| = 0$ ($\because$ the system does not have a unique solution)

Now, $(\text{Adj } A) = \begin{bmatrix} -3 & -6 & 3 \\ 1 & 2 & -1 \\ 2 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 6 \\ k \end{bmatrix}$

For $k = 15$, $(\text{Adj } A)B = \begin{bmatrix} -24-36+15 \\ 8+12-15 \\ 16+24-30 \end{bmatrix} = \begin{bmatrix} -45 \\ 5 \\ 10 \end{bmatrix} \neq 0$

($\because$ system is inconsistent i.e., it has no solution)

For $k = 20$, $(\text{Adj } A)B = \begin{bmatrix} -24-36+60 \\ 8+12-20 \\ 16+24-40 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$

($\because$ system has infinitely many solutions)

192. (d) Here, $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$|A| = 3 - (-2) = 5$ and $|B| = -4 - (-3) = -1$

$\Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ and $B^{-1} = -1\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}$ and $A^{-1}B^{-1} = \frac{1}{5}\begin{bmatrix} 5 & 7 \\ -5 & 8 \end{bmatrix}$

$\Rightarrow AB(A^{-1}B^{-1}) = \frac{1}{5}\begin{bmatrix} -10 & -61 \\ 5 & 7 \end{bmatrix} \neq 1.$

$|AB| = 0 - 5 = -5$

$\therefore (AB)^{-1} = \frac{-1}{5}\begin{bmatrix} 0 & -5 \\ -1 & 3 \end{bmatrix} \neq A^{-1}B^{-1}$

193. (b) $A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$

$(\text{Adj } A) = \begin{bmatrix} 3\lambda+6 & 15-3\lambda & -21 \\ -(7\lambda+4) & 2\lambda-10 & 39 \\ 15 & 0 & -15 \end{bmatrix}$

For infinitely many solutions :

$(\text{Adj } A)B = 0 \Rightarrow \begin{bmatrix} 27\lambda+54+120-24\lambda-21\mu \\ -63\lambda-36+16\lambda-80+39\mu \\ 135+0-15\mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Hence, $\mu = 9$ and $\lambda = 5$.

194. (c) For unique solution :

$|A| \neq 0 \Rightarrow 2(3\lambda+6) - 3(7\lambda+4) + 5(21-6) \neq 0$

$\Rightarrow \lambda \neq 5$.

and μ can have any real value.

195. (c) $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$, det $(A^3) = 125$

$|A^3| = 125 \Rightarrow |A| = 5$

$\therefore \alpha 2 - 4 = 5 \Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$

196. (b) $|B^{-1}AB| = |B^{-1}|\,|A|\,|B|$

$= \frac{1}{|B|}|A||B| \qquad \because \quad |B^{-1}| = \frac{1}{|B|}$

$= |A|$

197. (d) $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$

We know, the value of symmetric matrix's determinant is 0 $\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

In the given matrix, determinant is 0, if $x = 0$

198. (b) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$AA^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

199. (a) $x + 2y + 3z = 1$
$2x + y + 3z = 2$
$5x + 5y + 9z = 4$
Writing in matrix form, $Ax = B$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{bmatrix}$

$|A| = 1\,(9 - 15) - 2\,(18 - 15) + 3\,(10 - 5)$
$= -6 - 2\,(3) + 3\,(5) = -12 + 15 = 3 \neq 0$

So, these equations have unique solution

200. (d) $A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}, B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, c = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$AB = C$

$\Rightarrow \begin{bmatrix} 3x+3y-2y \\ 3x-2x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$\Rightarrow 3x + y = 4\,;\; x + 2y = -2$

Solving these equations,
$$\begin{array}{r} 3x + y = 4 \\ 3x + 6y = -6 \\ (-)\;(-)\;\;(+) \\ \hline -5y = 10 \end{array}$$

$\Rightarrow y = -2$

So, $3x - 2 = 4 \Rightarrow 3x = 6 \Rightarrow x = 2$

$\therefore\; A = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}, \quad A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

201. (c) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix} \quad \begin{matrix} c_2 \to c_2 - c_1 \\ c_3 \to c_3 - c_1 \end{matrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & xyz & 0 \\ 1 & 0 & xyz \end{vmatrix} = 1\left(x^2y^2z^2 - 0\right) = x^2y^2z^2$

202. (d) $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = 0 \Rightarrow x\left(x^2 - 0\right) - y\left(0 - y^2\right) + 0 = 0$

$\Rightarrow x^3 + y^3 = 0$
$\Rightarrow x^3 = -y^3$

$\Rightarrow \frac{x^3}{y^3} = -1 \Rightarrow \left(\frac{x}{y}\right)^3 = -1 \Rightarrow \frac{x}{y} = \sqrt{-1}$

203. (b) determinant of B = 1
Let 'B' be identity matrix

So, $B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

If we interchange any 2 rows, determinant will be – 1

Let, $C = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}, |C| = -1$

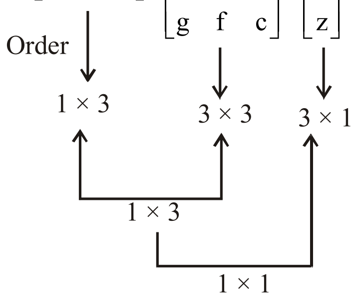
Here, number of elements in B and C are equal.

204. (a) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

We know, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$\therefore\; A^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

205. (b) $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$



206. (a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^4 = A^2 . A^2$

$= \underline{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} . \underline{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

207. (c) Given, Matrix A has x rows and x + 5 columns
Matrix B has y rows and 11 – y columns.
Also given AB and BA exists.
If AB exists, then the number of rows in A most be equal to number of columns in B.
i.e., $x = 11 - y$ ....(1)
If BA exists, then the number of rows in B must be equal to number of rows in A.
i.e., $x + 5 = y$
$\Rightarrow 11 - y + 5 = y$ (from (1))
$\Rightarrow 2y = 16$
$\Rightarrow y = 8.$
$(1) \Rightarrow x = 11 - 8 = 3.$
So, $x = 3, y = 8.$

208. (c) We know, $\text{Adj } A^T = (\text{adj } A)^T$
$\therefore \text{adj } A^T - (\text{adj } A)^T = \text{adj } A^T - \text{adj } A^T = 0.$

209. (b) $\begin{vmatrix} \cos^2\frac{\theta}{2} & \sin^2\frac{\theta}{2} \\ \sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{vmatrix} = \left(\cos^4\frac{\theta}{2} - \sin^4\frac{\theta}{2}\right)$

$= \left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)$

$= (1)\left(\cos 2\left(\frac{\theta}{2}\right)\right)$

$= \cos\theta.$

210. (a) $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix} = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

211. (d) $kx + y + z = 1$
$x + ky + z = k$
$x + y + kz = k^2$

These equations will have no solution of $\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0.$

$\Rightarrow k(k^2 - 1) - 1(k - 1) + (1 - k) = 0$
$\Rightarrow k(k + 1)(k - 1) - 1(k - 1) - (k - 1) = 0$
$\Rightarrow (k - 1)[k(k + 1) - 1 - 1] = 0$
$\Rightarrow (k - 1)(k^2 + k - 2) = 0$
$\Rightarrow k = 1$ or -2.
For k = 1, all equations are same and have infinite solution. So, for k = –2, equations have no solution.

212. (b) $\begin{vmatrix} 1-\alpha & \alpha - \alpha^2 & \alpha^2 \\ 1-\beta & \beta - \beta^2 & \beta^2 \\ 1-\gamma & \gamma - \gamma^2 & \gamma^2 \end{vmatrix}$

$c_1 \to c_1 + c_2 + c_3$

$= \begin{vmatrix} 1 & \alpha - \alpha^2 & \alpha^2 \\ 1 & \beta - \beta^2 & \beta^2 \\ 1 & \gamma - \gamma^2 & \gamma^2 \end{vmatrix}$

$c_2 \to c_2 + c_3$

$= \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

213. (b) $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

$A_{11} = 1 - 0 = 1; A_{12} = -(2 - 0) = -2; A_{13} = 6 - 0 = 6$
$A_{21} = -(0 - 6) = 6; A_{22} = 1 - 0 = 1; A_{23} = -3 - 0 = -3$
$A_{31} = 0 - 2 = -2; A_{32} = -(0 - 4) = 4; A_{33} = 1 - 0 = 1$

$\therefore$ Adj $A = \begin{bmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{bmatrix}$

214. (b) $A = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

$A^2 = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

$= \begin{pmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{pmatrix} = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$

$= -4\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} = -4A$

215. (a) $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pa(rqa^2 - p^2bc) - qb(q^2ac - prb^2) +$ $rc(qpc^2 - r^2ab)$

$= pqra^3 - p^3abc - q^3abc + pqrb^3 + pqrc^3 - r^3abc$

$= pqr(a^3 + b^3 + c^3) - abc(p^3 + q^3 + r^3)$

Given, $p + q + r = a + b + c = 0$

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$ and $a^3 + b^3 + c^3 = 3abc$.

$\therefore pqr(a^3 + b^3 + c^3) - abc(p^3 + q^3 + r^3) = pqr(3abc) - abc(3pqr) = 0$.

216. (a) $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We know, $AA^{-1} = I$.

Let us take first option (a) as $\overline{A}^{\,1}$.

$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta+\sin^2\theta & -\sin\theta\cos\theta+\cos\theta\sin\theta & 0 \\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \sin^2\theta+\cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

217. (a) A is 2×3 matrix
AB is 2×5 matrix
Let 'B' be $m \times n$ matrix

$[A]_{2\times3}\,[B]_{m\times n} = [AB]_{2\times5}$
number of columns of A = number of rows of B.
$\therefore m = 3$
we can observe that n = 5 from the product.
So, B is 3×5 matrix.

218. (a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1+4 & 2+6 \\ 2+6 & 4+9 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

$A^2 - kA - I_2 = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5-k-1 & 8-2k-0 \\ 8-2k-0 & 13-3k-1 \end{bmatrix}$

$= \begin{bmatrix} 4-k & 8-2k \\ 8-2k & 12-3k \end{bmatrix}$

Given, $A^2 - kA - I_2 = 0$
$\therefore 4 - k = 0 \Rightarrow k = 4$

219. (b) If $A^{-1} = A^T$, then A is orthogonal matrix.

220. (a) Statement 1 and 2 are correct.
Statement 3 is incorrect because

$(\lambda A)^{-1} = \frac{1}{\lambda}A^{-1}, \lambda \neq 0$

221. (a) On Applying
$C_1 \to C_1 - C_3$

$\begin{vmatrix} x-3 & y & 3 \\ x^2-9 & 5y^3 & 9 \\ x^3-27 & 10y^5 & 27 \end{vmatrix}$

$(x-3)\begin{vmatrix} 1 & y & 3 \\ x+3 & 5y^3 & 9 \\ x^2+9-3x & 10y^5 & 27 \end{vmatrix}$

222. (a) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

($\because$ $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$)

adj $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

223. (a) $(AB)^{-1} = B^{-1}A^{-1}$

224. (d) $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$-\begin{vmatrix} x-(a+b+c) & c & b \\ x-(a+b+c) & b-x & a \\ x-(a+b+c) & a & c-x \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & c & b \\ x & b-x & a \\ x & a & c-x \end{vmatrix} = 0$$

{Applying $C_1 \to C_1 + C_2 + C_3$}

$$\begin{vmatrix} x & c & b \\ 0 & c+x-b & b-a \\ 0 & c-a & b+x-c \end{vmatrix} = 0$$

{Applying $R_2 \to R_1 - R_2$ and $R_3 \to R_1 - R_3$}

$x\{(c+x-b)(b+x-c)-(b-a)(c-a)\} = 0$

$x\{(x^2-(b-c)^2 - bc + ac + ab - a^2\} = 0$

$x(x^2 - a^2 - b^2 - c^2 + ab + bc + ca) = 0$

$x\{x^2 - (a-b)^2 - (b-c)^2 - (c-a)^2\} = 0$

$\therefore$ $\boxed{x = 0}$

225. (b) A matrix does not have an inverse if $|A| = 0$

$$\begin{vmatrix} 2 & 4 \\ -8 & x \end{vmatrix} = 0$$

$\Rightarrow x = -16$

226. (b) Since $\begin{vmatrix} 2 & 1 & -3 \\ 3 & -2 & 2 \\ 5 & -3 & -1 \end{vmatrix} = 26 \neq 0$

$\Rightarrow$ System is consistent with unique solution.

227. (a) Let u, v and w are p^{th}, q^{th} and r^{th} term of the G.P. with first term 'a' and common ratio 'd'.

then,

$u = a.(d)^{p-1} \Rightarrow \text{In } u = \text{In}(a) + (p-1)\,\text{In}(d)$

$v = a.(d)^{q-1} \Rightarrow \text{In } v = \text{In}(a) + (q-1)\,\text{In}(d)$

$w = a.(d)^{r-1} \Rightarrow \text{In } w = \text{In}(a) + (r-1)\,\text{In}(d)$

Now, $\text{In } u - \text{In } v = (p-q)\,\text{In }(d)$

$\text{In } u - \text{In } w = (p-r)\,\text{In }(d)$

$$\begin{vmatrix} \text{In } u & p & 1 \\ \text{In } v & q & 1 \\ \text{In } w & r & 1 \end{vmatrix}$$

$$\begin{vmatrix} \text{In } u & p & 1 \\ \text{In } u - \text{In } v & (p-q) & 0 \\ \text{In } u - \text{In } w & (p-r) & 0 \end{vmatrix}$$

{Applying $R_2 \to R_1 - R_2$ and $R_3 \to R_1 - R_3$}

or, $\begin{vmatrix} \text{In } u & p & 1 \\ (p-q)\text{In}(d) & (p-q) & 0 \\ (p-r)\text{In}(d) & (p-r) & 0 \end{vmatrix}$

$$= (p-q)(p-r)[\text{In}(d) - \text{In}(d)] = 0$$

228. (c) By property, statement 1 and 3 are correct.

229. (b) B = adj A, I = Identity matrix, $|A| = k$

AB = A(adj A) = $|A|$I = kI.

230. (d) $\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{vmatrix} = R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$= (x+y+z)(z+x)^2$, (replacing z by x)

231. (a) $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix}$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\sin A & \sin A - \sin B & \sin A - \sin C \\ \sin A + \sin^2 A & \sin^2 A - \sin^2 B & \sin^2 A - \sin^2 C \end{vmatrix} + 0 = 0$$

{Applying $C_2 \to C_1 - C_2$ and $C_3 \to C_1 - C_3$}

$= (\sin A - \sin B) \times (\sin A - \sin C)$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\sin A & 1 & 1 \\ \sin A + \sin^2 A & (\sin A + \sin B) & (\sin A + \sin C) \end{vmatrix} = 0$$

$(\sin A - \sin B)(\sin A - \sin C)(\sin B - \sin C) = 0$

$\therefore$ sin A = sin B or sin A = sin C or sin B = sin C either A = B or B = C or A = C.

232. (b) Matrix product is commutative if both are diagonal matrices of same order.

$\Rightarrow$ $A^2 - B^2 = (A+B)(A-B)$ is not true.

Next, $(A-I)(A+I) = 0$

$\Rightarrow$ $A^2 + AI - IA - I^2 = 0 (\because AI = IA)$

$\Rightarrow$ $A^2 = I$ is correct.

233. (d)

$$\text{Area} = \frac{1}{2}\left| x_1\left(\frac{1}{x_2} - \frac{1}{x_3}\right) + x_2\left(\frac{1}{x_3} - \frac{1}{x_1}\right) + x_3\left(\frac{1}{x_1} - \frac{1}{x_2}\right) \right|$$

$$= \frac{1}{2}\left| \frac{x_1(x_3 - x_2)}{x_2 x_3} + \frac{x_2(x_1 - x_3)}{x_1 x_3} + \frac{x_3(x_2 - x_1)}{x_1 x_2} \right|$$

$$= \frac{1}{2}\left| \frac{-x_1^2(x_2 - x_3) - x_2^2(x_3 - x_1) - x_3^2(x_1 - x_2)}{x_1 x_2 x_3} \right|$$

$$= \left| \frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2(x_1 x_2 x_3)} \right|$$

234. (a) $B = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

C factor matrix of

$$B = \begin{bmatrix} (4)(0)-(0)(1) & -[(2)(0)-(0)(1)] & (2)(1)-(4)(1) \\ -[2(0)-(0)(1)] & 3(0)-(0)(1) & -[(3)(1)-(2)(1)] \\ 2(0)-0(4) & -[3(0)-2(0)] & 3(4)-2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & 8 \end{bmatrix}$$

Adjoint $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 8 \end{bmatrix}$

235. (b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(0)(0) & (1)(0)+(0)(1) \\ (0)(1)+(1)(0) & (0)(0)+(1)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$$

$\therefore$ It is involuntary matrix.

236. (b) Given, A is an identity matrix.

$\therefore A = I$

We know, $I^{-1} = I$

$\therefore A^{-1} = A$

237. (d) The determinant of transpose will not change. So, the determinant is equal to 4.

238. (b) A is square matrix of order $n > 1$.

$\det(-A) = (-1)^n \det A$

239. (a) A and B are (3×3) matrices

$\det A = 4$

$\det B = 3$

$\therefore \det(2AB) = (2)^3 |A|\,|B|$

$= 2^3 (4)(3) = 8(4)(3) = 96$

240. (c) $\det(3AB^{-1}) = (3)^3 |A|\,|B^{-1}|$

$$= 27\frac{|A|}{|B|} = \overset{9}{\cancel{27}} \times \frac{4}{\underset{1}{\cancel{3}}} = 36$$

Probability and Probability Distribution 21

1. From past experience it is known that an investor will invest in security A with a probability of 0.6, will invest in security B with a probability 0.3 and will invest in both A and B with a probability of 0.2. What is the probability that an investor will invest neither in A nor in B ?
(a) 0.7 (b) 0.28
(c) 0.3 (d) 0.4 *[2006-I]*

2. Five coins whose faces are marked 2, 3 are thrown. What is the probability of obtainining a total of 12 ?
(a) $\frac{1}{16}$ (b) $\frac{3}{16}$
(c) $\frac{5}{16}$ (d) $\frac{7}{16}$ *[2006-I]*

3. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.
Assertion (A) : If $P(A) = \frac{3}{4}$ and $P(B) = \frac{3}{8}$, then $P(A \cup B) \geq \frac{3}{4}$
Reason (R) : $P(A) \leq P(A \cup B)$ and $P(B) \leq P(A \cup B)$; hence $P(A \cup B) \geq \max. \{P(A), P(B)\}$
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2006-II]*

4. An aircraft has three engines A, B and C. The aircraft crashes if all the three engines fail. The probabilities of failure are 0.03, 0.02 and 0.05 for engines A, B and C respectively. What is the probability that the aircraft will not crash?
(a) 0.00003 (b) 0.90
(c) 0.99997 (d) 0.90307 *[2006-II]*

5. A coin is tossed three times. What is the probability of getting head and tail (HTH) or tail and head (THT) alternatively ?
(a) 1/4 (b) 1/5
(c) 1/6 (d) 1/8 *[2006-II]*

6. The probability that a student passes in mathematics is 4/9 and that he passes in physics is 2/5. Assuming that passing in mathematics and physics are independent of each other, what is the probability that he passes in mathematics but fails in physics ?
(a) $\frac{4}{15}$ (b) $\frac{8}{45}$
(c) $\frac{26}{45}$ (d) $\frac{19}{45}$ *[2006-II]*

7. From a pack of 52 cards, two cards are drawn, the first being replaced before the second is drawn. What is the probability that the first is a diamond and the second is a king ?
(a) $\frac{1}{4}$ (b) $\frac{4}{13}$
(c) $\frac{1}{52}$ (d) $\frac{4}{15}$ *[2006-II]*

8. What is the probability of having a knave and a queen when two cards are drawn from a pack of 52 cards ?
(a) $\frac{16}{663}$ (b) $\frac{2}{663}$
(c) $\frac{4}{663}$ (d) $\frac{8}{663}$ *[2006-II]*

9. Consider the following statement:
"The mean of a binomial distribution is 3 and variance is 4."
Which of the following is correct regarding this statement?
(a) It is always true
(b) It is sometimes true
(c) It is never true
(d) No conclusion can be drawn *[2006-II]*

10. In throwing of two dice, what is the number of exhaustive events ?
(a) 6 (b) 12
(c) 36 (d) 18 *[2006-II]*

11. What is the probability of getting five heads and seven tails in 12 flips of a balanced coin?
(a) $C(12, 5)/(2^5)$ (b) $C(12, 5)/(2^7)$
(c) $C(12, 5)/(2^{12})$ (d) $C(12, 7)/(2^6)$ *[2007-I]*

12. In a lottery, 16 tickets are sold and 4 prizes are awarded. If a person buys 4 tickets,what is the probability of his winning a prize?
(a) $\frac{4}{16^4}$ (b) $\frac{175}{256}$
(c) $\frac{1}{4}$ (d) $\frac{81}{256}$ *[2007-I]*

13. If A and B are any two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$, where $\bar{A}$ stands for the complementary event of A, then what is P(B)?
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{9}$ (d) $\frac{2}{9}$ *[2007-I]*

14. A card is drawn from a pack of 52 cards and a gambler bets that it is a spade or an ace. Which one of the following are the odds against his winning this bet?
(a) 13 to 4 (b) 4 to 13
(c) 9 to 4 (d) 4 to 9 *[2007-I]*

15. A can hit a target 4 times in 5 shots;
B can hit a target 3 times in 4 shots;
C can hit a target 2 times in 3 shots;
All the three fire a shot each. What is the probability that two shots are at least hit?
(a) 1/6 (b) 3/5
(c) 5/6 (d) 1/3 *[2007-I]*

16. A box contains 10 identical electronic components of which 4 are defective. If 3 components are selected at random from the box in succession, without replacing the units already drawn, what is the probability that two of the selected components are defective?
(a) 1/5 (b) 5/24
(c) 3/10 (d) 1/40 *[2007-I]*

17. Each of A and B tosses two coins. What is the probability that they get equal number of heads?
(a) $\frac{3}{16}$ (b) $\frac{5}{16}$
(c) $\frac{4}{16}$ (d) $\frac{6}{16}$ *[2007-II]*

18. Examples of some random variables are given below :
1. Number of sons among the children of parents with five children
2. Number of sundays in some randomly selected months with 30 days
3. Number of apples in some 3 kg packets, purchased from a retail shop

Which of the above is expected to follow binomial distribution?
(a) Variable 1 (b) Variable 2
(c) Variable 3 (d) None of these *[2007-II]*

19. A, B are two events and $\bar{A}$ denotes the complements of A. Consider the following statements *[2007-II]*
1. $P(A \cup B) \le P(B) + P(A)$
2. $P(A) + P(\bar{A} \cup B) \le 1 + P(B)$

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

20. Six text books numbered 1, 2, 3, 4, 5 and 6 are arranged at random. What is the probability that the text books 2 and 3 will occupy consecutive places ?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$ *[2007-II]*

21. What is the probability that in a family of 4 children there will be at least one boy?
(a) $\frac{15}{16}$ (b) $\frac{3}{8}$
(c) $\frac{1}{16}$ (d) $\frac{7}{8}$ *[2008-I]*

22. In a school there are 40% science students and the remaining 60% are arts students. It is known that 5% of the science students are girls and 10% of the arts students are girls. One student selected at random is a girl. What is the probability that she is an arts student?
(a) $\frac{1}{3}$ (b) $\frac{3}{4}$
(c) $\frac{1}{5}$ (d) $\frac{3}{5}$ *[2008-I]*

23. Given $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$. What is $P(\bar{A})$?
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{1}{2}$ *[2008-I]*

24. The outcomes of 5 tosses of a coin are recorded in a single sequence as H (head) and T (tail) for each toss. What is the number of elementary events in the sample space?
(a) 5 (b) 10
(c) 25 (d) 32 *[2008-I]*

25. Which of the following numbers is nearest to the probability that three randomly selected persons are born on three different days of the week?
(a) 0.7 (b) 0.6
(c) 0.5 (d) 0.4 *[2008-I]*

26. One bag contains 5 white balls and 3 black balls and a second bag contains 2 white balls and 4 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?
(a) $\frac{15}{56}$ (b) $\frac{35}{56}$
(c) $\frac{37}{56}$ (d) $\frac{25}{48}$ *[2008-I]*

27. If $P(A) = 0.8, P(B) = 0.9, P(AB) = p$, which one of the following is correct? *[2008-II]*
(a) $0.72 \le p \le 0.8$ (b) $0.7 \le p \le 0.8$
(c) $0.72 < p < 0.8$ (d) $0.7 < p < 0.8$

28. The following questions consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.
Assertion (A) : For a binomial distribution B(n, p), Mean > Variance *[2008-II]*
Reason (R) : Probability is less than or equal to 1
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

29. The chance of winning the race of the horse *A* is 1/5 and that of horse *B* is 1/6. What is the probability that the race will be won by *A* or *B*? *[2008-II]*
(a) 1/30 (b) 1/3
(c) 11/30 (d) 1/15

30. What is the probability of two persons being born on the same day (ignoring date)? *[2008-II]*
(a) 1/49 (b) 1/365
(c) 1/7 (d) 2/7

31. A coin is tossed. If a head is observed, a number is randomly selected from the set {1, 2, 3} and if a tail is observed, a number is randomly selected from the set {2, 3, 4, 5}. If the selected number be denoted by *X*, what is the probability that $X = 3$? *[2008-II]*
(a) 2/7 (b) 1/5 (c) 1/6 (d) 7/24

32. Consider the following statements related to the nature of Bayes' theorem *[2008-II]*
1. Bayes' theorem is a formula for computation of a conditional probability.
2. Bayes' theorem modifies an assumed probability of an event in the light of a related event which is observed.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

33. The outcomes of an experiment classified as success *A* or failure *A* will follow a binomial distribution, if *[2008-II]*
(a) $P(A) = 1/2$
(b) $P(A) = 0$
(c) $P(A) = 1$
(d) $P(A)$ remains constant in all trials

34. If *A*, *B*, *C* are any three arbitrary events, then which one of the following expressions shows that both *A* and *B* occur but not *C*? *[2009-I]*
(a) $A \cap \bar{B} \cap \bar{C}$ (b) $A \cap B \cap \bar{C}$
(c) $A \cap B \cap C$ (d) $A \cap \bar{B} \cap C$

35. By Baye's theorem, which one of the following probabilities is calculated? *[2009-I]*
(a) Prior probability
(b) Likelihood probability
(c) Posterior probability
(d) Conditional probability

36. Given that $P(A) = 1/3$, $P(B) = 1/4$, $P(A/B) = 1/6$, then what is $P(B / A)$ equal to? *[2009-I]*
(a) 1/4 (b) 1/8
(c) 3/4 (d) 1/2

37. If *A* and *B* are two mutually exclusive and exhaustive events with $P(B) = 3P(A)$, then what is the value of $P(\bar{B})$? *[2009-I]*
(a) 3/4 (b) 1/4 (c) 1/3 (d) 2/3

38. Two dice are thrown. What is the probability that the sum of the faces equals or exceeds 10? *[2009-I]*
(a) 1/12 (b) 1/4
(c) 1/3 (d) 1/6

39. For a binomial distribution $B(n, p)$, $np = 4$ and variance $npq = 4/3$. What is the probability $P(x \geq 5)$ equal to? *[2009-I]*
(a) $(2/3)^6$ (b) $(1/3)^6$
(c) $(1/3)^6$ (d) $(2^8/3^6)$

40. When a card is drawn from a well shuffled pack of cards, what is the probability of getting a Queen? *[2009-I]*
(a) 2/13 (b) 1/13
(c) 1/26 (d) 1/52

41. *The following questions consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*
Assertion (A) : The probability of drawing either an ace or a king from a deck of card in a single draw is $\frac{2}{13}$.
Reason (R) : For two events E_1 and E_2, which are not mutually exclusive probability is given by *[2009-I]*
$$P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

42. Three letters are randomly selected from the 26 capital letters of the English Alphabet. What is the probability that the letter '*A*' will not be included in the choice? *[2009-II]*
(a) 1/2 (b) 23/26
(c) 12/13 (d) 25/26

43. A coin is tossed 10 times. The number of heads minus the number of tails in 10 tosses is considered as the outcome of the experiment. What is the number of points in the sample space? *[2009-II]*
(a) 10 (b) 11
(c) 21 (d) 99

44. Two numbers are successively drawn from the set {1, 2, 3, 4, 5, 6, 7} without replacement and the outcomes recorded in that order. What is the number of elementary events in the random experiment? *[2009-II]*
(a) 49 (b) 42
(c) 21 (d) 14

45. The probabilities of two events *A* and *B* are given as $P(A) = 0.8$ and $P(B) = 0.7$. What is the minimum value of $P(A \cap B)$? *[2009-II]*
(a) 0 (b) 0.1 (c) 0.5 (d) 1

46. Two numbers *X* and *Y* are simultaneously drawn from the set { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. What is the conditional probability of exactly one of the two numbers *X* and *Y* being even, given $(X + Y) = 15$? *[2009-II]*
(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

47. Given that $P(A) = 1/3$, $P(B) = 3/4$ and $P(A \cup B) = 11/12$ then what is $P(B/A)$? [2009-II]
(a) 1/6 (b) 4/9
(c) 1/2 (d) 1/3

48. The mean and variance of a binomial distribution are 8 and 4 respectively. What is $P(X = 1)$ equal to? [2010-I]
(a) $\frac{1}{2^{12}}$ (b) $\frac{1}{2^8}$
(c) $\frac{1}{2^6}$ (d) $\frac{1}{2^4}$

49. An observed event B can occur after one of the three events A_1, A_2, A_3. If [2010-I]
$P(A_1) = P(A_2) = 0.4$, $P(A_3) = 0.2$ and $P(B/A_1) = 0.25$, $P(B/A_2) = 0.4$, $P(B/A_3) = 0.125$, what is the probability of A_1 after observing B?
(a) $\frac{1}{3}$ (b) $\frac{6}{19}$
(c) $\frac{20}{57}$ (d) $\frac{2}{5}$

50. The probability distribution of random variable X with two missing probabilities p_1 and p_2 is given below [2010-I]

X	$P(X)$
1	k
2	p_1
3	$4k$
4	p_2
5	$2k$

It is further given that $P(X \le 2) = 0.25$ and $P(X \ge 4) = 0.35$.
Consider the following statements
1. $p_1 = p_2$
2. $p_1 + p_2 = P(X = 3)$
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

51. Consider the following statements: [2010-I]
1. The probability that there are 53 Sundays in a leap year is twice the probability that there are 53 Sundays in a non-leap year.
2. The probability that there are 5 Mondays in the month of March is thrice the probability that there are 5 Mondays in the month of April.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

52. In tossing three coins at a time, what is the probability of getting at most one head? [2010-I]
(a) $\frac{3}{8}$ (b) $\frac{7}{8}$
(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

53. Two balls are selected from a box containing 2 blue and 7 red balls. What is the probability that at least one ball is blue? [2010-I]
(a) $\frac{2}{9}$ (b) $\frac{7}{9}$
(c) $\frac{5}{12}$ (d) $\frac{7}{12}$

54. The probability of guessing a correct answer is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{2}{3}$, then what is x equal to? [2010-I]
(a) 2 (b) 3
(c) 4 (d) 6

55. Consider the following statements related to a variable X having a binomial distribution $b_x(n, p)$
1. If $p = \frac{1}{2}$, then the distribution is symmetrical.
2. p remaining constant, $P(X = r)$ increases as n increases.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only [2010-I]
(c) Both 1 and 2 (d) Neither 1 nor 2

56. What is the probability of having 53 Sundays or 53 Mondays in a leap year? [2010-II]
(a) 2/7 (b) 3/7
(c) 4/7 (d) 5/7

57. Three digital numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that the number has the same digits? [2010-II]
(a) 1/16 (b) 1/25
(c) 16/25 (d) 1/645

58. A lot of 4 white and 4 red balls is randomly divided into two halves. What is the probability that there will be 2 red and 2 white balls in each half? [2010-II]
(a) 18/35 (b) 3/35
(c) 1/2 (d) None of these

59. Consider the following statements : [2010-II]
If A and B are independent events, then
1. A and $\overline{B}$ are independent.
2. $\overline{A}$ and B are independent.
3. $\overline{A}$ and $\overline{B}$ are independent.
Which of the above statements is/are correct? [2010-II]
(a) 3 only (b) 1 and 2 only
(c) 1,2 and 3 (d) None of these

60. An experiment consists of flipping a coin and then flipping it a second time if head occurs. If a tail occurs on the first flip, then a six-faced die is tossed once. Assuming that the outcomes are equally likely, what is the probability of getting one head and one tail? [2011-I]
(a) 1/4 (b) 1/36
(c) 1/6 (d) 1/8

61. A box contains 6 distinct dolls. From this box, 3 dolls are randomly selected one by one with replacement. What is the probability of selecting 3 distinct dolls? [2011-I]
(a) 5 / 54 (b) 12/25
(c) 1/20 (d) 5/9

62. If A and B are events such that $P(A \cup B) = 0.5$, $P(\overline{B}) = 0.8$ and $P(A/B) = 0.4$, then what is $P(A \cap B)$ equal to?
(a) 0.08 (b) 0.02 *[2011-I]*
(c) 0.8 (d) 0.2

63. In an examination, there are 3 multiple choice questions and each question has 4 choices. If a student randomly selects answer for all the 3 questions, what is the probability that the student will not answer all the 3 questions correctly? *[2011-I]*
(a) 1/64 (b) 63/64
(c) 1/12 (d) 11/12

64. If A and B are two mutually exclusive events, then what is $P(AB)$ equal to? *[2011-I]*
(a) 0 (b) $P(A) + P(B)$
(c) $P(A)\,P(B)$ (d) P(A) $P\left(\frac{B}{A}\right)$

65. There are 4 letters and 4 directed envelopes. These 4 letters are randomly inserted into the 4 envelopes. What is the probability that the letters are inserted into the corresponding envelopes? *[2011-I]*
(a) $\frac{11}{12}$ (b) $\frac{23}{24}$
(c) $\frac{1}{24}$ (d) None of these

66. Two letters are drawn at random from the word 'HOME'. What is the probability that both the letters are vowels? *[2011-II]*
(a) 1/ 6 (b) 5/ 6
(c) 1/ 2 (d) 1/ 3

67. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/5 and that of wife's selection is 1/3. What is the probability that only one of them will be selected? *[2011-II]*
(a) 1/5 (b) 2/5
(c) 3/5 (d) 4/5

68. There is a point inside a circle. What is the probability that this point is close to the circumference than to the centre? *[2011-II]*
(a) 3/4 (b) 1/2
(c) 1/4 (d) 1/3

69. In a random arrangement of the letters of the word 'UNIVERSITY', what is the probability that two I's do not come together? *[2011-II]*
(a) 4/5 (b) 1/5 (c) 1/10 (d) 9/10

70. In a class of 125 students 70 passed in Mathematics, 55 passed in Statistics and 30 passed in both. What is the probability that a student selected at random from the class has passed in only one subject? *[2011-II]*
(a) 13/25 (b) 3/25 (c) 17/25 (d) 8/25

71. Three dice are thrown. What is the probability that the same number will appear on each of them? *[2012-I]*
(a) $\frac{1}{6}$ (b) $\frac{1}{18}$ (c) $\frac{1}{24}$ (d) $\frac{1}{36}$

72. What is the probability that a leap year selected at random contains 53 Mondays? *[2012-I]*
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$
(c) $\frac{7}{366}$ (d) $\frac{26}{183}$

73. If four dice are thrown together, then what is the probability that the sum of the numbers appearing on them is 25?
(a) 0 (b) 1/2 *[2012-I]*
(c) 1 (d) 1/1296

74. If P(E) denotes the probability of an event E, then E is called certain event if : *[2012-II]*
(a) P(E) = 0 (b) P(E) = 1
(c) P(E) is either 0 or 1 (d) P(E) = 1/2

75. What is the probability that a leap year selected at random will contain 53 Mondays ? *[2012-II]*
(a) 2/5 (b) 2/7 (c) 1/7 (d) 5/7

76. If A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\overline{A}) = \frac{2}{3}$ where $\overline{A}$ is the complement of A, then what is P(B) equal to ? *[2012-II]*
(a) 1/3 (b) 2/3 (c) 1/9 (d) 2/9

77. Three coins are tossed simultaneously. What is the probability that they will fall two heads and one tail ?
(a) 1/3 (b) 1/2 *[2012-II]*
(c) 1/4 (d) 3/8

78. Which one of the following is correct ? *[2012-II]*
(a) An event having no sample point is called an elementary event.
(b) An event having one sample point is called an elementary event.
(c) An event having two sample points is called an elementary event.
(d) An event having many sample points is called an elementary event.

79. What is the most probable number of successes in 10 trials with probability of success 2/3 ? *[2012-II]*
(a) 10 (b) 7
(c) 5 (d) 4

DIRECTIONS (Qs. 80-81): *For the next two (02) questions that follow*

An urn contains one black ball and one green ball. A second urn contains one white and one green ball. One ball is drawn at random from each urn. *[2012-II]*

80. What is the probability that both balls are of same colour ?
(a) 1/2 (b) 1/3 (c) 1/4 (d) 2/3

81. What is the probability of getting at least one green ball ?
(a) 1/2 (b) 1/3 (c) 2/3 (d) 3/4

DIRECTIONS (Qs. 82-83): *For the next two (02) questions that follow*

Two dice each numbered from 1 to 6 are thrown together. Let A and B be two events given by

A : even number on the first die.
B : number on the second die is greater than 4.

82. What is $P(A \cup B)$ equal to ? [2012-II]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{6}$

83. What is $P(A \cap B)$ equal to ? [2012-II]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{6}$

84. Consider a random experiment of throwing together a die and two coins. The associated sample space has [2013-I]
(a) 8 points (b) 12 points
(c) 24 points (d) 36 points

85. In throwing a six faced die, let A be the event that an even number occurs, B be the event that an odd number occurs and C be the event that a number greater than 3 occurs. Which one of the following is correct? [2013-I]
(a) A and C are mutually exclusive
(b) A and B are mutually exclusive
(c) B and C are mutually exclusive
(d) A, B and C mutually exclusive.

86. What is the probability of getting a sum of 7 with two dice?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ [2013-I]

(c) $\frac{1}{12}$ (d) $\frac{5}{36}$

87. Four coins are tossed simultaneously. What is the probability of getting exactly 2 heads ? [2013-I]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{3}{8}$

88. A bag contains 5 black and 3 white balls. Two balls are drawn at random one after the other without replacement. What is the probability that both are white? [2013-I]

(a) $\frac{1}{28}$ (b) $\frac{1}{14}$

(c) $\frac{3}{28}$ (d) None of these above

89. If A and B are any two events such that $P(\overline{A}) = 0.4,\ P(\overline{B}) = 0.3,\ P(A \cup B) = 0.9,$ then what is the value of $P(\overline{A} \cup \overline{B})$ equal to ? [2013-I]
(a) 0.2 (b) 0.5
(c) 0.6 (d) 0.7

90. A fair coin is tossed repeatedly. The probability of getting a result in the fifth toss different from those obtained in the first four tosses is: [2013-I]

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{16}$

91. If X follows a binomial distribution with parameters n = 100 and p = 1/3, then P(X = r) is maximum when [2013-I]
(a) r = 16 (b) r = 32
(c) r = 33 (d) r = 34

92. Two numbers are successively drawn from the set U = {1, 2, 3, 4, 5, 6, 7, 8}, the second being drawn without replacing the first. The number of elementary events in the sample is:
(a) 64 (b) 56 [2013-I]
(c) 32 (d) 14

93. The binomial distribution has: [2013-I]
(a) only one parameter (b) two parameters
(c) three parameters (d) four parameters

94. A bag contains balls of two colours, 3 black and 3 white. What is the smallest number of balls which must be drawn from the bag, without looking, so that among these three are two of the same colour? [2013-I]
(a) 2 (b) 3
(c) 4 (d) 5

95. If three events A, B, C are mutually exclusive, then which one of the following is correct ? [2013-II]

(a) $P(A \cup B \cup C) = 0$ (b) $P(A \cup B \cup C) = 1$

(c) $P(A \cap B \cap C) = 0$ (d) $P(A \cap B \cap C) = 1$

96. If A and B are independent events such that $P(A) = \frac{1}{5}, P(A \cup B) = \frac{7}{10}$, then what is $P(\overline{B})$ equal to ? [2013-II]

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{3}{8}$ (d) $\frac{7}{9}$

97. In a binomial distribution, the occurrence and the non-occurrence of an event are equally likely and the mean is 6. The number of trials required is [2013-II]
(a) 15 (b) 12
(c) 10 (d) 6

98. A die is tossed twice. What is the probability of getting a sum of 10 ? [2013-II]

(a) $\frac{1}{18}$ (b) $\frac{1}{6}$

(c) $\frac{1}{12}$ (d) $\frac{5}{12}$

99. Three dice are thrown. What is the probability of getting a triplet ? [2013-II]

(a) $\frac{1}{6}$ (b) $\frac{1}{18}$

(c) $\frac{1}{36}$ (d) $\frac{1}{72}$

100. Consider the following statements : *[2013-II]*
 1. If A and B are exhaustive events, then their union is the sample space.
 2. If A and B are exhaustive events, then their intersection must be an empty event.

 Which of the above statements is/are correct ?
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2
101. Which one of the following may be the parameter of a binomial distribution ? *[2013-II]*
 (a) $np = 2, npq = 4$ (b) $n = 4,\ p = \frac{3}{2}$
 (c) $n = 8, p = 1$ (d) $np = 10, npq = 8$
102. What is the number of outcomes when a coin is tossed and then a die is rolled only in case a head is shown on the coin ? *[2013-II]*
 (a) 6 (b) 7
 (c) 8 (d) None of these
103. If $P(A) = \frac{2}{3}, P(B) = \frac{2}{5}$ and $P(A \cup B) - P(A \cap B) = \frac{2}{5}$, then what is $P(A \cap B)$ equal to ? *[2013-II]*
 (a) $\frac{3}{5}$ (b) $\frac{5}{11}$
 (c) $\frac{1}{3}$ (d) None of these
104. What is the propability that there are 5 Mondays in the month of February 2016 ? *[2013-II]*
 (a) 0 (b) $\frac{1}{7}$
 (c) $\frac{2}{7}$ (d) None of these
105. In a relay race, there are six teams *A, B, C, D, E* and *F.* What is the probability that *A, B, C* finish first, second, third respectively ? *[2013-II]*
 (a) $\frac{1}{2}$ (b) $\frac{1}{12}$ (c) $\frac{1}{60}$ (d) $\frac{1}{120}$
106. A box contains 3 white and 2 black balls. Two balls are drawn at random one after the other. If the balls are not replaced, what is the probability that both the balls are black? *[2014-I]*
 (a) 2/5 (b) 1/5
 (c) 1/10 (d) None of these
107. It has been found that if A and B play a game 12 times, A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. The probability that they win alternately, is : *[2014-I]*
 (a) 5/12 (b) 5/36
 (c) 19/27 (d) 5/27

DIRECTIONS (Qs. 108-111) : *For the next four (04) items that follow*

Number of X is randomly selected from the set of odd numbers and Y is randomly selected from the set of even numbers of the set $\{1, 2, 3, 4, 5, 6, 7\}$. Let $Z = (X + Y)$.. *[2014-I]*

108. What is $P(Z = 5)$ equal to ?
 (a) 1/2 (b) 1/3
 (c) 1/4 (d) 1/6
109. What is $P(Z = 10)$ equal to ?
 (a) 0 (b) 1/2
 (c) 1/3 (d) 1/5
110. What is $P(Z = 11)$ equal to ?
 (a) 0 (b) 1/4
 (c) 1/6 (d) 1/12
111. What is $P(Z$ is the product of two prime numbers) equal to ?
 (a) 0 (b) 1/2
 (c) 1/4 (d) None of these
112. Suppose A and B are two events. Event B has occurred and it is known that $P(B) < 1$. What is $P(A|B^c)$ equal to ? *[2014-II]*
 (a) $\frac{P(A) - P(B)}{1 - P(B)}$ (b) $\frac{P(A) - P(AB)}{1 - P(B)}$
 (c) $\frac{P(A) + P(B^c)}{1 - P(B)}$ (d) None of these

DIRECTIONS (Qs. 113-116) : *For the next four (04) items that follow*

Consider events A, B, C, D, E of the sample space S = {n : n is an integer such that $10 \le n \le 20$} given by : *[2014-II]*
A is the set of all even numbers.
B is the set of all prime numbers.
C = (15).
D is the set of all integers ≤ 16.
E is the set of all double digit numbers expressible as a power of 2.

113. A, B and D are
 (a) Mutually exclusive events but not exhaustive events
 (b) Exhaustive events but not mutually exclusive events
 (c) Mutually exclusive and exhaustive events
 (d) Elementary events
114. A, B and C are
 (a) Mutually exclusive events but not exhaustive events
 (b) Exhaustive events but not mutually exclusive events
 (c) Mutually exclusive and exhaustive events
 (d) Elementary events
115. B and C are
 (a) Mutually exclusive events but not exhaustive events
 (b) Compound events
 (c) Mutually exclusive and exhaustive events
 (d) Elementary events
116. C and E are
 (a) Mutually exclusive events but not elementary events
 (b) Exhaustive events but not mutually exclusive events
 (c) Mutually exclusive and exhaustive events
 (d) Elementary and mutually exclusive events
117. For any two events A and B, which one of the following holds ? *[2014-II]*
 (a) $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$
 (b) $P(A \cup B) \le P(A) \le P(A \cap B) \le P(A) + P(B)$
 (c) $P(A \cup B) \le P(B) \le P(A \cap B) \le P(A) + P(B)$
 (d) $P(A \cap B) \le P(B) \le P(A) + P(B) \le P(A \cup B)$

118. The probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two I's do not come together is [2014-II]
(a) 4/5 (b) 1/5
(c) 1/10 (d) 9/10

119. Threre are 4 white and 3 black balls in a box. In another box, there are 3 white and 4 black balls. An unbiased dice is rolled. If it shows a number less than or equal to 3, then a ball is drawn from the second box, otherwise from the first box. If the ball drawn is black then the possibility that the ball was drawn from the first box is [2014-II]
(a) 1/2 (b) 6/7
(c) 4/7 (d) 3/7

120. Two students X and Y appeared in an examination. The probability that X will qualify the examination is 0.05 and Y will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. What is the probability that only one of them will qualify the examination ? [2014-II]
(a) 0.15 (b) 0.14
(c) 0.12 (d) 0.11

121. A fair coin is tossed four times. What is the probability that at most three tails occur ? [2014-II]
(a) 7/9 (b) 15/16
(c) 13/16 (d) 3/4

122. Two men hit at a target with probabilities $\frac{1}{2}$ and $\frac{1}{3}$ respectively. What is the probability that exactly one of them hits the target? [2015-I]
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

123. Two similar boxes B_i $(i = 1, 2)$ contain $(i+1)$ red and $(5-i-1)$ black balls. One box is chosen at random and two balls are drawn randomly. What is the probability that both the balls are of different colours? [2015-I]
(a) $\frac{1}{2}$ (b) $\frac{3}{10}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

124. In an examination, the probability of a candidate solving a question is $\frac{1}{2}$. Out of given 5 questions in the examination, what is the probability that the candidate was able to solve at least 2 questions? [2015-I]
(a) $\frac{1}{64}$ (b) $\frac{3}{16}$
(c) $\frac{1}{2}$ (d) $\frac{13}{16}$

125. If $A \subseteq B$, then which one of the following is not correct? [2015-I]
(a) $P(A \cap \overline{B}) = 0$
(b) $P(A \mid B) = \frac{P(A)}{P(B)}$
(c) $P(B \mid A) = \frac{P(B)}{P(A)}$
(d) $P(A \mid (A \cup B)) = \frac{P(A)}{P(B)}$

126. The mean and the variance in a binomial distribution are found to be 2 and 1 respectively. The probability $P(X = 0)$ is [2015-I]
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

127. If A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$, then what is $P(B)$ equal to? [2015-I]
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{8}$ (d) $\frac{2}{9}$

128. In throwing of two dice, the number of exhaustive events that '5' will never appear on any one of the dice is [2015-I]
(a) 5 (b) 18 (c) 25 (d) 36

129. Two cards are drawn successively without replacement from a wellshuffled pack of 52 cards. The probability of drawing two aces is [2015-I]
(a) $\frac{1}{26}$ (b) $\frac{1}{221}$
(c) $\frac{4}{223}$ (d) $\frac{1}{13}$

130. Three digits are chosen at random from 1, 2, 3, 4, 5, 6, 7, 8 and 9 without repeating any digit. What is the probability that the product is odd ? [2015-II]
(a) $\frac{2}{3}$ (b) $\frac{7}{48}$
(c) $\frac{5}{42}$ (d) $\frac{5}{108}$

131. Two events A and B are such that P(not B) = 0.8, $P(A \cup B) = 0.5$ and $P(A|B) = 0.4$. Then P(A) is equal to [2015-II]
(a) 0.28 (b) 0.32
(c) 0.38 (d) None of the above

132. If mean and variance of a Binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than 1 is [2015-II]
(a) $\frac{2}{3}$ (b) $\frac{4}{5}$
(c) $\frac{7}{8}$ (d) $\frac{11}{16}$

133. Seven unbiased coins are tossed 128 times. In how many throws would you find at least three heads ? [2015-II]
(a) 99 (b) 102
(c) 103 (d) 104

134. A coin is tossed five times. What is the probability that heads are observed more than three times ? [2015-II]
(a) $\frac{3}{16}$ (b) $\frac{5}{16}$
(c) $\frac{1}{2}$ (d) $\frac{3}{32}$

135. An unbiased coin is tossed until the first head appears or until four tosses are completed, whichever happens earlier. Which of the following statements is/are correct ? *[2015-II]*

1. The probability that no head is observed is $\frac{1}{16}$.
2. The probability that the experiment ends with three tosses is $\frac{1}{8}$.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

136. If $x \in [0, 5]$, then what is the probability that $x^2 - 3x + 2 \geq 0$? *[2015-II]*
(a) $\frac{4}{5}$ (b) $\frac{1}{5}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

137. A bag contains 4 white and 2 black balls and another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, then the probability that one ball is white and one ball is black is *[2015-II]*
(a) $\frac{5}{24}$ (b) $\frac{13}{24}$
(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

138. A problem in statistics is given to three students A, B and C whose chances of solving it independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that the problem will be solved is *[2015-II]*
(a) $\frac{1}{12}$ (b) $\frac{11}{12}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

139. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a scooter driver, car driver and a truck driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. The probability that the person is a scooter driver is *[2015-II]*
(a) $\frac{1}{52}$ (b) $\frac{3}{52}$
(c) $\frac{15}{52}$ (d) $\frac{19}{52}$

140. A coin is tossed 5 times. The probability that tail appears an odd number of times, is *[2015-II]*
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

141. What is the probability that the sum of any two different single digit natural numbers is a prime number ? *[2015-II]*
(a) $\frac{5}{27}$ (b) $\frac{7}{18}$
(c) $\frac{1}{3}$ (d) None of the above

142. Three dice are thrown simultaneously. What is the probability that the sum on the three faces is at least 5? *[2016-I]*
(a) $\frac{17}{18}$ (b) $\frac{53}{54}$
(c) $\frac{103}{108}$ (d) $\frac{215}{216}$

143. Two independent events A and B have $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$. What is the probability that exactly one of the two events A or B occurs? *[2016-I]*
(a) $\frac{1}{4}$ (b) $\frac{5}{6}$
(c) $\frac{5}{12}$ (d) $\frac{7}{12}$

144. A coin is tossed three times. What is the probability of getting head and tail alternately? *[2016-I]*
(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

145. A card is drawn from a wel-shuffled deck of 52 cards. What is the probability that it is queen of spade? *[2016-I]*
(a) $\frac{1}{52}$ (b) $\frac{1}{13}$
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

146. If two dice are thrown, then what is the probability that the sum on the two faces is greater than or equal to 4? *[2016-I]*
(a) $\frac{13}{18}$ (b) $\frac{5}{6}$
(c) $\frac{11}{12}$ (d) $\frac{35}{36}$

147. A certain type of missile hits the target with probability $p = 0.3$. What is the least number of missiles should be fired so that there is at least an 80% probability that the target is hit? *[2016-I]*
(a) 5 (b) 6
(c) 7 (d) None of the above

148. For two mutually exclusive events A and B, $P(A) = 0.2$ and $P(\bar{A}\text{Ç}B) = 0.3$. What is $P(A \mid (A \cup B))$ equal to? *[2016-I]*
(a) $\frac{1}{2}$ (b) $\frac{2}{5}$
(c) $\frac{2}{7}$ (d) $\frac{2}{3}$

149. What is the probability of 5 Sundays in the month of December? *[2016-I]*
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$
(c) $\frac{3}{7}$ (d) None of the above

150. A point is chosen at random inside a rectangle measuring 6 inches by 5 inches. What is the probability that the randomly selected point is at least one inch from the edge of the rectangle? *[2016-I]*
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

151. A fair coin is tossed 100 times. What is the probability of getting tails an odd number of times? *[2016-I]*
(a) $\frac{1}{2}$ (b) $\frac{3}{8}$
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

152. A special dice with numbers 1, –1, 2, –2, 0 and 3 is thrown thrice. What is the probability that the sum of the numbers occurring on the upper face is zero? *[2016-II]*
(a) 1 / 72 (b) 1 / 8
(c) 7 / 72 (d) 25 / 216

153. There is 25% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 7 days? *[2016-II]*
(a) $1-\left(\frac{1}{4}\right)^7$ (b) $\left(\frac{1}{4}\right)^7$
(c) $\left(\frac{3}{4}\right)^7$ (d) $1-\left(\frac{3}{4}\right)^7$

154. A salesman has a 70% chance to sell a product to any customer. The behaviour of successive customers is independent. If two customers A and B enter, what is the probability that the salesman will sell the product to customer A or B? *[2016-II]*
(a) 0.98 (b) 0.91
(c) 0.70 (d) 0.49

155. A student appears for tests I, II and III. The student is considered successful if he passes in tests I, II or III or all the three. The probabilities of the student passing in tests I, II and III are m, n and 1/2 respectively. If the probability of the student to be successful is 1/2, then which one of the following is correct? *[2016-II]*
(a) $m(1+n) = 1$ (b) $n(1+m) = 1$
(c) $m = 1$ (d) $mn = 1$

156. Three candidates solve a question. Odds in favour of the correct answer are 5:2, 4:3 and 3:4 respectively for the three candidates. What is the probability that at least two of them solve the question correctly? *[2016-II]*
(a) 209/343 (b) 134/343
(c) 149/343 (d) 60/343

157. A medicine is known to be 75% effective to cure a patient. If the medicine is given to 5 patients, what is the probability that at least one patient is cured by this medicine? *[2016-II]*
(a) $\frac{1}{1024}$ (b) $\frac{243}{1024}$
(c) $\frac{1023}{1024}$ (d) $\frac{781}{1024}$

158. For two events, A and B, it is given that $P(A) = \frac{3}{5}$, $P(B) = \frac{3}{10}$, and $P(A \mid B) = \frac{2}{3}$. If $\bar{A}$ and $\bar{B}$ are the complementary events of A and B, then $P(\bar{A} \mid \bar{B})$ equal to? *[2016-II]*
(a) $\frac{3}{7}$ (b) $\frac{3}{4}$
(c) $\frac{1}{3}$ (d) $\frac{4}{7}$

159. A machine has three parts, A, B and C, whose chances of being defective are 0.02, 0.10 and 0.05 respectively. The machine stops working if any one of the parts becomes defective. What is the probability that the machine will not stop working? *[2016-II]*
(a) 0.06 (b) 0.16
(c) 0.84 (d) 0.94

160. Three independent events, A_1, A_2 and A_3 occur with probabilities $P(A_i) = \frac{1}{1+i}$, $i = 1, 2, 3$. What is the probability that at least one of the three events occurs? *[2016-II]*
(a) $\frac{1}{4}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) $\frac{1}{24}$

161. In a series of 3 one-day cricket matches between teams A and B of a college, the probability of team A winning or drawing are 1/3 and 1/6 respectively. If a win, loss or draw gives 2, 0 and 1 point respectively, then what is the probability that team A will score 5 points in the series? *[2016-II]*
(a) $\frac{17}{18}$ (b) $\frac{11}{12}$
(c) $\frac{1}{12}$ (d) $\frac{1}{18}$

162. Let the random variable X follow B (6, p). If 16 P(X=4)= P(X=2), then what is the value of p? *[2016-II]*

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{5}$ (d) $\frac{1}{6}$

163. A committee of two persons is contituted from two men and two women. What is the probability that the committee will have only women? *[2017-I]*

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

164. A question is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the question will be solved? *[2017-I]*

(a) $\frac{1}{24}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{23}{24}$

165. For two dependent events A and B, it is given that P(A) = 0.2 and P(B) = 0.5. If $A \subseteq B$, then the values of conditional probabilites P(A|B) and P(B|A) are respectively *[2017-I]*

(a) $\frac{2}{5}, \frac{3}{5}$

(b) $\frac{2}{5}, 1$

(c) $1, \frac{2}{5}$

(d) Information is insufficient

166. A point is chosen at random inside a circle. What is the probability that the point is closer to the centre of the circle than to its boundary? *[2017-I]*

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

167. A card is drawn from a well-shuffled ordinary deck of 52 cards. What is the probability that it is an ace? *[2017-I]*

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$

(c) $\frac{3}{13}$ (d) $\frac{1}{52}$

168. Consider the following statements : *[2017-I]*
1. Two events are mutually exclusive if the occurrence of one event prevents the occurrence of the other.
2. The probability of the union of two mutually exclusive events is the sum of their individual probabilities.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

169. If two fair dice are thrown, then what is the probability that the sum is neither 8 nor 9? *[2017-I]*

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{5}{6}$

170. Let A and B are two mutually exclusive events with P(A) = $\frac{1}{3}$ P(B) = $\frac{1}{4}$. What is the value of $P(\bar{A} \cap \bar{B})$? *[2017-I]*

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

171. The mean and standard deviation of a binomial distribution are 12 and 2 respectively. What is the number of trials? *[2017-I]*

(a) 2 (b) 12
(c) 18 (d) 24

172. A committee of two persons is selected from two men and two women. The probability that the committee will have exactly one woman is *[2017-II]*

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

173. Let a die be loaded in such a way that even faces are twice likely to occur as the odd faces. What is the probability that a prime number will show up when the die is tossed? *[2017-II]*

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

174. Let the sample space consist of non-negative integers up to 50, X denote the numbers which are multiples of 3 and Y denote the odd numbers. Which of the following is/are correct? *[2017-II]*

1. $P(X) = \frac{8}{25}$ 2. $P(Y) = \frac{1}{2}$

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

175. For two events A and B, let $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. What is $P(\bar{A} \cap B)$ equal to? *[2017-II]*

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

176. Let A and B be two events with $P(A)=\frac{1}{3}, P(B)=\frac{1}{6}$ and $P(A\cap B)=\frac{1}{12}$. What is $P(B|\overline{A})$ equal to? *[2017-II]*

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{1}{8}$ (d) $\frac{1}{10}$

177. In a binomial distribution, the mean is $\frac{2}{3}$ and the variance is $\frac{5}{9}$. What is the probability that $X = 2$? *[2017-II]*

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$

(c) $\frac{25}{216}$ (d) $\frac{25}{54}$

178. The probability that a ship safely reaches a port is $\frac{1}{3}$. The probability that out of 5 ships, at least 4 ships would arrive safely is *[2017-II]*

(a) $\frac{1}{243}$ (b) $\frac{10}{243}$

(c) $\frac{11}{243}$ (d) $\frac{13}{243}$

179. What is the probability that at least two persons out of a group of three persons were born in the same month (disregard year)? *[2017-II]*

(a) $\frac{33}{144}$ (b) $\frac{17}{72}$

(c) $\frac{1}{144}$ (d) $\frac{2}{9}$

180. If $P(B)=\frac{3}{4}, P(A\cap B\cap\overline{C})=\frac{1}{3}$ and $P(\overline{A}\cap B\cap\overline{C})=\frac{1}{3}$, then what is $P(B\cap C)$ equal to? *[2017-II]*

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

181. In a multiple-choice test, an examinee either knows the correct answer with probability p, or guesses with probability 1 – p. The probability of answering a question correctly is $\frac{1}{m}$, if he or she merely guesses. If the examinee answers a question correctly, the probability that he or she really knows the answer is *[2017-II]*

(a) $\frac{mp}{1+mp}$ (b) $\frac{mp}{1+(m-1)p}$

(c) $\frac{(m-1)p}{1+(m-1)p}$ (d) $\frac{(m-1)p}{1+mp}$

182. Five sticks of length 1, 3, 5, 7 and 9 feet are given. Three of these sticks are selected at random. What is the probability that the selected sticks can from a triangle? *[2017-II]*

(a) 0.5 (b) 0.4

(c) 0.3 (d) 0

183. Consider the following statements: *[2018-I]*

1. $P(\overline{A}\cup B)=P(\overline{A})+P(B)-P(\overline{A}\cap B)$
2. $P(A\cap\overline{B})=P(B)-P(A\cap B)$
3. $P(A\cap B)=P(B)P(A\,|\,B)$

Which of the above statements are correct?

(a) 1 and 2 only (b) 1 and 3 only

(c) 2 and 3 only (d) 1, 2 and 3

184. The probabilities that a student will solve Question A and Question B are 0.4 and 0.5 respectively. What is the probability that he solves at least one of the two questions? *[2018-I]*

(a) 0.6 (b) 0.7

(c) 0.8 (d) 0.9

185. Two fair dice are rolled. What is the probability of getting a sum of 7? *[2018-I]*

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$

(c) $\frac{7}{12}$ (d) $\frac{5}{12}$

186. If A and B are two events such that $2P(A) = 3P(B)$, where $0 < P(A) < P(B) < 1$, then which one of the following is correct? *[2018-I]*

(a) $P(A|B) < P(B|A) < P(A\cap B)$
(b) $P(A\cap B) < P(B|A) < P(A|B)$
(c) $P(B|A) < P(A|B) < P(A\cap B)$
(d) $P(A\cap B) < P(A|B) < P(B|A)$

187. A box has ten chits numbered 0, 1, 2, 3, ..., 9. First, one chit is drawn at random and kept aside. From the remaining, a second chit is drawn at random. What is the probability that the second chit drawn is "9"? *[2018-I]*

(a) $\frac{1}{10}$ (b) $\frac{1}{9}$

(c) $\frac{1}{90}$ (d) None of the above

188. One bag contains 3 white and 2 black balls, another bag contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the chance that it is white? *[2018-I]*

(a) $\frac{3}{8}$ (b) $\frac{49}{80}$

(c) $\frac{8}{13}$ (d) $\frac{1}{2}$

189. Consider the following in respect of two events A and B: *[2018-I]*
 1. P(A occurs but not B) = P(A) – P(B) if $B \subset A$
 2. P(A alone or B alone occurs) = P(A) + P(B) – $P(A \cap B)$
 3. $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive

 Which of the above is/are correct?
 (a) 1 only (b) 1 and 3 only
 (c) 2 and 3 only (d) 1 and 2 only

190. A committee of three has to be chosen from a group of 4 men and 5 women. If the selection is made at random, what is the probability that exactly two members are men? *[2018-I]*
 (a) $\frac{5}{14}$ (b) $\frac{1}{21}$
 (c) $\frac{3}{14}$ (d) $\frac{8}{21}$

191. If two dice are thrown and at least one of the dice shows 5, then the probability that the sum is 10 or more is
 (a) $\frac{1}{6}$ (b) $\frac{4}{11}$
 (c) $\frac{3}{11}$ (d) $\frac{2}{11}$

192. Let A, B and C be three mutually exclusive and exhaustive events associated with a random experiment. If P(B) = 1.5 P(A) and P(C) = 0.5P(B), then P(A0 is equal to *[2018-II]*
 (a) $\frac{3}{4}$ (b) $\frac{4}{13}$
 (c) $\frac{2}{3}$ (b) $\frac{1}{2}$

193. In a bolt factory, machines X, Y, Z manufacture bolts that are respectively 25%, 35% and 40% of the factory's total output. The machines X, Y, Z respectively produce 2%, 4% and 5% defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine X? *[2018-II]*
 (a) $\frac{5}{39}$ (b) $\frac{11}{39}$
 (c) $\frac{20}{39}$ (d) $\frac{34}{39}$

194. 8 coins are tossed simultaneously. The probability of getting at least 6 heads is *[2018-II]*
 (a) $\frac{7}{64}$ (b) $\frac{57}{64}$
 (c) $\frac{37}{256}$ (d) $\frac{229}{256}$

195. Three groups of children contain 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group.The probability that the three selected consist of 1 girl and 2 boys is *[2018-II]*
 (a) $\frac{13}{32}$ (b) $\frac{9}{32}$
 (c) $\frac{3}{32}$ (d) $\frac{1}{32}$

196. If the probability of simultaneous occurrence of two events A and B is p and the probability that exactly one of A, B occurs is q, then which of the following is/are correct? *[2018-II]*
 1. $P(\overline{A}) + P(\overline{B}) = 2 - 2p - q$
 2. $P(\overline{A} \cap \overline{B}) = 1 - p - q$

 Select the correct answer using the code given below:
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2

197. Two integers x and y are chosen with replacement from the set (0, 1, 2, ..., 10). The probability that $|x - y| > 5$ is *[2018-II]*
 (a) $\frac{6}{11}$ (b) $\frac{35}{121}$
 (c) $\frac{30}{121}$ (d) $\frac{25}{121}$

198. From a deck of cards, cards are taken out with replacement. What is the probability that the fourteenth card taken out is an ace ? *[2019-I]*
 (a) $\frac{1}{51}$ (b) $\frac{4}{51}$
 (c) $\frac{1}{52}$ (d) $\frac{1}{13}$

199. If A and B are two events such that P(A) = 0.5, P(B) = 0.6 and $P(A \cap B) = 0.4$, then what is $P(\overline{A \cup B})$ equal to ? *[2019-I]*
 (a) 0.9 (b) 0.7
 (c) 0.5 (d) 0.3

200. A problem is given to three students A, B and C whose probabilities of solving the problem are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if they all solve the problem independently ? *[2019-I]*
 (a) $\frac{29}{32}$ (b) $\frac{27}{32}$
 (c) $\frac{25}{32}$ (d) $\frac{23}{32}$

201. A pair of fair dice is rolled. What is the probability that the second dice lands on a higher value than does the first ? *[2019-I]*
 (a) $\frac{1}{4}$ (b) $\frac{1}{6}$
 (c) $\frac{5}{12}$ (d) $\frac{5}{18}$

202. A fair coin is tossed and an unbiased dice is rolled together. What is the probability of getting a 2 or 4 or 6 along with head ? *[2019-I]*
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

203. If A, B, C are three events, then what is the probability that at least two of these events occur together? [2019-I]
(a) $P(A \cap B) + P(B \cap C) + P(C \cap A)$
(b) $P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$
(c) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
(d) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

204. If two variables X and Y are independent, then what is the correlation coefficient between them ? [2019-I]
(a) 1 (b) –1
(c) 0 (d) None of the above

205. Two independent events A and B are such that $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. If $P(B) < P(A)$, then what is P(B) equal to ? [2019-I]
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

206. If two fair dice are rolled then what is the conditional probability that the first dice lands on 6 given that the sum of numbers on the dice is 8 ? [2019-I]
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{5}$ (d) $\frac{1}{6}$

207. Two symmetric dice flipped with each dice having two sides painted red, two painted black, one painted yellow and the other painted white. What is the probability that both land on the same colour ? [2019-I]
(a) $\frac{3}{18}$ (b) $\frac{2}{9}$
(c) $\frac{5}{18}$ (d) $\frac{1}{3}$

208. There are n socks in a drawer, of which 3 socks are red. If 2 of the socks are chosen randomly and the probability that both selected socks are red is $\frac{1}{2}$, then what is the value of n? [2019-I]
(a) 3 (b) 4
(c) 5 (d) 6

209. Two cards are chosen at random from a deck of 52 playig cards. What is the probability that both of them have the same value? [2019-I]
(a) $\frac{1}{17}$ (b) $\frac{3}{17}$
(c) $\frac{5}{17}$ (d) $\frac{7}{17}$

210. In eight throws of a die, 5 or 6 is considered a success. The mean and standard deviation of total number of successes is respectively given by [2019-I]
(a) $\frac{8}{3}, \frac{16}{9}$ (b) $\frac{8}{3}, \frac{4}{3}$
(c) $\frac{4}{3}, \frac{4}{3}$ (d) $\frac{4}{3}, \frac{16}{9}$

211. A and B are two events such that $\overline{A}$ and $\overline{B}$ are mutually exclusive. If P(A) = 0.5 and P(B) = 0.6, then what is the value of P(A|B)? [2019-I]
(a) $\frac{1}{5}$ (b) $\frac{1}{6}$
(c) $\frac{2}{5}$ (d) $\frac{1}{3}$

212. What is the probability that an interior point in a circle is closer to the centre than to the circumference ? [2019-I]
(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$
(d) It cannot be determined

213. If A and B are two events, then what is the probability of occurrence of either event A or event B? [2019-I]
(a) $P(A) + P(B)$ (b) $P(A \cup B)$
(c) $P(A \cap B)$ (d) $P(A)\,P(B)$

ANSWER KEY

1	(c)	23	(b)	45	(c)	67	(b)	89	(c)	111	(c)	133	(a)	155	(a)	177	(c)	199	(d)
2	(c)	24	(b)	46	(a)	68	(a)	90	(d)	112	(b)	134	(a)	156	(a)	178	(c)	200	(a)
3	(b)	25	(b)	47	(c)	69	(a)	91	(c)	113	(b)	135	(c)	157	(c)	179	(b)	201	(c)
4	(c)	26	(b)	48	(a)	70	(a)	92	(b)	114	(c)	136	(a)	158	(a)	180	(a)	202	(c)
5	(a)	27	(b)	49	(c)	71	(d)	93	(b)	115	(a)	137	(b)	159	(c)	181	(b)	203	(c)
6	(a)	28	(b)	50	(d)	72	(b)	94	(c)	116	(d)	138	(d)	160	(c)	182	(c)	204	(c)
7	(c)	29	(c)	51	(a)	73	(a)	95	(c)	117	(a)	139	(a)	161	(d)	183	(b)	205	(b)
8	(c)	30	(b)	52	(c)	74	(b)	96	(b)	118	(a)	140	(a)	162	(c)	184	(b)	206	(c)
9	(c)	31	(d)	53	(a)	75	(b)	97	(c)	119	(d)	141	(b)	163	(a)	185	(b)	207	(c)
10	(c)	32	(c)	54	(c)	76	(b)	98	(c)	120	(d)	142	(b)	164	(c)	186	(b)	208	(b)
11	(c)	33	(d)	55	(c)	77	(d)	99	(a)	121	(b)	143	(d)	165	(b)	187	(c)	209	(a)
12	(c)	34	(b)	56	(b)	78	(b)	100	(d)	122	(a)	144	(b)	166	(b)	188	(b)	210	(b)
13	(b)	35	(d)	57	(b)	79	(b)	101	(b)	123	(d)	145	(a)	167	(a)	189	(b)	211	(b)
14	(c)	36	(b)	58	(a)	80	(c)	102	(c)	124	(d)	146	(c)	168	(c)	190	(a)	212	(a)
15	(c)	37	(b)	59	(c)	81	(d)	103	(b)	125	(c)	147	(a)	169	(c)	191	(c)	213	(b)
16	(c)	38	(d)	60	(d)	82	(c)	104	(d)	126	(d)	148	(b)	170	(d)	192	(b)		
17	(b)	39	(d)	61	(d)	83	(d)	105	(b)	127	(b)	149	(c)	171	(c)	193	(a)		
18	(b)	40	(b)	62	(a)	84	(c)	106	(b)	128	(c)	150	(d)	172	(b)	194	(c)		
19	(c)	41	(b)	63	(b)	85	(b)	107	(b)	129	(b)	151	(a)	173	(c)	195	(a)		
20	(b)	42	(b)	64	(a)	86	(a)	108	(d)	130	(c)	152	(d)	174	(d)	196	(c)		
21	(a)	43	(b)	65	(c)	87	(d)	109	(a)	131	(c)	153	(d)	175	(a)	197	(c)		
22	(b)	44	(b)	66	(c)	88	(c)	110	(d)	132	(d)	154	(b)	176	(c)	198	(d)		

HINTS & SOLUTIONS

1. (c) As given $P(A) = 0.6, P(B) = 0.3$ and $P(A \cap B) = 0.2$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.6 + 0.3 - 0.2 = 0.9 - 0.2 = 0.7$$

So, P (neither in A nor in B) $= 1 - P(A \cup B) = 1 - 0.7 = 0.3$

2. (c) Let E be the event of total of 12.

E = (2, 2, 2, 3, 3), (2, 2, 3, 3, 2), (2, 3, 3, 2, 2), (3, 3, 2, 2, 2), (3, 2, 3, 2, 2), (3, 2, 2, 3, 2), (3, 2, 2, 2, 3), (2, 3, 2, 3, 2), (2, 3, 2, 2, 3), (2, 2, 3, 2, 3)

n (E) = 10

Sample sapce contain total possibility $= 2^5 = 32$

Hence, n(s) = 32

So, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{10}{32} = \dfrac{5}{16}$

3. (b) A and R true but R is not correct explanation of A.

4. (c) Since, probabilities of failure for engines A, B and C P(A), P(B) and P(C) are 0.03, 0.02 and 0.05 respectively. The aircraft will crash only when all the three engine fail. So, probability that it crashes = P(A). P(B). P(C)

$= 0.03 \times 0.02 \times 0.05$

$= 0.00003$

Hence, the probability that the aircraft will not crash,

$= 1 - 0.00003$

$= 0.99997$

5. (a) Total possible outcomes, S = {HHH, HHT, HTH, THT, TTH, THH, TTT, HTT} and desired outcomes E = {HTH, THT}

$\Rightarrow$ n(E) = 2 and n(S) = 8

Hence, required probability $= P(E) = \dfrac{n(E)}{n(S)} = \dfrac{2}{8} = \dfrac{1}{4}$

6. (a) Probability of passing in mathematics $= \dfrac{4}{9}$

Probability of passing in physics $= \dfrac{2}{5}$

Probability of failure in physics $= 1 - \dfrac{2}{5} = \dfrac{3}{5}$

Given that both the events are independent.

Required probability $= \dfrac{4}{9} \times \dfrac{3}{5} = \dfrac{4}{15}$

7. (c) Probability of getting a diamond, $P(D) = \dfrac{13}{52} = \dfrac{1}{4}$

and probability to king, $P(k) = \dfrac{4}{52} = \dfrac{1}{13}$

So, required probability = P(D).P(K)

$= \dfrac{1}{4} \times \dfrac{1}{13} = \dfrac{1}{52}$

8. (c) Required probability $= \frac{4}{52} \times \frac{4}{51}$

[Since, card is not replaced after first draw]

$= \frac{4}{13 \times 51} = \frac{4}{663}$

9. (c) Given, that np = 3 and npq = 4
where p is the probability of success in one trial and q is the probability of failure and n is the number of trials

$\Rightarrow q = \frac{4}{3}$

and this is not possible.
Thus , the given statement is never true.

10. (c) A dice has six faces. So, in throwing of two dice, the number of exhaustive events is $6 \times 6 = 36$.

11. (c) Number of ways of selecting 5 heads cut of total 12 flips $= {}^{12}C_5$.

Probability of getting one head in a coin $= \frac{1}{2}$

Also, probability of getting one tail in a coin $= \frac{1}{2}$

Probability of getting 5 head $= \left(\frac{1}{2}\right)^5$

Probability of getting 7 tails $= \left(\frac{1}{2}\right)^7$

So, required probability

$= {}^{12}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^7 = {}^{12}C_5 \left(\frac{1}{2}\right)^{12} = \frac{{}^{12}C_5}{2^{12}}$

12. (c) 16 tickets are sold and 4 prizes are awarded. A person buys 4 tickets, then required probability $= \frac{4}{16} = \frac{1}{4}$

13. (b) As given, $P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$

and $P(\overline{A}) = \frac{2}{3}$

$P(\overline{A}) = 1 - P(A) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$

$\Rightarrow 1 - \frac{1}{3} = P(B) \Rightarrow \frac{2}{3} = P(B)$

14. (c) Probability of getting a spade $= \frac{13}{52}$

Probability of an ace $= \frac{4}{52}$

and probability of getting a spade ace $= \frac{1}{52}$

$\therefore$ Required probability $= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Odds against his winning $= \frac{1 - \frac{4}{13}}{\frac{4}{13}} = \frac{\frac{9}{13}}{\frac{4}{13}} = \frac{9}{4}$

15. (c) Probability of no one hitting the target

$= \frac{1}{5 \times 4 \times 3} = \frac{1}{60}$

Probability of one hitting the target

$= \frac{4+3+2}{60} = \frac{9}{60}$

$\therefore$ Probability of maximum one hit

$= \frac{1}{60} + \frac{9}{60} = \frac{10}{60} = \frac{1}{6}$

Probability that two shots are hit at least is the required probability $= 1 - \frac{1}{6} = \frac{5}{6}$

16. (c) Total number of selecting 3 components out of 10 = ${}^{10}C_3$. Out of 3 selected components two defective pieces can be selected in ${}^4C_2$ ways and one non-defective piece will be selected in ${}^6C_1$ ways, hence,

Required probability $= \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3} = \frac{6 \times 6 \times 6}{10 \times 9 \times 8} = \frac{3}{10}$

17. (b) If both get one head then it is $\frac{1}{4} \times \frac{1}{4}$

and if both get two heads then it is $\frac{1}{2} \times \frac{1}{2}$

$\Rightarrow$ Prob (getting same number of heads) $= \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{16} + \frac{1}{4} = \frac{5}{16}$

18. (b) Number of Sundays in some randomly selected months with 30 days follow binomial distribution.

19. (c) (1) $P(A \cup B) \le P(A) + P(B)$ is correct

and (2) $P(A) + P(\overline{A} \cup B) \le 1 + P(B)$ is also correct

$\Rightarrow$ Both the given statement are correct.

20. (b) Total number of possible arrangements n(s) = 6!.
Since 2 and 3 occupy consecutive places, so, they are grouped together.
So, there will be 5! such arrangements. But 2 and 3 can be arranged in themselves in 2! ways

Required Probability $= \frac{5!\times 2!}{6!} = \frac{2}{6} = \frac{1}{3}$

21. (a) Total possibility of 4 children, either girl or boy is $2^4 = 16$. Out of these there is one possibility in which there will be no boy and only girls. So, total possibility of at least one boy is 16 – 1 = 15

$\Rightarrow$ P (at least one boy) $= \frac{15}{16}$.

22. (b) Let there be 100 students.
So, there are 40 students of science and 60 students of arts.
5% of 40 = 2 science students (girls)
10% of 60 = 6 science students (girls)
Total girls students = 8
If a girl is chosed then

P (arts) $= \frac{6}{8} = \frac{3}{4}$

23. (b) As given : $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$

and $P(\overline{B}) = \frac{1}{2}$

$P(B) = 1 - P(\overline{B}) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$

$\Rightarrow \frac{5}{6} = 1 - P(\overline{A}) + \frac{1}{2} - \frac{1}{3}$

$\Rightarrow P(\overline{A}) = 1 + \frac{1}{2} - \frac{1}{3} - \frac{5}{6}$

$= \frac{6+3-2-5}{6} = \frac{2}{6} = \frac{1}{3}$

24. (b) A coin has two faces and is tossed 5 times. So, number of elements in the sample space = 10.

25. (b) There are 7 days in a week. If 1st person's birth day falls on any day out of 7. So, probability is $\frac{7}{7}$. Since birthday of second person will fall on any of the remaining six days then its probability $= \frac{6}{7}$. And, birthday of 3rd person will fall on any of the remaining 5 days so, its probability $= \frac{5}{7}$

$\Rightarrow$ Probability that all three persons will have different day as their birthday

$= \frac{7}{7} \times \frac{6}{7} \times \frac{5}{7} = \frac{30}{49} = 0.612 \approx 0.60$

26. (b) Bag I. has 5 white + 3 black balls.
Bag II. has 2 white + 4 black balls.

$P\,(\text{Black})_{1\text{st bag}} = \frac{3}{8}$ & $P\,(\text{White})_{1\text{st bag}} = \frac{5}{8}$

If one ball is drawn from bag I & placed in bag II, bag II will have 7 balls.
If black ball is drawn, then; bag II contains,
2W + 5 Black balls = 7 balls
P(black ball from bag 1 and black ball from bag 2)

$= \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

If ball is white then bag II has 3w + 4 black balls
P (white ball from bag 1 and black ball from bag 2)

$= \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

$\Rightarrow \text{Prob(blackball)}_{\text{bagII}} = \frac{15}{56} + \frac{20}{56} = \frac{35}{56}$

27. (b) We know,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow 0.8 + 0.9 - p \le 1$
$\Rightarrow 1.7 - p \le 1$
$\Rightarrow 0.7 \le p$
Now, $P(A) < P(B)$
$\therefore P(A \cap B) \le P(A)$
$\Rightarrow p \le 0.8$
Hence, $0.7 \le p \le 0.8$

28. (b) Mean = np and Variance = npq < np ($\because$ q < 1)

29. (c) Let P (A) be the probability that the race will be won by A and P(B) be the probability that the race will be won by B.

$\therefore P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{6}$

$\therefore$ Probability that the race will be won by

A or B $= P(A) + P(B) = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$

30. (b) Required probability $= \frac{365}{365} \times \frac{1}{365} = \frac{1}{365}$

31. (d) Probability that $(X = 3) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{4} = \frac{7}{24}$

32. (c) Baye's theorem says
If $A_1, A_2, ..., A_n$ are (n) mutually exclusive and exhaustive events in sample space S and E is any event is S intersecting events
A_i (iz 1, 2,...., n) such that P(E) $\neq$ 0 then

$$P(A_i / E) = \frac{P(E / A_i)}{P(E)}$$

$$= \frac{P(A_i)P(E/A_i)}{\sum_{|2|}^{n} P(A_i)P(E/A_i)}$$

Thus, both statement 1 and 2 are correct.

33. (d) Given, the outcomes of an experiment classified as success A will follow a binomial distribution if P(A) remains constant in all trials.

34. (b) If A, B, C are any three arbitary events then only expression $A \cap B \cap \overline{C}$ will show that both A and B occur but not C.

35. (d) By Baye's theorem, we know that, conditional probability is calculated.

36. (b) Given, $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{6}$

But $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \frac{1}{6} = \frac{P(A \cap B)}{\frac{1}{4}}$

$\Rightarrow P(A \cap B) = \frac{1}{24}$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/24}{1/3} = \frac{1}{8}$

37. (b) A and B are mutually exclusive and exhaustive events with
$P(A \cap B) = 0, P(A \cup B) = 1$
we know that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow 1 = P(A) + 3P(A)$

$\Rightarrow P(A) = \frac{1}{4}$ $\quad\therefore P(B) = \frac{3}{4}$

Hence, $P(\overline{B}) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$

38. (d) Let E be the sum of the faces equals or exceeds
Then, E = $\{(5, 5), (4, 6), (6, 4), (5, 6), (6, 5), (6, 6)\}$
$\therefore n(E) = 6$

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

39. (d) Given, $np = 4$ and $npq = \frac{4}{3}$

$\therefore 4q = \frac{4}{3} \Rightarrow q = \frac{1}{3}$

$\therefore p = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow n = \frac{4 \times 3}{2} = 6$

Now, $P(X \geq 5) = {}^6C_5 (p)^5 (q)^1 + {}^6C_6 p^6 q^0$

$= {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + {}^6C_6 \left(\frac{2}{3}\right)^6$

$= \frac{6 \times 32}{3^6} + \frac{64}{3^6} = \frac{256}{3^6} = \frac{2^8}{3^6}$

40. (b) Since, $n(S) = 52$ and $n(E) = 4$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

41. (b) (A) Total no. of cards = 52
Total no. of ace cards = 4
Total no. of king cards = 4

P(drawn an ace) = $\frac{4}{52}$, P(drawn an king) = $\frac{4}{52}$

Thus, P(drawing either an ace or a king)
= P(an ace) + P(a king)

$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{4}{26} = \frac{2}{13}$

Both A and R are true but R is not the correct explanation of A.

42. (b) Total no. of letters = 26
No. of selected letters = 3
$\therefore$ No. of ways to select 3 letters out of 26 letters $= {}^{26}C_3$
Since, A will not be include in our choice therefore,
Total no. of letters = 26 – 1 = 25
Now, No. of ways to select 3 letters out of 25 letters ${}^{25}C_3$

$\therefore$ Required Prob $= \frac{{}^{25}C_3}{{}^{26}C_3} = \frac{25!}{3!22!} \times \frac{3!23!}{26!} = \frac{23}{26}$

43. (b)

Head	Tail
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

Hence, total number of points in the sample space is 11.

44. (b) Given set is $\{1, 2, 3, 4, 5, 6, 7\}$
Total numbers in set = 7
Since, two numbers are drawn without replacement
$\therefore$ Total number of elementary events $= {}^7C_1 \times {}^6C_1$
$= 7 \times 6$
$= 42$

45. (c) As we know $P(A \cup B) \le 1$
$\therefore\ P(A) + P(B) - P(A \cap B) \le 1$
$\Rightarrow\ 0.8 + 0.7 - P(A \cap B) \le 1$
$\Rightarrow\ P(A \cap B) \ge 1.5 - 1$
$\Rightarrow\ P(A \cap B) \ge 0.5$
Hence, the minimum value of $P(A \cap B)$ is 0.5.

46. (a) Given, $X + Y = 15$
The total number of ordered pairs which satisfies $X + Y = 15$ is
$\therefore$ (5, 10), (6, 9), (7, 8), (8, 7), (9, 6), (10, 5)
$\therefore\ n(S) = 6$ where S denotes the sample space.
In each above pairs exactly one of the two numbers is even number.
Therefore $E = \{(5, 10), (6, 9), (7, 8), (8, 7), (9, 6), (10, 5)\}$ where E is an event.
$\therefore\quad n(E) = 6$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{6}{6} = 1$

47. (c) We know $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = \dfrac{1}{3} + \dfrac{3}{4} - \dfrac{11}{12} = \dfrac{2}{12} = \dfrac{1}{6}$

Consider $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1/6}{1/3} = \dfrac{1}{2}$

48. (a) We know that mean and variance of Binomial distribution are np and npq respectively therefore $np = 8$ and $npq = 4$
On dividing we get

$q = \dfrac{npq}{np} = \dfrac{4}{8} = \dfrac{1}{2}$ and $p + q = 1 \Rightarrow p = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

$\Rightarrow\ n\left(\dfrac{1}{2}\right) = 8 \Rightarrow n = 16$

We know that $P(X = r) = {}^nC_r\, p^r q^{n-r}$
therefore

$P(X = 1) = {}^{16}C_1 \left(\dfrac{1}{2}\right)^{16-1} \left(\dfrac{1}{2}\right)^1$

$= \dfrac{16}{2^{15}.2} = \dfrac{1}{2^{12}}$

49. (c) By Baye's theorem
Required probability $= P(A_1/B)$

$$= \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$$

$$= \frac{0.4 \times 0.25}{0.4 \times 0.25 + 0.4 \times 0.4 + 0.2 \times 0.125}$$

$$= \frac{0.1}{0.1 + 0.16 + 0.025} = \frac{0.1}{0.285} = \frac{20}{57}$$

50. (d) Let $P(X \le 2) = 0.25$
$\Rightarrow\ P(X = 1) + P(X = 2) = 0.25$
$\Rightarrow\ k + p_1 = 0.25$ (from the table)
$\Rightarrow\ p_1 = 0.25 - k$ ...(1)
and $P(X \ge 4) = 0.35$
$\Rightarrow\ P(X = 4) + P(X = 5) = 0.35$
$\Rightarrow\ p_2 + 2k = 0.35$ (from the table)
$\Rightarrow\ p_2 = 0.35 - 2k$ ...(2)
From (1) and (2)
$p_1 \ne p_2$
and $p_1 + p_2 = 0.25 - k + 0.35 - 2k = 0.6 - 3k$
$\ne P(X = 3)$
Hence, neither 1 nor 2 is correct.

51. (a) **Statement 1 :**
A non leap year has 365 days. i.e., 52 weeks and 1 day.
1 day can be {Sunday}, {Monday}, {Tuesday}, {Wednesday}, {Thursday}, {Friday}, {Saturday}
In total, there are 7 possibilities and 1 possibility is Sunday.

$\therefore$ Required probability $= \dfrac{1}{7}$

A leap year has 366 days. i.e., 52 weeks and 2 days.
2 days can be {Sun, Mon}, {Mon, Tue}, {Tue, Wed}, {Wed, Thu}, {Thu, Fri}, {Fri, Sat}, {Sat, Sun}.
In total, there are 7 possibilities and 2 possibilities have Sundays.

$\therefore$ Required probability $= \dfrac{2}{7}$

So, statement 1 is correct.
Statement 2 :
March has 31 days. i.e., 4 complete weeks and 3 days.
3 days can be {S, M, T}, {M, T, W}, {T, W, Th}, {W, Th, F}, {Th, F, Sa}, {F, Sa, S}, {Sa, S, M}.
In total 7 possibilities, Monday can come in 3 possibilities

$\therefore$ Required probabilities $= \dfrac{3}{7}$

April has 30 days. i.e., 4 complete weeks and 2 days.
2 days can be {S, M}, {M, T}, {T, W}, {W, Th}, {Th, F}, {F, Sa}, {Sa, S}.
In total 7 possibilities, Monday can come in 2 possibilities.

$\therefore$ Required probability $= \dfrac{2}{7}$

$\therefore$ Statement 2 is wrong.

52. (c) Possible samples are as follows
$\{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT\}$
Let A be the event of getting one head.
Let B be the event of getting no head.
Favourable outcome for

$A = \{TTH, THT, HTT\}$

Favourable outcome for

$B = \{TTT\}$

Total no. of outcomes = 8

$\therefore \; P(A) = \frac{3}{8},\; P(B) = \frac{1}{8}$

$\therefore$ Required probability = Probability of getting one head + Probability of getting no head

$= P(A) + P(B) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

53. (a) No. of blue balls = 2
No. of red balls = 7
Total no. of balls = 9
Required probability
= P (one ball is blue) + P (both ball is blue)

$= \frac{2}{9} \times \frac{7}{8} + \frac{2}{9} \times \frac{1}{8} = \frac{14}{72} + \frac{2}{72} = \frac{16}{72} = \frac{2}{9}$

54. (c) Given Probability of guessing a correct answer $= \frac{x}{12}$

and probability of not guessing the correct answer $= \frac{2}{3}$

As we know

P (occurence of an event) + P (non-occurence of an event) = 1

$\therefore \; \frac{x}{12} + \frac{2}{3} = 1 \Rightarrow \frac{x+8}{12} = 1 \Rightarrow x = 12 - 8 = 4$

55. (c) Both (1) and (2) statements are correct.

56. (b) A leap year has 366 days, in which 2 days may be any one of the following pairs.
(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday) (Friday, Saturday) (Saturday, Sunday).

$\therefore$ Required probability $= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

(By using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$)

57. (b) Given digits are 0, 2, 4, 6, 8.
Total number of cases = 4× 4× 4
($\because$ We have 4 choices for each number)

$\therefore$ Required probability $= \frac{4}{64} = \frac{1}{16}$

58. (a) Required probability $= \frac{{}^4C_2 \times {}^4C_2}{{}^8C_2} = \frac{36}{70} = \frac{18}{35}$

59. (c) Let A and B are independent events.

$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$ ...(A)

1. Consider $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
$= P(A) - P(A)\,P(B)$ (from A)
$= P(A)\,[1 - P(B)] = P(A)P(\bar{B})$

Hence, A and $\bar{B}$ are independent.

2. Similarly, $\bar{A}$ and B are independent.

3. Consider $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$
$= 1 - P(A \cap B)$
$= 1 - [P(A) + P(B) - P(A \cup B)]$
$= 1 - P(A) - P(B) + P(A).P(B)$
$= [1 - P(A)]\;[1 - P(B)]$
$= P(\bar{A}).\,P(\bar{B})$

Hence, $\bar{A}$ and $\bar{B}$ are independent.

60. (d) Events when a coin is flipped and head occurs are {HT, HH}
Events when a coin is flipped and tail occurs are {T1, T2, T3, T4, T5, T6}
($\because$ dice are rolled after tail appears)
So, Total number of events = 8
Favourable event = {H T} = 1

$\therefore$ Required probability $= \frac{1}{8}$

61. (d) Required probability $= \frac{{}^6C_1 \times {}^5C_1 \times {}^4C_1}{{}^6C_1 \times {}^6C_1 \times {}^6C_1}$

$= \frac{6 \times 5 \times 4}{6 \times 6 \times 6} = \frac{5}{9}$

62. (a) Let $P(A \cup B) = 0.5,\; P(\bar{B}) = 0.8,\; P\left(\frac{A}{B}\right) = 0.4$

$P(\bar{B}) = 1 - P(B)$
$\Rightarrow 0.8 = 1 - P(B)$
$\Rightarrow P(B) = 1 - 0.8 = 0.2$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(B) \times P\left(\frac{A}{B}\right) = P(A \cap B)$

$\Rightarrow 0.4 \times 0.2 = P(A \cap B)$

$\Rightarrow 0.08 = P(A \cap B)$

63. (b) Since, probability of answering all the three questions correctly $= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

$\therefore$ Probability of not answering all the three questions correctly $= 1 - \frac{1}{64} = \frac{63}{64}$

64. (a) Since, A and B are mutually exclusive events.
$\therefore$ P (AB) = P (A $\cap$ B) = 0

65. (c) Since, 4 letters are randomly inserted into the 4 envelopes therefore

Required probability $= \frac{1}{4!} = \frac{1}{4 \times 3 \times 2} = \frac{1}{24}$

66. (c) Total number of letters = 4
Total number of vowels = 2 (O and E)

Required Probability $= \frac{2}{4} = \frac{1}{2}$

67. (b) Let A = Event of husband's selection
and B = Event of wife's selection

Given : $P(A) = \frac{1}{5} \Rightarrow P(\bar{A}) = 1 - \frac{1}{5} = \frac{4}{5}$

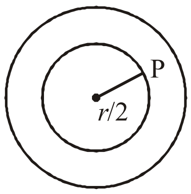
$P(B) = \frac{1}{3} \Rightarrow P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

P(Only one of them selected)

$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$

$= \frac{1}{5} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{3} = \frac{2}{15} + \frac{4}{15} = \frac{6}{15} = \frac{2}{5}$

68. (a) Let radius of circle be 'r'.
Total possible outcomes = Area of circle = πr^2
Observe the figure, we have to find the probability of point, P in the ring which will be closer to circumference.



Area of ring = Area of outer circle – Area of inner circle

$= \pi r^2 - \pi\left(\frac{r}{2}\right)^2 = \pi r^2 - \frac{\pi r^2}{4} = \frac{3\pi r^2}{4}$

So, favourable outcome $= \frac{3\pi r^2}{4}$.

$\therefore$ Required probability $= \frac{\frac{3}{4}\pi r^2}{\pi r^2} = \frac{3}{4}$

69. (a) Total number of words formed by letters of UNIVERSITY

$= \frac{10!}{2!}$ ($\because$ I is repeated)

Taking two I_s together, number of ways to arrange letters of UNIVERSITY = 9!

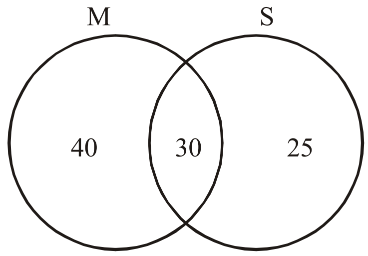
$\therefore$ Probability of two I_s coming together

$= \frac{9!}{\frac{10!}{2!}} = \frac{9! \times 2!}{10!} = \frac{2}{10} = \frac{1}{5}$

$\therefore$ Probability of two I_s not coming together

$= 1 - \frac{1}{5} = \frac{4}{5}$

70. (a)



Required probability

$= \frac{40}{125} + \frac{25}{125} = \frac{65}{125} = \frac{13}{25}$

71. (d) Total no. of case = $6^3 = 216$
Favourable cases = {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}.

Probability $= \frac{6}{216} = \frac{1}{36}$

72. (b) No. of days in leap year = 366
No. of complete week = 52

($\because 366 \div 7$ gives 2 as remainder)

$\therefore$ No. of days left = 2

Required probability $= \frac{2}{7}$

73. (a) Maximum sum of numbers appearing on four dice together = 24
$\therefore$ Required probability = 0

74. (b) Prob. (certain event) = 1
$\Rightarrow$ P (E) = 1

75. (b) Total no. of days in leap year = 366
Favourable cases = 2

Required prob $= \frac{2}{7}$

76. (b) Given $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$,

$P(\bar{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$

As we know

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$

$\Rightarrow \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = P(B)$

$\Rightarrow P(B) = \frac{2}{3}$

77. (d) Three coins tossed simultaneously
$\therefore$ Total outcomes = $2^3 = 8$
Now, S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

Favourbale cases = two heads one tail
= HHT, HTH, THH

$\therefore$ Required prob = $\frac{3}{8}$.

78. (b) An event having one sample point is called an elementary event.

79. (b) This is the question of Binomial Distribution.
Number of r success in n trial is
$P_r = {}^nC_r\, p^r\, q^{n-r}$.
Where p= prob of success
q = prob. of failure = 1 – p

Given : n = 10, $p = \frac{2}{3}, q = \frac{1}{3}$

Fo $r = 10, P_{10} = {}^{10}C_{10} \left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{0} = \frac{2^{10}}{3^{10}}$

For $r = 7, P_7 = 120. \frac{2^7}{3^{10}}$

For $r = 5, P_5 = 252 \times \frac{2^5}{3^{10}}$

For $r = 4, P_4 = 210. \frac{2^4}{3^{10}}$

It is maximum for r = 7

Solutions for 80 and 81

Total number of balls in urn – I = 1 Black + 1 Green = 2 Balls
Total number of balls in urn – II =1 White + 1 Green = 2 Balls

80. (c) Required prob = $(1\,G)_I \times (1G)_{II} = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

81. (d) Required prob = $(1\,G)_I \times (1G)_{II} + (1\,G)_I \times (1B)\,I + (1\,G)_{II} \times (1W)_{II}$.

$= \left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right) = \frac{1}{4}+\frac{1}{4}+\frac{1}{4} = \frac{3}{4}$

82. (c) In A there are 18 events possible.

$\therefore \quad P(A) = \frac{18}{36} = \frac{1}{2}$

In B there are 12 events are possible

$\therefore \quad P(B) = \frac{12}{36} = \frac{1}{3}$

In $A \cap B$ there are 6 events are possible.
Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2}+\frac{1}{3}-\frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$

83. (d) $P(A \cap B) = \frac{6}{36} = \frac{1}{6}$

84. (c) $6 \times 2 \times 2 = 24$ sample points
($\because$ no. of points in sample space of a die = 6 and no. of points in sample space of a coin = 2)

85. (b) A = {2, 4, 6}, B = {1, 3, 5}, C = {4, 5, 6}
Hence, A and B are mutually exclusive.

86. (a) Total case = 6 × 6 = 36
Favourable = (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

Probability = $\frac{6}{36} = \frac{1}{6}$

87. (d) From Binomial Distribution, we have
$P(X = r) = {}^nC_r\,(p)^r\,(q)^{n-r}, r \le n$.
In the given quesiton

n = 4, r = 2, p = prob. of head = $\frac{1}{2}$, q = prob. of tail = $\frac{1}{2}$

$P(X = 2) = {}^4C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$

88. (c) Total number of balls = 5 + 3 = 8.
Prob (both ball are white)

$= \frac{3}{8}\times\frac{2}{7}$ ($\because$ No. of white ball = 3)

$= \frac{3}{28}$

89. (c) We have $P(\overline{A} \cup \overline{B}) = P(\overline{A}) + P(\overline{B}) - P(\overline{A} \cap \overline{B})$

$= P(\overline{A}) + P(\overline{B}) - P(\overline{A \cup B})$

By (De-Morgan's law)

$= P(\overline{A}) + P(\overline{B}) - (1 - P(A \cup B)) = .4 + .3 - (1 - .9) = .6$

90. (d) P(HHHHT or TTTTH) = P (HHHHT) + P (TTTTH)

$= \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5 = \frac{1}{16}$

91. (c) $P(X = r)$ will be maximum when r is mode. There are 2 cases.
(i) If $(n + 1)P$ is an integer, binomial distribution is bimodal and two modal values are $(n + 1)P$ and $(n + 1)P - 1$.
(ii) If $(n + 1)P$ is not an integer, then modal value is integral part of $(n + 1)P$.

Here, $(n + 1)P = (100 + 1)\frac{1}{3} = 101 \times \frac{1}{3} = 33.69$, which is not integer.

$\therefore P(X = r)$ is maximum at integral part of 33.69 i.e., 33.

92. (b) Required no. of elementary events = ${}^8C_2 \times 2! = 56$

93. (b) Binomial distribution has two parameters n and p, where n is number of trials and p is probability of success.

94. (c) Minimum 4 balls have to be drawn so that among these there are two of the same colour

95. (c) A, B and C are mutually exclusive
$A \cap B \cap C = 0$
$P(A \cap B \cap C) = 0$

96. (c) As A and B are independent event
So, $P(A \cap B) = P(A) \cdot P(B)$

$$P(A \cap B) = \frac{1}{5} \cdot P(B)$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{7}{10} = \frac{1}{5} + P(B) - \frac{1}{5} \cdot P(B)$$

$$P(B)\left(1 - \frac{1}{5}\right) = \frac{7}{10} - \frac{1}{5}$$

$$\frac{4}{5} \cdot P(B) = \frac{1}{2} \Rightarrow P(B) = \frac{5}{8}$$

Now, $P(\bar{B}) = 1 - P(B) = 1 - \frac{5}{8} = \frac{3}{8}$.

97. (b) $P = q = \frac{1}{2}$, $nP = 6$

$$\frac{n}{2} = 6 \Rightarrow n = 12$$

98. (c) Number of possible outcomes = 36
When sum is 10, samples are (5, 5), (4, 6) and (6, 4)

Required probability $= \frac{3}{36} = \frac{1}{12}$

99. (c) Number of possible outcomes = 216
triplets = (1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)

Required probability $= \frac{6}{216} = \frac{1}{36}$

100. (a)

101. (d) For binomial distribution
$p < 1$, $q < 1$ and $p + q = 1$

102. (b) Possible outcomes are (Head, 1), (Head, 2), (Head, 3), (Head, 4), (Head, 5), (Head, 6), Tail

103. (c) $P(A \cup B) - P(A \cap B) = \frac{2}{5}$
We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore P(A) + P(B) - P(A \cap B) - P(A \cap B) = \frac{2}{5}$$

$$\frac{2}{3} + \frac{2}{5} - 2P(A \cap B) = \frac{2}{5}$$

$$-2P(A \cap B) = \frac{2}{5} - \frac{2}{3} - \frac{2}{5}$$

$$-2P(A \cap B) = \frac{6 - 16}{15} \Rightarrow \frac{-10}{15} = -\frac{2}{3} \times \frac{-1}{2} = \frac{1}{3}$$

104. (b) February 2016 has 29 days.
= 4 weeks + 1 odd day.

Now, probability that 1 odd day is monday $= \frac{1}{7}$.

105. (d) Probability of $A = \frac{1}{6}$

Probability of $B = \frac{1}{5}$

Probability of $C = \frac{1}{4}$

Hence, required probability $= \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{120}$

106. (b) Total number of balls = 5
Number of black balls = 2
Required probability

$$= \frac{n(E)}{n(S)} = \frac{{}^2C_1 \times {}^1C_1}{{}^5C_2} = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$

107. (b) $P(A) = \frac{6}{12} = \frac{1}{2}$, $P(B) = \frac{4}{12} = \frac{1}{3}$

Req. probability $= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$

Sol. (108–111)

Set A = {1, 2, 3, 4, 5, 6, 7} and z = x + y
x = set of odd numbers
y = set of even numbers

108. (d) $n(S) = 12$
$n(E_1) = 2$

$$P(Z = 5) = \frac{n(E_1)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

109. (a) $P(Z = 10) = \frac{n(E_2)}{n(S)} = \frac{0}{12} = 0$

110. (d) $Z > 11$ is only possible when x = 7 and y = 6

$$P(> 11) = \frac{n(E_3)}{n(S)} = \frac{1}{12}$$

111. (c) Z = product of two prime numbers
$Z = x + y = 7 + 6 = 13$
$n(E_4) = 3$

$$P(Z = 9) = \frac{n(E_4)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

112. (b) $P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(AB)}{1 - P(B)}$

113. (b) A, B and D are exhaustive events but not mutually exclusive events.

114. (c) A, B and C are exhaustive events and mutually exclusive events.

115. (a) B and C are mutually exclusive events but not exhaustive events.

116. (d) C and E are mutually exclusive and elementary events.

117. (a) Clearly, $A \cap B \subseteq A$
$\Rightarrow P(A \cap B) \le P(A)$ ...(i)
$A \subseteq A \cup B$
$\Rightarrow P(A) \le P(A \cup B)$ ...(ii)
We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(A \cup B) \le P(A) + P(B)$ ...(iii)
From eqs. (i), (ii) and (iii), we get
$P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$

118. (a) $\because$ 1U, 1N, 2I, 1V, 1E, 1R, 1S, 1T, 1Y

$\therefore$ Total number of possible arrangements $= \frac{10!}{2!}$

and favourable arrangements $= \frac{10!}{2!} - 9!$

$\therefore$ Required probability $= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}}$

$= \frac{9!(5-1)}{9! \times 10} \times 2 = \frac{4}{5}$

119. (d) Box I $\to$ 4 W; 3 B
Box II $\to$ 3 W; 4 B

Probability for choosing first box $= \frac{3}{6} = \frac{1}{2}$

Probability for choosing second box $= \frac{1}{2}$

$\therefore$ Required probability $= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{4}{7}}$

$= \frac{\frac{3}{14}}{\frac{3}{14} + \frac{4}{14}} = \frac{\frac{3}{14}}{\frac{7}{14}} = \frac{3}{7}$

120. (d) Let A and B be the events

$= P(A \cap \overline{B}) + P(B \cap \overline{A})$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

$= 0.05 + 0.1 - 2(0.02)$

$= 0.15 - 0.04 = 0.11$

Hence the pobability that only one of them will qualify the examination is 0.11.

121. (b) $n(S) = 2^4 = 16$

and $n(E) = {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$

$= 1 + 4 + \frac{4 \times 3}{2 \times 1} + 4 = 1 + 4 + 6 + 4 = 15$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{15}{16}$

122. (a) $P = P(E_1)\, P(\overline{E_2}) + P(\overline{E_1})\, P(E_2)$

$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \left(1 - \frac{1}{2}\right)\frac{1}{3}$

$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$\therefore$ Option (a) is correct.

124. (d) $P = {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$+ {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^5 \left[{}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5\right]$

$= \frac{1}{3^2}[10 + 10 + 5 + 1]$

$= \frac{1}{3^2} \times 26 = \frac{13}{16}.$

126. (d) $nP = 2$

$n\,Pq = 1$

$q = \frac{1}{2}$

$P + q = 1$

$P = 1 - \frac{1}{2} = \frac{1}{2}$

$n\left(\frac{1}{2}\right) = 2$

$\therefore$ $n = 4$

${}^4C_0 = \left(\frac{1}{2}\right)^{4-0} = \frac{1}{16}.$

127. (b) $P(A \cup B) = \frac{3}{4}$

$P(A \cap B) = \frac{1}{4}$

$P(\overline{A}) = \frac{2}{3},\ P(A) = \frac{1}{3}$

$\therefore$ $P(B) = \frac{3}{4} - \frac{1}{3} + \frac{1}{4} = \frac{9 - 4 + 3}{12} = \frac{8}{12} = \frac{2}{3}$

$\therefore$ Option (b) is correct.

128. (c) $n(S) = 6 \times 6 = 36$

1,1	1, 2	1, 3	1, 4	1, 5	1,6
2,1	2, 2	2, 3	2, 4	2, 5	2,6
3,1	3,2	3, 3	3, 4	3, 5	3,6
4,1	4,2	4, 3	4, 4	4, 5	4,6
5,1	5,2	5, 3	5, 4	5, 5	5,6
6,1	6,2	6, 3	6, 4	6, 5	6,6

$\therefore$ Required number of exhausistance events $= (6-1) \times (6-1) = 5 \times 5 = 25$

$\therefore$ Option (c) is correct.

129. (b) $P(A) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$P(B/A) = \frac{^3C_1}{^{51}C_1} = \frac{3}{51} = \frac{1}{17}$

$\therefore$ Required probability $= \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

$\therefore$ Option (b) is correct.

130. (c) Total no. of 3-digit numbers $= 9 \times 8 \times 7 = 504$
For product to be odd, we have to choose only from odd numbers.
$\therefore$ Total no. of 3-digit no. whose product are odd $= 5 \times 4 \times 3 = 60$

$\therefore$ Required probability $= \frac{60}{504} = \frac{5}{42}$

131. (c) $\because P(\overline{B}) = 0.8 \Rightarrow P(B) = 0.2$

$P(A \cup B) = 0.5$ & $P(A|B) = 0.4$

$\because P(A \cap B) = P(B) \times P(A \mid B) = 0.2 \times 0.4 = 0.08$

& $P(A) = P(A \cup B) - P(B) + P(A \cap B)$

$P(A) = 0.5 - 0.2 + 0.08 = 0.38$

132. (d) We have; np = 2 = mean
npq = 1 = variance

$\Rightarrow p = \frac{1}{2}; q = \frac{1}{2}$ & $n = 4$

Required probability $= P(x > 1)$
$= 1 - P(x \le 1)$
$= 1 - [P(x = 0) + P(x = 1)]$
$= 1 - [^4C_0 q^4 + {^4C_1} q^3 p^1]$
$= 1 - \frac{5}{16} = \frac{11}{16}$

133. (a) Let X denote the no. of coins showing 3 or more heads in a set of 7 coins.
X follows binomial distribution with n = 7
p = probability of getting a head in a single toss of a coin

$\Rightarrow p = \frac{1}{2}$; thus $q = 1 - p = \frac{1}{2}$.

$\therefore$ Probability of getting at least 3 heads
$= P(x \ge 3)$
$= 1 - P(x < 3)$
$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

$= 1 - [^7C_0 + {^7C_1} + {^7C_2}] \frac{1}{2^7}$

$= \frac{128}{128} - \frac{29}{128} = \frac{99}{128}$

$\therefore$ No. of throws $= \frac{99}{128} \times 128 = 99$

134. (a) Let P denote the probability of getting head in a single toss of a coin.

$\therefore p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

Let X denote the no. of heads in 5 tosses of a coin. then, X is a binomial variate with parameters; n = 5 & $p = \frac{1}{2}$.

$\therefore$ Req. probability $= P(x > 3)$
$= 1 - P(x \le 3)$
$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$= 1 - [^5C_0 + {^5C_1} + {^5C_2} + {^5C_3}] \frac{1}{2^5}$

$= 1 - [1 + 5 + 10 + 10] \frac{1}{32}$

$= \frac{32}{32} - \frac{26}{32} = \frac{6}{32} = \frac{3}{16}$

135. (c) S = {H, TH, TTH, TTTH, TTTT}

$P(T) = P(H) = \frac{1}{2}$

[Probability of getting head or tail in a single toss]
$\therefore$ Probability that no head is observed = P(TTTT)
= P(T) P(T) P(T) P(T)

$= \frac{1}{2^4} = \frac{1}{16}$

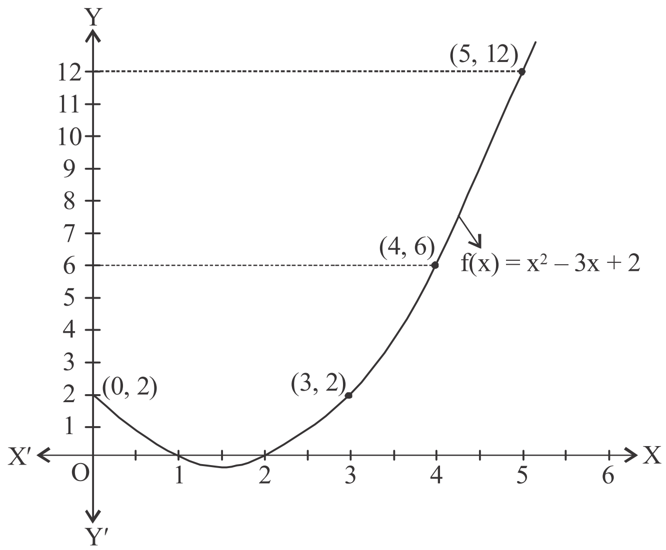
And the probability that the experiment ends with 3 tosses
= P (TTH)
= P(T) P(T) P(H)

$= \frac{1}{8}$

Hence, both statements are correct.

136. (a) Let $x^2 - 3x + 2 = 0$
$\Rightarrow x = 1, 2$
$\therefore x^2 - 3x + 2 \ge 0$ for $x \in [0, 1] \cup [2, 3] \cup [3, 4] \cup [4, 5]$.
It is given that :
$x \in [0, 1] \cup [1, 2] \cup [2, 3] \cup [3, 4] \cup [4, 5]$

$\therefore$ Required probability $= \frac{4}{5}$



137. (b) P (one white ball & one black ball)
= P {[black from 1st bag & white from 2nd] or [white from 1st & black from 2nd]}
$= P([B_1 \cap W_2) \cup (W_1 \cap B_2)]$
$= P(B_1 \cap W_2) + P(W_1 \cap B_2)$
(By addition theorem for mutually exclusive events)
$= P(B_1)P(W_2) + P(W_1)P(B_2)$
($\because B_2$ & W_2; B_2 & W_1 are pairs of independent events)

$$= \left[\frac{2}{6}\times\frac{3}{8}\right] + \left[\frac{4}{6}\times\frac{5}{8}\right]$$

$$= \frac{13}{24}$$

138. (d) Let $P(A) = \frac{1}{2},\ P(B) = \frac{1}{3}$ & $P(C) = \frac{1}{4}$

$$P(A \cup B \cup C) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$

$$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$$

$$= 1 - \left[\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right] = 1 - \frac{1}{4} = \frac{3}{4}$$

139. (a) Let E_1, E_2, E_3 & A be events defined as follows.
E_1 = person chosen is a scooter driver
E_2 = person chosen is a car driver.
E_3 = person chosen is a truck driver &
A = person meets with an accident

$$P(E_1) = \frac{2000}{12000} = \frac{1}{6};\ P(E_2) = \frac{1}{3}\ \&\ P(E_3) = \frac{1}{2}$$

$\because$ Probability that a person meets with an accident given that he is a scooter driver $= P\left(A/E_1\right) = 0.01$

$$P\left(A/E_2\right) = 0.03\ \&\ P\left(A/E_3\right) = 0.15$$

$\because$ the person meets with an accident.
$\therefore$ the probability that he was a scooter driver

$$= P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$$

$$= P\left(E_1/A\right) = \frac{1/6 \times 0.01}{\left(\frac{1}{6}\times 0.01\right) + \left(\frac{1}{3}\times 0.03\right) + \left(\frac{1}{2}\times 0.15\right)} = \frac{1}{52}$$

140. (a) Let p denote the probability of getting tail in a single of a coin.

$$\therefore p = \frac{1}{2} \Rightarrow q = \frac{1}{2}\ \&\ n = 5$$

Let X denot no. of tails in 5 tosses of coin.
$\therefore$ Required probability $= P(x=1) + P(x=3) + P(x=5)$

$$= \frac{1}{2^5}\left[{}^5C_1 + {}^5C_3 + {}^5C_5\right]$$

$$= \frac{1}{2^5}[5+10+1] = \frac{16}{32} = \frac{1}{2}$$

141. (b) Total no. of two different single digit natural number $= {}^9C_2 = 36$
The number of prime number which is sum of two different single digit number. (3, 5, 7, 11, 13 & 17) = 14

$$\therefore \text{Required probability} = \frac{14}{36} = \frac{7}{18}$$

142. (b) As we know that 3 dice are thrown. We want prob. of sum on three faces at least 5 i.e. some may be 5 or more.
We will find prob. of sum on three faces not 5 or less. i.e. sum on faces is 3 and 4 (1, 2 is not possible because of 3 dice).
No. of ways for sum on faces not 5 or more = 4
[(1, 1, 1), (1, 2, 1), (1, 1, 2), (2, 1, 1)]
Total out comes = 216

$$\text{Prob. of not 5 or more} = \frac{4}{216}$$

Prob. of sum on three faces at least 5

$$= 1 - \frac{4}{216} = \frac{212}{216} = \frac{53}{54}$$

143. (d) A and B are independent.

$$P(A) = \frac{1}{3} \qquad P(B) = \frac{3}{4}$$

We want to find probability that exactly one of the two events A or B occurs i.e. when A occurs B does not and vice-versa.
Lets take desired prob. is P.
$\therefore P = P(A)(1 - P(B)) + P(B)(1 - P(A))$

$$= \frac{1}{3}\left(1-\frac{3}{4}\right) + \frac{3}{4}\left(1-\frac{1}{3}\right)$$

$$= \frac{1}{3}\times\frac{1}{4} + \frac{3\times 2}{12}$$

$$\boxed{P = \frac{7}{12}}$$

144. (b) Coin is tossed three times i.e. total outcomes $= 2^3 = 8$
[(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)]
Alternate head and tail are coming two times only.

$$\text{Thus prob. of getting head and tail alternately} = \frac{2}{8} = \frac{1}{4}$$

145. (a) Prob. of getting queen of spade $= \frac{{}^1C_1}{{}^{52}C_1} = \frac{1}{52}$

146. (c) Since two dice are thrown so number of outcomes are 36.
No. of ways when sum on two faces less than 4 = 3.
[(1, 1), (1, 2), (2, 1)]
Hence prob of getting sum on two faces less than 4

$$= \frac{3}{36} = \frac{1}{12}$$

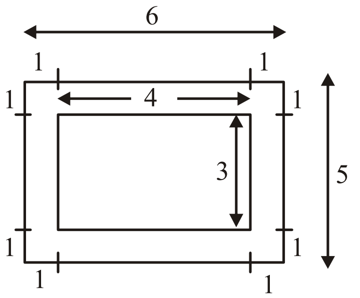
Thus required prob. that sum on the two faces is greater than or equal to 4 $= 1 - \frac{1}{12} = \frac{11}{12}$

147. (a) Probability of hitting the target = 0.3
If 'n' is the no. of times that the Missile is fired.
$\therefore$ Probability of hitting at least once = $1-[1–0.3]^n = 0.8$
$0.7^n = 0.2$
$n \log 0.7 = \log 0.2$
$\Rightarrow n = 4.512$
for n = 4; p < 0.8
take n = 5
$\boxed{n = 5}$
Hence 5 missiles should be fired so that there is at least 80% prob. that the target is hit.

148. (b) Events A and B are mutually exclusive.
Hence $P(A\cap B) = \phi = 0$
$\therefore P(A\cup B) = P(A) + P(B)$ ...(1)
$P(A) = 0.2$ [given]
$P(B) = P(\bar{A}\cap B) + P(A\cap B)$
$P(B) = P(\bar{A}\cap B)$ $[\because P(A\cap B) = 0]$
$= 0.3$
$P(A\cup B) = 0.2 + 0.3 = 0.5$
$$P(A\,|\,(A\cup B)) = \frac{P(A)}{P(A\cup B)} = \frac{0.2}{0.5} = \frac{2}{5}$$
$$P(A\,|\,(A\cup B)) = \frac{2}{5}$$

149. (c) In month of December 31 days i.e. (28 + 3) days.
In 28 days will get 4 Sundays.
If we get any Sunday in first 3 days of December than only we can get 5 Sundays in month.
n (5th Sunday) = 3 [4 weeks + 3 days]
$n(5) = 7$
Hence prob. of 5 Sundays in month of December = $\frac{3}{7}$.

150. (d)



Probability that the randomly selected point is at least one inch from the edge of the rectangle
$$= \frac{4\times 3}{6\times 5} = \frac{12}{30} = \frac{2}{5}$$

151. (a) Let x denote number of tails. Then, X is a binomial variate with parameters:
$x = 100 \;\&\; p = \frac{1}{2}$
$\therefore\; p(x=r) = {}^{100}C_r \left(\frac{1}{2}\right)^{100}$; (r = 0, 1, 2, ... 100)
Req. probability = P (x = 1) + P (x = 3) + ...+ P(x = 99)
$$= \left[\frac{1}{2}\right]^{100}\left[{}^{100}C_1 + {}^{100}C_3 + ... + {}^{100}C_{99}\right]$$
$$= \frac{1}{2^{100}}2^{99} = \frac{1}{2}$$

152. (d) Total no. of elementary events = 6^3.
Favourable no. of elementary events
= coefficient of x^0 in $[x + x^{-1} + x^0 + x^{-2} + x^2 + x^3]^3$
$$= \text{coeff. of } x^0 \text{ in } \left[\frac{1+x+x^2+x^3+x^4+x^5}{x^2}\right]^3$$
$$= \text{coeff. of } x^6 \text{ in } [1+x+x^2+x^3+x^4+x^5]^3$$
$$= \text{coeff. of } x^6 \text{ in } \left[\frac{1-x^6}{1-x}\right]^3$$
$$= \text{coeff. of } x^6 \text{ in } [1-x^6]^3[1-x]$$
$$= \text{coeff. of } x^6 \text{ in } [1 - {}^3C_1x^6 + ...][1-x]^{-3}$$
$$= \text{coeff. of } x^6 \text{ in } (1-x)^{-3} \cdot {}^3C_1 \text{ coeff. of } x^0 \text{ in } (1-x)^{-3}$$
$$= {}^{6+3-1}C_{3-1} - {}^3C_1$$
$$= {}^8C_2 - {}^3C_1 = \frac{8!}{6!2!} - \frac{3!}{2!}$$
$$= \frac{8\times 7}{2} - 3 = 25.$$
Required probability $= \frac{25}{216}$

153. (d) The probability of rain in one day
$$= \frac{25}{100} = \frac{1}{4}$$
Probability of getting at least one rainy day within a period of 7 days
$$= 1 - \left[1 - \frac{1}{4}\right]^7 = 1 - \left[\frac{3}{4}\right]^7$$

154. (b) $P(A) = \frac{70}{100} = \frac{7}{10} = P(B)$
A and B are independent.
$\Rightarrow P(A\cap B) = P(A)P(B)$
$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$$= \frac{7}{10} + \frac{7}{10} - \frac{7}{10}\times\frac{7}{10} = 0.91$$

155. (a) $\frac{1}{2}=m\times n\times\frac{1}{2}+m\times\frac{1}{2}\times(1-n)+m\times n\times 1-\frac{1}{2}$

$\Rightarrow \quad 1=m(n+1)$

156. (a) Odd in fav. for student $(A)=\frac{5}{2}=\frac{P(A)}{P(A')}$

Odd in fav. for student $(B)=\frac{4}{3}=\frac{P(B)}{P(B')}$

Odd in fav. for student $(C)=\frac{3}{4}=\frac{P(C)}{P(C')}$

$\Rightarrow \quad P(A')=\frac{2}{5}P(A),\ P(B')=\frac{3}{4}P(B),\ P(C')=\frac{4}{3}P(C)$

Now $P(A)+P(A')=1 \Rightarrow P(A)+\frac{2}{5}P(A)=1 \Rightarrow P(A)=\frac{5}{7}$

Also $P(B)+P(B')=1 \Rightarrow P(B)+\frac{3}{4}P(B)=1 \Rightarrow P(B)=\frac{4}{7}$

And $P(C)+P(C')=1 \Rightarrow P(C)+\frac{4}{3}P(C)=1 \Rightarrow P(C)=\frac{3}{7}$

$\therefore \quad P(A')=\frac{2}{5}\times\frac{5}{7}=\frac{2}{7}, \qquad P(B')=\frac{3}{4}\times\frac{4}{7}=\frac{3}{7},$

$P(C')=\frac{4}{3}\times\frac{3}{7}=\frac{4}{7}$

Req. Prob. $= P(A)\times P(B)\times P(C')+P(A)\times P(B')\times P(C)$
$+P(A')\times P(B)\times P(C)+P(A)\times P(B)\times P(C)$

$=\frac{5}{7}\times\frac{4}{7}\times\frac{4}{7}+\frac{5}{7}\times\frac{3}{7}\times\frac{3}{7}+\frac{2}{7}\times\frac{4}{7}\times\frac{3}{7}+\frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}$

$=\frac{209}{343}$

157. (c) Probabilty of medicine to cure a patient $=\frac{75}{100}=\frac{3}{4}$

Probability of curing at least one patient

$=1-\left[1-\frac{3}{4}\right]^5=1-\left(\frac{1}{4}\right)^5=1-\frac{1}{1024}=\frac{1023}{1024}$

158. (a) $P(A)=\frac{3}{5}$; $P(B)=\frac{3}{10}$

$\Rightarrow P(\bar{B})=1-P(B)=\frac{7}{10}$

$P\left(\frac{A}{B}\right)=\frac{2}{3}$

$\Rightarrow P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{2}{3}$

$\Rightarrow P(A\cap B)=\frac{2}{3}\times\frac{3}{10}=\frac{1}{5}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{3}{5}+\frac{3}{10}-\frac{1}{5}=\frac{7}{10}$

$\Rightarrow P(\overline{A\cup B})=1-\frac{7}{10}=\frac{3}{10}$

$\Rightarrow P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=\frac{3}{10}$

$\Rightarrow P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\frac{\frac{3}{10}}{\frac{7}{10}}=\frac{3}{10}\times\frac{10}{7}$

$\Rightarrow P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{3}{7}.$

159. (c) Probability that machine stops working

$=P(A\cup B\cup C)$

$\Rightarrow P(A\cup B\cup C)=P(A)+P(B)+P(C)$
$-P(A\cap B)-P(A\cap C)-P(B\cap C)$
$+P(A\cap B\cap C)$

$\Rightarrow P(A\cup B\cup C)=P(A)+P(B)+P(C)$
$-P(A)P(B)-P(A)P(C)$
$-P(B)P(C)+P(A)P(B)P(C)$

($\because$ *A, B & C* are independent events)

$\Rightarrow P(A\cup B\cup C)=0.02+0.1+0.05-(0.02\times 0.1)$
$-(0.02\times 0.05)-(0.1\times 0.05)$
$+(0.02\times 0.05\times 0.1)$

$\Rightarrow P(A\cup B\cup C)=0.16$

$\therefore$ Probability that the machine will not stop working

$=1-P(A\cup B\cup C)=1-0.16=0.84$

160. (c) $P(A_1)=\frac{1}{1+1}=\frac{1}{2}$

$P(A_2)=\frac{1}{3}$

$P(A_3)=\frac{1}{4}$

$\therefore$ Probability that at least one of these events occur is $P(A_1\cup A_2\cup A_3)$. Also A_1, A_2 & A_3 are independent events.

$P(A_1\cup A_2\cup A_3)=P(A_1)+P(A_2)+P(A_3)$
$-P(A_1\cap A_2)-P(A_1\cap A_3)$
$-P(A_2\cap A_3)+P(A_1\cap A_2\cap A_3)$

$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\left(\frac{1}{2}\times\frac{1}{3}\right)-\left(\frac{1}{2}\times\frac{1}{4}\right)$
$-\left(\frac{1}{3}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}\right)$

$=\frac{3}{4}.$

161. (d) Req. Prob. = P(5 points) = P(two wins and one draw)
= P(WWD) + P(WDW) + P(DWW)

$$= \frac{1}{3}\times\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{6}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{3}\times\frac{1}{3}$$

$$= \frac{1}{18}$$

162. (c) X follows $B(6, p) = 16P(x=4) = P(x=2)$

$$\Rightarrow 16\ {}^6C_4 \cdot p^4(1-p)^{6-4} = {}^6C_2 \cdot p^2(1-p)^{6-2}$$

$$\Rightarrow 16\times\frac{6!}{4!\,2!}p^4(1-p)^2 = \frac{6!}{2!\,4!}p^2(1-p)^4$$

$$\Rightarrow 16p^2 = (1-p)^2 \Rightarrow 16p^2 = 1+p^2-2p$$

$$\Rightarrow 15p^2+2p-1=0 \Rightarrow 15p^2+5p-3p-1=0$$

$$\Rightarrow p = \frac{1}{5}, -\frac{1}{3}$$

As $P > 0 \Rightarrow P = \frac{1}{5}$

163. (a) 2 Men + 2 Women

Required probability $= \frac{{}^2C_0 \times {}^2C_2}{{}^4C_2} = \frac{1\times 1}{6} = \frac{1}{6}$

164. (c) Required probability = 1 – (P' (A). P' (B). P' (C))

$$= 1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$$

$$= \left(1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right)$$

$$= 1-\frac{1}{4} = \frac{3}{4}$$

165. (b) $A \subseteq B \Rightarrow A\cap B = A$

$$\therefore\ P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{P(A)}{P(A)} = \frac{0.2}{0.2} = 1$$

166. (b) Probability $= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2}$

$$= \frac{\not{\pi}\left(\frac{r^2}{4}\right)}{\not{\pi} r^2}$$

$$= \frac{r^2}{4r^2} = \frac{1}{4}$$

167. (a) Number of ace cards = 4

So, probability $= \frac{4}{52} = \frac{1}{13}$

169. (c) Total **no** of outcomes when two dice are thrown
$= 6\times 6 = 36$
outcomes when sum is 8 = (2, 6) (6, 2) (3, 5) (5, 3) (4, 4)
= 5
Outcomes when sum is 9
= (3, 6) (6, 3) (4, 5) (5, 4)
= 4
Possible number of outcomes
$= 36-(5+4) = 36-9 = 27$

$\therefore$ Required probability $= \frac{27}{36} = \frac{3}{4}$

170. (d) A & B are mutualy exclusive events. i.e.,
$P(A\cap B) = 0$

We know, $P\left(\overline{A}\cap\overline{B}\right) = P\left(\overline{A\cup B}\right)$
$= 1 - P(A\cup B)$
$= 1 - [P(A) + P(B) - P(A\cap B)]$

$$= 1-\left[\frac{1}{3}+\frac{1}{4}-0\right]$$

$$= 1-\left[\frac{7}{12}\right] = \frac{5}{12}.$$

171. (c) Mean, $\overline{x} = np = 12$ ...(i)
Standard deviation
$\Rightarrow \sqrt{npq} = 2$
$\Rightarrow npq = 4$ ...(ii)

$$\frac{(ii)}{(i)} \Rightarrow \frac{\not{n}\not{p}q}{\not{n}\not{p}} = \frac{4}{12} \Rightarrow q = \frac{1}{3}$$

$$p = 1-\frac{1}{3} = \frac{2}{3}$$

Now, (i) $\Rightarrow np = 12$

$$\Rightarrow n\left(\frac{2}{3}\right) = 12 \Rightarrow 2n = 36 \Rightarrow n = 18$$

172. (b) We know, $P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

Number of possible outcomes $= {}^4C_2 = \frac{4\times 3}{2\times 1} = 6$
(selecting 2 people from 4 people)
Number of favourable outcomes
$= {}^2C_1 \times {}^2C_1 = 2\times 2 = 4$
(selecting 1 from 2 men, 1 from 2 women)

$$P(E) = \frac{4}{6} = \frac{2}{3}.$$

173. (c) Possible prime numbers on the dice are 2, 3 and 5.

Probability of getting prime number $= \frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}.$

$$= \frac{2}{9}+\frac{2}{9}$$

$$= \frac{4}{9}.$$

174. (d) Given S = 51 (includes 0)
x denotes the multiples of 3 upto 51.
n(x) = 16
y denotes the odd numbers upto 51.
n(y) = 25

$\therefore P(x) = \frac{16}{51}, P(y) = \frac{25}{51}$

175. (a) $P(A) = \frac{1}{2}, P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{2}{3} = \frac{1}{2} + P(B) - \frac{1}{6}$

$\Rightarrow P(B) = \frac{2}{3} - \frac{1}{2} + \frac{1}{6} = \frac{1}{3}$.

$P(\bar{A} \cap B) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$.

176. (c) $P(A) = \frac{1}{3}, P(B) = \frac{1}{6}, P(A \cap B) = \frac{1}{12}$

$P(B|\bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})}$

Now, $P(B \cap \bar{A}) = P(B) - P(A \cap B) = \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$

$P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore P(B|\bar{A}) = \frac{\frac{1}{12}}{\frac{2}{3}} = \frac{1}{12} \times \frac{3}{2} = \frac{1}{8}$.

177. (c) Mean $= \frac{2}{3}$, variance $= \frac{5}{9}$

$np = \frac{2}{3}, npq = \frac{5}{9}$

$\Rightarrow \frac{2}{3}q = \frac{5}{9} \Rightarrow q = \frac{5}{9} \times \frac{3}{2} = \frac{5}{6}$

So, $p = 1 - \frac{5}{6} = \frac{1}{6}$.

Now, $np = \frac{2}{3} \Rightarrow n\left(\frac{1}{6}\right) = \frac{2}{3} \Rightarrow n = \frac{2}{3} \times 6 = 4$.

$\therefore p(x = 2) = {}^{n}c_r \cdot p^r \cdot q^{n-r} = {}^{4}c_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{4-2}$

$= 6 \times \frac{1}{36} \times \frac{25}{36}$

$= \frac{25}{216}$.

178. (c) Probability that a ship reaches port $= \frac{1}{3}$.

Probability that a ship not reaching port $= 1 - \frac{1}{3} = \frac{2}{3}$

Number of ships (n) = 5.
r = 4, 5

$p(4) + p(5) = {}^{5}c_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{5-4} + {}^{5}c_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{5-5}$

$= 5\left(\frac{2}{3^5}\right) + \frac{1}{3^5}$

$= \frac{10}{243} + \frac{1}{243} = \frac{11}{243}$

179. (b) Probability that no one born in the same month

$= \frac{12 \times 11 \times 10}{12 \times 12 \times 12}$

Probability that atleast two are born in same month

$= 1 - \frac{12 \times 11 \times 10}{12 \times 12 \times 12}$

$= \frac{144 - 110}{144} = \frac{17}{72}$

180. (a) $P(B) = \frac{3}{4}, P(A \cap B \cap \bar{C}) = \frac{1}{3}, P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$.

We know,

$P(B \cap \bar{C}) = P(A \cap B \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C})$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$P(B) = P(B \cap C) + P(B \cap \bar{C})$

$\therefore P(B \cap C) = P(B) - P(B \cap \bar{C})$

$= \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$.

181. (b) Probability of knowing correct answer = p

Probability to guess correct answer $= (1-p)\left(\frac{1}{m}\right)$

Probability to answer correctly $= p + \frac{1-p}{m}$

So, required probability $= \frac{p}{p + \frac{1-p}{m}} = \frac{mp}{mp + 1 - p}$.

$= \frac{mp}{1 + p(m-1)}$.

182. (c) 3 sticks can be selected from 5 sticks in ${}^{5}c_3$ ways

${}^{5}c_3 = 10$.

Probability that selected sticks from a triangle is

$n(E) = {}^{4}c_3 - 1 = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} - 1 = 4 - 1 = 3$.

$\therefore p(E) = \frac{3}{10} = 0.3$

183. (b) The statements (1) and (3) are true.

184. (b) Probability of solving Question A, P(A) = 0.4
Probability of solving Question B, P(B) = 0.5
$\therefore$ $P(A') = 1 - P(A) = 1 - 0.4 = 0.6$
$P(B') = 1 - P(B) = 1 - 0.5 = 0.5$
Probability to solve atleast one question = $P(A \cup B)$
$P(A \cup B) = 1 - P(A' \cap B')$
$= 1 - (0.6 \times 0.5)$
$= 1 - 0.3$
$= 0.7$

185. (b) When two dice are rolled, the events where we get sum of 7 is
E = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
$\therefore$ n(E) = 6
Total number of events, n(S) = 36.

$$\therefore \text{Probability} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

186. (b) Given, 2.P(A) = 3.P(B)

$$\Rightarrow \frac{2P(A)}{P(A \cap B)} = \frac{3P(B)}{P(A \cap B)}$$

$$\Rightarrow \frac{P(A \cap B)}{2.P(A)} = \frac{P(A \cap B)}{3P(B)}$$

$$\Rightarrow \frac{1}{2}.P\left(\frac{B}{A}\right) = \frac{1}{3}.P\left(\frac{A}{B}\right)$$

$$\Rightarrow P\left(\frac{B}{A}\right) < P\left(\frac{A}{B}\right).$$

187. (c) First chit can be drawn in 10 ways.
Second chit can be drawn in 9 ways.
Total number of events, n(S) = 10 × 9 = 90.
Number of events of drawing chit numbered 9 = n(E) = 1

$$\text{Probability} = \frac{n(E)}{n(S)} = \frac{1}{90}$$

188. (b) Probability of choosing one bag from two bags $= \frac{1}{2}$

Probability of choosing white ball from first bag $= \frac{3}{5}$

Probability of choosing white ball from second bag $= \frac{5}{8}$

$$\therefore \text{Required probability} = \frac{1}{2}\left(\frac{3}{5} + \frac{5}{8}\right)$$

$$= \frac{1}{2}\left(\frac{24+25}{40}\right) = \frac{1}{2} \times \frac{49}{40} = \frac{49}{80}$$

189. (b) Statement 1 : If $B \subset A$, then
$P(A - B) = P(A) - P(A \cap B)$
$= P(A) - P(B)$
$\therefore$ It is correct.
Statement 2 : P(A alone or B alone)
$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= P(A) + P(B) - 2P(-A \cap B)$
$\therefore$ It is false.
Statement 3 : If A, B are mutually exclusive events, then $P(A \cap B) = 0$
$\Rightarrow P(A \cup B) = P(A) + P(B)$
It is correct.

190. (a) $n(E) = {}^4C_2 \times {}^5C_1 = 6 \times 5 = 30.$

$n(S) = {}^9C_3 = 84$

$$\therefore \text{Probability, } P(E) = \frac{n(E)}{n(S)} = \frac{30}{84} = \frac{5}{14}$$

191. (c) A = Event of showing 5 and atleast one dice
= (1, 5), (2, 5), (3, 5), (4, 5) (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)
$n(A) = 11$
and $n(s) = 6 \times 6 = 36$
B = Event of showing sum 10 or more when atleast one dice shows 5
= (5, 5), (5, 6), (6, 5)
$n(B) = 3$
$\Rightarrow$ $n(A \cap B) = 3$

$$\text{Now, } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(A)}{n(S)}} = \frac{\frac{3}{36}}{\frac{11}{36}} = \frac{3}{11}$$

192. (b) As A, B and C are mutually exclusive and exhaustive event.
$\therefore$ $P(A) + P(B) + P(C) = 1$

$$\Rightarrow P(A) + \frac{3}{2}P(A) + \frac{1}{2} \times \frac{3}{2}P(A) = 1$$

$$\Rightarrow \frac{13}{4}P(A) = 1$$

$$\Rightarrow P(A) = \frac{4}{13}$$

193. (a) Required probability

$$= \frac{25 \times 2}{25 \times 2 + 35 \times 4 + 40 \times 5} = \frac{5}{39}$$

194. (c) In tossing of coin getting r head out of n tossing

$$= {}^nC_r.\left(\frac{1}{2}\right)^n$$

$\therefore$ Required probability

$$= ({}^8C_6 + {}^8C_7 + {}^8C_8)\left(\frac{1}{2}\right)^8$$

$$= (28 + 8 + 1) \times \frac{1}{256} = \frac{37}{256}$$

195. (a) Required probability

$$= \frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}$$

$$= \frac{26}{64}=\frac{13}{32}$$

196. (c) $P(A)+P(B)-2P(A\cap B)=q$

$P(A\cap B)=p$

$P(A)+P(B)=2p+q$

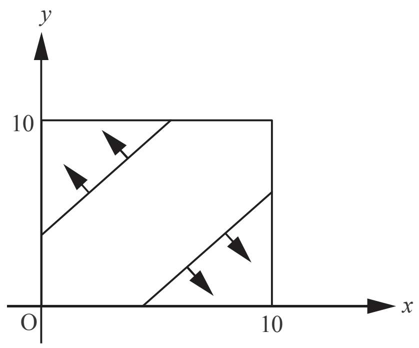
$1-P(\overline{A})+1-P(\overline{B})=p+q$

$P(\overline{A})+P(\overline{B})=2-2p-q$

$P(\overline{A}\cap\overline{B})=1-P(A\cup B)$

$=1-(q+p)=1-p-q$

197. (c) Probability $=\dfrac{2\times\frac{1}{2}\times 5\times 6}{11\times 11}=\dfrac{30}{121}$



198. (d) A deck of cards has 52 cards.
Probability of taking fourteenth card as Ace

$$=\frac{{}^4C_1}{{}^{52}C_1}=\frac{4}{52}=\frac{1}{13}$$

199. (d) $P(A)=0.5,\ P(B)=0.6,\ P(A\cap B)=0.4$

$P(\overline{A\cup B})=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=1-[0.5+0.6-0.4]$

$=1-0.7=0.3$

200. (a) Given, $P(A)=\frac{1}{2}, P(B)=\frac{3}{4}, P(C)=\frac{1}{4}$

$P(\overline{A})=1-\frac{1}{2}=\frac{1}{2}$

$P(\overline{B})=1-\frac{3}{4}=\frac{1}{4}$

$P(\overline{C})=1-\frac{1}{4}=\frac{3}{4}$

$\therefore$ Probability that problem will be solved if they solve independently is

$1-(P(\overline{A})\cdot P(\overline{B})\cdot P(\overline{C}))$

$$=1-\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)=1-\frac{3}{32}=\frac{29}{32}$$

201. (c) Possibilities of having higher number on second dice.

First dice	**Second dice**
1	2, 3, 4, 5, 6 $\rightarrow$ 5 possibilities
2	3, 4, 5, 6 $\rightarrow$ 4 possibilities
3	4, 5, 6 $\rightarrow$ 3 possibilities
4	5, 6 $\rightarrow$ 2 possibilities
5	6 $\rightarrow$ 1 possibility

Total number of possibilities = 15

Total number of events = 36

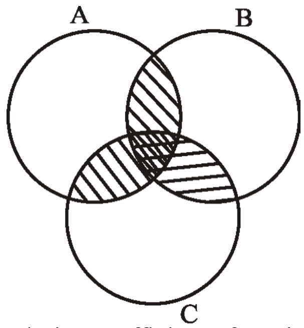
$\therefore$ Probability $=\frac{15}{36}=\frac{5}{12}$

202. (c) Total number of events with dice and coin = 6 × 2 = 12

Number of possibilities = (2, H), (4, H) and (6, H) i.e., 3

$\therefore$ Probability $=\frac{3}{12}=\frac{1}{4}$

203. (c) $P(A\cap B)+P(B\cap C)+P(A\cap C)-2P(A\cap B\cap C)$



204. (c) The correlation coefficient of two independent events is zero.

205. (b) $P(A\cup B)=\frac{2}{3}$

$P(A\cap B)=\frac{1}{6}$

Since, A and B are independent events,

$P(A\cap B)=P(A)\cdot P(B)=\frac{1}{6}$ ...(1)

$P(A\cup B)=\frac{2}{3}\Rightarrow P(A)+P(B)-P(A\cap B)=\frac{2}{3}$

$\Rightarrow P(A)+P(B)-\frac{1}{6}=\frac{2}{3}$

$\Rightarrow P(A)+P(B)=\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$ ...(2)

from (1), (2), $P(B)=\frac{1}{3}$ or $\frac{1}{2}$

$\because P(B)<P(A), P(B)=\frac{1}{3}$

206. (c) Total number of cases that sum is 8 are
(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)
favourable case = (6, 2)

$\therefore$ Probability $= \frac{1}{5}$

207. (c) Sides of dice = R, R, B, B, Y, W
Total events with dice = 6 × 6 = 36
Favourable events = ${}^2C_1 . {}^2C_1 + {}^2C_1 . {}^2C_1 + {}^1C_1 . {}^1C_1 + {}^1C_1 . {}^1C_1$
= 4 + 4 + 2 + 2 = 10

$\therefore$ Probability $= \frac{10}{36} = \frac{5}{18}$

208. (b) Number of socks = n
Number of red socks = 3

Given, $\frac{{}^3C_2}{{}^nC_2} = \frac{1}{2}$

$\Rightarrow {}^nC_2 = 3 \times 2 = 6$
$\Rightarrow n = 4$

209. (a) Number of ways of selecting 2 cards from deck of cards $= {}^{52}C_2$
favourable cases $= {}^{13}C_2$

Probability $= \frac{{}^{13}C_2}{{}^{52}C_2} = \frac{\frac{13\times12}{2}}{\frac{52\times51}{2}} = \frac{13\times12}{52\times51} = \frac{1}{17}$

210. (b) 5 or 6 is success

$\therefore p = \frac{2}{6} = \frac{1}{3}$

$\therefore q = 1 - \frac{1}{3} = \frac{2}{3}$

n = 8

$\therefore$ Mean = np = $8\left(\frac{1}{3}\right) = \frac{8}{3}$

Standard deviation $= \sqrt{npq} = \sqrt{8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)}$

$= \sqrt{\frac{16}{9}} = \frac{4}{3}$

211. (b) $\bar{A}$ and $\bar{B}$ are mutually exclusive

$\therefore P(\bar{A} \cap \bar{B}) = 0$

Given, P(A) = 0.5 $\Rightarrow P(\bar{A}) = 1 - 0.5 = 0.5$

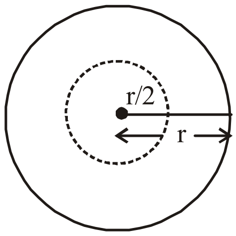
P(B) = 0.6 $\Rightarrow P(\bar{B}) = 1 - 0.6 = 0.4$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1 - P(\bar{A} \cup \bar{B})}{P(B)}$

$= \frac{1 - (P(\bar{A}) + P(\bar{B}))}{P(B)} = \frac{1 - (0.5 + 0.4)}{0.6}$

$= \frac{1 - 0.9}{0.6} = \frac{0.1}{0.6} = \frac{1}{6}$

212. (a) If r is the radius of circle, A = πr^2



Probability $= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{\pi r^2}{4} \times \frac{1}{\pi r^2} = \frac{1}{4}$

213. (b) Probability of occurence of either event A or event B = P (A $\cup$ B).

22 Vectors

1. Let $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar vectors and $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q} = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r} = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$.

   What is the value of

   $(\vec{a} - \vec{b} - \vec{c}).\vec{p} + (\vec{b} - \vec{c} - \vec{a}).\vec{q} + (\vec{c} - \vec{a} - \vec{b}).\vec{r}$?

   (a) 0 (b) -3

   (c) 3 (d) -9 *[2006-I]*

2. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of corners A, B, C of a parallelogram ABCD, then what is the position vector of the corner D ?

   (a) $\vec{a} + \vec{b} + \vec{c}$ (b) $\vec{a} + \vec{b} - \vec{c}$

   (c) $\vec{a} - \vec{b} + \vec{c}$ (d) $-\vec{a} + \vec{b} + \vec{c}$ *[2006-I]*

3. In a ΔABC, angle B is obtuse and D, E, F are the middle points of sides BC, CA, AB respectively. Which one of the following vectors has the greatest magnitude ?

   (a) $\overrightarrow{BC}$ (b) $\overrightarrow{CA}$

   (c) $\overrightarrow{AB}$ (d) $\overrightarrow{AD}$ *[2006-I]*

4. If $\vec{p} \neq \vec{0}$ and the conditions $\vec{p}.\vec{q} = \vec{p}.\vec{r}$ and $\vec{p} \times \vec{q} = \vec{p} \times \vec{r}$ hold simultaneously, then which one of the following is correct ?

   (a) $\vec{q} \neq \vec{r}$ (b) $\vec{q} = -\vec{r}$

   (c) $\vec{q}.\vec{r} = 0$ (d) $\vec{q} = \vec{r}$ *[2006-I]*

5. If two unit vectors $\vec{p}$ and $\vec{q}$ make an angle $\dfrac{\pi}{3}$ with each other, what is the magnitude of $\vec{p} - \dfrac{1}{2}\vec{q}$?

   (a) 0 (b) $\dfrac{\sqrt{3}}{2}$

   (c) 1 (d) $\dfrac{1}{\sqrt{2}}$ *[2006-I]*

6. What are the values of x for which the two vectors $(x^2 - 1)\hat{i} + (x+2)\hat{j} + x^2\hat{k}$ and $2\hat{i} - x\hat{j} + 3\hat{k}$ are orthogonal ? *[2006-I]*

   (a) No real value of x (b) $x = \dfrac{1}{2}$ and $x = -1$

   (c) $x = -\dfrac{1}{2}$ and $x = 1$ (d) $x = -1$ and $x = 2$

7. What is the moment about the point $\hat{i} + 2\hat{j} + 3\hat{k}$, of a force represented by $\hat{i} + \hat{j} + \hat{k}$, acting through the point $-2\hat{i} + 3\hat{j} + \hat{k}$? *[2006-I]*

   (a) $2\hat{i} + \hat{j} + 2\hat{k}$ (b) $\hat{i} - \hat{j} + 3\hat{k}$

   (c) $3\hat{i} + 2\hat{j} - \hat{k}$ (d) $3\hat{i} + \hat{j} - 4\hat{k}$

8. A particle is acted upon by following forces :

   (i) $2\hat{i} + 3\hat{j} + 5\hat{k}$, (ii) $-5\hat{i} + 4\hat{j} - 3\hat{k}$ and (iii) $3\hat{i} - 7\hat{k}$

   In which plane does it move ? *[2006-I]*

   (a) x-y plane
   (b) y-z plane
   (c) z-x plane
   (d) any arbitrary plane

9. What is the vector whose magnitude is 3, and is perpendicular to $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$? *[2006-I]*

   (a) $3(\vec{i} + \vec{j} - \vec{k})$

   (b) $\sqrt{3}(\vec{i} - \vec{j} + \vec{k})$

   (c) $\sqrt{3}(\vec{i} + \vec{j} + \vec{k})$

   (d) $3(\vec{i} - \vec{j} + \vec{k})$

10. If α, β, γ be angles which the vector $\vec{r} = \lambda\vec{i} + 2\vec{j} - \vec{k}$ makes with the coordinate axes, then what is the value of

    $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$? *[2006-I]*

    (a) 2 (b) 1

    (c) $\lambda^2 + 1$ (d) $1 - \lambda^2$

11. The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.

Assertion (A) : If $\vec{a} = 2\vec{i} + \vec{j} - 2\vec{k}, \vec{b} = \vec{i} + \vec{j} - \vec{k}$,

then $|\vec{a}| \neq |\vec{b}|$

Reason (R) : Two unequal vectors can never have same magnitude.

(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2006-I]*

12. OAB is a given triangle such that $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$. Also C is a point on $\overrightarrow{AB}$ such that $\overrightarrow{AB} = 2\,\overrightarrow{BC}$. What is $\overrightarrow{AC}$ equal to ?

(a) $\frac{1}{2}(\vec{b} - \vec{a})$ (b) $\frac{1}{2}(\vec{b} + \vec{a})$
(c) $\frac{3}{2}(\vec{a} - \vec{b})$ (d) $\frac{3}{2}(\vec{b} - \vec{a})$ *[2006-I]*

13. Let ABCD be a parallelogram whose diagonals intersect at P and let O be the origin, then what is $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ equal to ?

(a) $\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$
(c) $3\overrightarrow{OP}$ (d) $4\overrightarrow{OP}$ *[2006-II]*

14. If $\vec{r_1}, \vec{r_2}, \vec{r_3}$ are the position vectors of three collinear points and scalars m and n exist such that $\vec{r_3} = m\vec{r_1} + n\vec{r_2}$, then what is the value of (m + n) ?

(a) 0 (b) 1
(c) – 1 (d) 2 *[2006-II]*

15. Let α be the angle which the vector $\vec{V} = 2\hat{i} - \hat{j} + 2\hat{k}$ makes with the z-axis,. Then, what is the value of sin α ?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{\sqrt{5}}{3}$ (d) $\frac{\sqrt{5}}{9}$ *[2006-II]*

16. If $\vec{m}, \vec{n}, \vec{r}$ are three vectors, θ is the angle between the vectors $\vec{m}$ and $\vec{n}$, what is mnr cos θ equal to ?*[2006-II]*

(a) $(\vec{m}.\vec{n})(\vec{r}.(\vec{r}/r))$ (b) $(\vec{m}.\vec{n})(\vec{r}.\vec{r})$
(c) $(\vec{m}.\vec{r})(\vec{n}.(\vec{n}/n))$ (d) $(\vec{m}.\vec{n})\,\vec{r}$

17. If the vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then what is the locus of the point (x, y) ?

(a) A circle (b) An ellipse
(c) A parabola (d) A hyperbola *[2006-II]*

18. If the components of $\vec{b}$ along and perpendicular to $\vec{a}$ are $\lambda\vec{a}$ and $\vec{b} - \lambda\vec{a}$ respectively, what is λ equal to ?

(a) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$
(c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}$ (d) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$ *[2006-II]*

19. A force $m\hat{i} - 3\hat{j} + \hat{k}$ acts on a point and so the point moves from (20, 3m, 0) to (0, 0, 7). If the work done by the force is – 48 unit, what is the value of m ?

(a) 5 (b) 3
(c) 2 (d) 1 *[2006-II]*

20. For any two vectors $\vec{a}$ and $\vec{b}$ consider the following statement:

1. $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = |\vec{\mathbf{a}} - \vec{\mathbf{b}}| \Leftrightarrow \vec{\mathbf{a}}, \vec{\mathbf{b}}$ are orthogonal.
2. $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = |\vec{\mathbf{a}}| + |\vec{\mathbf{b}}| \Leftrightarrow \vec{\mathbf{a}}, \vec{\mathbf{b}}$ are orthogonal.
3. $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 \Leftrightarrow \vec{\mathbf{a}}, \vec{\mathbf{b}}$ are orthogonal.

Which of the above statements is/are correct?

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3 *[2007-I]*

21. Two vector $2\hat{\mathbf{i}} + m\hat{\mathbf{j}} - 3n\hat{\mathbf{k}}$ and $5\hat{\mathbf{i}} + 3m\hat{\mathbf{j}} + n\hat{\mathbf{k}}$ are such that their magnitudes are respectively $\sqrt{14}$ and $\sqrt{35}$, where m, n are integers. Which one of the following is correct ?

(a) m takes 1 value, n takes 1 value
(b) m takes 1 value, n takes 2 values
(c) m takes 2 value, n takes 1 value
(d) m takes 2 value, n takes 2 values *[2007-I]*

22. Two vectors $\vec{a}$ and $\vec{b}$ are non-zero and non-collinear. What is the value of x for which the vectors $\vec{p} = (x - 2)\,\vec{a} + \vec{b}$ and $\vec{q} = (x + 1)\,\vec{a} - \vec{b}$ are collinear?

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) 2 *[2007-I]*

23. If $\vec{a}$ and $\vec{b}$ are position vectors of the points A and B respectively, then what is the position vector of a point C on AB produced such that $\overline{AC} = \overline{2AB}$?

(a) $2\vec{a}-\vec{b}$ (b) $2\vec{b}-\vec{a}$

(c) $\vec{a}-2\vec{b}$ (d) $\vec{a}-\vec{b}$ [2007-I]

24. If $|\vec{a}|=3, |\vec{b}|=4$, then for what value of l is $(\vec{a}+\lambda\vec{b})$ perpendicular to $(\vec{a}-\lambda\vec{b})$?

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{9}{16}$ (d) $\frac{3}{5}$ [2007-I]

25. What is the magnitude of the moment of the couple consisting of the force $\vec{F}=3\hat{i}+2\hat{j}-\hat{k}$ acting through the point $\hat{i}-\hat{j}+\hat{k}$ and and $-\vec{F}$ acting through the point $2\hat{i}-3\hat{j}-\hat{k}$?

(a) $2\sqrt{5}$ (b) $3\sqrt{5}$

(c) $5\sqrt{5}$ (d) $7\sqrt{5}$ [2007-I]

26. Let $\vec{a}=2\hat{j}-3\hat{k}, \vec{b}=\hat{j}+3\hat{k}$ and $\vec{c}=-3\hat{i}+3\hat{j}+\hat{k}$. Let $\hat{n}$ be a unit vector such that $\vec{a}.\hat{n}=\vec{b}.\hat{n}=0$. What is the value of $\vec{c}.\hat{n}$?

(a) 1 (b) $\sqrt{19}$

(c) 3 (d) –3 [2007-I]

27. Let $\vec{u}=\hat{i}-\hat{j}$, $\vec{v}=2\hat{i}+5\hat{j}$, $\vec{w}=4\hat{i}+3\hat{j}$ and $\vec{p}=\vec{u}+\vec{v}+\vec{w}$. Which one of the following is correct?

(a) $-3\vec{u}+2\vec{v}=\vec{p}$ (b) $3\vec{u}-2\vec{v}=\vec{p}$

(c) $3\vec{u}+2\vec{v}=\vec{p}$ (d) $-3\vec{u}-2\vec{v}=\vec{p}$ [2007-I]

28. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle of 30° to each other, then which one of the following is correct ?

(a) $|\vec{a}+\vec{b}|>1$ (b) $1<|\vec{a}+\vec{b}|<2$

(c) $|\vec{a}+\vec{b}|=2$ (d) $|\vec{a}+\vec{b}|>2$ [2007-I]

29. Which one of the following is correct ? If the vector $\vec{c}$ is normal to the vectors $\vec{a}$ and $\vec{b}$, then $\vec{c}$, is :

(a) parallel to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

(b) normal to $\vec{a}-\vec{b}$ and parallel to $\vec{a}+\vec{b}$

(c) normal to $\vec{a}+\vec{b}$ and parallel to $\vec{a}-\vec{b}$

(d) normal to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ [2007-II]

30. Which one of the following statements is not correct ?

(a) Vector product is commutative

(b) Vector product is not associative

(c) Vector product is distributive over addition

(d) Scalar product is commutative [2007-II]

31. If $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}$, and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar vectors, then what is the value of $a+b+c-abc$?

(a) 0 (b) 1

(c) 2 (d) -2 [2007-II]

32. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero vectors and $|(\vec{a}\times\vec{b}).\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$, then which one of the following is correct ?

(a) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}\neq 0$

(b) $\vec{a}.\vec{b}=0$ only

(c) $\vec{b}.\vec{c}=0$ only

(d) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ [2007-II]

33. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+\lambda\hat{k}$, and $(\vec{a}+\vec{b})$ is perpendicular to $\vec{a}-\vec{b}$, then what is the value of λ ?

(a) -2 only (b) ± 2

(c) 3 only (d) ± 3 [2007-II]

34. The vectors $\overline{AB}=\vec{c}, \overline{BC}=\vec{a}, \overline{CA}=\vec{b}$, are the sides of a triangle ABC. Which of the following vectors represent (s) the median $\overline{AD}$?

1. $\frac{1}{2}\vec{a}+\vec{c}$

2. $-\frac{1}{2}\vec{b}+\frac{1}{2}\vec{c}$

3. $\frac{1}{2}\vec{a}+\vec{b}$

Select the correct answer using the code given below

(a) 1 and 2 (b) 1 and 3

(c) 1 only (d) 2 only [2007-II]

35. If $\vec{a}$ is a position vector of a point (1, – 3) and A is another point (– 1, 5), then what are the coordinates of the point B such that $\overline{AB}=\vec{a}$?

(a) (2, 0) (b) (0, 2)

(c) (–2, 0) (d) (0, –2) [2008-I]

36. If $\vec{a}=2\hat{i}-3\hat{j}-\hat{k}, \vec{b}=\hat{i}+4\hat{j}-2\hat{k}$; then what is $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$ equal to ?

(a) $2(\vec{a}\times\vec{b})$ (b) $-2(\vec{a}\times\vec{b})$

(c) $(\vec{a}\times\vec{b})$ (d) $-(\vec{a}\times\vec{b})$ [2008-I]

37. If $\vec{a}$ is a non-zero vector of modulus a and λ is a non-zero scalar and $\lambda\,\vec{a}$ is a unit vector then
(a) $\lambda = \pm 1$ (b) $a = |\lambda|$
(c) $a = \frac{1}{|\lambda|}$ (d) $a = \frac{1}{\lambda}$ only [2008-I]

38. Let $\vec{a}$ and $\vec{b}$ be the position vectors of A and B respectively. If C is the point $3\vec{a} - 2\vec{b}$, then which one of the following is correct?
(a) C is in between A and B
(b) A is in between C and B
(c) B is in between A and C
(d) A, B, C are not collinear [2008-I]

39. Consider the following
If $\vec{a}$ and $\vec{b}$ are the vectors forming consecutive sides of a regular hexagon ABCDEF, then
1. $\overrightarrow{CE} = \vec{b} - 2\vec{a}$ 2. $\overrightarrow{AE} = 2\vec{b} - \vec{a}$
3. $\overrightarrow{FA} = \vec{a} - \vec{b}$
Which of the above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3 [2008-I]

40. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}$ is perpendicular to the plane of $\vec{b}, \vec{c}$; and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.
Then, what is $|\vec{a} + \vec{b} + \vec{c}|$?
(a) 1 (b) 2
(c) 3 (d) 4 [2008-I]

41. What is the locus of the point (x, y) for which the vectors $(\hat{i} - x\hat{j} - 2\hat{k})$ and $(2\hat{i} + \hat{j} + y\hat{k})$ are orthogonal ?
(a) A circle (b) An ellipse
(c) A parabola (d) A straight line [2008-I]

42. What is the number of vectors of length 5 unit perpendicular to the vectors $\vec{a} = (1, 1, 0)$ and $\vec{b} = (0, 1, 1)$?
(a) 1 (b) 2
(c) 3 (d) 4 [2008-I]

43. What is the area of the rectangle of which $\vec{r} = a\vec{i} + b\vec{j}$ is a semidiagonal ?
(a) $a^2 + b^2$ (b) $2(a^2 + b^2)$
(c) $4(a^2 + b^2)$ (d) 4ab [2008-I]

44. If $(3\vec{a} - \vec{b}) \times (\vec{a} + 3\vec{b}) = k\,\vec{a} \times \vec{b}$ then what is the value of k? [2008-II]
(a) 10 (b) 5
(c) 8 (d) –8

45. What is the value of λ if the triangle whose vertices are $\hat{i}, \hat{j}$ and $\hat{i} + \hat{j} + \lambda\hat{k}$ will be right angled? [2008-II]
(a) 2 (b) 0
(c) –1 (d) 1

46. The scalar triple product $(\vec{A} \times \vec{B}) \cdot \vec{C}$ of three vectors $\vec{A}, \vec{B}, \vec{C}$ determines [2008-II]
(a) Volume of a parallelopiped
(b) Volume of a tetrahedron
(c) Volume of an ellipsoid
(d) None of the above

47. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the value of $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$? [2008-II]
(a) 0 (b) 2
(c) 1 (d) 1/2

48. Two forces are equal to $2\overrightarrow{OA}$ and $3\overrightarrow{BO}$, their resultant being $\lambda\overrightarrow{OG}$, where G is the point on AB such that $\frac{BG}{AG} = -\frac{2}{3}$. What is the value of λ ? [2008-II]
(a) 1 (b) –1
(c) 2 (d) None of the above

49. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle 60° to each other, then which one of the following is correct? [2008-II]
(a) $|\vec{a} + \vec{b}| < 1$ (b) $|\vec{a} + \vec{b}| > 1$
(c) $|\vec{a} - \vec{b}| < 1$ (d) $|\vec{a} - \vec{b}| > 1$

50. Let $\vec{a} = (1, -2, 3)$ and $\vec{b} = (3, 1, 2)$ be two vectors and $\vec{c}$ be a vector of length l and parallel to $(\vec{a} + \vec{b})$. What is $\vec{c}$ equal to? [2008-II]
(a) $\frac{1}{\sqrt{4}}(-2, -3, 1)$ (b) $\frac{1}{\sqrt{2}}(1, 0, 1)$
(c) $\frac{1}{\sqrt{42}}(-5, -4, -1)$ (d) None of these

51. If $\vec{r}_1 = \lambda\hat{i} + 2\hat{j} + \hat{k}$, $\vec{r}_2 = \hat{i} + (2 - \lambda)\hat{j} + 2\hat{k}$ are such that $|\vec{r}_1| > |\vec{r}_2|$, then λ satisfies which one of the following? [2008-II]
(a) $\lambda = 0$ only (b) $\lambda = 1$
(c) $\lambda < 1$ (d) $\lambda > 1$

52. If P, Q, R are the mid points of the sides AB, BC, CA, respectively of a triangle ABC and if $\vec{a}$, $\vec{p}$, $\vec{q}$ are the position vector of A, P, Q respectively, then what is the position vector of R? *[2008-II]*
(a) $2\vec{a}-(\vec{p}-\vec{q})$ (b) $(\vec{p}-\vec{q})-2\vec{a}$
(c) $\vec{a}-(\vec{p}-\vec{q})$ (d) $\vec{a}/2-(\vec{p}-\vec{q})/2$

53. What is the length of the vector (1, 1)? *[2008-II]*
(a) 0 (b) 1
(c) $\sqrt{2}$ (d) $\frac{1}{2}$

54. Which one of the following vectors of magnitude $\sqrt{51}$ makes equal angles with three vectors
$$\vec{a}=\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}, \vec{b}=\frac{-4\hat{i}-3\hat{k}}{5} \text{ and } \vec{c}=\hat{j}\text{ ?}$$ *[2009-I]*
(a) $5\hat{i}-\hat{j}-5\hat{k}$ (b) $5\hat{i}+\hat{j}+5\hat{k}$
(c) $-5\hat{i}-\hat{j}+5\hat{k}$ (d) $5\hat{i}+5\hat{j}-\hat{k}$

55. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, then what is the value of $\vec{a}\cdot\vec{b}$? *[2009-I]*
(a) 4 (b) 6
(c) 8 (d) 10

56. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then which one of the following is correct? *[2009-I]*
(a) $\vec{a}$ is parallel to $\vec{b}$
(b) $\vec{a}$ is perpendicular to $\vec{b}$
(c) $\vec{a}=\vec{b}$
(d) Both $\vec{a}$ and $\vec{b}$ are unit vectors

57. If $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-3\hat{k}$, then what is $(\vec{b}-\vec{a})\cdot(3\vec{a}+\vec{b})$ equal to? *[2009-I]*
(a) 106 (b) –106
(c) 53 (d) –53

58. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of points A, B, C respectively. Under which one of the following conditions are the points A, B, C collinear? *[2009-I]*
(a) $\vec{a}\times\vec{b}=\vec{0}$
(b) $\vec{b}\times\vec{c}$ is parallel to $\vec{a}\times\vec{b}$
(c) $\vec{a}\times\vec{b}$ is perpendicular to $\vec{b}\times\vec{c}$
(d) $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\vec{0}$

59. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$, then what is $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$ equal to? *[2009-I]*
(a) $2\hat{i}+3\hat{j}-\hat{k}$ (b) $2\hat{i}-3\hat{j}-\hat{k}$
(c) $3\hat{i}+\hat{j}+\hat{k}$ (d) $\vec{0}$

60. The following item consists of two statements, one labelled the Assertion (A) and the other labelled the Reason (R). You are to examine these two statements carefully and decide if the Assertion (A) and Reason (R) are individually true and if so, whether the reason is a correct explanation of the Assertion. Select your answer using the codes given below.
Assertion (A) : The work done when the force and displacement are perpendicular to each other is zero.
Reason (R) : the dot product $\vec{A}\cdot\vec{B}$ vanishes, if the vector $\vec{A}$ and $\vec{B}$ are perpendicular. *[2009-I]*
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

61. If $\hat{a}$ and $\hat{b}$ are the unit vectors along $\vec{a}$ and $\vec{b}$ respectively, then what is the projection of $\vec{b}$ on $\vec{a}$? *[2009-II]*
(a) $\vec{a}\cdot\vec{b}$ (b) $\hat{a}\cdot\hat{b}$
(c) $\hat{a}\cdot\vec{b}$ (d) $|\vec{a}\times\vec{b}|$

62. What are the unit vectors parallel to xy-plane and perpendicular to the vector $4\hat{i}-3\hat{j}+\hat{k}$? *[2009-II]*
(a) $\pm(3\hat{i}+4\hat{j})/5$
(b) $\pm(4\hat{i}+3\hat{j})/5$
(c) $\pm(3\hat{i}-4\hat{j})/5$
(d) $\pm(4\hat{i}-3\hat{j})/5$

63. What is the vector in the xy-plane through origin and perpendicular to the vector $\vec{r}=a\hat{i}+b\hat{j}$ and of the same length? *[2009-II]*
(a) $-a\hat{i}-b\hat{j}$ (b) $a\hat{i}-b\hat{j}$
(c) $-a\hat{i}+b\hat{j}$ (d) $b\hat{i}-a\hat{j}$

64. Given $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\hat{b}$ is a unit vector codirectional with $\hat{a}$. If m is a scalar such that $\hat{b}=m\vec{a}$, then what is the value of m? *[2009-II]*
(a) 1/5 (b) $1/\sqrt{5}$
(c) 1/29 (d) $1/\sqrt{29}$

65. The magnitude of the vectors $\vec{a}$ and $\vec{b}$ are equal and the angle between them is 60°. If the vectors $\lambda\vec{a}+\vec{b}$ and $\vec{a}-\lambda\vec{b}$ are perpendicular to each other, then what is the value of λ? *[2009-II]*
(a) 1 (b) 2
(c) 3 (d) 4

66. If $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{a}-\vec{b}|=7$, then what is the value of $|\vec{a}+\vec{b}|$? *[2009-II]*
(a) 3 (b) 2
(c) 1 (d) 0

67. Consider the diagonals of a quadrilateral formed by the vectors $3\hat{i}+6\hat{j}-2\hat{k}$ and $4\hat{i}-\hat{j}+3\hat{k}$. The quadrilateral must be a *[2009-II]*
(a) Square (b) Rhombus
(c) Rectangle (d) None of these

68. What is the area of the triangle with vertices (0, 2, 2), (2, 0, – 1) and (3, 4, 0)? *[2010-I]*
(a) $\frac{15}{2}$ sq unit (b) 15 sq unit
(c) $\frac{7}{2}$ sq unit (d) 7 sq unit

69. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, what is the angle between $-5\vec{a}$ and $6\vec{b}$? *[2010-I]*
(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$
(c) $\frac{2\pi}{5}$ (d) $\frac{3\pi}{7}$

70. Consider the following statements *[2010-I]*
1. For any three vectors $\vec{a},\vec{b},\vec{c}$;
$\vec{a}.\{(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})\}=0$
2. For any three coplanar unit vectors
$\vec{d},\vec{e},\vec{f};(\vec{d}\times\vec{e}).\vec{f}=1$

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

71. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them. If $(\vec{a}+\vec{b})$ is also the unit vectors, then what is the value of α? *[2010-I]*
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

72. What is the value of λ for which the vectors $\hat{i}-\hat{j}+\hat{k}, 2\hat{i}+\hat{j}-\hat{k}$ and $\lambda\hat{i}-\hat{j}+\lambda\hat{k}$ are co-planar? *[2010-I]*
(a) 1 (b) 2
(c) 3 (d) 4

73. What is the geometric interpretation of the identity $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2(\vec{a}\times\vec{b})$? *[2010-I]*
1. If the diagonals of a given parallelogram are used as sides of a second parallelogram, then the area of the second parallelogram is twice that of the given parallelogram.
2. If the semi-diagonals of a given parallelogram are used as sides of a second parallelogram, then the area of the second parallelogram is half that of the given parallelogram.

Select the correct answer using the code given below
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

74. A vector $\vec{b}$ is collinear with the vector $\vec{a}$ = (2, 1, –1) and satisfies the condition $\vec{a}.\vec{b}$ =3. What is $\vec{b}$ equal to? *[2010-I]*
(a) (1, 1/2, –1/2)
(b) (2/3, 1/3, –1/3)
(c) (1/2, 1/4, –1/4)
(d) (1, 1, 0)

75. The vectors $\vec{a}=x\vec{i}+y\vec{j}+z\vec{k}, \vec{b}=\hat{k}, \vec{c}$ are such that they form a right handed system. What is $\vec{c}$ equal to? *[2010-I]*
(a) $\hat{j}$ (b) $y\hat{j}-x\hat{k}$
(c) $y\hat{i}-x\hat{j}$ (d) $x\hat{i}-y\hat{j}$

76. If the position vector of a point *P* with respect to origin *O* is $\hat{i}+3\hat{j}-2\hat{k}$ and that of a point *Q* is $3\hat{i}+\hat{j}-2\hat{k}$, then what is the position vector of a point on the bisector of the angle *POQ*? *[2010-II]*
(a) $\hat{i}-\hat{j}-\hat{k}$ (b) $\hat{i}+\hat{j}-\hat{k}$
(c) $\hat{i}+\hat{j}+\hat{k}$ (d) None of these

77. Let *a*, *b* and *c* be the distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}, c\hat{i}+c\hat{j}+b\hat{k}$ lie on a plane, then which one of the following is correct ? *[2010-II]*
(a) *c* is the arithmetic mean of a and *b*
(b) *c* is the geometric mean of a and *b*
(c) *c* is the harmonic mean of a and *b*
(d) *c* is equal to zero

78. If $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ [2010-II]

$\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$.

then $\vec{a}.\left(\vec{b} \times \vec{c}\right)$ depends on

(a) x only (b) y only
(c) Both x and y (d) Neither x nor y

79. PQRS is a parallelogram, where $\overrightarrow{PQ} = 3\hat{i} + 2\hat{j} - m\hat{k}$, $\overrightarrow{PS} = \hat{i} + 3\hat{j} + \hat{k}$ and the area of the parallelogram is $\sqrt{90}$. What is the value of m? [2010-II]

(a) 1 (b) – 1
(c) 2 (d) – 2

80. What is the vector equally inclined to the vectors $\hat{i} + 3\hat{j}$ and $3\hat{i} + \hat{j}$? [2010-II]

(a) $\hat{i} + \hat{j}$ (b) $2\hat{i} - \hat{j}$
(c) $2\hat{i} + \hat{j}$ (d) None of these

81. ABCD is a quadrilateral. Forces $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}$ and $\overrightarrow{DA}$ act along its sides. What is their resultant? [2010-II]

(a) $2\overrightarrow{CD}$ (b) $2\overrightarrow{DA}$
(c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{CB}$

82. What is the area of a triangle whose vertices are at (3, – 1, 2), (1, – 1, – 3) and (4, – 3, 1)? [2010-II]

(a) $\frac{\sqrt{165}}{2}$ (b) $\frac{\sqrt{135}}{2}$
(c) 4 (d) 2

83. What is the value of b such that the scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and b $\hat{i} + 2\hat{j} + 3\hat{k}$ is unity?

(a) – 2 (b) – 1 [2010-II]
(c) 0 (d) 1

84. Let p, q, r and s be respectively the magnitudes of the vectors $3\hat{i} - 2\hat{j}$, $2\hat{i} + 2\hat{j} + \hat{k}$, $4\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$. Which one of the following is correct? [2011-I]

(a) $r > s > q > p$ (b) $s > r > p > q$
(c) $r > s > p > q$ (d) $s > r > q > p$

85. If $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector and x : y : z = $\sqrt{3} : 2 : 3$, then what is the value of z? [2011-I]

(a) $\frac{3}{16}$ (b) 3
(c) $\frac{3}{4}$ (d) 2

86. Which one of the following is the unit vector perpendicular to the vectors $4\hat{i} + 2\hat{j}$ and $-3\hat{i} + 2\hat{j}$?

(a) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ (b) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ [2011-I]
(c) $\hat{k}$ (d) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

87. Consider the following statements in respect of the vectors $\vec{u}_1 = (1, 2, 3)$, $\vec{u}_2 = (2, 3, 1)$, $\vec{u}_3 = (1, 3, 2)$ and $\vec{u}_4 = (4, 6, 2)$ [2011-I]

I. $\vec{u}_1$ is parallel to $\vec{u}_4$.

II. $\vec{u}_2$ is parallel to $\vec{u}_4$.

III. $\vec{u}_2$ is parallel to $\vec{u}_3$.

Which of the statements given above is/are correct?

(a) Only I (b) Only II
(c) Only III (d) Both I and III

88. The points with position vectors [2011-I] $10\hat{i} + 3\hat{j}$, $12\hat{i} - 5\hat{j}$, $a\hat{i} + 11\hat{j}$ are collinear, if the value of a is

(a) – 8 (b) 4
(c) 8 (d) 12

89. What is the sine of angle between the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$? [2011-I]

(a) $\sqrt{\frac{13}{7}}$ (b) $\frac{\sqrt{13}}{7}$
(c) $\frac{13}{\sqrt{7}}$ (d) None of these

90. The vector $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$. Which one of the following is correct? [2011-I]

(a) $\vec{a}.\left(\vec{b} \times \vec{c}\right) = 0$ (b) $\vec{a}.\left(\vec{b} \times \vec{c}\right) = 1$
(c) $\vec{a}.\left(\vec{b} \times \vec{c}\right) = -1$ (d) $\vec{a}.\left(\vec{b} \times \vec{c}\right) = 3$

91. What is the projection of the vector $\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ on the vector $4\hat{\mathbf{i}}-4\hat{\mathbf{j}}+7\hat{\mathbf{k}}$? [2011-I]

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{19}{9}$

(c) $\frac{\sqrt{5}}{4}$ (d) $\frac{11}{3}$

92. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \bullet \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then which one of the following is correct?

(a) $\vec{a}$ is parallel to $\vec{b}$ [2011-II]

(b) $\vec{a}$ is perpendicular to $\vec{b}$

(c) Either $\vec{a}$ or $\vec{b}$ is a null vector

(d) None of the above

93. If the vectors $-\hat{i}-2x\hat{j}-3y\hat{k}$ and $\hat{i}-3x\hat{j}-2y\hat{k}$ are orthogonal to each other, then what is the locus of the point (x, y)?

(a) a straight line (b) an ellipse [2011-II]

(c) a parabola (d) a circle

94. If $\vec{c}$ is the unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, then what is another unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$? [2011-II]

(a) $\vec{c} \times \vec{a}$ (b) $\vec{c} \times \vec{b}$

(c) $-\frac{\left(\vec{a}\times\vec{b}\right)}{\left|\vec{a}\times\vec{b}\right|}$ (d) $\frac{\left(\vec{a}\times\vec{b}\right)}{\left|\vec{a}\times\vec{b}\right|}$

95. For what value of m are the points with position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$ and $m\hat{i}+11\hat{j}$ collinear?

(a) – 8 (b) 4 [2011-II]

(c) 8 (d) 12

96. For what value of m are the vectors $2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $m\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ coplanar? [2011-II]

(a) 0 (b) 5/3

(c) 1 (d) 8/5

97. What is the area of the triangle with vertices (1,2,3),(2,5,–1) and (–1,1,2)? [2011-II]

(a) $\frac{\sqrt{155}}{2}$ square units (b) $\frac{\sqrt{175}}{2}$ square units

(c) $\frac{\sqrt{155}}{4}$ square units (d) $\frac{\sqrt{175}}{4}$ square units

98. What is the area of the rectangle having vertices A, B, C and D with positive vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$? [2012-I]

(a) 1/2 square unit (b) 1 square unit

(c) 2 square unit (d) 4 square unit

99. If $\vec{a}=(2,1,-1)$, $\vec{b}=(1,-1,0)$, $\vec{c}=(5,-1,1)$, then what is the unit vector parallel to $\vec{a}+\vec{b}-\vec{c}$ in the opposite direction? [2012-I]

(a) $\frac{\hat{i}+\hat{j}-2\hat{k}}{3}$ (b) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

(c) $\frac{2\hat{i}-\hat{j}+2\hat{k}}{3}$ (d) None of the above

100. If the magnitudes of two vectors a and b are equal then which one of the following is correct? [2012-I]

(a) $\left(\vec{a}+\vec{b}\right)$ is parallel to $\left(\vec{a}-\vec{b}\right)$

(b) $\left(\vec{a}+\vec{b}\right)\bullet\left(\vec{a}-\vec{b}\right)=1$

(c) $\left(\vec{a}+\vec{b}\right)$ is perpendicular to $\left(\vec{a}-\vec{b}\right)$

(d) None of the above

101. Let O be the origin and P, Q, R be the points such that $\overline{PO}+\overline{OQ}=\overline{QO}+\overline{OR}$. Then which one of the following is correct? [2012-I]

(a) P, Q, R are the vertices of an equilateral triangle

(b) P, Q, R are the vertices of an isosceles triangle

(c) P, Q, R are collinear

(d) None of the above

102. What is the value of m if the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+m\hat{j}+5\hat{k}$ are coplanar? [2012-I]

(a) – 2 (b) 2

(c) – 4 (d) 4

103. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \bullet \vec{b} = 12$, then what is the value of $|\vec{a} \times \vec{b}|$? [2012-I]
(a) 12 (b) 16
(c) 20 (d) 24

104. If the vectors $\hat{i} - x\hat{j} - y\hat{k}$ and $\hat{i} + x\hat{j} + y\hat{k}$ are orthogonal to each other, then what is the locus of the point (x, y) ?
(a) a parabola (b) an ellipse [2012-I]
(c) a circle (d) a straight line

105. EFGH is a rhombus such that the angle EFG is 60°. The magnitude of vectors $\overline{FH}$ and $\{m\,\overline{EG}\}$ are equal where m is a scalar. What is the value of m ? [2012-II]
(a) 3 (b) 1.5
(c) $\sqrt{2}$ (d) $\sqrt{3}$

106. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$ then which one of the following is correct ? [2012-II]
(a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a}$ is perpendicular to $\vec{b}$
(c) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ (d) None of the above

107. The vector $\vec{a} \times (\vec{b} \times \vec{a})$ is coplanar with : [2012-II]
(a) $\vec{a}$ only (b) $\vec{b}$ only
(c) Both $\vec{a}$ and $\vec{b}$ (d) Neither $\vec{a}$ nor $\vec{b}$

108. Consider the following :
1. $4\hat{i} \times 3\hat{i} = \vec{0}$ 2. $\dfrac{4\hat{i}}{3\hat{i}} = \dfrac{4}{3}$
Which of the above is/are correct ? [2012-II]
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

109. What is the value of λ for which
$(\lambda\hat{i} + \hat{j} - \hat{k}) \times (3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 11\hat{j} - 7\hat{k})$? [2012-II]
(a) 2 (b) –2
(c) 1 (d) 7

110. The magnitude of the scalar p for which the vector $p(-3\hat{i} - 2\hat{j} + 13\hat{k})$ is of unit length is : [2012-II]
(a) 1/8 (b) 1/64
(c) $\sqrt{182}$ (d) $\dfrac{1}{\sqrt{182}}$

111. The vector $2\hat{j} - \hat{k}$ lies : [2012-II]
(a) in the plane of XY (b) in the plane of YZ
(c) in the plane of XZ (d) along the X-axis

112. ABCD is a parallelogram. If $\overline{AB} = \vec{a}$, $\overline{BC} = \vec{b}$, then what is $\overline{BD}$ equal to ? [2012-II]
(a) $\vec{a} + \vec{b}$ (b) $\vec{a} - \vec{b}$
(c) $-\vec{a} - \vec{b}$ (d) $-\vec{a} + \vec{b}$

113. If $\vec{\beta}$ is perpendicular to both $\vec{\alpha}$ and $\vec{\gamma}$ where $\vec{\alpha} = \hat{k}$ and $\vec{\gamma} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, then what is $\vec{\beta}$ equal to? [2013-I]
(a) $3\hat{i} + 2\hat{j}$ (b) $-3\hat{i} + 2\hat{j}$
(c) $2\hat{i} - 3\hat{j}$ (d) $-2\hat{i} + 3\hat{j}$

114. For any vector $\vec{\alpha}$, what is $(\vec{\alpha} \cdot \hat{i})\hat{i} + (\vec{\alpha} \cdot \hat{j})\hat{j} + (\vec{\alpha} \cdot \hat{k})\hat{k}$ equal to ? [2013-I]
(a) $\vec{\alpha}$ (b) $3\vec{\alpha}$
(c) $-\vec{\alpha}$ (d) $\vec{0}$

115. If the magnitude of $\vec{a} \times \vec{b}$ equals to $\vec{a} \cdot \vec{b}$, then which one of the following is correct? [2013-I]
(a) $\vec{a} = \vec{b}$
(b) The angle between $\vec{a}$ and $\vec{b}$ is 45°
(c) $\vec{a}$ is parallel to $\vec{b}$
(d) $\vec{a}$ is perpendicular to $\vec{b}$

116. If $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = \sqrt{3}$ and $|\vec{a} + \vec{b}| = \sqrt{6}$, then what is $|\vec{a} - \vec{b}|$ equal to? [2013-I]
(a) 1 (b) 2
(c) 3 (d) 4

117. Which one of the following vectors is normal to the vector $\hat{i} + \hat{j} + \hat{k}$? [2013-I]
(a) $\hat{i} + \hat{j} - \hat{k}$ (b) $\hat{i} - \hat{j} + \hat{k}$
(c) $\hat{i} - \hat{j} - \hat{k}$ (d) None of the above

118. If θ is the angle between the vectors is $4(\hat{i} - \hat{k})$ and $\hat{i} + \hat{j} + \hat{k}$, then what is (sin θ + cos θ) equal to ? [2013-I]
(a) 0 (b) $\dfrac{1}{2}$
(c) 1 (d) 2

119. If the angle between the vectors $\hat{i} - m\hat{j}$ and $\hat{j} + \hat{k}$ is $\dfrac{\pi}{3}$, then what is the value of m ? [2013-II]
(a) 0 (b) 2
(c) –2 (d) None of these

120. What is the vector perpendicular to both the vectors $\hat{i} - \hat{j}$ and $\hat{i}$? *[2013-II]*

(a) $\hat{i}$ (b) $-\hat{j}$

(c) $\hat{j}$ (d) $\hat{k}$

121. The position vectors of the points *A* and *B* are respectively $3\hat{i} - 5\hat{j} + 2\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$. What is the length of *AB* ? *[2013-II]*

(a) 11 (b) 9

(c) 7 (d) 6

122. The vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other. Then the locus of the point (*x, y*) is *[2013-II]*

(a) hyperbola (b) ellipse

(c) parabola (d) circle

123. What is the value of *P* for which the vector $p(2\hat{i} - \hat{j} + 2\hat{k})$ is of 3 units length ? *[2013-II]*

(a) 1 (b) 2

(c) 3 (d) 6

124. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are three vectors such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$, then what is *t* equal to ? *[2013-II]*

(a) 8 (b) 6

(c) 4 (d) 2

DIRECTIONS (Qs. 125 - 127): *(For the next three (03) items that follow) :*

The vertices of a triangle ABC are A (2, 3, 1), B (–2, 2, 0), and C (0, 1, –1). *[2014-I]*

125. What is the cosine of angle *ABC* ?

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{\sqrt{6}}$ (d) None of these

126. What is the area of the triangle ?

(a) $6\sqrt{2}$ square unit (b) $3\sqrt{2}$ square unit

(c) $10\sqrt{3}$ square unit (d) None of these

127. What is the magnitude of the line joining mid points of the sides *AC* and *BC* ?

(a) $\frac{1}{\sqrt{2}}$ unit (b) 1 unit

(c) $\frac{3}{\sqrt{2}}$ unit (d) 2 unit

DIRECTIONS (Qs. 128-129) : *For the next two (02) items that follow.*

Consider the vectors $\bar{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\bar{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$. *[2014-I]*

128. What is the scalar projection of $\bar{a}$ on $\bar{b}$?

(a) 1 (b) 19/9

(c) 17/9 (d) 23/9

129. What is the vector perpendicular to both the vectors ?

(a) $-10\hat{i} - 3\hat{j} + 4\hat{k}$ (b) $-10\hat{i} + 3\hat{j} + 4\hat{k}$

(c) $10\hat{i} - 3\hat{j} + 4\hat{k}$ (d) None of these

DIRECTIONS (Qs. 130-131): *For the next two (02) items that follow.*

Let a vector $\bar{r}$ make angle 60°, 30° with x and y-axes respectively. *[2014-I]*

130. What angle does $\bar{r}$ make with *z*-axis ?

(a) 30° (b) 60°

(c) 90° (d) 120°

131. What are the direction cosines of $\bar{r}$?

(a) $\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$ (b) $\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$ (d) $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$

DIRECTIONS (Qs. 132-133) : *For the next two (02) items that follow.*

Let $|\bar{a}| = 7$, $|\bar{b}| = 11$, $|\bar{a} + \bar{b}| = 10\sqrt{3}$ *[2014-I]*

132. What is $|\bar{a} - \bar{b}|$ equal to ?

(a) $2\sqrt{2}$ (b) $2\sqrt{10}$

(c) 5 (d) 10

133. What is the angle between $(\bar{a} + \bar{b})$ and $(\bar{a} - \bar{b})$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) None of these

134. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}|$, then what is $\vec{a}.\vec{b}$ equal to ? *[2014-II]*

(a) 6 (b) 7

(c) 8 (d) 9

135. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then which one of the following is correct? *[2014-II]*

(a) $|\vec{a}| = |\vec{b}|$.

(b) $\vec{a}$ is parralel to $\vec{b}$.

(c) $\vec{a}$ is perpendicular to $\vec{b}$.

(d) $\vec{a}$ is a unit vector .

136. What is the area of the triangle OAB where O is the origin, $\overrightarrow{OA} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 3\hat{k}$? *[2014-II]*
(a) $5\sqrt{6}$ square unit (b) $\frac{5\sqrt{6}}{2}$ square unit
(c) $\sqrt{6}$ square unit (d) $\sqrt{30}$ square unit

137. Which one of the following is the unit vector perpendicular to both $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$? *[2014-II]*
(a) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ (b) $\hat{k}$
(c) $\frac{\hat{j} + \hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$

138. What is the interior acute angle of the parallelogram whose sides are represented by the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$? *[2014-II]*
(a) 60° (b) 45°
(c) 30° (d) 15°

139. For what value of λ are the vectors
$\lambda\hat{i} + (1+\lambda)\hat{j} + (1+2\lambda)\hat{k}$ and $(1-\lambda)\hat{i} + \lambda\hat{j} + 2\hat{k}$
perpendicular ? *[2014-II]*
(a) –1/3 (b) 1/3
(c) 2/3 (d) 1

DIRECTIONS (Qs. 140-143) : *For the next four (04) items that follow.*

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$.
[2014-II]

140. What is the angle between $|\vec{a}|$ and $|\vec{b}|$?
(a) $\pi/6$ (b) $\pi/4$
(c) $\pi/3$ (d) $\pi/2$

141. What is $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ equal to ?
(a) –83 (b) –83/2
(c) 75 (d) –75/2

142. What is cosine of the angle between $\vec{b}$ and $\vec{c}$?
(a) 11/12 (b) 13/14
(c) –11/12 (d) –13/14

143. What is $|\vec{a} + \vec{b}|$ equal to ?
(a) 7 (b) 8
(c) 10 (d) 11

144. The adjacent sides AB and AC of a triangle ABC are represented by the vectors $-2\hat{i} + 3\hat{j} + 2\hat{k}$ and $-4\hat{i} + 5\hat{j} + 2\hat{k}$ respectively. The area of the triangle ABC is *[2015-I]*
(a) 6 square units (b) 5 square units
(c) 4 square units (d) 3 square units

145. A force $\vec{F} = 3\hat{i} + 4\hat{j} - 3\hat{k}$ is applied at the point P, whose position vector is $\vec{r} = 2\hat{i} - 2\hat{j} - 3\hat{k}$. What is the magnitude of the moment of the force about the origin? *[2015-I]*
(a) 23 units (b) 19 units
(c) 18 units (d) 21 units

146. Given that the vectors $\vec{\alpha}$ and $\vec{\beta}$ are non-collinear. The values of x and y for which $\vec{u} - \vec{v} = \vec{w}$ holds true if $\vec{u} = 2x\vec{\alpha} + y\vec{\beta}, \vec{v} = 2y\vec{\alpha} + 3x\vec{\beta}$ and $\vec{w} = 2\vec{\alpha} - 5\vec{\beta}$, are *[2015-I]*
(a) $x = 2, y = 1$ (b) $x = 1, y = 2$
(c) $x = -2, y = 1$ (d) $x = -2, y = -1$

147. If $|\vec{a}| = 7$, $|\vec{b}| = 11$ and $|\vec{a} + \vec{b}| = 10\sqrt{3}$, then $|\vec{a} - \vec{b}|$ is equal to *[2015-I]*
(a) 40 (b) 10
(c) $4\sqrt{10}$ (d) $2\sqrt{10}$

148. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ and $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$ *[2015-I]*
(a) are collinear
(b) form an equilateral triangle
(c) form a scalene triangle
(d) form a right-angled triangle

149. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then which of the following is/are correct? *[2015-I]*
1. $\vec{a}, \vec{b}, \vec{c}$ are coplanar.
2. $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
Select the correct answer using the code given below.
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

150. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then which one of the following is correct? *[2015-I]*
(a) $\vec{a} = \lambda\vec{b}$ for some scalar λ
(b) $\vec{a}$ is parallel to $\vec{b}$
(c) $\vec{a}$ is perpendicular to $\vec{b}$
(d) $\vec{a} = \vec{b} = \vec{0}$

151. The area of the square, one of whose diagonals is $3\hat{i} + 4\hat{j}$ is *[2015-II]*
(a) 12 square unit
(b) 12.5 square unit
(c) 25 square unit
(d) 156.25 square unit

152. ABCD is a parallelogram and P is the point of intersection of the diagonals. If O is the origin, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ is equal to *[2015-II]*

(a) $4\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$

(c) $\overrightarrow{OP}$ (d) Null vector

153. If $\vec{b}$ and $\vec{c}$ are the position vectors of the points B and C respectively, then the position vector of the point D such that $\overrightarrow{BD}=4\overrightarrow{BC}$ is *[2015-II]*

(a) $4(\vec{c}-\vec{b})$ (b) $-4(\vec{c}-\vec{b})$

(c) $4\vec{c}-3\vec{b}$ (d) $4\vec{c}+3\vec{b}$

154. If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}|=13$, then the value/values of *n* an be *[2015-II]*

(a) ± 8 (b) ± 12

(c) 8 only (d) 12 only

155. If $|\vec{a}|=2$ and $|\vec{b}|=3$, then $|\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2$ is equal to *[2015-II]*

(a) 72 (b) 64

(c) 48 (d) 36

156. Consider the following inequalities in respect of vectors $\vec{a}$ and $\vec{b}$: *[2015-II]*

1. $|\vec{a}+\vec{b}|$ £ $|\vec{a}|+|\vec{b}|$

2. $|\vec{a}-\vec{b}|$ ³ $|\vec{a}|-|\vec{b}|$

Which of the above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

157. If the magnitude of difference of two unit vectors is $\sqrt{3}$, then the magnitude of sum of the two vectors is *[2015-II]*

(a) $\frac{1}{2}$ unit (b) 1 unit

(c) 2 unit (d) 3 unit

158. If the vectors $\alpha\hat{i}+\alpha\hat{j}+\gamma\hat{k}, \hat{i}+\hat{k}$ and $\gamma\hat{i}+\gamma\hat{j}+\beta\hat{k}$ lie on a plane, where α, β and γ are distinct non-negative numbers, then γ is *[2015-II]*

(a) Arithmetic mean of α and β

(b) Geometric mean of α and β

(c) Harmonic mean of α and β

(d) None of the above

159. The vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are such that $\vec{a}\times\vec{b}=\vec{c}\times d$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$. Which of the following is/ are correct ? *[2015-II]*

1. $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$

2. $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$

Select the correct answer using the code given below :

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 160-161) : *For the next two (2) items that follow:*

Let $\hat{a},\hat{b}$ be two unit vectors and θ be the angle between them. *[2016-I]*

160. What is $\cos\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a}-\hat{b}|}{2}$ (b) $\frac{|\hat{a}+\hat{b}|}{2}$

(c) $\frac{|\hat{a}-\hat{b}|}{4}$ (d) $\frac{|\hat{a}+\hat{b}|}{4}$

161. What is $\sin\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a}-\hat{b}|}{2}$ (b) $\frac{|\hat{a}+\hat{b}|}{2}$

(c) $\frac{|\hat{a}-\hat{b}|}{4}$ (d) $\frac{|\hat{a}+\hat{b}|}{4}$

162. What is a vector of unit length orthogonal to both the vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}-\hat{k}$? *[2016-I]*

(a) $\frac{-4\hat{i}+3\hat{j}-\hat{k}}{\sqrt{26}}$ (b) $\frac{-4\hat{i}+3\hat{j}+\hat{k}}{\sqrt{26}}$

(c) $\frac{-3\hat{i}+2\hat{j}-\hat{k}}{\sqrt{14}}$ (d) $\frac{-3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{14}}$

163. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct? *[2016-I]*

(a) $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ (b) $\vec{a}+\vec{b}+\vec{c}=$ unit vector

(c) $\vec{a}+\vec{b}=\vec{c}$ (d) $\vec{a}=\vec{b}+\vec{c}$

164. What is the area of the parallelogram having diagonals $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$? *[2016-I]*

(a) $5\sqrt{5}$ square units (b) $4\sqrt{5}$ square units

(c) $5\sqrt{3}$ square units (d) $15\sqrt{2}$ square units

DIRECTIONS (Qs. 165-166) : *Consider the following for the next two (02) items that follow:*

Let $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=3\hat{i}+4\hat{k}$ and $\vec{b}=\vec{c}+\vec{d}$, where $\vec{c}$ is parallel to $\vec{a}$ and $\vec{d}$ is perpendicular to $\vec{a}$. *[2016-II]*

165. What is $\vec{c}$ equal to?

(a) $\frac{3(\hat{i}+\hat{j})}{2}$ (b) $\frac{2(\hat{i}+\hat{j})}{3}$

(c) $\frac{(\hat{i}+\hat{j})}{2}$ (d) $\frac{(\hat{i}+\hat{j})}{3}$

166. If $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$, then which of the following equations is/are correct?
1. $y - x = 4$
2. $2z - 3 = 0$

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 167-168) : *Consider the following for the next two (02) items that follow.*

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 10, |\vec{b}| = 6$ and $|\vec{c}| = 14$. *[2016-II]*

167. What is $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ equal to?
(a) –332 (b) –166
(c) 0 (d) 166

168. What is the angle between $\vec{a}$ and $\vec{b}$?
(a) 30° (b) 45°
(c) 60° (d) 75°

169. In a right-angled triangle ABC, if the hypotenuse AB = p, then what is $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$ equal to? *[2016-II]*
(a) p (b) p^2
(c) $2p^2$ (d) $\frac{p^2}{2}$

170. A force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point (1, –1, 2). What is the moment of the force about the point (2, –1, 3)? *[2016-II]*
(a) $\hat{i} + 4\hat{j} + 4\hat{k}$ (b) $2\hat{i} + \hat{j} + 2\hat{k}$
(c) $2\hat{i} - 7\hat{j} - 2\hat{k}$ (d) $2\hat{i} + 4\hat{j} - \hat{k}$

171. If $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{c} = \hat{i} - m\hat{j} + n\hat{k}$ are three coplanar vectors and $|\vec{c}| = \sqrt{6}$, then which one of the following is correct? *[2017-I]*
(a) m = 2 and n = ± 1 (b) m = ± 2 and n = –1
(c) m = 2 and n = –1 (d) m = ± 2 and n = 1

172. Let ABCD be a parallelogram whose diagonals intersect at P and let O be the origin. What is $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ equal to? *[2017-I]*
(a) $2\overrightarrow{OP}$ (b) $4\overrightarrow{OP}$
(c) $6\overrightarrow{OP}$ (d) $8\overrightarrow{OP}$

173. ABCD is a quadrilateral whose diagonals are AC and BD. Which one of the following is correct? *[2017-I]*
(a) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{AC} + \overrightarrow{DB}$ (b) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} + \overrightarrow{CA}$
(c) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{AC} + \overrightarrow{BD}$ (d) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BC} + \overrightarrow{AD}$

174. If $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then which one of the following is correct? *[2017-I]*
(a) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{a}| = |\vec{c}|$ and $|\vec{b}| = 1$
(b) $\vec{a}, \vec{b}, \vec{c}$ are non-orthogonal to each other
(c) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{a}| \neq |\vec{c}|$
(d) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal is pairs but $|\vec{b}| \neq 1$

175. If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$ are perpendicular, then what is the value of λ? *[2017-I]*
(a) 2 (b) 3
(c) 4 (d) 5

176. If α, β and γ are the angles which the vector $\overrightarrow{OP}$ (O being the origin) makes with positive direction of the coordinate axes, then which of the following are correct? *[2017-II]*
1. $\cos^2\alpha + \cos^2\beta = \sin^2\gamma$
2. $\sin^2\alpha + \sin^2\beta = \cos^2\gamma$
3. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Select the correct answer using the code given below.
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

177. Let $\vec{\alpha} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$ be three vectors. If $\vec{\alpha}$ and $\vec{\beta}$ are both perpendicular to the vector $\vec{\delta}$ and $\vec{\delta} \cdot \vec{\gamma} = 10$, then what is the magnitude of $\vec{\delta}$? *[2017-II]*
(a) $\sqrt{3}$ units (b) $2\sqrt{3}$ units
(c) $\frac{\sqrt{3}}{2}$ unit (d) $\frac{1}{\sqrt{3}}$ unit

178. If $\hat{a}$ and $\hat{b}$ are two unit vectors, then the vector $(\hat{a} + \hat{b}) \times (\hat{a} \times \hat{b})$ is parallel to *[2017-II]*
(a) $(\hat{a} - \hat{b})$ (b) $(\hat{a} + \hat{b})$
(c) $(2\hat{a} - \hat{b})$ (d) $(2\hat{a} + \hat{b})$

179. A force $\vec{F} = \hat{i} + 3\hat{j} + 2\hat{k}$ acts on a particle to displace it from the point A $(\hat{i} + 2\hat{j} - 3\hat{k})$ to the point B$(3\hat{i} - \hat{j} + 5\hat{k})$. The work done by the force will be *[2017-II]*
(a) 5 units (b) 7 units
(c) 9 units (d) 10 units

180. For any vector $\vec{a}$ $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is equal to *[2017-II]*
(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$
(c) $3|\vec{a}|^2$ (d) $4|\vec{a}|^2$

181. If the vectors $a\hat{i}+\hat{j}+\hat{k},\ \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}=\hat{j}=c\hat{k}(a,b,c\neq 1)$ are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is equal to *[2017-II]*
(a) 0 (b) 1
(c) a + b + c (d) abc

182. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}|=2$, $|\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$, then what is the acute angle between $\vec{a}$ and $\vec{b}$? *[2018-I]*
(a) 30° (b) 45°
(c) 60° (d) 90°

183. Let $\vec{p}$ and $\vec{q}$ be the position vectors of the points P and Q respectively with respect to origin O. The points R and S divide PQ internally and externally respectively in the ratio 2 : 3. If $\overline{OR}$ and $\overline{OS}$ are perpendicular, then which one of the following is correct? *[2018-I]*
(a) $9p^2=4q^2$ (b) $4p^2=9p^2$
(c) $9p=4q$ (d) $4p=9q$

184. What is the moment about the point $\hat{i}+2\hat{j}-\hat{k}$ of a force represented by $3\hat{i}+\hat{k}$ acting through the point $2\hat{i}-\hat{j}+3\hat{k}$? *[2018-I]*
(a) $-3\hat{i}+11\hat{j}+9\hat{k}$ (b) $3\hat{i}+2\hat{j}+9\hat{k}$
(c) $3\hat{i}+4\hat{j}+9\hat{k}$ (d) $\hat{i}+\hat{j}+\hat{k}$

185. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\lambda(\vec{b}\times\vec{c})$, then what is the value of λ? *[2018-I]*
(a) 2 (b) 3
(c) 4 (d) 6

186. If the vectors $\vec{k}$ and $\vec{A}$ are parallel to each other, then what is $k\vec{k}\times\vec{A}$ equal to? *[2018-I]*
(a) $k^2\vec{A}$ (b) $\vec{0}$
(c) $-k^2\vec{A}$ (d) $\vec{A}$

187. Let $|\vec{a}|\neq 0, |\vec{b}|\neq 0$ *[2018-II]*
$(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$ holds if and only if
(a) $\vec{a}$ and $\vec{b}$ are perpendicular
(b) $\vec{a}$ and $\vec{b}$ are parallel
(c) $\vec{a}$ and $\vec{b}$ are inclined at an angle of 45°
(d) $\vec{a}$ and $\vec{b}$ are anti-parallel

188. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then what is $\vec{r}.(\hat{i}+\hat{j}+\hat{k})$ equal to? *[2018-II]*
(a) x (b) x + y
(c) –(x + y + z) (d) (x + y + z)

189. A unit vector perpendicular to each of the vectors $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}-4\hat{j}-\hat{k}$ is *[2018-II]*
(a) $\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$ (b) $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}$
(c) $\frac{1}{\sqrt{3}}\hat{i}-\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$ (d) $\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$

190. If $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{a}-\vec{b}|=5$, then what is the value of $|\vec{a}+\vec{b}|=$? *[2018-II]*
(a) 8 (b) 6
(c) $5\sqrt{2}$ (d) 5

191. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three mutually perpendicular vectors each of unit magnitud. If $\vec{A}=\vec{a}+\vec{b}+\vec{c}$, $\vec{B}=\vec{a}-\vec{b}+\vec{c}$ and $\vec{C}=\vec{a}-\vec{b}-\vec{c}$, then which one of the following is correct? *[2018-II]*
(a) $|\vec{A}|>|\vec{B}|>|\vec{C}|$ (b) $|\vec{A}|=|\vec{B}|\neq|\vec{C}|$
(c) $|\vec{A}|=|\vec{B}|=|\vec{C}|$ (d) $|\vec{A}|\neq|\vec{B}|\neq|\vec{C}|$

192. What is $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$ equal to? *[2018-II]*
(a) $\vec{0}$ (b) $\vec{a}\times\vec{b}$
(c) $2(\vec{a}\times\vec{b})$ (d) $|\vec{a}|^2-|\vec{b}|^2$

193. A spacecraft at $\hat{i}+2\hat{j}+3\hat{k}$ is subjected to a force $\lambda\hat{k}$ by firing a rocket. The spacecraft is subjected to a moment of magnitude *[2018-II]*
(a) λ (b) $\sqrt{3}\lambda$
(c) $\sqrt{5}\lambda$ (d) None of these

194. In a triangle ABC, if taken in order, consider the following statements: *[2018-II]*
1. $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$
2. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$
3. $\overrightarrow{AB}-\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$
4. $\overrightarrow{BA}-\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

How many of the above statements are correct?

(a) One (b) Two
(c) Three (d) Four

195. If $\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ then what is $(\vec{b} - \vec{a}).(3\vec{a} + \vec{b})$ equal to? [2019-I]

(a) 106 (b) –106 (c) 53 (d) –53

196. If the position vectors of points A and B are $3\hat{i} - 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} - 3\hat{k}$ respectively, then what is the length of $\overrightarrow{AB}$? [2019-I]

(a) $\sqrt{14}$ (b) $\sqrt{29}$ (c) $\sqrt{43}$ (d) $\sqrt{53}$

197. If in a right-angled triangle ABC, hypotenuse AC = p, then what is $\overrightarrow{AB} \cdot \overrightarrow{AC} + \overrightarrow{BC} \cdot \overrightarrow{BA} + \overrightarrow{CA} \cdot \overrightarrow{CB}$ equal to ? [2019-I]

(a) p^2 (b) $2p^2$ (c) $\frac{p^2}{2}$ (d) p

198. The sine of the angle between vectors [2019-I]
$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$

(a) $\frac{1}{\sqrt{26}}$ (b) $\frac{5}{\sqrt{26}}$
(c) $\frac{5}{26}$ (d) $\frac{1}{26}$

199. What is the value of λ for which the vectors $3\hat{i} + 4\hat{j} - \hat{k}$ and $-2\hat{i} + \lambda\hat{j} + 10\hat{k}$ are perpendicular? [2019-I]

(a) 1 (b) 2 (c) 3 (d) 4

ANSWER KEY

1	(c)	21	(d)	41	(d)	61	(a)	81	(d)	101	(c)	121	(c)	141	(b)	161	(a)	181	(b)
2	(c)	22	(b)	42	(b)	62	(a)	82	(a)	102	(c)	122	(d)	142	(d)	162	(b)	182	(a)
3	(b)	23	(b)	43	(d)	63	(d)	83	(d)	103	(b)	123	(a)	143	(a)	163	(a)	183	(a)
4	(d)	24	(a)	44	(a)	64	(d)	84	(c)	104	(c)	124	(a)	144	(d)	164	(c)	184	(a)
5	(b)	25	(c)	45	(b)	65	(a)	85	(c)	105	(d)	125	(a)	145	(a)	165	(a)	185	(d)
6	(c)	26	(d)	46	(a)	66	(c)	86	(c)	106	(c)	126	(b)	146	(a)	166	(d)	186	(b)
7	(d)	27	(c)	47	(c)	67	(b)	87	(b)	107	(d)	127	(c)	147	(d)	167	(b)	187	(a)
8	(b)	28	(b)	48	(b)	68	(a)	88	(c)	108	(a)	128	(b)	148	(b)	168	(c)	188	(d)
9	(b)	29	(d)	49	(b)	69	(b)	89	(b)	109	(a)	129	(a)	149	(c)	169	(b)	189	(a)
10	(a)	30	(a)	50	(d)	70	(a)	90	(a)	110	(d)	130	(c)	150	(c)	170	(c)	190	(d)
11	(c)	31	(c)	51	(d)	71	(c)	91	(b)	111	(b)	131	(a)	151	(b)	171	(d)	191	(c)
12	(a)	32	(d)	52	(c)	72	(a)	92	(c)	112	(d)	132	(b)	152	(a)	172	(b)	192	(c)
13	(d)	33	(b)	53	(c)	73	(c)	93	(d)	113	(b)	133	(d)	153	(c)	173	(b)	193	(c)
14	(b)	34	(c)	54	(a)	74	(a)	94	(d)	114	(a)	134	(a)	154	(b)	174	(a)	194	(a)
15	(c)	35	(b)	55	(b)	75	(c)	95	(c)	115	(b)	135	(c)	155	(d)	175	(b)	195	(b)
16	(d)	36	(b)	56	(b)	76	(b)	96	(d)	116	(b)	136	(b)	156	(c)	176	(c)	196	(d)
17	(a)	37	(c)	57	(b)	77	(b)	97	(a)	117	(d)	137	(a)	157	(b)	177	(b)	197	(a)
18	(c)	38	(b)	58	(d)	78	(d)	98	(c)	118	(c)	138	(a)	158	(b)	178	(a)	198	(b)
19	(a)	39	(d)	59	(d)	79	(a)	99	(c)	119	(d)	139	(a)	159	(c)	179	(c)	199	(d)
20	(b)	40	(b)	60	(b)	80	(a)	100	(c)	120	(d)	140	(c)	160	(b)	180	(b)		

HINTS & SOLUTIONS

1. (c) As given $\vec p = \frac{\vec b\times\vec c}{[\vec a\vec b\vec c]}$, $\vec q = \frac{\vec c\times\vec a}{[\vec a\vec b\vec c]}$, and $\vec r = \frac{\vec a\times\vec b}{[\vec a\vec b\vec c]}$

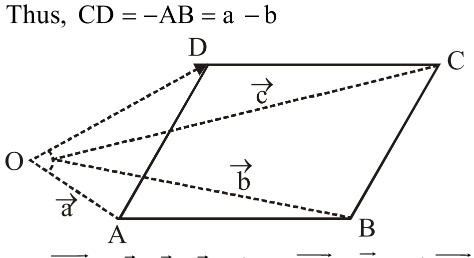
$\therefore\ (\vec a-\vec b-\vec c).\vec p+(\vec b-\vec c-\vec a).\vec q+(\vec c-\vec a-\vec b).\vec r$

$= \frac{\vec a.(\vec b\times\vec c)}{[\vec a\vec b\vec c]}+\frac{\vec b.(\vec c\times\vec a)}{[\vec a\vec b\vec c]}+\frac{\vec c.(\vec a\times\vec b)}{[\vec a\vec b\vec c]}$

[Since $\vec b.(\vec b\times\vec c)=0$, $\vec c.(\vec b\times\vec c)=0$, $\vec c.(\vec c\times\vec a)=0$, $\vec a.(\vec c\times\vec a)=0$, $\vec a.(\vec a\times\vec b)=0$ and $\vec b.(\vec a\times\vec b)=0$]

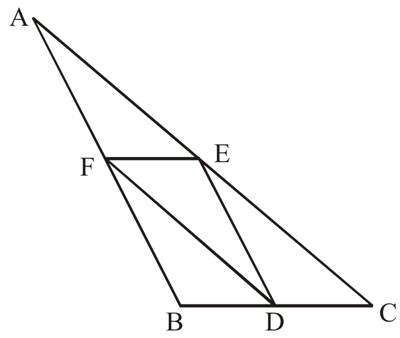
$= \frac{[\vec a\,\vec b\,\vec c]}{[\vec a\,\vec b\,\vec c]}+\frac{[\vec a\,\vec b\,\vec c]}{[\vec a\,\vec b\,\vec c]}+\frac{[\vec a\,\vec b\,\vec c]}{[\vec a\,\vec b\,\vec c]}=3$

2. (c) Let O be the origin and ABCD be the parallelogram.
In Δ ODC,
$\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{CD}$
$\overrightarrow{CD}=-\overrightarrow{AB}$
and, In Δ AOB, $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\vec b-\vec a$
Thus, $\overrightarrow{CD}=-\overrightarrow{AB}=\vec a-\vec b$



So, $\overrightarrow{OD}=\vec c+\vec a-\vec b$ [since, $\overrightarrow{OC}=\vec C$ and $\overrightarrow{CD}=\vec a-\vec b$]

3. (b) Since, side opposite to greatest angle is longest and $\angle$B is greatest angle in ΔABC. Thus, $\overrightarrow{CA}$ has greatest magnitude.



4. (d) Given that $\vec p.\vec q=\vec p.\vec r$

$\Rightarrow\ \vec p.\ (\vec q-\vec r)=0$

$\Rightarrow\ \vec p$ is perpendicular to $\vec q-\vec r$

Also, $\vec p\times\vec q=\vec p\times\vec r$ (given).

$\Rightarrow\ \vec p\times(\vec q-\vec r)=0$

$\Rightarrow\ \vec p$ is parallel to $\vec q-\vec r$

Which is not possible simultaneously unless either $\vec p$ or $\vec q-\vec r$ is zero, since $\vec p\neq 0$, $\Rightarrow \vec q-\vec r=0$
Thus, the given conditions hold simultaneously if $\vec q=\vec r$.

5. (b) If $\vec{p}$ and $\vec{q}$ are unit vectors which make an angle $\frac{\pi}{3}$ with each other.

Then, $\vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\frac{\pi}{3} = \frac{1}{2}$

Now, $\left|\vec{p} - \frac{1}{2}\vec{q}\right|^2 = |\vec{p}|^2 + \frac{1}{4}|\vec{q}|^2 - \frac{2}{2}\,\vec{p}\cdot\vec{q}$

$= 1 + \frac{1}{4} - \frac{1}{2}$ [since $|\vec{p}| = |\vec{q}| = 1$]

$= \frac{5}{4} - \frac{1}{2} = \frac{5-2}{4} = \frac{3}{4}$

So, $\left|\vec{p} - \frac{1}{2}\vec{q}\right| = \frac{\sqrt{3}}{2}$

6. (c) If vectors $(x^2-1)\,\hat{i} + (x+2)\,\hat{j} + x^2\,\hat{k}$ and $2\,\hat{i} - x\,\hat{j} + 3\,\hat{k}$ are orthogonal, then

$2(x^2-1) - x(x+2) + 3x^2 = 0$

$\Rightarrow 2x^2 - 2 - x^2 - 2x + 3x^2 = 0$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow (2x^2 - x - 1) = 0$

$\Rightarrow (2x+1)(x-1) = 0$

$\Rightarrow x = -\frac{1}{2}$ and $x = 1$

7. (d) Force, $\vec{F}$ is given by $\vec{F} = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -2\hat{i} + 3\hat{j} + \hat{k}$

$\therefore$ $\vec{r} = \overrightarrow{AB} = -2\hat{i} + 3\hat{j} + \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$

$= -3\hat{i} + \hat{j} - 2\hat{k}$

Moment $\overrightarrow{M}$ about the point $\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{M} = \vec{r} \times \vec{F} = (-3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + \hat{j} + \hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1+2) - \hat{j}(-3+2) + \hat{k}(-3-1)$

$= 3\hat{i} + \hat{j} - 4\hat{k}$

8. (b) Three forces are given by, say, F_1, F_2 and F_3

$\vec{F}_1 = 2\hat{i} + 3\hat{j} + 5\hat{k}, \vec{F}_2 = -5\hat{i} + 4\hat{j} - 3\hat{k}$

and $\vec{F}_3 = 3\hat{i} - 7\hat{k}$

Total resultant force, $\vec{F}$ is given by

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$= 2\hat{i} + 3\hat{j} + 5\hat{k} - 5\hat{i} + 4\hat{j} - 3\hat{k} + 3\hat{i} - 7\hat{k} = 7\hat{j} - 5\hat{k}$

This show that the resultant force is in the $y-z$ plane. Thus, it moves in the $y-z$ plane.

9. (b) Let the vector be $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Since, $\vec{r}$ and $\hat{i} + \hat{j}$ are perpendicular to each other.

Hence, $\vec{r}.(\hat{i} + \hat{j}) = 0 \Rightarrow x + y = 0$ ...(i)

also $\vec{r}$ and $\hat{j} + \hat{k}$ are perpendicular to each other. so,

$\vec{r}.(\hat{j} + \hat{k}) = 0$

So, $y + z = 0$ ...(ii)

and $x^2 + y^2 + z^2 = 9$ ...(iii)

$\Rightarrow (-y^2) + y^2 + (-y)^2 = 9$

$\Rightarrow 3y^2 = 9$

$\Rightarrow y = \pm\sqrt{3}$

$\therefore$ $x = \mp\sqrt{3}$ [from (i)]

and $z = \pm\sqrt{3}$ [from (ii)]

So, vector is $\sqrt{3}(\hat{i} - \hat{j} + \hat{k})$

10. (a) We know that for any line that makes α, β and γ angle with axis

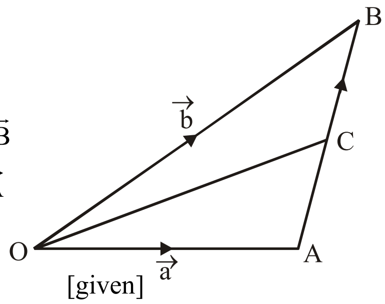
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

11. (d) A is true, but R is false.

12. (a) In Δ OAB,



Let $\overrightarrow{OA} = \vec{a}$

$\overrightarrow{OB} = \vec{b}$

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

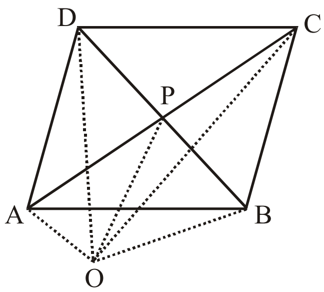
$= \vec{b} - \vec{a}$

$\overrightarrow{AB} = 2\,\overrightarrow{BC}$ [given]

$\Rightarrow$ C is mid point of $\overrightarrow{AB}$

$\therefore$ $\overrightarrow{AC} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\vec{b} - \vec{a})$

13. (d) Diagonals of a parallelogram bisect each other. Therefore, P is the mid point of AC and BD both.



So, in ΔOAC, $\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OP}$

and in ΔODB, $\overrightarrow{OB} + \overrightarrow{OD} = 2\overrightarrow{OP}$

$\Rightarrow \overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OP}$ and $\overrightarrow{OB} + \overrightarrow{OD} = 2\overrightarrow{OP}$

$\Rightarrow \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$

14. (b) Since $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ are the position vector of three collinear points. Thus $\vec{r}_3$ is the position vector of the point which divides the joining of points whose position vectors are $\vec{r}_1$ and $\vec{r}_2$ in the ratio m : n.

So, $\vec{r}_3 = \dfrac{m\vec{r}_1 + n\vec{r}_2}{m+n}$

But as given, $\vec{r}_3 = m\vec{r}_1 + n\vec{r}_2$

So, $\dfrac{\overrightarrow{mr_1} + \overrightarrow{nr_2}}{m+n} = mr_1 + nr_2$

$\Rightarrow$ $m + n = 1$

15. (c) The given vector is $\vec{V} = 2\hat{i} - \hat{j} + 2\hat{k}$
and for z-axis x = 0 and y = 0, so the vector equation is
$\vec{A} = 0\hat{i} + 0\hat{j} + \hat{k}$

$\cos\alpha = \dfrac{\vec{V}.\vec{A}}{|\vec{V}|.|\vec{A}|}$

$\cos\alpha = \dfrac{2.0 + (-1).(0) + 2.(1)}{\sqrt{4+1+4}\sqrt{0+0+1}} = \dfrac{2}{3}$

Hence, $\sin\alpha = \sqrt{1-\cos^2\alpha}$

$= \sqrt{1-\dfrac{4}{9}} = \sqrt{\dfrac{5}{9}} = \dfrac{\sqrt{5}}{3}$

16. (d) Given that $\vec{m}, \vec{n}$ and $\vec{r}$ are three vectors and θ is the angle between $\vec{m}$ and $\vec{n}$,

We get, $\cos\theta = \dfrac{\vec{m}.\vec{n}}{|\vec{m}||\vec{n}|}$

$\Rightarrow$ $mn\cos\theta = \vec{m}.\vec{n}$

[where, $|\vec{m}| = m$ and $|\vec{n}| = n$]

$\Rightarrow$ $mn\cos\theta = (\vec{m}.\vec{n})\dfrac{\vec{r}}{|\vec{r}|}$

$\Rightarrow$ $mn\cos\theta = (\vec{m}.\vec{n})\dfrac{\vec{r}}{r}$

[where $|\vec{r}| = r$]

$\Rightarrow$ $mn\,r\cos\theta = (\vec{m}.\vec{n})\vec{r}$

17. (a) Let the given vectors
$\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ be $\vec{A}$ and $\vec{B}$ respectively, and θ be the angle between

them, so, $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

These are orthogonal to each other, θ = π/2

so, $\vec{A}.\vec{B} = 0$

$\Rightarrow$ $1 - (2x)(3x) - (3y)(2y) = 0$

$\Rightarrow$ $1 - 6x^2 - 6y^2 = 0$

$\Rightarrow$ $x^2 + y^2 = \dfrac{1}{6}$

This equation represents an equation of a circle which is the locus of the point (x, y).

18. (c) We know that the components of $\vec{b}$ along $\vec{a}$ is $\left\{\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right\}\vec{a}$ and perpendicular to $\vec{a}$ is $\vec{b} - \left\{\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right\}\vec{a}$

As given : $\left\{\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right\}\vec{a} = \lambda\vec{a}$

and $\vec{b} - \left\{\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right\}.\vec{a} = \vec{b} - \lambda\vec{a}$

$\Rightarrow$ $\lambda = \left\{\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right\}$

19. (a) Force, $\vec{F} = m\hat{i} - 3\hat{j} + \hat{k}$
Due to this force, point moves from A (20, 3m, 0) to B (0, 0, 7).

So, the displacement vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = -20\hat{i} - 3m\hat{j} + 7\hat{k}$

Work done = $\vec{F}.\overrightarrow{AB}$

$= (m\hat{i} - 3\hat{j} + \hat{k}).(-20\hat{i} - 3m\hat{j} + 7\hat{k})$

$= (-20m + 9m + 7)$ unit

But work done = –48 unit, as given

$\Rightarrow$ $-11m + 7 = -48$

$\Rightarrow$ $-11m = -55$

$\Rightarrow$ $m = 5$

20. (b) (i) $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$
Squaring both the sides

$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$

or, $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

or, $4\vec{a}.\vec{b} = 0$

$\Rightarrow$ $\vec{a}$ and $\vec{b}$ are orthogonal.

(ii) $|\vec{a} + \vec{b}|^2 = |\vec{a}| + |\vec{b}|$

Squaring both the sides

$|\vec{a} + \vec{b}| = (|\vec{a}| + |\vec{b}|)^2$

or, $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|$

or, $\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|$

or, $|\vec{a}|.|\vec{b}|.\cos\theta = |\vec{a}||\vec{b}|$

$\cos\theta = 1 \Rightarrow \theta = 0$

$\vec{a}$ and $\vec{b}$ are parallel, and not orthogonal.

(iii) $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2$

$2\vec{a}.\vec{b} = 0$

$\Rightarrow \vec{a}$ and $\vec{b}$ are orthogonal.
Statements 1 and 3 are correct.

21. (d) Vectors are $2i + mj - 3nk$ and $5i + 3mj + nk$

$|2i + mj - 3nk| = \sqrt{14}$ ...(i)

and $|5i + 3mj + nk| = \sqrt{35}$ ...(ii)

$\sqrt{2^2 + m^2 + (-3n)^2} = \sqrt{14}$ From (i)

or, $4 + m^2 + 9n^2 = 14$
or, $m^2 + 9n^2 = 10$ ...(iii)
From (ii)

$\sqrt{5^2 + (3m)^2 + n^2} = \sqrt{35}$

or, $25 + 9m^2 + n^2 = 35$
or, $9m^2 + n^2 = 10$ ...(iv)
From (iii) and (iv)
$m^2 + 9n^2 = 9m^2 + n^2$
or, $8n^2 = 8m^2$
or, $n^2 = m^2$
$\Rightarrow n = \pm m$
n takes 2 values and m takes 2 values.

22. (b) Since, $\vec{p}$ and $\vec{q}$ are collinear, then

$\vec{p} = k\vec{q}$ [where k is a scalar]

$\Rightarrow (x-2)\,\vec{a} + \vec{b} = k(x+1)\vec{a} - k\vec{b}$
On equating the coefficients
$x - 2 = k(x+1)$ and $-k = 1$,
putting value of k
we get, $x - 2 = -(x+1)$
$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

23. (b) Let $\vec{c}$ be the position vector of point C on AB produced.
From the laws of vector addition,

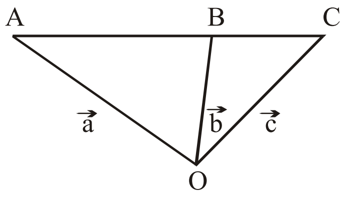
$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$

and similarly in ΔAOC, $\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$

$\Rightarrow \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \vec{c} - \vec{a}$

As given, $\overrightarrow{AC} = 2\overrightarrow{AB} = 2\vec{b} - 2\vec{a}$



$\Rightarrow 2\vec{b} - 2\vec{a} = \vec{c} - \vec{a}$

$\Rightarrow \vec{c} = 2\vec{b} - 2\vec{a} + \vec{a} = 2\vec{b} - \vec{a}$

24. (a) $\because (\vec{a} + \lambda\vec{b})$ is perpendicular to $(\vec{a} - \lambda\vec{b})$, their dot product is zero, so, $(\vec{a} + \lambda\vec{b}).(\vec{a} - \lambda\vec{b}) = 0$

$\Rightarrow |\vec{a}|^2 - \lambda^2|\vec{b}|^2 - \lambda\vec{a}.\vec{b} + \lambda\vec{b}.\vec{a} = 0$

$\Rightarrow |\vec{a}|^2 - \lambda^2|\vec{b}|^2 = 0$ $(\because \vec{a}.\vec{b} = \vec{b}.\vec{a})$

$\Rightarrow 9 - 16\lambda^2 = 0$

$\Rightarrow \lambda = \pm\frac{3}{4}$ $\lambda = \frac{3}{4}$ matches with the given option.

25. (c) Here, $\vec{F} = 3\hat{i} + 2\hat{j} - \hat{k}$

and $\vec{r_1} - \vec{r_2} = \hat{i} - \hat{j} + \hat{k} - 2\hat{i} + 3\hat{j} + \hat{k}$

$= -\hat{i} + 2\hat{j} + 2\hat{k}$

Moment of couple $= (\vec{r_1} - \vec{r_2}) \times \vec{F}$

$= (-\hat{i} + 2\hat{j} + 2\hat{k}) \times (3\hat{i} + 2\hat{j} - \hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ 3 & 2 & -1 \end{vmatrix} = \hat{i}(-2-4) - \hat{j}(1-6) + \hat{k}(-2-6)$

$= -6\hat{i} + 5\hat{j} - 8\hat{k}$

Magnitude of the moment $= |-6\hat{i} + 5\hat{j} - 8\hat{k}|$

$= \sqrt{36 + 25 + 64} = 5\sqrt{5}$

26. (d) As given $\vec{a} = 2\hat{j} - 3\hat{k}$ and $\vec{b} = \hat{j} + 3\hat{k}$

and $\vec{c} = -3\hat{i} + 3\hat{j} + \hat{k}$

Let $\vec{n} = x\hat{i} + y\hat{j} + z\hat{k}$

Since $\vec{a}$ and $\hat{n}$ are perpendicular to each other.

$\vec{a}.\hat{n} = 0 \Rightarrow (2\hat{j} - 3\hat{k}).(x\hat{i} + y\hat{j} + z\hat{k}) = 0$

$\Rightarrow 2y - 3z = 0$ ...(1)

and $\vec{b}.\hat{n} = 0$

$\Rightarrow (\hat{j} + 3\hat{k}).(x\hat{i} + y\hat{j} + z\hat{k}) = 0$

$\Rightarrow y + 3z = 0$ ...(2)
On solving Eqs. (1) and (2)
$y = z = 0$

Since $\hat{n}$ is a unit vector,

$\sqrt{x^2 + y^2 + z^2} = 1 \Rightarrow x = 1$ [since, $y = z = 0$]

hence, $\hat{n} = \hat{i}$

This gives, $\vec{c}.\hat{n} = (-3\hat{i} + 3\hat{j} + \hat{k}).(\hat{i}) = -3$

27. (c) Since, $\vec{p} = \vec{u} + \vec{v} + \vec{w}$

$\Rightarrow \vec{p} = (\hat{i} - \hat{j}) + (2\hat{i} + 5\hat{j}) + (4\hat{i} + 3\hat{j}) = 7\hat{i} + 7\hat{j}$

Now, $3\vec{u} + 2\vec{v} = 3(\hat{i} - \hat{j}) + 2(2\hat{i} + 5\hat{j}) =$

$3\hat{i} - 3\hat{j} + 4\hat{i} + 10\hat{j} = 7\hat{i} + 7\hat{j}$

$\Rightarrow 3\vec{u} + 2\vec{v} = \vec{p}$

28. (b) $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}\cos\theta$

$\Rightarrow |\vec{a} + \vec{b}|^2 = 1 + 1 + 2|\vec{a}||\vec{b}|\cos 30°$

$= 1 + 1 + 2 \times \frac{\sqrt{3}}{2}$

$\Rightarrow |\vec{a} + \vec{b}|^2 = 2 + \sqrt{3}$

$\Rightarrow |\vec{a} + \vec{b}| = \sqrt{2 + \sqrt{3}}$

$1 < \sqrt{2 + \sqrt{3}} < 2$

$\Rightarrow 1 < |\vec{a} + \vec{b}| < 2$

29. (d) As given

$\vec{c}$, is normal to the vectors $\vec{a}$ and $\vec{b}$

$\Rightarrow \vec{c}.\vec{a} = 0$ and $\vec{c}.\vec{b} = 0$

$\Rightarrow \vec{c}.(\vec{a} + \vec{b}) = \vec{c}.\vec{a} + \vec{c}.\vec{b} = 0$

$\Rightarrow$ Also $\vec{c}.(\vec{a} - \vec{b}) = \vec{c}.\vec{a} - \vec{c}.\vec{b} = 0$

$\vec{c}$ is normal to $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

30. (a) Since vector product is not commutative. So, option (a) is correct.

31. (c) Vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar vectors.

$$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$\Rightarrow$ a (bc – 1) –1 (c – 1) + 1 (1 –b) = 0

$\Rightarrow$ abc – a – c + 1 + 1 – b = 0

$\Rightarrow$ a + b + c – abc = 2

32. (d) Refer to the figure.

$|(\vec{a} \times \vec{b}).\vec{c}|$ is triple dot product and is volume (V) of the parallelepiped whose adjacent edges are a, b, and c.

i.e., $V = |(\vec{a} \times \vec{b})|.\vec{OC}$

$\Rightarrow V = |\vec{a}||\vec{b}|(\sin\theta)(\cos\phi)|\vec{c}|$

where $0 \le \theta \le \pi$ is the angle between $\vec{a}$ and $\vec{b}$.

As given

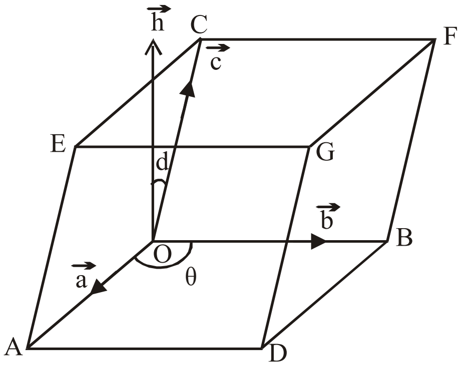
$|(\vec{a} \times \vec{b}).\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$

$\Rightarrow |\sin\theta\cos\phi| . = 1$

$\Rightarrow \sin\theta = 1, \cos\phi = 1$

$\Rightarrow \theta = \frac{\pi}{2}, \phi = 0$

$\Rightarrow \vec{a}.\vec{b}. = 0 = \vec{b}.\vec{c} = \vec{c}.\vec{a}$



33. (b) As given :

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + \lambda\hat{k}$

$\vec{a} + \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - \hat{j} + \lambda\hat{k}$

$= 4\hat{i} + \hat{j} + (\lambda - 3)\hat{k}$

and $\vec{a} - \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} - 3\hat{i} + \hat{j} - \lambda\hat{k}$

$= -2\hat{i} + 3\hat{j} - (3 + \lambda)\hat{k}$

$(\vec{a} + \vec{b})$ is perpendicular to $(\vec{a} - \vec{b})$

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$

$\Rightarrow \{4\hat{i} + \hat{j} + (\lambda - 3)\hat{k}\}\{-2\hat{i} + 3\hat{j} - (3 - \lambda)\hat{k}\} = 0$

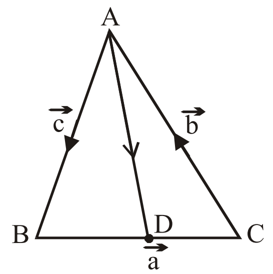
$\Rightarrow -8 + 3 + (3^2 - \lambda^2) = 0$

$\Rightarrow 4 - \lambda^2 = 0$

$\Rightarrow \lambda = \pm 2$

34. (c) Refer to the figure which is self explanatory

In ΔABD,



$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$

$\vec{c} = \overrightarrow{AD} - \frac{1}{2}\vec{a}$ $\quad [\because \overrightarrow{DB} = \frac{1}{2}\overrightarrow{CB}]$

$\overrightarrow{AD} = \frac{1}{2}\vec{a} + \vec{c}$

Also in ΔACD,

$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{CA}$

$\Rightarrow \overrightarrow{AD} + \frac{1}{2}\vec{a} = \vec{b}$

$\Rightarrow \overrightarrow{AD} = \vec{b} - \frac{1}{2}\vec{a}$

But this is not in the statement.

Hence, only (1) statement represent median $\overrightarrow{AD}$.

35. (b) Let the coordinates of B be (x, y).

$\vec{a} = \hat{i} - 3\hat{j}$

P.V. of A is (–1, 5) so, $\overrightarrow{OA} = \hat{i} + 5\hat{j}$, $\overrightarrow{OB} = x\hat{i} + y\hat{j}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{a}$

$\Rightarrow (x+1)\hat{i} + (y-5)\hat{j} = \hat{i} - 3\hat{j}$

$\Rightarrow x + 1 = 1$ and $y - 5 = -3$

$\Rightarrow x = 0$ and $y = 2$

$\therefore$ Coordinates of B are (0, 2).

36. (b) Given vectors are :

$\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} = (2\hat{i} - 3\hat{j} - \hat{k}) + (\hat{i} + 4\hat{j} - 2\hat{k})$

$= 3\hat{i} + \hat{j} - 3\hat{k}$

and $\vec{a} - \vec{b} = (2\hat{i} - 3\hat{j} - \hat{k}) - (\hat{i} + 4\hat{j} - 2\hat{k})$

$= \hat{i} - 7\hat{j} + \hat{k}$

$$\therefore (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix}$$

$= \hat{i}(1 - 21) - \hat{j}(3 + 3) + \hat{k}(-21 - 1)$

$= -20\hat{i} - 6\hat{j} - 22\hat{k}$

$= -2(10\hat{i} + 3\hat{j} + 11\hat{k})$

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -3 & -1 \\ 4 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$$

$= \hat{i}(6 + 4) - \hat{j}(-4 + 1) + \hat{k}(8 + 3)$

$= 10\hat{i} + 3\hat{j} + 11\hat{k}$

Hence, $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) = -2\left(\vec{a} \times \vec{b}\right)$

37. (c) As given, $\lambda\vec{a}$ is a unit vector.

$\Rightarrow |\lambda\vec{a}| = 1$

$\Rightarrow |\lambda|\,|\vec{a}| = 1$

$\Rightarrow a = \dfrac{1}{|\lambda|}$ $\left[\because |\vec{a}| = a\right]$

38. (b) As given : $\vec{a}$ and $\vec{b}$ are position vectors of A and B respectively and position vector of C is $3\vec{a} - 2\vec{b}$

$\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$, where O is the origin and

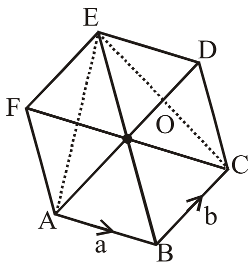
$\overrightarrow{OC} = 3\vec{a} - 2\vec{b}$, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 3\vec{a} - 2\vec{b} - \vec{a} = 2\vec{a} - 2\vec{b}$

$\Rightarrow \overrightarrow{AC} = 2\left(\vec{a} - \vec{b}\right) = -2\left(\vec{b} - \vec{a}\right) = -2\overrightarrow{AB}$

So, $\overrightarrow{AC}$ is opposite to $\overrightarrow{AB}$ so A is between C and B and position vector of C shows an external division by C.

39. (d) Let ABCDEF be the regular hexagen as shown in the figure.



Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$

Join AD, FC and EB. They meet at a common point O, which is the centre of hexagon.

AO || BC so, $\overrightarrow{AO} = \overrightarrow{BC} = \vec{b}$

OC || AB so, $\overrightarrow{OC} = \overrightarrow{AB} = \vec{a}$

OAB forms a triangle, $\overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{AO}$

$\Rightarrow \overrightarrow{BO} = \overrightarrow{AO} - \overrightarrow{AB} = \vec{b} - \vec{a}$

BO = OE and they are on the same line,

So, $\overrightarrow{BO} = \overrightarrow{OE} = \vec{b} - \vec{a}$

In ΔOCE, $\overrightarrow{CO} + \overrightarrow{OE} = \overrightarrow{CE}$

$\Rightarrow \overrightarrow{CE} = -\overrightarrow{OC} + \text{OE} = -\vec{a} + \vec{b} - \vec{a} = \vec{b} - 2\vec{a}$

So, (1) is correct.

$\overrightarrow{BE} = 2\overrightarrow{OB}$ In ΔAEB, $\overrightarrow{AB} + \overrightarrow{BE} = \overrightarrow{AE}$

$\Rightarrow \overrightarrow{AE} = \overrightarrow{AB} + 2\overrightarrow{BO} = \vec{A} + 2\left(\vec{b} - \vec{a}\right)$

$\Rightarrow \overrightarrow{AE} = \vec{a} + 2\vec{b} - 2\vec{a} = 2\vec{b} - \vec{a}$

So, (2) is also correct.

FA||OB $\Rightarrow \overrightarrow{FA} = -\overrightarrow{BO} = -(\vec{b} - \vec{a}) = \vec{a} - \vec{b}$

So, (3) is also correct.

So, (1), (2) & (3) are correct.

40. (b) As given : $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$

$\Rightarrow$ $\vec{a}.\vec{b} = 0$ & $\vec{a}.\vec{c} = 0$

and angle between $\vec{b}$ and $\vec{c}$ $= \frac{\pi}{3}$

$\therefore \vec{b}.\vec{c} = |\vec{b}||\vec{c}|\cos\frac{\pi}{3} = 1.1.\frac{1}{2}$

$= \frac{1}{2}$ ($\because \vec{b}$ and $\vec{c}$ are unit vectors)

Now, $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$

$= 1+1+1+2.\left(0+\frac{1}{2}+0\right)$

$= 1 + 1 + 1 + 1 = 4$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = 2$

41. (d) As given : $(\hat{i} - x\hat{j} - 2\hat{k})$ and $(2\hat{i}+\hat{j}+y\hat{k})$ are orthogonal.

So, there dot product = 0

$\Rightarrow (\hat{i} - x\hat{j} - 2\hat{k}).(2\hat{i}+\hat{j}+y\hat{k}) = 0$

$\Rightarrow 2 - x - 2y = 0$

$\Rightarrow x + 2y = 2$

Which is an equation of straight line.

Thus, the locus of the point (x, y) is a straight line.

42. (b) As given, vectors are : $\vec{a} = \hat{i}+\hat{j}$ and $\vec{b} = \hat{j}+\hat{k}$

So, $\vec{a}\times\vec{b}$ is perpendicular to $\vec{a}$ and $\vec{b}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$= \hat{i} - \hat{j} + \hat{k}$

and $|\vec{a}\times\vec{b}| = \sqrt{1+1+1} = \sqrt{3}$

Unit vector perpendicular to $\vec{a}$ and $\vec{b}$

$= \pm\frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|} = \pm\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

Thus, the number of vectors perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is 2. This is true for vectors of any length. So, it is true for vector of length 5 unit.

43. (d) As given Semidiagonal is $\vec{r} = a\vec{i} + b\vec{j}$

So diagonal is $2\vec{r} = 2a\vec{i} + 2b\vec{j}$

$\Rightarrow$ Sides of rectangle are 2a and 2b

Hence, area of rectangle = 2a × 2b = 4ab.

44. (a) $(3\vec{\mathbf{a}} - \vec{\mathbf{b}})\times(\vec{\mathbf{a}} + 3\vec{\mathbf{b}})$

$= (3\vec{\mathbf{a}} - \vec{\mathbf{b}})\times\vec{\mathbf{a}} + (3\vec{\mathbf{a}} - \vec{\mathbf{b}})\times 3\vec{\mathbf{b}}$

$= 3\vec{\mathbf{a}}\times\vec{\mathbf{a}} - \vec{\mathbf{b}}\times\vec{\mathbf{a}} + 3\vec{\mathbf{a}}\times 3\vec{\mathbf{b}} - \vec{\mathbf{b}}\times 3\vec{\mathbf{b}}$

$= 0 - (-\vec{\mathbf{a}}\times\vec{\mathbf{b}}) + 9\vec{\mathbf{a}}\times\vec{\mathbf{b}} - 0$

$= 10\vec{\mathbf{a}}\times\vec{\mathbf{b}}$

$\therefore k = 10$

45. (b) Let us consider triangle ABC. Suppose $\hat{i}$, $\hat{j}$ and $\hat{i}+\hat{j}+\lambda\hat{k}$ are the position vector of A , B and C.

Then $\overrightarrow{AB} = \hat{j} - \hat{i}$, $\overrightarrow{AC} = \hat{j} + \lambda\hat{k}$, $\overrightarrow{BC} = \hat{i} + \lambda\hat{k}$

$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

$|\overrightarrow{BC}| = \sqrt{(1)^2 + (\lambda)^2} = \sqrt{1+\lambda^2}$

$|\overrightarrow{AC}| = \sqrt{(1)^2 + (\lambda)^2} = \sqrt{1+\lambda^2}$

To be ΔABC is a right angled triangle, $\angle C$ should be right angle,

i.e., $\overrightarrow{BC}.\overrightarrow{AC} = 0$

$\Rightarrow (\hat{i}+\lambda\hat{k}).(\hat{j}+\lambda\hat{k}) = 0$

$\Rightarrow 0 + 0 + \lambda^2 = 0$

$\therefore \lambda = 0$

46. (a) The scalar triple product $(\vec{\mathbf{A}}\times\vec{\mathbf{B}}).\vec{\mathbf{C}}$ of three vectors $\vec{\mathbf{A}},\vec{\mathbf{B}},\vec{\mathbf{C}}$ determines volume of a parallelopiped.

47. (c) $|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|^2 + (\vec{\mathbf{a}}.\vec{\mathbf{b}})^2$

$= (|\vec{\mathbf{a}}|.|\vec{\mathbf{b}}|.\sin\theta)^2 + (|\vec{\mathbf{a}}|.|\vec{\mathbf{b}}|.\cos\theta)^2$

$= (1.1.\sin\theta)^2 + (1.1.\cos\theta)^2$

$= \sin^2\theta + \cos^2\theta = 1$

48. (b) $\overrightarrow{OG} = \frac{2\overrightarrow{OA} - 3\overrightarrow{OB}}{2-3}$

$\overrightarrow{OG} = \frac{2\overrightarrow{OA} - 3\overrightarrow{OB}}{-1}$

$-\overrightarrow{OG} = 2\overrightarrow{OA} - 3\overrightarrow{OB}$ ... (1)

$\lambda\overrightarrow{OG} = 2\overrightarrow{OA} + 3\overrightarrow{OB}$ ... (2)

Adding (1) and (2)

$(\lambda - 1)\overrightarrow{OG} = 4\overrightarrow{OA}$

$\Rightarrow \overrightarrow{OA} = \left(\frac{\lambda-1}{4}\right)\overrightarrow{OG}$ ...(3)

Subtracting (2) from (1)

$(-1-\lambda)\overrightarrow{OG} = -6\overrightarrow{OB}$

$\overrightarrow{OB} = \frac{(1+\lambda)}{6}\overrightarrow{OG}$ ...(4)

From equ (2), (3) and (4)

$\lambda\overrightarrow{OG} = 2\left(\frac{\lambda-1}{4}\right)\overrightarrow{OG} + 3\left(\frac{\lambda+1}{6}\right)\overrightarrow{OG}$

$\Rightarrow \lambda = \frac{\lambda-1}{2} = \frac{\lambda+1}{3}$

$\therefore\ \lambda = \frac{\lambda-1}{2}$ or $\lambda = \frac{\lambda+1}{3}$

$\Rightarrow 2\lambda - \lambda = -1$ or $3\lambda - \lambda = 1$

$\Rightarrow \lambda = -1$ or $\lambda = \frac{1}{2}$

49. (b) $|\vec{\mathbf{a}}+\vec{\mathbf{b}}| = \sqrt{|\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2}$

$= \sqrt{|\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2|\vec{\mathbf{a}}|.|\vec{\mathbf{b}}|\cos 60°}$

$= \sqrt{1^2 + 1^2 + 2.1.1.\frac{1}{2}} = \sqrt{3}$

$\therefore |\vec{\mathbf{a}}+\vec{\mathbf{b}}| > 1$

50. (d) Given $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{a}}+\vec{\mathbf{b}} = 4\hat{\mathbf{i}} - \mathbf{j} + 5\hat{\mathbf{k}}$

Then, $\vec{\mathbf{c}} = \lambda(\vec{\mathbf{a}}+\vec{\mathbf{b}})$

$= \lambda(4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$\Rightarrow l = \sqrt{16\lambda^2 + \lambda^2 + 25\lambda^2}$

$\Rightarrow l = \sqrt{42}\,\lambda$

$\lambda = \frac{l}{\sqrt{42}}$

$\therefore\ \vec{\mathbf{c}} = \frac{l}{\sqrt{42}}(4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = \frac{1}{\sqrt{42}}(4,-1,5)$

51. (d) Given, $\vec{\mathbf{r}}_1 = \lambda\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $\vec{\mathbf{r}}_1 = \hat{\mathbf{i}} + (2-\lambda)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore |\vec{\mathbf{r}}_1| > |\vec{\mathbf{r}}_2|$

$\Rightarrow \sqrt{\lambda^2 + (2)^2 + (1)^2} > \sqrt{(1)^2 + (2-\lambda)^2 + (2)^2}$

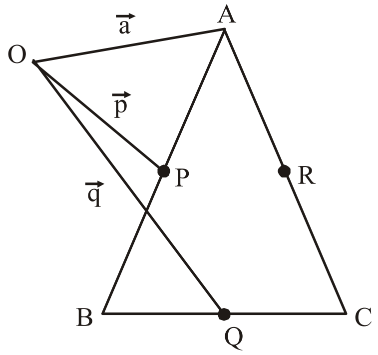
$\Rightarrow \lambda^2 + 4 + 1 > 1 + 4 + \lambda^2 - 4\lambda + 4$

$\Rightarrow 5 > 9 - 4\lambda$

$\Rightarrow 4\lambda > 4$

$\Rightarrow \lambda > 1$

52. (c) Let the position vectors of B, C and R are $\vec{\mathbf{b}}$, $\vec{\mathbf{c}}$ and $\vec{\mathbf{r}}$ respectively.



$\therefore\ \vec{p} = \frac{\vec{a}+\vec{b}}{2} \Rightarrow \vec{b} = 2\vec{p} - \vec{a}$

$\vec{q} = \frac{\vec{b}+\vec{c}}{2}$

$\Rightarrow \vec{c} = 2\vec{q} - \vec{b}$

$\Rightarrow\ \vec{c} = 2\vec{q} - (2\vec{p} - \vec{a}) = 2\vec{q} - 2\vec{p} + \vec{a}$

and $\vec{r} = \frac{\vec{a}+\vec{c}}{2}$

$= \frac{2\vec{q} - 2\vec{p} + \vec{a} + \vec{a}}{2} = \vec{q} - \vec{p} + \vec{a} = \vec{a} - (\vec{p} - \vec{q})$

53. (c) Given, vector is (1, 1).

$\therefore$ Length of vector = $\sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$.

54. (a) Let $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is a vector of magnitude $\sqrt{51}$

$\therefore\ \sqrt{x^2 + y^2 + z^2} = \sqrt{51}$

$\Rightarrow\ x^2 + y^2 + z^2 = 51$ ...(i)

Let $\vec{p}$ makes equal angle θ with and $\vec{a}, \vec{b}$ and $\vec{c}$.

$\therefore\ \vec{p}.\vec{a} = |\vec{p}|.|\vec{a}|\cos\theta$

$\therefore\ \cos\theta = \frac{\vec{p}.\vec{a}}{|\vec{p}||\vec{a}|}$

Similarly, $\cos\theta = \frac{\vec{p}.\vec{b}}{|\vec{p}||\vec{b}|}$

and $\cos\theta = \frac{\vec{p}.\vec{c}}{|\vec{p}||\vec{c}|}$

$\therefore\ \frac{\vec{p}.\vec{a}}{|\vec{p}||\vec{a}|} = \frac{\vec{p}.\vec{b}}{|\vec{p}||\vec{b}|} = \frac{\vec{p}.\vec{c}}{|\vec{p}||\vec{c}|}$

$$\Rightarrow \frac{\frac{1}{3}(x-2y+2z)}{\sqrt{x^2+y^2+z^2}\,\frac{1}{3}\sqrt{1+4+4}} = \frac{\frac{1}{5}(-4x-3z)}{\sqrt{x^2+y^2+z^2}\,\frac{1}{5}\sqrt{16+9}}$$

$$= \frac{y}{\sqrt{x^2+y^2+z^2}\sqrt{1}}$$

$$\Rightarrow \frac{x-2y+2z}{3\sqrt{x^2+y^2+z^2}} = \frac{-4x-3z}{5\sqrt{x^2+y^2+z^2}} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\Rightarrow \frac{x-2y+2z}{3} = \frac{-4x-3z}{5} = y$$

$\therefore$ $5(x-2y+2z) = -3(4x+3z) = 15y$

$\therefore$ $5x-10y+10z = 15y$ and $-12x-9z = 15y$

$\Rightarrow$ $5x-25y+10z = 0$ and $-12x-15y-9z = 0$

$\Rightarrow$ $x-5y+2z = 0$ and $4x+5y+3z = 0$

$x-5y+2z=0$

$4x+5y+3z=0$

$$\begin{vmatrix} x & y & z \\ 1 & -5 & 2 \\ 4 & 5 & 3 \end{vmatrix} = \frac{x}{-15-10} = \frac{y}{8-3} = \frac{z}{5+20}$$

$$\frac{x}{-25} = \frac{y}{5} = \frac{z}{25}$$

$$\frac{x}{-5} = \frac{y}{1} = \frac{z}{5} = k \text{ (let)}$$

$\therefore$ $x = -5k, y = k, z = 5k$

Now, $x^2+y^2+z^2 = 51$

$\therefore$ $(-5k)^2 + k^2 + (5k)^2 = 51$

$\Rightarrow$ $25k^2 + k^2 + 25k^2 = 51$

$\Rightarrow$ $51k^2 = 51$

$\therefore$ $k = \pm 1$

When $k = 1$, then $x = -5, y = 1, z = 5$

and $\vec{p} = 5\hat{i} + \hat{j} + 5\hat{k}$

when $k = -1$, then $x = 5, y = -1, z = -5$

and $\vec{p} = 5\hat{i} - \hat{j} - 5\hat{k}$

55. (b) We know that

$|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + |\vec{\mathbf{a}}.\vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 \times |\vec{\mathbf{b}}|^2$

$\therefore$ $64 + |\vec{\mathbf{a}}.\vec{\mathbf{b}}|^2 = (4 \times 25)$

$\Rightarrow$ $|\vec{\mathbf{a}}.\vec{\mathbf{b}}|^2 = 36$

$\Rightarrow$ $\vec{\mathbf{a}}.\vec{\mathbf{b}} = 6$

56. (b) Given, $|\vec{\mathbf{a}}+\vec{\mathbf{b}}| = |\vec{\mathbf{a}}-\vec{\mathbf{b}}|$

$\Rightarrow$ $|\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}-\vec{\mathbf{b}}|^2$

$\Rightarrow$ $|\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2|\vec{\mathbf{a}}|.|\vec{\mathbf{b}}| = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}|.|\vec{\mathbf{b}}|$

$\Rightarrow$ $4|\vec{\mathbf{a}}|.|\vec{\mathbf{b}}| = 0$

$\Rightarrow$ $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$.

57. (b) Given, $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\therefore$ $\vec{\mathbf{b}} - \vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}} - \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

and $(3\vec{\mathbf{a}} + \vec{\mathbf{b}}) = (3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 15\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

$= 5\hat{i} - 5\hat{j} + 12\hat{k}$

Hence, $(\vec{\mathbf{b}} - \vec{\mathbf{a}}).(3\vec{\mathbf{a}} + \vec{\mathbf{b}}) = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}).(5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$

$= 5 - 15 - 96$

$= -106$

58. (d) Points A, B and C are collinear, if

$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) = \vec{0}$

59. (d) Since, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\Rightarrow$ $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} + \vec{\mathbf{c}}) + \vec{\mathbf{b}} \times (\vec{\mathbf{c}} + \vec{\mathbf{a}}) + \vec{\mathbf{c}} \times (\vec{\mathbf{a}} + \vec{\mathbf{b}})$

$= (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + (\vec{\mathbf{a}} \times \vec{\mathbf{c}}) + (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + (\vec{\mathbf{b}} \times \vec{\mathbf{a}}) + (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) + (\vec{\mathbf{c}} \times \vec{\mathbf{b}})$

$= (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) - (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) + (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) - (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) - (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = 0$

60. (b) (A) We know that

Work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = |\vec{\mathbf{F}}| \cdot |\vec{\mathbf{d}}| \cos\theta$

Since, $\theta = 90°$

$\Rightarrow$ work done $= |\vec{\mathbf{F}}| \cdot |\vec{\mathbf{d}}| \cos 90° = 0$

(R) $\vec{\mathbf{A}}.\vec{\mathbf{B}} = 0$

$\Rightarrow$ $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular.

Both A and R are true but R is not correct explanation of A.

61. (a) The projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|}$

Since, $\vec{a}$ is the unit vector

$\therefore$ $|\vec{a}| = 1$

Hence, projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}.\vec{b}}{1} = \vec{a}.\vec{b}$

62. (a) A vector whose dot product with the vector $4\hat{i} - 3\hat{j} + \hat{k}$ is zero and magnitude is 1, will be the required vectors.

By taking option (a)

$$\pm\frac{(3\hat{i} + 4\hat{j})}{5}.(4\hat{i} - 3\hat{j} + \hat{k}) = \frac{1}{5}(12-12) = 0$$

Hence, the vector given in option ‘a’ is the required vector.

63. (d) Let $\vec{r_1} = b\hat{i} - a\hat{j}$ be the required vector

Given, $\vec{r} = a\vec{i} + b\vec{j}$

Now, $\vec{r_1}.\vec{r} = (b\hat{i} - a\hat{j}).(a\hat{i} + b\hat{j})$
$= ab - ab = 0$
Hence, option (d) is the correct answer

64. (d) Given, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

Also, $\hat{b} = m\vec{a} = m(2\hat{i} - 3\hat{j} + 4\hat{k})$

As $\hat{b}$ is a unit vector therefore $|\vec{b}| = 1$

Now, $|2\hat{i} - 3\hat{j} + 4\hat{k}| = \sqrt{4+9+16} = \sqrt{29}$

Therefore, m should be $\frac{1}{\sqrt{29}}$.

65. (a) Since, vectors $\lambda\vec{a} + \vec{b}$ and $a - \lambda\vec{b}$ are perpendicular to each other therefore $(\lambda\vec{a} + \vec{b}).(\vec{a} - \lambda\vec{b}) = 0$

$\Rightarrow \lambda\vec{a}.\vec{a} - \lambda^2\vec{a}.\vec{b} + \vec{b}.\vec{a} - \lambda\vec{b}.\vec{b} = 0$

$\Rightarrow \lambda|\vec{a}|^2 + (1-\lambda^2)\vec{a}.\vec{b} - \lambda|\vec{b}|^2 = 0$

$\Rightarrow \lambda|\vec{a}|^2 + (1-\lambda^2)\vec{a}.\vec{b} - \lambda|\vec{a}|^2 = 0 \quad (\because |\vec{a}| = |\vec{b}|)$

$\Rightarrow (1-\lambda^2)\vec{a}.\vec{b}$

Since, $\cos 60^\circ = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{a}|} = \frac{\vec{a}.\vec{b}}{|\vec{a}|^2} = \vec{a}.\vec{b}$

$\therefore \quad (1-\lambda^2)\vec{a}.\vec{b} = (1-\lambda^2)\cos 60^\circ$

$\therefore \quad (1-\lambda^2)\frac{1}{2} = 0 \Rightarrow \lambda = \pm 1$

66. (c) Given, $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} - \vec{b}| = 7$

Since, $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2[|\vec{a}|^2 + |\vec{b}|^2]$

$\therefore$ By putting the values of $|\vec{a}|, |\vec{b}|$ and $|\vec{a} - \vec{b}|$ we get

$|\vec{a}+\vec{b}|^2 + 7^2 = 2[3^2 + 4^2]$

$|\vec{a}+\vec{b}|^2 = 50 - 49 \Rightarrow |\vec{a}+\vec{b}|^2 = 1 \Rightarrow |\vec{a}+\vec{b}| = 1$

67. (b) Let $\vec{d_1}$ and $\vec{d_2}$ be the two diagonals of a quadrilateral such that

$\vec{d_1} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

and $\vec{d_2} = 4\hat{i} - \hat{j} + 3\hat{k}$

Now, Dot product of $\vec{d_1}$ and $\vec{d_2}$ is

$\vec{d_1}.\vec{d_2} = 3(4) + 6(-1) - 2(3) = 0$

Now, $|\vec{d_1}| = \sqrt{3^2 + 6^2 + 2^2} = 7$

$|\vec{d_2}| = \sqrt{4^2 + 1^2 + 3^2} = \sqrt{26}$

Since, $|\vec{d_1}| \neq |\vec{d_2}|$

Hence, given quadrilateral is a rhombus,

68. (a) Let $A = (0, 2, 2)$, $B = (2, 0, -1)$ and $C = (3, 4, 0)$

$\overrightarrow{AB} = (2-0, 0-2, -1-2)$ and $\overrightarrow{AC} = (3-0, 4-2, 0-2)$

$\Rightarrow \overrightarrow{AB} = (2, -2, -3)$ and $\overrightarrow{AC} = (3, 2, -2)$

$\therefore$ Area of triangle $= \frac{1}{2} \times$ magnitude of $\overrightarrow{AB} \times \overrightarrow{AC}$

$$= \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$$

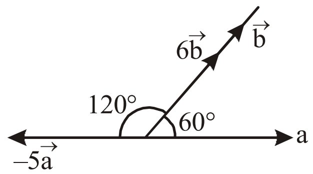
$$= \frac{1}{2}\left\|\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -2 & -3 \\ 3 & 2 & -2 \end{vmatrix}\right\|$$

$= \frac{1}{2}\left|\left[\hat{\mathbf{i}}(4+6) - \hat{\mathbf{j}}(-4+9) + \hat{\mathbf{k}}(4+6)\right]\right|$

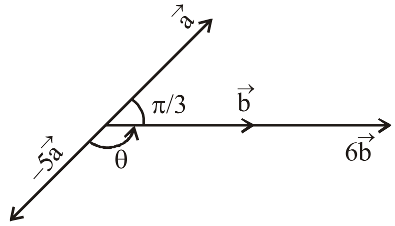
$= \frac{1}{2}\left|10\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 10\hat{\mathbf{k}}\right|$

$= \frac{1}{2}\sqrt{(10)^2 + (5)^2 + (10)^2} = \frac{1}{2}\sqrt{225} = \frac{15}{2}$

69. (b) From the figure it is clear that the angle between $6\vec{\mathbf{b}}$ and $-5\vec{\mathbf{a}}$ is $120°$ or $\frac{2\pi}{3}$.



ALTERNATE SOLUTION:



$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

70. (a) Consider statement 1

$\vec{a}.\left\{\left(\vec{b}+\vec{c}\right) \times \left(\vec{a}+\vec{b}+\vec{c}\right)\right\} = 0$

$= \vec{a}.\{\vec{b}\times\vec{a} + \vec{b}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{c}\times\vec{b} + \vec{c}\times\vec{c}\}$

$= 0 + 0 + \vec{a}.\left(\vec{b}\times\vec{c}\right) + 0 + \vec{a}.\left(\vec{c}\times\vec{b}\right) + 0 \quad \left(\because \vec{a}\times\vec{a} = 0\right)$

$= \vec{a}.\left(\vec{b}\times\vec{c}\right) - \vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

and for any three coplanar vectors $\vec{d}, \vec{e}, \vec{f}$,

$\left(\vec{d}\times\vec{e}\right).\vec{f} = 0$

Hence, statement (1) is correct and statement-2 is incorrect.

71. (c) Let $\vec{a}$ and $\vec{b}$ be two unit vectors.

$\therefore\ |a| = 1$ and $|b| = 1$

Since, α is the angle between $\vec{a}$ and $\vec{b}$

$$\therefore\ \cos\alpha = \frac{\vec{a}.\vec{b}}{|a||b|}$$

$$\cos\alpha = \frac{\vec{a}.\vec{b}}{1}$$

$$\cos\alpha = \vec{a}.\vec{b}$$

Now, $\left|\vec{a}+\vec{b}\right| = 1 \quad \left(\because \vec{a}+\vec{b} \text{ is unit vector}\right)$

Squaring both sides

$$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} = 1$$

$$\Rightarrow 1+1+2\cos\alpha = 1$$

$$\Rightarrow 2\cos\alpha = -1$$

$$\Rightarrow \cos\alpha = -\frac{1}{2} = \cos\frac{2\pi}{3}$$

$$\Rightarrow \alpha = \frac{2\pi}{3}$$

72. (a) Given vectors are

$\hat{i}-\hat{j}+\hat{k},\ 2\hat{i}+\hat{j}-\hat{k}$ and $\lambda\,\hat{i}-\hat{j}+\lambda\,\hat{k}$

We know given vectors are coplanar, if

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(\lambda-1)+1(2\lambda+\lambda)+1(-2-\lambda) = 0$$

$$\Rightarrow \lambda-1+3\lambda-2-\lambda = 0$$

$$\Rightarrow 3\lambda = 3 \Rightarrow \lambda = 1$$

73. (c) Both statements (1) and (2) are correct.

74. (a) Let $\vec{b} = x\,\vec{i}+y\,\vec{j}+z\,\vec{k}$

Since, $\vec{b}$ is collinear with vetor $\vec{a}$

therefore $\vec{a} = k\,\vec{b}$ where k is a scalar.

Given $\vec{a} = (2, 1, -1)$

$$\therefore\ (2, 1, -1) = k(x, y, z)$$

$$\Rightarrow x = \frac{2}{k},\ y = \frac{1}{k},\ z = \frac{-1}{k}$$

Also, $\vec{a}.\vec{b} = 3$

$$\Rightarrow 2x+y-z = 3$$

$$\Rightarrow 2\left(\frac{2}{k}\right)+\frac{1}{k}+\frac{1}{k} = 3$$

$$\Rightarrow \frac{4}{k}+\frac{1}{k}+\frac{1}{k} = 3$$

$$\Rightarrow \frac{6}{k} = 3 \Rightarrow k = 2$$

$$\therefore\ x = 1,\ y = \frac{1}{2} \text{ and } z = \frac{-1}{2}$$

Hence $\vec{b} = \left(1, \frac{1}{2}, \frac{-1}{2}\right)$

75. (c) We know, scalar triple product $\left(\vec{a}\times\vec{b}\right).\,\vec{c}$ is positive or negative according as $\vec{a}, \vec{b}, \vec{c}$ form a right handed or left handed system respectively.

consider option (a)

Let $\vec{c} = \vec{j}$

$$\therefore\ \left[\vec{a}\,\vec{b}\,\vec{c}\right] = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = x(-1)-y(0)+z(0)$$

$$= -x$$

option (b)

Let $\vec{c} = y\,\hat{j}-x\,\hat{k}$

$$\therefore\ \left[\vec{a}\,\vec{b}\,\vec{c}\right] = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ 0 & y & -x \end{vmatrix} = x(-y)-y(0)+z(0)$$

$$= -xy$$

option (c)

Let $\vec{c} = y\,\hat{i}-x\,\hat{j}$

$$\therefore\ \left[\vec{a}\,\vec{b}\,\vec{c}\right] = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ y & -x & 0 \end{vmatrix}$$

$$= x(x)-y(-y)+z(0) = x^2+y^2$$

Since, scalar triple product is positive when $\vec{c} = y\,\hat{i}-x\,\hat{j}$

$\therefore$ option (c) is correct.

76. (b) Let $\overrightarrow{OP} = \hat{i}+3\hat{j}-2\hat{k}$ and $\overrightarrow{OQ} = 3\hat{i}+\hat{j}-2\hat{k}$

Let $\hat{i}+\hat{j}-\hat{k}$ be required position vector of the bisector of the angle POQ since, it is the bisector of $\angle POQ$ therefore. It will make equal angles with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.

Let Angle between $\hat{i}+3\hat{j}-2\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$ is

$\theta = \cos^{-1}\left(\frac{1+3+2}{\sqrt{1+9+4}\sqrt{1+1+1}}\right)$

$= \cos^{-1}\left(\frac{6}{\sqrt{14}\sqrt{3}}\right)$

and angle between $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$, is

$\phi = \cos^{-1}\left(\frac{1+3+2}{\sqrt{9+1+4}\sqrt{1+1+1}}\right) = \cos^{-1}\left(\frac{6}{\sqrt{14}\sqrt{3}}\right)$

Hence, $\theta = \phi$

77. (b) Given $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$. and $c\hat{i}+c\hat{j}+b\hat{k}$ lie on a plane.

$\Rightarrow$ vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar.

$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow a(-c) - a(b-c) + c(c) = 0$
$\Rightarrow -ac - ab + ac + c^2 = 0$
$\Rightarrow c^2 = ab$
$\Rightarrow$ c is the geometric mean of a and b.

78. (d) Let $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$

and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

Now, $\left(\vec{b}\times\vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 1 & (1-x) \\ y & x & (1+x-y) \end{vmatrix}$

$= \hat{i}(1+x-y-x+x^2) - \hat{j}(x+x^2-xy-y)$
$+ \hat{k}(x^2-y)$

$= \hat{i}(1-y+x^2) - \hat{j}(x+x^2-xy-y) + \hat{k}(x^2-y)$

Now, $\vec{a}.\left(\vec{b}\times\vec{c}\right) = 1(1-y+x^2) + 0(x+x^2-xy-y)$
$-1(x^2-y)$

$= 1 - y + x^2 - x^2 + y$

$= 1$ which shows that $\vec{a}.\left(\vec{b}\times\vec{c}\right)$ does not depend on x and y.

79. (a) Let $\overrightarrow{PQ} = 3\hat{i}+2\hat{j}-m\hat{k}$ and $\overrightarrow{PS} = \hat{i}+3\hat{j}+\hat{k}$ where PQRS is a parallelogram.

$\therefore$ Area of parallelogram $= \left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -m \\ 1 & 3 & 1 \end{matrix}\right\|$

$= |\hat{i}(2+3m) - \hat{j}(3+m) + \hat{k}(9-2)|$

$= \sqrt{(2+3m)^2 + (3+m)^2 + 7^2}$

$\Rightarrow 90 = 4 + 9m^2 + 12m + 9 + m^2 + 6m + 49$
$\Rightarrow 10m^2 + 18m - 28 = 0$
$\Rightarrow 5m^2 + 9m - 14 = 0$
$\Rightarrow 5m^2 + 14m - 5m - 14 = 0$
$\Rightarrow m(5m+14) - 1(5m+14) = 0$
$\Rightarrow (5m+14)(m-1) = 0$

$\Rightarrow$ m = 1 or $\frac{-14}{5}$

80. (a) Let the required vector be $\hat{i}+\hat{j}$

Since the vector $\hat{i}+\hat{j}$ is equally inclined to the vectors $\hat{i}+3\hat{j}$ and $3\hat{i}+\hat{j}$ therefore

Angle b/w $\hat{i}+\hat{j}$ and $\hat{i}+3\hat{j} = \theta_1$ is equal to angle between $\hat{i}+\hat{j}$ and $3\hat{i}+\hat{j} = \theta_2$

$\therefore$ Angle between $\hat{i}+\hat{j}$ and $\hat{i}+3\hat{j}$

$= \cos^{-1}\left[\frac{(1)(1)+(1)(3)}{\sqrt{(1)^2+(1)^2}\sqrt{(1)^2+(3)^2}}\right]$

$= \cos^{-1}\left[\frac{1+3}{\sqrt{2}\sqrt{10}}\right] = \cos^{-1}\left[\frac{4}{\sqrt{2}\sqrt{10}}\right]$

$= \cos^{-1}\left[\frac{2}{\sqrt{5}}\right]$ and

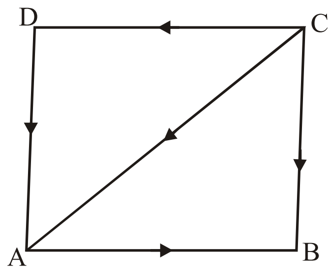
angle between $\hat{i}+\hat{j}$ and $(3\hat{i}+\hat{j})$

$= \cos^{-1}\left|\frac{1+3}{\sqrt{10}\sqrt{2}}\right|$

$= \cos^{-1}\left(\frac{4}{\sqrt{2}\sqrt{10}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Hence required vector is $\hat{i}+\hat{j}$

81. (d) Let ABCD be a quadrilateral



$\overrightarrow{AB}+\overrightarrow{CB}+\overrightarrow{CD}+\overrightarrow{DA}$

$= \overrightarrow{AB}+\overrightarrow{CB}+\overrightarrow{CA}\ \left(\because \overrightarrow{CD}+\overrightarrow{DA}=\overrightarrow{CA}\right)$

$= \overrightarrow{AB}+\overrightarrow{CA}+\overrightarrow{CB}$

$= \overrightarrow{CB}+\overrightarrow{CB}\qquad \left(\because \overrightarrow{AB}+\overrightarrow{CA}=\overrightarrow{CB}\right)$

$= 2\overrightarrow{CB}$

82. (a) Let the vertices of the Δ ABC are A (3,–1,2), B (1,–1,–3) and C (4,–3,1).

Let $\overline{OA}=3\hat{i}-\hat{j}+2\hat{k}$,

$\overline{OB}=\hat{i}-\hat{j}-3\hat{k}$ and

$\overline{OC}=4\hat{i}-3\hat{j}+k$

Area of $\Delta ABC=\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

Now, $\overline{AB}=\overline{OA}-\overline{OB}=2\hat{i}+5\hat{k}$

$\overline{AC}=\overline{OA}-\overline{OC}=-\hat{i}+2\hat{j}+\hat{k}$

$\therefore\quad$ Required Area $=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 5\\ -1 & 2 & 1\end{matrix}\right\|$

$=\frac{1}{2}\left|\hat{i}(-10)-\hat{j}(2+5)+\hat{k}(4)\right|$

$=\frac{1}{2}\left|-10\hat{i}-7\hat{j}+4\hat{k}\right|$

$=\frac{1}{2}\sqrt{100+49+16}=\frac{1}{2}\sqrt{165}$ sq unit

83. (d) Let $\vec{A}=\hat{i}+\hat{j}+\hat{k}, \vec{B}=2\hat{i}+4\hat{j}-5\hat{k}$

and $\vec{C}=b\hat{i}+2\hat{j}+3\hat{k}$

Now, $\vec{B}+\vec{C}=2\hat{i}+4\hat{j}-5\hat{k}+b\hat{i}+2\hat{j}+3\hat{k}$

$=(2+b)\ \hat{i}+6\hat{j}-2\hat{k}$

The unit vector parallel to $\vec{B}+\vec{C}$ is

$$\hat{n}=\frac{(2+b)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+b)^2+6^2+(-2)^2}}=\frac{(2+b)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{b^2+4b+44}}$$

Now, $\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\hat{n}=1\Rightarrow\frac{2+b+6-2}{\sqrt{b^2+4b+44}}=1$

$\Rightarrow\quad 2+b+6-2=\sqrt{b^2+4b+44}$

$\Rightarrow\quad 8b=8$

$\Rightarrow\quad b=1$

84. (c) Let $p=$ Magnitude of $3\hat{i}-2\hat{j}=\sqrt{9+4}=\sqrt{13}$

$q=$ Magnitude of $2\hat{i}+2\hat{j}+\hat{k}=\sqrt{4+4+1}=3$

$r=$ Magnitude of $4\hat{i}-\hat{j}+\hat{k}=\sqrt{16+1+1}=\sqrt{18}=3\sqrt{2}$

$s=$ Magnitude of $2\hat{i}+2\hat{j}+3\hat{k}=\sqrt{4+4+9}=\sqrt{17}$

$\therefore\quad r>s>p>q$

85. (c) Let $x\hat{i}+y\hat{j}+z\hat{k}$ is a unit vector.

$\therefore\quad x^2+y^2+z^2=1$

Given $x:y:z=\sqrt{3}:2:3$

$\Rightarrow x=\sqrt{3}\,k, y=2k$ and $z=3k$

$\therefore\quad (\sqrt{3}\,k)^2+(2k)^2+(3k)^2=1$

$\Rightarrow 3k^2+4k^2+9k^2=1$

$\Rightarrow k^2=\frac{1}{16}\Rightarrow k=\frac{1}{4}$

Hence, $z=3k=3\times\frac{1}{4}=\frac{3}{4}$

86. (c) Let vector $x\hat{i}+y\hat{j}+z\hat{k}$ be perpendicular to vectors $4\hat{i}+2\hat{j}$ and $-3\hat{i}+2\hat{j}$.

Their dot product is zero.

$\therefore\quad 4x+2y=0$ ... (i)

and $-3x+2y=0$ ... (ii)

From eqs. (i) and (ii),

$x=0, y=0$

Hence, required vector is $\hat{k}$.

87. (b) Since, $\vec{u}_4=2\vec{u}_2$

$\therefore\quad \vec{u}_2$ is parallel to $\vec{u}_4$.

Hence, only statement II is correct.

88. (c) Since, the points with position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$, $a\hat{i}+11\hat{j}$ are collinear.

$$\therefore\quad \begin{vmatrix}10 & 3 & 1\\ 12 & -5 & 1\\ a & 11 & 1\end{vmatrix}=0$$

$\Rightarrow 10(-5-11)-3(12-a)+1(132+5a)=0$

$\Rightarrow -160-36+3a+132+5a=0$

$\Rightarrow -196+132+8a=0$

$\Rightarrow 8a=64$

$\Rightarrow a=8$

89. (b) We know that, the angle between the vectors $a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ is given by

$$\cos\theta=\left[\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right]$$

$\therefore$ Angle between the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $-\hat{i}+2\hat{j}+3\hat{k}$ is given by

$$\cos\theta=\left[\frac{1\times(-1)+2\times2+3\times3}{\sqrt{1+4+9}\sqrt{1+4+9}}\right]$$

$$=\frac{-1+4+9}{14}=\frac{12}{14}=\frac{6}{7}$$

Now, $\sin\theta=\sqrt{1-\cos^2\theta}$

$$=\sqrt{1-\frac{36}{49}}=\sqrt{\frac{49-36}{49}}=\sqrt{\frac{13}{49}}=\frac{\sqrt{13}}{7}$$

90. (a) Let the vector $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$.

$\Rightarrow \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

$\therefore\ \vec{a}.(\vec{b}\times\vec{c})=0$

91. (b) Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}-4\hat{j}+7\hat{k}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

Now, $\vec{a}.\vec{b}=4(1)+4(2)+1(7)=19$

and $|\vec{b}|=\sqrt{(4)^2+(4)^2+(7)^2}=\sqrt{81}=9$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}=\dfrac{19}{9}$

92. (c) Let $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b}=0$ and $\vec{a}\times\vec{b}=0$ then either $\vec{a}$ or $\vec{b}$ is a null vector.

93. (d) Given vectors $-\hat{i}-2x\hat{j}-3y\hat{k}$ and $\hat{i}-3x\hat{j}-2y\hat{k}$ are orthogonal to each other.

$\therefore\ \left(-\hat{i}-2x\hat{j}-3y\hat{k}\right).\left(\hat{i}-3x\hat{j}-2y\hat{k}\right)=0$

$\Rightarrow (-1)(1)+(-2x)(-3x)+(-3y)(-2y)=0$

$\Rightarrow -1+6x^2+6y^2=0$

$\Rightarrow 6x^2+6y^2=1$

$$\Rightarrow x^2+y^2=\left(\frac{1}{\sqrt{6}}\right)^2$$

Hence, locus of (x, y) is a circle.

94. (d) Let $\vec{c}$ is the unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$.

So, A unit vector which is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is $\dfrac{\left(\vec{a}\times\vec{b}\right)}{\left|\vec{a}\times\vec{b}\right|}$

95. (c) Let $\vec{A}=10\hat{i}+3\hat{j}$, $\vec{B}=12\hat{i}-5\hat{j}$

and $\vec{C}=m\hat{i}+11\hat{j}$

Now, $\vec{AB}=2\hat{i}-8\hat{j}$ and $\vec{BC}=(m-12)\hat{i}+16\hat{j}$

$$\vec{AB}\times\vec{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -8 & 0\\ m-12 & 16 & 0\end{vmatrix}=0$$

$=\hat{i}(0)-\hat{j}(0)+\hat{k}(32+8m-96)$

$=k(-64+8m)=0\Rightarrow 8m=64$

$\Rightarrow m=8$

96. (d) Since the three vectors are coplanar, so one of them is expressible as a linear combination of the other two.

$\therefore (m,-1,2)=x(2,-3,4)+y(1,2,-1)$

$\Rightarrow 2x+y=m$ ...(i)

$-3x+2y=-1$ ...(ii)

and $4x-y=2$ ...(iii)

on solving equation (ii) and (iii) we get

$x=\dfrac{3}{5}$ and $y=\dfrac{2}{5}$

$\therefore$ from (i), $2\left(\dfrac{3}{5}\right)+\dfrac{2}{5}=m$

$\Rightarrow \dfrac{6}{5}+\dfrac{2}{5}=m\Rightarrow\dfrac{8}{5}=m$

97. (a) Let $\vec{OA}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{OB}=2\hat{i}+5\hat{j}-\hat{k}$ and $\vec{OC}=-\hat{i}+\hat{j}+2\hat{k}$ be three position vectors.

Now, $\vec{AB}=\vec{OB}-\vec{OA}=\hat{i}+3\hat{j}-4\hat{k}$

and $\vec{AC}=\vec{OC}-\vec{OA}=-2\hat{i}-\hat{j}-\hat{k}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|$

Now, $\vec{AB}\times\vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -4 \\ -2 & -1 & -1 \end{vmatrix}$

$= \hat{i}(-3-4)-\hat{j}(-1-8)+\hat{k}(-1+6)$

$= -7\hat{i}+9\hat{j}+5\hat{k}$

Now, $\left|\vec{AB}\times\vec{AC}\right| = \sqrt{155}$

$\therefore$ Required Area $= \frac{\sqrt{155}}{2}$

98. (c) From the given vectors we can conclude that

$A\left(-1,\frac{1}{2},4\right), B\left(1,\frac{1}{2},4\right), C\left(1,\frac{-1}{2},4\right), D\left(-1,-\frac{1}{2},4\right)$

Length = AB = 2, BC = 1

Area = AB × BC = 2

99. (c) Let $\vec{a} = (2,1,-1)$, $\vec{b} = (1,-1,0)$, $\vec{c} = (5,-1,1)$

$\therefore\ \vec{a}+\vec{b}-\vec{c} = (-2,1,-2)$

Let $\vec{n} = x\hat{i}+y\hat{j}+z\hat{k}$ be the unit vector which is || to (–2, 1, – 2) in the opposite direction.

$\therefore x^2+y^2+z^2 = 1$ and $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ -2 & 1 & -2 \end{vmatrix} = 0$

$\Rightarrow x = -2y, y = y, z = -2y$

$x^2+y^2+z^2 = 1 \Rightarrow 4y^2+y^2+4y^2 = 1 \Rightarrow y = \pm\frac{1}{3}$

Hence, the Required vector

$\hat{n} = \frac{2}{3}\hat{i}-\frac{\hat{j}}{3}+\frac{2}{3}\hat{k}$

100. (c) Given $|\vec{a}| = |\vec{b}|$

Consider $(\vec{a}+\vec{b})\bullet(\vec{a}-\vec{b})$

$= \vec{a}.\vec{a}-\vec{a}.\vec{b}+\vec{b}.\vec{a}-\vec{b}.\vec{b}$

$= |\vec{a}|^2-|\vec{b}|^2 = |\vec{a}|^2-|\vec{a}|^2 = 0$

Hence $(\vec{a}+\vec{b})$ is perpendicular to $(\vec{a}-\vec{b})$.

101. (c) Consider $\overline{PO}+\overline{OQ} = \overline{QO}+\overline{OR}$

$\Rightarrow \overline{OQ}-\overline{QO} = -\overline{PO}+\overline{OR}$

$\Rightarrow \overline{OQ}+\overline{OQ} = \overline{OR}+\overline{OP}$

$\Rightarrow \overline{OQ} = \frac{1}{2}\left(\overline{OP}+\overline{OR}\right)$

Hence Q is the mid-point of P and R.

$\therefore$ P, Q, R are collinear.

102. (c) Three vectors $x_1\vec{a}+y_1\vec{b}+z_1\vec{c}$, $x_2\vec{a}+y_2\vec{b}+z_2\vec{c}$ and $x_3\vec{a}+y_3\vec{b}+z_3\vec{c}$ will be coplanar iff

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

Here, $x_1 = 2, y_1 = -1, z_1 = 1$

$x_2 = 1, y_2 = 2, z_2 = -3$

$x_3 = 3, y_3 = m, z_3 = 5$

$\therefore \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & m & 5 \end{vmatrix} = 0$

$\Rightarrow 2(10+3m)+1(5+9)+1(m-6) = 0$

$\Rightarrow 7m+28 = 0 \Rightarrow m = -4$

103. (b) We have

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ and $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

Given: $|\vec{a}| = 10$, $|\vec{b}| = 2$, $\vec{a}\cdot\vec{b} = 12$

$\therefore \cos\theta = \frac{12}{20}$ and $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{20}$

Now, By squaring and adding, we get

$\sin^2\theta+\cos^2\theta = 1$

$\Rightarrow \frac{|\vec{a}\times\vec{b}|^2}{400}+\frac{144}{400} = 1 \Rightarrow |\vec{a}\times\vec{b}|^2 = 256$

$\Rightarrow |\vec{a}\times\vec{b}| = 16$

104. (c) Since both vectors are orthogonal $\therefore$ their dot product is zero.

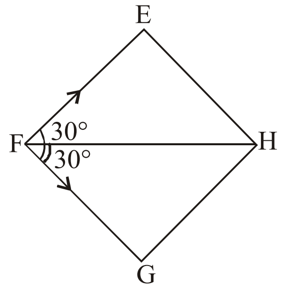
$\therefore 1(1)+(-x)(x)+(-y)(y) = 0$

$\Rightarrow 1-x^2-y^2 = 0$

$\Rightarrow x^2+y^2 = 1$

Which is a circle.

105. (d)



Rhombus EFGH, $\angle EFG = 60°$

$\angle EFH = 30° = \angle HFG$

From parallelogram of forces

$\overrightarrow{FE} + \overrightarrow{FG} = \overrightarrow{FH}$

Given $|\overrightarrow{FE}| + |\overrightarrow{FG}| = a$ (say)

$\therefore\ \overrightarrow{FH} = 2.\frac{\sqrt{3}}{2}\vec{a} = \sqrt{3}\,\vec{a}$

$\overrightarrow{EG} = \overrightarrow{EF} + \overrightarrow{EH}$

$= a \sin 30° + a \sin 30° = a.\frac{1}{2} + a.\frac{1}{2} = a$

Thus, $\frac{\overrightarrow{FH}}{\overrightarrow{EG}} = \frac{\sqrt{3}a}{a} = \sqrt{3}$

So, $\overrightarrow{FH} = \sqrt{3}\ \overrightarrow{EG}$

$\Rightarrow\quad m = \sqrt{3}$

106. (c) $\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$ .... (i)

$\vec{a}\times\vec{b} = 0 \Rightarrow \vec{a} \parallel \vec{b}$ .... (ii)

From (i) and (ii) it is clear that $\vec{a} = 0$ or $\vec{b} = 0$

107. (d) $\vec{a}\times\left(\vec{b}\times\vec{a}\right)$ is coplanar with neither $\vec{a}$ nor $\vec{b}$.

108. (a) **Statement (1) :** $4\hat{i}\times 3\hat{i} = 0$

It is a true statement

$\left(\because \hat{i}\times\hat{i} = 0\right)$

Statement (2) : $\frac{4\hat{i}}{3\hat{i}} = \frac{4}{3}$

It is a incorrect statement

$\because\ \frac{4\hat{i}}{3\hat{i}} = \frac{4}{3}\hat{i}$

109. (a) Given $\left(\lambda\hat{i} + \hat{j} - \hat{k}\right)\times\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right) = \left(2\hat{i} - 11\hat{j} - 7\hat{k}\right)$

$\Rightarrow\quad 2\hat{i} - \hat{j}(4\lambda + 3) + \hat{k}(-2\lambda - 3) = 2\hat{i} - 11\hat{j} - 7\hat{k}$

$\Rightarrow\quad -4\lambda - 3 = -11$

$\Rightarrow\quad 4\lambda = 8 \Rightarrow \lambda = 2$

110. (d) Let $\vec{a} = p\left(-3\hat{i} - 2\hat{j} + 13\hat{k}\right)$

$= (-3p)\hat{i} + (-2p)\hat{j} + (13p)\hat{k}$

It is given that $\vec{a}$ is of unit length

$\therefore\ |\vec{a}| = 1 \Rightarrow |\vec{a}|^2 = 1$

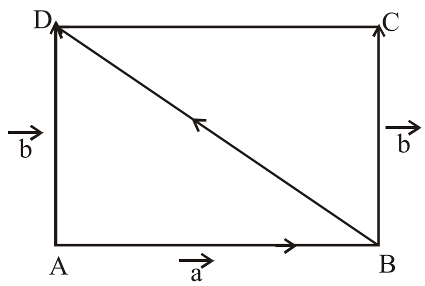
$\Rightarrow\quad (-3p))(-3p) + (-2p)(-2p) + (13p)(13p) = 1$

$9p^2 + 4p^2 + 169p^2 = 1$

$\Rightarrow\quad p^2 = \frac{1}{182} \Rightarrow p = \frac{1}{\sqrt{182}}$

111. (b) The vector $2\hat{j} - \hat{k}$ lies in the plane of YZ.

112. (d)



$\overrightarrow{BD} = -\vec{a} + \vec{b}$

113. (b) Let $\vec{\beta} = a\hat{i} + b\hat{j} + c\hat{k}$

Since $\vec{\beta}\cdot\vec{\alpha} = 0$ and $\vec{\beta}\cdot\vec{\gamma} = 0$

$\therefore\quad c = 0$ and $2a + 3b = 0 \Rightarrow a = -\frac{3b}{2}$

Hence, $\vec{\beta} = -\frac{3b}{2}\hat{i} + b\hat{j} = 0 \Rightarrow -3\hat{i} + 2\hat{j} = 0$

114. (a) Let $\vec{\alpha} = \vec{a}\,\hat{i} + \vec{b}\,\hat{j} + \vec{c}\,\hat{k}$

Now, $\vec{\alpha}.\hat{i} = \left(\vec{a}\,\hat{i} + \vec{b}\hat{j} + \vec{c}\hat{k}\right).\hat{i} = \vec{a}$

$\vec{\alpha}.\hat{j} = \left(\vec{a}\,\hat{i} + \vec{b}\,\hat{j} + \vec{c}\,\hat{k}\right).\hat{j} = \vec{b}$

$\vec{\alpha}.\hat{k} = \left(\vec{a}\,\hat{i} + \vec{b}\,\hat{j} + \vec{c}\hat{k}\right).\hat{k} = \vec{c}$

Now, $\vec{a}\,\hat{i} + \vec{b}\hat{j} + \vec{c}\hat{k} = \vec{\alpha}$

Thus, Required expression $= \vec{\alpha}$.

115. (b) Since magnitude of $\vec{a}\times\vec{b}$ = magnitude of $\vec{a}\cdot\vec{b}$

$\therefore |\vec{a}\times\vec{b}| = |\vec{a}\cdot\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = |\vec{a}||\vec{b}|\cos\theta$ (By Definition)

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

116. (b) Consider $\left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}-\vec{b}\right|^2=2\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2\right)$

By putting the values of $\left|\vec{a}\right|,\left|\vec{b}\right|$ and $\left|\vec{a}+\vec{b}\right|$, we get

$6+\left|\vec{a}-\vec{b}\right|^2=2(2+3)$

$\Rightarrow \quad \left|\vec{a}-\vec{b}\right|=2$

117. (d) One vector will be normal to the other vector if their dot product will be zero.
Since none option satisfies the condition of normality. Therefore option (d) is correct.

118. (c) Angle b/w the vectors is $\cos\theta=\dfrac{4(1)-4(1)}{\sqrt{32}\sqrt{3}}=0$

$\Rightarrow \theta=\dfrac{\pi}{2}$

Hence, $\cos\theta+\sin\theta=\cos\dfrac{\pi}{2}+\sin\dfrac{\pi}{2}=0+1=1.$

119. (d) $\cos\dfrac{\pi}{3}=\dfrac{(\hat{i}-m\hat{j}).(\hat{j}+\hat{k})}{\left|\sqrt{1+m^2}\right|\left|\sqrt{1^2+1^2}\right|}$

$\dfrac{1}{2}=\dfrac{-m}{\sqrt{1+m^2}.\sqrt{2}}$

$\dfrac{1}{2}=\dfrac{m^2}{1+m^2}$

$m=\pm 1$

120. (d) $(\hat{i}-\hat{j})\times(\hat{j})$

$=\hat{i}\times\hat{i}-\hat{j}\times\hat{j}=\hat{k}$

121. (c) $\overrightarrow{AB}=-2\hat{i}+6\hat{j}-3\hat{k}$

$|\overrightarrow{AB}|=\sqrt{(-2)^2+6^2+(-3)^2}=\sqrt{49}=7$

122. (d) $(\hat{i}-2x\hat{j}-3y\hat{k}).(\hat{i}+3x\hat{j}+2y\hat{k})=0$

$1-6x^2-6y^2=0$

$-6x^2-6y^2=-1$

$x^2+y^2=\dfrac{1}{6}$

$x^2+y^2=\left(\sqrt{\dfrac{1}{6}}\right)^2$

Hence, locus of the point i.e. a circle.

123. (a) $|P(2\hat{i}-\hat{j}+2\hat{k})|=3$

$P\left|\sqrt{2^2+(-1)^2+2^2}\right|=3$

$3P=3\Rightarrow P=1$

124. (a) $\vec{a}+t\vec{b}=(2-t)\hat{i}+(2+2t)\hat{j}+(3+t)\hat{k}$

$(\vec{a}+t\vec{b})$ and $\vec{c}$ is perpendicular. Therefore,

$(\vec{a}+t\vec{b}).\vec{c}=0$

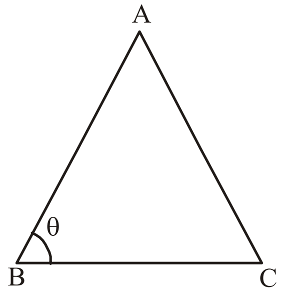
$3(2-t)+2+2t=0$

$6-3t+2t+2=0$

$t=8$

125. (a) $\overrightarrow{BA}=4\hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{BC}=2\hat{i}-\hat{j}-\hat{k}$



$\cos B=\dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$

$=\dfrac{(4\hat{i}+\hat{j}+\hat{k}).(2\hat{i}-\hat{j}-\hat{k})}{|\sqrt{4^2+1^2+1^2}||\sqrt{2^2+(-1)^2+(-1)^2}|}$

$=\dfrac{6}{\sqrt{18}\sqrt{6}}=\dfrac{1}{\sqrt{3}}$

126. (b) Area of triangle ABC $=\dfrac{1}{2}|\overrightarrow{BA}\times\overrightarrow{BC}|$

$\overrightarrow{BA}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 1 & 1\\ 2 & -1 & -1\end{vmatrix}$

$=\hat{i}(0)-\hat{j}(-6)+\hat{k}(-6)$

$|\overrightarrow{BA}\times\overrightarrow{BC}|=|6\hat{j}-6\hat{k}|=\sqrt{6^2+(-6)^2}$

$=6\sqrt{2}$

Area of triangle $=\dfrac{1}{2}\times 6\sqrt{2}=3\sqrt{2}$

127. (c) Mid-point of A and C, $\left(\dfrac{2+0}{2},\dfrac{3+1}{2},\dfrac{1-1}{2}\right)=(1,2,0)$

Mid-point of B and C, $\left(\dfrac{-2+0}{2},\dfrac{2+1}{2},\dfrac{0-1}{2}\right)$

$=\left(-1,\dfrac{3}{2},\dfrac{-1}{2}\right)$

Magnitude $=\sqrt{(1+1)^2+\left(2-\dfrac{3}{2}\right)^2+\left(\dfrac{1}{2}\right)^2}$

$=\sqrt{4+\dfrac{1}{4}+\dfrac{1}{4}}=\dfrac{3}{\sqrt{2}}$ units

128. (b) Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{(\hat{i}-2\hat{j}+\hat{k}).(4\hat{i}-4\hat{j}+7\hat{k})}{|\sqrt{4^2+(-4)^2+7^2}|}$

$= \frac{19}{9}$

129. (a) Vector perpendicular to $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 4 & -4 & 7 \end{vmatrix} = \hat{i}(-14+4) - j(7-4) + \hat{k}(-4+8)$

$= -10\hat{i} - 3\hat{j} + 4\hat{k}$

130. (c) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 60° + \cos^2 30° + \cos^2\gamma = 1$

$\frac{1}{4} + \frac{3}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = 0 \Rightarrow \gamma = 90°$

131. (a) $r = < l, m, n >$; $r = < \cos 60°, \cos 30°, \cos 90° >$

Direction cosines of $\vec{r} = < \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$

132. (b) Let angle between $\vec{a}$ and $\vec{b}$ be θ.

$|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta|}$

$10\sqrt{3} = \sqrt{49+121+2\times7\times11\cos\theta}$

$300 = 170 + 154\cos\theta$

$154\cos\theta = 130$

$|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta}$

$|\vec{a}-\vec{b}| = \sqrt{170-154\cos\theta}$

$|\vec{a}-\vec{b}| = \sqrt{170-130} = \sqrt{40}$ or $2\sqrt{10}$

133. (d) Let angle between $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ be α

$\cos\alpha = \frac{(\vec{a}+\vec{b})(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$

$= \frac{(7)^2-(11)^2}{10\sqrt{3}\times2\sqrt{10}} = \frac{(7+11)(7-11)}{20\sqrt{3}\times\sqrt{10}} = \frac{-18}{5\sqrt{30}}$

$= \frac{-6\times3}{5\sqrt{30}}\times\frac{\sqrt{30}}{\sqrt{30}} = -\frac{3\sqrt{30}}{25}$

$\alpha = \cos^{-1}\left(\frac{-3}{5}\sqrt{\frac{6}{5}}\right)$

134. (a) Given $|a| = 2$, $|b| = 5$ and $|a\times b| = 8$

Also $|a\times b| = |a| . |b| . |\sin\theta|$

$\Rightarrow |\sin\theta| = \frac{8}{2\times5} = \frac{4}{5}$

$\Rightarrow |\cos\theta| = \frac{3}{5} \Rightarrow \cos\theta = \pm\frac{3}{5}$

$\therefore a.b = |a| . |b| \cos\theta = 2\times5\times\frac{3}{5} = 6$

135. (c) Since, $|a+b| = |a-b|$

$\Rightarrow [|a+b|]^2 = [a-b]^2$

$\Rightarrow a.a + b.b + a.b + b.a = a.a + b.b - a.b - b.a$

$\Rightarrow 4a.b = 0$ $(\because a.b = b.a)$

$\Rightarrow a.b = 0$

Hence, a is perpendicular to b.

136. (b) Area of ΔOAB = $\frac{1}{2}|\overrightarrow{OA}\times\overrightarrow{OB}|$

$\therefore \overrightarrow{OA}\times\overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix}$

$= \hat{i}[3-1] - \hat{j}[-9-2] + \hat{k}[3+2]$

$= 2\hat{i} + 11\hat{j} + 5\hat{k}$

$\therefore |\overrightarrow{OA}\times\overrightarrow{OB}| = \sqrt{2^2+11^2+5^2} = \sqrt{150} = 5\sqrt{6}$

$\therefore$ Required area = $\frac{1}{2}\times5\sqrt{6} = \frac{5\sqrt{6}}{2}$ sq. units

137. (a) According to question $\vec{a} = -\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+\hat{k}$

Then, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}[1+1] - \hat{j}[-1-1] + \hat{k}[1-1]$

$= 2\hat{i} + 2\hat{j} + 0 = 2(\hat{i}+\hat{j})$

and $|\vec{a}\times\vec{b}| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Required unit vector $= \pm\frac{2(\hat{i}+\hat{j})}{2\sqrt{2}} = \pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

138. (a) Let $\vec{a} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$

and $\vec{b} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$

$\therefore \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$= \frac{\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\hat{k}\right).\left(\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}+\hat{k}\right)}{\sqrt{\frac{1}{2}+\frac{1}{2}+1}\sqrt{\frac{1}{2}+\frac{1}{2}+1}}$

$= \frac{1}{2}\left[\frac{1}{2}-\frac{1}{2}+1\right] = \frac{1}{2} = \cos 60°$

$\therefore \theta = 60°$

139. (a) Let $\vec{a} = \lambda\hat{i} + (1+\lambda)\hat{j} + (1+2\lambda)\hat{k}$

and $\vec{b} = (1-\lambda)\hat{i} + \lambda\hat{j} + 2\hat{k}$

For a and b to be perpendicular, we should have

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

$\Rightarrow [\lambda\hat{i} + (1+\lambda)\hat{j} + (1+2\lambda)\hat{k}].[(1-\lambda)\hat{i} + \lambda\hat{j} + 2\hat{k}] = 0$

$\Rightarrow \lambda - \lambda^2 + \lambda + \lambda^2 + 2 + 4\lambda = 0$

$\Rightarrow 6\lambda = -2$

$\therefore \lambda = -\frac{2}{6} = -\frac{1}{3}$

Sol. (Qs. 140–143)

We have, $\vec{a} + \vec{b} + \vec{c} = 0$ ...(i)

On squaring both sides request.

$\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a} = 0$

($\because$ $\vec{a}.\vec{b} = \vec{b}.\vec{a}$, $\vec{b}.\vec{c} = \vec{c}.\vec{b}$ and $\vec{c}.\vec{a} = \vec{a}.\vec{c}$)

$\Rightarrow |a|^2 + |b|^2 + |c|^2 = -2$ [a.b + b.c + c.a]

$\Rightarrow (3)^2 + (5)^2 + (7)^2 = -2$ [a.b + b.c + c.a]

$\Rightarrow a.b + b.c + c.a = \frac{9+25+49}{-2} = -\frac{83}{2}$

Now a + b + c = 0 [using eq. (i)]

$\Rightarrow a + b = -c$

On squaring both sides, we get

$\Rightarrow a^2 + b^2 + 2a.b = c^2$

$\Rightarrow (3)^2 + (5)^2 + 2ab = (7)^2$

$\Rightarrow \vec{a}\cdot\vec{b} = \frac{15}{2}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = \frac{15}{2} \Rightarrow 3.5\cos\theta = \frac{15}{2}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

From eq. (i),

$\vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow b^2 + c^2 + 2b.c = a^2$

$\Rightarrow 2b.c = a^2 - b^2 - c^2 = 9 - 25 - 49 = -65$

$\Rightarrow b.c = -\frac{65}{2} \Rightarrow |b|\,|c|\cos\theta = -\frac{65}{2}$

$\Rightarrow \cos\theta = \frac{65}{2}\times\frac{1}{5}\times\frac{1}{7} = -\frac{13}{14}$

Also, $|\vec{a} + \vec{b}| = |-\vec{c}| = |\vec{c}| = 7$

140. (c)
141. (b)
142. (d)
143. (a)

144. (d) Area of $\Delta ABC = \frac{1}{2}\left(\overline{AB}\times\overline{AC}\right)$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 2 \\ -4 & 5 & 2 \end{vmatrix}$

$= \frac{1}{2}\left[\hat{i}(6-10) - \hat{j}(-4+8) + \hat{k}(-10+12)\right]$

$= \frac{1}{2}\left[-4\hat{i} - 4\hat{j} + 2\hat{k}\right]$

$= \frac{1}{2}\sqrt{16+16+4} = \frac{1}{2}\sqrt{36} = \frac{1}{2}\times 6$

= 3 square units

$\therefore$ Option (d) is correct.

145. (a) Moment of force, $m = \vec{r}\times\vec{F}$

$m = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & -3 \\ 3 & 4 & -3 \end{vmatrix}$

$= \hat{i}(6+12) - \hat{j}(-6+9) + \hat{k}(8+6)$

$= 18\hat{i} - 3\hat{j} + 14\hat{k}$

$= \sqrt{(18)^2 + (-3)^2 + (14)^2}$

$= \sqrt{529} = 23$ units.

146. (a) $\vec{u} - \vec{v} = \vec{w}$

$\left(2x\vec{\alpha} + y\vec{\beta}\right) - \left(2y\vec{\alpha} + 3x\vec{\beta}\right) = 2\vec{\alpha} - 5\vec{\beta}$

$(2x-2y)\vec{\alpha} + (y-3x)\vec{\beta} = 2\vec{\alpha} - 5\vec{\beta}$

$\therefore$ $2x - 2y = 2$ ... (i)

and $3x - y = 5$ ... (ii)

Solving equations (i) and (ii), we get

x = 2 and y = 1

$\therefore$ Option (a) is correct.

147. (d) $|\vec{a}| = 7$, $|\vec{b}| = 11$ and $|\vec{a}+\vec{b}| = 10\sqrt{3}$

Now $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

$\therefore$ $\left(10\sqrt{3}\right)^2 = 49 + 121 + 2\times 7\times 7\cos\theta$

$\therefore$ $300 = 170 + 154\cos\theta$

$\frac{300-170}{154} = \cos\theta$

$\therefore$ $\frac{65}{77} = \cos\theta$

Now, $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$= (7)^2 + (11)^2 - 2\times 7\times 11\times\frac{65}{77}$

$= 49 + 121 - 2\times 65$

$= 170 - 130 = 40$

$\therefore$ $|\vec{a}-\vec{b}| = \sqrt{40} = 2\sqrt{10}$

$\therefore$ Option (d) is correct.

148. (b) α, β and γ be distinct real numbers

$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{b} = \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$

$\vec{c} = \gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

Let $\alpha = 1, \beta = 2$ and $\gamma = 3$

then, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

Now, $|\overrightarrow{ab}| = \sqrt{(2-1)^2 + (3-2)^2 + (1-3)^2} = \sqrt{6}$

Similarly $|\overrightarrow{bc}| = |\overrightarrow{ac}| = \sqrt{6}$

Hence, point for equilateral triangle.

149. (c) According to statements (1) and (2),

when $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then both statements are correct.

If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = 0$, then $\vec{a}$, $\vec{b}$ and $\vec{c}$ are collinear.

Therefore, option (c) is correct.

150. (c) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

We know that when $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$, then $\vec{a}$ is perpendicular to $\vec{b}$.

$= \vec{a}.\vec{a} - \vec{a}.\vec{b} + \vec{b}.\vec{a} - \vec{b}.\vec{b} = 0$

$\vec{a}$ is perpendicular to $\vec{b}$

$\therefore$ Option (c) is correct.

151. (b) Length of diagonal

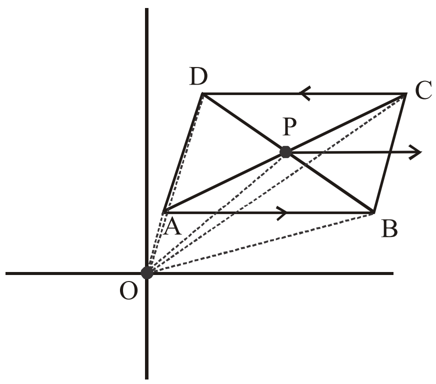
$= D = \sqrt{3^2 + 4^2}$

$\Rightarrow D = 5$

$\therefore$ Area $= \frac{1}{2}(D)^2 = \frac{25}{2}$

$= 12.5$ units

152. (a)



$\because \overrightarrow{OA} = \overrightarrow{OP} - \overrightarrow{AP}$; $\overrightarrow{OB} = \overrightarrow{OP} + \overrightarrow{PB}$;

$\overrightarrow{OC} = \overrightarrow{OP} + \overrightarrow{PC}$ & $\overrightarrow{OD} = \overrightarrow{OP} - \overrightarrow{DP}$;

Also, $\overrightarrow{AP} = \overrightarrow{PC}$ & $\overrightarrow{DP} = \overrightarrow{PB}$

$\therefore \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$

153. (c) $\overrightarrow{OB} = \vec{b}$

$\overrightarrow{OC} = \vec{c}$

$\overrightarrow{BD} = 4\overrightarrow{BC}$

$\overrightarrow{BO} + \overrightarrow{OD} = 4(\overrightarrow{BO} + \overrightarrow{OC})$

$\overrightarrow{OD} = 3\overrightarrow{BO} + 4\overrightarrow{OC}$

$\overrightarrow{OD} = 4\overrightarrow{OC} - 3\overrightarrow{OB}$

$\overrightarrow{OD} = 4\vec{c} - 3\vec{b}$

154. (b) $\sqrt{(5-0)^2 + (n-0)^2} = 13$

$25 + n^2 = 169$

$n^2 = 144$

$n = \pm 12$

155. (d) $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$

$\therefore |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 6\sin\theta$

& $|\vec{a}.\vec{b}| = |\vec{a}||\vec{b}|\cos\theta = 6\cos\theta$

$|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = (6\sin\theta)^2 + (6\cos\theta)^2$

$= 36(\sin^2\theta + \cos^2\theta)$

$= 36$

156. (c) $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

$|\vec{a} - \vec{b}| \ge |\vec{a}| - |\vec{b}|$

both are correct.

157. (b) $|\hat{a} - \hat{b}| = \sqrt{3}$

$\Rightarrow |\hat{a} - \hat{b}|^2 = (\sqrt{3})^2$

$\Rightarrow \hat{a}.\hat{a} + \hat{b}.\hat{b} - 2\hat{a}.\hat{b} = 3$

$\Rightarrow 2\hat{a}.\hat{b} = -1$

Now; $|\hat{a} + \hat{b}|^2 = \hat{a}.\hat{a} + \hat{b}.\hat{b} + 2\hat{a}.\hat{b} = 1 + 1 - 1$

$\Rightarrow |\hat{a} + \hat{b}|^2 = 1$

$\Rightarrow |\hat{a} + \hat{b}| = 1$

158. (b) If three vectors are co-planar.

$\Rightarrow \begin{vmatrix} \alpha & \alpha & \gamma \\ 1 & 0 & 1 \\ \gamma & \gamma & \beta \end{vmatrix} = 0$

$\Rightarrow \alpha[0-\gamma] - \alpha[\beta-\gamma] + \gamma[\gamma-0] = 0$

$\Rightarrow -\alpha\gamma - \alpha\beta + \alpha\gamma + \gamma^2 = 0$

$\Rightarrow \gamma^2 = \alpha\beta$

$\Rightarrow$ So α, γ, β are in G.P.

159. (c) $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$= \vec{a}\times\vec{b}-\vec{d}\times\vec{b}-\vec{a}\times\vec{c}+\vec{d}\times\vec{c}$

$= \vec{c}\times\vec{d}-\vec{d}\times\vec{b}-\vec{b}\times\vec{d}-\vec{c}\times\vec{d}$

$= -\vec{d}\times\vec{b}+\vec{d}\times\vec{b}$

$= 0$

again $(\vec{a}\times\vec{b})=(\vec{c}\times\vec{d})$ given

$\Rightarrow (\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=(\vec{c}\times\vec{d})\times(\vec{c}\times\vec{d})=0$ $(\text{as } \vec{a}\times\vec{a}=0)$

So both (1) and (2) are correct.

160. (b) $\vec{a}$ and $\vec{b}$ are two unit vectors.

$\because$ Hence,

$|\vec{a}+\vec{b}|^2=(\hat{a}+\hat{b}).(\hat{a}+\hat{b})$

$=\hat{a}.\hat{a}+\hat{a}.\hat{b}+\hat{b}.\hat{a}+\hat{b}.\hat{b}$

$=\hat{a}.\hat{a}+\hat{b}.\hat{b}+2\hat{a}.\hat{b}$

$=|\hat{a}|^2+2|\hat{a}||\hat{b}|\cos\theta+|\hat{b}||\hat{b}|$

$=1+2\cos\theta+1$ $[\because \hat{b}, \hat{a}$ are unit vector$]$

$=2(1+\cos\theta)$

$|\hat{a}+\hat{b}|^2=2.2\cos^2\frac{\theta}{2}$

$\cos\frac{\theta}{2}=\frac{|\hat{a}+\hat{b}|}{2}$

161. (a) $|\hat{a}-\hat{b}|^2=(\hat{a}-\hat{b}).(\hat{a}-\hat{b})$

$=\hat{a}.\hat{a}-\hat{b}.\hat{a}-\hat{a}.\hat{b}+\hat{b}.\hat{b}$

$=|\hat{a}|^2-2|\hat{a}||\hat{b}|\cos\theta+|\hat{b}|^2$

$=2-2\cos\theta$

$=2(1-\cos\theta)$

$|\hat{a}-\hat{b}|^2=2.2\sin^2\frac{\theta}{2}$

$\sin\frac{\theta}{2}=\frac{|\hat{a}-\hat{b}|}{2}$

162. (b) $\vec{A}=\hat{i}+\hat{j}+\hat{k}$

$\vec{B}=2\hat{i}+3\hat{j}-\hat{k}$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & 3 & -1\end{vmatrix}$$

$=\hat{i}(-1-3)-\hat{j}(-1-2)+\hat{k}(3-2)$

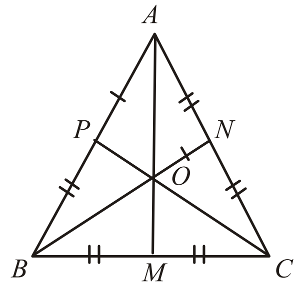
$=-4\hat{i}+3\hat{j}+\hat{k}$

Vector of unit length orthogonal to both the vectors $\vec{A}$ and $\vec{B}$

$=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$=\frac{-4i+3j+k}{\sqrt{16+9+1}}=\frac{-4\hat{i}+3\hat{j}+\hat{k}}{\sqrt{26}}$

163. (a) Position vectors of vertices A, B and C are $\vec{a}, \vec{b}$ and $\vec{c}$.



$\because$ triangle is equilateral.

$\therefore$ Centroid and orthocenter will coincide.

Centroid $\equiv$ orthocenter position vector

$=\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$

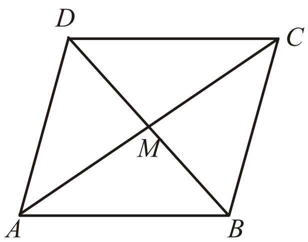
$\because$ given in question orthocenter is at origin.

Hence $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})=0$

$\boxed{\vec{a}+\vec{b}+\vec{c}=0}$

164. (c) Diagonal d_1, $\overrightarrow{AC}=3i+j-2k$

Diagonal d_2, $\overrightarrow{BD}=i-3j+4k$



Area of parallelogram is $\frac{1}{2}|\vec{d_1}\times\vec{d_2}|$

$$\text{Hence area} = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}$$

$=\frac{1}{2}|[\hat{i}(4-6)-\hat{j}(12+2)+\hat{k}(-9-1)]|$

$=\frac{1}{2}|-2\hat{i}-14\hat{j}-10\hat{k}|$

$=\frac{1}{2}\sqrt{4+196+100}$

$=\frac{10\sqrt{3}}{2}=5\sqrt{3}$ square units

165. (a) Since $\vec{c}$ is parallel to $\vec{a}$

$\vec{c} = \lambda\vec{a}$

Now $\vec{b} = \vec{c} + \vec{d} = \lambda\vec{a} + \vec{d}$

$= \lambda(\hat{i} + \hat{j}) + x\hat{i} + y\hat{j} + z\hat{k}$

$3\hat{i} + 4\hat{k} = (\lambda + x)\hat{i} + (\lambda + y)\hat{j} + z\hat{k}$

Comparing we get

$z = 4, \lambda + y = 0, \lambda + x = 3 \Rightarrow -y + x = 3$ (From (1))

$\Rightarrow \lambda = -y$ ...(1) $\Rightarrow x - y = 3$ ...(2)

Now $\vec{d}$ is $\perp$r to $\vec{a}$

So, $\cos\theta = 0$

$\Rightarrow x + y = 0$ ...(3)

Solving (2) and (3) we get

$2x = 3$

$\Rightarrow x = \frac{3}{2},\ y = -\frac{3}{2}$

$\Rightarrow \vec{c} = \lambda(\vec{a}) = \frac{3}{2}(\hat{i} + \hat{j})$

166. (d) Since $z = 4$ and $x = \frac{3}{2},\ y = -\frac{3}{2}$.

So, neither 1 nor 2 is correct.

167. (b) We have $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

So $|\vec{a} + \vec{b} + \vec{c}| = 0$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow 0 = (10)^2 + (6)^2 + (14)^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow 0 = 100 + 36 + 196 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow -\frac{332}{2} = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$

$\Rightarrow -166 = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$

168. (c) Since $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow |\vec{a} + \vec{b}| = |-\vec{c}| = |\vec{c}|$

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}\cdot\vec{b}) = |\vec{c}|^2$

$\Rightarrow (10)^2 + (6)^2 + 2(\vec{a}\cdot\vec{b}) = (14)^2$

$\Rightarrow 2(\vec{a}\cdot\vec{b}) = 60$

$\Rightarrow \vec{a}\cdot\vec{b} = 30$

$\Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{30}{10\times 6} = \frac{1}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = 60°$

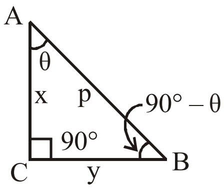
169. (b) $\overrightarrow{AB}\cdot\overrightarrow{AC} + \overrightarrow{BC}\cdot\overrightarrow{BA} + \overrightarrow{CA}\cdot\overrightarrow{CB}$

$= (AB\cdot AC\cdot\cos\theta) + (BC\cdot BA\cdot\cos(90-\theta))$

$+ (CA\cdot CB\cdot\cos 90)$

$= (p\cdot x\cdot\cos\theta) + (y\cdot p\cdot\sin\theta) + 0$

$= p[x\cos\theta + y\sin\theta]$



By projection formula :

$p = x\cos\theta + y\cos(90-\theta)$

$= x\cos\theta + y\sin\theta$

$\therefore\ p[x\cos\theta + y\sin\theta] = p\times p = p^2.$

170. (c) Let point P is $(1, -1, 2)$

and point Q is $(2, -1, 3)$

$\Rightarrow$ Position vector of P w.r.t. Q is

$\vec{r} = (1-2)\hat{i} + (-1+1)\hat{j} + (2-3)\hat{k}$

$\Rightarrow \vec{r} = -\hat{i} + 0\hat{j} - \hat{k}$ and $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\Rightarrow$ Moment $= \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$

$= \hat{i}(0+2) - \hat{j}(4+3) + \hat{k}(-2+0) = 2\hat{i} - 7\hat{j} - 2\hat{k}$

171. (d) $\vec{a} = \hat{i} - \hat{j} + \hat{k},\ \vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

and $\vec{c} = \hat{i} + m\hat{j} + n\hat{k}$; $|\vec{c}| = \sqrt{6}$

Given, $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

So, $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$

$c_1 \to c_1 + c_2$ $\qquad c_3 \to c_3 + c_2$

$\Rightarrow \begin{vmatrix} 0 & 1 & 0 \\ 5 & 3 & 5 \\ 1+m & m & m+n \end{vmatrix} = 0$

$\Rightarrow 0 + 1\,(5m + 5n) - (5 + 5m) = 0$

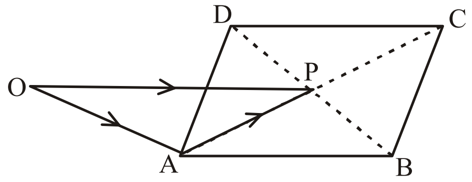
$\Rightarrow 5m + 5n - 5 - 5m = 0 \Rightarrow 5n = 5 \Rightarrow n = 1$

$|\vec{c}| = 6 \Rightarrow \sqrt{1 + m^2 + n^2} = \sqrt{6}$

$\Rightarrow 1 + m^2 + n^2 = 6 \Rightarrow 2 + m^2 = 6 \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

172. (b) From the figure, observe that

$\overline{OA} + \overline{AP} = \overline{OP}$



Similarly, $\overrightarrow{OB}+\overrightarrow{BP}=\overrightarrow{OP}$

$\overrightarrow{OC}+\overrightarrow{CP}=\overrightarrow{OP}$

$\overrightarrow{OD}+\overrightarrow{DP}=\overrightarrow{OP}$

$\Rightarrow \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{AP}$

$+\overrightarrow{BP}+\overrightarrow{CP}+\overrightarrow{DP}=4\overrightarrow{OP}$ $\rightarrow$ (i)

In a parallelogram, diagonals bisect each other.

$\therefore \overrightarrow{AP}=\overrightarrow{PC}$ $(\therefore$ Since AP = CP)

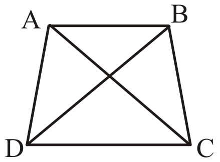
$\Rightarrow \overrightarrow{AP}=-\overrightarrow{PC}$

$\Rightarrow \overrightarrow{AP}+\overrightarrow{PC}=O$

Similarly $\overrightarrow{BP}+\overrightarrow{DP}=O$

$\therefore$ (i) $\Rightarrow \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=4\overrightarrow{OP}$

173. (b)



$\overrightarrow{BA}+\overrightarrow{AD}=\overrightarrow{BD}$ ...(i)

$\overrightarrow{CD}+\overrightarrow{DA}=\overrightarrow{CA}$ ...(ii)

(i) + (ii) $\Rightarrow \overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{BD}+\overrightarrow{CA}$ $\left(\because \overrightarrow{AD},\overrightarrow{DA} \text{ cancel}\right)$

174. (a) $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow \vec{c}\perp\vec{a}$ & $\vec{c}\perp\vec{b}$

$\Rightarrow \vec{a}\perp\vec{b}\perp\vec{c}$

$\vec{a},\vec{b},\vec{c}$ are orthogonal.

Now, $|\vec{a}\times\vec{b}|=|\vec{c}|$

$\Rightarrow |\vec{a}||\vec{b}|\sin 90^\circ=|\vec{c}|$

$\Rightarrow |\vec{a}||\vec{b}|=|\vec{c}|$

$\Rightarrow |\vec{b}|\,\cancel{|\vec{c}|}\,|\vec{b}|=\cancel{|\vec{c}|}$

$\Rightarrow |\vec{b}|^2=1$

$\Rightarrow |\vec{b}|=1$ ...(i)

$\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow \vec{a}\perp\vec{b}\perp\vec{c}$

Also, $|\vec{b}\times\vec{c}|=|\vec{a}|$

$\Rightarrow |\vec{b}||\vec{c}|\sin 90^\circ=|\vec{a}|$

$\Rightarrow |\vec{b}|\cdot|\vec{c}|=|\vec{a}|$

$\Rightarrow |\vec{c}|=|\vec{a}|$ (from (i))

175. (b) $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-\lambda\hat{k}$

Given, $\vec{a}$, $\vec{b}$ are $\perp^r$.

$\therefore \vec{a}.\vec{b}=0$

$\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 6+6-4\lambda=0$

$\Rightarrow 4\lambda=12 \Rightarrow \lambda=3.$

176. (c) We know, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \cos^2\alpha+\cos^2\beta=1-\cos^2\gamma=\sin^2\gamma$

$\therefore$ Statement 1 is correct.

Now, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow 1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma=1$

$\Rightarrow 3-(\sin^2\alpha+\sin^2\beta+\sin^2\gamma)=1 \Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=2.$

$\therefore$ Statement 3 is correct.

177. (b) $\vec{\alpha}=\hat{i}+2\hat{j}-\hat{k}$

$\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$

$\vec{\gamma}=2\hat{i}+\hat{j}+6\hat{k}$

Let $\vec{\delta}=a\hat{i}+b\hat{j}+c\hat{k}$

Since $\vec{\alpha}$ and $\vec{\delta}$ are perpendicular, $\vec{\alpha}\cdot\vec{\delta}=0$

$\Rightarrow a+2b-c=0$ ....(1)

$\vec{\beta}$ and $\vec{\delta}$ are perpendicular, $\vec{\beta}\cdot\vec{\delta}=0$

$\Rightarrow 2a-b+3c=0$ ....(2)

from (1), (2), $\frac{a}{5}=\frac{b}{-5}=\frac{c}{-5}\Rightarrow\frac{a}{1}=\frac{b}{-1}=\frac{c}{-1}=x(\text{say})$

So, $a=x$, $b=-x$, $c=-x$

Also, it is given $\vec{\gamma}\cdot\vec{\delta}=10.$

$\Rightarrow 2a+b+6c=10$

$\Rightarrow 2x-x-6x=10$

$\Rightarrow -5x=10$

$\Rightarrow x=-2.$

$\therefore \vec{\delta}=-2\hat{i}+2\hat{j}+2\hat{k}.$

$\therefore |\vec{\delta}| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}.$

178. (a) $\left(\hat{a}+\hat{b}\right)\times\left(\hat{a}\times\hat{b}\right)$

$= \hat{a}\times\left(\hat{a}\times\hat{b}\right)+\hat{b}\times\left(\hat{a}\times\hat{b}\right)$

$= \left(\hat{a}\cdot\hat{b}\right)\hat{a} - (\hat{a}\cdot\hat{a})\hat{b} + \left(\hat{b}\cdot\hat{b}\right)\hat{a} - \left(\hat{b}\cdot\hat{a}\right)\hat{b}$

$= \lambda\hat{a} - \hat{b} + \hat{a} - \lambda\hat{b}$

$= \lambda\left(\hat{a}-\hat{b}\right)+1\left(\hat{a}-\hat{b}\right).$

$= (\lambda+1)\left(\hat{a}-\hat{b}\right)$

So, it is parallel to $\hat{a}-\hat{b}$.

179. (c) $\vec{F} = \hat{i}+3\hat{j}+2\hat{k}$

$\overline{AB} = (3-1)\hat{i}+(-1-2)\hat{j}+(5-(-3))\hat{k}$

$= 2\hat{i}-3\hat{j}+8\hat{k}$

Work done $= \vec{F}\cdot\overline{AB} = \left(\hat{i}+3\hat{j}+2\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+8\hat{k}\right)$

$= (1\times 2)+(3\times(-3))+(2\times 8)$

$= 2-9+16 = 9$ units.

180. (b) Let $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$

$\left|\vec{a}\times\hat{i}\right|^2 = y^2+z^2$

$\left|\vec{a}\times\hat{j}\right|^2 = x^2+z^2$

$\left|\vec{a}\times\hat{k}\right|^2 = x^2+y^2$

$\therefore \left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2$

$= y^2+z^2+x^2+z^2+x^2+y^2$

$= 2x^2+2y^2+2z^2$

$= 2(x^2+y^2+z^2)$

$= 2|\vec{a}|^2$

181. (b) $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar.

i.e., $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$c_2 \to c_2 - c_3;\ c_3 \to c_3 - c_1$

$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$

$\Rightarrow a(b-1)(c-1)-(1-a)(c-1)-(1-a)(b-1) = 0$

Divide both sides by $(1-a)(1-b)(1-c)$

$$\Rightarrow \frac{a(b-1)(c-1)}{(1-a)(1-b)(1-c)} - \frac{(1-a)(c-1)}{(1-a)(1-b)(1-c)} - \frac{(1-a)(b-1)}{(1-a)(1-b)(1-c)} = \frac{0}{(1-a)(1-b)(1-c)}$$

$\Rightarrow \frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c} = 0$

$\Rightarrow \frac{1}{1-b}+\frac{1}{1-c} = -\frac{a}{1-a}.$

Add $\frac{1}{1-a}$ on both sides.

$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c} = \frac{1}{1-a}-\frac{a}{1-a} = \frac{1-a}{1-a} = 1.$

182. (a) $|\vec{a}| = 2, |\vec{b}| = 7.$

$\vec{a}\times\vec{b} = 3\hat{i}+2\hat{j}+6\hat{k}$

We know, $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta.\hat{n}$ where $\hat{n}$ is unit vector.

$\Rightarrow |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow \left|3\hat{i}+2\hat{j}+6\hat{k}\right| = (2)(7)\sin\theta$

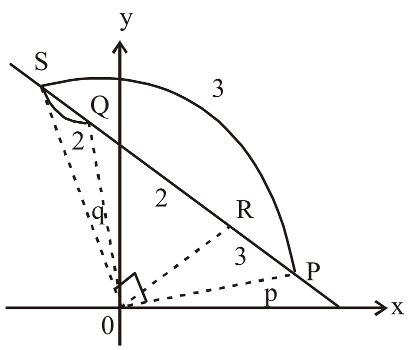
$\Rightarrow \sqrt{9+4+36} = (2)(7)\sin\theta.$

$\Rightarrow \pm 7 = (2)(7)\sin\theta.$

$\therefore \sin\theta = \pm\frac{1}{2}.$

sin θ is acute angle, $\theta = 30°$

183. (a) R divides PQ internally in ratio 2:3



$\therefore \overline{OR} = \frac{2\vec{q}+3\vec{p}}{5}$ ....(1)

S divides PQ externally in ratio 2:3

$\overline{OS} = \frac{2\vec{q}-3\vec{p}}{2-3} = 3\vec{p}-2\vec{q}$ ....(2)

Given, $\overline{OR}$ and $\overline{OS}$ are perpendicular.

$\therefore \left(\frac{3\vec{p}+2\vec{q}}{5}\right)(3\vec{p}-2\vec{q})=0$

$\Rightarrow 9p^2-4q^2=0 \Rightarrow 9p^2=4q^2$

184. (a) $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})-(\hat{i}+2\hat{j}-\hat{k})$

$=(\hat{i}-3\hat{j}+4\hat{k})$

Moment $(\tau)=\vec{r}\times\vec{f}$

$=(\hat{i}-3\hat{j}+4\hat{k})\times(3\hat{i}+\hat{k})$

$=-3\hat{i}+11\hat{j}+9\hat{k}$

185. (d) $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ ....(1)

$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\lambda(\vec{b}\times\vec{c})$ ....(2)

$(1)\times\vec{b}\Rightarrow\vec{b}\times\vec{a}+2\vec{b}\times\vec{b}+3\vec{b}\times\vec{c}=0$

$\Rightarrow-\vec{a}\times\vec{b}+0+3\vec{b}\times\vec{c}=0$

$\Rightarrow\vec{a}\times\vec{b}=3\vec{b}\times\vec{c}$ ....(3)

$(1)\times\vec{c}\Rightarrow\vec{c}\times\vec{a}+2\vec{c}\times\vec{b}+3\vec{c}\times\vec{c}=0$

$\Rightarrow\vec{c}\times\vec{a}-2\vec{b}\times\vec{c}+0=0$

$\Rightarrow\vec{c}\times\vec{a}=2\vec{b}\times\vec{c}$ ....(4)

$\therefore(2)\Rightarrow\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\lambda(\vec{b}\times\vec{c})$

$\Rightarrow 3\vec{b}\times\vec{c}+\vec{b}\times\vec{c}+2(\vec{b}\times\vec{c})=\lambda(\vec{b}\times\vec{c})$

(from (3), (4))

$\Rightarrow 6(\vec{b}\times\vec{c})=\lambda(\vec{b}\times\vec{c})$

$\therefore\ \lambda=6$

186. (b) Given, $\vec{k}$ and $\vec{A}$ are parallel vectors.

we know, cross product of parallel vectors is $\vec{O}$.

187. (a) $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = a^2+2\vec{a}.\vec{b}+b^2 = a^2+b^2$

$\Rightarrow$ $\vec{a}.\vec{b}=0\Rightarrow\vec{a}\perp\vec{b}$

188. (d) $(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}+\hat{k})=x+y+z$

189. (a) $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -4 & -1\end{vmatrix}$

$=\hat{i}(1+4)-\hat{j}(-2-3)+\hat{k}(-8+3)$

$=5\hat{i}+5\hat{j}-5\hat{k}$

Unit vector $=\frac{5}{5\sqrt{3}}\hat{i}+\frac{5}{5\sqrt{3}}\hat{j}-\frac{5}{5\sqrt{3}}\hat{k}$

$=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$

190. (d) $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\{|\vec{a}|^2+|\vec{b}|^2\}=2\times25=50$

$\Rightarrow$ $|\vec{a}+\vec{b}|^2+25=50$

$\Rightarrow$ $|\vec{a}+\vec{b}|^2=25$

$\Rightarrow$ $|\vec{a}+\vec{b}|=5$

191. (c) For simplicity let us take $\vec{a},\vec{b},\vec{c}$ as $\hat{i},\hat{j},\hat{k}$

Now magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ will be $\sqrt{3}$.

192. (c) $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$

$=\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$=0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0$

$=2(\vec{a}\times\vec{b})$

193. (c) $\vec{\tau}=\vec{r}\times\vec{F}=(\hat{i}+2\hat{j}+3\hat{k})\times\lambda\hat{k}$

$=2\lambda\hat{i}-\lambda\hat{j}$

$\Rightarrow$ $|\vec{\tau}|=\sqrt{5}\lambda$

194. (a) From triangle law of vector addition

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

Only statement (1) is correct.

195. (b) $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$

$\vec{b}=2\hat{i}+\hat{j}-3\hat{k}$

$\vec{b}-\vec{a}=2\hat{i}+\hat{j}-3\hat{k}-(\hat{i}-2\hat{j}+5\hat{k})$

$=\hat{i}(2-1)+\hat{j}(1+2)+\hat{k}(-3-5)=\hat{i}+3\hat{j}-8\hat{k}$

$3\vec{a}+\vec{b}=3(\hat{i}-2\hat{j}+5\hat{k})+(2\hat{i}+\hat{j}-3\hat{k})$

$=3\hat{i}-6\hat{j}+15\hat{k}+2\hat{i}+\hat{j}-3\hat{k}$

$=\hat{i}(3+2)+\hat{j}(-6+1)+\hat{k}(15-3)$

$=5\hat{i}-5\hat{j}+12\hat{k}$

$(\vec{b}-\vec{a}).(3\vec{a}+\vec{b})=(\hat{i}+3\hat{j}-8\hat{k}).(5\hat{i}-5\hat{j}+12\hat{k})$

$=(1)(5)+(3)(-5)+(-8)(12)$

$=5-15-96=-106$

196. (d) Given, $\overrightarrow{OA}=3\hat{i}-2\hat{j}+\hat{k}$

$\overrightarrow{OB}=2\hat{i}+4\hat{j}-3\hat{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

$=(2\hat{i}+4\hat{j}-3\hat{k})-(3\hat{i}-2\hat{j}+\hat{k})$

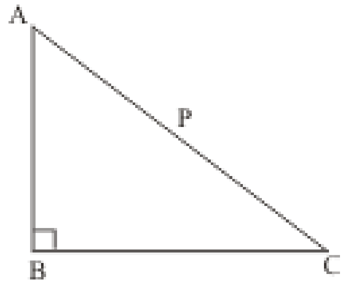
$=\hat{i}(2-3)+\hat{j}(4+2)+\hat{k}(-3-1)$

$=-\hat{i}+6\hat{j}-4\hat{k}$

Length of AB $=\sqrt{(-1)^2+(6)^2+(-4)^2}$

$=\sqrt{1+36+16}=\sqrt{53}$

197. (a) Hypotenuse, AC = P
BC is perpendicular to AB.



$\therefore\ \overrightarrow{BC}\cdot\overrightarrow{BA}=0$

$\therefore\ \overrightarrow{AB}\cdot\overrightarrow{AC}+\overrightarrow{BC}\cdot\overrightarrow{BA}+\overrightarrow{CA}\cdot\overrightarrow{CB}$

$=\overrightarrow{AB}\cdot\overrightarrow{AC}+0+\overrightarrow{AC}.\overrightarrow{BC}=\overrightarrow{AC}\left(\overrightarrow{AB}+\overrightarrow{BC}\right)$

$=\overrightarrow{AC}\cdot\overrightarrow{AC}=\overrightarrow{AC}^2=P^2$

198. (b) Given, $\vec{a}=2\hat{i}-6\hat{j}-3\hat{k}$

$\vec{b}=4\hat{i}+3\hat{j}-\hat{k}$

$\vec{a}.\vec{b}=(2\hat{i}-6\hat{j}-3\hat{k}).(4\hat{i}+3\hat{j}-\hat{k})$

$=(2)(4)+(-6)(3)+(-3)(-1)$
$=8-18+3=-7$

$\Rightarrow|\vec{a}||\vec{b}|\cos\theta=-7\Rightarrow\cos\theta=\dfrac{-7}{|\vec{a}||\vec{b}|}$

$=\dfrac{-7}{\sqrt{49}\sqrt{26}}=\dfrac{-1}{\sqrt{26}}$

$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\left(\dfrac{-1}{\sqrt{26}}\right)^2}=\sqrt{1-\dfrac{1}{26}}$

$=\sqrt{\dfrac{25}{26}}=\dfrac{5}{\sqrt{26}}$

199. (d) Given vectors are $3\hat{i}+4\hat{j}-\hat{k}$ and $-2\hat{i}+\lambda\hat{j}+10\hat{k}$.
If these are perpendicular, dot product is 0.

$(3\hat{i}+4\hat{j}-\hat{k}).(-2\hat{i}+\lambda\hat{j}+10\hat{k})=0$
$\Rightarrow(3)(-2)+(4)(\lambda)+(-10)=0$
$\Rightarrow-6+4\lambda-10=0$
$\Rightarrow4\lambda=16$
$\Rightarrow\lambda=4$

3-D Geometry 23

1. Consider the points (a – 1, a, a + 1), (a, a + 1, a – 1) and (a + 1, a – 1, a).
   1. These points always form the vertices of an equilateral triangle for any real value of a.
   2. The area of the triangle formed by these points is independent of a.

   Which of the statement(s) given above is/are correct ?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2 *[2006-I]*
2. What are coordinates of the point equidistant from the points (a, 0, 0), (0, a, 0), (0, 0, a) and (0, 0, 0) ?
   (a) $\left(\frac{a}{3},\frac{a}{3},\frac{a}{3}\right)$ (b) $\left(\frac{a}{2},\frac{a}{2},\frac{a}{2}\right)$
   (c) (a, a, a) (d) (2a, 2a, 2a) *[2006-I]*
3. A line makes 45° with positive x-axis and makes equal angles with positive y, z axes, respectively. What is the sum of the three angles which the line makes with positive x, y and z axes ?
   (a) 180° (b) 165°
   (c) 150° (d) 135° *[2006-I]*
4. What is the angle between the two lines whose direction numbers are ($\sqrt{3}-1, -\sqrt{3}-1$, 4) and ($-\sqrt{3}-1, \sqrt{3}-1$, 4) ?
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
   (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ *[2006-I]*
5. Consider the following statements:
   1. Equations ax + by + cz + d = 0, a'x + b'y + c'z + d' = 0 represent a straight line.
   2. Equation of the form
   $$\frac{x-\alpha}{\ell}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

   represent a straight line passing through the point (α, β, γ) and having direction ratio proportional to *l*, m, n.
   Which of the statements given above is/are correct ?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2
   [2006-II]
6. If the centre of the sphere $ax^2 + by^2 + cz^2 - 2x + 4y + 2z - 3 = 0$ is (1/2, – 1, – 1/2), what is the value of b ?
   (a) 1 (b) – 1
   (c) 2 (d) – 2 *[2006-II]*
7. What is the length of the perpendicular from the origin to the plane $ax + by + \sqrt{2ab}\,z = 1$?
   (a) 1/(ab) (b) 1/(a + b)
   (c) a + b (d) ab *[2006-II]*
8. If the direction ratios of the normal to a plane are < *l*, m, n > and the length of the normal is p, then what is the sum of intercepts cut-off by the plane from the coordinate axes ?
   (a) $p\left(\frac{1}{\ell}+\frac{1}{m}+\frac{1}{n}\right)$
   (b) $p\sqrt{(\ell^2+m^2+n^2)}$
   (c) $p\sqrt{(\ell^2+m^2+n^2)}\left(\frac{1}{\ell}+\frac{1}{m}+\frac{1}{n}\right)$
   (d) $\frac{p}{\sqrt{(\ell^2+m^2+n^2)}}\left(\frac{1}{\ell}+\frac{1}{m}+\frac{1}{n}\right)$ *[2006-II]*
9. How many arbitrary constants are there in the equation of a plane ?
   (a) 2 (b) 3
   (c) 4 (d) Any finite number
   [2006-II]
10. If P, Q are (2, 5, –7), (–3, 2, 1) respectively, then what are the direction ratios of the line PQ?
    (a) < 10, 6, –16 > (b) < 5, 3, 8 >
    (c) < – 5, – 3, – 8 > (d) None of these
    [2006-II]
11. If O, P are the points (0, 0, 0), (2, 3, –1) respectively, then what is the equation to the plane through P at right angles to OP ?
    (a) 2x + 3y + z = 16 (b) 2x + 3y – z = 14
    (c) 2x + 3y + z = 14 (d) 2x + 3y – z = 0
    [2006-II]

12. The four points (0, 4, 1), (2, 3, –1), (4, 5, 0) (2, 6, 2) are the vertices of which one of the following figures?
(a) Rhombus (b) Rectangle
(c) Square (d) Parallelogram *[2006-II]*

13. If the sum of the squares of the distance of the point (x, y, z) from the points (a, 0, 0) and (–a , 0, 0) is $2c^2$, then which one of the following is correct?
(a) $x^2 + a^2 = 2c^2 - y^2 - z^2$ (b) $x^2 + a^2 = c^2 - y^2 - z^2$
(c) $x^2 - a^2 = c^2 - y^2 - z^2$ (d) $x^2 + a^2 = c^2 + y^2 + z^2$
[2007-I]

14. Which one of following is correct?
The three planes $2x + 3y - z - 2 = 0$, $3x + 3y + z - 4 = 0$, $x - y + 2z - 5 = 0$ intersect
(a) at a point (b) at two points
(c) at three points (d) in a line *[2007-I]*

15. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*
Assertion(A): If $\langle l, m, n \rangle$ are direction cosines of a line, there can be a line whose direction cosines are
$$\left\langle \sqrt{\frac{l^2+m^2}{2}}, \sqrt{\frac{m^2+n^2}{2}}, \sqrt{\frac{n^2+l^2}{2}} \right\rangle.$$
Reason(R): The sum of direction cosines of a line is unity.
(a) Both A and R individually true, and R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true. *[2007-I]*

16. Which one of the following is the plane containing the line $\frac{x-2}{2} = \frac{y-3}{3} = \frac{z-4}{5}$ and parallel to z-axis?
(a) $2x - 3y = 0$ (b) $5x - 2z = 0$
(c) $5y - 3z = 0$ (d) $3x - 2y = 0$ *[2007-I]*

17. What is the centre of the sphere $ax^2 + by^2 + cz^2 - 6x = 0$ if the radius is 1 unit?
(a) (0, 0, 0)
(b) (1, 0, 0)
(c) (3, 0, 0)
(d) cannot be determined as values of a,b, c are unknown
[2007-I]

18. Under what condition do $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{2}, K \right\rangle$ represent direction cosines of a line?
(a) $k = \frac{1}{2}$ (b) $k = -\frac{1}{2}$
(c) $k = \pm\frac{1}{2}$ (d) k can take any value
[2007-I]

19. If $x = a \sec\theta \cos\phi$, $y = b \sec\theta \sin\phi$, $z = c \tan\theta$, then what is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ equal to?
(a) 1 (b) 0
(c) – 1 (d) $a^2 + b^2$ *[2007-II]*

20. A line makes angles θ, ϕ and ψ with x, y, z axes respectively. Consider the following
1. $\sin^2\theta + \sin^2\phi = \cos^2\psi$
2. $\cos^2\theta + \cos^2\phi = \sin^2\psi$
3. $\sin^2\theta + \cos^2\phi = \cos^2\psi$
Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) 3 only (d) 2 and 3 *[2007-II]*

21. What is the equation of the plane passing through (x_1, y_1, z_1) and normal to the line with $\langle a, b, c \rangle$ as direction ratios ?
(a) $ax + by + cz = ax_1 + by_1 + cz_1$
(b) $a(x + x_1) + b(y + y_1) + c(z + z_1) = 0$
(c) $ax + by + cz = 0$
(d) $ax + by + cz = x_1 + y_1 + z_1 = 0$ *[2007-II]*

22. What are the direction cosines of the line represented by $3x + y + 2z = 7$, $x + 2y + 3z = 5$?
(a) (– 1, – 7, 5) (b) (– 1, 7, 5)
(c) $\left(-\frac{1}{\sqrt{75}}, -\frac{7}{\sqrt{75}}, \frac{5}{\sqrt{75}}\right)$ (d) $\left(-\frac{1}{\sqrt{75}}, \frac{7}{\sqrt{75}}, \frac{5}{\sqrt{75}}\right)$
[2007-II]

23. The equation of a sphere is $x^2 + y^2 + z^2 - 10z = 0$. If one end point of a diameter of the sphere is (– 3, – 4, 5), what is the other end point ?
(a) (– 3, – 4, – 5) (b) (3, 4, 5)
(c) (3, 4, – 5) (d) (– 3, 4, – 5) *[2007-II]*

24. O (0, 0), A (0, 3), B (4, 0) are the vertices of triangle OAB. A force $10\hat{i}$ acts at B. What is the magnitude of moment of force about the vertex A?
(a) 0 (b) 30 unit
(c) 40 unit (d) 50 unit *[2007-II]*

25. What is the ratio in which the line joining the points (2,4, 5) and (3, 5, – 4) is internally divided by the xy-plane?
(a) 5 : 4 (b) 3 : 4
(c) 1 : 2 (d) 7 : 5 *[2007-II]*

26. Under which one of the following conditions will the two planes $x + y + z = 7$ and $\alpha x + \beta y + \gamma z = 3$, be parallel (but not coincident) ?
(a) $\alpha = \beta = \gamma = 1$ only (b) $\alpha = \beta = \gamma = \frac{3}{7}$ only
(c) $\alpha = \beta = \gamma$ (d) None of the above
[2008-I]

27. The straight line $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ is parallel to which one of the following ?
(a) $4x+3y-5z=0$ (b) $4x+5y-4z=0$
(c) $4x+4y-5z=0$ (d) $5x+4y-5z=0$ *[2008-I]*

28. If θ is the acute angle between the diagonals of a cube, then which one of the following is correct?
(a) $\theta<30°$ (b) $\theta=60°$
(c) $30°<\theta<60°$ (d) $\theta>60°$ *[2008-I]*

29. Which one of the following planes contains the z-axis?
(a) $x-z=0$ (b) $z+y=0$
(c) $3x+2y=0$ (d) $3x+2z=0$ *[2008-I]*

30. Under what condition are the two lines
$y=\frac{m}{\ell}x+\alpha, z=\frac{n}{\ell}x+\beta;$ and $y=\frac{m'}{\ell'}x+\alpha', z=\frac{n'}{\ell'}x+\beta'$
orthogonal ?
(a) $\alpha\alpha'+\beta\beta'+1=0$
(b) $(\alpha+\alpha')+(\beta+\beta')=0$
(c) $\ell\ell'+mm'+nn'=1$
(d) $\ell\ell'+mm'+nn'=0$ *[2008-I]*

31. What are the coordinates of the point equidistant from the four points (0, 0, 0), (2, 0, 0), (0, 4, 0), (0, 0, 6)? *[2008-II]*
(a) (1, 2, 3) (b) (2, 3, 1)
(c) (3, 1, 2) (d) (1, 3, 2)

32. The angle between the lines with direction ratios $(1, 0, \pm\cos\alpha)$ is 60°. What is the value of α? *[2008-II]*
(a) $\cos^{-1}\left(1/\sqrt{2}\right)$ (b) $\cos^{-1}\left(1/\sqrt{3}\right)$
(c) $\cos^{-1}(1/3)$ (d) $\cos^{-1}(1/2)$

33. The line passing through (1, 2, 3) and having direction ratios given by < 1, 2, 3 > cuts the *x*-axis at a distance *k* from origin. What is the value of *k*? *[2008-II]*
(a) 0 (b) 1
(c) 2 (d) 3

34. The equation $by + cz + d = 0$ represents a plane parallel to which one of the following? *[2008-II]*
(a) *x*-axis (b) *y*-axis
(c) *z*-axis (d) None of these

35. Which one of the following planes is normal to the plane $3x+y+z=5$? *[2008-II]*
(a) $x+2y+z=6$ (b) $x-2y+z=6$
(c) $x+2y-z=6$ (d) $x-2y-z=6$

36. If the radius of the sphere $x^2+y^2+z^2-6x-8y+10z+\lambda=0$ is unity, what is the value of λ? *[2008-II]*
(a) 49 (b) 7
(c) –49 (d) –7

37. Curve of intersection of two spheres is *[2008-II]*
(a) an ellipse (b) a circle
(c) a parabola (d) None of these

38. The points (1, 3, 4), (–1, 6, 10), (–7, 4, 7) and (–5, 1, 1) are the vertices of a *[2009-I]*
(a) rhombus (b) rectangle
(c) parallelogram (d) square

39. What is the number of planes passing through three non-collinear points? *[2009-I]*
(a) 3 (b) 2
(c) 1 (d) 0

40. What is the angle between the lines $x+z=0$, $y=0$ and $20x = 15y = 12z$? *[2009-I]*
(a) $\cos^{-1}(1/5)$ (b) $\cos^{-1}(1/7)$
(c) $\cos^{-1}\frac{45}{7\sqrt{61}}$ (d) $\sin^{-1}(1/7)$

41. Under what condition does the equation $x^2+y^2+z^2+2ux+2uy+2wz+d=0$ represent a real sphere? *[2009-I]*
(a) $u^2+v^2+w^2=d^2$ (b) $u^2+v^2+w^2>d$
(c) $u^2+v^2+w^2<d$ (d) $u^2+v^2+w^2<d^2$

42. What is the angle between the planes $2x-y+z=6$ and $x+y+2z=3$? *[2009-I]*
(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

43. What is the equation of a plane through the *x*-axis and passing through the point (1, 2, 3)? *[2009-I]*
(a) $x+y+z=6$ (b) $x=1$
(c) $y+z=5$ (d) $z+y=1$

44. What is the value of *n* so that the angle between the lines having direction ratios (1, 1, 1) and (1, –1, *n*) is 60°? *[2009-II]*
(a) $\sqrt{3}$ (b) $\sqrt{6}$
(c) 3 (d) None of these

45. The direction cosines of a line are proportional to (2,1,2) and the line intersects a plane perpendicularly at the point (1, –2, 4). What is the distance of the plane from the point (3, 2, 3)? *[2009-II]*
(a) $\sqrt{3}$ (b) 2
(c) $2\sqrt{2}$ (d) 4

46. The foot of the perpendicular drawn from the origin to a plane is the point (1, –3, 1). What is the intercept cut on the *x*-axis by the plane? *[2009-II]*
(a) 1 (b) 3
(c) $\sqrt{11}$ (d) 11

47. A line makes the same angle α with each of the *x* and *y* axes. If the angle θ, which it makes with the *z*-axis, is such that $\sin^2\theta = 2\sin^2\alpha$, then what is the value of α ? *[2009-II]*
(a) $\pi/4$ (b) $\pi/6$
(c) $\pi/3$ (d) $\pi/2$

48. What is the equation of the sphere which has its centre at (6, –1, 2) and touches the plane $2x - y + 2z - 2 = 0$? [2009-II]
(a) $x^2 + y^2 + z^2 + 12x - 2y + 4z + 16 = 0$
(b) $x^2 + y^2 + z^2 + 12x - 2y + 4z - 16 = 0$
(c) $x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$
(d) $x^2 + y^2 + z^2 - 12x + 2y - 4z + 25 = 0$

49. What are the direction ratios of the line determined by the planes $x - y + 2z = 1$ and $x + y - z = 3$? [2009-II]
(a) (–1, 3, 2) (b) (–1, –3, 2)
(c) (2, 1, 3) (d) (2, 3, 2)

50. Under what condition do the planes [2010-I]
$bx - ay = n$, $cy - bz = l$, $az - cx = m$ intersect in a line?
(a) $a + b + c = 0$ (b) $a = b = c$
(c) $al + bm + cn = 0$ (d) $l + m + n = 0$

51. The planes $px + 2y + 2z - 3 = 0$ and $2x - y + z + 2 = 0$ intersect at an angle $\frac{\pi}{4}$. What is the value of p^2?
(a) 24 (b) 12 [2010-I]
(c) 6 (d) 3

DIRECTIONS (Qs. 52-54) : The vertices of a cube are (0, 0, 0), (2, 0, 0), (0, 2, 0), (0, 0, 2), (2, 2, 0), (2, 0, 2), (0, 2, 2), (2, 2, 2) respectively.

52. What is the angle between any two diagonals of the cube?
(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(1/3)$ [2010-I]
(c) $\cos^{-1}(1/\sqrt{3})$ (d) $\cos^{-1}(2/\sqrt{3})$

53. What is the angle between one of the edges of the cube and the diagonal of the cube intersecting the edge of the cube?
(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(1/3)$ [2010-I]
(c) $\cos^{-1}(1/\sqrt{3})$ (d) $\cos^{-1}(2/\sqrt{3})$

54. What is the angle between the diagonal of one of the faces of the cube and the diagonal of the cube intersecting the diagonal of the face of the cube? [2010-I]
(a) $\cos^{-1}(1/\sqrt{3})$ (b) $\cos^{-1}(2/\sqrt{3})$
(c) $\cos^{-1}(\sqrt{2/3})$ (d) $\cos^{-1}(\sqrt{2}/3)$

55. What is the equation of the plane through z-axis and parallel to the line $\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$? [2010-I]
(a) $x \cot\theta + y = 0$ (b) $x \tan\theta - y = 0$
(c) $x + y \cot\theta = 0$ (d) $x - y \tan\theta = 0$

56. If the line through the points A (k, 1, –1) and B (2k, 0, 2) is perpendicular to the line through the points B and C (2 + 2k , k, 1), then what is the value of k? [2010-I]
(a) –1 (b) 1
(c) –3 (d) 3

57. The two planes $ax + by + cz + d = 0$ and $ax + by + cz + d_1 = 0$, where $d \neq d_1$, have [2010-II]
(a) one point only in common
(b) three points in common
(c) infinite points in common
(d) no points in common

58. What is the distance of the origin from the plane $2x + 6y - 3z + 7 = 0$? [2010-II]
(a) 1 (b) 2
(c) 3 (d) 6

59. What is the acute angle between the planes $x + y + 2z = 3$ and $-2x + y - z = 11$? [2011-I]
(a) $\pi/5$ (b) $\pi/4$
(c) $\pi/6$ (d) $\pi/3$

60. What is the radius of the sphere $x^2 + y^2 + z^2 - x - y - z = 0$? [2011-I]
(a) $\sqrt{\frac{3}{4}}$ (b) $\sqrt{\frac{1}{2}}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{1}{3}$

61. Consider the following relations among the angles α, β and γ made by a vector with the coordinate axes
I. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ [2011-I]
II. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$
Which of the above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

62. Which one of the following points lies on the plane $2x + 3y - 6z = 21$? [2011-I]
(a) (3, 2, 2) (b) (3, 7, 1)
(c) (1, 2, 3) (d) (2, 1, – 1)

63. What is the angle between the lines whose direction cosines are proportional to (2, 3, 4) and (1, – 2, 1) respectively? [2011-I]
(a) 90° (b) 60°
(c) 45° (d) 30°

64. What is the locus of points of intersection of a sphere and a plane? [2011-II]
(a) Circle (b) Ellipse
(c) Parabola (d) Hyperbola

65. What is the angle between two planes $2x - y + z = 4$ and $x + y + 2z = 6$? [2011-II]
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

66. What is the equation of the plane passing through the point (1, – 1, – 1) and perpendicular to each of the planes $x - 2y - 8z = 0$ and $2x + 5y - z = 0$? [2011-II]
(a) $7x - 3y + 2z = 14$ (b) $2x + 5y - 3z = 12$
(c) $x - 7y + 3z = 4$ (d) $14x - 5y + 3z = 16$

67. The equation to sphere passing through origin and the points (–1, 0, 0), (0, – 2, 0) and (0, 0, – 3) is $x^2 + y^2 + z^2 + f(x, y, z) = 0$. What is $f(x, y, z)$ equal to? [2011-II]
(a) $-x - 2y - 3z$ (b) $x + 2y + 3z$
(c) $x + 2y + 3z - 1$ (d) $x + 2y + 3z + 1$

68. If a line makes the angles α, β, γ with the axes, then what is the value of 1+ cos 2α + cos 2β + cos 2γ equal to [2012-I]
(a) – 1 (b) 0
(c) 1 (d) 2

69. What are the direction ratios of normal to the plane $2x - y + 2z + 1 = 0$ [2012-I]
(a) $\langle 2, 1, 2 \rangle$ (b) $\left\langle 1, -\frac{1}{2}, 1 \right\rangle$
(c) $\langle 1, -2, 1 \rangle$ (d) None of the above

70. What is the cosine of angle between the planes $x + y + z + 1 = 0$ and $2x - 2y + 2z + 1 = 0$? [2012-I]
(a) 1/2 (b) 1/3
(c) 2/3 (d) None of the above

71. What is the sum of the squares of direction cosines of the line joining the points (1, 2, – 3) and (– 2, 3, 1)? [2012-I]
(a) 0 (b) 1
(c) 3 (d) $\frac{2}{\sqrt{26}}$

72. What is the diameter of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z - 7 = 0$? [2012-I]
(a) 4 units (b) 5 units
(c) 6 units (d) 12 units.

73. If the distance between the points (7, 1, – 3) and (4, 5, λ) is 13 units, then what is one of the values of λ ? [2012-II]
(a) 20 (b) 10
(c) 9 (d) 8

74. If a line OP of length r (where 'O' is the origin) makes an angle α with x-axis and lies in the xz-plane, then what are the coordinates of P ? [2012-II]
(a) (r cos α, 0, r sin α) (b) (0, 0, r sin α)
(c) (r cos α, 0, 0) (d) (0, 0, r cos α)

75. What is the distance of the point (1, 2, 0) from yz-plane is: [2012-II]
(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

76. What are the direction cosines of a line which is equally inclined to the positive directions of the axes ? [2012-II]
(a) $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$ (b) $\left\langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$
(c) $\left\langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$ (d) $\left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$

77. What is the angle between the lines $\frac{x-2}{1} = \frac{y+1}{-2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$? [2012-II]
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) None of the above

78. What is the equation to the plane through (1, 2, 3) parallel to $3x + 4y - 5z = 0$? [2012-II]
(a) $3x + 4y + 5z + 4 = 0$ (b) $3x + 4y - 5z + 14 = 0$
(c) $3x + 4y - 5z + 4 = 0$ (d) $3x + 4y - 5z - 4 = 0$

79. What are the direction ratios of the line of intersection of the planes $x = 3z + 4$ and $y = 2z - 3$? [2012-II]
(a) $\langle 1, 2, 3 \rangle$ (b) $\langle 2, 1, 3 \rangle$
(c) $\langle 3, 2, 1 \rangle$ (d) $\langle 1, 3, 2 \rangle$

80. What is the equation to the straight line passing through (a, b, c) and parallel to z-axis ? [2012-II]
(a) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$ (b) $\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$
(c) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{0}$ (d) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{1}$

81. The sum of the direction cosines of z-axis is [2013-I]
(a) 0 (b) 1/3
(c) 1 (d) 3

82. What is the area of the triangle whose vertices are (0, 0, 0), (1, 2, 3) and (–3, –2, 1)? [2013-I]
(a) $3\sqrt{5}$ square unit (b) $6\sqrt{5}$ square unit
(c) 6 square unit (d) 12 square unit

83. What is the distance between the planes $x - 2y + z - 1 = 0$ and $-3x + 6y - 3z + 2 = 0$? [2013-I]
(a) 3 unit (b) 1 unit
(c) 0 (d) None of the above

84. If a line makes 30° with the positive direction of x-axis, angle β with the positive direction of y-axis and angle γ with the positive direction of z-axis, then what is $\cos^2\beta + \cos^2\gamma$ equal to ? [2013-I]
(a) 1/4 (b) 1/2
(c) 3/4 (d) 1

85. What should be the value of k for which the equation $3x^2 + 3y^2 + (k + 1) z^2 + x - y + z = 0$ represents the sphere? [2013-I]
(a) 3 (b) 2
(c) 1 (d) – 1

86. What is the angle between the planes $2x - y - 2z + 1 = 0$ and $3x - 4y + 5z - 3 = 0$? [2013-I]
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

87. If the straight line $\frac{x - x_0}{\ell} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$ is parallel to the plane $ax + by + cz + d = 0$ then which one of the following is correct? [2013-I]
(a) $\ell + m + n = 0$ (b) $a + b + c = 0$
(c) $\frac{a}{\ell} + \frac{b}{m} + \frac{c}{n} = 0$ (d) $a\ell + bm + cn = 0$

88. If θ is the acute angle between the diagonals of a cube, then which one of the following is correct ? [2013-II]
(a) $\theta = 30°$ (b) $\theta = 45°$
(c) $2\cos\theta = 1$ (d) $3\cos\theta = 1$

89. What is the equation of the sphere with unit radius having centre at the origin ? [2013-II]
(a) $x^2 + y^2 + z^2 = 0$ (b) $x^2 + y^2 + z^2 = 1$
(c) $x^2 + y^2 + z^2 = 2$ (d) $x^2 + y^2 + z^2 = 3$

90. What is the sum of the squares of direction cosines of x-axis ? [2013-II]
(a) 0 (b) $\frac{1}{3}$
(c) 1 (d) 3

91. What is the distance of the line $2x + y + 2z = 3$ from the origin ? [2013-II]
(a) 1 units (b) 1.5 units
(c) 2 units (d) 2.5 units

92. If the projections of a straight line segment on the coordinate axes are 2, 3, 6, then the length of the segment is [2013-II]
(a) 5 units (b) 7 units
(c) 11 units (d) 49 units

DIRECTIONS (Qs. 93-95): *For the next three (03) items that follow*

A straight line passes through (1, –2, 3) and perpendicular to the plane $2x + 3y - z = 7$.

93. What are the direction ratios of normal to plane ? [2014-I]
(a) $\langle 2, 3, -1\rangle$ (b) $\langle 2, 3, 1\rangle$
(c) $\langle -1, 2, 3\rangle$ (d) None of these

94. Where does the line meet the plane ? [2014-I]
(a) (2, 3, –1) (b) (1, 2, 3)
(c) (2, 1, 3) (d) (3, 1, 2)

95. What is the image of the point (1, –2, 3) in the plane ?
(a) (2, –1, 5) (b) (–1, 2, –3) [2014-I]
(c) (5, 4, 1) (d) None of these

DIRECTIONS (Qs. 96-97): *For the next two (02) items that follow*

Consider the spheres $x^2 + y^2 + z^2 - 4y + 3 = 0$ and $x^2 + y^2 + z^2 + 2x + 4z - 4 = 0$.

96. What is the distance between the centres of the two spheres ? [2014-I]
(a) 5 units (b) 4 units
(c) 3 units (d) 2 units

97. Consider the following statements : [2014-I]
1. The two spheres intersect each other.
2. The radius of first sphere is less than that of second sphere.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

98. A line passes through the points (6, –7, –1) and (2, –3, 1). What are the direction ratios of the line ? [2014-I]
(a) $\langle 4, -4, 2\rangle$ (b) $\langle 4, 4, 2\rangle$
(c) $\langle -4, 4, 2\rangle$ (d) $\langle 2, 1, 1\rangle$

DIRECTIONS (Qs. 99-101): *For the next three (03) items that follow*

Consider a sphere passing through the origin and the points (2, 1, –1), (1, 5, –4), (–2, 4, –6).

99. What is the radius of the sphere ? [2014-II]
(a) $\sqrt{12}$ (b) $\sqrt{14}$
(c) 12 (d) 14

100. What is the centre of the sphere ? [2014-II]
(a) (–1, 2, –3) (b) (1, –2, 3)
(c) (1, 2, –3) (d) (–1, –2, –3)

101. Consider the following statements : [2014-II]
1. The sphere passes through the point (0, 4, 0).
2. The point (1, 1, 1) is at a distance of 5 unit from the centre of the sphere.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 102-103): *For the next two (02) items that follow*

The line joining the points (2, 1, 3) and (4, –2, 5) cuts the plane $2x + y - z = 3$.

102. Where does the line cut the plane ? [2014-II]
(a) (0, –4, –1) (b) (0, –4, 1)
(c) (1, 4, 0) (d) (0, 4, 1)

103. What is the ratio in which the plane divideds the line ?
(a) 1 : 1 (b) 2 : 3 [2014-II]
(c) 3 : 4 (d) None of these

DIRECTIONS (Qs. 104-105): *For the next two (02) items that follow*

Consider the plane passing through the points A (2, 2, 1), B (3, 4, 2) and C (7, 0, 6).

104. Which one of the following points lines on the plane ?
(a) (1, 0, 0) (b) (1, 0, 1) [2014-II]
(c) (0, 0, 1) (d) None of these

105. What are the direction ratios of the normal to the plane ?
(a) $\langle 1, 0, 1\rangle$ (b) $\langle 0, 1, 0\rangle$ [2014-II]
(c) $\langle 1, 0, -1\rangle$ (d) None of these

DIRECTIONS (Qs. 106-107): *For the next two (02) items that follow*

The projections of a directed line segment on the coordinate axes are 12, 4, 3 respectively.

106. What is the length of the line segment? [2015-I]
(a) 19 units (b) 17 units
(c) 15 units (d) 13 units

107. What are the direction cosines of the line segment?
(a) $\left\langle \frac{12}{13}, \frac{4}{13}, \frac{3}{13} \right\rangle$ (b) $\left\langle \frac{12}{13}, -\frac{4}{13}, \frac{3}{13} \right\rangle$ [2015-I]
(c) $\left\langle \frac{12}{13}, -\frac{4}{13}, -\frac{3}{13} \right\rangle$ (d) $\left\langle -\frac{12}{13}, -\frac{4}{13}, \frac{3}{13} \right\rangle$

DIRECTIONS (Qs. 108-109): *For the next two (02) items that follow*

From the point $P(3, -1, 11)$, a perpendicular is drawn on the line L given by the equation $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Let Q be the foot of the perpendicular.

108. What are the direction ratios of the line segment PQ? *[2015-I]*
(a) $\langle 1,6,4 \rangle$ (b) $\langle -1,6,-4 \rangle$
(c) $\langle -1,-6,4 \rangle$ (d) $\langle 2,-6,4 \rangle$

109. What is the length of the line segment PQ? *[2015-I]*
(a) $\sqrt{47}$ units (b) 7 units
(c) $\sqrt{53}$ units (d) 8 units

DIRECTIONS (Qs. 110-111): *For the next two (02) items that follow*

A triangular plane ABC with centroid (1, 2, 3) cuts the coordinate axes at A, B, C respectively.

110. What are the intercepts made by the plane ABC on the axes? *[2015-I]*
(a) 3, 6, 9 (b) 1, 2, 3
(c) 1, 4, 9 (d) 2, 4, 6

111. What is the equation of plane ABC? *[2015-I]*
(a) $x + 2y + 3z = 1$ (b) $3x + 2y + z = 3$
(c) $2x + 3y + 6z = 18$ (d) $6x + 3y + 2z = 18$

DIRECTIONS (Qs. 112-113): *For the next two (02) items that follow*

A point $P(1, 2, 3)$ is one vertex of a cuboid formed by the coordinate planes and the planes passing through P and parallel to the coordinate planes.

112. What is the length of one of the diagonals of the cuboid? *[2015-I]*
(a) $\sqrt{10}$ units (b) $\sqrt{14}$ units
(c) 4 units (d) 5 units

113. What is the equation of the plane passing through $P(1, 2, 3)$ and parallel to xy–plane? *[2015-I]*
(a) $x + y = 3$ (b) $x - y = -1$
(c) $z = 3$ (d) $x + 2y + 3z = 14$

114. The radius of the sphere *[2015-II]*
$3x^2 + 3y^2 + 3z^2 - 8x + 4y + 8z - 15 = 0$ is
(a) 2 (b) 3
(c) 4 (d) 5

115. The direction ratios of the line perpendicular to the lines with direction ratios $< 1, -2, -2 >$ and $< 0, 2, 1 >$ are *[2015-II]*
(a) $< 2, -1, 2 >$ (b) $< -2, 1, 2 >$
(c) $< 2, 1, -2 >$ (d) $< -2, -1, -2 >$

116. What are the co-ordinates of the foot of the perpendicular drawn from the point (3, 5, 4) on the plane z = 0? *[2015-II]*
(a) (0, 5, 4) (b) (3, 5, 0)
(c) (3, 0, 4) (d) (0, 0, 4)

117. The lengths of the intercepts on the co-ordinate axes made by the plane $5x + 2y + z - 13 = 0$ are *[2015-II]*
(a) 5, 2, 1 unit (b) $\frac{13}{5}, \frac{13}{2}$, 13 unit
(c) $\frac{5}{13}, \frac{2}{13}, \frac{1}{13}$ unit (d) 1, 2, 5 unit

DIRECTIONS (Qs. 118-120): *For the next three (03) items that follow*

A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point (1, 0, 1).

118. What are the direction ratios of the line of intersection of the given planes? *[2016-I]*
(a) $\langle 2,-5,-3 \rangle$ (b) $\langle 1,-5,-3 \rangle$
(c) $\langle 2,5,3 \rangle$ (d) $\langle 1, 3, 5 \rangle$

119. What is the equation of the plane P? *[2016-I]*
(a) $2x + 5y - 2 = 0$ (b) $5x + 2y - 5 = 0$
(c) $x + z - 2 = 0$ (d) $2x - y - 2z = 0$

120. If the plane P touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is r equal to? *[2016-I]*
(a) $\frac{2}{\sqrt{29}}$ (b) $\frac{4}{\sqrt{29}}$
(c) $\frac{5}{\sqrt{29}}$ (d) 1

DIRECTIONS (Qs. 121-122): *For the next two (02) items that follow*

Let Q be the image of the point P (–2, 1, –5) in the plane $3x - 2y + 2z + 1 = 0$.

121. Consider the following: *[2016-II]*
1. The coordinates of Q are (4, –3, –1).
2. PQ is of length more than 8 units.
3. The point (1, –1, –3) is the mid-point of the line segment PQ and lies on the given plane.

Which of the above statements are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

122. Consider the following: *[2016-II]*
1. The direction ratios of the line segment PQ are <3, –2, 2 >.
2. The sum of the squares of direction cosines of the line segment PQ is unity.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS (Qs. 123-124): *For the next two (02) items that follow*

A line L passes through the point P(5, – 6, 7) and is parallel to the planes $x + y + z = 1$ and $2x - y - 2z = 3$.

123. What are the direction ratios of the line of intersection of the given planes? *[2016-II]*
(a) $< 1, 4, 3 >$ (b) $< -1, -4, 3>$
(c) $<1, -4, 3>$ (d) $<1, -4, -3>$

124. What is the equation of the line L ? *[2016-II]*
(a) $\frac{x-5}{-1} = \frac{y+6}{4} = \frac{z-7}{-3}$
(b) $\frac{x+5}{-1} = \frac{y-6}{4} = \frac{z+7}{-3}$
(c) $\frac{x-5}{-1} = \frac{y+6}{-4} = \frac{z-7}{3}$
(d) $\frac{x-5}{-1} = \frac{y+6}{-4} = \frac{z-7}{-3}$

125. A straight line with direction cosines (0, 1, 0) is *[2017-I]*
(a) parallel to x-axis
(b) parallel to y-axis
(c) parallel to z-axis
(d) equally inclined to all the axes

126. (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c) are four distinct points. What are the coordinates of the point which is equidistant from the four points? *[2017-I]*
(a) $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right)$
(b) (a, b, c)
(c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
(d) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

127. The points P(3, 2, 4), Q(4, 5, 2), R(5, 8, 0) and S(2, –1, 6) are
(a) vertices of a rhombus which is not a square *[2017-I]*
(b) non-coplanar
(c) collinear
(d) coplanar but not collinear

128. The line passing through the points (1, 2, –1) and (3, –1, 2) meets the yz-plane at which one of the following points? *[2017-I]*
(a) $\left(0, -\frac{7}{2}, \frac{5}{2}\right)$ (b) $\left(0, \frac{7}{2}, \frac{1}{2}\right)$
(c) $\left(0, -\frac{7}{2}, -\frac{5}{2}\right)$ (d) $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

129. Under which one of the following conditions are the lines $x = ay + b; z = cy + d$ and $x = ey + f; z = gy + h$ perpendicular? *[2017-I]*
(a) $ae + cg - 1 = 0$ (b) $ae + bf - 1 = 0$
(c) $ae + cg + 1 = 0$ (d) $ag + ce + 1 = 0$

130. The length of the normal from origin to the plane $x + 2y - 2z = 9$ is equal to *[2017-II]*
(a) 2 units (b) 3 units
(c) 4 units (d) 5 units

131. The point of intersection of the line joining the points (–3, 4, –8) and (5, – 6, 4) with the XY-plane is *[2017-II]*
(a) $\left(\frac{7}{3}, -\frac{8}{3}, 0\right)$ (b) $\left(-\frac{7}{3}, -\frac{8}{3}, 0\right)$
(c) $\left(-\frac{7}{3}, \frac{8}{3}, 0\right)$ (d) $\left(\frac{7}{3}, \frac{8}{3}, 0\right)$

132. If the angle between the lines whose direction ratios are (2, –1, 2) and (x, 3, 5) is $\frac{\pi}{4}$, then the smaller value of x is *[2017-II]*
(a) 52 (b) 4
(c) 2 (d) 1

133. A variable plane passes through a fixed point (a, b, c) and cuts the axes in A, B and C respectively. The locus of the centre of the sphere OABC, O being the origin, is *[2017-II]*
(a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (b) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$
(c) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$ (d) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$

134. The equation of the plane passing through the line of intersection of the planes $x + y + z = 1$, $2x + 3y + 4z = 7$, and perpendicular to the plane $x - 5y + 3z = 5$ is given by *[2017-II]*
(a) $x + 2y + 3z - 6 = 0$ (b) $x + 2y + 3z + 6 = 0$
(c) $3x + 4y + 5z - 8 = 0$ (d) $3x + 4y + 5z + 8 = 0$

135. Let the coordinates of the points A, B, C be (1, 8, 4), (0, –11, 4) and (2, –3, 1) respectively. What are the coordinates of the point D which is the foot of the perpendicular from A on BC? *[2018-I]*
(a) (3, 4, –2) (b) (4, –2, 5)
(c) (4, 5, –2) (d) (2, 4, 5)

136. What is the equation of the plane passing through the points (–2, 6, –6), (–3, 10, –9) and (–5, 0, –6)? *[2018-I]*
(a) $2x - y - 2z = 2$ (b) $2x + y + 3z = 3$
(c) $x + y + z = 6$ (d) $x - y - z = 3$

137. A sphere of constant radius r through the origin intersects the coordinate axes in A, B and C. What is the locus of the centroid of the triangle ABC? *[2018-I]*
(a) $x^2 + y^2 + z^2 = r^2$
(b) $x^2 + y^2 + z^2 = 4r^2$
(c) $9(x^2 + y^2 + z^2) = 4r^2$
(d) $3(x^2 + y^2 + z^2) = 2r^2$

138. The coordinates of the vertices P, Q and R of a triangle PQR are (1, –1, 1), (3, –2, 2) and (0, 2, 6) respectively. If $\angle RQP = \theta$, then what is $\angle PRQ$ equal to? *[2018-I]*
(a) $30° + \theta$ (b) $45° - \theta$
(c) $60° - \theta$ (d) $90° - \theta$

139. What is the equation to the sphere whose centre is at (–2, 3, 4) and radius is 6 units? *[2018-I]*
(a) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$
(b) $x^2 + y^2 + z^2 + 6x - 4y - 8z = 7$
(c) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 4$
(d) $x^2 + y^2 + z^2 + 4x + 6y + 8z = 4$

140. What is the distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = ?$ *[2018-II]*
(a) 1 unit (b) 2 unit
(c) 3 unit (d) 4 units

141. Coordinates of the points O, P, Q and R are respectively (0, 0, 0), (4, 6, 2m), (2, 0, 2n) and (2, 4, 6). Let L, M, N and K be points on the sides OR, OP, PQ and QR respectively such that LMNK is a parallelogram whose two adjacent sides side LK are each of length $\sqrt{2}$. What are the values of m and n respectively? *[2018-II]*

(a) 6, 2 (b) 1, 3
(c) 3, 1 (d) None of the above

142. The line $\frac{x-1}{2}-\frac{y-2}{3}=\frac{z-3}{3}$ is given by *[2018-II]*
(a) $x+y+z=6, x+2y-3z=-4$
(b) $x+2y-2z=-1, 4x+4y-5z-3=0$
(c) $3x+2y-3z=0, 3x-6y+3z=-2$
(d) $3x+2y-3z=-2, 3x-6y+3z=0$

143. Consider the following statements: *[2018-II]*
1. The angle between the planes
$2x-y+z=1$ and $x+y+2z=3$ is $\frac{\pi}{3}$.
2. The distance between the planes
$6x-3y+6z+2=0$ and $2x-y+2z+4=0$ is $\frac{10}{9}$
Which of the above statements is/are correct
(a) 1 only (b) 2 only
(c) Both and 2 (d) Neither 1 nor 2

144. What is the radius of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$? *[2019-I]*
(a) 5 (b) 2
(c) 7 (d) 3

145. The equation of the plane passing through the intersection of the planes $2x+y+2z=9$, $4x-5y-4z=1$ and the point (3, 2, 1) is *[2019-I]*
(a) $10x-2y+2z=28$ (b) $10x+2y+2z=28$
(c) $10x+2y-2z=28$ (d) $10x-2y-2z=24$

146. The distance between the parallel planes $4x-2y+4z+9=0$ and $8x-4y+8z+21=0$ is *[2019-I]*
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{7}{4}$

147. What are the direction cosines of z-axis ? *[2019-I]*
(a) $<1, 1, 1>$ (b) $<1, 0, 0>$
(c) $<0, 1, 0>$ (d) $<0, 0, 1>$

ANSWER KEY

1	(c)	16	(d)	31	(a)	46	(d)	61	(a)	76	(a)	91	(a)	106	(d)	121	(d)	136	(a)
2	(b)	17	(d)	32	(b)	47	(a)	62	(b)	77	(a)	92	(b)	107	(a)	122	(c)	137	(c)
3	(b)	18	(c)	33	(a)	48	(c)	63	(a)	78	(c)	93	(a)	108	(b)	123	(c)	138	(d)
4	(c)	19	(a)	34	(a)	49	(a)	64	(a)	79	(c)	94	(d)	109	(c)	124	(a)	139	(a)
5	(c)	20	(b)	35	(d)	50	(c)	65	(b)	80	(b)	95	(c)	110	(a)	125	(b)	140	(a)
6	(c)	21	(a)	36	(a)	51	(a)	66	(d)	81	(c)	96	(c)	111	(d)	126	(c)	141	(c)
7	(b)	22	(c)	37	(b)	52	(b)	67	(b)	82	(a)	97	(c)	112	(b)	127	(c)	142	(d)
8	(a)	23	(b)	38	(a)	53	(d)	68	(b)	83	(d)	98	(c)	113	(c)	128	(d)	143	(c)
9	(c)	24	(b)	39	(c)	54	(c)	69	(b)	84	(a)	99	(b)	114	(b)	129	(c)	144	(c)
10	(d)	25	(a)	40	(a)	55	(b)	70	(b)	85	(b)	100	(a)	115	(a)	130	(b)	145	(a)
11	(b)	26	(c)	41	(b)	56	(d)	71	(b)	86	(d)	101	(a)	116	(b)	131	(a)	146	(a)
12	(c)	27	(c)	42	(b)	57	(d)	72	(d)	87	(d)	102	(d)	117	(b)	132	(b)	147	(d)
13	(b)	28	(d)	43	(b)	58	(a)	73	(c)	88	(d)	103	(d)	118	(a)	133	(c)		
14	(d)	29	(c)	44	(b)	59	(d)	74	(a)	89	(b)	104	(a)	119	(b)	134	(a)		
15	(c)	30	(d)	45	(b)	60	(a)	75	(a)	90	(c)	105	(c)	120	(c)	135	(c)		

HINTS & SOLUTIONS

1. (c) Let A(a –1, a, a + 1), B (a, a + 1, a – 1) and C (a + 1, a – 1, a) be the vertices of a triangle ABC.
Length of

$AB = \sqrt{(a-a+1)^2 + (a+1-a)^2 + (a-1-a-1)^2}$

$= \sqrt{1+1+4} = \sqrt{6}$

Length of

$BC = \sqrt{(a+1-a)^2 + (a-1-a-1)^2 + (a-a+1)^2}$

$= \sqrt{1+4+1} = \sqrt{6}$

and $CA = \sqrt{(a-1-a-1)^2 + (a-a+1)^2 + (a+1-a)^2}$

$= \sqrt{4+1+1} = \sqrt{6}$

$\therefore$ AB = BC = CA

$\therefore$ Triangle ABC is an equilateral triangle and these given points are vertices of an equilateral triangle for any real value of a.

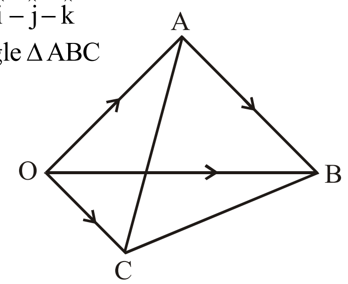
Position vector of A, $\overrightarrow{OA} = (a-1)\hat{i} + a\hat{j} + (a+1)\hat{k}$

Position vector of B, $\overrightarrow{OB} = a\hat{i} + (a+1)\hat{j} + (a-1)\hat{k}$

Position vector of C, $\overrightarrow{OC} = (a+1)\hat{i} + (a-1)\hat{j} + a\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$



Now, area of a triangle ΔABC

$= \frac{1}{2} | \overrightarrow{AB} \times \overrightarrow{AC} |$

$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix}$

$= \frac{1}{2} | -3\hat{i} - 3\hat{j} - 3\hat{k} | = \frac{1}{2}\sqrt{9+9+9} = \frac{\sqrt{27}}{2}$ sq units.

Since, a does not appear so, area of triangle formed by these points is independent of a.

2. (b) Let the point A(x, y, z) is equidistant from the points B(a, 0, 0), C (0,a, 0), D (0, 0, a) and E (0, 0, 0).
Hence, $(x-a)^2 + y^2 + z^2 = x^2 + (y-a)^2 + z^2$
$= x^2 + y^2 + (z-a)^2$
$= x^2 + y^2 + z^2$

$\Rightarrow (x-a)^2 + y^2 + z^2 = x^2 + (y-a)^2 + z^2$

$\Rightarrow x^2 + a^2 - 2ax + y^2 + z^2 = x^2 + y^2 + a^2 - 2ay + z^2$

$\Rightarrow -2ax = -2ay$

$\Rightarrow ax = ay \Rightarrow x = y$

Similarly, $ay = az \Rightarrow y = z$

$\Rightarrow x = y = z$

$\therefore (x-a)^2 + x^2 + x^2 = x^2 + x^2 + x^2$

$\Rightarrow x^2 + a^2 - 2ax + x^2 + x^2 = 3x^2$

$\Rightarrow a^2 = 2ax \Rightarrow x = \frac{a}{2}$

$\therefore$ Point is $\left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$.

3. (b) We know that sum of square of direction cosines = 1
i.e. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2 45° + \cos^2\beta + \cos^2\beta = 1$
(As given $\alpha = 45°$ and $\beta = \gamma$)

$\Rightarrow \frac{1}{2} + 2\cos^2\beta = 1$

$\Rightarrow \cos^2\beta = \frac{1}{4}$

$\Rightarrow \cos\beta = \pm\frac{1}{2}$, Negative value is discarded, since the line makes angle with positive axes.

Hence, $\cos\beta = \frac{1}{2}$

$\Rightarrow \cos\beta = \cos 60°$

$\Rightarrow \beta = 60°$

$\therefore$ Required sum $= \alpha + \beta + \gamma = 45° + 60° + 60° = 165°$

4. (c) Let position vectors of two points be

$\overrightarrow{OA} = (\sqrt{3}-1)\hat{i} - (\sqrt{3}+1)\hat{j} + 4\hat{k}$ and

$\overrightarrow{OB} = -(\sqrt{3}+1)\hat{i} + (\sqrt{3}-1)\hat{j} + 4\hat{k}$

$|\overrightarrow{OA}| = \sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 + 4^2}$

$= \sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}+16} = \sqrt{24}$

Also, $|\overrightarrow{OB}| = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 + 4^2} = \sqrt{24}$

$\overrightarrow{OA}.\overrightarrow{OB} = |\overrightarrow{OA}|.|\overrightarrow{OB}|.\cos\theta$, where θ is the angle between $\overrightarrow{OA}$ & $\overrightarrow{OB}$.

$\overrightarrow{OA}.\overrightarrow{OB} = -(\sqrt{3}-1)(\sqrt{3}+1) - (\sqrt{3}+1)(\sqrt{3}-1) + 16$

$= -3 + 1 - 3 + 1 + 16 = 12$

So, $\cos\theta = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|} = \frac{12}{24} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

5. (c) Equations
$ax + by + cz + d = 0,\ a'x + b'y + c'z + d' = 0$
represent a straight line and equation of the form

$\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-y}{n}$

represent a straight line passing through the point (α, β, γ) and having direction ratios proportional to l, m, n. Thus, both statements are correct.

6. (c) The given equation of sphere is
$ax^2 + by^2 + cz^2 - 2x + 4y + 2z - 3 = 0$
This equation represents a equation of sphere, if coefficient of x^2, y^2 and z^2 is same.
i.e., $a = b = c$
$\therefore$ Equation of sphere can be re-written as
$bx^2 + by^2 + bz^2 - 2x + 4y + 2z - 3 = 0$

$$\Rightarrow x^2 + y^2 + z^2 - \frac{2x}{b} + \frac{4y}{b} + \frac{2z}{b} - \frac{3}{b} = 0$$

The centre of this sphere is $\left(\frac{1}{b}, \frac{-2}{b}, \frac{-1}{b}\right)$

Given that the centre of sphere is $\left(\frac{1}{2}, -1, -\frac{1}{2}\right)$

$$\frac{1}{b} = \frac{1}{2} \Rightarrow b = 2$$

7. (b) Length of perpendicular from the origin to the plane $ax + by + \sqrt{2ab}\, z - 1 = 0$ is

$$= \left|\frac{0+0+0-1}{\sqrt{a^2+b^2+2ab}}\right| = \frac{1}{\sqrt{(a+b)^2}} = \frac{1}{(a+b)}$$

8. (a) Since, the direction ratio's of normal to a plane are $< l, m, n >$ and the length of normal is p, then intercept on x-axis is p/l and that on y-axis is p/m and on z-axis it is p/n, hence sum of intercepts cut off by the plane from the coordinate axes

$$= p\left(\frac{1}{\ell} + \frac{1}{m} + \frac{1}{n}\right)$$

9. (c) Since, the general equation of a plane is $ax + by + cz + d = 0$. Where a, b, c show direction ratio and d is a parameter.
So, the number of arbitrary constants in equation of a plane = 4.

10. (d) Since, coordinates of points P and Q are (2, 5, –7) and (– 3, 2, 1) respectively.
Direction ratios of PQ are
$< -3 - 2, 2 - 5, 1 + 7 >$ i.e., $< -5, -3, 8 >$.

11 (b) Since, coordinates of points O and P are (0,0,0) and (2, 3, – 1) respectively.
Direction ratios of OP are $< 2, 3, -1 >$.
The plane is perpendicular to OP. So, its equation is
$2x + 3y - z + d = 0$ .....(i)
Since, this plane passes through (2, 3, – 1); $2 \times 2 + 3 \times 3 - 1 \times -1 + d = 0$
$\Rightarrow 4 + 9 + 1 + d = 0$
$\Rightarrow d = -14$
On putting the value of d in Eq. (i)
$2x + 3y - z - 14 = 0$
$\Rightarrow 2x + 3y - z = 14$
which is required equation of plane.

12. (c) Let the coordinates of A, B, C and D are (0, 4. 1), (2, 3, – 1) (4, 5, 0) and (2, 6, 2) respectively.
We find its sides and diagonals as below

$$AB = \sqrt{(2-0)^2 + (3-4)^2 + (-1-1)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$BC = \sqrt{(4-2)^2 + (5-3)^2 + (0+1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$CD = \sqrt{(2-4)^2 + (6-5)^2 + (2-0)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$DA = \sqrt{(0-2)^2 + (4-6)^2 + (1-2)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$AC = \sqrt{(4-0)^2 + (5-4)^2 + (0-1)^2} = \sqrt{16+1+1} = \sqrt{18}$$

and $$BD = \sqrt{(2-2)^2 + (6-3)^2 + (2+1)^2} = \sqrt{9+9} = \sqrt{18}$$

Since, AB = BC = CD = DA, sides are equal and AC = BD, diagonals are also equal.
Hence, A, B, C and D are the vertices of a square.

13. (b) Let the point be P(x, y, z) and two points,(a, 0, 0) and (– a, 0, 0) be A and B
As given in the problem,
$PA^2 + PB^2 = 2c^2$
so, $(x + a)^2 + (y - 0)^2 + (z - 0)^2 + (x - a)^2 + (y - 0)^2 + (z - 0)^2 = 2c^2$
or, $(x + a)^2 + y^2 + z^2 + (x - a)^2 + y^2 + z^2 = 2c^2$
$x^2 + 2ax + a^2 + y^2 + z^2 + x^2 - 2ax + a^2 + y^2 + z^2 = 2c^2$
$= 2(x^2 + y^2 + z^2 + a^2) = 2c^2$
$= x^2 + y^2 + z^2 + a^2 = c^2$
$= x^2 + a^2 = c^2 - y^2 - z^2$

14. (d) Planes always intersect in a line.

15. (c) Sum of directions cosines of a line i.e. $\ell + m + n \neq 1$.
So, R is false.
Since sum of squares of direction cosines is unity

$$= \left(\sqrt{\frac{\ell^2+m^2}{2}}\right)^2 + \left(\sqrt{\frac{m^2+n^2}{2}}\right)^2 + \left(\sqrt{\frac{n^2+\ell^2}{2}}\right)^2$$

$$= \frac{\ell^2+m^2}{2} + \frac{m^2+n^2}{2} + \frac{n^2+\ell^2}{2}$$

$$\Rightarrow \frac{2(\ell^2+m^2+n^2)}{2} = \ell^2 + m^2 + n^2 = 1$$

Hence, assertion A is true.
So, A is true but R is false.

16. (d) The equation of the line is

$$\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{5}=r$$

where r is a constant. Any point on this line, is given by $x = 2r + 2$, $y = 3r + 2$ and $z = 5r + 4$

Since, a plane that is parallel to z-axis will have no z-co-ordinate, $z = 0$

$$z=0 \Rightarrow 5r+4=0 \text{ or, } r=\frac{-4}{5}$$

Putting this value of r for x and y co-ordinates.

$$x=2r+2=2\times\left(-\frac{4}{5}\right)+2$$

or, $5x = -8 + 10 = 2$

$$x=\frac{2}{5}, \text{ or } \frac{2}{x}=5 \quad ...(1)$$

Similarly, $y=3r+3=3\times\left(-\frac{4}{5}\right)+3$

or, $5y = -12 + 15 = 3$

$$y=\frac{3}{5} \Rightarrow \frac{3}{y}=5 \quad ...(2)$$

From equations (1) and (2)

$$\frac{2}{x}=\frac{3}{y} \Rightarrow 3x-2y=0$$

17. (d) In the given equation, there are three unknown parameters and no equation has been given to evaluate those, hence centre of sphere cannot be determined.

18. (c) For $\left(\frac{1}{\sqrt{2}},\frac{1}{2},k\right)$ to represent direction cosines

$$\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2+k^2=1$$

or, $\frac{1}{2}+\frac{1}{4}+k^2=1$

$$\frac{3}{4}+k^2=1 \Rightarrow k=\pm\frac{1}{2}$$

19. (a) As given :

$x = a \sec\theta \cos\phi$, $y = b \sec\theta \sin\phi$, $z = c \tan\theta$

$$\text{So, } \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=\frac{a^2\sec^2\theta\cos^2\phi}{a^2}$$

$$+\frac{b^2\sec^2\theta\sin^2\phi}{b^2}-\frac{c^2\tan^2\theta}{c^2}$$

$$=\sec^2\theta(\cos^2\phi+\sin^2\phi)-\tan^2\theta=\sec^2\theta-\tan^2\theta=1$$

20. (b) If a line makes angle θ, ϕ and ψ with x, y, z axes respectively, then

$\cos^2\theta + \cos^2\phi + \cos^2\psi = 1$

$\Rightarrow \cos^2\theta + \cos^2\phi = 1 - \cos^2\psi \Rightarrow \cos^2\theta + \cos^2\phi = \sin^2\psi$

∴ Statement (2) is correct.

21. (a) The equation of the plane passing through (x_1, y_1, z_1) and normal to the line with $\langle a, b, c\rangle$ as direction ratios, is given by :

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\Rightarrow ax - ax_1 + by - by_1 + cz - cz_1 = 0$

$\Rightarrow ax + by + cz = ax_1 + by_1 + cz_1$

which is required equation of plane.

22. (c) Let the direction ratio of the line be, a, b,c,

This line is contained by both plane,

$3x + y + 2z = 7$ and $x + 2y + 3z = 5$.

$\Rightarrow 3a + b + 2c = 0 \quad ....(1)$

and $a + 2b + 3c = 0 \quad ....(2)$

Solving these two equations :

$$\frac{a}{\begin{vmatrix}1 & 2\\2 & 3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}3 & 2\\1 & 3\end{vmatrix}}=\frac{c}{\begin{vmatrix}3 & 1\\1 & 2\end{vmatrix}}=k$$

Let

$$\Rightarrow \frac{a}{-1}=\frac{-b}{7}=\frac{c}{5}=k$$

$a = -k$, $b = -7k$, $c = 5k$.

Direction cosines are :

$$\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\sqrt{a^2+b^2+c^2}=\sqrt{(-k)^2+(-7k)^2+(5k)^2}$$

$$=\sqrt{k^2+49k^2+25^2}=k\sqrt{75}$$

So, Direction cosines are $\left(\frac{-k}{k\sqrt{75}},\frac{-7k}{k\sqrt{75}},\frac{5k}{k\sqrt{75}}\right)$

$$=\left(-\frac{1}{\sqrt{75}},-\frac{7}{\sqrt{75}},\frac{5}{\sqrt{75}}\right).$$

23. (b) The equation of the given sphere is

$x^2 + y^2 + z^2 - 10z = 0$.

∴ Its centre is (0, 0, 5).

Coordinates of one end point of a diameter of the sphere is given as (– 3, – 4, 5).

Let Coordinates of another end point of this diameter (x_1, y_1, z_1)

$$\therefore \frac{-3+x_1}{2}=0 \Rightarrow x_1=3$$

$$\frac{-4+y_1}{2}=0 \Rightarrow y_1=4$$

and $\frac{5+z_1}{2} = 5 \Rightarrow z_1 = 5$

$\therefore$ Required coordinates are (3, 4, 5).

24. (b) As given :
O (0,0), A (0, 3), B (4, 0) are the vertices of triangle OAB.

$\therefore$ $\overrightarrow{OA} = \vec{r} = 3\hat{j}$ and $\vec{F} = 10\hat{i}$

Movement of force about the vertex

$A = \vec{r} \times \vec{F} = 3\hat{j} \times 10\hat{i} = -30\hat{k}$

$\therefore$ Magnitude of moment $= |30\hat{k}| = 30$ unit

25. (a) Let the line joining the points (2, 4, 5) and (3, 5, – 4) is internally divided by the xy - plane in the ratio k : 1.

$\therefore$ For xy plane, z = 0

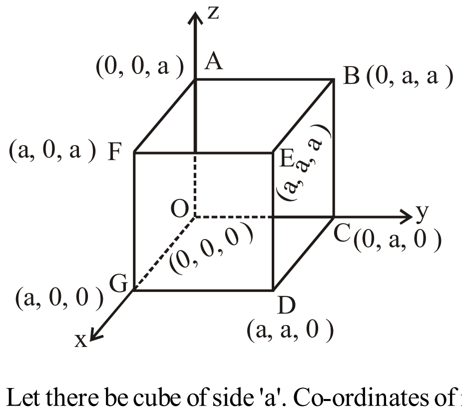
$\Rightarrow 0 = \frac{-k \times 4 + 5}{k+1} \Rightarrow 4k = 5 \Rightarrow k = \frac{5}{4}.$

$k = \frac{5}{4}$ so, ratio is 5 : 4

26. (c) Given equation of planes are :
$x + y + z = 7$
and $\alpha x + \beta y + \gamma z = 3$
For these planes to be parallel, coefficients of x, y and z should be same i.e.
$\Rightarrow \alpha = \beta = \gamma$

27. (c) A plane ax + by + cz = 0 is parallel to, a straight line having direction ratios a', b', c'.
If aa' + bb' + cc' = 0
In the given problem, dr_s of line is 2, 3, 4.
We check the equations of plane in the given choices, one by one.
(a) $4 \times 2 + 3 \times 3 + (-5) \times 4 = 8 + 9 - 20 \neq 0$
(b) $4 \times 2 + 5 \times 3 + (-4) \times 4 = 8 + 15 - 16 \neq 0$
(c) $4 \times 2 + 4 \times 3 + (-5) \times 4 = 8 + 12 - 20 = 0$
Further checking is not needed.

28. (d)



Let there be cube of side 'a'. Co-ordinates of its vertices O, A, B, C, D, E, F have be marked in the figure. Diagonals are OE, FC, GB and AD. Direction ratios (dr_3) of these diagonals are : OE $\langle (a-0), (a-0), (a-0) \rangle = (a, a, a)$

FC $\langle(-a, a, -a)\rangle$; GB $\langle(-a, a, a)\rangle$ and AD $\langle(a, a, -a)\rangle$
Their dcs are :

OE, $\langle \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}} \rangle$

$= \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

AD, $\langle \frac{a}{\sqrt{\Sigma a^2}}, \frac{a}{\sqrt{\Sigma a^2}}, \frac{-a}{\sqrt{\Sigma a^2}} \rangle = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$

Angle, θ, between AD and OE is given by

$$\cos\theta = \pm \frac{\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}}{\sqrt{\left\{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2\right\}\left\{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2\right\}}}$$

$$= \frac{\frac{1}{3}}{1 \times 1} = \pm\frac{1}{3}$$

Since the cube is in positive octant, we take $+\frac{1}{3}$.

So, $\cos\theta = \frac{1}{3} \Rightarrow \theta > 60'$

[Since value of cos θ decreases as θ increases in 0 to 90°. cos θ = 1 when θ = 0° and cos θ = 0 when θ = 90°]

29. (c) The equation of plane which contains z-axis is 3x + 2y = 0 as z is absent in this equation.

30. (d) Given two lines are : $y = \frac{mx}{\ell} + \alpha$, $z = \frac{n}{\ell}x + \beta$ and

$y = \frac{m'}{\ell'}x + \alpha', z = \frac{n'}{\ell'}x + \beta'$

These two lines can be represented as :

$\frac{y-\alpha}{m/\ell} = \frac{x-0}{1} = \frac{z-\beta}{n/\ell}$ and $\frac{y-\alpha'}{m'/\ell'} = \frac{x-0}{1} = \frac{z-\beta'}{n'/\ell'}$

They are orthogonal, if

$\frac{m}{\ell} \times \frac{m'}{\ell'} + 1 \times 1 + \frac{n}{\ell} \times \frac{n'}{\ell'} = -1 \Rightarrow \ell\ell' + mm' + nn' = 0$

31. (a) The equation of sphere passing through the given points is
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$
$\therefore (0)^2 + (0)^2 + (0)^2 + 2u(0) + 2v(0) + 2w(0) + d = 0$
$\Rightarrow d = 0$
$(2)^2 + (0)^2 + (0)^2 + 2u(2) + 2v(0) + 2w(0) + 0 = 0$
$\Rightarrow u = -1$
$(0)^2 + (4)^2 + (0)^2 + 2u(0) + 2v(4) + 2w(0) + 0 = 0$
$\Rightarrow v = -2$

and

$(0)^2+(0)^2+(6)^2+2u(0)+2v(0)+2w(6)+0=0$

$\Rightarrow w=-3$

$\therefore$ The centre of sphere = (1, 2, 3)

It will be the point equidistant from the four points (0, 0, 0), (2, 0,0), (0, 4, 0) and (0, 0, 6).

32. (b) Let the angle between two lines whose direction ratios are a_1, b_1, c_1 and a_2, b_2, c_2 respectively is 0.

Then, $\cos\theta = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\therefore \cos 60° = \dfrac{1\times1+0\times0+(\cos\alpha)(-\cos\alpha)}{\sqrt{1^2+(0)^2+\cos^2\alpha}\sqrt{1^2+(0)^2+(-\cos\alpha)^2}}$

$\Rightarrow \dfrac{1}{2}=\dfrac{1-\cos^2\alpha}{\sqrt{1+\cos^2\alpha}\sqrt{1+\cos^2\alpha}}$

$\Rightarrow \dfrac{1}{2}=\dfrac{1-\cos^2\alpha}{1+\cos^2\alpha} \Rightarrow \dfrac{1+2}{1-2}=\dfrac{2}{-2\cos^2\alpha}$

$\Rightarrow \dfrac{3}{-1}=\dfrac{1}{-\cos^2\alpha}$

$\Rightarrow \cos\alpha=\dfrac{1}{\sqrt{3}} \Rightarrow \alpha=\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

33. (a) Let the equation of line which is passing through (1, 2, 3) and having direction ratios (1, 2, 3) is

$\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}=a$

$\therefore\ x-1=a$

$y-2=2a$ and $z-3=3a$

$\Rightarrow x=a+1,\ y=2a+2$ and $z=3a+3$

At x-axis, $y=0$ and $z=0$

$\Rightarrow 2a+2=0$ and $3a+3=0$

$\Rightarrow a=-1$ and $a=-1$

$\therefore\ x=(-1)+1=0$

34. (a) Direction cosines of the normal to the given plane $by+cz+d=0$ are 0, b, c

Direction cosines of the x-axis are 1, 0, 0

Since, $0\times1+b\times0+c\times0=0$

Hence x-axis is perpendicular to normal to the given plane. Therefore x-axis is parallel to the given plane.

35. (d) Direction cosines of the normal to the plane $3x+y+z=5$ are 3, 1, 1

Direction cosines of the normal to the plane $x-2y-z=6$ are 1, – 2, – 1

Sum of the product of direction cosines

$=3\times1+1\times(-2)+1\times(-1)=0$

Hence, normals to the two planes are perpendicular to each other. Therefore two planes are also perpendicular.

36. (a) Given sphere : $x^2+y^2+z^2-6x-8y+10z+\lambda=0$

Its radius = 1

$\Rightarrow \sqrt{(-3)^2+(-4^2)+(5)^2-\lambda}=1$

$\Rightarrow 9+16+25-\lambda=1$

$\therefore\ \lambda=49$

37. (b) a circle (obviously)

38. (a) Let (1, 3, 4), (–1, 6, 10), (–7, 4, 7) and (–5, 1, 1) be the coordinates of points A, B, C and D respectively.

$\therefore\ AB=\sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}$

$=\sqrt{4+9+36}=7$

$BC=\sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}$

$=\sqrt{36+4+9}=7$

$CD=\sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}$

$=\sqrt{4+9+36}=7$

$DA=\sqrt{(1+5)^2+(3-1)^2+(4-1)^2}$

$=\sqrt{36+4+9}=7$

$AC=\sqrt{(-7-1)^2+(4-3)^2+(7-4)^2}$

$=\sqrt{64+1+9}=\sqrt{74}$

and $BD=\sqrt{(-5+1)^2+(1-6)^2+(1-10)^2}$

$=\sqrt{16+25+81}=\sqrt{122}$

$\because\ AB=BC=CD=DA$

But $BD\neq AC$

$\therefore$ Points A, B, C and D are the vertices of a rhombus.

39. (c) We know that the number of planes passing through the non-collinear points is 1.

40. (a) Given, $x+z=0, y=0$ and $20x=15y=12z$

$\Rightarrow \dfrac{x}{1}=\dfrac{y}{0}=\dfrac{z}{-1}$ and $\dfrac{x}{3}=\dfrac{y}{4}=\dfrac{z}{5}$

Let θ be angle between two lines.

$\therefore\ \cos\theta=\left|\dfrac{(1)(3)+(0)(4)+(-1)(5)}{\sqrt{1+0+1}\sqrt{9+16+25}}\right|=\left|\dfrac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$

$=\dfrac{2}{\sqrt{2}\cdot5\sqrt{2}}=\dfrac{1}{5}$

$\therefore\ \theta=\cos^{-1}\left(\dfrac{1}{5}\right)$

41. (b) Equation $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ represent a real sphere if $u^2+v^2+w^2>d$

42. (b) We know, if
$a_1 x + b_1 y + c_1 z = d_1$ and
$a_2 x + b_2 y + c_2 z = d_2$
are two planes then angle between them is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Let θ be the angle between given planes
Here, $a_1 = 2, b_1 = -1, c_1 = 1$
$a_2 = 1, b_2 = 1, c_2 = 2$

$$\therefore \cos\theta = \frac{2\times1+1\times(-1)+1\times2}{\sqrt{4+1+1}\sqrt{1+1+4}}$$

$$= \frac{3}{6} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

43. (b) The equation of plane which is passing through x-axis is
$x = a.$
This plane also passes through (1, 2, 3)
∴ By putting $x = 1$ in eqn of plane, we get $a = 1$
Hence $x = 1$ is required equation of plane.

44. (b) If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction ratios then angle between the lines is

$$\cos\theta = \frac{l_1l_2 + m_1m_2 + n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}$$

Here $l_1 = 1, m_1 = 1, n_1 = 1$ and
$l_2 = 1, m_2 = -1, n_2 = n$
and $\theta = 60°$

$$\therefore \cos 60° = \frac{1\times1+1\times(-1)+1\times n}{\sqrt{1^2+1^2+1^2}\times\sqrt{1^2+1^2+n^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{n}{\sqrt{3}\sqrt{2+n^2}} \Rightarrow 3(2+n^2) = 4n^2$$

$$\Rightarrow n^2 = 6 \Rightarrow n = \pm\sqrt{6}$$

45. (b) Equation of plane passing through (1, –2, 4) and whose normal (2, 1, 2) is
$2(x-1) + 1(y+2) + 2(z-4) = 0$
$\Rightarrow 2x - 2 + y + 2 + 2z - 8 = 0$
$\Rightarrow 2x + y + 2z - 8 = 0$
∴ So, distance of the plane $2x + y + 2z - 8 = 0$ from the point (3, 2, 3) is

$$= \frac{2(3)+1(2)+(2)(3)-8}{\sqrt{4+1+4}} = \frac{6}{3} = 2$$

46. (d) Equation of plane passing through (1, –3, 1) and whose normal (1, –3, 1) is
$1(x-1) - 3(y+3) + 1(z-1) = 0$
$\Rightarrow x - 3y + z - 11 = 0$

$$\Rightarrow \frac{x}{11} - \frac{y}{11/3} + \frac{z}{11} = 0$$

The above plane intercept the x-axis at 11.

47. (a) Since $l^2 + m^2 + n^2 = 1$
$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\theta = 1$ .... (i)
(∵ A line makes the same angle α with x and y-axes and θ with z-axis)
Also, $\sin^2\theta = 2\sin^2\alpha$
$\Rightarrow 1 - \cos^2\theta = 2(1 - \cos^2\alpha)$ (∵ $\sin^2 A + \cos^2 A = 1$)
$\Rightarrow \cos^2\theta = 2\cos^2\alpha - 1$ .... (ii)
∴ From Eq. (i) and (ii)
$2\cos^2\alpha + 2\cos^2\alpha - 1 = 1$

$$\Rightarrow 4\cos^2\alpha = 2 \Rightarrow \cos^2\alpha = \frac{1}{2}$$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \alpha = \frac{\pi}{4}, \frac{3\pi}{4}$$

48. (c) Given centre of sphere is (6, – 1, 2) and eq^n of plane is $2x - y + 2z - 2 = 0$
Since, sphere touches the plane therefore ⊥ distance from centre to the plane is radius of the sphere.

$$\therefore \text{Radius} = \frac{2(6)-1(-1)+2(2)-2}{\sqrt{4+1+4}} = \frac{15}{3} = 5$$

∴ Required equation of sphere is
$(x-6)^2 + (y+1)^2 + (z-2)^2 = 5^2$
$\Rightarrow x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

49. (a) The intersection of given plane is
$x - y + 2z - 1 + \lambda(x + y - z - 3) = 0$
$\Rightarrow x(1+\lambda) + y(\lambda - 1) + z(2-\lambda) - 3\lambda - 1 = 0$
DR's of normal to the above plane is
$(1+\lambda, \lambda-1, 2-\lambda)$
By taking option (a)
$-1(1+\lambda) + 3(\lambda-1) + 2(2-\lambda) = 0$
$\Rightarrow -1 - \lambda + 3\lambda - 3 + 4 - 2\lambda = 0$
$\Rightarrow 0 = 0$ which is true.
Hence, option (a) is correct.

50. (c) The planes $bx - ay = n$, $cy - bz = l$ and $az - cx = m$ intersect in a line, if $al + bm + cn = 0$.

51. (a) We know that the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Given equation of planes are $px + 2y + 2z - 3 = 0$ and $2x - y + z + 2 = 0$
On comparing with standard equations, we get

$a_1 = p, a_2 = 2, b_1 = 2, b_2 = -1, \quad c_1 = 2, c_2 = 1$

Also, $\theta = \frac{\pi}{4}$ (given)

$\therefore \cos\frac{\pi}{4} = \left|\frac{p\times 2 + 2\times(-1) + 2\times 1}{\sqrt{p^2+4+4}\sqrt{4+1+1}}\right|$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2p}{\sqrt{p^2+8}\sqrt{6}} \Rightarrow \frac{1}{2} = \frac{4p^2}{(p^2+8)6}$

$\Rightarrow \frac{3}{4} = \frac{p^2}{p^2+8}$

$\Rightarrow 3p^2 + 24 = 4p^2 \Rightarrow p^2 = 24$

52. (b) Required angle $= \cos^{-1}\left(\frac{1}{3}\right)$

53. (d) Required angle $= \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

54. (c) Required angle $= \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

55. (b) Let equation of plane through z-axis is

$ax + by = 0$

It is given that this plane is parallel to the line

$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$

Since the plane parallel to the line

$\therefore a\cos\theta + b\sin\theta = 0$

$\Rightarrow a\cos\theta = -b\sin\theta \Rightarrow a = -b\tan\theta$

$\therefore -b\tan\theta x + by = 0$

$\Rightarrow x\tan\theta - y = 0 \ (\because b \neq 0)$

which is required equation of plane.

56. (d) Given points are $A(k, 1, -1), B(2k, 0, 2)$ and $C(2+2k, k, 1)$

Let r_1 = length of line

$AB = \sqrt{(2k-k)^2 + (0-1)^2 + (2+1)^2} = \sqrt{k^2+10}$

and r_2 = length of line $BC = \sqrt{(2)^2 + k^2 + (-1)^2}$

$= \sqrt{k^2+5}$

Now, let ℓ_1, m_1, n_1 be direction-cosines of line AB and ℓ_2, m_2, n_2 be the direction cosines of BC.

Since AB is perpendicular to BC

$\therefore \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$

Now, $\ell_1 = \frac{k}{\sqrt{k^2+10}}, m_1 = \frac{-1}{\sqrt{k^2+10}}, n_1 = \frac{3}{\sqrt{k^2+10}}$

and $\ell_2 = \frac{2}{\sqrt{k^2+5}}, m_2 = \frac{k}{\sqrt{k^2+5}}, n_2 = \frac{-1}{\sqrt{k^2+5}}$

So, $\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$

$\Rightarrow \frac{2k}{\sqrt{k^2+10}\sqrt{k^2+5}} - \frac{k}{\sqrt{k^2+10}\sqrt{k^2+5}} - \frac{3}{\sqrt{k^2+10}\sqrt{k^2+5}} = 0$

$\Rightarrow 2k - k - 3 = 0$

$\Rightarrow k = 3$

For $k = 3$, AB is perpendicular to BC.

57. (d) Two planes $ax + by + cz + d = 0$ and $ax + by + cz + d_1 = 0$ are parallel to each other.

$\therefore$ They have no common point.

58. (a) Required distance $= \frac{|2(0)+6(0)-3(0)+7|}{\sqrt{(2)^2+(6)^2+(-3)^2}}$

$= \frac{|7|}{\sqrt{4+36+9}} = \left|\frac{7}{7}\right| = 1$

59. (d) The given equation of the planes are $x + y + 2z = 3$ and $-2x + y - z = 11$.

We know that, the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

Here, $a_1 = 1, b_1 = 1, c_1 = 2, a_2 = -2, b_2 = 1, c_2 = -1$

$\therefore \cos\theta = \left|\frac{1\times(-2) + 1\times 1 + 2\times(-1)}{\sqrt{1+1+4}\sqrt{4+1+1}}\right|$

$= \left|\frac{-2+1-2}{\sqrt{6}\sqrt{6}}\right| = \left|\frac{3}{6}\right| = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$

60. (a) The given equation of sphere is

$x^2 + y^2 + z^2 - x - y - z = 0$

On comparing with

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

we get, $u = -\frac{1}{2}, v = -\frac{1}{2}, w = -\frac{1}{2}, d = 0$

$\therefore$ Radius of sphere $= \sqrt{u^2+v^2+w^2-d}$

$= \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{3}{4}}$

61. (a) We have,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ... (i)

$\Rightarrow 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma = 2$

$\Rightarrow 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 = 2 - 3$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Hence statement - I is correct.

and now from (i),

$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Hence, only statement I is correct.

62. (b) Only point (3, 7, 1) satisfy the equation of plane

$2x + 3y - 6z = 21$

Hence, (3, 7, 1) lies on the plane.

63. (a) Since, direction cosines are proportional to (2,3,4) and (1,–2,1) respectively

$\therefore \quad 2 \times 1 + 3 \times (-2) + 4 \times 1 = 0$

$\therefore$ Angle between the lines is 90°.

64. (a) Locus of points of intersection of a sphere and a plane is circle.

65. (b) Given equations of two planes are $2x - y + z = 4$ and $x + y + 2z = 6$

So, angle between them is

$$\cos\theta = \frac{2(1) + (-1)(1) + (1)(2)}{\sqrt{4+1+1}\sqrt{4+1+1}}$$

$$= \frac{2 - 1 + 2}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}.$$

66. (d) Since, the required plane passing through (1, – 1, – 1) therefore only equation given in option 'd' satisfied by the point (1, – 1, – 1).

Hence, Required equation of plane is

$14x - 5y + 3z = 16$

67. (b) As we know, general equation of sphere is given as

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ...(1)

Given equation of sphere is

$x^2 + y^2 + z^2 + f(x, y, z) = 0$ ...(2)

On comparing both the equations (1) and (2), we get

$f(x, y, z) = 2ux + 2vy + 2wz + d$

Since, sphere passing through (0, 0, 0), (– 1, 0, 0), (0,–2,0), (0, 0, – 3)

$\therefore$ we have from (1),

$$d = 0,\ 1 - 2u = 0 \Rightarrow u = \frac{1}{2},$$

$$4 - 4v = 0 \Rightarrow v = 1,\ 9 - 6w = 0 \Rightarrow w = \frac{3}{2}$$

Hence,

$$f(x, y, z) = 2x\left(\frac{1}{2}\right) + 2y(1) + 2z\left(\frac{3}{2}\right) + 0$$

$= x + 2y + 3z$

68. (b) We have

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Consider $1 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$$= 1 + \left(2\cos^2\alpha - 1\right) + \left(2\cos^2\beta - 1\right) + \left(2\cos^2\gamma - 1\right)$$

$$= 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma - 2$$

$$= 2\left[\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right] - 2 = 2(1) - 2 = 0$$

69. (b) Given equation of plane is $2x - y + 2z + 1 = 0$

$\Rightarrow a = 2, b = -1, c = 2$

Hence d.R $\langle 2, -1, 2\rangle$ i.e., $\left\langle 1, -\frac{1}{2}, 1\right\rangle$

70. (b) If θ be the angle between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$

then $$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}.\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{1(2) + 1(-2) + 1(2)}{\sqrt{1^2 + 1^2 + 1^2}.\sqrt{2^2 + (-2)^2 + 2^2}}$$

$$= \frac{2}{\sqrt{3}\,2\sqrt{3}} = \frac{1}{3}$$

71. (b) Let the direction cosines be ℓ, m, n.

Let P (x_1, y_1, z_1) and Q(x_2, y_2, z_2) be two points which joins a line.

$\therefore x_1 = 1, y_1 = 2, z_1 = -3$

$x_2 = -2, y_2 = 3, z_2 = 1$

Now, $$\ell = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$= \frac{-3}{\sqrt{26}}$$

$$m = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}} = \frac{1}{\sqrt{26}}$$

$n = \frac{4}{\sqrt{26}}$

$\therefore \ell^2 + m^2 + n^2 = \frac{9}{26} + \frac{1}{26} + \frac{16}{26} = \frac{26}{26} = 1$

Always equal to 1

72. (d) General equation of sphere is

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

On comparing with the given equation we have

$u = -2, v = 3, w = -4, d = -7$

Radius $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{4+9+16+7} = \sqrt{36} = 6$

Diameter $= 2 \times 6 = 12$

73. (c) We have,

$13 = \sqrt{(4-7)^2 + (5-1)^2 + (\lambda+3)^2}$

$169 = 9 + 16 + \lambda^2 + 9 + 6\lambda$

$\Rightarrow \lambda^2 + 6\lambda - 135 = 0$

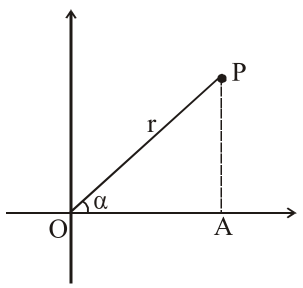
$\Rightarrow \lambda^2 + 15\lambda - 9\lambda - 135 = 0$

$\Rightarrow \lambda(\lambda+15) - 9(\lambda+15) = 0$

$\Rightarrow (\lambda+15)(\lambda-9) = 0$

$\Rightarrow \lambda = -15$ or $\lambda = 9$

74. (a)



Since line OP of length 'r' which makes an angle 'α' with x-axis lies in xz – plane.

Therefore y-coordinate of P is zero.

Now, from ΔOAP, we have

$OA = r\cos\alpha$, $PA = r\sin\alpha$.

$\therefore P = (r\cos\alpha, 0, r\sin\alpha)$

75. (a) Equation of plane is $x = 0$

$\therefore$ Required distance from (1, 2, 0) is

$= \frac{1.1 + 2.0 + 0.0 - 0}{\sqrt{1^2 + 0^2 + 0^2}} = \frac{1}{\sqrt{1}} = 1$ unit

76. (a) Let ℓ, m, n are the dc's of a line that is inclined equally at α to the +ve direction of axes.

Now, $\ell = \cos\alpha$

$m = \cos\alpha$

$n = \cos\alpha$.

Also, $\ell^2 + m^2 + n^2 = 1$

$3\cos^2\alpha = 1$

$\cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore$ dc's of the line are : $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

77. (a) The given lines are:-

$\frac{x-2}{1} = \frac{y-(-1)}{-2} = \frac{z-(-2)}{1}$ and

$\frac{x-1}{1} = \frac{y-\left(-\frac{3}{2}\right)}{\frac{3}{2}} = \frac{z-(-5)}{2}$

dr's of Ist line are:-

$a_1 = 1, b_1 = -2, c_1 = 1$

dr's of IInd line are:-

$a_2 = 2, b_2 = 3, c_2 = 4$

Let 'θ' be the angle b/w two lines, then,

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}.\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

78. (c) The equation of a plane parallel to the plane

$3x + 4y - 5z = 0$ is given by,

$3x + 4y - 5z = d$ .... (i)

Since plane (i) passes through (1,2, 3) then, $3 + 8 - 15 = d$

$\Rightarrow d = -4$

$\therefore$ from (i),

$3x + 4y - 5z + 4 = 0$

79. (c) Let $P_1 : x - 3z - 4 = 0$ and $P_2 : y - 2z + 3 = 0$ be two planes . Let $ax + by + cz = d$ be the equation of line.

Since, the line of intersection will be perpendicular to the normal of both the planes

$\therefore a(1) + b(0) + c(-3) = 0$

$\Rightarrow a - 3c = 0$ .... (i)

and $a(0) + b(1) + c(-2) = 0$

$\Rightarrow b - 2c = 0$ .... (ii)

From (1) and (2) we have

$\frac{a}{3} = \frac{b}{2} = \frac{c}{1}$

Hence, d.Rs = <3, 2, 1>.

80. (b) We know that dr's of z-axis are (0, 0, 1)

So, dr's of the required line are 0, 0 and 1

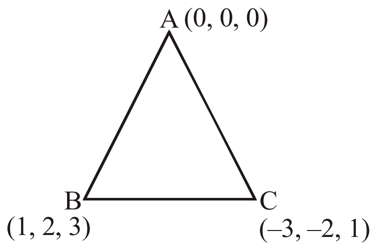
Now, equation of the line passing through (a, b, c) and having dr's 0, 0 and 1 is

$$\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$$

81. (c) Direction cosines of z-axis are 0, 0, 1

sum $=0+0+1=1$

82. (a) Let A(0, 0, 0), B (1, 2, 3) and C (–3, –2, 1) be the vertices of a triangle.



$$\text{Area of } \Delta ABC=\frac{1}{2}\left\|\begin{matrix} i & j & k \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{matrix}\right\|$$

$$=\frac{1}{2}\left[\hat{i}(2+6)-\hat{j}(1+9)+\hat{k}(-2+6)\right]$$

$$=\frac{1}{2}\left|8\hat{i}-10\hat{j}+4\hat{k}\right|=\frac{1}{2}\sqrt{64+16+100}$$

$$=\frac{1}{2}\left(6\sqrt{5}\right)=3\sqrt{5}$$

83. (d) Given planes are

$x-2y+z=1$ .... (i)

and $-3x+6y-3z=-2$

$\equiv x-2y+z=\frac{2}{3}$ .... (ii)

Since both planes are parallel and a = 1, b = –2, c = 1

and $d_1=-1,\ d_2=\frac{-2}{3}$

$$\therefore\ \text{Distance}=\left|\frac{d_2-d_1}{\sqrt{a^2+b^2+c^2}}\right|$$

$$\text{Distance}=\left|\frac{1-\frac{2}{3}}{\sqrt{1+4+1}}\right|=\frac{1}{3\sqrt{6}}$$

84. (a) Direction cosines are

$\cos 30°$, $\cos\beta$ and $\cos\gamma$.

Since we know

$\cos^2 30+\cos^2\beta+\cos^2\gamma=1$

$$\Rightarrow\ \cos^2\beta+\cos^2\gamma=\frac{1}{4}\quad\left(\because \cos 30°=\frac{\sqrt{3}}{2}\right)$$

85. (b) Given equation is

$3x^2+3y^2+(k+1)z^2+x-y+z=0$

which will represents a sphere if

coeff of x^2 = coeff of y^2 = coeff of z^2.

$\Rightarrow\ 3=k+1$

$\Rightarrow\ k=2$

86. (d) Given equation of plane are

$2x-y-2z+1=0$

$\Rightarrow a_1=2, b_1=-1, c_1=-2,\ d_1=1$

and $3x-4y+5z-3=0$

$\Rightarrow a_2=3, b_2=-4, c_2=5, d_2=-3$

$\therefore$ Required angle is

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

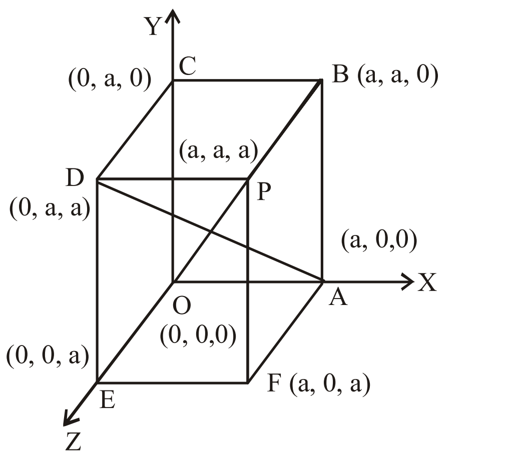
$$\Rightarrow \cos\theta=\frac{2(3)+(-1)(-4)+5(-2)}{\sqrt{2^2+1^2+2^2}\sqrt{3^2+4^2+5^2}}=0$$

$$\Rightarrow \theta=\frac{\pi}{2}$$

87. (d) If the line is parallel to the plane then

$a\ell+bm+cn=0$

88. (d)



diagonals are OP and AD and Acute anngle = θ

$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$=\left|\frac{a(-a)+(a)(a)+(a)(a)}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+a^2}}\right|$$

$= \left|\frac{-a^2+a^2+a^2}{\sqrt{3a^2}\sqrt{3a^2}}\right| = \left|\frac{a^2}{3a^2}\right| = \frac{1}{3}$

$\Rightarrow 3\cos\theta = 1$

89. (b) $(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = r^2$
Centre (0, 0, 0) and radius = 1
$(x-0)^2 + (y-0)^2 + (z-0)^2 = (1)^2$
$x^2 + y^2 + z^2 = 1$

90. (c) Sum of squares of direction cosines
$= (1)^2 + (0)^2 + (0)^2 = 1$

91. (a) $d = \left|\frac{2\times 0 + 0\times 1 + 2\times 0 - 3}{\sqrt{2^2+1^2+2^2}}\right| = \frac{3}{3} = 1$ unit

92. (b) Position vector of line segment = $2\hat{i} + 3\hat{j} + 6\hat{k}$

length = $\sqrt{2^2+3^2+6^2} = \sqrt{49} = 7$ units

93. (a) Direction ratios of normal to plane $2x + 3y - z = 7$ is $< 2, 3, -1 >$

94. (d) Equation of line, $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-1}$
Let P $(2r+1, 3r-2, -r+3)$ of the line meets the plane.
Then, $2(2r+1) + 3(3r-2) - (-r+3) = 0$
$4r + 2 + 9r - 6 + r - 3 = 7$
$14r = 14$
$r = 1$
P (3, 1, 2) meets the plane.

95. (c) Let Q(x, y, z) is the image of (1, –2, 3) in the plane

$\frac{x+1}{2} = 3 \Rightarrow x = 5$

$\frac{y-2}{2} = 1 \Rightarrow y = 4$

$\frac{z+3}{2} = 2 \Rightarrow z = 1$

∴ Image of (1, –2, 3) are (5, 4, 1)

For (96-97)

$x^2 + y^2 + z^2 - 4y + 3 = 0$
$x^2 + y^2 - 4y + 4 - 4 + z^2 + 3 = 0$
$x^2 + (y-2)^2 + z^2 = 1$ ...(i)
Sphere with centre (0, 2, 0) and radius 1 unit.
$x^2 + y^2 + z^2 + 2x + 4z - 4 = 0$
$x^2 + 2x + 1 - 1 + y^2 + z^2 + 4z + 4 - 4 - 4 = 0$
$(x+1)^2 + y^2 + (z+2)^2 = 3^2$ ...(ii)
Sphere with centre (–1, 0, –2) and radius 3 units.

96. (c) $C_1C_2 = \sqrt{(0+1)^2 + (2-0)^2 + (0+2)^2} = 3$ units

97. (c) $r_1 + r_2 = 3 + 1 = 4$
$C_1C_2 < r_1 + r_2$
∴ Two spheres intersect each other.

98. (c) Direction ratios $< (2-6), (-3+7), (1+1) >$
$= < -4, 4, 2 >$

99. (b) Equation of sphere passing through origin is
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$
which passes through the points (2, 1, –1), (1, 5, –4), and (–2, 4, –6)

$\therefore 4u + 2v - 2w = -6$ ...(i)
$2u + 10v - 8w = -42$ ...(ii)
and $-4u + 8v - 12w = -56$ ...(iii)
From eqns (i), (ii) and (iii), we get
$u = 1, v = -2$ and $w = 3$

∴ Radius of sphere = $\sqrt{u^2+v^2+w^2}$

$= \sqrt{1+4+9} = \sqrt{14}$

100. (a) From explanation 54
Centre of sphere,
$(-u, -v, -w) = (-1, 2, -3)$

101. (a) (1) Equation of sphere is
$x^2 + y^2 + z^2 + 2x - 4y + 6z = 0$
Put the value (0, 4, 0),we get
$0 + 16 + 0 + 0 - 16 + 0 = 0$
So, the sphere passes through the point (0, 4, 0).
Hence, Statement 1 is correct.
2. Distance between the point (1, 1, 1) and centre of sphere (–1, 2, –3)

$= \sqrt{(1+1)^2 + (1-2)^2 + (1+3)^2}$

$= \sqrt{4+1+16} = \sqrt{21} \neq 5$

Hence, statement 2 is not correct.

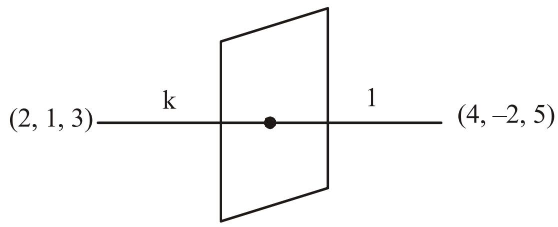
102. (d) Equation of line passing through the points (2, 1, 3) and (4, –2, 5) is

$\frac{x-2}{4-2} = \frac{y-1}{-2-1} = \frac{z-3}{5-3} = \lambda$

$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-3} = \frac{z-3}{2} = \lambda$

$\Rightarrow x = 2\lambda + 2, y = -3\lambda + 1$ and $z = 2\lambda + 3$
Since, this line cuts the plane $2x + y - z = 3$
So, $(2\lambda + 2, -3\lambda + 1, 2\lambda + 3)$ satisfies the equation of plane
$\therefore 2\lambda + 2 - 3\lambda + 1 - 2\lambda - 3 = 3$
$\Rightarrow -3\lambda = 3$
$\Rightarrow \lambda = -1$
Hence, points are $[2(-1) + 2, -3(-1) + 1, 2(-1) + 3]$ i.e., (0, 4, 1).

103. (d) Let the ratio is k : 1



Then, $0 = \frac{4k+2}{k+1}$

$\Rightarrow 4k + 2 = 0 \Rightarrow k = -\frac{1}{2}$

and $4 = \frac{-2k+1}{k+1} \Rightarrow 4k + 4 = -2k + 1 \Rightarrow k = -\frac{1}{2}$

Hence, plane divides the line in ratio 1 : 2 externally.

104. (a) We know that, equation of plane passing through three non-collinear points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Put the value of $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) we get

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & 2 & 1 \\ 5 & -2 & 5 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(10+2) - (y-2)(5-5) + (z-1)(-2-10) = 0$

$\Rightarrow 12x - 12z = 12 \Rightarrow x - z = 1$

Hence the equation of plane parses through (1, 0, 0)

105. (c) Direction ratios of the nomal to the plane $x - z = 1$ are $(1, 0, -1)$.

106. (d) The projection of a directed line segment on the co-ordinate axes are 12, 4, 3, respectively.

$\therefore$ Length of the line segment $= \sqrt{12^2 + 4^2 + 3^2}$

$= \sqrt{144 + 16 + 9} = \sqrt{169} = 13$ units

$\therefore$ Option (d) is correct.

107. (a) Direction cosine of line segment $= \left(\frac{12}{13}, \frac{4}{13}, \frac{3}{13}\right)$

$\therefore$ Option (a) is correct.

108. (b) Equation of line passing through P(3, –1, 11) and perpendicular to $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is:

$\frac{x-3}{-1} = \frac{y-1}{6} = \frac{z-1}{-4}$

The direction ratio are (–1, 6, – 4)

$\therefore$ Option (b) is correct.

109. (c) Now $x_2 - x_1 = -1$

$x_2 - 3 = -1$

$\therefore$ $x_2 = 2$

Similarly,

$y_2 - y_1 = 6$

$y_2 + 1 = 6$

$\therefore$ $y_2 = 5$

and $z_2 - z_1 = -4$

$z_2 - 11 = -4$

$z_2 = -4 + 11 = 7$

$\therefore$ Co-ordinate of Q is (2, 5, 7)

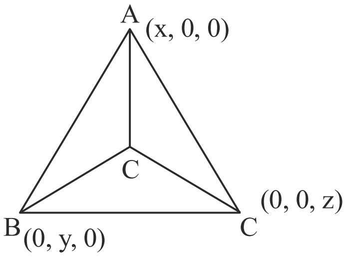
$\therefore$ Length of segment PQ

$= \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2}$

$= \sqrt{1 + 36 + 16} = \sqrt{53}$ units.

$\therefore$ Option (c) is correct.

110. (a) Centroid = (1, 2, 3)



$(1, 2, 3) = \left(\frac{x+0+0}{3}, \frac{0+y+0}{3}, \frac{0+0+z}{3}\right)$

$\therefore$ $x = 3, y = 6$ and $z = 9$

$\therefore$ Intercept made by plane on the axes are 3, 6 and 9, respectively.

$\therefore$ Option (a) is correct.

111. (d) The plane passes through the point A (3, 0, 0), B(0, 6, 0) and C(0, 0, 9). So, it should satisfy the equation given in option for all the three points.

From option (a)

For point A (3, 0, 0)

$x + 2y + 3z = 1$

$\Rightarrow$ $3 + 0 + 0 \neq 1$

$\therefore$ option (a) is wrong.

From option (b)

For point A(3, 0, 0)

$3x + 2y + z = 3$

$\therefore$ $3(3) + 0 + 0 \neq 3$

$\therefore$ option (b) is wrong.

From option (c)

For point A(3, 0, 0)

$2x + 3y + 6z = 18$

$\therefore$ $2(3) + 0 + 0 \neq 18$

$\therefore$ option (c) is wrong.

From option (d)

For point A (3, 0, 0)

$6x + 3y + 2z = 18$

$\Rightarrow$ $6(3) + 0 + 0 = 18$

For point B(0, 6, 0)

$6x + 3y + 2z = 18$

$\therefore$ $0 + 3(6) + 0 = 18$

For point C (0, 0, 9)

$6x + 3y + 2z = 18$

$0 + 0 + 2 \times 9 = 18$

$\therefore$ Option (d) is correct.

112. (b) Length of one of the diagonal of cube

$= \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{1 + 4 + 9} = \sqrt{14}$ units

$\therefore$ Option (b) is correct.

113. (c) Equation of plane passing through (1, 2, 3) and parallel to xy-plane is z = 3.

$\therefore$ Option (c) is correct.

114. (b) $3x^2 + 3y^2 + 3z^2 - 8y + 4y + 8z - 15 = 0$

$$\Rightarrow x^2 + y^2 + z^2 - \frac{8}{3}x + \frac{4}{3}y + \frac{8}{3}z - 5 = 0$$

$$\Rightarrow \left(x - \frac{4}{3}\right)^2 - \frac{16}{9} + \left(y + \frac{2}{3}\right)^2 - \frac{4}{9} + \left(z + \frac{4}{3}\right)^2 - \frac{16}{9} - 5 = 0$$

$$\Rightarrow \left(x - \frac{4}{3}\right)^2 + \left(y + \frac{2}{3}\right)^2 + \left(z + \frac{4}{3}\right)^2 = (3)^2$$

So radius is 3.

115. (a) Let the direction ratio be <a, b, c>

$$\cos 90° = \frac{(a)(1) + (b)(-2) + (c)(-2)}{\sqrt{a^2 + b^2 + c^2}\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$a - 2b - 2c = 0$ ...(1)

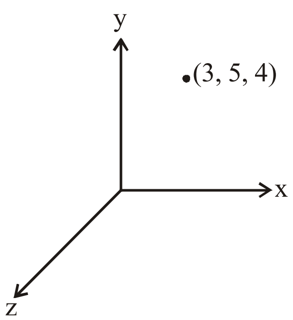
$$\cos 90° = \frac{(a)(0) + b(2) + (c)(1)}{\sqrt{a^2 + b^2 + c^2}\sqrt{(0)^2 + (-2)^2 + (1)^2}}$$

$2b + c = 0$ ...(2)

From eq. (1) & (2)

$a = -2b;\ c = -2b$

116. (b) Plane z = 0 is simply xy plane, so z quadrant value will be zero.



So, options (b) is correct option.

117. (b) $5x + 2y + z - 13 = 0$

Putting y = 0 & z = 0

$$\boxed{x = \frac{13}{5}}$$

Putting z = 0 & x = 0

$$\boxed{y = \frac{13}{2}}$$

Putting x = 0 & y = 0

$$\boxed{z = 13}$$

118. (a) To find, intersection point first put $z = 0$

$2x - y = 2$

$x + y = 1$

$3x = 3$

$x = 1$

$x + y = 1$

$y = 1 - x$

At $x = 1, y = 0$

$(x, y, z) \equiv (1, 0, 0)$

Putting $x = 0$

$y - z = 1$

$-y + 3z = 2$

$2z = 3$

$$z = \frac{3}{2}$$

$$y = 1 + z = 1 + \frac{3}{2} = \frac{5}{2}$$

$$(x, y, z) \equiv \left(0, \frac{5}{2}, \frac{3}{2}\right)$$

Point of intersection $(x_1, y_1, z_1) \equiv (1, 0, 0)$

$$(x_2, y_2, z_2) \equiv \left(0, \frac{5}{2}, \frac{3}{2}\right)$$

Hence direction ratios of the line of intersection of given plane < 2, –5, –3 >.

119. (b) Eq. of plane through two given planes is :

$(2x - y + 3z - 2) + \lambda\ (x + y - z - 1) = 0$

$\because$ It passes through (1, 0, 1)

$\therefore 3 - \lambda = 0 \Rightarrow \lambda = 3$

$\therefore$ Eq. of plane is:

$5x + 2y - 5 = 0$

120. (c) Plane P touches the sphere $x^2 + y^2 + z^2 = r^2$ then r = Distane between centre of sphere (0, 0, 0) to plance P.

$$\Rightarrow r = \left|\frac{5(0) + 2(0) - 5}{\sqrt{5^2 + 2^2 + (0)^2}}\right|$$

$$= \frac{5}{\sqrt{25 + 4}}$$

$$\boxed{r = \frac{5}{\sqrt{29}}}$$

121. (d) Let $Q\,(x_1, y_1, z_1)$ be the image of the point P.

The direction ratios of PQ are 3, –2, 2. ...(i)

The Equation of line PQ is $\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z+5}{2} = r$

Coordinates of any point on the line PQ is $3r - 2, -2r + 1$ and $2r - 5$.

Let $Q\,(3r - 2, -2r + 1, 2r - 5)$ be such a point.

Let L be the mid point of PQ, $L = \left(\frac{3r}{2} - 2,\ 1 - r,\ r - 5\right)$

Since L lies on the plane $3x - 2y + 2z + 1 = 0$

So, $3\left(\frac{3r}{2}-2\right)-2(1-r)+2(r-5)+1=0$

$\Rightarrow \frac{17}{2}r-17=0 \Rightarrow r=2$

So, coordinates of Q are $(3\times2-2, -2\times2+1, 2\times2-5)$
$= (4, -3, -1)$ ...(ii)

Also the mid point of PQ is $L=\left(\frac{3\times2}{2}-2,\ 1-2,\ 2-5\right)$
$= (1, -1, -3)$ ...(iii)

$\therefore PQ=\sqrt{(-2-4)^2+(1+3)^2+(-5+1)^2}=\sqrt{68}$

$\Rightarrow PQ=2\sqrt{17}>8$

$\therefore$ Option (d) is correct.

122. (c) From (i) above, 1 is correct.
We know that,
Sum of direction cosines of the line segment PQ = 1.

123. (c) Let a, b, c be the direction ratios of the line.
Then its equation is

$$\frac{x-5}{a}=\frac{y+6}{b}=\frac{z-7}{c} \quad ...(i)$$

Since (i) is parallel to the planes $x+y+z=1$ and $2x-y-2z=3$ then
$a(1)+b(1)+c(1)=0$ and $a(2)+b(-1)+c(-2)=0$
By cross multiplication

$$\frac{a}{-1}=\frac{b}{4}=\frac{c}{-3}=\lambda$$

$\Rightarrow a=-\lambda,\ b=4\lambda,\ c=-3\lambda$
$\Rightarrow$ Direction ratios of the line are
$<-1, 4, -3> = <1, -4, 3>$

124. (a) Substituting a, b, c in (i), we get

$$\frac{x-5}{-1}=\frac{y+6}{4}=\frac{z-7}{-3} \quad ...(ii)$$

Hence, equation of the line is

$$\frac{x-5}{-1}=\frac{y+6}{4}=\frac{z-7}{-3}$$

125. (b) D.C's (0, 1, 0)
Since x and z are zero, the straight line is parallel to y – axis

126. (c) A (0, 0, 0), B (a, 0, 0), C (0, b, 0), D (0, 0, c)
Let the equidistant point be P(x, y, z)
i.e, AP = BP, AP = CP, AP = DP

$\Rightarrow \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$

$=\sqrt{(x-a)^2+y^2+z^2}$

$\Rightarrow x^2+\cancel{y^2}+\cancel{z^2}=(x-a)^2+\cancel{y^2}+\cancel{z^2}$
$\Rightarrow x^2=x^2-2ax+a^2$
$\Rightarrow a^2-2ax=0$
$\Rightarrow a(a-2x)=0$

Since $a\neq0$, $a=2x \Rightarrow x=\frac{a}{2}$

Similarly, we will get $y=\frac{b}{2}$, $z=\frac{c}{2}$

127. (c) P (3, 2, 4), Q (4, 5, 2), R (5, 8, 0), S (2, –1, 6)

$PQ=\sqrt{(4-3)^2+(5-2)^2+(2-4)^2}$

$=\sqrt{1+9+4}=\sqrt{14}$

$QR=\sqrt{(5-4)^2+(8-5)^2+(0-2)^2}$

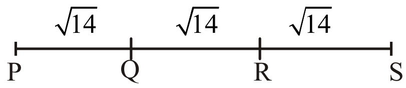
$=\sqrt{1+9+4}=\sqrt{14}$

$RS=\sqrt{(2-5)^2+(-1-8)^2+(6-0)^2}$

$=\sqrt{9+81+36}=\sqrt{126}=3\sqrt{14}$

$PS=\sqrt{(2-3)^2+(-1-2)^2+(6-4)^2}$

$=\sqrt{1+9+4}=\sqrt{14}$



Since, PQ + QR + PS = RS, points are collinear.

128. (d) Eqn. of line

$\Rightarrow \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

i.e, $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+1}{3}=K(\text{say})$
$\Rightarrow x-1=2K$; $y-2=-3K$; $z+1=3K$
$\Rightarrow x=2K+1$; $y=-3K+2$; $z=3K-1$
Since the line meets yz plane, x = 0

$\therefore\ 2K+1=0 \Rightarrow K=\frac{-1}{2}$

$\therefore\ y=-3\left(\frac{-1}{2}\right)+2=\frac{3}{2}+2=\frac{7}{2}$

$z=3\left(\frac{-1}{2}\right)-1=\frac{-3}{2}-1=\frac{-5}{2}$

129. (c) Given, lines x = ay + b and z = cy + d are perpendicular.

$\Rightarrow \frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ ...(i)

Also, x = ey + f and z = gy + h are perpendicular.

$\Rightarrow \frac{x-f}{e}=\frac{y}{1}=\frac{z-h}{g}$ ...(ii)

We know, for $\perp^r$ lines $a_1a_2+b_1b_2+c_1c_2=0$
$\Rightarrow ae+1+cg=0$

130. (b) Given plane, $x + 2y - 2z = 9$.
Length of normal from origin to plane $ax + by + cz = d$ is

$$\frac{d}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore \text{ length of the normal } = \frac{9}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}$$

$$= \frac{9}{\sqrt{9}} = \frac{9}{3} = 3 \text{ units}$$

131. (a) The equation of the line joining the points (–3, 4, –8) and (5, –6, 4) is

$$\frac{x+3}{8} = \frac{y-4}{-10} = \frac{z+8}{12} = k(\text{say})$$

$\Rightarrow x + 3 = 8k;\ y - 4 = -10k;\ z + 8 = 12k$

$\Rightarrow x = 8k - 3;\ y = -10k + 4;\ z = 12k - 8$

Given that this line intersects with xy plane. So, $z = 0$

$$\therefore 12k - 8 = 0 \Rightarrow 12k = 8 \Rightarrow k = \frac{8}{12} = \frac{2}{3}.$$

$$\therefore x = 8\left(\frac{2}{3}\right) - 3;\ y = -10\left(\frac{2}{3}\right) + 4;\ z = 12\left(\frac{2}{3}\right) - 8$$

$$\Rightarrow x = \frac{16}{3} - 3;\ y = \frac{-20}{3} + 4;\ z = \frac{24}{3} - 8$$

$$\Rightarrow x = \frac{7}{3};\ y = \frac{-8}{3};\ z = 0$$

$$\therefore (x, y, z) = \left(\frac{7}{3}, \frac{-8}{3}, 0\right)$$

132. (b) Given direction ratios are (2, –1, 2) and (x, 3, 5)
We know that the angle between the lines whose direction ratios are (a_1, b_1, c_1) and (a_2, b_2, c_2) is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \cos\frac{\pi}{4} = \frac{2x - 3 + 10}{\sqrt{4+1+4}\sqrt{x^2 + 9 + 25}} = \frac{2x + 7}{\sqrt{9}\cdot\sqrt{x^2 + 34}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2x + 7}{3\sqrt{x^2 + 34}} \Rightarrow 2x + 7 = 3\sqrt{\frac{x^2 + 34}{2}}$$

$$\Rightarrow 4x^2 + 49 + 28x = \frac{9(x^2 + 34)}{2}$$ (Squaring on both sides)

$\Rightarrow 2(4x^2 + 49 + 28x) = 9x^2 + 306$

$\Rightarrow 8x^2 + 98 + 56x = 9x^2 + 306$

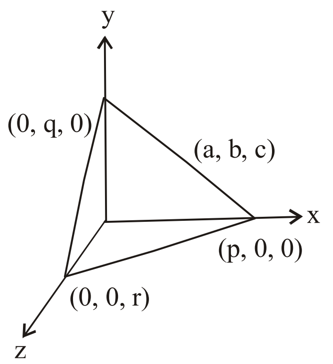
$\Rightarrow x^2 - 56x + 208 = 0$

$$\therefore x = \frac{56 \pm \sqrt{3136 - 812}}{2} = \frac{56 \pm 48}{2} = 28 \pm 24$$

$= 4, 52.$

Smaller value = 4.

133. (c)



Equation of plane passing through points

(p, 0, 0), (0, q, 0) and (0, 0, r) is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$

Given that this plane passes through (a, b, c).

$$\therefore \frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 1$$

Equation of sphere is $x^2 + y^2 + z^2 - px - qy - rz = 0$.

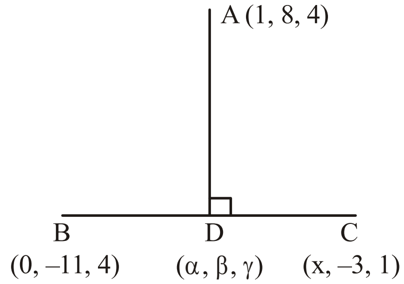
Centre of the sphere = $(l, m, n) = \left(\frac{p}{2}, \frac{q}{2}, \frac{r}{2}\right)$

$\Rightarrow p = 2\ell,\ q = 2m,\ r = 2n$

$$\therefore \text{ locus of the centre } \Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2.$$

134. (a) Given planes, $p_1 : x + y + z = 1$
$p_2 : 2x + 3y + 4z = 7$
So, equation of plane passing through intersection of planes p_1 and p_2 is
$x + y + z - 1 + k(2x + 3y + 4z - 7) = 0.$
$\Rightarrow x + y + z - 1 + 2kx + 3ky + 4kz - 7k = 0$
$\Rightarrow x(1 + 2k) + y(1 + 3k) + z(1 + 4k) - 1 - 7k = 0.$
This is perpendicular to $x - 5y + 3z = 5$.
$\Rightarrow x - 5y + 3z - 5 = 0.$
$\Rightarrow 1(1 + 2k) - 5(1 + 3k) + 3(1 + 4k) = 0$
$\Rightarrow 1 + 2k - 5 - 15k + 3 + 12k = 0$
$\Rightarrow -k - 1 = 0 \Rightarrow k = -1$
$\therefore$ Equation of plane is $x + y + z - 1 - 1(2x + 3y + 4z - 7) = 0$
$\Rightarrow x + y + z - 1 - 2x - 3y - 4z + 7 = 0$
$\Rightarrow -x - 2y - 3z + 6 = 0$
$\Rightarrow x + 2y + 3z - 6 = 0.$

135. (c) A(1, 8, 4), B(0, –11, 4), C(2, –3, 1)
Let $D = (\alpha, \beta, \gamma)$



Direction ratios of BC = $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$
Let, (a, b, c) = (2, 8, –3)
Direction ratios of AD = $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$
Let $(a', b', c') = (\alpha - 1, \beta - 8, \gamma - 4)$
Since, AD is perpendicular to BC,
$aa' + bb' + cc' = 0$
$\Rightarrow 2(\alpha - 1) + 8(\beta - 8) - 3(\gamma - 4) = 0.$
$\Rightarrow 2\alpha - 2 + 8\beta - 64 - 3\gamma + 12 = 0$
$\Rightarrow 2\alpha + 8\beta - 3\gamma - 54 = 0$ ....(1)
On substituting the options, we find option (c) is correct.
when $(\alpha, \beta, \gamma) = (4, 5, -2)$
$(1) \Rightarrow 2(4) + 8(5) - 3(-2) - 54 = 0$
$\Rightarrow 8 + 40 + 6 - 54 = 0$
$\Rightarrow 0 = 0.$

136. (a) We know, the equation of plane passing through 3 points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

So, the plane passing through points (–2, 6, –6), (–3, 10, –9) and (–5, 0, –6) is

$$\begin{vmatrix} x + 2 & y - 6 & z + 6 \\ -1 & 4 & -3 \\ -3 & -6 & 0 \end{vmatrix} = 0$$

$\Rightarrow (x + 2)(-18) - (y - 6)(-9) + (z + 6)(6 + 12) = 10$
$\Rightarrow -18x - 36 + 9y - 54 + 18z + 108 = 0$
$\Rightarrow -18x + 9y + 18z + 18 = 0$
$\Rightarrow 2x - y - 2z - 2 = 0$
$\Rightarrow 2x - y - 2z = 2.$

137. (c) Let the sphere passing through points A(a, 0, 0), B(0, b, 0), C(0, 0, c)
Equation of sphere is $x^2 + y^2 + z^2 - ax - by - cz = 0$

radius, $r = \frac{1}{2}\sqrt{a^2 + b^2 + c^2}$

$\Rightarrow a^2 + b^2 + c^2 = 4r^2$ ....(1)
(Squaring on both sides)
Let (α, β, γ) be centroid of sphere.

$$\therefore (\alpha, \beta, \gamma) = \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$$

$$= \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

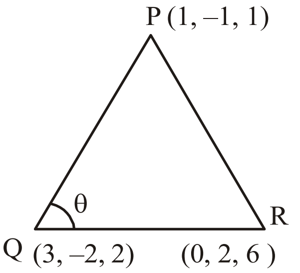
$$\therefore \alpha^2 + \beta^2 + \gamma^2 = \frac{a^2}{9} + \frac{b^2}{9} + \frac{c^2}{9}$$

$$= \frac{a^2 + b^2 + c^2}{9}$$

$$= \frac{4r^2}{9} \quad \text{(from (1))}$$

$\Rightarrow 9(\alpha^2 + \beta^2 + \gamma^2) = 4r^2$
So, Locus is $9(x^2 + y^2 + z^2) = 4r^2$.

138. (d) Given,
P = (1, –1, 1)
Q = (3, –2, 2)
R = (0, 2, 6)



Direction ratios of PQ(a_1, b_1, c_1) = (3 – 1, –2 + 1, 2 – 1)
= (2, –1 ,1)
Direction ratios of PR(a_2, b_2, c_2) = (0 – 1, 2 + 1, 6 – 1)
= (–1, 3, 5)
Let us calculate, $a_1a_2 + b_1b_2 + c_1c_2$
= (2) (–1) + (–1) (3) + (1) (5)
= – 2 –3 + 5
= 0.
$\therefore PQ \perp PR$ i.e., $\angle QPR = 90°$
In $\angle PQR$, $\angle P + \angle Q + \angle R = 180° \Rightarrow 90° + \theta + \angle R = 180°$
$\Rightarrow \angle R = 90° - \theta$

139. (a) Given, centre of sphere (h, k, l) = (–2, 3, 4)
radius (r) = 6 units.
Equation of sphere is $(x - h)^2 + (y - k)^2 + (z - \ell)^2 = r^2$
$\Rightarrow (x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 6^2$
$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 + z^2 - 8z + 16 = 36$
$\Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$

140. (a) Distance $= \left|\frac{3 \times 2 - 6 \times 3 + 2 \times 4 + 11}{\sqrt{3^2 + (-6)^2 + (2)^2}}\right|$

$$= \left|\frac{6 - 18 + 8 + 11}{\sqrt{49}}\right| = 1$$

141. (c) Go through the option (c)

142. (d) drs of line is 2, 3, 4
going through option,
2(1) + 3(2) + 4(–2) = 0
2(4) + 3(4) – 4(5) = 0

143. (c) Angle between planes

$$= \cos^{-1}\left(\frac{2 - 1 + 2}{6}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Distance between planes

$$= \frac{\left|\frac{2}{3} - 4\right|}{\sqrt{9}} = \frac{10}{3 \times 3} = \frac{10}{9}$$

144. (c) The equation of sphere is
$x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$.
Comparing the equation with general form of sphere,
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - d - 0$,

we get, $u = \frac{-6}{2} = -3, v = \frac{8}{2} = 4, w = \frac{-10}{2} = 5, d = 1$

Radius $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{(-3)^2 + (4)^2 + (5)^2 - 1}$

$= \sqrt{9 + 16 + 25 - 1} = \sqrt{49} = 7$

145. (a) The equation of plane passing through the intersection of planes $2x + y + 2z = 9$ and $4x - 5y - 4z = 1$ is
$(2x + y + 2z - 9) + \lambda(4x - 5y - 4z - 1) = 0$
Given that this plane passes through (3, 2, 1)
$\Rightarrow 2(3) + 2 + 2(1) - 9 + \lambda[4(3) - 5(2) - 4(1) - 1] = 0$
$\Rightarrow 1 + \lambda(-3) = 0$

$\Rightarrow \lambda = \frac{1}{3}$

$\therefore$ Equation is $(2x + y + 2z - 9)$

$+ \frac{1}{3}(4x - 5y - 4z - 1) = 0$

$\Rightarrow 6x + 3y + 6z - 27 + 4x - 5y - 4z - 1 = 0$
$\Rightarrow 10x - 2y + 2z - 28 = 0$
$\Rightarrow 10x - 2y + 2z = 28$

146. (a) Given planes : $4x - 2y + 4z + 9 = 0$
$\Rightarrow 8x - 4y + 8z + 18 = 0$ ...(1)
$8x - 4y + 8z + 21 = 0$ (2)

Distance $= \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|18 - 21|}{\sqrt{64 + 16 + 64}}$

$= \frac{3}{\sqrt{144}} = \frac{3}{12} = \frac{1}{4}$

147. (d)

Statistics 24

1. The production of food grains in Maharashtra is given for the 12 years from 1992 to 2003. Which one of the following representations is most suitable to depict the data ?
   (a) A simple bar diagram
   (b) A pie diagram
   (c) A component bar diagram with the components arranged in chronological order
   (d) A broken line graph *[2006-I]*
2. In a manufacture of ready-made garments, which average is used to find the most frequent size ?
   (a) Arithmetic mean (b) Geometric mean
   (c) Mode (d) Harmonic mean *[2006-I]*
3. Under what condition will the angle between two regression lines become zero ?
   (a) $r = 0$ (b) Only when $r = +1$
   (c) Only when $r = -1$ (d) $r = \pm 1$ *[2006-I]*
4. What is the arithmetic mean of the series $^nC_0, {}^nC_1, \ldots\ldots {}^nC_n$, ?
   (a) $\frac{2^n}{n}$ (b) $\frac{2^n}{(n+1)}$
   (c) $\frac{2^{(n+1)}}{n}$ (d) $\frac{2^{(n+1)}}{(n+1)}$ *[2006-I]*
5. The standard deviation of n observations $x_1, x_2, \ldots\ldots x_n$ is 6. The standard deviation of another set of n observations $y_1, y_2, \ldots\ldots, y_n$ is 8. What is the standard deviation of n observations $x_1 - y_1, x_2 - y_2, \ldots\ldots, x_n - y_n$?
   (a) 10 (b) 7
   (c) 14 (d) 2 *[2006-I]*
6. Following is the frequency distribution of life length in hours of 100 electric bulbs :

Life length of bulbs (in hrs)	8.5 - 13.5	13.5 - 18.5	18.5 - 23.5	23.5 - 28.5	28.5 - 33.5	33.5 - 38.5
No. of bulbs	7	x	40	y	10	2

   If the median of life length is 20 hours, then what are the missing frequencies (x, y) ?
   (a) (28, 13) (b) (23, 18)
   (c) (31, 10) (d) (25, 16) *[2006-I]*
7. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*

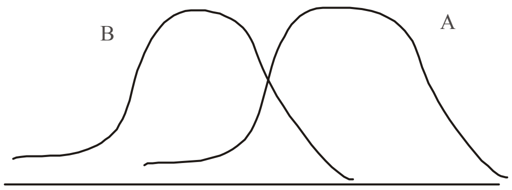
   Assertion (A) : We cannot find out the regression of x on y from that of y on x.
   Reason (R) : In one equation x is dependent variable and y is independent whereas in other equation y is dependent variable and x is independent.
   (a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
   (b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
   (c) **A** is true but **R** is false.
   (d) **A** is false but **R** is true. *[2006-I]*
8. If from the point of intersection of two ogives, a perpendicular is drawn on the x-axis, what does the x-coordinate give?
   (a) Arithmetic Mean (b) Mode
   (c) Median (d) Geometric Mean *[2006-II]*
9. The marks scored by two students A and B in six subjects are given below:

A	71	56	45	89	54	44
B	55	74	83	54	38	52

   Which one of the following statements is correct ?
   (a) The average scores of A and B are same but A is consistent
   (b) The average scores of A and B are not same but A is consistent
   (c) The average scores of A and B are same but B is consistent
   (d) The average scores of A and B are not same but B is consistent *[2006-II]*
10. If we join the mid points of the upper horizontal sides of each rectangle of a histogram by straight lines, what is the figure so obtained known as ?
   (a) Frequency curve (b) Frequency polygon
   (c) Ogive (>) (d) Ogive (<) *[2006-II]*
11. The definition of Mode fails if:
   (a) the maximum frequency is repeated
   (b) the maximum frequency is not repeated
   (c) the maximum frequency occurs in the middle
   (d) the curve drawn with the help of given data is symmetrical *[2006-II]*
12. A firm employing 30 workers and paying on an average Rs 500 is combined with another firm employing 20 workers paying on an average Rs 600. What is the average pay of the workers of the combined firm ?
   (a) Rs 540 (b) Rs 550
   (c) Rs 560 (d) Rs 580 *[2006-II]*

13. Which one of the following statement is ***not*** correct?
(a) Median divides distributions into two equal subgroups
(b) The third quartile is the same as the 75th percentile
(c) The 5th decile is the same as the 50th percentile
(d) The 50th decile is the same as the 5th percentile *[2007-I]*

14. The mean weight of all the students in a certain class is 60 kg. The mean weight of the boys from the class is 70 kg. while that of the girls is 55 kg. What is the ratio of number of boys to that of girls?
(a) 2 : 1 (b) 1 : 2
(c) 1:4 (d) 4 : 1 *[2007-I]*

15. 

Frequency curves for the distribution of blood pressure readings of certain athletes before exercise (A) and after exercise(B) are plotted together as shown in the figure above. From the frequency curves, which one of the following can be concluded?
(a) Both distributions are identical
(b) Both distributions have the same mean value
(c) Both distributions have the same mean value but different variance
(d) Both distributions have the same variance but different mean values *[2007-I]*

16. If the slopes of the line of regression of Y and X and of X and Y are 30° and 60° respectively, then r(X , Y) is :
(a) –1 (b) 1
(c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$ *[2007-I]*

17. If you want to measure the intelligence of a group of students,which one of the following measures will be more suitable?
(a) Arithmetic mean (b) Mode
(c) Median (d) Geometric mean *[2007-I]*

18. In a binomial distribution, the mean is 4 and the variance is 3. What is the mode?
(a) 6 (b) 5
(c) 4 (d) 3 *[2007-I]*

19. If X is changed to a + hU and Y to b + kV, then which one of the following is the correct relation between the regression coefficients b_{XY} and b_{UV}?
(a) $h\, b_{XY} = k\, b_{UV}$ (b) $k\, b_{XY} = h\, b_{UV}$
(c) $b_{XY} = b_{UV}$ (d) $k^2 b_{XY} = h^2 b_{UV}$ *[2007-I]*

20. Students of two schools appeared for a common test carrying 100 marks. The arithmetic means of their marks for school I and II are 82 and 86 respectively. If the number of students of school II is 1.5 times the number of students of school I, what is the arithmetic mean of the marks of all the students of both the schools? *[2007-II]*
(a) 84.0
(b) 84.2
(c) 84.4
(d) This cannot be calculated with the given data

21. If AM of numbers $x_1, x_2 .... x_n$ is μ, then what is the AM of the numbers which are increased by 1, 2, 3, ...n respectively?
(a) $\mu+\left(\frac{n+1}{2}\right)$ (b) μ *[2007-II]*
(c) $\mu+\frac{n(n+1)}{2}$ (d) $\mu-\left(\frac{n+1)}{2}\right)$

22. In computing a measure of the central tendency for any set of 51 numbers, which one of the following measures is well-defined but uses only very few of the numbers of the set?
(a) Arithmetic mean (b) Geometric mean *[2007-II]*
(c) Median (d) Mode

23. The data below record the itemwise quarterly expenditure of a private organization :

Item of expenditure	**Amount (in lakh rupees)**
1. Salaries	6.0
2. TA & DA	4.9
3. House rent and postage	3.6
4. All other expenses	5.5
Total :	20.0

The data is represented by a pie diagram. What is the sectorial angle of the sector with largest area?
(a) 120° (b) 108°
(c) 100° (d) 90° *[2007-II]*

24. *The following question consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answer.*
While constructing the cumulative frequency column of a frequency distribution, it is noticed that these cumulative frequencies are in arithmetic progression.
Assertion (A) : All the class frequencies are equal.
Reason (R) : When all the class frequencies are equal, the cumulative frequencies are in arithmetic progression.
(a) Both **A** and **R** are individually true, and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true. *[2007-II]*

25. If in a frequency distribution table with 12 classes, the width of each class is 2.5 and the lowest class boundary is 6.1, then what is the upper class boundary of the highest class?
(a) 30.1 (b) 27.6
(c) 30.6 (d) 36.1 *[2007-II]*

26. Consider the following statements :
The appropriate number of classes while constructing a frequency distribution should be chosen such that
1. the class-frequency first increases to a peak and then declines.
2. the class-frequency should cluster around the class mid point.
Which of the statements given is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2 *[2008-I]*

27. The populations of four towns A, B, C and D as on 2001 are as follows :

Town	Population
A	6863
B	519
C	12185
D	1755

What is the most appropriate diagram to present the above data?
(a) Pie diagram (b) Bar chart
(c) Cubic chart (d) Histogram *[2008-I]*

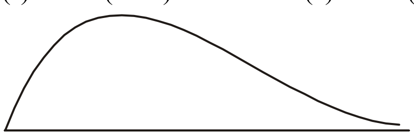
28. Consider the two series of observations A and B as follows:

Series A	1019	1008	1015	1006	1002
Series B	1.9	0.8	1.5	0.6	0.2

If the standard deviation of the Series A is $\sqrt{38}$, then what is the standard deviation of the Series B?
(a) 3.8 (b) $\sqrt{0.38}$
(c) 0.38 (d) $\sqrt{38}$ *[2008-I]*

29. If n_1 and n_2 are the sizes, G_1 and G_2 the geometric means of two series respectively, then which one of the following expresses the geometric mean (G) of the combined series?
(a) $\log G = \frac{n_1G_1 + n_2G_2}{n_1 + n_2}$
(b) $\log G = \frac{n_2 \log G_1 + n_1 \log G_2}{n_1 + n_2}$
(c) $G = \frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$
(d) None of the above *[2008-I]*

30. Let $\bar{x}$ be the mean of n observations $x_1, x_2, ..., x_n$. If (a – b) is added to each observation, then what is the mean of new set of observations?
(a) 0 (b) $\bar{x}$
(c) $\bar{x}$ – (a – b) (d) $\bar{x}$ + (a – b) *[2008-I]*

31. 

The frequency curve for the distribution of income in a region is positively skewed as shown in the figure above. Then, for this distribution
(a) Mean < Mode < Median
(b) Mode < Median < Mean
(c) Mode < Mean < Median
(d) Median < Mean < Mode *[2008-I]*

32. What is the value of *n* for which the numbers 1, 2, 3, ..., *n* have variance 2? *[2008-II]*
(a) 4 (b) 5
(c) 6 (d) 8

33. What is the arithmetic mean of the series ${}^nC_1, {}^nC_2, {}^nC_3 ... {}^nC_n$? *[2008-II]*
(a) $(2^n - 1)/n$ (b) $2^n/(n+1)$
(c) $(2^n)/n$ (d) $2^{(n+1)}/(n+1)$

34. The average age of 20 students in a class is 15 yr. If the teacher's age is included, the average increases by one. What is the teacher's age? *[2008-II]*
(a) 30 yr (b) 21 yr
(c) 42 yr (d) 36 yr

35.

X	1	2	3	4
Frequency	2	3	*f*	5

The frequency distribution of a discrete variable *X* with one missing frequency *f* is given above. If the arithmetic mean of *X* is $\frac{23}{8}$, what is the value of the missing frequency? *[2008-II]*
(a) 5 (b) 6
(c) 8 (d) 10

36. For a set of discrete numbers, three measures of central tendency are given below *[2008-II]*
1. Arithmetic mean
2. Median
3. Geometric mean

Which of the above measures may not have a meaningful definition?
(a) 1 only
(b) 2 only
(c) 3 only
(d) All of them are meaningfully defined

37. Consider the following three methods of collecting data
(1) collecting data from government offices *[2008-II]*
(2) collecting data from public libraries
(3) collecting data by telephonic interview

Select the correct answer using the code given below
(a) All the three methods give secondary data
(b) 1 and 2 give secondary and 3 gives primary data
(c) 1 and 3 give secondary and 2 gives primary data
(d) 2 and 3 give secondary and 1 gives primary data

38. The arithmetic mean of 4 numbers is 15. The arithmetic mean of another 6 numbers is 12. What is the arithmetic mean of the combined 10 numbers? *[2008-II]*
(a) 12.2 (b) 12.8
(c) 13.2 (d) 13.8

39. The average sales and standard deviation of sales for four months for a company are as follows :

	Month 1	**Month 2**	**Month 3**	**Month 4**
Average sales	30	57	82	28
Standard deviation of sales	2	3	4	2

During which month are the sales most consistent? *[2009-I]*
(a) Month 1 (b) Month 2
(c) Month 3 (d) Month 4

40. The marks scored by two students *A* and *B* in six subjects are given below

A	71	56	55	75	54	49
B	55	74	83	54	38	52

Which one of the following statements is most appropriate? *[2009-I]*
(a) The average scores of *A* and *B* are same but *A* is consistent
(b) The average scores of *A* and *B* are not same but *A* is consistent
(c) The average scores of *A* and *B* are same but *B* is consistent
(d) The average scores of *A* and *B* are not same but *B* is consistent

41. In a factory, there are 30 men and 20 women employees. If the average salary of men is Rs 4050 and the average salary of all the employees is Rs 3550, then what is the average salary of women? [2009-I]
(a) Rs 3800 (b) Rs 3300
(c) Rs 3000 (d) Rs 2800

42. What is the standard deviation of numbers 7, 9, 11, 13, 15? [2009-I]
(a) 2.2 (b) 2.4
(c) 2.6 (d) 2.8

43. If the monthly expenditure pattern of a person who earns a monthly salary of Rs 15000 is represented in a pie diagram, then the sector angle of an item on transport expenses measures 15°. What is his monthly expenditure on transport? [2009-I]
(a) Rs 450
(b) Rs 625
(c) Rs 675
(d) Cannot be computed from the given data

44. If $\sum_{i=1}^{n}(x_i-2)=110,\ \sum_{i=1}^{n}(x_i-5)=20,$ then what is the mean? [2009-I]
(a) 11/2 (b) 2/11
(c) 17/3 (d) 17/9

45. A class consists of 3 sections A, B and C with 35, 35 and 30 students respectively. The arithmetic means of the marks secured by students of sections A and B, who appeared for a test of 100 marks are 74 and 70 respectively. The arithmetic mean of the marks secured by students of section C, who appeared for a test in the same subject which carried 75 marks is 51. What is the average percentage of marks secured by all the 100 student of the three sections? [2009-II]
(a) 70.0 (b) 70.8
(c) 65.0 (d) 67.5

46. In a study on the relationship between investment (X) and profit (Y), the following two regression equations were obtained based on the data on X and Y [2009-II]
$3X+Y-12=0$
$X+2Y-14=0$
What is the mean $\overline{X}$?
(a) 6 (b) 5
(c) 4 (d) 2

47. Following table gives the mean and variance of monthly demand for four products A, B, C and D in a supermarket

Product	A	B	C	D
Mean demand	60	90	80	120
Variance	12	25	36	16

For which product the demand is consistent? [2009-II]
(a) Product A (b) Product B
(c) Product C (d) Product D

48. What is the least value of the standard deviation of 5 integers, no two of which are equal? [2009-II]
(a) $\sqrt{5}$
(b) 2
(c) $\sqrt{2}$
(d) No such least value can be computed

49. Correlation between two variable is said to be perfect if [2009-II]
(a) one variable increases, the other also increases
(b) one variable increases, the other decreases
(c) one variable increases, the other also increases proportionally
(d) one variable increases, the other decreases proportionally

50. Consider the following statements
I. The data, which are collected from the unit or individual respondents directly for the purpose of certain study or information are known as primary data.
II. The data obtained in a census study are primary data.
Which of the above statements is/are correct? [2009-II]
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

DIRECTIONS (Qs. 51-53) : *The table below gives an incomplete frequency distribution with two missing frequencies f_1 and f_2*

Value of x	Frequency
0	f_1
1	f_2
2	4
3	4
4	3

The total frequency is 18 and the arithmetic mean of x is 2.

51. What is the value of f_2? [2010-I]
(a) 4 (b) 3
(c) 2 (d) 1

52. What is the standard deviation? [2010-I]
(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{3}$
(c) $\frac{4}{3}$ (d) $\frac{16}{9}$

53. What is the coefficient of variance? [2010-I]
(a) $\frac{200}{3}$ (b) $\frac{50\sqrt{5}}{9}$
(c) $\frac{600}{\sqrt{5}}$ (d) 150

54. What is the mean deviation of the data 2, 9, 9, 3, 6, 9, 4?
(a) 2.23 (b) 2.57 [2010-II]
(c) 3.23 (d) 3.57

55. A set of n values $x_1, x_2, ..., x_n$ has standard deviation σ. What is the standard deviation of n values $x_1+k, x_2+k....,x_n+k$? [2010-II]
(a) σ (b) σ+k
(c) σ−k (d) kσ

56. The two lines of regression are $8x - 10y = 66$ and $40x - 18y = 214$ and variance of x series is 9. What is the standard deviation of y series? [2010-II]
(a) 3 (b) 4
(c) 6 (d) 8

57. The standard deviation of some consecutive integers is found to be 2. Which of the following statements best describes the nature of the consecutive integers? [2010-II]

(a) The integers are any set of eight consecutive integers
(b) The integers are any set of eight consecutive positive integers
(c) The integers are any set of seven consecutive integers
(d) None of the above

58. Consider the following data : *[2010-II]*

	Factory - A	Factory - B
Mean wage of workers	₹ 540	₹ 620
Standard deviation of wages	₹ 40.50	₹ 31

What is the variability in the wages of the workers in Factory - *A*?
(a) 100 % more than the variability in the wages of the workers in Factory - *B*
(b) 50% more than the variability in the wages of the workers in Factory -*B*
(c) 50% less than the variability in the wages of the workers in Factory-*B*
(d) 150% more than the variability in the wages of the workers in Factory-*B*

59. The distributions *X* and *Y* with total number of observations 36 and 64, and mean 4 and 3 respectively are combined. What is the mean of the resulting distribution $X + Y$? *[2010-II]*
(a) 3.26 (b) 3.32
(c) 3.36 (d) 3.42

60. Consider the following data : *[2010-II]*

x	5	7	8	4	6
y	2	4	3	2	4

What is the regression equation of *y* on *x*?
(a) $y = 0.6 + 0.4x$ (b) $y = 0.7 + 0.3x$
(c) $y = 6 + 5x$ (d) $y = 4 + 9x$

DIRECTIONS (Qs. 61-63) : *The frequency distribution of life of 90 TV tubes whose median life is 17 months is as follows*

Life of TV tubes (in months)	No. of TV tubes
0-5	3
5-10	12
10-15	x
15-20	35
20-25	y
25-30	4

$\because$ n = 90

$\because$ $\frac{n}{2} = 45$

(For qs. 61-63)

Calss	Frequency	*cf*
0-5	3	3
5-10	12	15
10-15	x	15 + x
15-20	35	50 + x
20-25	y	50 + x + y
25-30	4	54 + x + y

61. What is the lower limit of the median class ? *[2010-II]*
(a) 10 (b) 15
(c) 20 (d) 25

62. What is the missing frequency *y* ? *[2010-II]*
(a) 20 (b) 16
(c) 15 (d) 12

63. What is the cumulative frequency of the modal class ? *[2010-II]*
(a) 31
(b) 35
(c) 66
(d) Cannot be determined as the given data is insufficient.

64.

Class Interval	1-5	6-10	11-15	16-20
Frequency	3	7	6	5

Consider the following statements in respect of the above frequency distribution.
I. The median is contained in the modal class.
II. The distribution is bell-shaped.
Which of the above statements is/are correct? *[2011-I]*
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

DIRECTIONS (Qs. 65-66) : *The following table gives the continuous frequency distribution of a continuous variable X*

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	10	20	5	10

65. What is the median of the above frequency distribution? *[2011-I]*
(a) 23 (b) 24
(c) 25 (d) 26

66. What is the mean of the above frequency distribution?
(a) 25 (b) 26 *[2011-I]*
(c) 27 (d) 28

67. Consider the following statements with regard to correlation coefficient *r* between random variables *x* and *y*. *[2011-I]*
I. $r = +1$ or -1 means there is a linear relationship between the variables.
II. $-1 \le r \le 1$ and r^2 is a measure of the linear relationship between the variables.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

68. If the values of a set are measured in cm, what will be the unit of variance? *[2011-I]*
(a) cm (b) cm^2
(c) cm^3 (d) No unit

69. What is the cumulative frequency curve of statistical data commonly called? *[2011-I]*
(a) Cartogram (b) Histogram
(c) Ogive (d) Pictogram

70. The average daily income of workers of a factory including that of the owner is ₹ 110. However, if the income of the owner is excluded, the average daily income of the remaining 9 workers is ₹ 76. What is the daily income of the owner?
(a) ₹ 300 (b) ₹ 316 *[2011-I]*
(c) ₹ 322 (d) ₹ 416

71. Which one of the following is the mean of the data given below? *[2011-II]*

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

(a) 17 (b) 18
(c) 19 (d) 20

72. Students of three sections of a class, having 30, 30 and 40 students appeared for a test of 100 marks. The arithmetic means of the marks of the three sections are 72.2, 69.0 and 64.1 in that order. What is the arithmetic mean of the marks of all the students of the three sections? *[2011-II]*
(a) 66.6 (b) 67.3
(c) 68.0 (d) 70.6

73. If the variance of the data 2, 4, 5, 6, 17 is v, then what is the variance of the data 4, 8, 10, 12, 34? *[2011-II]*
(a) v (b) $4v$
(c) v^2 (d) $2v$

74. The mean of 7 observations is 10 and that of 3 observations is 5. What is the mean of all the 10 observations? *[2011-II]*
(a) 15 (b) 10
(c) 8.5 (d) 7.5

75. Some measures of central tendency for n discrete observations are given below: *[2011-II]*
1. Arithmetic mean 2. Geometric mean
3. Harmonic mean 4. Median

A desirable property of a measure of central tendency is if every observation is multiplied by c, then the measure of central tendency is also multiplied by c, where $c > 0$. Which of the above measures satisfy the property?
(a) 1, 2 and 3 only
(b) 1, 2 and 4 only
(c) 3 and 4 only
(d) 1, 2, 3 and 4

76. A variate X takes values 2, 3, 4, 2, 5, 4, 3, 2, 1. What is the mode? *[2011-II]*
(a) 2 (b) 3
(c) 4 (d) 5

DIRECTIONS (Qs. 77- 84) :

Note : *Study the following Table and Answer the next 08 (Eight) Questions that follow:*

Year	Male			Female			Total
	Urban	Rural	Total	Urban	Rural	Total	
1995	280	350			310		1350
1996	370		670	180		450	
1997		130	440		190		
1998	400	280		290			
Total				1060	850		

77. What is the total population for the year 1997? *[2011-II]*
(a) 810 (b) 830
(c) 970 (d) 1030

78. What is the female urban population in the year 1995? *[2011-II]*
(a) 390 (b) 410
(c) 430 (d) 470

79. What is the urban population in the year 1997? *[2011-II]*
(a) 400 (b) 460
(c) 490 (d) 510

80. What is the total population in the year 1998? *[2011-II]*
(a) 1000 (b) 1020
(c) 1040 (d) 1050

81. What is the difference between the number of females and the number of males in the year 1995? *[2011-II]*
(a) 90 (b) 100
(c) 110 (d) 120

82. In which year is the male population minimum? *[2011-II]*
(a) 1995 (b) 1996
(c) 1997 (d) 1998

83. In which year is the female population maximum?
(a) 1995 (b) 1996 *[2011-II]*
(c) 1997 (d) 1998

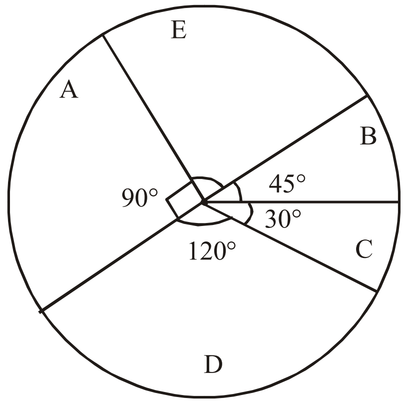
84. What is the percentage of rural male population (over the whole population) in the year 1998? *[2011-II]*
(a) $\frac{80}{3}\%$ (b) $\frac{100}{3}\%$
(c) 35% (d) 40%

DIRECTIONS (Qs. 85- 88) :

Note : *Study the pie chart given below and answer the next 04 (four) questions that follow :*

The following pie chart gives the distribution of funds in a five year plan under the major heads of development expenditures: Agriculture (A), Industry (B), Education (C), Employment (D) and Miscellaneous (E)

The total allocation is 36,000 (in crores of rupees).



85. Which head is allocated maximum funds? *[2011-II]*
(a) Agriculture (b) Industry
(c) Employment (d) Miscellaneous

86. How much money (in crores) is allocated to Education? *[2011-II]*
(a) 3000 (b) 6000
(c) 9000 (d) 10800

87. How much money (in crores) is allocated to both Agriculture and Employment? *[2011-II]*
(a) 20000 (b) 21000
(c) 24000 (d) 27000

88. How much excess money (in crores) is allocated to Miscellaneous over Education? *[2011-II]*
(a) 3600 (b) 4200
(c) 4500 (d) 4800

89. What is the median of the distribution 3, 7, 6, 9, 5, 4, 2 ?
(a) 5 (b) 6 *[2011-II]*
(c) 7 (d) 8

90. What is the arithmetic mean of first 16 natural numbers with weights being the number itself ? *[2012-I]*
(a) $\frac{17}{2}$ (b) $\frac{33}{2}$ (c) 11 (d) $\frac{187}{2}$

91. What is the mode for the data 20, 20, 20, 21, 21, 21, 21, 21, 22, 22, 22, 22, 22, 22, 22, 23, 23, 23, 23, 23, 24, 24, 25?
(a) 7 (b) 21 *[2012-I]*
(c) 22 (d) 25

92. Consider the following statements:
    1. A continuous random variable can take all values in an interval.
    2. A random variable which takes a finite number of values is necessarily discrete.
    3. Construction of a frequency distribution is based on data which are discrete.

    Which of the above statements are correct? *[2012-I]*
    (a) 1 and 2 only (b) 2 and 3 only
    (c) 1 and 3 only (d) 1, 2 and 3
93. Consider the following statements:
    1. Two independent variables are always uncorrelated.
    2. The coefficient of correlation between two variables X and Y is positive when X decreases then Y decreases.

    Which of the above statements is/are correct? *[2012-I]*
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2
94. A variate X takes values 2, 9, 3, 7, 5, 4, 3, 2, 10. What is the median? *[2012-I]*
    (a) 2 (b) 4
    (c) 7 (d) 9
95. The mean of 10 observations is 5. If 2 is added to each observation and then multiplied by 3, then what will be the new mean ? *[2012-II]*
    (a) 5 (b) 7
    (c) 15 (d) 21
96. What is the mean of first n odd natural numbers ? *[2012-II]*
    (a) n (b) $\frac{(n+1)}{2}$
    (c) $\frac{n(n+1)}{2}$ (d) n + 1
97. The arithmetic mean of numbers a, b, c, d, e is M. What is the value of (a – M) + (b – M) + (c – M) + (d – M) + (e – M) ? *[2012-II]*
    (a) M (b) a + b + c + d + e
    (c) 0 (d) 5 M
98. The algebraic sum of the deviations of 20 observations measured from 30 is 2. What would be the mean of the observations? *[2012-II]*
    (a) 30 (b) 32
    (c) 30.2 (d) 30.1
99. The median of 27 observations of a variable is 18. Three more observations are made and the values of these observations are 16, 18 and 50. What is the median of these 30 observations ? *[2012-II]*
    (a) 18
    (b) 19
    (c) 25.5
    (d) Can not be determined due to insufficient data
100. Frequency curve may be: *[2012-II]*
    (a) symmetrical (b) positive skew
    (c) negative skew (d) all the above
101. The monthly family expenditure (in percentage) on different items are as follows : *[2012-II]*

Food	Rent	Cloth	Transport	Education	Others
38	19	18	–	9	6

    If the total monthly expenditure is ₹ 9000, then what is the expenditure on transport ?
    (a) ₹ 180 (b) ₹ 1000
    (c) ₹ 900 (d) ₹ 360
102. If the mean of few observations is 40 and standard deviation is 8, then what is the coefficient of variation ? *[2012-II]*
    (a) 1% (b) 10%
    (c) 20% (d) 30%
103. What is the standard deviation of 7, 9, 11, 13, 15 ? *[2012-II]*
    (a) 2.4 (b) 2.5
    (c) 2.7 (d) 2.8
104. Which one of the following is a measure of dispersion ? *[2012-II]*
    (a) Mean (b) Median
    (c) Mode (d) Standard deviation
105. Let X and Y be two related variables. The two regression lines are given by x – y + 1 = 0 and 2x – y + 4 = 0. The two regression lines pass through the point : *[2012-II]*
    (a) (–4, –3) (b) (– 6, – 5)
    (c) (3, – 2) (d) (– 3, – 2)
106. The marks obtained by 13 students in a test are 10, 3, 10, 12, 9, 7, 9, 6, 7, 10, 8, 6, 7. The median of this data is ? *[2013-I]*
    (a) 7 (b) 8
    (c) 9 (d) 10
107. Consider the following statements:
    1. Both variance and standard deviation are measures of variability in the population.
    2. Standard deviation is the square of the variance.

    Which of the above statements is/are correct? *[2013-I]*
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2
108. Consider the following frequency distribution :

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	14	x	27	y	15

    If the total of the frequencies is 100 and mode is 25, then which one of the following is correct? *[2013-I]*
    (a) x = 2y (b) 2x = y
    (c) x = y (d) x = 3y
109. The average marks obtained by the students in a class are 43. If the average marks obtained by 25 boys are 40 and the average marks obtained by the girl students are 48, then what is the number of girl students in the class ? *[2013-I]*
    (a) 15 (b) 17
    (c) 18 (d) 20
110. Marks obtained by 7 students in a subject are 30, 55, 75, 90, 50, 60, 39. The number of students securing marks less than the mean marks is *[2013-I]*
    (a) 7 (b) 6
    (c) 5 (d) 4
111. Variance is always independent of the change of *[2013-I]*
    (a) origin but not scale
    (b) scale only
    (c) both origin and scale
    (d) None of the above
112. If two lines of regression are perpendicular, then the correlation coefficient r is *[2013-I]*
    (a) 2 (b) $\frac{1}{2}$
    (c) 0 (d) None of the above
113. The standard deviation of the observations 5, 5, 5, 5, 5 is *[2013-I]*
    (a) 0 (b) 5
    (c) 20 (d) 25

114. The mean of 20 observations is 15. On checking, it was found that two observations were wrongly copied as 3 and 6. If wrong observations are replaced by correct values 8 and 4, then the correct mean is [2013-II]
(a) 15 (b) 15.15
(c) 15.35 (d) 16

115. The arithmetic mean of the squares of the first n natural numbers is [2013-II]
(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)(2n+1)}{2}$
(c) $\frac{(n+1)(2n+1)}{6}$ (d) $\frac{(n+1)(2n+1)}{3}$

116. Consider the following statements : [2013-II]
1. Both the regression coefficients have same sign.
2. If one of the regression coefficients is greater than unity, the other must be less than unity.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

117. Which one of the following measures is determined only after the construction of cumulative frequency distribution ? [2013-II]
(a) Arithmetic mean (b) Mode
(c) Median (d) Geometric mean

118. Coefficient of correlation is the measure of [2013-II]
(a) central tendency
(b) dispersion
(c) both central tendency and dispersion
(d) neither central tendency nor dispersion

119. What is the variance of the first 11 natural numbers ? [2013-II]
(a) 10 (b) 11
(c) 12 (d) 13

120. Consider the following statements : [2013-II]
1. The algebraic sum of the deviations of a set of n values from its arithmetic mean is zero.
2. In the case of frequency distribution, mode is the value of variable which corresponds to maximum frequency.
Which of the statements above given is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

121. Consider the following statements : [2013-II]
1. Pie diagrams are suitable for categorical data.
2. The arc length of a sector of a pie diagram is proportional to the value of the component represented by the sector.
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

122. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the new variance of the resulting observations ? [2013-II]
(a) 5 (b) 10
(c) 20 (d) 40

123. For two variables x and y, the two regression coefficients are $b_{yx} = -3/2$ and $b_{xy} = -1/6$. The correlation coefficient between x and y is : [2014-I]
(a) –1/4 (b) 1/4
(c) –1/2 (d) 1/2

124. The variance of numbers $x_1, x_2, x_3, ...x_n$ is V. Consider the following statements : [2014-I]
1. If every x_1 is increased by 2, the variance of the new set of the new set of numbers is V.
2. If the numbers x_i is squared, the variance of the new set is V^2.
Which of the following statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1nor 2

125. What is the mean of the squares of the first 20 natural numbers ? [2014-I]
(a) 151.5 (b) 143.5
(c) 65 (d) 72

126. The cumulative frequency of the largest observed value must always be : [2014-I]
(a) Less than the total number of observations
(b) Greater than the total number of observations
(c) Equal to total number of observations
(d) Equal to mid point of the last class interval

127. Let X denote the number of scores which exceed 4 in 18, tosses of a symmetrical die. Consider the following statements : [2014-I]
1. The arithmetic mean of X is 6.
2. The standard deviation of X is 2.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS: (Qs. 128 - 130) *For the next three (03) items that follow* **:**

Number of telephone calls received in 245 succesive one minute intervals at an exchange is given below in the following frequency distribution. [2014-I]

Number of calls	0	1	2	3	4	5	6	7
Frequency	14	21	25	43	51	40	39	12

128. What is the mean of the distribution ?
(a) 3.76 (b) 3.84
(c) 3.96 (d) 4.05

129. What is the median of the distribution ?
(a) 3.5 (b) 4
(c) 4.5 (d) 5

130. What is the mode of the distribution ?
(a) 3 (b) 4
(c) 5 (d) 6

DIRECTIONS: (Qs. 131-133) *For the next three (03) items that follow :*

The mean and standard deviation of 100 items are 50, 5 and that of 150 items are 40, 6 respectively. [2014-I]

131. What is the combined mean of all 250 items ?
(a) 43 (b) 44
(c) 45 (d) 46

132. What is the combined standard deviation of all 250 items ?
(a) 7.1 (b) 7.3
(c) 7.5 (d) 7.7

133. What is the variance of all 250 items ?
(a) 50.6 (b) 53.3
(c) 55.6 (d) 59.6

134. Consider the following statements in respect of histogram : [2014-II]
   1. The histogram is a suitable representation of a frequency distribution of a continuous variable.
   2. The area included under the whole histogram is the total frequency.

   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2
135. The regression lines will be perpendicular to each other if the coefficient of correlation *r* is equal to [2014-II]
   (a) 1 only (b) 1 or –1
   (c) –1 only (d) 0
136. If $\overline{x}$ and $\overline{y}$ are the means of two distrubutions such that $\overline{x} < \overline{y}$ and $\overline{z}$ is the mean of the combined distrubution, then which one of the following statements is correct ? [2014-II]
   (a) $\overline{x} < \overline{y} < \overline{z}$ (b) $\overline{x} > \overline{y} > \overline{z}$
   (c) $\overline{z} = \frac{\overline{x} + \overline{y}}{2}$ (d) $\overline{x} < \overline{z} < \overline{y}$
137. What is the mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 ? [2014-II]
   (a) 2.5 (b) 3
   (c) 3.5 (d) 4
138. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the new varianve of the resulting observations ? [2014-II]
   (a) 5 (b) 10
   (c) 20 (d) 40
139. The mean and the variance 10 observations are given to be 4 and 2 respectively. If every observation is multiplied by 2, the mean and the variance of the new series will be respectively [2015-I]
   (a) 8 and 20 (b) 8 and 4
   (c) 8 and 8 (d) 80 and 40
140. Which one of the following measures of central tendency is used in construction of index numbers? [2015-I]
   (a) Harmonic mean (b) Geometric mean
   (c) Median (d) Mode
141. The correlation coefficient between two variables *X* and *Y* is found to be $0 \cdot 6$. All the observations on *X* and *Y* are transformed using the transformations $U = 2 - 3X$ and $V = 4Y + 1$. The correlation coefficient between the transformed variables *U* and *V* will be [2015-I]
   (a) $-0 \cdot 5$ (b) $+0 \cdot 5$
   (c) $-0 \cdot 6$ (d) $+0 \cdot 6$
142. Which of the following statements is/are correct in respect of regression coefficients? [2015-I]
   1. It measures the degree of linear relationship between two variables.
   2. It gives the value by which one variable changes for a unit change in the other variable.

   Select the correct answer using the code given below.
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2
143. A set of annual numerical data, comparable over the years, is given for the last 12 years. [2015-I]
   1. The data is best represented by a broken line graph, each corner (turning point) representing the data of one year.
   2. Such a graph depicts the chronological change and also enables one to make a short-term forecast.

   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2
144. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is [2015-I]
   (a) 28 (b) 30
   (c) 35 (d) 38
145. The 'less than' ogive curve and the 'more than' ogive curve intersect at [2015-I]
   (a) median (b) mode
   (c) arithmetic mean (d) None of these
146. The geometric mean of the observations $x_1, x_2, x_3, \ldots\ldots x_n$ is G_1, The geometric mean of the observations $y_1, y_2, y_3, \ldots. y_n$ is G_2. The geometric mean of observations $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \ldots. \frac{x_n}{y_n}$ is [2015-II]
   (a) G_1G_2 (b) $\ln(G_1G_2)$
   (c) $\frac{G_1}{G_2}$ (d) $\ln\left(\frac{G_1}{G_2}\right)$
147. The arithmetic mean of 1, 8, 27, 64,..... up to n terms is given by [2015-II]
   (a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$
   (c) $\frac{n(n+1)^2}{4}$ (d) $\frac{n^2(n+1)^2}{4}$
148. The regression coefficients of a bivariate distribution are –0.64 and –0.36. Then the correlation coefficient of the distribution is [2015-II]
   (a) 0.48 (b) – 0.48
   (c) 0.50 (d) – 0.50
149. What is the mean deviation from the mean of the numbers 10, 9, 21, 16, 24 ? [2016-I]
   (a) 5·2 (b) 5·0 (c) 4·5 (d) 4·0
150. If the total number of observations is 20, $\Sigma x_i = 1000$ and $\Sigma x_i^2 = 84000$, then what is the variance of the distribution? [2016-I]
   (a) 1500 (b) 1600
   (c) 1700 (d) 1800
151. The mean of the series $x_1, x_2, \ldots, x_n$ is $\overline{X}$. If x_2 is replaced by λ, then what is the new mean? [2016-I]
   (a) $\overline{X} - x_2 + \lambda$ (b) $\frac{\overline{X} - x_2 - \lambda}{n}$
   (c) $\frac{\overline{X} - x_2 + \lambda}{n}$ (d) $\frac{n\overline{X} - x_2 + \lambda}{n}$
152. For the data

   3, 5, 1, 6, 5, 9, 5, 2, 8, 6

   the mean, median and mode are x, y and z respectively. Which one of the following is correct? [2016-I]
   (a) $x = y \neq z$ (b) $x \neq y = z$
   (c) $x \neq y \neq z$ (d) $x = y = z$

153. Consider the following statements in respect of a histogram: *[2016-I]*

1. The total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequnecy polygon and the x-axis.
2. When class intervals are unequal in a frequency distribution, the area of the rectangle is proportional to the frequency.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

154. Consider the following statements: *[2016-II]*

1. The mean and median are equal in symmetric distribution.
2. The range is the difference between the maximum value and the minimum value in the data.
3. The sum of the areas of the rectangles in the histogra is equal to the total area bounded by the frequency polygon and the horizontal axis.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

155. The scores of 15 students in an examination were recorded as 10, 5, 8, 16, 18, 20, 8, 10, 16, 20, 18, 11, 16, 14 and 12. After calculating the mean, median and mode, an error is found. One of the values is wrongly written as 16 instead of 18. Which of the following measures of central tendency will change? *[2016-II]*

(a) Mean and median (b) Median and mode
(c) Mode only (d) Mean and mode

156. For 10 observations on price (x) and supply (y), the following data was obtained : *[2016-II]*

$\sum x = 130, \sum y = 220,$

$\sum x^2 = 2288, \sum y^2 = 5506$ and $\sum xy = 3467$.

What is line of regression of y on x?

(a) $y = 0.91x + 8.74$ (b) $y = 1.02x + 8.74$
(c) $y = 1.02x - 7.02$ (d) $y = 0.91x - 7.02$

157. In a study of two groups, the following results were obtained: *[2016-II]*

	Group A	Group B
Sample Size	20	25
Sample mean	22	23
Sample standard deviation	10	12

Which of the following statements is correct?

(a) Group A is less variable than Group B because Group A's standard deviation is smaller.
(b) Group A is less variable than Group B because Group A's sample size is smaller.
(c) Group A is less variable than Group B because Group A's sample mean is smaller.
(d) Group A is less variable than group B because Group A's coefficient of variation is smaller.

158. Consider the following statements in respect of class intervals of grouped frequency distribution: *[2016-II]*

1. Class intervals need not be mutually exclusive.
2. Class intervals should be exhaustive.
3. Class intervals need not be of equal width.

Which of the above statements are correct?

(a) 1 only 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

159. Two variates, x and y, are uncorrelated and have standard deviations σ_x and σ_y respectively. What is the correlation coefficient between x+y and x – y? *[2016-II]*

(a) $\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$ (b) $\frac{\sigma_x+\sigma_y}{2\sigma_x\sigma_y}$

(c) $\frac{\sigma_x^2-\sigma_y^2}{\sigma_x^2+\sigma_y^2}$ (d) $\frac{\sigma_y-\sigma_x}{\sigma_x\sigma_y}$

160. A random sample of 20 people is classified in the following table according to their ages: *[2016-II]*

Age	Frequency
15 – 25	2
25 – 35	4
35 – 45	6
45 – 55	5
55 – 65	3

What is the mean age of this group of people?

(a) 41.0 (b) 41.5
(c) 42.0 (d) 42.5

161. If the covariance between x and y is 30, variance of x is 25 and variance of y is 144, then what is the correlation coefficient? *[2016-II]*

(a) 0.4 (b) 0.5
(c) 0.6 (d) 0.7

162. The variance of 20 observations is 5. If each observation is multiplied by 3, then what is the new variance of the resulting observations? *[2017-I]*

(a) 5 (b) 10
(c) 15 (d) 45

163. The mean of a group of 100 observations was found to be 20. Later it was found that four observations were incorrect, which were recorded as 21, 21, 18 and 20. What is the mean if the incorrect observations are omitted? *[2017-I]*

(a) 18 (b) 20
(c) 21 (d) 22

164. If two regression lines between height (x) and weight (y) are $4y - 15x + 410 = 0$ and $30x - 2y - 825 = 0$, then what will be the correlation coefficient between height and weight? *[2017-I]*

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

165. In an examination, 40% of candidates got second class. When the data are represented by a pie chart, what is the angle corresponding to second class? *[2017-I]*

(a) 40° (b) 90°
(c) 144° (d) 320°

166. Consider the following statements : *[2017-I]*

Statement 1 : Range is not a good measure of dispersion.

Statement 2 : Range is highly affected by the existence of extreme values.

Which one of the following is correct in respect of the above statements?

(a) Both Statement 1 and Statement 2 are correct and Statement 2 is the correct explanation of Statement 1

(b) Both Statement 1 and Statement 2 are correct but Statement 2 is not the correct explanation of Statement 1
(c) Statement 1 is correct but Statement 2 is not correct
(d) Statement 2 is correct but Statement 1 is not correct

167. If the data are moderately non-symmetrical, then which one of the following empirical relationships is correct? *[2017-I]*
(a) 2 × Standard deviation = 5 × Mean deviation
(b) 5 × Standard deviation = 2 × Mean deviation
(c) 4 × Standard deviation = 5 × Mean deviation
(d) 5 × Standard deviation = 4 × Mean deviation

168. Data can be represented in which of the following forms?
1. Textual form 2. Tabular form
3. Graphical form
Select the correct answer using the code given below. *[2017-I]*
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

169. For given statistical data, the graphs for less than ogive and more than ogive are drawn. If the point at which the two curves intersect is P, then abscissa of point P gives the value of which one of the following measures of central tendency? *[2017-I]*
(a) Median (b) Mean
(c) Mode (d) Geometric mean

170. If the regression coefficient of x on y and y on x are $-\frac{1}{2}$ and $-\frac{1}{8}$ respectively, then what is the correlation coefficient between x and y? *[2017-I]*
(a) $-\frac{1}{4}$ (b) $-\frac{1}{16}$
(c) $\frac{1}{16}$ (d) $\frac{1}{4}$

171. A sample of 5 observations has mean 32 and median 33. Later it is found that an observation was recorded incorrectly as 40 instead of 35. If we correct the data, then which one of the following is correct? *[2017-I]*
(a) The mean and median remain the same
(b) The median remains the same but the mean will decrease
(c) The mean and median both will decrease
(d) The mean remains the same but median will decrease

172. Consider the following statements : *[2017-II]*
1. Coefficient of variation depends on the unit of measurement of the variable.
2. Range is a measure of dispersion.
3. Mean deviation is least when measured about median.
Which of the above statements are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

173. Given that the arithmetic mean and standard deviation of a sample of 15 observations are 24 and 0 respectively. Then which one of the following is the arithmetic mean of the smallest five observations in the data? *[2017-II]*
(a) 0 (b) 8
(c) 16 (d) 24

174. Which one of the following can be considered as appropriate pair of values of regression coefficient of y on x and regression coefficient of x on y? *[2017-II]*
(a) (1, 1) (b) (–1, 1)
(c) $\left(-\frac{1}{2}, 2\right)$ (d) $\left(\frac{1}{3}, \frac{10}{3}\right)$

175. It is given that $\bar{X} = 10, \bar{Y} = 90, \sigma_X = 3, \sigma_Y = 12$ and $r_{XY} = 0.8$. The regression equation of X on Y is *[2017-II]*
(a) $Y = 3.2X + 58$ (b) $X = 3.2Y + 58$
(c) $X = -8 + 0.2Y$ (d) $Y = -8 + 0.2X$

176. The following table gives the monthly expenditure of two families :

	Expenditure (in ₹)	
Items	**Family A**	**Family B**
Food	3,500	2,700
Clothing	500	800
Rent	1,500	1,000
Education	2,000	1,800
Miscellaneous	2,500	1,800

In constructing a pie diagram to the above data, the radii of the circles are to be chosen by which one of the following ratios? *[2017-II]*
(a) 1 : 1 (b) 10 : 9
(c) 100 : 91 (d) 5 : 4

177. If a variable takes values 0, 1, 2, 3, ..., n with frequencies 1, C(n, 1), C(n, 2), C(n, 3), ..., C(n, n) respectively, then the arithmetic mean is *[2017-II]*
(a) 2n (b) n + 1
(c) n (d) $\frac{n}{2}$

178. Consider the following statements : *[2017-II]*
1. Variance is unaffected by change of origin and change of scale.
2. Coefficient of variance is independent of the unit of observations.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

179. The coefficient of correlation when coefficients of regression are 0.2 and 1.8 is *[2017-II]*
(a) 0.36 (b) 0.2
(c) 0.6 (d) 0.9

180. In a Binominal distribution, the mean is three times its variance. What is the probability of exactly 3 successes out of 5 trials? *[2018-I]*
(a) $\frac{80}{243}$ (b) $\frac{40}{243}$
(c) $\frac{20}{243}$ (d) $\frac{10}{243}$

181. If the correlation coefficient between x and y is 0.6, covariance is 27 and variance of y is 25, then what is the variance of x? *[2018-I]*
(a) $\frac{9}{5}$ (b) $\frac{81}{25}$
(c) 9 (d) 81

182. Let $\overline{x}$ be the mean of $x_1, x_2, x_3, ..., x_n$. If $x_i = a + cy_i$ for some constants a and c, then what will be the mean of $y_1, y_2, y_3, ..., y_n$? *[2018-I]*

(a) $a + c\overline{x}$ (b) $a - \frac{1}{c}\overline{x}$

(c) $\frac{1}{c}\overline{x} - a$ (d) $\frac{\overline{x} - a}{c}$

183. Consider the following statements: *[2018-I]*

1. If the correlation coefficient $r_{xy} = 0$, then the two lines of regression are parallel to each other.
2. If the correlation coefficient $r_{xy} = +1$, then the two lines of regression are perpendicular to each other.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

184. If $4x - 5y + 33 = 0$ and $20x - 9y = 107$ are two lines of regression, then what are the values of $\overline{x}$ and $\overline{y}$ respectively? *[2018-I]*

(a) 12 and 18 (b) 18 and 12
(c) 13 and 17 (d) 17 and 13

185. Consider the following statements: *[2018-I]*

1. Mean is independent of change in scale and change in origin.
2. Variance is independent of change in scale but not in origin.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

186. Consider the following statements: *[2018-I]*

1. The sum of deviations from mean is always zero.
2. The sum of absolute deviations is minimum when taken around median.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

187. What is the median of the numbers 4.6, 0, 9.3, –4.8, 7.6, 2.3, 12.7, 3.5, 8.2, 6.1, 3.9, 5.2? *[2018-I]*

(a) 3.8 (b) 4.9
(c) 5.7 (d) 6.0

188. In a test in Mathematics, 20% of the students obtained "first class". If the data are represented by a Pie-Chart, what is the central angle corresponding to "first class"? *[2018-I]*

(a) 20° (b) 36°
(c) 72° (d) 144°

189. The mean and standard deviation of a set of values are 5 and 2 respectively. If 5 is added to each value, then what is the coefficient of variation for the new set of values? *[2018-I]*

(a) 10 (b) 20
(c) 40 (d) 70

190. The standard deviation σ of the first N natural numbers can be obtained using which one of the following formulae? *[2018-I]*

(a) $\sigma = \frac{N^2 - 1}{12}$ (b) $\sigma = \sqrt{\frac{N^2 - 1}{12}}$

(c) $\sigma = \sqrt{\frac{N - 1}{12}}$ (d) $\sigma = \sqrt{\frac{N^2 - 1}{6N}}$

191. The correlation coefficient computed from a set of 30 observations is 0.8. Then the percentage of variation not explained by linear regression is *[2018-II]*

(a) 80% (b) 20%
(c) 64% (d) 36%

192. The average age of a combined group of men and women is 25 years. If the average age of the group of men is 26 years and the of the group of women is 21 years, then the percentage of men and women in the group is respectively *[2018-II]*

(a) 20, 80 (b) 40, 60
(c) 60, 40 (d) 80, 20

193. Consider the following statements: *[2018-II]*

1. If 10 is added to each entry on a list then the average increase by 10.
2. If 10 is added to each entry on a list, then the standard deviation increase by 10.
3. If each entry on a list is doubled, then the average doubles.

Which of the above statement are correct?

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

194. The variance of 25 observations is 4. If 2 is added to each observation, then the new variance of the resulting observations is *[2018-II]*

(a) 2 (b) 4
(c) 6 (d) 8

195. If the regression coefficient of Y on X is – 6, and the correlation coefficient between X and Y $-\frac{1}{2}$, then the regression coefficient of X on Y would be *[2018-II]*

(a) $\frac{1}{24}$ (b) $-\frac{1}{24}$

(c) $-\frac{1}{6}$ (d) $\frac{1}{6}$

196. The set of bivariate observation $(x_1, y_1), (x_2, y_2)$, ... (xn, yn) are such that all the values are distinct and all the observations fall on a straight line with non-zero slope. Then the possible values of the correlation coefficient between x and y are *[2018-II]*

(a) 0 and 1 only (b) 0 and – 1 only
(c) 0, 1 and – 1 (d) – 1 and 1 only

197. An analysis of monthly wages paid to the workers in two firms A and B belonging to the same industry the following result: *[2018-II]*

	Firm A	**Firm B**
Number of workers	500	600
Average monthly wage	₹ 1860	₹ 1750
Variance of distribution of wages	81	100

The average of monthly wages and variance of distribution of wages of all the workers in the firms A and B taken together are *[2018-II]*

(a) ₹ 1860, 100 (b) ₹ 1750, 100

(c) ₹ 1800m, 81 (d) None of above

198. Which one of the following can be obtained from an ogive? *[2018-II]*

(a) Mean (b) Median

(c) Geometric mean (d) Mode

199. In any discrete series (when all values are not same) if x represents mean deviation about mean and y represents standard deviation, then which one of the following is correct? *[2018-II]*

(a) $y \geq x$ (b) $y \leq x$

(c) $x = y$ (d) $x < y$

200. In which one of the following cases would you except to get a negative correlation? *[2018-II]*

(a) The ages of husbands and wives

(b) Shoe size and intelligence

(c) Insurance companies profits and the number of claims they have to pay

(d) Amount of rainfall and yield of crop

201. The mean of 100 observations is 50 and the standard deviation is 10. If 5 is subtracted from each observation and then it is divided by 4, then what will be the new mean and the new standard deviation respectively ? *[2019-I]*

(a) 45, 5 (b) 11.25, 1.25

(c) 11.25, 2.5 (d) 12.5, 2.5

202. Consider the following statements : *[2019-I]*

1. The algebraic sum of deviations of a set of values from their arithmetic mean is always zero.
2. Arithmetic mean > Median > Mode for a symmetric distribution.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

203. Let the correlation coefficient between X and Y be 0.6. Random variables Z and W are defined as Z = X + 5 and W $= \frac{Y}{3}$. What is the correlation coefficient between Z and W? *[2019-I]*

(a) 0.1 (b) 0.2

(c) 0.36 (d) 0.6

204. If all the natural numbers between 1 and 20 are multiplied by 3, then what is the variance of the resulting series ? *[2019-I]*

(a) 99.75 (b) 199.75

(c) 299.25 (d) 399.25

ANSWER KEY

1	(a)	22	(d)	43	(b)	64	(d)	85	(c)	106	(b)	127	(c)	148	(b)	169	(a)	190	(b)
2	(c)	23	(b)	44	(c)	65	(c)	86	(a)	107	(d)	128	(a)	149	(a)	170	(a)	191	(d)
3	(d)	24	(a)	45	(b)	66	(b)	87	(b)	108	(c)	129	(b)	150	(c)	171	(b)	192	(d)
4	(b)	25	(d)	46	(d)	67	(c)	88	(c)	109	(a)	130	(b)	151	(d)	172	(b)	193	(c)
5	(d)	26	(b)	47	(d)	68	(d)	89	(a)	110	(d)	131	(b)	152	(d)	173	(d)	194	(b)
6	(c)	27	(b)	48	(c)	69	(c)	90	(a)	111	(a)	132	(c)	153	(c)	174	(a)	195	(b)
7	(a)	28	(b)	49	(c)	70	(d)	91	(c)	112	(c)	133	(c)	154	(d)	175	(c)	196	(d)
8	(c)	29	(b)	50	(c)	71	(c)	92	(b)	113	(a)	134	(a)	155	(d)	176	(b)	197	(d)
9	(d)	30	(d)	51	(a)	72	(c)	93	(a)	114	(b)	135	(d)	156	(b)	177	(b)	198	(b)
10	(b)	31	(d)	52	(c)	73	(d)	94	(b)	115	(c)	136	(d)	157	(d)	178	(b)	199	(d)
11	(d)	32	(b)	53	(a)	74	(c)	95	(d)	116	(c)	137	(b)	158	(b)	179	(c)	200	(c)
12	(a)	33	(a)	54	(b)	75	(b)	96	(a)	117	(c)	138	(c)	159	(c)	180	(a)	201	(c)
13	(d)	34	(d)	55	(a)	76	(a)	97	(c)	118	(d)	139	(c)	160	(b)	181	(d)	202	(a)
14	(b)	35	(b)	56	(b)	77	(a)	98	(d)	119	(a)	140	(b)	161	(b)	182	(d)	203	(d)
15	(d)	36	(d)	57	(c)	78	(b)	99	(b)	120	(c)	141	(c)	162	(d)	183	(d)	204	(c)
16	(c)	37	(b)	58	(b)	79	(c)	100	(d)	121	(c)	142	(b)	163	(b)	184	(c)		
17	(b)	38	(c)	59	(c)	80	(d)	101	(c)	122	(c)	143	(c)	164	(b)	185	(d)		
18	(c)	39	(c)	60	(a)	81	(a)	102	(c)	123	(c)	144	(d)	165	(c)	186	(c)		
19	(b)	40	(b)	61	(b)	82	(c)	103	(d)	124	(a)	145	(a)	166	(a)	187	(b)		
20	(c)	41	(d)	62	(a)	83	(a)	104	(d)	125	(b)	146	(c)	167	(c)	188	(c)		
21	(a)	42	(d)	63	(c)	84	(a)	105	(d)	126	(c)	147	(c)	168	(d)	189	(b)		

HINTS & SOLUTIONS

1. (a) A simple bar diagram is most suitable for this.
2. (c) Mode is most suitable for this.
3. (d) The angle between two regression lines becomes zero if $r = \pm 1$.
4. (b) Since, expansion contains $(n + 1)$ terms,

$$\text{Required mean} = \frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n}{(n+1)}$$

$$= \frac{2^n}{n+1}$$

5. (d) The standard deviation of n observation $x_1, x_2, \ldots\ldots, x_n$ is 6 and of $y_1, y_2, \ldots\ldots, y_n$ is 8, then the standard deviation of n observation $x_1 - y_1, x_2 - y_2, x_3 - y_3, \ldots\ldots, x_n - y_n$ is $8 - 6 = 2$.
6. (c) Let $x = 31$ and $y = 10$

C.I.	x	f	cf
8.5 – 13.5	11	7	7
13.5 – 18.5	16	31	38
18.5 – 23.5	21	40	78
23.5 – 28.5	26	10	88
28.5 – 33.5	31	10	98
33.5 – 38.5	36	2	100

$\because$ $N = 100$, $\therefore \frac{N}{2} = 50$

$\therefore$ Median group is 18.5 – 23.5

$\therefore$ $L_1 = 18.5, L_2 = 23.5, C = 38, h = 5, f = 40$

$$\therefore \text{Median} = L_1 + \frac{\left(\frac{N}{2} - C\right)}{f} \times h$$

$$= 18.5 + \frac{50 - 38}{40} \times 5 = 18.5 + \frac{12 \times 5}{40} = 18.5 + 1.5 = 20$$

Thus, our assumption is correct. Therefore missing numbers are 31 and 10 respectively.

7. (a) All are correct statement and R is correct explanation of A.
8. (c) The x-coordinate of the point of intersection of two ogives, gives median.
9. (d) Average score of $A = \frac{71 + 56 + 45 + 89 + 54 + 44}{6}$

$$= \frac{359}{6} = 59.83$$

and average score of

$$B = \frac{55 + 74 + 83 + 54 + 38 + 52}{6} = \frac{356}{6} = 59.33$$

Variation is lesser in case of B than A. So, the average scores of A and B are not same but B is consistent.

10. (b) Joining the mid points of the upper horizontal sides of each rectangle of a histogram by straight lines, the figure so obtained is known as frequency polygon.
11. (d) The definition of Mode fails if the curve drawn with the help of given data is symmetrical.
12. (a) Let n denote number of workers and x, the pay. $n_1 = 30, n_2 = 20, x_1 = ₹\,500, x_2 = ₹\,600$

$$\therefore \text{Combined average} = \frac{n_1x_1 + n_2x_2}{n_1 + n_2}$$

$$= \frac{30 \times 500 + 20 \times 600}{30 + 20} = \frac{15000 + 12000}{50}$$

$$= \frac{27000}{50} = 540$$

Combined average pay = ₹ 540

13. (d) 50_{th} decile $= \frac{50}{10} = 5$

and 5_{th} percentile $= \frac{5}{100}$ $\quad 5 \neq \frac{5}{100}$

14. (b) Let there be x number of boys and y number of girls.

Total students = x + y

Total weight of the students = (x + y)60

Total weight for boys = x × 70

Total weight for girls = y × 55

Hence, (x + y)60 = 70x + 55y

60x + 60y = 70x + 55y

$5y = 10x \Rightarrow y = 2x$

$\frac{x}{y} = \frac{1}{2} \Rightarrow x : y = 1 : 2$

15. (d) From observation of the graph it is noted the nature is similar, but are centered around different values, hence, they have same variance, but different mean values.
16. (c) Slope of line of regression of Y and X, is 30°. So

$b_{yx} = \tan 30° = \frac{1}{\sqrt{3}}$ and for X and Y it is 60°.

Hence, $\frac{1}{b_{xy}} = \tan 60° = \sqrt{3}$

$b_{yx} = \frac{1}{\sqrt{3}}$ and $b_{xy} = \frac{1}{\sqrt{3}}$

$r(x, y) = r^2 = b_{yx} \,.\, b_{xy} = \frac{1}{3}$

so, $r = \pm \frac{1}{\sqrt{3}}$

Since, b_{yx} and b_{xy} are both positive, $r = + \frac{1}{\sqrt{3}}$

17. (b) To measure the intelligence of a group of students mode will be more suitable.
18. (c) As give, np = 4 and npq = 3

[where p is the probability of success and q is the probability of failure for an event to occur, and 'n' is the number of trials]

$$\Rightarrow q = \frac{npq}{np} = \frac{3}{4}$$

Also, $p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore$ n = 16

In a binomial distribution, the value of r for which P (X = r) is maximum is the mode of binomial distribution.

hence, $(n+1)p - 1 \le r \le (n+1)p$

$\Rightarrow \frac{17}{4} - 1 \le r \le \frac{17}{4}$

$\Rightarrow \frac{13}{4} \le r \le \frac{17}{4}$

$\Rightarrow 3.25 \le r \le 4.25$

$\Rightarrow r = 4$

19. (b) If X is changed to a + hU and Y to b + kV, then

$b_{XY} = \left|\frac{h}{k}\right| b_{UV}$

$\Rightarrow kb_{XY} = h.b_{UV}$

20. (c) Let the number of students of school I = x

$\therefore$ Number of students of School II = 1.5 x

As given :

Mean of marks for school I = 82

and mean of marks for school II = 86

$\therefore$ Combined mean $= \frac{x \times 82 + 1.5x \times 86}{x + 1.5x}$

$= \frac{x(82 + 129)}{2.5x} = \frac{211}{2.5} = 84.4$

21. (a) Since, AM of number $x_1, x_2, x_3, ...x_n$ is μ

$\therefore$ $n\mu = x_1 + x_2 + ... x_n$

Sum of new numbers

$= (x_1 + 1) + (x_2 + 2) + (x_3 + 3) + ... + (x_n + n)$

$= (x_1 + x_2 + ... + x_n) + (1 + 2 + 3 + ... + n)$

$= n\mu + \frac{n(n+1)}{2}$

$\therefore$ $AM = \mu + \frac{(n+1)}{2}$

22. (d) Mode is the required measure.

23. (b) In a pie chart

largest amount occupies largest area. So, the salaries occupies largest area.

$\Rightarrow$ Sectorial angle $= \frac{6}{20} \times 360° = 108°$

24. (a) From the given statement

$\Rightarrow$ Both (A) and (R) are true and R is the correct explanation of A.

25. (d) Given : lowest class boundary = 6.1

Class width = 2.5, Number of classes = 12

$\Rightarrow$ Upper class boundary of the highest class $= 6.1 + (2.5 \times 12) = 6.1 + 30 = 36.1$

26. (b) The appropriate number of classes while constructing a frequency distribution should be chosen such that the class frequency should cluster around the class mid point

27. (b) Bar chart is most appropriate.

28. (b) Standard deviation (series B) :

$$= \sqrt{\frac{1}{5}(1.9^2 + 0.8^2 + 1.5^2 + 0.6^2 + 0.2^2) - \left(\frac{1.9 + 0.8 + 1.5 + 0.6 + 0.2}{5}\right)^2}$$

$$= \sqrt{\frac{6.9}{5} - 1} = \sqrt{1.38 - 1}$$

$$= \sqrt{0.38}$$

29. (b) Geometric Mean of combined series is given by the expression

$$\log G = \frac{n_2 \log G_1 + n_1 \log G_2}{n_1 + n_2}$$

30. (d) Let $\bar{x}$ is the mean of n observation $x_1, x_2, ..., x_n$.

$$\Rightarrow \bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

Now, (a – b) is added to each term.

$\therefore$ New mean

$$= \frac{x_1 + (a-b) + x_2 + (a-b) + ... + x_n + (a-b)}{n}$$

$$= \frac{x_1 + x_2 + ... + x_n}{n} + \frac{n(a-b)}{n}$$

$$= \bar{x} + (a-b)$$

31. (d) For the given distribution which is positively seowed, Median < Mean < Mode

32. (b) Mean of the numbers $= \frac{\frac{n(n+1)}{2}}{n}$

$$= \frac{n+1}{2}$$

$\therefore$ Variance

$$= \frac{\left(1 - \frac{n+1}{2}\right)^2 + \left(2 - \frac{n+1}{2}\right)^2 + \left(3 - \frac{n+1}{2}\right)^2 + ...}{n}$$

$$2 = \frac{(1^2 + 2^2 + 3^2 + ...) + n\left(\frac{n+1}{2}\right)^2 - 2\left(\frac{n+1}{2}\right)[1 + 2 + 3 + ...]}{n}$$

$$2n = \frac{1}{6}n(n+1)(2n+1) + \frac{n(n+1)^2}{4} - 2\left(\frac{n+1}{2}\right)\left\{\frac{n(n+1)}{2}\right\}$$

$$2n = n(n+1)\left[\frac{2n+1}{6} + \frac{n+1}{4} - \frac{n+1}{2}\right]$$

$$2 = (n+1)\left[\frac{4n + 2 - 3n - 3}{12}\right]$$

$\Rightarrow 24 = (n+1)(n-1)$ $\quad \Rightarrow n^2 - 1 = 24$

$\Rightarrow n^2 = 25$ $\quad \Rightarrow n = \pm 5$

33. (a) Arithmetic mean of the series

$$= \frac{^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n}{n} = \frac{2^n - 1}{n}$$

34. (d) Let the teacher's age is x years
According to question

$$15 + 1 = \frac{20 \times 15 + x}{21}$$

$\Rightarrow 16 \times 21 = 300 + x$

$\Rightarrow x = 336 - 300 = 36$ years

35. (b) Arithmetic mean $= \frac{2 \times 1 + 3 \times 2 + 3f + 4 \times 5}{2 + 3 + f + 5}$

$\Rightarrow \frac{23}{8} = \frac{28 + 3f}{10 + f}$

$\Rightarrow 230 + 23f = 224 + 24f$

$\Rightarrow f = 6$

36. (d) (i) Arithmetic mean $= \frac{\text{Sum of all observations}}{\text{Total no. of observation}}$

(ii) Median = The midpoint of the data after being ranked (arranged in ascending order).

(iii) Geometric mean = If $x_1, x_2, x_3, ..., x_n$ are n values of a variate x, none of them being zero, then the geometric mean G is defined as $G = (x_1 x_2 x_3 .... x_n)^{1/n}$.

Thuse, all of them are meaningfully defined.

37. (b) Collecting data from government offices is secondary. collecting data from public libraries is also secondary but collecting data by telephonic interview is primary data.

38. (c) Arithmetic mean of

10 numbers $= \frac{4 \times 15 + 6 \times 12}{10} = \frac{60 + 72}{10} = 13.2$

39. (c) From visual observation of given table we can say that during month 3, the sales are most consistent.

40. (b) Average of marks of A

$$= \frac{71 + 56 + 55 + 75 + 54 + 49}{6} = \frac{360}{6} = 60$$

and SD

$$= \sqrt{\frac{121 + 16 + 25 + 225 + 36 + 121}{6}} = \sqrt{\frac{544}{6}} = 9.52$$

Average of marks of B

$$= \frac{55 + 74 + 83 + 54 + 38 + 52}{6} = \frac{356}{6} = 59.33$$

Thus, the average scores of A and B are not same but A is consistent.

41. (d) Let average salary of women be x.
According to question,

$\Rightarrow 50 \times 3550 = 30 \times 4050 + 20x$

$\Rightarrow 177500 - 121500 = 20x$

$\Rightarrow x = 2800$

Hence, average salary of women = Rs 2800

42. (d) Mean of given numbers

$$= \frac{7 + 9 + 11 + 13 + 15}{5} = \frac{55}{5} = 11$$

Now,

$$SD = \sqrt{\frac{(7-11)^2 + (9-11)^2 + (11-11)^2 + (13-11)^2 + (15-11)^2}{5}}$$

$$= \sqrt{\frac{16 + 4 + 0 + 4 + 16}{5}} = \sqrt{8} = 2.8 \text{ (aprox)}$$

43. (b) Since, monthly salary = Rs 15000
and sector angle of expenses = 15°

$\therefore$ Amount $= \frac{15°}{360°} \times 15000 = \text{Rs } 625$

44. (c) $\sum_{t=1}^{n} (x_i - 2) = 110$

$\therefore x_1 + x_2 + \ldots + x_n - 2n = 110$

$\Rightarrow x_1 + x_2 + \ldots + x_n = 2n + 110$ ...(i)

and $\sum_{i=1}^{n} (x_i - 5) = 20$

$\Rightarrow x_1 + x_2 + \ldots + x_n - 5n = 20$

$\Rightarrow x_1 + x_2 + \ldots + x_n = 5n + 20$ ...(ii)

From equations (i) and (ii), we get

$5n + 20 = 2n + 110$

$\Rightarrow 3n = 90 \Rightarrow n = 30$

Now, mean $= \frac{x_1 + x_2 + \ldots x_n}{n}$

$$= \frac{5 \times 30 + 20}{30} = \frac{170}{30} = \frac{17}{3}$$

45. (b) Total no. of students in section A
$= 74 \times 35 = 2590$

Total no. of students in section B
$= 70 \times 35 = 2450$

Now, total no. of students in section C

$= \frac{51}{75} \times 100 \times 30 = 2040$

Thus, total students in all $= 2590 + 2450 + 2040$
$= 7080$

Thus, Required percentage $= 7080 \div 100$
$= 70.8$

46. (d) Given lines of regression are $3X + Y - 12 = 0$ and $X + 2Y - 14 = 0$

Since, lines of regression passes through $(\bar{X}, \bar{Y})$.

therefore $(\bar{X}, \bar{Y})$ satisfies the given equations.

$\therefore 3\bar{X} + \bar{Y} - 12 = 0$ ...(i)

and $\bar{X} + 2\bar{Y} - 14 = 0$ ...(ii)

Multiply equation (ii) by 3 and subtract from (i), we get

$(3\bar{X} + \bar{Y} - 12) - (3\bar{X} + 6\bar{Y} - 42) = 0$

$\Rightarrow 5\bar{Y} + 30 = 0 \Rightarrow \bar{Y} = 6$

Thus, $\bar{X} = 14 - 2\bar{Y} = 14 - 12 = 2$

Hence, mean, X = 2

47. (d) To find the consistent demand we will calculate coeff of variance.

We know coefficient of variance $= \frac{\sqrt{SD}}{\text{mean}}$

Also, we know, S.D $= \sqrt{\text{variance}}$

Coefficient of variance of $A = \frac{\sqrt{12}}{60} = \frac{3.46}{60} = 0.057$

Coefficient of variance of $B = \frac{\sqrt{25}}{90} = \frac{5}{90} = 0.055$

Coefficient of variance of $C = \frac{\sqrt{36}}{80} = \frac{6}{80} = 0.075$

Coefficient of variance of $D = \frac{\sqrt{16}}{120} = \frac{4}{120} = 0.033$

We see that minimum coefficient of variance is of D, hence product D is consistent.

48. (c) Let us consider any five integers which are 3, 4, 5, 6, 7.

$\therefore$ mean $= \frac{3+4+5+6+7}{5} = \frac{25}{5} = 5$

$\therefore SD = \sqrt{\frac{(5-3)^2+(5-4)^2+(5-5)^2+(5-6)^2+(5-7)^2}{5}}$

$= \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{2}$

Hence, the least value of the standard deviation of 5 integers is $\sqrt{2}$

49. (c) Correlation between two variables is said to be perfect, if one variable increases, the other also increases proportionally.

50. (c) Both the given statements I and II are true.

Sol. (51 - 53):

x	f	xf
0	f_1	0
1	f_2	f_2
2	4	8
3	4	12
4	3	12
Total	f_1+f_2+11	$32+f_2$

Since, total frequency is 18

$\therefore f_1+f_2+11=18$

$\Rightarrow f_1+f_2=7$ ...(*i*)

As we have, Mean $= \frac{\Sigma xf}{\Sigma f} = 2$

$\therefore \frac{32+f_2}{18} = 2 \Rightarrow f_2 = 36-32 = 4$

On putting the value of f_2 in Eq. (i), we get

$f_1 = 7-4 = 3$

51. (a) $f_2 = 4$

52. (c) Since, mean $= \bar{x} = 2(\text{given})$

x	$x-\bar{x}$	$(x-\bar{x})^2$	f	$f(x-\bar{x})^2$
0	−2	4	3	12
1	−1	1	4	4
2	0	0	4	0
3	1	1	4	4
4	2	4	3	12
Total			18	32

Now, $SD = \sqrt{\frac{\sum f(x-\bar{x})^2}{N}}$

where N = sum of all frequencies

$= \sqrt{\frac{32}{18}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

53. (a) Coefficient of variance $= \frac{\sigma}{\bar{x}} \times 100$ where $\sigma = S.D$

$= \frac{4}{3} \times \frac{1}{2} \times 100 = \frac{200}{3}$

54. (b) Given data is 2, 9, 9, 3, 6, 9, 4.

We know,

Mean $= \frac{\text{Sum of all observations}}{\text{Total number of observations}}$

$\therefore$ Mean $= \frac{2+9+9+3+6+9+4}{7} = \frac{42}{7} = 6$

$\therefore$ Mean deviation

$= \frac{|2-6|+3|9-6|+|3-6|+|6-6|+|4-6|}{7}$

$= \frac{4+9+3+0+2}{7} = \frac{18}{7} = 2.57$

55. (a) We know that, if a number is added in values, then the standard deviation remains unaltered.

$\therefore$ Standard deviation of new values $= \sigma$

56. (b) Let us consider lines

$8x-10y=66$ and $40x-18y=214$

$\Rightarrow 10y = 8x-66 \Rightarrow 40x = 18y+214$

$\Rightarrow b_{yx} = \frac{8}{10} = \frac{4}{5} \Rightarrow b_{xy} = \frac{18}{40} = \frac{9}{20}$

Thus, $r = \pm\sqrt{\frac{4}{5} \times \frac{9}{20}} = \frac{3}{5}$

Also, $\sigma_x = \sqrt{9} = 3$

$\therefore \sigma_y = \frac{byx \times \sigma_x}{r} = \frac{\frac{4}{5}\times 3}{r} = \frac{12}{5r} = \frac{12}{5} \times \frac{5}{3} = 4$

57. (c) Since, the standard deviation of same consecutive integers is 2, these integers are any set of seven consecutive integers.

58. (c) Coefficient of variation = $\frac{\text{S.D}}{\text{Mean}} \times 100$

For factory A = $\frac{40 \cdot 50}{540} \times 100 = 7.5$

For factory B = $\frac{31}{620} \times 100 = 5$

$\therefore$ Variability in wages of A is 50% more than the variability in wages of B.

59. (c) Required mean $= \frac{36 \times 4 + 64 \times 3}{36 + 64} = \frac{144 + 192}{100} = \frac{336}{100}$

$= 3.36$

60. (a) Given table can be rewritten as

x	y	x^2	y^2	xy
5	2	25	4	10
7	4	49	16	28
8	3	64	9	24
4	2	16	4	8
6	4	36	16	24
$\sum x = 30$	$\sum y = 15$	$\sum x^2 = 190$	$\sum y^2 = 49$	$\sum xy = 94$

$\bar{x} = \frac{30}{5} = 6$ and $\bar{y} = \frac{15}{5} = 3$

$\therefore \quad b_{yx} = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$

$= \frac{5 \times 94 - 30 \times 15}{5 \times 190 - (30)^2} = \frac{2}{5} = 0.4$

Hence, line of regression is

$y - 3 = 0.4(x - 6) \Rightarrow y = 0.4x + 0.6$

Sol. (61 - 63):

Class	Frequency	C. F
0 - 5	3	3
5 - 10	12	15
10 - 15	x	15 + x
15 - 20	35	50 + x
20 - 25	y	50 + x + y
25 - 30	4	54 + x + y

Since, $n = 90 \therefore \frac{n}{2} = 45$

61. (b) $\because$ Lower limit of median class is 15.
62. (a) Missing frequency y is 20.
63. (c) Cumulative frequency of model class is 66.

64. (d)

Class Interval	f	cf
0.5-5.5	3	3
5.5-10.5	7	10
10.5-15.5	6	16
15.5-20.5	5	21
Total	21	50

$N = 21$

$\therefore \quad \frac{N}{2} = \frac{21}{2} = 10.5$

$\therefore$ Median class is 10.5-15.5

Hence, Median = $10.5 + \frac{10.5 - 10}{6} \times 5$

$= 10.5 + 0.417 = 10.917$

Thus, median is not contained in the modal class and the distribution is not bell-shaped.

Sol. (65 - 66):

Class Interval	f	cf	x	fx
0-10	5	5	5	25
10-20	10	15	15	150
20-30	20	35	25	500
30-40	5	40	35	175
40-50	10	50	45	450
Total	50	145	125	1300

$\therefore \quad \frac{N}{2} = \frac{50}{2} = 25$

65. (c) Median group is 20-30.

$\Rightarrow$ Median $= 20 + \frac{25 - 15}{20} \times 10 = 20 + 5 = 25$

66. (b) Mean $= \frac{\sum fx}{\sum f} = \frac{1300}{50} = 26$

67. (c) Both the given statements which is related to correlation coefficient r between variables x and y are correct.

68. (d) If the values of a set are measured in cm, then there will not be unit of variance.

69. (c) The cumulative frequency curve of statistical data is called Ogive.

70. (d) Daily income of owner
$= 10 \times 110 - 9 \times 76 = 1100 - 684 =$ ₹416

71. (c) Given table can be rewritten as

x_i	f_i	$f_i x_i$
6	2	12
10	4	40
14	7	98
18	12	216
24	8	192
28	4	112
30	3	90
Total	40	760

So, required mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{760}{40} = 19$

72. (c) Required Arithmetic mean

$$= \frac{(30 \times 72.2) + (30 \times 69.0) + (40 \times 64.1)}{100}$$

$$= \frac{6800}{100} = 68$$

73. (d) Given, variance of the data 2, 4, 5, 6, 17 is v.
Variance of the data 4, 8, 10, 12, 34 is $2v$.
because when each observation is multiplied by 2, then variance is also multiplied by 2.

74. (c) Given mean of 7 observations is 10.

$$\therefore \frac{x_1 + x_2 + ... + x_7}{7} = 10$$

$$\Rightarrow x_1 + x_2 + ... + x_7 = 70 \qquad ...(1)$$

Also, mean of 3 observations is 5.

$$\therefore \frac{x_8 + x_9 + x_{10}}{3} = 5$$

$$\Rightarrow x_8 + x_9 + x_{10} = 15 \qquad ...(2)$$

So, from (1) and (2)
Required mean

$$= \frac{x_1 + x_2 + ... + x_7 + x_8 + x_9 + x_{10}}{10}$$

$$= \frac{70+15}{10} = \frac{85}{10} = 8.5$$

75. (b) "If every observation is multiplied by c, then the measure of central tendency is also multiplied by c, where $c > 0$. Arithmetic mean, Geometric mean and median satisfies above property.

76. (a) Given data is 2, 3, 4, 2, 5, 4, 3, 2, 1
Mode = 2
[∵ 2 occurs maximum number of times].

Sol. (77 - 84):
Complete table is

Year	Male			Female			Total
	Urban	Rural	Total	Urban	Rural	Total	
1995	280	350	630	410	310	720	1350
1996	370	300	670	180	270	450	1120
1997	310	130	440	180	190	370	810
1998	400	280	680	290	80	370	1050
Total	1360	1060	2420	1060	850	1910	4330

77. (a) Total population for the year 1997 = 440 + 370 = 810
78. (b) Female urban population in the year 1995 = 410.
79. (c) Urban population in the year 1997 = 310 + 180 = 490.
80. (d) Total population in the year 1998 = 1050.
81. (a) Required difference = 720 – 630 = 90.
82. (c) In 1997, the male population is minimum.
83. (a) In 1995, the female population is maximum.
84. (a) Total rural male population in the year 1998 = 280

$$\text{Required \%} = \frac{280}{1050} \times 100\% = \frac{28}{105} \times 100\%$$

$$= \frac{28 \times 20}{21}\% = \frac{4 \times 20}{3}\% = \frac{80}{3}\%$$

85. (c) Agriculture : $\frac{90}{360} \times 36000 = 9000$

Miscellaneous : $\frac{75}{360} \times 36000 = 7500$

Industry : $\frac{45}{360} \times 36000 = 4500$

Education : 3000 Employment : 12000
Hence, Employment is allocated maximum funds.

86. (a) Education : 3000
87. (b) Required = 9000 + 12000 = 21000
88. (c) Required = 7500 – 3000 = 4500
89. (a) Ascending order is 2, 3, 4, 5, 6, 7, 9
Since, $n = 7$ (odd)

$$\therefore \text{Required Median} = \left(\frac{n+1}{2}\right)^{th} \text{obs} = 4^{th} \text{obs} = 5$$

90. (a) Given natural numbers are
1, 2, 3, 4, 5, ..., 16
This is an A.P. with first term = 1
and common difference = 1, n = 16
∴ By using sum of 16 natural numbers

$$\text{i.e., } S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

We have

$$S_{16} = \frac{16}{2}\left[2(1) + 15(1)\right] = 8(17) = 136$$

$$\therefore \text{AM.} = \frac{136}{16} = \frac{17}{2}$$

91. (c) Since observation 22 occurs maximum time.
∴ mode = 22
92. (b) Statement 2 and 3 are correct.
93. (a) It is a property.
94. (b) First we arrange the data in ascending order
2, 2, 3, 3, 4, 5, 7, 9, 10
Since number of observation is odd

$$\therefore \text{median} = \left(\frac{9+1}{2}\right)^{th} \text{observation} = 5\text{th observation}$$

= 4

95. (d) Given : Mean of 10 observations is 5.

$$\Rightarrow \frac{\sum_{i=1}^{10} x_i}{10} = 5$$

According to the Question

$$\frac{\sum_{i=1}^{10} 3(x_i + 2)}{10} = \text{New mean}$$

$$\Rightarrow \frac{3\sum_{i=1}^{10} x_i}{10} + \frac{3 \times 2 \times 10}{10} = \text{New mean}$$

$$\Rightarrow 3 \times 5 + 6 = \text{New mean} \qquad \Rightarrow 21 = \text{New mean}$$

96. (a) Sum of first n odd natural numbers = n^2.

Now, mean = $\frac{n^2}{n} = n$

97. (c) Given $M = \frac{a+b+c+d+e}{5}$

$\Rightarrow$ a + b + c + d + e = 5 M

$\Rightarrow$ a + b + c + d + e – 5 M = 0

$\Rightarrow$ (a – M) + (b – M) + (c – M) + (d – M) + (e – M) = 0

Hence, Required value = 0

98. (d) Given $\sum_{i=1}^{20}(x_i - 30) = 2$

$\Rightarrow \sum_{i=1}^{20} x_i - \sum_{i=1}^{20} 30 = 2$

$\Rightarrow \sum_{i=1}^{20} x_i = 2 + \sum_{i=1}^{20} 30 = 2 + 30 \times 20 = 602$

Now, mean = $\frac{\sum_{i=1}^{20} x_i}{20}$

$\therefore$ Mean = $\frac{602}{20} = 30.1$

99. (b) Median is middle of data. Observations are 27 and median is 18. So, sum of all the observation are 18 × 27 = 486.

Now, 16, 18 and 50 are additional three observations.

So, Total = 486 + 16 + 18 + 50 = 570.

and number of obs. are 30.

$\therefore$ Median = $\frac{570}{30} = 19$

100. (d) Frequency curve may be symmetrical, positive skew and negative skew.

101. (c) Required expenditure $= 9000 \times \frac{10}{100}$

= ₹ 900.

102. (c) Mean = 40, S.D = 8

$\therefore$ Coeft of variation = $\frac{8}{40} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

103. (d) Given observations are

7, 9, 11, 13, 15

$\bar{x} = \frac{7+9+11+13+15}{5} = \frac{55}{5} = 11$

Now, Variance = $\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

$= \frac{(7-11)^2 + (9-11)^2 + (11-11)^2 + (13-11)^2 + (15-11)^2}{5}$

$= \frac{16+4+4+16}{5} = 8$

$\therefore$ S.D $= \sqrt{v} = \sqrt{8} = 2\sqrt{2} = 2.8$

$\left(\because \sqrt{2} = 1.414\right)$

104. (d) Standard Deviation is a measure of dispersion.

105. (d) Given regression lines are x – y + 1 = 0 and 2x – y + 4 = 0 only point (–3, –2) given in option (d) satisfies the both equations.

Hence, The two regression lines pass through the point (–3, –2).

106. (b) First we arrange the data in ascending order.

3, 6, 6, 7, 7, 7, 8, 9, 9, 10, 10, 10, 12.

Since no. of observations is odd

$\therefore$ Required median $= \left(\frac{13+1}{2}\right)^{th}$ observation

$= \left(\frac{14}{2}\right)^{th} = 7^{th}$ observation

= 8

107. (d) Both statements are wrong. Also, S . D = $\sqrt{\text{variance}}$

108. (c) Since the total of the frequencies = 100

$\therefore$ 14 + x + 27 + y + 15 = 100

$\Rightarrow$ x + y = 44

Now, to exist the mode x and y should be equal

$\therefore$ x = y is only possibility

109. (a) Let Number of girls student be x

Sum of marks = 25 × 40 + x × 48

Total students = 25 + x

$\therefore 43 = \frac{25 \times 40 + x \times 48}{x + 25}$

$\Rightarrow$ 43 x + 43 × 25 = 25 × 40 + x × 48

$\Rightarrow$ 5x = 3 × 25

$\Rightarrow$ x = 15

110. (d) Given marks are 30, 55, 75, 90, 50, 60, 39.

Mean marks = $\bar{x}$

$\bar{x} = \frac{30+55+75+90+50+60+39}{7} = \frac{399}{7} = 57$

Hence, 4 students secured marks less than the mean marks.

111. (a) Variance is always independent of change of the origin but not scale.

112. (c) It is a property

113. (a) Given observation are

5, 5, 5, 5, 5.

$\therefore \bar{x} = \frac{5+5+5+5+5}{5} = \frac{25}{5} = 5$

Now, $\sigma^2 = \sum_{i=1}^{5}\frac{\left(x_i - \bar{x}\right)^2}{N}$

Where N = total number of observations.

$\therefore$ Variance = σ^2

$$= \frac{(5-5)+(5-5)+(5-5)+(5-5)+(5-5)}{5} = 0$$

Hence, standard deviation = $\sqrt{\text{var}} = 0$

114. (b) Sum of all observations = 20 × 15 = 300
Sum of correct observations
$= 300 - (3+6) + (8+4) = 303$

Correct mean = $\frac{303}{20} = 15.15$

115. (c) Sum of squares of first 'n' natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

Mean of the squares of first 'n' natural numbers

$$= \frac{(n+1)(2n+1)\times n}{6\times n} = \frac{(n+1)(2n+1)}{6}$$

116. (c)
117. (c) After construction of cumulative frequency distribution, we can find out median easily.
118. (d)
119. (a) Variance of 11 natural numbers = $\frac{11^2-1}{12} = 10$
120. (c)
121. (c)
122. (c) **Detailed Method:**

Variance = $\frac{\sum d^2}{n}$

$5 = \frac{\sum d^2}{20} \Rightarrow \sum d^2 = 100$

According to question

$\sum (d_1)^2 = \sum (2d)^2 = 4\sum d^2 = 4\times 100 = 400$

New variance = $\frac{400}{20} = 20$

Shortcut Method:
If each observation is multiplied by 2
New variance = $2^2 \times 5 = 20$

123. (c) $r = \sqrt{b_{xy}.b_{yx}}$

$$= \sqrt{\left(-\frac{1}{6}\right)\times\left(-\frac{3}{2}\right)}$$

$$= \sqrt{\frac{1}{2}\times\frac{1}{2}} = \pm\frac{1}{2}$$

b_{xy} and b_{yx} both have negative sign. Therefore we have to take negative sign

Hence, correlation coefficient (r) = $-\frac{1}{2}$.

124. (a) I : Variance is not dependent on change of origin. Therefore, if every x_i is increased by 2, the variance of the new set of numbers is not changed.

125. (b) Mean of the squares of the first 20 natural number

$$= \frac{(n+1)(2n+1)}{6} = \frac{21\times 41}{6} = 143.5$$

126. (c) The cumulative frequency of the largest observed value must always be equal to the total number of observations.

127. (c) Statement 1 :
$n(X) = 2$

$p = \frac{n(X)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

arithmetic mean of X = np = $18 \times \frac{1}{3} = 6$

Statement 2 : Standard deviation of

$X = \sqrt{\text{variance of X}} = \sqrt{18\times\frac{1}{3}\times\frac{2}{3}} = \sqrt{4} = 2$

Hence, statements 1 and 2 both are correct.

Sol. (128-130):

Numbers (x)	Frequency (f)	c.f.	Σ fx
0	14	14	0
1	21	35	21
2	25	60	50
3	43	103	129
4	51	154	204
5	40	194	200
6	39	233	234
7	12	245	84
	N = 245		Σ fx = 922

128. (a) $\frac{\Sigma fx}{N} = \frac{922}{245} = 3.76$

129. (b) $\frac{N}{2} = \frac{245}{2} = 122.5$
Required mean = 4

130. (b) The higher frequency is 51
∴ mode = value of the variable corresponding to the higher frequency 154 = 4

Sol. (131-133):

Mean of 100 items = $\bar{x}_{100} = 50$

Mean of 150 items = $\bar{x}_{150} = 40$

Standard deviation of 100 items = $\sigma_{100} = 5$
Standard deviation of 150 items = $\sigma_{150} = 6$

131. (b) $\bar{x}_{250} = \frac{n_1.\bar{x}_{100} + n_2.\bar{x}_{150}}{n_1+n_2} = \frac{(100\times 50)+(150\times 40)}{100+150}$

$$= \frac{11000}{250} = 44$$

132. (c) $d_1 = 50 - 44 = 6$ $\quad d_1^2 = 36$
$d_2 = 40 - 44 = -4$ $\quad d_2^2 = 16$

$$\sigma_{250} = \frac{\sqrt{n_1(\sigma_{100}^2 + d_1^2) + n_2(\sigma_{150}^2 + d_2^2)}}{n_1 + n_2}$$

$$= \frac{\sqrt{390}}{5} = \frac{37.28}{5} = 7.456 = 7.5$$

133. (c) Variance of all 250 items $= (\sigma_{250})^2 = (7.456)^2 = 55.6$

134. (a) 1. It is true that, the histogram is a suitable representation of a frequency distribution of a continuous variable.
Hence, Statement 1 is correct.
2. We know that, the area of histogram is proportional to the frequency, so it is not true statement.

135. (d) When regression lines perpendicular to each other then angle will be :

$$\tan\theta = \left\{\left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x.\sigma_y}{\sigma_x^2 + \sigma_y^2}\right)\right\}$$

$$\Rightarrow \tan\frac{\pi}{2} = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x.\sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$$

$$\Rightarrow r.(\sigma_x^2 + \sigma_y^2) = 0$$

$\therefore r = 0$

136. (d) It is obvious that, $\bar{x} < \bar{z} < \bar{y}$.

137. (b) Mean $= (\bar{x}) = \frac{\sum x_i}{N}$

Here $\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = 10$

$\therefore$ Mean deviation about mean $= \frac{\sum |x_i - \bar{x}|}{N}$

$$= \frac{|4-10| + |7-10| + |8-10| + |9-10| + |10-10| + |12-10| + |13-10| + |17-10|}{8}$$

$$= \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$$

138. (c) Let $x_1, x_2, \ldots, x_{20}$ be the given observations.

Given, $\frac{1}{20}\sum_{i=1}^{20} (x_i - \bar{x})^2 = 5$

To find variance of $2x_1, 2x_2, 2x_3, \ldots, 2x_{20}$,
Let $\bar{x}$ denotes the mean of new observation,

Clearly, $\bar{x} = \frac{\sum_{i=1}^{20} 2x_i}{20} = \frac{2\sum_{i=1}^{20} x_i}{20} = 2\bar{x}$

Now, variance of new observation

$$= \frac{1}{20}\sum_{i=1}^{20}(2x_i - \bar{x})^2 = \frac{1}{20}\sum_{i=1}^{20}(2x_i - 2\bar{x})^2$$

$$= \frac{1}{20}\sum_{i=1}^{20} 4(x_i - \bar{x})^2 = 4\left(\frac{1}{20}\sum_{i=1}^{20}(x_i - \bar{x})^2\right) = 4 \times 5 = 20$$

139. (c) Mean $(\bar{X}) = 4$
Variance $(X) = 2$
Number of observations = 10

New average $= \frac{(4 \times 10) \times 2}{10} = 8$

New variance $= (2)^2 \times$ Variance (X)
$= 4 \times 2 = 8$
$\therefore$ Option (c) is correct.

140. (b) Geometric mean is used in construction of index numbers.
$\therefore$ option (b) is correct.

141. (c)

142. (b) It gives the value by which one variable changes for a unit change in the other variable.
$\therefore$ option (b) is correct.

143. (c) The annual numerical data for comparable for last 12 years is represented by broken line graph, where each turning point represent the data of a particular year, while such graph do not depict the chronological change.
$\therefore$ Option (c) is correct.

144. (d) Mean of 5 numbers = 30
$\therefore$ Total sum of 5 numbers $= 30 \times 5 = 150$
After excluded one number
Mean of 4 numbers will be = 28
$\therefore$ Total sum of 4 numbers $= 4 \times 28 = 112$
Thus, excluded number
= (sum of 5 numbers – sum of 4 numbers)
$= 150 - 112 = 38$
$\therefore$ Option (d) is correct.

145. (a) The 'less than' ogive curve and the 'more than' ogive curve intersect at median.
$\therefore$ Option (a) is correct.

146. (c) $G_1 = [x_1 \times x_2 \times x_3 \times \ldots \times x_n]^{1/n}$

$G_2 = [y_1 \times y_2 \times y_3 \times \ldots \times y_n]^{1/n}$

$$\Rightarrow \frac{G_1}{G_2} = \left[\frac{x_1}{y_1} \times \frac{x_2}{y_2} \times \frac{x_3}{y_3} \times \ldots \times \frac{x_n}{y_n}\right]^{1/n}$$

$\therefore \frac{G_1}{G_2}$ is the G.M of $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \ldots, \frac{x_n}{y_n}$

147. (c) Sum of the given terms $= 1 + 8 + 27 + 64 \ldots +$ upto n terms
Sum $= 1^3 + 2^3 + 3^3 + 4^3 + \ldots +$ upto n terms

$$\text{Sum} = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$$

$$\text{AM} = \frac{n^2(n+1)^2}{4} \times \frac{1}{n} = \frac{n(n+1)^2}{4}$$

148. (b) Let $b_1 = -0.64$
$b_2 = -0.36$
$r = \pm\sqrt{-0.64 \times -0.36} = \pm 0.48$
$\because b_1 < 0$ & $b_2 < 0$
$\therefore r < 0$
$\Rightarrow r = -0.48$

149. (a) Given numbers - 10, 9, 21, 16, 24

Mean $= \dfrac{10+9+21+16+24}{5} = \dfrac{80}{5} = 16$

Numbers	Distance (d) from mean (16)
10	6
9	7
21	5
16	0
24	8

$\Sigma d = 26$

Mean deviation $= \dfrac{\Sigma d}{5} = \dfrac{26}{5} = 5.2$

150. (c) Total no. of observation $(n) = 20$

$\Sigma x_i = 1000$

$\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{1000}{20} = 50$

Variance $= sd^2$

$$sd = \sqrt{\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2}$$

$$(sd)^2 = \frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = \frac{84000}{20} - (50)^2$$

$= 4200 - 2500 = 1700.$

Variance = 1700

151. (d) Mean of series $(x_1, x_2, x_3 \ldots\ldots x_n)$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots\ldots x_n}{n}$$

$\Rightarrow x_1 + x_2 + x_3 + \ldots\ldots x_n = n\bar{x}$

Now we will replace x_2 by λ so no. of elements in series will not change.
New series will include λ and exclude x_2
Hence new series sum :

$(x_1 + x_2 + \ldots\ldots x_n) - x_2 + \lambda = n\bar{x} + \lambda - x_2$

Now new mean $= \dfrac{n\bar{x} + \lambda - x_2}{n} = \dfrac{n\bar{x} - x_2 + \lambda}{n}$

152. (d) Given data 3, 5, 1, 6, 5, 9, 5, 2, 8, 6 and mean, median and mode are *x, y, z* respectively.
Rearranging data
1, 2, 3, 5, 5, 5, 6, 6, 8, 9

Mean $= x = \dfrac{1+2+3+5+5+5+6+6+8+9}{10} = \dfrac{50}{10} = 5$

$$\text{Median} = y = \frac{\frac{n}{2}^{\text{th}} \text{ term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ term}}{2}$$

$y = \dfrac{5+5}{2} = 5$

Mode (z) = most frequently occuring value = 5
Hence $x = y = z$.

153. (c) Statement (1) is correct because total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequency polygon and *x*-axis. Statement (2) is also correct.

154. (d) Mean = Median (in symmetric distribution)
Range = (Max. value – Min. value)
And sum of areas of rectangles in the histogram is always equal to the total area bounded by frequency polygon and the horizontal axis.

155. (d) Mean of the scores $= \dfrac{202}{15}$

Mean of the correct scores $= \dfrac{200}{15}$

i.e., Mean changes.
Median is same for both cases i.e., 14.
Mode is proportional to mean.

156. (b) Line of regression of *y* on *x* is :

$y - \bar{y} = b_{yx}(x - \bar{x})$

$\bar{y} = \dfrac{\Sigma y}{n}$; $\bar{x} = \dfrac{\Sigma x}{n} \Rightarrow \bar{y} = \dfrac{220}{10} = 22$; $\bar{x} = \dfrac{130}{10} = 13$

$b_{yx} = r \cdot \dfrac{\sigma_y}{\sigma_x}$

$$r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{10(3467) - (130)(220)}{\sqrt{[(10 \times 2288) - 130^2][(10 \times 5506) - (220^2)]}}$$

$r = 0.962$

$\sigma_y = \sqrt{\dfrac{\Sigma y^2}{n} - \left(\dfrac{\Sigma y}{n}\right)^2} \Rightarrow \sigma_y = 8.2$; $\sigma_x = 7.73.$

$\Rightarrow b_{yx} = 0.962 \times \dfrac{8.2}{7.73} = 1.02$

$\Rightarrow$ Line of regression of *y* on *x* is :
$y - 22 = 1.02\,(x - 13)$
$\Rightarrow y = 1.02x + 8.74$

157. (d) For Group A :
Coefficient of variation

$CV_A = \dfrac{\text{S.D.}}{\text{Mean}} = \dfrac{10}{22} = 0.4545.$

For Group B :

$CV_B = \frac{12}{23} = 0.522$

$\Rightarrow$ Group A is less variable.

158. (b)

159. (c) Let $u = (x+y);\ v = (x-y)$

$\therefore \bar{u} = (\bar{x}+\bar{y});\ \bar{v} = (\bar{x}-\bar{y})$

$\text{cov}(u,v) = E\{(u-\bar{u})(v-\bar{v})\}$

$= E\{(x-\bar{x})+(y-\bar{y})\}\cdot\{(x-\bar{x})-(y-\bar{y})\}$

$= E\{(x-\bar{x})^2-(y-\bar{y})^2\} = \sigma_x^2 - \sigma_y^2$

$\text{var}(u) = E(u-\bar{u})^2 = E\{(x-\bar{x})+(y-\bar{y})\}^2 = \sigma_x^2+\sigma_y^2$

Therefore *x* and *y* are uncorrelated.

$E(x-\bar{x})(y-\bar{y}) = 0$

Similarly, $\text{var}(v) = \sigma_x^2 + \sigma_y^2$

Thus, $r(u,\ v) = \frac{\text{cov}(u,v)}{\sqrt{\text{var}(u)\cdot\text{var}(v)}} = \frac{\sigma_x^2-\sigma_y^2}{\sigma_x^2+\sigma_y^2}$

160. (b)

Age	Mid value x_i	Frequency f_i	$f_i x_i$
15 – 25	20	2	40
25 – 35	30	4	120
35 – 45	40	6	240
45 – 55	50	5	250
55 – 65	60	3	180
		$\Sigma f_i = 20$	**$\Sigma x_i f_i = 830$**

$\Rightarrow$ Mean age $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{830}{20} = 41.5$

161. (b) $\text{cov}(x,y) = 30$

$\text{var}(x) = 25;\ \text{var}(y) = 144$

$\Rightarrow r(x,\ y) = \frac{\text{cov}(x,\ y)}{\sqrt{\text{var}(x)\cdot\text{var}(y)}}$

$\Rightarrow r(x,\ y) = \frac{30}{\sqrt{25\times 144}} = \frac{30}{5\times 12} = 0.5$

162. (d) New variance = k^2 (old variance)

$= 3^2 \times 5$

$= 9 \times 5 = 45$

163. (b) Mean of 100 observations = 20

Required mean $= \frac{20\times 100-(21+21+18+20)}{96}$

$= \frac{1920}{96} = 20$

164. (b) $4y - 15x + 410 = 0$

$\Rightarrow y - \frac{15}{4}x + \frac{410}{4} = 0 \Rightarrow y = \frac{15}{4}x - \frac{410}{4}$

$\therefore b_{yx} = \frac{15}{4}$

$30x - 2y - 825 = 0 \Rightarrow x = \frac{2}{30}y + \frac{825}{30}$

$\therefore b_{xy} = \frac{2}{30}$

Correlation coefficient $= \sqrt{(b_{yx})(b_{xy})}$

$= \sqrt{\frac{15}{4}\times\frac{2}{30}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

165. (c) First, convert 40 % into fraction

$40\% = \frac{40}{100} = \frac{2}{5}$

Angle $= \frac{2}{5}\times 360 = 144°$

166. (a)

167. (c) We know, $\frac{4}{5}$ standard deviation = Mean deviation

$\Rightarrow$ 4 × standard deviation = 5 × mean deviation

168. (d) Data can be represented in all three forms

169. (a)

170. (a) Correlation coefficient $= \sqrt{\frac{-1}{2}\times\frac{-1}{8}}$

$= \sqrt{\frac{1}{16}} = -\frac{1}{4}$

Since, both the regression coefficients are negative, correlation coefficient is negative.

171. (b) Mean = 32, Median = 33

It was recorded as 40 instead of 35. So, sum will decrease and so mean. Median remains same.

172. (b) Only statements 2 and 3 are correct

173. (d) Given, standard deviation = 0

Mean = 24

Since standard deviation is 0, all observations are equal to 24.

$\therefore$ Mean of smallest five observations = 24.

174. (a) The product of regression coefficient of y on x and regression coefficient of x on y is always less than or equal to 1.

Also, the signs of both coefficients should be same.

Here, |x| = 1

175. (c) $\bar{X} = 10, \bar{Y} = 90, \sigma_x = 3, \sigma_y = 12, r_{xy} = 0.8$

Regression equation x on y is

$x - 10 = r\cdot\frac{\sigma_x}{\sigma_y}(y-90)$

$\Rightarrow x - 10 = 0.8\times\frac{3}{12}(y-90)$

$\Rightarrow x - 10 = \frac{2.4}{12}(y-90)$

$\Rightarrow x - 10 = 0.2\,(y - 90)$
$\Rightarrow x - 10 = 0.2\,y - 18$
$\Rightarrow x = 0.2\,y - 8.$

176. (b) Total expenditure of A = 3,500 + 500 + 1,500 + 2,000 + 2,500
= 10,000
Total expenditure of B = 2,700 + 800 + 1,000 + 1,800 + 1,800
= 8,100
Area of A : Area of B = 10,000 : 8,100
= 100 : 81
$\Rightarrow$ radius of A : radius of B $= \sqrt{100} : \sqrt{81}$
= 10 : 9

177. (b) The arithmetic mean is always between minimum and maximum value.
So, $\frac{n}{2}$ is arithmetic mean.

178. (b) Only statement 2 is correct

179. (c) Coefficient of correlation $= \sqrt{0.2 \times 1.8}$
$= \sqrt{0.36}$
$= 0.6.$

180. (a) Mean = np
Variance = npq
Given, np = 3np q
$\Rightarrow q = \frac{1}{3}, p = \frac{2}{3}.$
Also, Given n = 5 trials.
r = 3
we know, $p(x = r) = {}^nc_r.p^r.q^{n-r}$
$p(x = 3) = {}^5c_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$
$= \frac{80}{243}.$

181. (d) Given, r = 0.6, covariance = 27, $\sigma^2_{(y)} = 25 \Rightarrow \sigma(y) = 5$
We know, $r = \frac{\text{covariance}(x, y)}{\sigma(x).\sigma(y)}$
$\Rightarrow \sigma(x) = \frac{\text{covariance}(x, y)}{r.\sigma(y)} = \frac{27}{\left(\frac{6}{10}\right).5}$
$= \frac{27 \times 2}{6} = 9.$
$\Rightarrow \sigma^2(x) = 81.$

182. (d) Given, Mean of $x_i = \bar{x}$
Also, Given $x_i = a + cy_i$
$\therefore$ Mean of $a + cy_i = \bar{x}$
$\Rightarrow$ Mean of $cy_i = \bar{x} - a$
$\Rightarrow$ Mean of $y_i = \frac{\bar{x} - a}{c}$

183. (d) We know, $\tan\theta = \left|\frac{1 - r_{xy}{}^2}{r_{xy}}\right|\left(\frac{\sigma_x.\sigma_y}{\sigma_x^2.\sigma_y^2}\right).$
If r_{xy} is 0, then $\tan\theta = \infty$ and lines are perpendicular.
If r_{xy} is 1, then the lines are parallel.

184. (c) $4x - 5y + 33 = 0$ (1)
$20x - 9y = 107$ (2)

$$\begin{aligned}(1)\times 5 \Rightarrow \quad & 20x - 25y + 165 = 0\\ (2) \Rightarrow \quad & 20x - 9y - 107 = 0\\ & \quad (-)\quad (+)\quad (+)\\ \hline & -16y + 272 = 0 \Rightarrow 16y = 272 \Rightarrow y = 17.\end{aligned}$$

$(1) \Rightarrow 4x - 5(17) + 33 = 0$
$\Rightarrow 4x - 85 + 33 = 0$
$\Rightarrow 4x = 52 \Rightarrow x = 13.$

185. (d) 1. Mean is dependent with change in origin.
2. Variance is independent with change in origin.

186. (c) Both the given statements are correct.

187. (b) By arranging the given numbers in ascending order,
$-4.8, 0, 2.3, 3.5, 3.9, \underline{4.6, 5.2}, 6.1, 7.6, 8.2, 9.3, 12.7$
↓ Middle terms
$\therefore$ Median $= \frac{4.6 + 5.2}{2}$
$= \frac{9.8}{2} = 4.9$

188. (c) Central angle = 20% of 360°
$= \frac{20 \times 60}{100}$
= 72°

189. (b) Given, Mean = 5
Standard deviation = 2
If 5 is added to each value, mean = 5 + 5 = 10.
Standard deviation will not change.
Coefficient of variation $= \frac{\text{Standard deviation}}{\text{Mean}} \times 100$
$= \frac{2}{10} \times 100 = 2 \times 10$
= 20

190. (b) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}.\sum_{i=1}^{n} x_i\right)^2$
$= \frac{1}{n}\left(1^2 + 2^2 + 3^2 + ... + n^2\right) - \left(\frac{1}{n}(1 + 2 + 3 + ... + n)\right)^2$
$= \frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6} - \left(\frac{1}{n}.\frac{n(n+1)}{2}\right)^2$
$= \frac{n^2 - 1}{12} \qquad \therefore \sigma = \sqrt{\frac{n^2 - 1}{12}}.$

192. (d) Let No. of Man = M
Let No. of Women = W
26 M + 21 W = 25(M + W)
M = 4 W
M : W = 4 : 1
$\therefore$ Percent of Men and Women = 80%, 20%

194. (b) Variance will not change by adding or subtracting a fixed value to all the elements.

195. (b) $b_{yx} = -6, r = -\frac{1}{2}$

$$\left(-\frac{1}{2}\right)^2 = -6 \times b_{xy}$$

$$\Rightarrow b_{xy} = -\frac{1}{24}$$

196. (d) By definition.

197. (d) Combined Average

$$= \frac{500 \times 1860 + 600 \times 1750}{1100} = 1800$$

Combined Variance

$$= \frac{500(81 + 3600) + 600(100 + 2500)}{1100}$$

$$= \frac{(5 \times 3681) + (6 \times 2600)}{1100} \approx 3092$$

198. (b) Median can be obtained from ogive.

199. (d) x = M.D., y = S.D.

M.D. = $\frac{4}{5}$ S.D. $\Rightarrow x < y$

200. (c) By definition.

201. (c) Number of observations (N) = 100

Mean $(\bar{x}) = 50$

Standard deviation $(\sigma) = 10$

New mean, $\bar{x}_1 = \frac{\Sigma \frac{x-5}{4}}{N} = \frac{1}{4}\left(\frac{\Sigma x}{N} - 5\right) = \frac{1}{4}(50 - 5)$

$$= \frac{45}{4} = 11.25$$

New standard deviation $(\sigma_1) = \sigma\left(\frac{x-5}{4}\right)$

$$= \frac{1}{4}\sigma(x-5) = \frac{1}{4}\sigma(x) = \frac{10}{4} = 2.5$$

202. (a) Only the statement 1 is correct.

203. (d) Given, $r_{xy} = 0.6$

$z = x + 5;\ w = \frac{y}{3}$

$\Rightarrow b_{zx} = 1 \Rightarrow b_{wy} = \frac{1}{3}$

$$b_{zx}\, b_{wy} = (1)\left(\frac{1}{3}\right) = \frac{1}{3}$$

$$\Rightarrow \frac{r_{zw}}{r_{xy}} = \frac{1}{3} \Rightarrow r_{zw} = \frac{r_{xy}}{3} = \frac{0.6}{3} = 0.2$$

204. (c) The series is 1, 2, 3, ... 20

Variance $(\sigma) = \frac{\Sigma x^2}{n} - \Sigma(\bar{x})^2$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= \frac{(n+1)}{12}(n-1)$$

$$= \frac{n^2 - 1}{12} = \frac{(20)^2 - 1}{12} = \frac{399}{12} = \frac{133}{4} = 33.25$$

$\because$ Numbers are multiplied by 3,
variance $(\sigma) = 33.25 \times 9 = 299.25$